State-of-the-art and challenges of contact-impact problems using multibody dynamics methodologies

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Abstract. Multibody dynamics deals with the modeling and analysis of mechanical systems comprising multiple bodies, whose interactions are governed by kinematic constraints and external applied forces. Contact-impact events represent one of the most critical and challenges issues when dealing with dynamical systems, because they can significantly affect the behavior of multibody systems. The accurate response of a collision in multibody dynamics is strongly dependent on some critical factors, including the geometry of contacting surfaces, material properties of the bodies involved, and the constitutive laws utilized to evaluate the contact forces. This work provides a comprehensive overview of the main aspects and state-of-the-art techniques for modeling contact-impact events in multibody dynamics. In the sequel of this process, contact detection and contact resolution tasks are presented. In addition, some of the most prominent force models available in the literature to compute normal and tangential forces developed within contact-impact events are discussed. Moreover, this work examines key numerical aspects related to contact-impact events that strongly influence the computational accuracy and efficiency. Finally, an application example is presented, whose results permit to discuss the key aspects related to the modeling process of friction contact-impact events in multibody dynamics.

Keywords: Contact-Impact, Frictional Contacts, Multibody Dynamics, Contact Detection, Contact Resolution.

1 Introduction

It has been recognized by many researchers that the application of multibody methodologies to actual systems requires the modeling and analysis of contact situations [1]. The dynamic response of contact-impact problems can be influenced by several different aspects, including the physical properties of the contacting surfaces and the approach used to model the contact process. By and large, the formulation for dynamic contact problems includes three main points: (*i*) identification of the potential contact points of the colliding bodies; (*ii*) computation of the contact-impact forces developed; and (*iii*) establishment of the transitions between different contact scenarios.

The mathematical representation of contact-impact problems in multibody dynamics can be strongly influenced by the method used to model the interaction of the contacting surfaces. In a broad sense, there are two key approaches to handling collisions within multi-body dynamics analysis: the “piecewise” formulation and the “continuous” methods. Both of these methods are suitable for computational analysis in multibody dynamics. Nevertheless, both techniques require precise determination of the instant of contact between the colliding surfaces [2].

In the piecewise formulation, also denominated discrete or impulse-momentum approach, the impact process is considered to occur instantaneously. The discontinuous nature of the collision results in an abrupt variation in velocities at the instant of impact. In the piecewise method, the rapid change in velocities is infinitesimal, while the position and orientation of the bodies in the system are unaltered. In this formulation, the dynamics of the mechanical system are based on the numerical resolution of the equations of motion until a discontinuous event associated with an impact occurs. At that point, the resolution of the system's equations of motion is halted, and a momentum balance is used to compute the post-impact velocities of the bodies involved in the collision event. Afterward, the resolution of the equations of motion resumes with the updated velocities until a new impact event occurs. The coefficient of restitution is the parameter used to quantify the energy dissipated during the contact-impact events. It must be highlighted that the piecewise formulation is quite effective, but its applicability can be limited by the unknown duration of the contact-impact events. It must be noticed that for long collisions, the configuration of the multibody model can change significantly [3].

In turn, the continuous method, also known as the compliant or force-based approach, the bodies are allowed to deform locally. Thus, the relative contact velocity of the contacting surfaces changes continuously. The contact-impact forces generated during collisions are typically represented by spring-damper elements, which mimic the stiffness and damping characteristics related to the resistive contact phenomenon. In practice, the contact forces are written as continuous functions of the penetration and penetration rate associated with contact-impact points. The penetration, indentation, or deformation is represented by the overlap of the bodies occurring at the local contact area. Over the last decades, several contact force solutions have been presented in different scientific domains, with many of them being based on Hunt and Crossley's cornerstone contact force model [4].

The present study revisits the fundamental aspects associated with the modeling and analysis of contact-impact events in multibody mechanical systems, aiming to characterize the state of the art. The organization of this work is structured as follows. Section 2 deals with the main aspects related to the contact kinematics, that permits the comprehensive determination of the local contact deformation, as well as the normal and tangential contact velocities between two contacting surfaces in a mechanical system. In section 3, the most fundamental and well-established contact force models to handle normal and tangential actions are revisited and compared. A representative example of application and corresponding results are provided in Section 4. Finally, Section 5 addresses the key conclusions, and discusses future directions in terms of potential developments in the area of contact-impact events in multibody systems.

2 Contact Kinematics

Contact kinematics is the dimension in which potential or candidate contact points, as well as the relative normal and tangential velocities, are determined [5]. "This set of information plays a key role in the response of the systems, as they are required to compute the contact-impact reaction forces generated during collisions. To better comprehend how these contact characteristics are evaluated within a multibody system, let's consider the contact scenario between two convex surfaces that can collide with each other, as depicted in Fig. 1. In the case illustrated in Fig. 1a, the two bodies, denoted as i and j, are separated and moving with absolute velocities $\dot{\mathbf{r}}_i$ and $\dot{\mathbf{r}}_j$ [6].

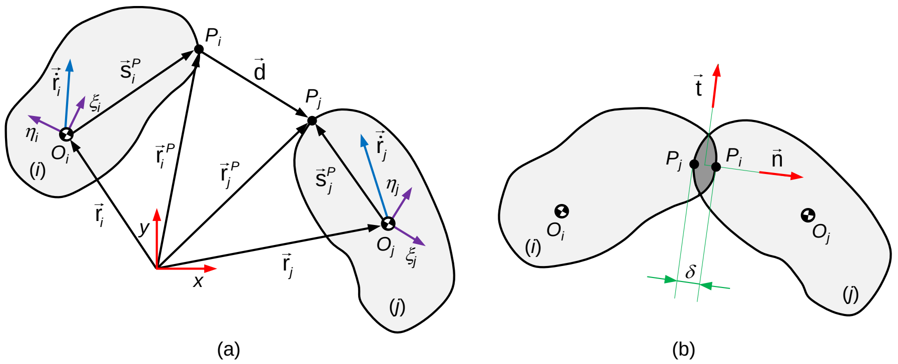

Fig. 1. (a) Convex surfaces separated; **(b)** Two convex surfaces in contact.

The vector connecting the candidate contact points, P_i and P_j, as represented in Fig. 1, signifies the gap distance, which can be expressed as [6]

$$\mathbf{d} = \mathbf{r}_j^P - \mathbf{r}_i^P \tag{1}$$

in which the terms $\mathbf{r}_i^P$ and $\mathbf{r}_j^P$ are written as

$$\mathbf{r}_k^P = \mathbf{r}_k + \mathbf{A}_k \mathbf{s}'^P_k \qquad (k = i, j) \tag{2}$$

where $\mathbf{r}_i$ and $\mathbf{r}_j$ denote the global position vectors of the bodies i and j, respectively and $\mathbf{s}'^P_i$ and $\mathbf{s}'^P_j$ are the local components of the candidate contact points with respect to local coordinates systems. It must be noted that $\mathbf{A}_i$ and $\mathbf{A}_j$ represent the transformation matrices associated with bodies i and j [6].

The normal vector depicted in Fig. 1b, is given by

$$\mathbf{n} = \frac{\mathbf{d}}{d} \tag{3}$$

It is clear that the tangential vector $\mathbf{t}$, represented in Fig. 1b, can be computed by rotating the normal vector $\mathbf{n}$ counterclockwise by 90 degrees.

The magnitude of the distance vector $\mathbf{d}$, shown in Fig. 1a, is evaluated as

$$d = \delta = \mathbf{d}^{\mathrm{T}} \mathbf{n} \tag{4}$$

At this stage, it is important to note that the condition expressed by Eq. (4) is not sufficient to detect candidate contact points, as it cannot account for all potential collision scenarios. Therefore, contact points are established as those corresponding to the maximum value of penetration, representing the points at which maximum deformations occur along the normal direction [5]. Thus, the effective conditions for the contact between two convex surfaces can be defined as (*i*) The distance between the two potential contact points, as represented by vector **d**, corresponds to the minimum distance; (*ii*) the vectors $\mathbf{n}_i$ and **d** have to be collinear with each other; (*iii*) the two normal vectors $\mathbf{n}_i$ and $\mathbf{n}_j$ must also be collinear. From the implementation point of view, conditions (*ii*) and (*iii*) can be expressed by two vector products [4]

$$\mathbf{d} \times \mathbf{n}_i = \mathbf{0} \tag{5}$$

$$\mathbf{n}_j \times \mathbf{n}_i = \mathbf{0} \tag{6}$$

It must be highlighted that the mathematical conditions associated with Eqs. (5) and (6) represent two nonlinear equations with two unknowns, which can be calculated using, for instance, the well-known Newton-Raphson numerical scheme [6]. The set of solutions that result from Eqs. (5) and (6) represent the effective candidate or potential contact points. After identifying the contact points, the next step involves evaluating the relative penetration between the contacting bodies.

The velocities associated with the potential contact points are computed as the time derivative of Eq. (2), resulting in

$$\dot{\mathbf{r}}_k^P = \dot{\mathbf{r}}_k + \dot{\mathbf{A}}_k \mathbf{s}'^P_k \qquad (k = i, j) \tag{7}$$

where the dot denotes the derivative with respect to time, being $\dot{\mathbf{A}}$ the time derivative of the transformation matrix [6]. The relative velocity of the contact points needs to be projected onto the normal and tangential directions of the contacting points, since they play a crucial role in identifying the type of contact dynamics problem.

The scalar contact velocities, in the normal and tangential directions, are evaluated using the following formulation

$$v_\mathrm{n} = \dot{\delta} = \left(\dot{\mathbf{r}}_j^P - \dot{\mathbf{r}}_i^P\right)^\mathrm{T} \mathbf{n} \tag{8}$$

$$v_\mathrm{t} = \left(\dot{\mathbf{r}}_j^P - \dot{\mathbf{r}}_i^P\right)^\mathrm{T} \mathbf{t} \tag{9}$$

This representation of the relative normal and tangential velocities is highly convenient, as it eliminates the need to derive the normal unit vector directly from the differentiation of Eq. (4) to obtain the velocity components. Furthermore, the application of fully rigid body velocity kinematics is straightforward, and the computational implementation of this method is highly effective. This formulation has some limitations, as it applies only to convex rigid bodies with smooth surfaces, at least within a vicinity of the potential contact points. In such cases, the contact area can be reduced to a single point that may move relative to the bodies' surfaces. This approach can be extended to accommodate more generalized contact geometries, provided that a common tangent plane of the contacting bodies is uniquely established [2].

3 Contact Force Models

The oldest and simplest force model is based on Hooke's law, which utilizes a spring to simulate a collision. This contact force model can be expressed as [2]

$$f_{\mathrm{n}} = k\delta \tag{10}$$

in which k denotes the stiffness, and δ is the local deformation. This model is represented in the diagram of Fig. 2a. A more advanced formulation was introduced by Hertz, which established a nonlinear relation between force and deformation as [2]

$$f_{\mathrm{n}} = K\delta^{n} \tag{11}$$

where n represents the nonlinear exponent. Figure 2b depicts the force-penetration diagram relative to the Hertz contact force model. The first force model that accounts for energy dissipation in contact interactions is due to Kelvin-Voigt solution. This approach combines a spring with a damper to compute the forces at the contact points [4]

$$f_{\mathrm{n}} = K\delta + D\dot{\delta} \tag{12}$$

in which the first parcel represents the elastic term, and the second parcel is the dissipative term, being D the damping coefficient, and $\dot{\delta}$ is the relative contact velocity in the normal direction. Figure 2c shows the diagram for the relation between force and deformation of the Kelvin-Voigt formulation. Hunt and Crossley presented the most prominent force model that can be expressed as [4]

$$f_{\mathrm{n}} = K\delta^{n}\left[1 + \frac{3(1-c_{\mathrm{r}})}{2}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{13}$$

in which the first parcel represents the nonlinear Hertz's force model, being the second parcel the dissipative term, c_{r} is the restitution coefficient, and $\dot{\delta}^{(-)}$ denotes the initial impact velocity. Figure 2d represents the force deformation diagram for Hunt and Crossley approach. The most well-known force model in multibody systems was offered by Lankarani and Nikravesh as [7]

$$f_{\mathrm{n}} = K\delta^{n}\left[1 + \frac{3(1-c_{\mathrm{r}}^{2})}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{14}$$

More recently, Flores et al. [8] presented an alternative formulation as

$$f_{\mathrm{n}} = K\delta^{n}\left[1 + \frac{8(1-c_{\mathrm{r}})}{5c_{\mathrm{r}}}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{15}$$

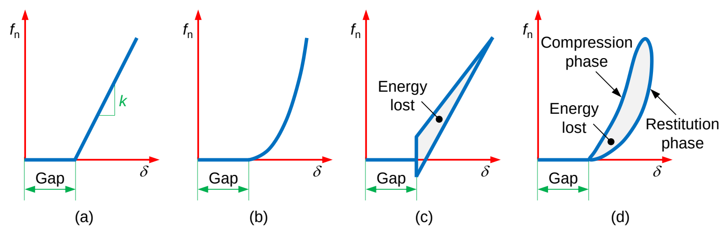

Fig. 2. Force-deformation diagrams for different force models: **(a)** Hooke's approach; **(b)** Hertz's model; **(c)** Kelvin-Voigt law; **(d)** Hunt and Crossley contact force solution.

The well-known friction force model proposed by Coulomb is given by [9]

$$f_{\mathrm{t}} = \begin{cases} [-\mu_{\mathrm{s}} f_{\mathrm{n}}, \mu_{\mathrm{s}} f_{\mathrm{n}}] & \text{if } v_{\mathrm{t}} = 0 \\ \mu_{\mathrm{d}} f_{\mathrm{n}} \operatorname{sgn}(v_{\mathrm{t}}) & \text{if } v_{\mathrm{t}} \neq 0 \end{cases} \tag{16}$$

where

$$\operatorname{sgn}(v_{\mathrm{t}}) = \begin{cases} -1 & \text{if } v_{\mathrm{t}} < 0 \\ 1 & \text{if } v_{\mathrm{t}} > 0 \end{cases} \tag{17}$$

in which μ_{s} and μ_{d} denote the static and dynamic friction coefficients, f_{n} represents the normal contact force, and v_{t} denotes the relative contact tangential velocity of the contact points. Figure 3a depicts a graphical representation of the Coulomb's friction force model. Threlfall [9] regularized the Coulomb's law to remove the discontinuities, as it is represented in the diagram of Fig. 3b. The Threlfall friction model is expressed as

$$f_{\mathrm{t}} = \begin{cases} \mu_{\mathrm{d}} f_{\mathrm{n}} \left(1 - \mathrm{e}^{-3\frac{v_{\mathrm{t}}}{v_{\varepsilon}}}\right) & \text{if } v_{\mathrm{t}} \leq v_{\varepsilon} \\ 0.95 \mu_{\mathrm{d}} f_{\mathrm{n}} & \text{if } v_{\mathrm{t}} > v_{\varepsilon} \end{cases} \tag{18}$$

where v_{ε} is a threshold velocity. Bengisu and Akay [9] presented an alternative formulation to determine the friction force as follows

$$f_{\mathrm{t}} = \begin{cases} \left[-\frac{\mu_{\mathrm{s}} f_{\mathrm{n}}}{v_0} \left(\|\mathbf{v}_{\mathrm{t}}\| - v_0\right)^2 + \mu_{\mathrm{s}} f_{\mathrm{n}} \right] \operatorname{sgn}(v_{\mathrm{t}}) & \text{if } v_{\mathrm{t}} \leq v_0 \\ \left[\mu_{\mathrm{d}} f_{\mathrm{n}} + \left(\mu_{\mathrm{s}} f_{\mathrm{n}} - \mu_{\mathrm{d}} f_{\mathrm{n}}\right) \mathrm{e}^{-\kappa\left(\|\mathbf{v}_{\mathrm{t}}\| - v_0\right)} \right] \operatorname{sgn}(v_{\mathrm{t}}) & \text{if } v_{\mathrm{t}} > v_0 \end{cases} \tag{19}$$

where κ is a positive parameter that represents the negative slope of the sliding state. Figure 3c shows the evolution of the Bengisu and Akay friction law. Ambrósio (cf. Fig. 3d) suggested an alternative solution for friction force as follows [9]

$$f_{\mathrm{t}} = c_{\mathrm{d}} \mu_{\mathrm{d}} f_{\mathrm{n}} \operatorname{sgn}(v_{\mathrm{t}}) \tag{20}$$

in which c_{d} represents a dynamic correction factor.

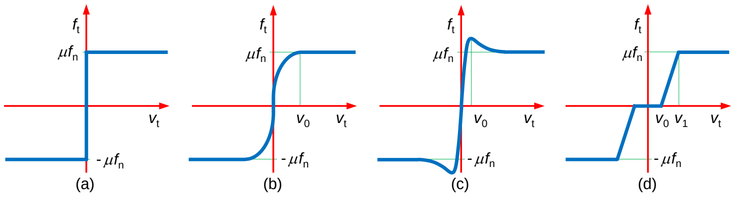

Fig. 3. (a) Coulomb's friction model; **(b)** Threlfall friction model; **(c)** Bengisu and Akay friction model; **(d)** Ambrósio friction model.

4 Example of Application

A hexapod walking robotic system that experiences both normal and tangential contact events between its feet and the ground surfaces and stairs, is utilized here as an application example [10, 11]. Figure 4 depicts a multibody model of the hexapod robotic system under analysis. Each leg is established using a four-bar mechanism connected to the main body via a spatial revolute joint. This robotic system is operated using six rotational motors and six linear actuators, which promote traction and elevation actions, respectively. In addition, a spherical foot is rigidly attached to each leg, allowing for the model of contact interactions with the ground and stairs. The normal and tangential contact forces are computed using the continuous formulation described in Section 3.

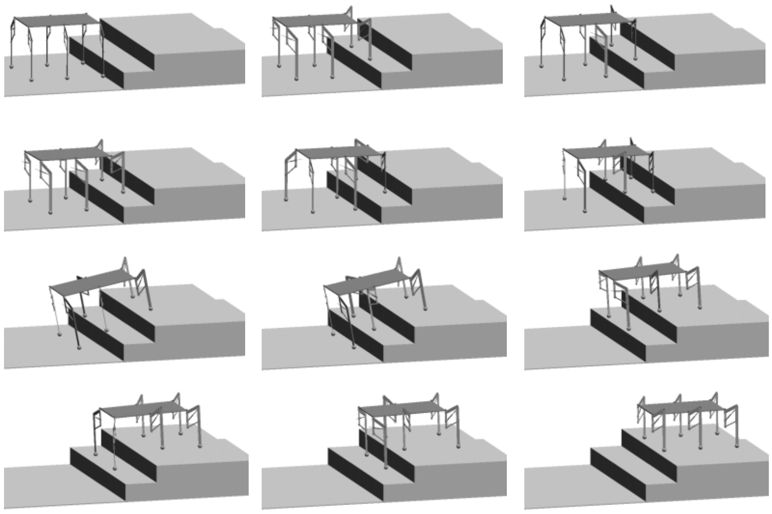

Fig. 4. Snapshots of the hexapod of a standard set of stairs climbing dynamic simulation.

In the present work, two representative simulations were conducted to assess the dynamic behavior of the hexapod system. Specifically, the simulations involved navigating a straight path on a planar horizontal surface and climbing a standard set of stairs. Figure 4 shows a series of snapshots from the simulation depicting the case of climbing stairs. The plots of the torques and forces generated in the rotational and linear actuators of a front leg for hexapod locomotion on flat terrain and during stairs climbing are depicted in Fig. 5. From the analysis of these diagrams, it can be observed that the worst case occurs in the stairs climbing simulation. Overall, this study allows to explore the critical role of the contact process in the success of hexapod motion simulations. Specifically, the adopted contact detection procedure and the smooth transition between different contact regimes are of paramount importance to ensure the dynamic stability of the hexapod robotic system.

5 Concluding Remarks

This work summarizes the state of the art related to main modeling aspects of contact-impact events in multibody mechanical systems. For this purpose, the fundamental kinematic aspects associated with collisions in dynamical systems are revisited, namely in what concerns the identification of the potential or candidate contact points, as well as the computation of the normal and tangential contact velocities. Furthermore, a short revision of the most relevant contact force models is offered. Finally, a hexapod system was considered as a demonstrative example of application, highlighting the key aspects related to modeling contact-impact events in dynamical systems.

Future directions for research within the framework of contact mechanics in multibody dynamics may include: (*i*) identification and estimation of contact parameters for complex scenarios; (*ii*) development of benchmark problems to assess the suitability of existing techniques for handling contact-impact events; (*iii*) analysis of contact problems with very large contact areas; (*iv*) development of techniques to accelerate contact detection with multiple potential contacts. These directions aim to address the challenges and enhance the capabilities of current methods in handling contact interactions within multibody mechanical systems.

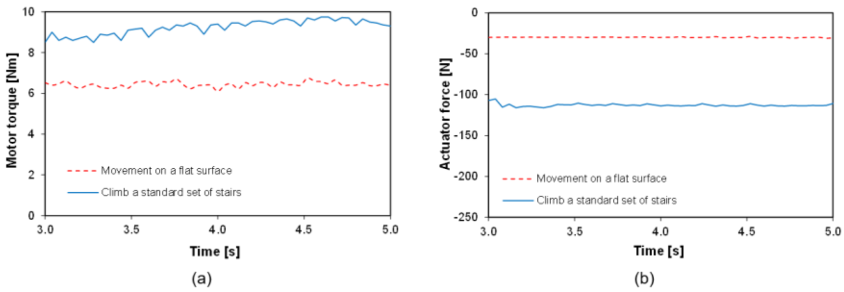

Fig. 5. (a) Torque developed in the rotational motor during the hexapod traction motion; **(b)** Force generated in the linear motor on the front leg during the traction motion.

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