Stochastic models of anomalous diffusion in incoherent radiation trapping

Eduardo J. Nunes-Pereira
Universidade do Minho, Escola de Ciências,
Departamento de Física, 4710-057 Braga, Portugal.
epereira@fisica.uminho.pt

(Dated: February 29, 2008)

This contribution is divided into four parts. In the first part the problem of incoherent resonance radiation trapping is defined, briefly stating both its fundamental character of a transport process as well as its main practical applications (discharge processes in gases, plasma and atomic vapour luminescence, electric discharge and electrodeless fluorescence lamps, plasma display panels, radiation transfer in stellar atmospheres, etc.). Radiation trapping is also known as *imprisonment of resonance radiation*, *reabsorption*, *self-absorption*, *line transfer*, *radiation diffusion* or *multiple scattering* of resonance radiation.

The second part constitutes the main focus of the talk. In it, the photonic superdiffusive motion is discussed making use of both the jump length and the single line opacity distribution functions (both defined from the single line spectral distribution). General expressions are given which relate the asymptotic behaviour of the three interconnected distribution functions. The problem of complete frequency redistribution for Doppler, Lorentz and Voigt line shapes is first addressed. Photonic trajectories are shown to be well characterized Lévy flights displaying characteristic long tails (superdiffusive transport properties). It is shown that the asymptotic form is a strict power-law $r^{-3/2}$ for Lorentz, while for Doppler the asymptotic is $r^{-2}\left(\ln r\right)^{-1/2}$, where r stands for the jump length. For the Voigt profile, the asymptotic form has always a Lorentz character, but the trajectory is a self-affine fractal with two characteristic Hausdorff scaling exponents. Some interesting but in this context marginal facts are briefly mentioned (incoherent transport is always superdiffusive irrespective of the spectral distribution, the asymptotic for a unidimensional formulation of radiation transfer, the one used in the standard plane-parallel stellar atmosphere problem, is the same as the asymptotic of the usual jump size distribution and that the deduced power-law asymptotics are indeed expected to hold in a large number of practical situations). The line opacity distribution function is then generalized for partial frequency redistribution (PFR) conditions and several possible redistribution mechanisms (pure Doppler broadening, combined natural and Doppler broadening and combined Doppler, natural and collisional broadening) discussed. With partial redistribution, there are two coexisting scales with a different behavior: the small scale is controlled by the intricate PFR details while the large scale is essentially given by the atom rest frame redistribution asymptotic. The pure Doppler and combined natural, Doppler and collisional broadening are characterized by both small and large scale superdiffusive Lévy flight behaviors while the combined natural and Doppler case has an anomalous small scale behavior but a diffusive large scale asymptotic. The conditions for the breakdown of superdiffusion in combined natural and Doppler broadening are defined.

Finally, in the last two parts the previous work is briefly put into perspective in a broader context. In the third part some stochastic models to quantify the dynamics of radiation trapping are presented giving particular emphasis to the actual experimentally accessible observables and the the conditions in which the previous presented asymptotics are expected to dominate the overall behaviour of real systems. In the last fourth part, the recent reevaluation of the importance of anomalous diffusion for radiation transport is put into historical perspective by comparing recent results with important seminal contributions dated from the 1930s (breakdown of standard diffusion), 1940s (the well-known Holstein-Biberman equation), and 1980s (breakdown of superdiffusion in some conditions for partial redistribution).

[1] M. N. Berberan-Santos, E. Pereira, and J. M. G. Martinho, *Dynamics of Radiative Transport*, in *Resonance Energy Transfer*, edited by D. L. Andrews and A. A. Demidov (John Wiley & Sons, Chichester, 1999).
[2] E. Pereira, J. M. G. Martinho, and M. N. Berberan-Santos, Phys. Rev. Lett. **93**, 120201 (2004).
[3] M. N. Berberan-Santos, E. J. Nunes-Pereira, and J. M. G. Martinho, J. Chem. Phys. **125**, 174308 (2006).
[4] A. R. Alves-Pereira, E. J. Nunes-Pereira, J. M. G. Martinho, and M. N. Berberan-Santos, J. Chem. Phys. **126**, 154505 (2007).
[5] A. R. Alves-Pereira, E. J. Nunes-Pereira, J. M. G. Martinho, and M. N. Berberan-Santos, Eur. J. Phys. **28**, 1105 (2007).