MODELING AND ANALYSIS OF THE ANKLE JOINT COMPLEX WITH MUSCLES

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1. INTRODUCTION

The ankle joint complex of the human foot is composed of the talocrural and the talocalcaneal joints, and it is the focal point of this work. In general, plantarflexion and dorsiflexion are allowed by the talocrural joint, while the talocalcaneal joint permits inversion and eversion of the human foot. In this study, a spatial biomechanical multibody model of the ankle joint complex is presented, which extends the authors' previous work [1] to incorporate the muscle behavior [2].

2. MUSCLE MODEL DESCRIPTION

The functional unit that produces joint motion is the muscle-tendon unit (MTU), which has an origin and an insertion, and it consists of a muscle and a tendon that interdigitates with the muscle by a musculotendinous junction. The muscle model utilized in this study is based on the Hill model [2], as schematized in Fig. 1, which is composed of an active contractile element (CE) arranged in parallel with a passive elastic element (PE). The MTU length, l^{mtu}, varies according to the kinematics of the movement under analysis and results from the sum of the tendon length, l^t, and the muscle length, l^m, considering the pennation angle, α, as

$$l^{mtu} = l^t + l^m \cos\alpha \tag{1}$$

Since the muscle and the tendon are assembled in series, as observed in Fig. 1, the resultant MTU force is the same as the tendon force, which is equal to the force produced by the muscle projected onto the line of action of the tendon. In this work, the tendon is assumed to be rigid, and, thus, to the MTU force contributes exclusively the muscle force as

$$f^{mtu} = f^t = f^m \cos\alpha \tag{2}$$

where f^{mtu}, f^t and f^m are the force of the MTU, tendon and muscle, respectively, and α is calculated as

$$\alpha = \arctan\left(\frac{l^w}{l^{mtu} - l^t}\right) = \arctan\left(\frac{l_0^m \sin\alpha_0}{l^{mtu} - l^t}\right) \tag{3}$$

in which l^w is the muscle width, l_0^m corresponds to the muscle resting length and α_0 is the optimal muscle pennation angle. Both the CE and the PE contribute to the total force produced by the MTU, f^{mtu}. Thus, Eq. (2) can be rewritten as

$$f^{mtu} = \left(f_{\mathrm{PE}}^m + f_{\mathrm{CE}}^m\right)\cos\alpha = \left(f_{\mathrm{PE}}^m + \hat{f}_{\mathrm{CE}}^m a^m\right)\cos\alpha = \left[f_{\mathrm{PE}}^m + \left(\frac{f_l^m f_v^m}{f_0^m}\right)a^m\right]\cos\alpha \tag{4}$$

where f_{PE}^m and f_{CE}^m are the PE and CE forces, respectively, f_l^m and f_v^m are the CE force-length and force-velocity relations, respectively, f_0^m is the maximum isometric force, and a^m is the muscle activation. The symbol '^' denotes the available CE force.

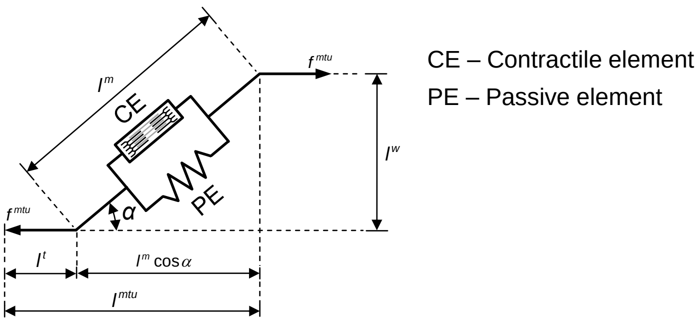


Figure 1. Representation of the Hill-type muscle model utilized in this work.

3. OPTIMIZATION PROCEDURE

In biomechanical systems, the joints are crossed by several muscles, which means that different muscle activation patterns can generate the same joint moments and, thus, result in the same movement of the human body. The central nervous system is responsible for selecting and activating the most appropriate muscles for the performed task and/or objectives to be achieved. From a mathematical point of view, the muscle redundancy problem results from the fact that the number of muscle activations exceeds the number of degrees-of-freedom of the model and, thus, a unique solution for the analytical determination of the muscle forces cannot be obtained. Thus, in this study, an optimization procedure is considered and can be expressed as

$$
\begin{aligned}
&\text{Given:} && \mathbf{x} = \{\boldsymbol{\lambda} \quad \mathbf{a}\}^{\mathrm{T}} \\
&\text{Minimize:} && \Im_0(\mathbf{x}) \\
&\text{Subject to:} && \begin{cases} \begin{bmatrix} \mathbf{D}^{\mathrm{T}} & -\boldsymbol{\chi}^{\mathrm{T}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{Bmatrix} + \mathbf{M}\dot{\mathbf{v}} - \left(\mathbf{g}^{\mathrm{ext}} + \mathbf{g}_{\mathrm{PE}}^{\mathrm{mtu}}\right) = \mathbf{0} \\ 0 \le a^m \le 1 \\ -\varepsilon \le \boldsymbol{\lambda}^* \le \varepsilon \\ -\infty < \boldsymbol{\lambda}^{\mathrm{R}} < +\infty \end{cases}
\end{aligned}
\tag{5}
$$

where $\mathbf{x}$ is the vector of the design variables, $\boldsymbol{\lambda}$ denotes the Lagrange multipliers vector, $\mathbf{a}$ is the muscle activations vector, $\mathbf{D}$ is the Jacobian matrix, $\mathbf{M}$ denotes the system mass matrix, $\dot{\mathbf{v}}$ is the accelerations vector, $\mathbf{g}^{\mathrm{ext}}$ is the generalized vector of externally applied forces and moments, $\mathbf{g}_{\mathrm{PE}}^{\mathrm{mtu}}$ is the generalized vector of PE forces and moments on the MTU, $\boldsymbol{\lambda}^*$ is the vector of Lagrange multipliers associated with the rotational driving constraints, $\boldsymbol{\lambda}^{\mathrm{R}}$ is the vector of Lagrange multipliers associated with joint and rigid body driving constraints, and ε is a user-specified tolerance. Vector $\mathbf{a}$ and matrix $\boldsymbol{\chi}$ are expressed, respectively, as

$$
\mathbf{a} = \left\{a^1 \quad \dots \quad a^{n_m}\right\}^{\mathrm{T}} \qquad \boldsymbol{\chi} = \left[\hat{\mathbf{g}}_{\mathrm{CE}}^1 \quad \dots \quad \hat{\mathbf{g}}_{\mathrm{CE}}^{n_{mtu}}\right]^{\mathrm{T}} \tag{6}
$$

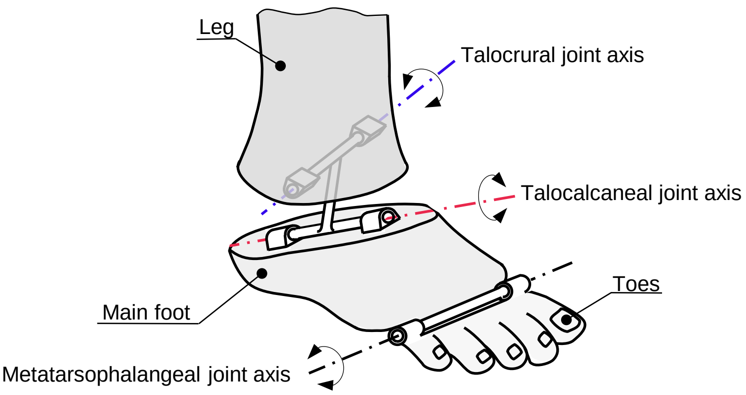
where n_m is the number of muscles. The cost function $\Im_0$ evaluates the sum of the cube of individual average muscle stresses

$$
\Im_0 = \sum_{m=1}^{n_m} \left(\bar{\sigma} \frac{f_l^m f_v^m}{\left(f_0^m\right)^2} a^m \right)^3 \tag{7}
$$

in which $\bar{\sigma}$ represents the specific muscle strength with a constant value of 31.39 N/cm^2 [3].

4. ANKLE JOINT MULTIBODY MODEL

A three-dimensional biomechanical multibody model is utilized in this work. The model is composed of rigid bodies, namely the toes, main foot, and leg. The bodies are kinematically connected by one revolute joint (metatarsophalangeal joint) and one modified universal joint (ankle joint complex) (Fig. 2, left) [1]. The model has nine degrees-of-freedom, which are guided using experimental data acquired at the Lisbon Biomechanics Laboratory. Skeletal muscles responsible for plantarflexion, dorsiflexion, inversion, and eversion of the foot are included (Fig. 2, right), and the obtained results are compared with the literature.



Muscle designation	Function
Gastrocnemius lateral head	Plantarflexion
Gastrocnemius medial head	Plantarflexion
Soleus	Plantarflexion
Tibialis Posterior	Plantarflexion; Inversion
Tibialis Anterior	Dorsiflexion; Inversion
Flexor Digitorum Longus	Plantarflexion; Inversion
Flexor Hallucis Longus	Plantarflexion; Inversion
Extensor Digitorum Longus	Dorsiflexion; Eversion
Extensor Hallucis Longus	Dorsiflexion; Inversion
Peroneus Brevis	Plantarflexion; Eversion
Peroneus Longus	Plantarflexion; Eversion

Figure 2. Representation of the biomechanical multibody model of the ankle joint complex (left) and muscle designation and function (right).

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