A nonlinear water level controller for reservoirs in cascade

Filipa N. Nogueira[1] and José L. S. Pinho[1]

***Abstract*— In this paper, a strategy for the control of the water levels of a cascade of reservoirs is presented. This is achieved by operating a pumping station or discharge structures. The system is disturbed by reservoirs inflows resulting from rainfall-runoff events. Some examples of these systems are reservoirs that must be operated to satisfy navigation or leisure requirements or their operation during flood events. The presented controller reveals a good performance and has the advantage of being simple to implement. Moreover, it was proven that the described control law ensures asymptotic reference tracking, reaches several convergence rates, by tuning, and it allows the changing of the desired water levels reference values during the control process.**

I. Introduction

The water levels of certain reservoirs must be managed under stringent ranges. The open water systems, used for navigation and recreation, should be operated in such a way that the water level is maintained near a desired predefined value, independently of their inflows. The rules for the operation of the reservoirs during flood events usually imply maintaining the water level as much as possible near a guide curve. A failure the control the water level can lead to dams over-topping during a flood event. On the other hand, inadequate low water levels may disrupt some water uses like navigation. Thus, an automatic controller that maintains an adequate water level in reservoirs constitutes an important tool to help the operational management of controlled reservoirs.

Many studies have been made in this context, see for instance [1], [2], [3]. A distributed linear quadratic Gaussian controller is presented in [4], and a model predictive controller (MPC) is used in [5] and in [6], however, the MPC is in general very time-consuming and highly dependent on the weather forecast quality. A proportional integral derivative (PID) controller is described in [7]. Although, a controller based on PID may lead to an oscillatory signal bringing forth an unstable output and it depends on a proper integral gain value, which could be also time-consuming to obtain.

In this paper, we propose a control law for automatic water level control through proper pumping/discharge values computation in a cascade of reservoirs, presenting rainfall-runoff inflows disturbances. Although this control law is designed with a linear function, it is considered to be nonlinear due to the positivity restriction. The controller is based on a state space representation, which makes it very easy to be implemented and very low computation time-consuming. It also has the advantage of allowing the change of the desired reference values during the control process, which is appropriate for reservoirs control during flood events, and it reaches several convergence rates by tuning.

The present paper is structured as follows. In Section 2 a cascade of reservoirs model is presented and in Section 3 a nonlinear water level controller for reservoirs in cascade is designed. In Section 4 several simulations are made under different circumstances. Conclusions follow in Section 5.

II. Model description

Here we consider a model of n reservoirs in cascade as illustrated in Fig. 1. The water level of the reservoir i, for $i = 1 \cdots n$, is represented by $h_i(k)$ and is measured in meters, m, with respect to a reference datum; the control flow that pumps/discharge water out of the reservoir i is represented by Q_c^i, whereas the disturbance rainfall-runoff inflow is represented by Q_d^i. Both Q_c^i and Q_d^i are measured in m^3/s. Mathematically, this model may be described as:

$$h(k+1) = h(k) + B_c\, Q_c(k) + B_d\, Q_d(k), \qquad (1)$$

where

$$h(k) = \begin{bmatrix} h_1(k) \\ \cdots \\ h_n(k) \end{bmatrix}, \qquad Q_c(k) = \begin{bmatrix} Q_c^1(k) \\ \cdots \\ Q_c^n(k) \end{bmatrix},$$

$$Q_d(k) = \begin{bmatrix} Q_d^1(k) \\ \cdots \\ Q_d^n(k) \end{bmatrix}, \quad B_d = \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & B_n \end{bmatrix}, \qquad (2)$$

*This work was financed by FCT / MCTES through national funds (PIDDAC) under the RD Unit The Centre for Territory, Environment, and Construction (CTAC), under reference UIDB / 04047/2020.

[1]CTAC - The Center for Territory, Environment and Construction, School of Engineering of University of Minho, 4800-048 Guimarães, Portugal. Corresponding author: Filipa N. Nogueira, filipan@civil.uminho.pt

$$B_c = \begin{bmatrix} B_c^1 \\ \cdots \\ B_c^n \end{bmatrix} = \begin{bmatrix} -B_1 & 0 & 0 & \cdots & 0 & 0 \\ B_2 & -B_2 & 0 & \cdots & 0 & 0 \\ 0 & B_3 & -B_3 & 0 & \cdots & 0 \\ & & & \cdots & & \\ 0 & 0 & 0 & \cdots & B_n & -B_n \end{bmatrix}, \tag{3}$$

$B_i = -\dfrac{T_c}{A_s^i}$, A_s^i is the average storage area (m^2) of the reservoir i, T_c is the control time step (s), $k \in \mathbb{N}$, and $h_i(k) < 0$, for $i = 1 \cdots n$.

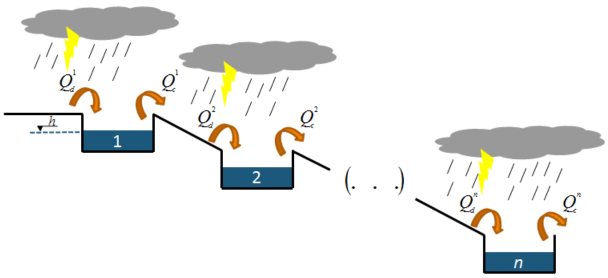

Fig. 1. Scheme of water reservoirs in cascade.

This model was designed taking into account the balance equation for the water volume, $Ah(t)$, in a reservoir:

$$\frac{d\,Ah(t)}{d\,t} = Q_{in}(t) - Q_{out}(t), \tag{4}$$

where A is the average storage area, h is the water level, Q_{in} is the inflow water, Q_{out} is the outflow water, and t represents the time. Moreover, the balance equation 4 may be discretized and approximated by the state space model

$$h(k+1) = h(k) + \frac{T}{A}(Q_{in}(k) - Q_{out}(k)), \tag{5}$$

where T is the control time step.

III. Controller description

The control law design for the automatic water pumping, in order to achieve and maintain a certain desired water level in each reservoir, may be regarded as an output reference tracking problem. This problem may be solved by designing first an auxiliary control law, $\tilde{Q}_c$, and then imposing a restriction of positivity to $\tilde{Q}_c$ such that a positive control input Q_c is obtained.

Let

$$\tilde{Q}_c(k) = \begin{bmatrix} \tilde{Q}_c^1 \\ \cdots \\ \tilde{Q}_c^n \end{bmatrix}$$

$$= B_c^{-1}\left((1-\alpha)\,(h^* - h(k)) - B_d\, Q_d(k)\right), \tag{6}$$

where

$$h^* = \begin{bmatrix} h_1^* \\ \cdots \\ h_n^* \end{bmatrix}, \tag{7}$$

$0 < \alpha < 1$ is a design parameter and h_i^* is the desired reference value for the water level in the reservoir i.

Applying this controller to the system we obtain:

$$h(k+1) = h(k) + B_d\, Q_d(k) + B_c\, B_c^{-1}\left((1-\alpha)\,(h^* - h(k)) - B_d\, Q_d(k)\right) \tag{8}$$

$$= \alpha(h(k) - h^*) + h^*. \tag{9}$$

Defining

$$\Delta h(k) = \begin{bmatrix} \Delta h_1(k) \\ \cdots \\ \Delta h_n(k) \end{bmatrix} = h(k) - h^*, \tag{10}$$

we get

$$\Delta h(k+1) = \alpha \Delta h(k), \tag{11}$$

which implies that

$$\Delta h(k) = \alpha^k \Delta h(0). \tag{12}$$

Hence,

$$\lim_{k\to\infty} \Delta h(k) = 0 \tag{13}$$

and

$$\lim_{k\to\infty} h(k) = h^*, \tag{14}$$

thus, the water levels of the reservoirs converge asymptotically for the desired references.

Hereafter, we prove that the system still converges also in the presence of the positivity constraint

$$Q_c^i = \max\left(0,\, \bar{Q}_i\right), \tag{15}$$

for $\bar{Q}_i := Q_c^{i-1} + \hat{Q}_i$, $Q_c^0 := 0$, and

$$\hat{Q}_i := -B_i^{-1}\left((1-\alpha)\,(h_i^* - h_i(k)) - B_i\, Q_d^i(k)\right). \tag{16}$$

This restriction was designed taking into account that

$$\tilde{Q}_c^i = \tilde{Q}_c^{i-1} + \hat{Q}_i, \tag{17}$$

for $i = 1 \cdots n$, considering $\tilde{Q}_c^0 := 0$, and replacing $\tilde{Q}_c^{i-1}$ for the control input Q_c^{i-1} in (17).

It may be proven that all trajectories converge to h^*, by applying the LaSalle's invariance principle (see [8]) to the Lyapunov function

$$V(h(k)) = \begin{bmatrix} V_1(h(k)) \\ \cdots \\ V_n(h(k)) \end{bmatrix} = \begin{bmatrix} (h_1(k) - h_1^*)^2 \\ \cdots \\ (h_n(k) - h_n^*)^2 \end{bmatrix} \tag{18}$$

for the system (1) on $\mathbb{R}_-^n$.

In conclusion, when the control law

$$Q_c(k) = \begin{bmatrix} max(0, \bar{Q}_1(k) \\ \cdots \\ max(0, \bar{Q}_n(k) \end{bmatrix}, \quad (19)$$

with

$$\begin{cases} \bar{Q}_i = Q_c^{i-1} + \hat{Q}_i \\ \hat{Q}_i = -B_i^{-1}\left((1-\alpha)\left(h_i^* - h_i\right) - B_i\, Q_d^i\right) \\ Q_c^0 := 0 \\ 0 < \alpha < 1 \end{cases} \quad (20)$$

is applied to the system (1), the water level h converges to the reference value h^*.

IV. Simulations

In this section, several simulations under different circumstances are presented in order to show the performance of the proposed controller. We consider two reservoirs in cascade, as illustrated in Fig. 2, subjected to two different disturbances. Moreover, an estimation, $\hat{Q}_d$, is used in (6) instead of Q_d, due to the fact that in practice the real value of the disturbance rainfall-runoff inflow is unknown. The value chosen for $\hat{Q}_d = \left[\begin{array}{cc} \hat{Q}_d^1 & \hat{Q}_d^2 \end{array}\right]^T$ was based on the rainfall-runoff flow event presented in [10]. On the other hand, the value for Q_d was computed by adding to each component of $\hat{Q}_d$ a Gaussian white noise with zero mean and standard deviation $\sigma = 0.5$. This noise is used to emulate the uncertainty associated with the rainfall-runoff forecasts when the controller is used in a water management forecast framework. We also assume that the desired reference values for the water level value of reservoir 1, h_1^*, and for the water level value of reservoir 2, h_2^*, are respectively $-0.40\,m$ (MSL) and $-0.30\,m$ (MSL). The maximum control flow for the reservoir 1, $Q_{c,max}^1$, is $100\,m^3/s$, and the maximum control flow for the reservoir 2, $Q_{c,max}^2$, is $70\,m^3/s$. The minimum control flow $Q_{c,min}$ is $0\,m^3/s$ for both reservoirs. The storage area of the reservoir 1, A_s^1, and of the reservoir 2, A_s^2, are, respectively, $7.3\times10^6\,m^2$ and $7\times10^5\,m^2$. The control time step, T_c, was set to be $900s$. Once these values are fixed, the control law only depends on the design parameter α $(0 < \alpha < 1)$. This parameter influences the speed of convergence to the reference value, as can be seen in Figures 3 and 4 where the values $\alpha = 0.1$, $\alpha = 0.7$, and $\alpha = 0.9$ were respectively taken.

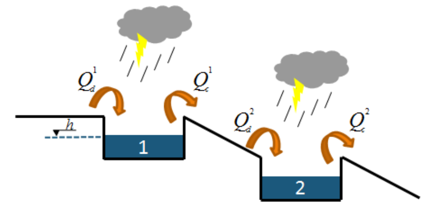

Fig. 2. Scheme of two water reservoirs in cascade.

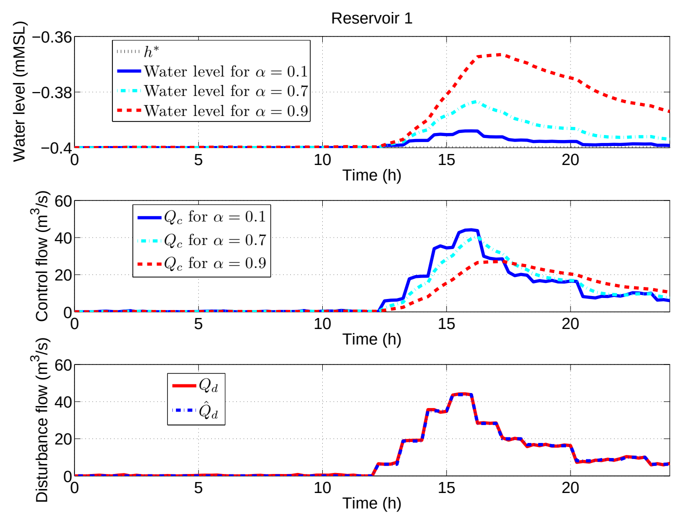

Fig. 3. Control of the water level of the reservoir 1 for different α parameter values. The evolution of the water level is illustrated on the top graph, where the dashed black line corresponds to the desired reference value. The pump control flow is represented on the second graph. On the bottom graph the rainfall-runoff inflow, Q_d, is marked in solid red line, whereas the estimated disturbance, $\hat{Q}_d$, is marked in dashed blue line.

Figures 5 and 6 illustrate the performance of the control algorithm in the presence of a change of the reference profile. For reservoir 1, in the first 6 hours it is intended that the water level follows the reference $h_1^* = -0.40\,m$, in the following 6 hours the reference for the water level is set to be $h_1^* = -0.45\,m$ and in the last twelve hours the water level should again follow the reference level $h_1^* = -0.40\,m$. For reservoir 2, in the first 6 hours it is intended that the water level follows the reference $h_2^* = -0.30\,m$, and in the last eight hours the water level should follow the reference level $h_2^* = -0.35\,m$. It may be seen that the controller has a good performance also in this case. As expected, for both reservoirs, when the reference decreases there is an increase of the pumped flow water, $Q_c^1 = Q_{c,max}^1$ for reservoir 1 and $Q_c^2 = Q_{c,max}^2$ for reservoir 2, until the reference value is achieved. On the other hand, when the reference values increase no water is pumped. Note that the speed with which the water level rises depends on the disturbance Q_d.

In Figures 7 and 8, we tested the performance of the controller assuming a minimum change in the control flow $\Delta Q_{c,min} = 2\,m^3/s$, that could be an operational re-

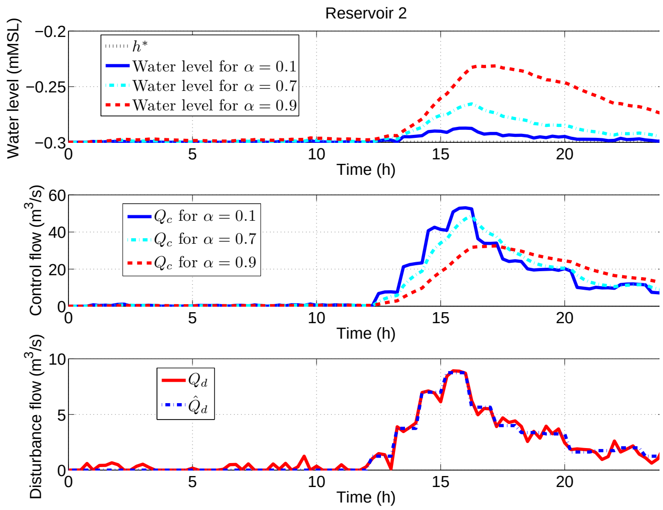

Fig. 4. Control of the water level of the reservoir 2 for different α parameter values. The evolution of the water level is illustrated on the top graph, where the dashed black line corresponds to the desired reference value. The pump control flow is represented on the second graph. On the bottom graph the rainfall-runoff inflow, Q_d, is marked in solid red line, whereas the estimated disturbance, $\hat{Q}_d$, is marked in dashed blue line.

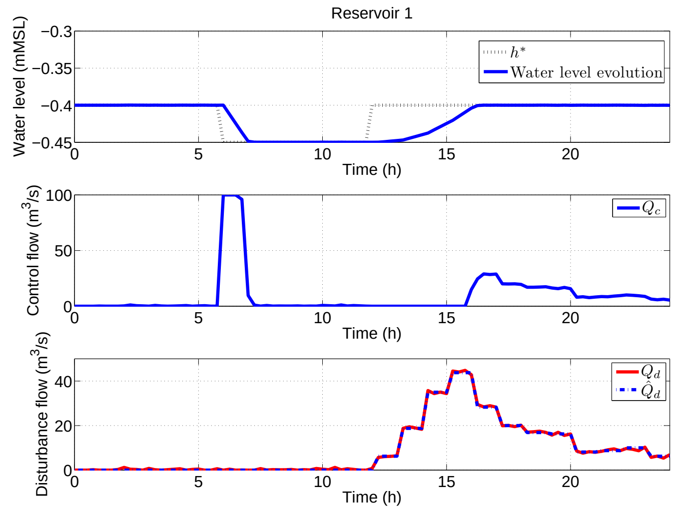

Fig. 5. Control of the water level of the reservoir 1 assuming changes in the reference level during the control period. ($h^* = -0.40$ m from the beginning till $t = 6$ hours, $h^* = -0.45$ m from $t = 6$ hours till $t = 12$ hours, and $h^* = -0.4$ m from then on), for $\alpha = 0.1$. The evolution of the water level is illustrated on the top graph, where the dashed black line corresponds to the desired reference value. The pump control flow is represented on the second graph. On the bottom graph the rainfall-runoff inflow, Q_d, is marked in solid red line, whereas the estimated disturbance, $\hat{Q}_d$, is marked in dashed blue line.

striction in real open water systems. In case the required flow change is smaller than this value, the control action is postponed, i.e. if $|Q_c^i(k+1) - Q_c^i(k)| < \Delta Q_{c,min}$ we make $Q_c^i(k+1) = Q_c^i(k)$, for $i = 1, 2$. As we may observe, even in these circumstances, the desired reference value is tracked.

V. Conclusion

A control law was designed and implemented for the operational management of open water systems, intended to maintain predefined water levels in a reservoirs cascade, based on a state space representation, and subject to rainfall-runoff inflow events.

This controller has the advantage of being simple to implement and its performance is not very time-consuming, it reaches several convergence rates, by tuning, and it allows the changing of the desired water levels reference values during the control process.

Several simulations were made under different circumstances, like considering errors associated with the disturbances uncertainties, changes in the water level reference profiles, consideration of a required minimum change in the control flow, and changes in the convergence speeds. The obtained results encourage the use of the proposed control strategy in real controlled open water systems.

References

[1] D. Schwanenberg, J. Talsma, J. L. S. Pinho, A. Reis, M. Bessa, J. Kuwa- jima, Short-term reservoir optimization for mitigation downstream flood., 8th International Conference of EWRA "Water Resources Man- agement in an Interdisciplinary and Changing Context" (2013) 1–8.

[2] D. Schwanenberg, I. Sheret, T. Rauschenbach, S. Galelli, J. M. P. Vieira, J. L. S. Pinho, Adjoint modeling framework for water resources models., 10th International Conference on Hydroinformatics (HIC 2012) (2012) 1–8.

[3] J. L. S. Pinho, J. M. P. Vieira, D. Schwanenberg, R. Montero, Modelling of reservoirs in management and control frameworks, First International Symposium on Energy Challenges and Mechanics. (2014) 1–8.

[4] J. M. Lemos, L. F. Pinto, L. M. Rato, M. Rijo, Multivariable and distributed lqg control of a water delivery canal., Irrigation and Drainage Engineering 139(10) (2013) 855–863.

[5] J. M. Maestre, P. J. van Overloop, M. Hashemy, A. Sadowska, E. F. Camacho, Human-in-the-loop model predictive control of an irrigation canal., IEEE control systems 35(4) (2015) 19–29.

[6] P. J. van Overloop, S. V. Weijs, S. Dijkstra, Multiple model predictive control on a drainage canal system., Control Engineering Practice 16(5) (2008) 531–540.

[7] A. Lacasta, M. Morales-Hernandez, P. Brufau, P. Garcia-Navarro, Simulation of pid control applied to irrigation channels., Proceedings of the 12th International Conference on Computing and Control for the Water Industry, Procedia Engineering 70 (2014) 978–987.

[8] V. Sundarapandian, An invariance principle for discrete-time nonlinear systems, Applied Mathematics Letters 16 (2003) 85–91.

[9] J. P. LaSalle, The Stability of Dynamical Systems, SIAM, Bristol, Eng- land, 1976.

[10] JP. J. van Overloop, Model predictive control on open water systems., Ph.D. dissertation. Delft University of Technology, The Netherlands. (2006).

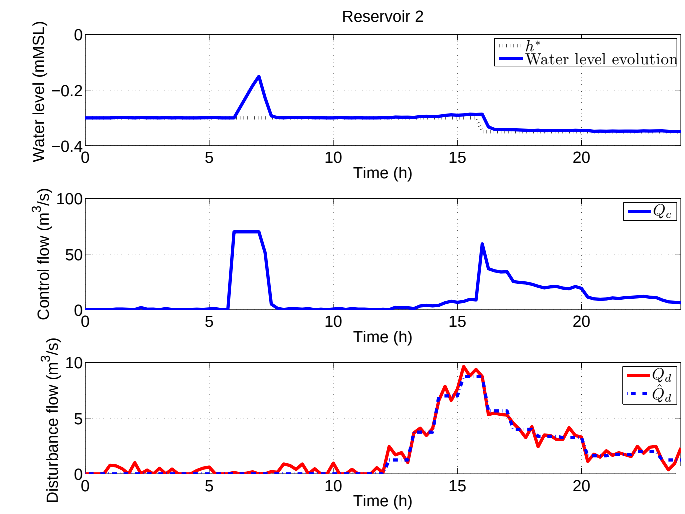

Fig. 6. Control of the water level of the reservoir 2 assuming changes in the reference level during the control period. ($h^* = -0.30\ m$ from the beginning till $t = 16$ hours, $h^* = -0.35\ m$ from $t = 16$ hours from then on), for $\alpha = 0.1$. The evolution of the water level is illustrated on the top graph, where the dashed black line corresponds to the desired reference value. The pump control flow is represented on the second graph. On the bottom graph the rainfall-runoff inflow, Q_d, is marked in solid red line, whereas the estimated disturbance, $\hat{Q}_d$, is marked in dashed blue line.

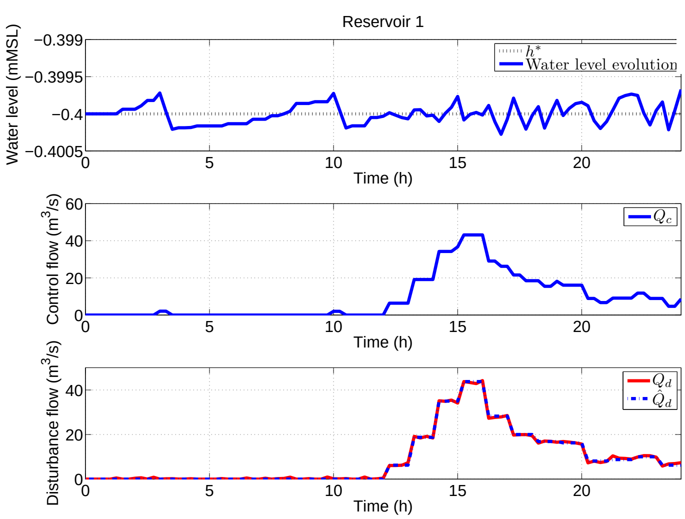

Fig. 7. Control of the water level of the reservoir 1 assuming $\Delta Q_{c,min} = 2\,m^3/s$, for $\alpha = 0.1$. The evolution of the water level is illustrated on the top graph, where the dashed black line corresponds to the desired reference value. The pump control flow is represented on the second graph. On the bottom graph the rainfall-runoff inflow, Q_d, is marked in solid red line, whereas the estimated disturbance, $\hat{Q}_d$, is marked in dashed blue line.

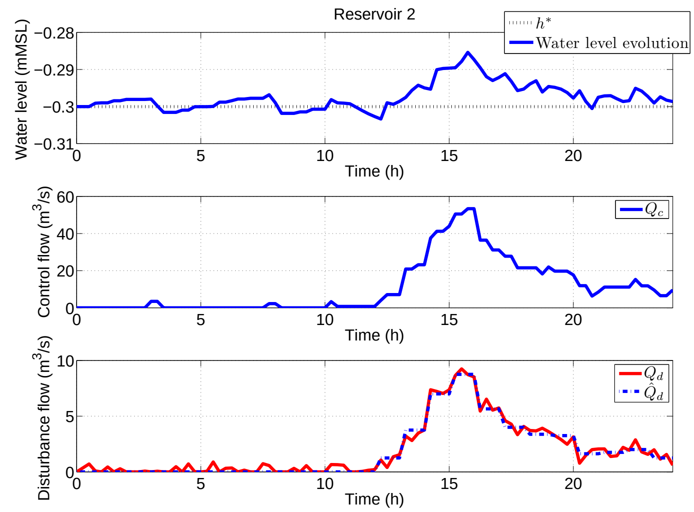

Fig. 8. Control of the water level of the reservoir 2 assuming $\Delta Q_{c,min} = 2\,m^3/s$, for $\alpha = 0.1$. The evolution of the water level is illustrated on the top graph, where the dashed black line corresponds to the desired reference value. The pump control flow is represented on the second graph. On the bottom graph the rainfall-runoff inflow, Q_d, is marked in solid red line, whereas the estimated disturbance, $\hat{Q}_d$, is marked in dashed blue line.