Escher and the Search for the Infinite: a Webquest for a K12 Class

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Abstract
In this research, it was created, implemented and evaluated teaching experiences about the extending and refining of concepts of infinity through one webquest applied to a 12th grade class. There were established three research questions. Will the webquest *Escher and the search for the infinite* allow the development of creative and critical thinking? Will it enhance a collaborative knowledge environment? Can it contribute for a better understanding of the concept of infinity? Through the analysis of the final work done by each group, we verified that students had developed the ability of organising and synthesizing information, that they had developed cooperative work, creativity, scientific reasoning and critical thinking. We detected a better understanding of the concept of infinity by comparing the pre-test with the pos-test results.

1. Introduction

Along the years, the concept of the infinite was target of a fervent discussion. Since Ancient Greece, Men have tried to come up with a definition for that concept. Demócrito, Zenão, Aristóteles, Arquimedes, Galilei, Bolzano, Dedekind, Cantor, Weierstrass, Poincaré, Hilbert, Borel, Russel, Robinson... these are just a few of the mathematicians that have dedicated themselves to this topic. The history of the infinite as a mathematical entity is long and controversial. The infinite was always associated to several paradoxes situations. Slowly, some of those paradoxes were eliminated, but nevertheless it remains as a very subjective and complex subject.

This theme is present in the Portuguese Mathematical National Curriculum and is the subject of a continuous evolution as students get older. There is an approach to the notion of the infinite in the 3rd cycle (7th, 8th and 9th grades) when students learn about sets of natural numbers, integers, rationales and reels. It's also connected to the notion of irrational number and to the notion of function, both taught in this cycle. The concept of the infinite is approached during the entire Mathematical journey of each student, and when they reach the Secondary education (10th grade), this notion becomes more formal. It has a more profound meaning of function and also an introduction to the notions of limit, continuity, derivate and sequence, enlarging the concept of number and working with the concepts of infinitely large, infinitely small, limit of a sequence …

The infinite was always present in Maurits Escher's work (1898-1972), but around 1956 his work showed more clearly the representation of the infinite in a bi-dimensional background. We can divide his work on this subject according with three categories: *cycles*, *filling in surfaces* and *limits*. In each one, there are pieces that even with different characteristics can still be considered important and well known. Deeply connected to the concepts of *cycles* and *filling in surfaces*, we find the notion of *Potential infinite,* represented by an endless process. There is a systematic idea that suggests an unlimited process. *Limits* are associated with an isomorphic reduction of the images. In the pieces where he performed a reduction from the inside to the outside (bigger shapes go into the centre and the infinite reduction is in the margin with a circular shape), he produced a series called *Circular Limits*, made of four

pieces. Amongst those we can find *Circular Limit III* (1959) as being Escher's best approach to the infinite, reaching the *Actual infinite*.

Bearing in mind that webquests are cognitive tools that aim a higher level of thinking, it was created a webquest called "Escher e a procura do infinito" (Escher and the search for the infinite) available on the web using the URL: http://patisampaio.no.sapo.pt. In order to make this webquest more appealing and having in mind that the infinite is a constant in Escher's work, we connected this concept to the artist himself. According to the complexity of the theme and the controversial history around it, the use of various sources allows us to understand better the past to face the present challenge. By creating a webquest related with Escher's art, we want students to discover the connection between Escher's world and the mathematical concept of the infinite! Therefore, this educational investigation aspires to analyse the implications that may occur when students try to understand this mathematical concept using the webquest.

2. Research Study

As the webquest refers (fig. 1), the concept of infinity has always been controversial, generating several paradoxes. We have decided to create a webquest about the infinite because it is a subject of the Mathematics school programme and it is complex enough to originate high thinking. In order to make a more attractive work, we have decided to include Maurits Escher's work, connecting this artist to the concept of the infinite, resulting in the title of the webquest: "Escher e a procura do infinito".

Figure 1: "Escher e a procura do infinito" introduction.

The notion of the infinite is in the national curricula of Mathematics, but it is in a more expressive way in Mathematics of the Sciences course. This is not an intuitive notion; it is a very complex one, so the webquest is for secondary students and not for the elementary ones. According to the National Curricula of Mathematics, secondary "students can realize individual or group work about the calculus history … and should also make some work about the extension of number (Silva et al, 2002: 5; 8) — notions intrinsically related to the infinite history.

According to the complexity of the theme and the students that is targeted for, this webquest is characterised as a long term one. Students should deeply analyse a quantity of information, transforming it into new knowledge and being able to create something new. Therefore, this *mindtool* should be applied between one and four weeks.

We have already explained some characteristics of the webquest, but it's still necessary to refer its structure. In the home page (fig. 2), there is information for teachers that clarifies what a webquest is, who are its mentors, when it should be used, to whom it concerns and some bibliography of the subject.

Figure 2: "Escher e a procura do infinito" home page.

By activating the link to go into this adventure, we reach the introduction and the menu (fig. 3), which is always available on the left side of the screen. There, we can find the six building blocks of any webquest: introduction, task, process, resources, evaluation and conclusion, as well as the students help.

Figure 3: "Escher e a procura do infinito" menu.

In the introduction, we do a problematic question and invite students to find out Maurits Escher's world, being the infinite a constant on his work. In the task we explain that his work around the subject can be divided into three categories: cycles, tessellations and limits. They must make a poster with these categories. The task is accomplishable and leads to deep thought of the theme.

The process is described with simple words, in such a way that students can understand exactly what is asked and what they should do. We have elaborated a small table that divided the sub themes of the research between the four elements of each group (tab. 1). We should notice that each group could make their own distribution of the work.

Task		**Students**
Find about the infinite history		A, B
Understand the concept of the infinite		C, D
Find about Escher's work		All
Connect Escher's work with the infinite	Cycles	A, C
	Tessellations	B, D
	Limits	A, D
Choose the images		B, C
Elaborate the poster		All

Table 1: Sample of the process of "Escher e a procura do infinito".

The process is sometimes associated with the resources, as in this case. Each resource has its importance and it is connected to the subject. According to the sub themes, the resources were divided into four categories: the infinite, M. C. Escher, images and tessellations. For each topic, there is a small description (tab. 2), followed by several web sites in Portuguese and in English.

Theme	Description
The infinite	Debates about the infinite were a constant on the Greek school, but only in the XIX century, with Georg Cantor, *actual infinite* was accepted as a subject of study by Mathematicians.
M. C. Escher	Maurits Cornelis Escher was born 17th July 1898, at Leeuwarden, Netherlands. The acknowledge of his work was not instantaneous, being only recognised in 1951! In a paper of 1959, Escher decided to write about limits.
Images	Escher's work represents a new vision of Mathematics.
Tessellations	We live around tessellations and in 1936, when Escher visited Spain, he decided to dedicate part of his work to this subject.

Table 2: Sample of "Escher e a procura do infinito" resources.

The evaluation has qualitative and quantitative standards to analyse what students have learned, their collective and individual work and the performance of the task. Finally, the conclusion indicates the advantages of making the webquest, challenging students to do a new research.

A webquest must be evaluated before being available on-line. To do that, two high school mathematics teachers were asked to analyse "Escher a procura do infinito" and fill in the evaluation form proposed by Bellofatto, Bohl, Casey, Krill e Dodge (2001) about the aesthetics, the introduction, the task, the process, the resources and the evaluation of the webquest. About the aesthetics component the searchers verified that this cognitive tool respects all the details mentioned in the fine points list proposed by Dodge (1999).

The conception and implementation of "Escher e a procura do infinito" are part of a investigation work that aims to evaluate the potential of the use of the webquest in the mathematic conception of the infinite. The activity begun on 18th April 2006 and was over on 18th May of the same year and 16 students of the 12th grade participated in it. The sample was of convenience once the students were taught by one of the researchers. On the first session the students were informed about the investigation work that was going to be developed and were requested to fill in a questionnaire that tends to evaluate their conception of the infinite. On the second session the webquest was introduced and till the seventh session students worked together doing all the tasks. On the eighth session they had to fill in the second questionnaire. Throughout these sessions each group made a poster about the conception of the infinite, connecting it to Escher's art and to the infinity history, and were evaluated by the standards described on the evaluation component of the webquest.

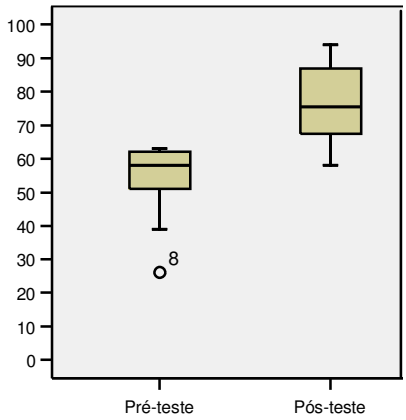
In this research were established three investigation questions. The webquest "Escher e a procura do infinito" will allow the development of creative and critical thinking? Will suggest one collaborative knowledge environment? Can it contribute for a better understanding of infinity?

The research group was formed by 16 students of the Escola Secundária de Paredes (secondary school of the north of Portugal). They formed a 12th grade class. There were 6 students with 17 years old, 8 with 18 years old, 1 with 19 years old and 1 with 25 years old. 75% were girls and 25% were boys. The sample was from convenience. They were divided into 4 working groups with 4 elements each. Now, we are going to analyse the final product made by each group (poster) and then compare the results obtained in the two tests (pre and pos).

The four groups worked in a cooperative way and showed a good research method. Only one group respected the distribution of the process. They got enough information from the internet and organize it in a proper way. The overall aesthetics of the posters is attractive, with well chosen colours and pictures; only in one poster it could be more attractive because there is a lack of colour. The language is simple and objective. It shows creativity but they could do better. It corresponds to scientific standards but there are some failures, they could explore more the concept of infinite. Now, we are going to see the posters made by the groups (fig. 4) and compare the results obtained in the pre and pos tests by each working group.

Figure 4: Posters.

To compare the results obtained by the 16 students before and after the webquest it was made a box plot (gra. 1). In the first test the extremes were 26% and 63% and increased in the second test to 58% and 94%, but the class was more homogenous before than after the webquest. The mean also increased from 54,25% to 77,13%.



Graphic 1: Box plot.

To confirm that the differences obtained in the two tests are statistical significant, we made a non-parametric test (tab. 3) since the group had only 16 elements and the distribution obtained in the tests wasn't symmetric.

	Pre/Pos-test
Exact Sig. (2-tailed)	,000 (Binomial distribution)

Table 3: Sign test.

After the webquest's accomplishment this 16 pupils were asked to make a small commentary on this type of strategy, focusing there sensations about the development of the work, like contentment, difficulties, advantages … Analyzing their opinion we conclude that, 14 (87.5%) had liked this strategy and 2 (12.5%) had been indifferent to the whole process. The main mentioned advantages were the interaction that the team work allowed (50%), the resources contained in the webquest (50%), the webquest menu that was always available showing all the components and giving a global perception of the work (50%) and the use of the Internet in the classroom (37.5%). They also referred the importance of the constant presence of the evaluation system that allows a great control, because they know exactly what it is asked, being able to perform better the tasks (2 pupils); the new technologies used were most attractive (2 pupils) and the poster elaboration was an idea unusual and imaginative (2 pupils). Finally, one pupil referred the originality and interest of this kind of strategy and the accessibility that a work like this has, because it can be consulted in any part.

The visual and graphic aspect of this *mindtool* was very successful, and 6 pupils (37.5%) mentioned that the use of Escher's images is marvelous and 3 (18.75%) believe that the site is very attractive and appellative. The disadvantages detected by this experimental group are arrested with the elaboration of a poster (2 pupils), the working group strategy (1 pupil), the Internet use (1 pupil) and the existence of foreign sites (1 pupil). To point out, 6 pupils considered infinite a very difficult and complex concept. It is also necessary to refer that 2 students considered that this type of strategy demands more effort and commitment, becoming more difficult, but simultaneously the learning is more significant.

3. Conclusion

Through the analysis of the final products, we verified that students have developed the ability of organisation and synthesis of information, cooperative work, creativity, scientific work and critical thinking. Comparing the results obtained in the two tests, we reach a statistical difference between the first and the second. Before doing the webquest the mean of the tests was 54,25% and in the second test the mean was 77,13%. Therefore, we can say that the use of webquests in class contexts may improve the learning process and, in particular, may link Mathematics to the technology and to the world.

This is a research study with only one 12^{th} grade class, formed by 16 students and the sample was from convenience. So we can't generalize the results. Despite these limitations, it's a contribution for the research with webquests in a learning environment, in particular, with the teaching of the infinite.

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