

## A GENERAL FORMULATION OF THE CONTACT INTERACTION BETWEEN A CIRCLE SURFACE AND A CONVEX-CONCAVE-CONVEX SURFACE

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### 1. INTRODUCTION

Representation of contact/impact mechanics in the context of multibody dynamics still poses a big challenge. The main problem lies in finding a balance between results accuracy and computational efficiency. With the purpose of describing this complex phenomenon, there exist different modelling techniques. A widespread approach consists of reducing the contact problem to a geometrically determined contact point and normal direction [1]. Under this assumption, two different procedures can be generally utilized to model collisions in the framework of multibody systems formulations, namely the non-smooth approaches and the continuous methods. The first type of solution considers contact interactions as additional geometrical constraints, whereas the latter implements these phenomena in the form of contact forces.

This work will focus on the use of continuous methods to propose a methodology that allows the characterization of the contact kinematics of planar bodies with both concave and convex surfaces. The results obtained will be compared with those provided by Finite Element Analysis (FEA) software. Subsequently, the presented solution will be applied to the modelling of a complex mechanical system, in which multiple contacts of different natures take place simultaneously. Continuous approaches have proved to deal with multiple-simultaneous contact scenarios successfully while containing the computational cost.

### 2. PROBLEM DESCRIPTION

To test the proposed methodology, the system described in Figure 1(a) is presented, in which a circle  $i$  with radius  $R$  can come into contact with a body  $j$  that has a convex-concave-convex surface (from now on, denoted as ccc-surface). This concave sector is defined by its curvature radius, denoted as  $R_{\text{center}}$ , and its centre, set at point  $M$ . The convex arcs are given by two distances, known as  $l_{\text{left}}$  and  $l_{\text{right}}$ , which are normal to the vertical direction established by the unit vector  $\mathbf{u}_{\text{center}}$ .

Contact/impact kinematics involves the evaluation of three fundamental quantities, namely the location of the potential contact points, the distance between them, and the corresponding relative contact velocity [2]. Regarding the computation of the normal contact force, energy dissipation has been neglected in this work. Thus, this value is defined solely by the Hertzian component:

$$f_{n, \text{hertz}} = K\delta^n \quad (1)$$

where  $K$  is the combined stiffness of the contact pair,  $n$  denotes the exponent that quantifies the degree of nonlinearity of the force-indentation, and  $\delta$  is the relative indentation between the bodies. The value of the contact stiffness is defined by [3]:

$$K = \frac{4C_\delta\pi}{3(\sigma_i + \sigma_j)\sqrt{A+B}} \quad (2)$$

where  $A$  and  $B$  are parameters that consider the curvature radii of the contacting bodies in the lateral and longitudinal directions, and  $C_\delta$  is a function of the ratio  $A/B$ . The radii of a certain principal direction have the same +/- sign if the contact in that specific direction is external (convex surfaces) and the opposite if it is an internal one. Consider the two examples shown in Figure 1(b) and Figure 1(c). In the first case, the respective radii of bodies  $i$  and  $j$ ,  $R_i$  and  $R_j$ , will have the same positive sign. However, for the second scenario, the surface with the largest radius (body  $j$ ) will be concave and its radius will have a negative value, whereas

body  $i$  will be convex and, subsequently, its radius will be positive.  $\sigma_i$  and  $\sigma_j$  are material parameters that depend on Poisson's ratio and Young's modulus of the bodies.

Thus, the contact modelling between the circle and the ccc-surface is divided into three interactions, depending on the location of the potential contact point, namely external circle-circle collisions for the convex sectors and an internal one for the central, concave arc. This results in two sudden changes in the stiffness values at the transition points of the different stages, as can be observed in Figure 2. To ensure a smooth passage between the external and internal contacts and avoid numerical singularities at these points, a suitable transition function has been employed to mitigate sudden changes in the contact stiffness between the different parts of the profile.

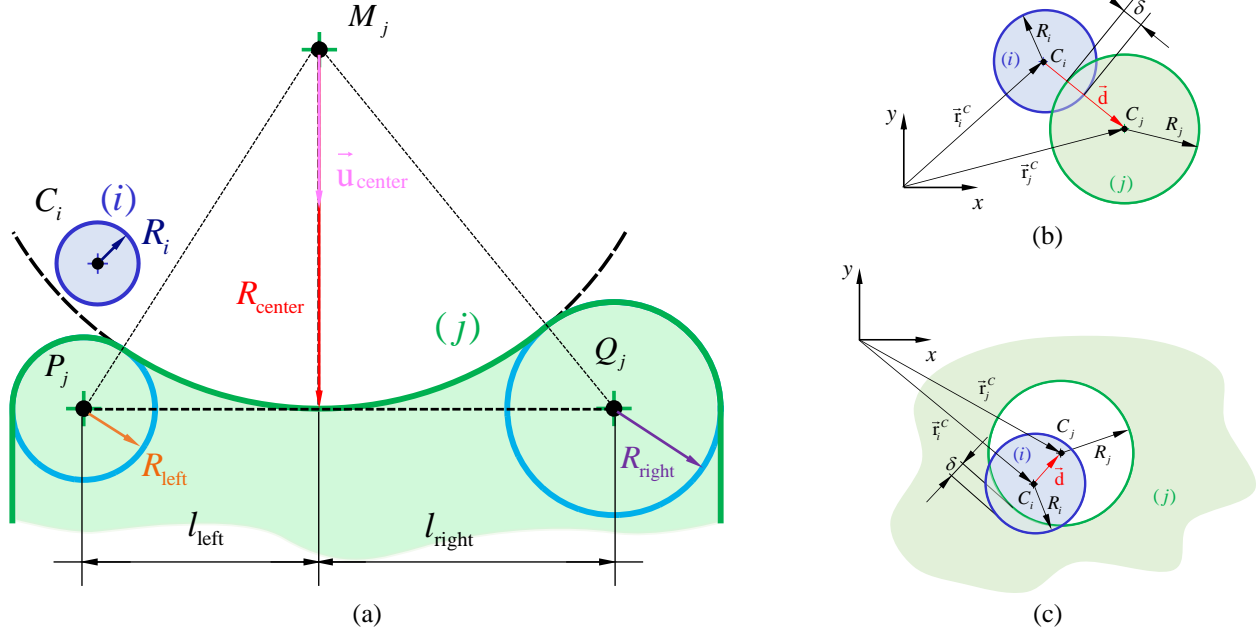


Figure 1. Contact kinematics description: (a) ccc-surface; (b) External circle-circle contact; (c) Internal circle-circle interaction.

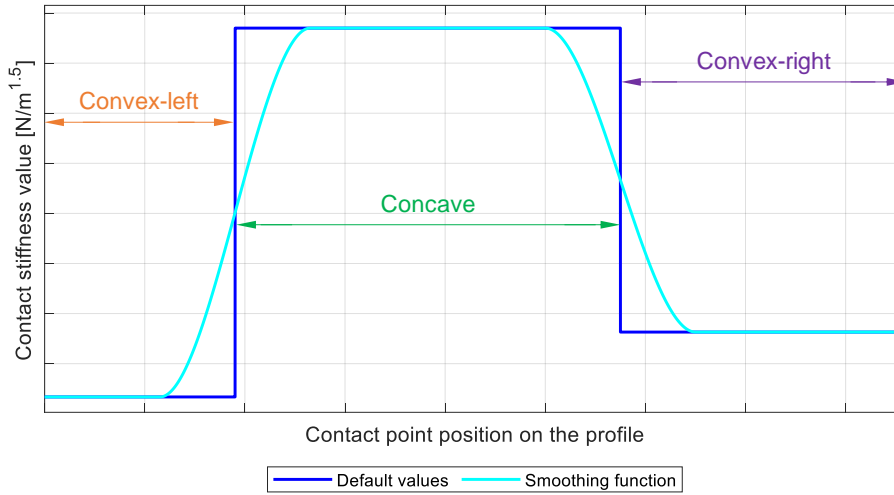


Figure 2. Evolution of the contact stiffness value along the profile of the ccc-surface.

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