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"Pricing with Customer Recognition"

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Pricing with Customer Recognition*

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Abstract

This article studies the dynamic effects of behaviour-based price discrimination and customer recognition in a duopolistic market where the distribution of consumers' preferences is discrete. In the static and first-period equilibrium firms choose prices with mixed strategies. When price discrimination is allowed, forward-looking firms have an incentive to avoid customer recognition, thus the probability that both will have positive first-period sales decreases as they become more patient. Furthermore, an asymmetric equilibrium sometimes exists, yielding a 100-0 division of the first-period sales. As a whole, price discrimination is bad for profits but good for consumer surplus and welfare.

1 Introduction

The rapid changes in technology, the increasing use of the Internet and the development of more sophisticated methods for acquiring, storing and analysing consumer information have dramatically improved the capability of sellers to learn the consumer types or preferences through the observation of their past choices, and to set prices accordingly in subsequent periods. As customer recognition and behaviour-based price discrimination (BBPD) are likely to become increasingly prevalent in the foreseeable future, a good economic understanding of the profit, consumer surplus and welfare implications of this price discrimination practice needs to be founded on a good economic understanding of the market in question.

The aim of this paper is to investigate the competitive and welfare effects of pricing with customer recognition in a duopolistic market where the distribution of consumer types is discrete. The use of a discrete distribution for consumer tastes brings new insights to the literature in the field and helps establish the idea that some of the competitive effects of BBPD and customer recognition do depend on what is learned about consumer demand, which in turn depends on the nature of preferences.

This paper considers a repeated interaction model, where firms A and B market their goods directly to consumers whose preferences are determined by a binary distribution: half of consumers prefer firm A by a fixed amount and half of consumers prefer firm B by the same fixed amount. In online markets, for instance, it is likely that consumers' loyalty is limited, meaning

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that even though firms may have some advantage over their competitors due to brand loyalty, all consumers may, nevertheless, be induced to switch.[1] There are only two periods so, being permitted, price discrimination can only occur in the second period when firms have learnt the consumer types by observing their first period choices.

In order to measure the dynamic effects of behaviour-based price discrimination two static benchmarks are analysed. In the first, price discrimination is not permitted, either because consumers are anonymous or because price discrimination is illegal (Section 4). In the other benchmark, consumers are non-anonymous and price discrimination is permitted. The results derived in both benchmarks show that firms are clearly worse off when they have the required information for price discrimination.[2]

The two-period interaction game where price discrimination can occur in the second period, if firms learn the consumers' types, is presented in Section 5. In contrast to the extant models with a continuous distribution of consumer types (e.g. Fudenberg and Tirole (2000) and Villas-Boas (1999)), where the equilibrium is in pure strategies, the model yields a first-period equilibrium where each firm follows a mixed pricing strategy as an attempt to prevent the rival from systematically predicting its price. This pricing strategy seems to be in accordance with several studies showing that random pricing is a feature of online markets (Brynjolfsson and Smith (2000a), Baye, Morgan and Sholten (2002), to name a few) and may be the result of retailers heterogeneity with respect to brand loyalty, trust, and awareness (Brynjolfsson and Smith (2000a)).

A relevant contribution of this paper is to highlight the fact that firms may eschew learning the consumer types as a way to avoid subsequent price discrimination and a less favourable competitive outcome. It is shown that when initial market shares are asymmetric (100-0 division) nothing is learnt about consumers, and subsequent prices and profits are higher. In contrast, with symmetric market shares, consumer types are fully revealed and second period prices and profits fall. Consequently, it is shown that forward looking firms have an incentive not to share the market in period 1, so as to avoid learning and therefore the negative effects of price discrimination (Proposition 5). In relation to this second profit effect consideration, it is shown that firms price *below* the static or no-discrimination levels in period 1 (Proposition 6).[3]

Given that the model predicts the existence of a bias towards asymmetric outcomes in period 1, Section 7 investigates the circumstances in which an *asymmetric* pure strategy equilibrium could exist in the initial period, where the same firm serves the entire market. Here it is found that there is sometimes an asymmetric equilibrium in the first period, where one firm sets a low price and captures all consumers while the rival finds it not profitable to match the low price firm because its profits will then be low in the second period (Proposition 8). This finding suggests that in a many period game the uniform pricing could be sustained without any explicit collective action.

Finally, the paper sheds light on the welfare effects of BBPD in markets where firms set prices randomly (Section 8). A common prediction in the existing literature on BBPD is that

[1] In a recent study Brynjolfsson and Smith (2000b) have found that Amazon customers are willing to pay up to 5-8 percent more before they consider switching to another seller.

[2] Note that in a static game, a monopolist does best with more information. In contrast, in competitive contexts, more information may act to intensify competition between firms and reduce profits.

[3] Fudenberg and Tirole (2000, p. 643) claim that the effects of behaviour-based price discrimination on first period prices in the brand preference approach are the reverse of those effects in the switching costs approach. Here, in contrast, we find that BBPD may lead to the same lower first period prices as in the switching costs approach.

price discrimination can be welfare reducing due to excessive switching (e.g. Fudenberg and Tirole (2000)). Here, in contrast, as random pricing tends to generate some inefficient shopping, price discrimination can act to increase efficiency.

The rest of the paper is organised as follows. Section 2 presents a brief review of the more relevant literature. The model is formulated in Section 3. Section 6 discusses the main implications of pricing with customer recognition. Conclusions are presented in Section 9.

2 Related literature

This model combines two strands of the literature. One is the literature on competitive price discrimination,[4] especially the literature on behaviour-based price discrimination and customer recognition (see Fudenberg and Villas-Boas (2006) for a comprehensive review). The other is the earliest literature on mixed pricing in oligopoly (e.g. Shilony (1977), Varian (1980) and Narasimhan (1988)).

The model addressed here is closely related to Shilony (1977), who studies a static oligopolistic market where the distribution of consumer preferences towards firms is discrete. In the location interpretation of the model, consumers can purchase costlessly from neighbourhood firms, but incur a transport cost when buying from more distant firms. Shilony shows that if the reservation value is high enough, even though each firm has an advantage in its local market, this segment of consumers is not captive.[5] In this setting, in equilibrium, firms set their prices randomly and therefore can attain high profits. This paper extends Shilony's duopoly analysis to a dynamic framework in which in the second period firms may be able to learn the consumer types, and so price differently towards them. To the best of our knowledge, this paper is one of the first models investigating the competitive and welfare effects of BBPD in markets where, due to the nature of brand preferences, firms set in equilibrium random prices.[6]

Regarding the literature on competitive price discrimination, this paper is related to those models where, in the terminology of Corts (1998), the market exhibits best-response asymmetry.[7] A relevant paper here is Thisse and Vives (1988), which examines price competition in a static Hotelling model, with two firms located at the extremes of the segment $[0,1]$. Consumers are

[4]See Stole (2003) and Armstrong (2006).

[5]An important difference between Shilony's model and those of Varian (1980) and Narasimhan (1988) is that while in these two latter models each firm has a captive group of customers (e.g. uninformed or loyal) and both compete for price-sensitive customers (e.g. informed or switchers), in Shilony neither firm has a group of captive customers. As a result, while in Varian and Narasimhan firms may sometimes set in equilibrium the monopoly price, the same is not true in Shilony.

[6]There are two other models with discrete types of consumers and mixed pricing. Chen and Zhang (2004) assume that each firm has an exogenous captive segment of the market, and that they compete for the remaining consumers, who are price-sensitive, with a reservation value lower than that of the loyal consumers'. They show that BBPD is only feasible if the first-period price of a firm is high enough, in which case it is not accepted by all consumers. In this context they show that BBPD might benefit competing firms. Esteves (2007) investigates BBPD in a model where, by investing in advertising, firms endogenously segment the market into captive and selective customers. When BBPD is permitted in period 2, only the highest-priced price has information to engage in price discrimination. As firms do best in the subsequent period when price discrimination can occur, they have an incentive to change their static pricing. Esteves shows that first-period prices are above the static levels due to the "race for discrimination effect".

[7]The market exhibits best response asymmetry when one firm's "strong" market is the other's "weak" market. In the literature of price discrimination, a market is designated as "strong" if in comparison to uniform pricing a firm wishes to increase its price there. The market is said to be "weak" if the reverse happens.

uniformly distributed in the line segment and firms can observe the "location" of each individual consumer and price accordingly. The strong (close) market for one firm is the weak (distant) market for the other firm. In this setting they show that price discrimination intensifies competition, *all* prices and profits fall. Although price discrimination is a dominant strategy for each firm, for given prices offered by its rival, when all firms follow the same strategy they find themselves trapped in the classic prisoner's dilemma.

Other relevant models with best-response asymmetry are those where firms and consumers interact more than once. In dynamic settings, firms may be able to learn the consumers' exogenous preferences by observing their past choices and price differently towards them in subsequent periods (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000) and Chen and Zhang (2004)).[8] In a monopoly market, Villas-Boas (2004) shows that when consumers behave strategically, a seller without commitment power may become worse off if it conditions prices on consumers purchase history.[9] New issues arise in competitive contexts due to the strategic behaviour of competitive forward looking firms. Fudenberg and Tirole (2000) analyse a two-period duopoly model where consumer preferences follow a symmetric continuous distribution. Consumers are anonymous in period 1, so price discrimination can only occur in period 2. After observing the consumers' past behaviour, firms are able to learn consumer relative preference between the two firms and so each firm will target the rival's previous customers with lower prices than those for its old customers. Fudenberg and Villas-Boas (2006) and Armstrong (2006) present a more detailed analysis of the second period game in FT model showing that, as in the present model, each firm's second period profit is minimized when first period pricing gives rise to the most informative outcome.[10] Thus, it is relevant to investigate whether forward looking firms change their static behaviour when they anticipate the negative effects of learning and price discrimination. Surprisingly, Fudenberg and Tirole find that with consumer tastes uniformly distributed and myopic consumers, forward looking firms do not change their static behaviour. In this specific case, the second period profit consideration has no effect on first-period pricing.[11] In their model, first period prices are above the static levels only as a result of consumers' foresight; i.e. when consumers anticipate BBPD, they become less sensitive to prices in period 1, thus permitting firms to raise first period prices.

An important contribution of this paper is to show that the nature of preferences plays an important role in the understanding of the economic effects of BBPD. By proposing a discrete distribution for consumer preferences this paper shows that the equilibrium is in mixed strategies and that forward looking firms do in fact change their static behaviour as a way to eschew the negative effects of learning and price discrimination in subsequent profits. (This result is valid even when consumers are myopic). Additionally, we will see that the use of a discrete distribution

[8] There is another approach in the literature on BBPD where purchase history discloses important information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)). In Esteves (2007) purchase history discloses information about consumer awareness of the firms' existence.

[9] See also Acquisti and Varian (2005).

[10] A previous version of the present paper is mentioned in Fudenberg and Villas-Boas (2006)'s survey on "Behaviour-Based Price Discrimination and Customer Recognition" and in Armstrong (2006)'s survey on "Recent Developments in the Economics of Price Discrimination".

[11] It is important to stress that with myopic consumers, the FT finding seems to be valid for other symmetric distributions for which the optimization problem is concave. Fudenberg and Tirole assume that the distribution of consumer types satisfies the MHR assumption. However, this assumption is not enough to imply concavity in all symmetric distributions. When the concavity condition is not satisfied, the equilibrium can be, as happens in the present paper, in mixed strategies.

for consumer types also raises new issues regarding the welfare effects of BBPD.

3 The model

Two firms, A and B, produce at zero marginal cost nondurable goods A and B.[12] There are two periods, 1 and 2. On the demand side, there are a large number of consumers, with mass normalized to one, each of whom wishes to buy a single unit of either good A or B in each of the two periods. Consumers have a common reservation price v. Consumer preferences are determined by a symmetric binary distribution, where half of consumers prefer A by a fixed amount and half of consumers prefer B by the same fixed amount. On the Hotelling line segment $[0, 1]$, placing firm A at zero and firm B at one, each consumer type is given by his "location" at one of the two points $\{x_A, x_B\}$ where the subscript $i = \{A, B\}$ in x means that the consumer is closer to firm i in the line segment. In the brand loyalty interpretation, consumers face a kind of "transport cost" when buying the brand they like less.[13] The assumption that x_A and x_B may take, respectively, any value in the interval $\left[0, \frac{1}{2}\right]$ and $\left[\frac{1}{2}, 1\right]$ will allow us to solve the model for different degrees of brand loyalty. In so doing, it can be said that a consumer located at x_A is loyal to brand A whilst a consumer located at x_B is loyal to brand B. Consumers are more loyal when their location is biased towards the two end points of the line. In addition, it is assumed that consumer preferences are fixed across periods.[14]

Consider the following assumptions:

Assumption 1 The location of consumers is symmetric, i.e. $x_A = 1 - x_B$. Given the locations of the two firms, tx_i is the "transport cost" of choosing brand A while $t(1 - x_i)$ is the "transport cost" of choosing brand B. The parameter t is the same for all consumers and over the two periods.

Assumption 2 Due to symmetry, when customers buy their most preferred brand they incur a transport cost equal to $\alpha = tx_A = t(1 - x_B)$. When they buy their less preferred brand they incur a transport cost equal to $\beta = t(1 - x_A) = tx_B$. Given assumption 1, it follows that $\alpha \leqq \beta$.

Assumption 3 (Degree of brand loyalty) The difference in transport costs between buying the less and the most preferred brand is given by

$$\gamma = \left\{ \begin{array}{ll} t(1 - x_A) - tx_A = t\,(1 - 2x_A) & \text{for consumers located at } x_A \\ tx_B - t(1 - x_A) = t\,(2x_B - 1) & \text{for consumers located at } x_B \end{array} \right\},$$

[12] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

[13] Even though we consider that the market is segmented according to brand loyalty, the model also accommodates other interpretations (e.g. search costs and switching costs).

[14] The assumption of fixed-preferences across periods is a requirement in models where firms can price discriminate based on past behaviour. If preferences were not fixed from period to period, the knowledge of a consumer's first period choice would provide no information about his second period preferences, so firms could not engage in price discrimination. For an analysis where preferences change over time, see for instance, Caminal and Matutes (1990) and Fudenberg and Tirole (2000, section 6).

where $\gamma \geq 0$ measures the degree of a consumer's brand loyalty, which can be defined as the minimum difference between the prices of the two competing brands necessary to induce consumers to buy the wrong brand.

Assumption 4 (Duopoly competition) It is assumed that v is sufficiently high so that the duopoly equilibrium exhibits competition. More precisely, $v > \left(2 + \sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma$.

3.1 Consumers' behaviour

To simplify, it is assumed that consumers are myopic (or naive). This means that in period 1, consumers do not anticipate that the next period's prices may depend on their current behaviour.[15] After firms have set their prices, consumers shop for the better bargain. In each period the net utility of a consumer located at x_i purchasing good A at price p_A is $v - tx_A - p_A$, while the net utility of purchasing good B at price p_B is $v - t(1 - x_A) - p_B$. Thus, a consumer located at x_A buys good A whenever

$$p_A < p_B + \gamma. \tag{1}$$

Similarly, a consumer located at x_B buys good A whenever

$$p_A < p_B - \gamma. \tag{2}$$

Reversing the above inequalities one gets the conditions under which consumers located at x_i buy brand B.

3.2 Firms' behaviour

In each period firms act simultaneously and non-cooperatively. In the first period, consumers are *anonymous* and firms quote the same price for all consumers. In the second period, whether or not a consumer bought from the firm in the initial period may reveal that consumer's brand preference. Thus when firms have the required information, they will set different prices to their own customers and to the rival's previous customers. When nothing is learned from the initial period, price discrimination is not feasible and firms again quote a single price to all consumers. Firms discount future profits using a common discount factor δ.

4 Benchmarks

Before proceeding further, two static benchmarks are examined. The first, considers the case where consumers are *non-anonymous* and firms can engage in price discrimination. The other, considers the case where there is no price discrimination, either because firms have no information to recognise customers or because price discrimination is illegal.

[15] It is important to stress that the assumption of myopic consumers does not make a difference in our framework. Forward looking consumers will behave in the same way.

4.1 Customer recognition and price discrimination

As in Thisse and Vives (1988), suppose that firms can observe a consumer's location x_i and price accordingly. (Here, the main difference is that there are only two types of consumers.) Let $p_i(x_i)$ denote the price charged by firm i to consumers located at x_i. In equilibrium firms compete for consumers at each point on the line. As a result, the lowest price the more distant firm may offer is the marginal cost price, which in this case is equal to zero. In order to prevent the consumer from being tempted by the rival's price, the closer firm needs to offer a price generating the same level of utility. It is straightforward to show that in equilibrium for those consumers located, for instance at x_A, firm B sets price $p_B(x_A) = 0$ and so $p_A(x_A)$ must satisfy $v - tx_A - p_A(x_A) = v - t(1 - x_A)$, or $p_A(x_A) = t(1 - 2x_A) = \gamma$. In sum, when firms are able to recognise customers and price discrimination is allowed, Bertrand competition in each segment of customers leads equilibrium prices to:

$$p_i(x_i) = \gamma, \tag{3}$$

and

$$p_j(x_i) = 0, \text{ for } i \neq j. \tag{4}$$

Equilibrium profit per firm when it recognises customers and prices accordingly, denoted π_D, is given by:

$$\pi_D = \frac{1}{2}\gamma. \tag{5}$$

4.2 No customer recognition

Suppose next that price discrimination cannot occur, either because consumers are anonymous and firms have no information to engage in price discrimination or because price discrimination is illegal. This benchmark is the special case of Shilony (1977)'s model when there are only two firms in the market. Using (1) and (2) firm i's demand is

$$D_i = \left\{ \begin{array}{lll} 0 & if & p_i > p_j + \gamma \\ \frac{1}{2} & if & p_j - \gamma \leqq p_i \leqq p_j + \gamma \\ 1 & if & p_i < p_j - \gamma \end{array} \right\}, \tag{6}$$

from which it follows that firm i's profit is $\pi_i(p_i, p_j) = p_i D_i$.

Corollary 1. *(i) There is no equilibrium where both firms quote the marginal cost price. Moreover, (ii) there is no equilibrium where both firms set the "monopoly" price v.*

Proof. The marginal cost price cannot be an equilibrium of this game since by charging price γ each firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$. Similarly, when both firms set price v, it is always profitable for a given firm to slightly decrease its price to $v - \varepsilon$ and capture the remaining customers. Any price lower than v but greater than or equal to $v - \gamma$ is dominated by v as it would give firm i the same demand and lower profits. However, if firm i chooses to undercut its rival charging a price $(v - \gamma - \varepsilon)$, it would capture all consumers and its profits would be equal to $(v - \gamma - \varepsilon)$. This deviation is profitable for firm i as long as v is high enough, i.e. $v > 2\gamma$. In other words, (v, v) is a pure strategy equilibrium if $\gamma > \frac{v}{2}$. However, this condition contradicts assumption 4.

Proposition 1. *When consumers are anonymous there can be no pure strategy equilibrium.*

Proof. See the Appendix.

The intuition is that although each firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$ by selling at price γ exclusively to its loyal segment, the presence of a positive fraction of disloyal customers creates a tension between the firm's incentives to price low in order to attract this latter set of customers and to price high as a way to extract rents from its loyal customers. Therefore, each firm follows a mixed pricing strategy as an attempt to prevent the rival from systematically predicting its price, which in turn makes undercutting less likely.[16]

Following Shilony (1977), I prove the existence of a mixed strategy equilibrium by construction. Suppose that firm i selects a price randomly from the cdf $F_i(p)$. In a symmetric mixed strategy equilibrium, both firms follow the same pricing strategy, thus, for the sake of simplicity write $F_i(p) = F_j(p) = F(p)$. Suppose further that the support of the equilibrium prices is $[p_{\min}, p_{\max}]$.

Claim 1. *In equilibrium,* $p_{\min} \geqq \gamma$ *and* $p_{\max} \leqq v$.

Setting a price higher than v is a dominated strategy as firms have no demand. Setting a price lower than γ is also a dominated strategy because each firm can guarantee itself a profit at least equal to $\frac{1}{2}\gamma$ by charging price γ. Determine next the expected profit for a representative firm i. When firm i chooses any price that belongs to the equilibrium support of prices, and firm j uses the cdf $F(p)$, firm i's expected profit is always equal to a constant, which is denoted K. Since a firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$, it immediately follows that $K \geq \frac{1}{2}\gamma$. When firm i charges price p, two events are relevant. Firstly, p is the lowest price so all consumers go to firm i. This event occurs with probability $[1 - F(p+\gamma)]$. Secondly, p is such that both firms share the market. This event occurs with probability $[F(p+\gamma) - F(p-\gamma)]$. The case where p is so high that it precludes firm i from getting any demand is not relevant because in that specific case firm i realises no profit. Hence, firm i's expected profit, denoted $E\pi(p)$, is

$$E\pi(p) = p[1 - F(p+\gamma)] + \frac{1}{2}p[F(p+\gamma) - F(p-\gamma)].$$

In equilibrium, expected profit satisfies:

$$p\left(1 - \frac{1}{2}F(p+\gamma) - \frac{1}{2}F(p-\gamma)\right) = K. \tag{7}$$

Claim 2. *Assume that*

$$0 \leqq p_{\min} < p_{\max} - \gamma \leqq p_{\min} + \gamma < p_{\max} \leqq v. \tag{8}$$

From claim 2 it follows that:

[16] In a recent empirical paper, Baye, Morgan and Sholten (2004) provide evidence that in internet markets those firms that adopted predictable pricing strategies were driven out of the market. For that reason, they claim that unpredictability in prices—i.e. setting prices at random—is widely used in those markets and is an effective way of avoiding aggressive price competition in online markets.

Lemma 1. *The cdf $F(p)$ is as follows:*

$$F(p) = \left\{ \begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2K}{p+\gamma} & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 1 - \frac{2K}{p_{\max}} & if & p_{\max} - \gamma \leqq p < p_{\min} + \gamma \\ 2 - \frac{2K}{p-\gamma} & if & p_{\min} + \gamma \leqq p < p_{\max} \\ 1 & if & p \geqq p_{\max} \end{array} \right\}. \tag{9}$$

Proof. See the Appendix.

Lemma 2. *$F(p)$ is continuous with no gap in the support of prices whenever $p_{\max} - p_{\min} = 2\gamma$.*

The proof of this lemma is straightforward. For any price p such that $p_{\max} - \gamma \leqq p < p_{\min} + \gamma$, one observes that $F(p)$ will be flat in this range. However, discontinuities cannot arise in equilibrium because if $F(p)$ were discontinuous, a firm could benefit by infinitesimally undercutting at the point of discontinuity. If $p_{\max} - \gamma = p_{\min} + \gamma$, F has no flat range and all prices in the support of F are charged with positive density. Therefore, we need to impose that $p_{\max} - p_{\min} = 2\gamma$, and so we are looking for a support of equilibrium prices that satisfies such condition.

Lemma 3. *$F(p)$ is an increasing function of p.*

Taking the derivative of $F(p)$ with respect to p it is easy to see that, at any p, $\frac{\partial F}{\partial p} = f(p) > 0$.

Lemma 4. *From $F(p_{\min}) = 0$ and $F(p_{\max}) = 1$ one gets*

$$p_{\min} = 2K - \gamma \text{ and } p_{\max} = 2K + \gamma. \tag{10}$$

Based on the above results, we may establish the following proposition.

Proposition 2. *Whenever firms cannot engage in price discrimination, either because they cannot recognise customers or because price discrimination is illegal the Nash equilibrium is as follows:*

(i) Each firm chooses a price randomly from the nondegenerate distribution function

$$F(p) = \left\{ \begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{p_{\min}+\gamma}{(p+\gamma)} & if & p_{\min} \leqq p \leqq p_{\min} + \gamma \\ 2 - \frac{p_{\min}+\gamma}{(p-\gamma)} & if & p_{\min} + \gamma \leqq p < p_{\max} \\ 1 & if & p \geqq p_{\max} \end{array} \right\}, \tag{11}$$

The minimum and maximum equilibrium prices are respectively equal to

$$p_{\min} = \sqrt{2}\gamma, \tag{12}$$

and

$$p_{\max} = \left(2 + \sqrt{2}\right)\gamma. \tag{13}$$

Because prices are bounded, i.e. $p_{\max} \leqq v$ *it follows that* $\left(2+\sqrt{2}\right)\gamma \leqq v$, *which is true under assumption 4.*

(ii) Firm i's expected profit is in equilibrium equal to

$$E\pi_{ND} = K = \frac{1}{2}\left(1+\sqrt{2}\right)\gamma. \tag{14}$$

Proof. See the proof of (ii) in the Appendix.

Corollary 2. (Level of price dispersion) *The variance in prices is equal to* $Var\,(p) = 0.288\gamma^2$.

Proof. See the Appendix.

This result predicts that price dispersion, measured by the variance of prices, tends to be greater in product-markets where customers are more loyal, and lends support to recent empirical research showing that observed price dispersion is mainly due to perceived differences among retailers related to branding, trust and awareness (e.g. Bynjolfsson and Smith (2000a)).[17]

From the comparison between (5) and (14) we may establish part (i) of the following proposition.

Proposition 3. *(i) Firms are better off when they cannot engage in price discrimination; either because customers are anonymous or because price discrimination is illegal.*

(ii) Consumers pay strictly lower prices under discrimination than under no-discrimination.

5 Pricing with customer recognition

Throughout this section it is assumed that, where feasible, price discrimination is permitted. Price discrimination can only occur in the second period if firms learn the consumers' preferences from the initial period. We will see that when both firms set (approximately) the same price in period 1, they will share equally the market, and consumer tastes are fully revealed. On the other hand, if firms set significantly different prices in period 1, one firm attracts all consumers and nothing is learned by period 2.

In period 1 each consumer type is a "mystery", so each firm sets a single first period price, denoted p_i^1. In period 2, price discrimination being feasible, firms quote a different price to loyal and disloyal customers (those that bought previously from the rival). Let p_{iL}^2 and p_{iD}^2 denote firm i's second period prices to loyal and disloyal customers, respectively. If nothing is learned from period 1, both firms charge again a single price in period 2, denoted p_i^2. In each period firms set prices simultaneously. Next, to derive the subgame perfect equilibrium, the game is solved working backward from the second period.

[17] Although γ has been used as a brand loyalty parameter, it may equally be interpreted as a switching cost or as a cost of acquiring information. In other words, if customers were perfectly informed about their most preferred firm but would have to bear a cost to obtain information from other firms (due to the existence of search costs) our model would predict that the level of price dispersion would be greater in markets where it is more difficult to acquire such information.

5.1 Second-period pricing

Depending on first period prices (and correspondent market shares), two scenarios are possible in period 2. In one scenario, firms share the market in period 1, i.e. $(D_i, D_j) = \left(\frac{1}{2}, \frac{1}{2}\right), i, j = A, B$, and both learn the consumer tastes in the subsequent period. In the other, the first period market is entirely served by the same firm, i.e. $(D_i, D_j) = (0, 1), i, j = A, B$, and nothing is learned about consumer tastes in the second period.

Subgame 1: Firms have information to price discriminate When p^1 is such that firms share the market equally in period 1, consumer tastes are fully revealed in the subsequent period. Let $G_i\left(p^1\right)$ denote firm i's cdf of first period prices in the overall game. In a symmetric equilibrium $G_i\left(p^1\right) = G_j\left(p^1\right) = G\left(p^1\right)$. For a given price p^1 chosen by firm i in period 1, assuming that firm j sets its price according to G, both firms share the first period market with probability $\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]$ and in period 2 both firms have the required information to price discriminate. This subgame is identical to the benchmark case with non-anonymous consumers and price discrimination. Hence, under symmetry, each firm's equilibrium price to loyal and disloyal customers is respectively given by

$$p_L^2 = \gamma \text{ and } p_D^2 = 0, \tag{15}$$

and the second period profit with discrimination, denoted π_D^2, is

$$\pi_D^2 = \frac{1}{2}\gamma. \tag{16}$$

Note that when first-period prices fully reveal consumer preferences, price discrimination intensifies competition in each "location", leading all prices to fall. According to Corts (1998), this all-out competition result is common in models where both firms have information to discriminate and where the market exhibits best-response asymmetry.

A special feature of this model is that even though firms offer lower second-period prices to their rival's customers (indeed, they quote the marginal cost price) in order to entice them to switch, there is no switching in equilibrium.[18]

Subgame 2: Firms learn nothing and price discrimination is unfeasible In this case all consumers buy from the same firm in period 1. With no loss of generality, take the behaviour of firm A in the first period. If firm A's price p^1 is much lower than the rival's price, all consumers go to firm A. This happens with probability $\left[1 - G\left(p^1+\gamma\right)\right]$. Otherwise, with probability $G\left(p^1+\gamma\right)$ firm A's first period price is much higher than firm B's price and no consumer goes to firm A. In either case, both firms learn nothing in the second period. Price discrimination cannot occur and the second period pricing is a replication of the benchmark with no-discrimination. Let $E\left(\pi_{NR}^2\right)$ denote the second period expected equilibrium profit with non recognition of customers. From (14) it follows that:

$$E\left(\pi_{NR}^2\right) = \frac{1}{2}\left(1+\sqrt{2}\right)\gamma. \tag{17}$$

In sum, the second period expected profit per firm is

$$E\pi^2 = \left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\pi_D^2 + \left\{1 - \left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\right\}E\left(\pi_{NR}^2\right)$$

[18] This no poaching result is a consequence of the two-point distribution of demand, and would not extend to a discrete model with more than two types.

5.2 First-period pricing

Consider next the choice of first-period prices. Because each firm is forward looking they take today's price decisions rationally anticipating *how* they will affect their subsequent profit. Note that firms take into account that their choice of first period prices will determine their knowledge of consumer tastes in the second period and thus the feasibility of price discrimination.

As in the benchmark case with anonymous consumers, it is straightforward to show that in the first-period of the repeated game there is no equilibrium in pure strategies. Rather, a mixed strategy equilibrium exists. I prove the existence of such an equilibrium by construction. As previously discussed, when firm i charges $p^1 \in \left[p^1_{\min}, p^1_{\max}\right]$, it captures the entire market if $p^1_j > p^1 + \gamma$. This happens with probability $\left[1 - G\left(p^1 + \gamma\right)\right]$, and yields a profit equal to p^1. It sells nothing when $p^1_j < p^1 - \gamma$. This occurs with probability $G\left(p^1 - \gamma\right)$ and yields a zero economic profit. Finally, it sells exclusively to its loyal segment if $p^1 - \gamma < p^1_j < p^1 + \gamma$, which in turn happens with probability $\left[G\left(p^1 + \gamma\right) - G\left(p^1 - \gamma\right)\right]$ and yields a profit equal to $\frac{1}{2}p^1$. Thus, each firm's first period expected profit is

$$E\pi^1 = p^1\left[1 - G\left(p^1 + \gamma\right)\right] + \frac{1}{2}p^1\left[G\left(p^1 + \gamma\right) - G\left(p^1 - \gamma\right)\right]. \tag{18}$$

Overall expected profit is:

$$E\pi = E\pi^1 + \delta\left[G\left(p^1 + \gamma\right) - G\left(p^1 - \gamma\right)\right]\left[\pi^2_D - E\left(\pi^2_{NR}\right)\right] + \delta E\left(\pi^2_{NR}\right) \tag{19}$$

For the sake of simplicity, suppose that the overall expected profit is equal to a constant C. In a mixed strategy equilibrium, any price chosen from a firm's price support should generate the same expected profit, thus using (16), (17) and (18) it follows that:

$$C = p^1\left[1 - G\left(p^1 + \gamma\right)\right] + \delta\frac{1}{2}\gamma\left(1 + \sqrt{2}\right) + \frac{1}{2}\left[G\left(p^1 + \gamma\right) - G\left(p^1 - \gamma\right)\right]\left(p^1 - \delta\sqrt{2}\gamma\right). \tag{20}$$

Lemma 5. *Given claim 2 and the condition that precludes the existence of any flat range on the cdf G, i.e. $p^1_{\max} - p^1_{\min} = 2\gamma$, it follows that,*

$$G(p^1) = \left\{\begin{array}{ccc} 0 & if & p^1 < p^1_{\min} \\ 1 - \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{p^1 + \left(1 - \delta\sqrt{2}\right)\gamma} & if & p^1_{\min} \leqq p^1 \leqq p^1_{\max} - \gamma \\ 2 - \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{p^1 - \left(1 - \delta\sqrt{2}\right)\gamma} & if & p^1_{\max} - \gamma \leqq p^1 < p^1_{\max} \\ 1 & if & p^1 \geqq p^1_{\max} \end{array}\right\} \tag{21}$$

Proof. See the Appendix.

Lemma 6. *The cdf G is increasing in p^1 whenever $C > \frac{1}{2}\delta\left(1 + \sqrt{2}\right)\gamma$.*

Proof. See the Appendix.

Lemma 7. *From $G\left(p^1_{\min}\right) = 0$ and $G\left(p^1_{\max}\right) = 1$ it follows that*

$$p^1_{\min} = 2C - (1 + \delta)\gamma \tag{22}$$

and

$$p^1_{\max} = 2C + (1 - \delta)\gamma \quad \textit{where} \quad p^1_{\max} \leqq v. \tag{23}$$

Lemma 8. *From the continuity of the cdf $G(\cdot)$ at $p_1 = p_{1\max} - \gamma$ we obtain that*

$$C = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma. \tag{24}$$

Proof. See the Appendix.

Note that $C = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma$ satisfies lemma 6. Hence, it is possible to provide a complete characterisation of the subgame perfect equilibrium of this game.

Proposition 4. *There is a symmetric mixed strategy subgame perfect Nash equilibrium in which,*

(i) each firm chooses a first-period price randomly chosen from the cdf

$$G(p^1) = \left\{ \begin{array}{ccc} 0 & if & p^1 < p^1_{\min} \\ 1 - \frac{p^1_{\min} + \left(1-\delta\sqrt{2}\right)\gamma}{p^1 + \left(1-\delta\sqrt{2}\right)\gamma} & if & p^1_{\min} \leqq p^1 \leqq p^1_{\min} + \gamma \\ 2 - \frac{p^1_{\min} + \left(1+\delta\sqrt{2}\right)\gamma}{p^1 - \left(1-\delta\sqrt{2}\right)\gamma} & if & p^1_{\min} + \gamma \leqq p^1 < p^1_{\max} \\ 1 & if & p^1 \geqq p^1_{\max} \end{array} \right\} \tag{25}$$

where the minimum and maximum prices are respectively given by

$$p^1_{\min} = \left(\sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma \tag{26}$$

and

$$p^1_{\max} = \left(2 + \sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma. \tag{27}$$

Because prices are bounded, $p^1_{\max} \leqq v$, implies that $\left(2 + \sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma \leqq v$ (which satisfies Assumption 4)

(ii) Each firm earns an expected overall equilibrium profit equal to

$$E\pi = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma. \tag{28}$$

Corollary 3. (Price Dispersion) *When price discrimination is introduced and $\delta = 1$, the variance of prices is $Var\,(p)_D = 0.373\gamma^2$.*

This result shows that when firms are allowed to price discriminate, first period prices remain dispersed. In fact, price dispersion only disappears when consumers are indifferent between both firms (i.e. when $\gamma = 0$). Comparing corollary 2 and 3, one can see that the level of first period price dispersion with discrimination is greater than if discrimination were banned. (All else equal, there is a 29.5% increase on the level of first-period price dispersion, when moving from no-discrimination to discrimination.) This suggests that although firms' heterogeneities (e.g.

brand loyalty) are in fact an important force behind observed price dispersion, some other factors may also account for some fraction of price dispersion.[19]

6 Competitive effects of price discrimination

This section investigates the competitive effects of price discrimination. Do firms have an incentive to distort their first-period prices? Will they set higher or lower initial prices in comparison to the static non-discrimination case? What is the impact of price discrimination on first-period demand for each firm? Do firms have an incentive to avoid learning and price discrimination?

6.1 First-period demand

Consider first the impact of price discrimination on first-period demand. Here I investigate whether a firm might have an incentive to *forgo* a positive demand in the initial period as an effective way to eschew learning and subsequent price discrimination.

Proposition 5. *(i) Either when firms are myopic or when price discrimination cannot occur firms share the first period market with probability equal to* 0.81*.*

(ii) When firms are forward looking they reduce the probability of sharing the first-period market. In particular, when $\delta = 1$*, firms share the first period demand with probability equal to* 0.71*.*

Proof. See the Appendix.

When firms foresee that they do better in period 2 when neither of them is able to learn the consumers' preferences, they may have an incentive to avoid learning, which in turn can be achieved by not sharing the initial market. Proposition 5 shows that forward looking firms share the initial market less frequently than if price discrimination were not permitted. Notice, however, that, as δ declines, the negative effects of price discrimination shrink, thus weakening the incentives to reduce the probability of sharing the first period demand (see Figure 1).

In the survey on "Behavior-Based Price Discrimination and Customer Recognition," Fudenberg and Villas-Boas (2006) argue that in the Fudenberg-Tirole model (i) a firm may have an incentive to distort its first-period price as a way to reduce its rival's information about consumer preferences and that (ii) firms are better off when neither has information about consumer tastes. Specifically, they analyse the FT model in more detail and show that, when consumer preferences are uniformly distributed, firms do better in the second period with less symmetric market shares in period 1. Armstrong (2006) follows a similar analysis for the FT model showing also that when initial market shares are very asymmetric, firms have less precise information about consumer tastes, which is good for subsequent profits. This suggests that firms may have an incentive to distort their first-period behaviour, when they foresee that by learning less (or even nothing) today, they do better tomorrow. Surprisingly, in the FT model with uniformly distributed preferences, if consumers were myopic, firms would not distort their

[19] Baye, Morgan and Sholten (2002), note that despite observed price dispersion being mainly due to retailers heterogeneities (i.e. branding, reputation and trust), 28% of the observed dispersion is not explained by these heterogeneities.

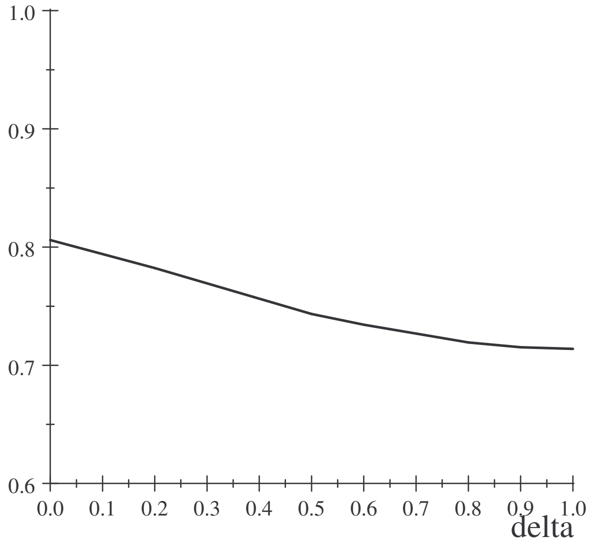

Figure 1: Probability of demand sharing as a function of δ.

first-period choices. (In the FT model first-period prices in the discrimination model are below the non-discrimination levels due to the strategic behaviour of forward looking consumers.)

An important contribution of the present model with discrete types is to show that, even when consumers are myopic, firms do in fact distort their first period behaviour when they realise that they do better tomorrow when both learn nothing about consumer preferences. Specifically, it shows that firms may strategically forgo any previous positive market share as an effective "weapon" to eschew the negative effects of price discrimination in subsequent periods.

6.2 First-period prices

Consider next the impact of the no-learning/no discrimination consideration on first-period prices.

Proposition 6. *When firms are forward looking, first-period prices are below the static or no-discrimination levels.*

Comparing equations (12) and (13) respectively with (26) and (27) it is easy to see that for $\delta \in [0, 1]$ the support of equilibrium prices falls.[20] In particular, if $\delta = 1$, the support of current prices falls from $[1.414\gamma, 3.414\gamma]$ in the no discrimination benchmark case to $[1.082\gamma, 3.082\gamma]$ when there is price discrimination. Likewise, comparing the equilibrium distribution functions with and without discrimination, respectively given by $G(\cdot)$ and $F(\cdot)$, one finds that, F first-order stochastically dominates G. Figure 2 is plotted for $\delta = 1$ and $\gamma = 1$. From the first order stochastic dominance of F over G it follows that the average first-period price under discrimination is below its static counterpart. In fact, while under non-discrimination the average price is approximately equal to 2.54γ (see proof of corollary 2 in the Appendix). Under discrimination the average first period price is 1.87γ (see proof of corollary 3 in the Appendix). From Figure 2,

[20] More precisely, this conclusion is valid for $\delta \leq \sqrt{2}$.

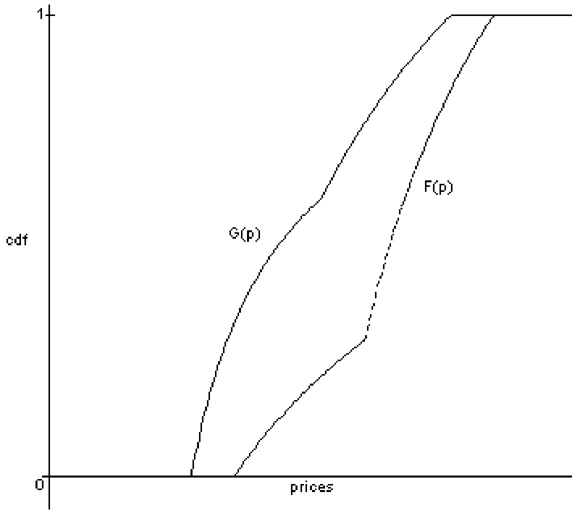

Figure 2: Cumulative distribution functions for prices

it can also be inferred that when price discrimination is permitted, firms charge low first period prices more frequently than high prices.

This result is in contrast with Fudenberg and Tirole's conclusion that the permission of price discrimination leads firms to raise (or at least to not reduce) their first period prices. Specifically, if consumers are forward looking they anticipate lower future prices due to poaching and become less price sensitive in the initial period. As a result, firms can choose higher first period prices than if poaching were not permitted. Fudenberg and Tirole show that if consumers are myopic and their tastes are uniformly distributed, firms do not change their first-period prices. As the assumption of myopic consumers does not make a difference in the present analysis, the discrete types' assumption is relevant to explain why forward-looking firms set first-period prices below the static levels. Additionally, Fudenberg and Tirole claim that the effects of BBPD on first period prices in the brand preference approach are the reverse of those effects in the switching costs approach. The present analysis challenges this view showing that first-period prices tend to be below the static levels, as in the switching cost approach.[21]

The results derived here and in the previous subsection highlight that conclusions about the first-period effects of behaviour-based price discrimination and customer recognition do depend on what is learned about consumers, and so on the distribution of consumer tastes.

[21] In Chen (1997), the presence of switching costs allows firms to lock-in their old customers and to offer new customers a lower price, in order to entice them to switch from the rival. Thus, firms have an incentive to increase first-period market share as a way to increase their base of locked-in customers. This explains why in Chen's model firms price below static levels in the first period and then raise their prices once consumers are locked-in.

6.3 Profits

Return now the profitability of behaviour-based price discrimination and customer recognition.

Proposition 7. *Price discrimination enabled by customer recognition hurts not only second period profit but also first period profit. Thus, overall expected profit decreases when firms learn the consumers types and become able to employ behaviour-based price discrimination.*

It was already shown that price discrimination is bad for second period profits. To prove the second part of proposition 7, note that if discrimination were not permitted, second period expected profit would be a replication of first period profit under anonymous consumers/no discrimination. Thus, in this case, overall expected profit with no discrimination would be equal to $\frac{1}{2}(1+\delta)\left(1+\sqrt{2}\right)\gamma$. In contrast, when discrimination is allowed, overall expected profit is equal to $\frac{1}{2}\left((1+\delta)+\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)}\right)\gamma$. It is, then, straightforward to prove that, apart from the special case where $\delta = 0$, expected overall profit when discrimination is permitted is always below its non-discrimination counterpart.

Expected first period profit, $E\pi^1$, can be derived from $E\pi$ given by equation (19) :

$$
\begin{aligned}
E\pi^1 &= E\pi - \delta E\left(\pi^2_{NR}\right) - \delta\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\left[\pi^2_D - E\left(\pi^2_{NR}\right)\right] \\
E\pi^1 &= \frac{1}{2}\left((1+\delta)+\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)}\right)\gamma - \frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma \\
&\quad -\frac{1}{2}\delta\,\mathrm{Pr}\left(\text{Demand sharing}\right)\left(-\sqrt{2}\right)\gamma
\end{aligned}
$$

If we let $q = \mathrm{Pr}\left(\text{Demand sharing}\right)$, where $q \in [0,1]$, it follows that

$$
E\pi^1 - E\pi_{ND} = \frac{1}{2}\gamma\left[\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)} - \left(1+\delta\left(1-q\right)\right)\left(\sqrt{2}\right)\right]
$$

Thus, it is easy to see that for $0 < \delta \leqq 1$, the above difference is negative and so first-period profit is clearly below its non-discrimination counterpart. This finding is the result of a smaller probability of positive market share in period 1 as well as lower first period prices.

7 Asymmetric equilibrium

As is by now a familiar theme in the present model, firms may have an incentive to avoid any learning, and as a result price discrimination, by not sharing the initial market. So far the model predicts that there is a bias towards asymmetric outcomes in period 1. A natural question is whether firms can easily sustain the no learning result and the consequent high future prices and profits in a many period model. The aim of this section is to investigate the circumstances in which an asymmetric pure strategy equilibrium may exist by which price discrimination is completely driven out of the market in subsequent periods.

For a first approach to this question, and to simplify, consider that $\delta > 1$ can be interpreted as a proxy for many future periods.

Proposition 8. *(i) If* $\delta < 1.06$ *there is no asymmetric pure strategy equilibrium.*

(ii) If $\delta \geqq 1.06$ *an asymmetric pure strategy equilibrium exists in the first period with prices* $\left(p_i^1, p_j^1\right)$ *such that firm i sets a low price and captures all the consumers whilst firm j gets nothing. For a given* p_i^1*, it must then be the case that* $p_j^1 > p_i^1 + \gamma$*. The equilibrium is defined as follows:*

(a) For $1.06 \leqq \delta < \sqrt{2}$ *and* $p_j^1 > p_i^1 + \gamma$*,*

$$p_i^1 = 1.414\delta\gamma - \gamma \tag{29}$$

and overall profits per firm are given by

$$\pi_i = 2.621\delta\gamma - \gamma, \tag{30}$$

$$\pi_j = 1.207\delta\gamma, \tag{31}$$

(b) For $\delta \geqq \sqrt{2}$ *and* $p_j^1 > p_i^1 + \gamma$*,*

$$p_i^1 = \gamma,$$

and overall profits per firm are now given by

$$\pi_i = (1 + 1.207\delta)\,\gamma, \tag{32}$$

$$\pi_j = (1.207\delta)\,\gamma. \tag{33}$$

Proof. See the Appendix.

Proposition 8 shows that even though firms are symmetric ex-ante, there is sometimes an asymmetric pure strategy equilibrium where one firm quotes a low price and captures all consumers in the initial period and where the rival firm has no incentive to match this low-price firm because its profits will then be low in the second period. However, we expect that the asymmetric equilibrium can only be sustained in a many-period model. Otherwise, firms would not have an incentive to sacrifice current profits; and thereby, the equilibrium would be the one defined in proposition 4.

The intuition is that when firms value future profits more than current profits, they are increasingly willing to sacrifice current profits as a way to fight against the drawbacks of price discrimination. When firms foresee that by learning today, the discrimination game will be played for many repeated periods, they may have an incentive to completely avoid the negative effects of discrimination, and in doing so, one of the firms might even be willing to forgo a positive market share and profit in period 1. Although there is no explicit agreement between firms to restrict discrimination practices in subsequent periods, the model suggests that, under certain conditions, when one firm quotes a low enough price it will not be undercut by its rival because the latter would not find it profitable to do so. This suggests that in certain circumstances *not* sharing the first period market is in fact an effective "weapon" for avoiding the all-out competition result in subsequent periods.

The literature on competitive price discrimination has pointed out that when profits are lower with discrimination, firms would become better off by colluding and committing not to discriminate. However, in the absence of any collective commitment, it is generally the case that even when one firm unilaterally commits to uniform pricing, the other firm finds it profitable to discriminate. In this regard, Thisse and Vives (1988) show that when one firm commits to uniform pricing, price discrimination is a dominant strategy for the other firm. The reason is that once one firm is committed to uniform pricing, the other firm is better off unconstrained in its pricing strategy. Thus, in this case, a unilateral commitment would not solve the prisoner's dilemma. One important implication of proposition 8 is that uniform pricing may arise without any collective action; it may arise on the basis of unilateral actions. Specifically, the choice of a low enough price by a firm could be interpreted as a credible kind of commitment to uniform pricing in the next stages of the game. For a high enough δ, the choice of a low price by a given firm would solve the prisoner's dilemma because at such a price, it would not be worth the other firm undercutting that price; and thus this latter firm would also be committed to uniform pricing in the subsequent period.

8 Welfare analysis

This section investigates the consumer and welfare effects of price discrimination through customer recognition. Given that price discrimination only occurs with some positive probability in the symmetric equilibrium analysed, the welfare analysis focuses only on this type of equilibrium. To simplify, throughout this section it is assumed that $\delta = 1$. Total welfare can be written as $v-$"expected transport cost". In the social optimal solution each consumer buys from the closer (preferred) firm, which happens exclusively when firms share the market in period 1. Otherwise, one group of consumers buy from the wrong firm thereby supporting an extra transport (i.e. $\gamma = \beta - \alpha$).

Look first at the second period welfare. Two situations are relevant. First, the case where price discrimination is permitted, and second, the case where there is no price discrimination, either because it is illegal or because firms do not recognise customers.

In the discrimination case, the second period outcome is efficient because all consumers buy from the right firm. Welfare is equal to

$$w_D^2 = v - \alpha. \tag{34}$$

Conversely, in the no discrimination case, due to the randomized nature of the equilibrium, the outcome may not be fully efficient as some consumers may buy from the least preferred firm. (Note that under non discrimination in period 2, all consumers buy efficiently with probability equal to 0.81. With probability equal to 0.19, half of consumers buy inefficiently.) Welfare is now equal to $w_{ND}^2 = v - ETC$, where ETC is the expected transport cost supported by all consumers.

$$\begin{aligned} ETC &= \alpha \Pr(\text{Demand sharing}) + \frac{1}{2}(\alpha + \beta)(1 - \Pr(\text{Demand sharing})) \\ &= 0.905\alpha + 0.095\beta. \end{aligned}$$

Thus, it ensues that

$$w_{ND}^2 = v - 0.905\alpha - 0.095\beta. \tag{35}$$

Look next at overall expected welfare when price discrimination is allowed, denoted W_D.

$$\begin{aligned} W_D &= w_D^1 + \Pr(\text{Demand sharing})\, w_D^2 \\ &\quad + (1 - \Pr(\text{Demand sharing}))\, w_{ND}^2. \end{aligned}$$

In period 1, being $\delta = 1$, $\Pr(\text{Demand sharing}) = 0.71$, and $ETC = 0.855\alpha + 0.145\beta$. Therefore, first-period welfare with discrimination is

$$w_D^1 = v - ETC = v - 0.855\alpha - 0.145\beta. \tag{36}$$

Using (34), (35) and (36) it follows that

$$W_D = 2v - 1.8275\alpha - 0.1725\beta. \tag{37}$$

Consider next that there is a ban on price discrimination. In this case, firms have no incentives to distort their first period behaviour, and so they do not reduce the probability of sharing the market in period 1, which is in this case equal to 0.81. Thus, first-period welfare with no discrimination is

$$w_{ND}^1 = v - 0.905\alpha - 0.095\beta. \tag{38}$$

Overall welfare when discrimination is not permitted, i.e. W_{ND}, is now equal to $w_{ND}^1 + w_{ND}^2$. Using (35) and (38) it follows that,

$$W_{ND} = 2v - 1.81\alpha - 0.19\beta. \tag{39}$$

Using (28), overall consumer surplus with discrimination, denoted CS_D, is given by

$$CS_D = W_D - 2E\pi = 2v + 1.2545\alpha - 3.2545\beta. \tag{40}$$

Similarly, using (14), overall consumer surplus with no discrimination, denoted CS_{ND}, is equal to:

$$CS_{ND} = W_{ND} - 2\left(2E\pi_{ND}\right) = 2v + 3.018\alpha - 5.018\beta. \tag{41}$$

The value of recognition Consider the following simple exercise. Compare the total price consumers are expected to pay when they buy anonymously with that they are expected to pay when firms can recognise them and price discriminate. The expected difference between these expected prices can be interpreted as the *value of recognition,* say VR, where

$$\begin{aligned} VR &= 2E\left(p\right)_{ND} - \left[E\left(p^1\right)_D + q\left(p_D\right) + \left(1-q\right)E\left(p\right)_{ND}\right] \\ &= 2\left(2.54\gamma\right) - \left[1.87\gamma + q\left(0.5\gamma\right) + \left(1-q\right)\left(2.54\gamma\right)\right] \\ &= \left(0.67 + 2.04q\right)\gamma \end{aligned}$$

and $q = \text{Prob(Demand sharing)}$. As $VR > 0$ consumers are expected to pay less when firms do recognise them and price accordingly. If say, $\delta = 1$, by behaving non-anonymously consumers are expected to save approximately 2.12γ.

It is now possible to establish the following proposition.

Proposition 9. *Price discrimination based on customer recognition is bad for profits, but good for consumer surplus and welfare.*

The proof of this result is trivial. One only needs to look at the variation in consumer surplus, industry profits and total welfare when moving from no discrimination to discrimination. For $\delta = 1$, it ensues that $CS_D - CS_{ND} = 1.764\gamma$, $\Pi_D - \Pi_{ND} = -1.746\gamma$ and $W_D - W_{ND} = 0.018\gamma$.

Compare next the welfare results derived in this paper with those in the existing literature. Here, both the static and the first-period equilibrium of the repeated game with discrimination are in mixed strategies. This implies that some consumers may buy inefficiently. In contrast, when firms learn the consumers' tastes and employ behaviour-based price discrimination, consumers buy from their preferred firm in period 2, which is efficient. (As previously said, even though firms try to poach their rival's previous customers, there is no switch in equilibrium.)[22] Note also that when discrimination is introduced, it is more likely that in period 1 some consumers buy inefficiently (due to the reduction on the probability of demand sharing). However, I have shown that the increase in efficiency in period 2 is greater than the decrease in efficiency in period 1. Particularly, price discrimination is welfare enhancing when it leads to more efficient shopping (i.e. if expected transport cost falls with discrimination). In fact, welfare increases with discrimination because $ETC_D - ETC_{ND} = -0.018\gamma$.

This paper highlights the importance of taking into account different forms of market competition when public policy tries to evaluate the welfare effects of price discrimination with customer recognition. In broad terms, in the two approaches considered in the literature so far, the equilibrium is in pure strategies, and price discrimination is welfare reducing, due to excessive inefficient switching (e.g. Chen (1997), Fudenberg and Tirole (2000)). In the Fudenberg-Tirole model, for instance, in period 1 all consumers buy from the closer firm, which is efficient. When firms recognise their own customers and their rival's customers, price discrimination in period 2 leads some consumers to buy from the more distant firm, which is inefficient. As in these models without discrimination consumers always buy from the right firm, a ban on price discrimination would be socially desirable. Here, in contrast, because random pricing tends to generate some inefficient shopping, price discrimination can increase efficiency.

The welfare results derived in this paper suggest that when the equilibrium market tends to generate random pricing, as is usually the case in internet markets, any attempt by firms to price discriminate based on customer recognition may be welfare enhancing provided that it leads to more efficient shopping. In this setting, public policy restricting the collection and use of consumers' private information would solve the industry prisoner's dilemma creating a mechanism in favour of uniform pricing rules and high industry profits. This would be a friendly competition world for firms looking for ways to boost their profits at the expense of consumer welfare. Thus, conclusions regarding the welfare implications of behaviour-based price discrimination and customer recognition do depend on the available information, which in turn depends on the form of consumer heterogeneity. Public policy in favour or against this form of price discrimination should be drawn up from a good economic understanding of each particular market.

9 Conclusions

This paper has studied the competitive effects of price discrimination based on customer recognition in a duopolistic market where the distribution of consumer types is discrete. The use of a discrete distribution for consumer tastes has raised issues not addressed in the literature so far. As far as I know this is one of the first papers to analyse the implications of behaviour-based price discrimination in markets where firms choose random prices.

[22] However, it is important to stress that with more than two types price discrimination could also be welfare reducing due to inefficient switching.

The present analysis has confirmed that more information leads to more intense competition and to a less favourable competitive outcome. As a result, it was shown that when firms foresee the negative effects of price discrimination they might have an incentive to distort their static behaviour. First, it was shown that firms may be willing to forgo a positive market share in period 1 as an effective way to eschew learning and price discrimination in the subsequent period. Specifically, it was proved that the probability of both firms having positive first period sales falls as they become more patient. When $\delta > 1$ is used as a proxy for many future periods, it was shown that firms can completely avoid the drawbacks of price discrimination. More precisely, it was proved that in this case there is an asymmetric pure strategy equilibrium in period 1, in which one firm sets a low price and captures all consumers, but it is not worth the competitor matching this low price-firm as its profits would then be low in the second period. Second, it was shown that first period prices are below their static or non-discrimination counterparts. This latter result is quite the reverse of that achieved in the extant models where purchase history discloses information about brand preferences (e.g. Villas-Boas (1999) and Fudenberg and Tirole (2000)).

Although this model is at best a crude approximation of real Internet markets, the stylised model addressed and the results derived herein suggest that conclusions regarding the economic and welfare effects of behaviour-based price discrimination and customer recognition do depend on what is learned about consumer demand, which in turn depends on the distribution of preferences. The welfare results obtained in this paper suggest that when the equilibrium market tends to generate random pricing, as is usually the case in Internet markets, any attempt by firms to price discriminate based on customer recognition may be welfare enhancing provided that it leads to more efficient shopping. Thus, in those markets that could be reasonably well represented by the features of the current model it seems that it is the consumers and not the firms that will benefit the most from price discrimination enabled by customer recognition.

Appendix

Proof of Proposition 1. By contradiction suppose that (p_A^*, p_B^*) is a Nash equilibrium with corresponding equilibrium profits π_A^* and π_B^*.

(i) With no loss of generality suppose $p_A^* > p_B^* + \gamma$. Hence, from (6) firm A has no demand and $\pi_A^* = 0$. Nonetheless, firm A can deviate and increase its profit by quoting a lower price $p_A^d = p_B^* + \gamma$ and thus making a profit $\pi_A^d = \frac{1}{2}\left(p_B^* + \gamma\right) > 0$. A contradiction.

(ii) With no loss of generality suppose $p_A^* < p_B^* + \gamma$. In this case, firm A can deviate and increase its profit by slightly increasing its price to $p_A^d = p_B^* + \gamma$. In this case $\pi_A^d = \frac{1}{2}\left(p_B^* + \gamma\right) > \pi_A^*$. A contradiction.

(iii) With no loss of generality suppose $p_A^* < p_B^* - \gamma$. Then , $p_B^* > p_A^* + \gamma$ and as firm A does in (i) and (ii) there is a profitable deviation for firm B. A contradiction. *Q.E.D.*

Proof of Lemma 1. When $p_{\min} + \gamma \leqq p < p_{\max}$, $F\left(p + \gamma\right) = 1$.Using (7) yields $p\left(\frac{1}{2} - \frac{1}{2}F\left(p - \gamma\right)\right) = K$, from which it follows that

$$F(p) = 1 - \frac{2K}{p + \gamma} \qquad \text{for} \qquad p_{\min} \leqq p < p_{\max} - \gamma. \tag{42}$$

Similarly, when $p_{\min} \leqq p < p_{\max} - \gamma$, $F(p-\gamma) = 0$.Using (7) yields $p\left(1 - \frac{1}{2}F(p+\gamma)\right) = K$, thus

$$F(p) = 2 - \frac{2K}{p-\gamma} \qquad \text{for} \qquad p_{\min} + \gamma \leqq p < p_{\max}. \tag{43}$$

Finally, when $p_{\max} - \gamma \leqq p < p_{\min} + \gamma$, one obtains

$$F(p) = 1 - \frac{2K}{p_{\max}}.\ Q.E.D. \tag{44}$$

Proof of part (ii) of Proposition 2. From the continuity of $F(p)$ at $p = p_{\max} - \gamma$, using the fact that $p_{\max} = 2K + \gamma$ one gets

$$2 - \frac{2K}{2K-\gamma} = 1 - \frac{2K}{2K+\gamma}$$

which yields

$$K = \frac{1}{2}\left(1+\sqrt{2}\right)\gamma.\ Q.E.D.$$

Proof of Corollary 2. The average price consumers expect to pay in the static or no-discrimination game is given by

$$E(p) = \int_{\text{sup}} pf(p)dp = \left(1+\sqrt{2}\right)\int_{\left(\sqrt{2}\right)\gamma}^{\left(1+\sqrt{2}\right)\gamma} pf(p)dp \cong 2.54\gamma.$$

Being the variance of prices given by $Var(p) = E\left(p^2\right) - \left[E(p)\right]^2$, where
$E\left(p^2\right) = \int_{\sqrt{2}\gamma}^{\left(2+\sqrt{2}\right)\gamma} p^2 f(p)dp \cong 6.74\gamma^2$, it follows that

$$Var(p) = 6.74\gamma^2 - (2.54\gamma)^2 = 0.288\gamma^2.\ Q.E.D.$$

Proof of Lemma 5. Considering claim 2, when $p^1_{\min} + \gamma \leqq p^1 < p^1_{\max}$, $G\left(p^1+\gamma\right) = 1$. Thus overall expected profit is

$$C = \delta\frac{1}{2}\gamma\left(1+\sqrt{2}\right) + \frac{1}{2}\left[1 - G\left(p^1-\gamma\right)\right]\left(p^1 - \delta\sqrt{2}\gamma\right),$$

which yields

$$G\left(p^1\right) = 1 - \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{p^1 + \left(1-\delta\sqrt{2}\right)\gamma} \qquad \text{for} \quad p^1_{\min} \leqq p^1 < p^1_{\max} - \gamma. \tag{45}$$

When $p^1_{\min} \leqq p^1 < p^1_{\max} - \gamma$, $G\left(p^1-\gamma\right) = 0$. Overall expected profit is now equal to

$$C = p^1\left[1 - G\left(p^1+\gamma\right)\right] + \frac{1}{2}\delta\gamma\left(1+\sqrt{2}\right) + \frac{1}{2}G\left(p^1+\gamma\right)\left(p^1 - \delta\sqrt{2}\gamma\right).$$

Then,

$$G\left(p^1\right) = 2 - \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{p^1 - \left(1-\delta\sqrt{2}\right)\gamma} \qquad \text{for} \quad p^1_{\min} + \gamma \leqq p^1 < p^1_{\max}. \tag{46}$$

In order to preclude the existence of any flat range in the cdf G, it must be true that $p^1_{\max} - p^1_{\min} = 2\gamma$. *Q.E.D.*

Proof of Lemma 6. When $p^1_{\min} \leqq p \leqq p^1_{\max} - \gamma$, $\frac{\partial G(p^1)}{\partial p^1} = \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{\left(p^1 + \left(1-\delta\sqrt{2}\right)\gamma\right)^2}$ is positive if and only if $2C > \delta\left(1+\sqrt{2}\right)\gamma$. Similarly, when $p^1_{\max} - \gamma \leqq p \leqq p^1_{\max}$, $\frac{\partial G(p^1)}{\partial p^1} = \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{\left(p^1 - \left(1-\delta\sqrt{2}\right)\gamma\right)^2}$ is positive if and only if $2C > \delta\left(1-\sqrt{2}\right)\gamma$. Both conditions are satisfied whenever $C > \frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma$. *Q.E.D.*

Proof of Lemma 8. From the continuity of $G(p_1)$ at $p_1 = p_{1\max} - \gamma$ it follows that

$$1 - \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{p_{1\max} - \gamma + \left(1-\delta\sqrt{2}\right)\gamma} = 2 - \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{p_{1\max} - \gamma - \left(1-\delta\sqrt{2}\right)\gamma}.$$

Using the fact that $p_{1\max} - \gamma = 2C - \delta\gamma$ yields

$$1 - \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{2C - \delta\gamma + \left(1-\delta\sqrt{2}\right)\gamma} = 2 - \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{2C - \delta\gamma - \left(1-\delta\sqrt{2}\right)\gamma},$$

from which it follows that

$$C = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma. \ Q.E.D. \tag{47}$$

Proof of Proposition 5. Because the model is symmetric both firms have the same support of prices. Then

$$\Pr\left(\text{Demand Sharing}\right) = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}}\left(\int_{p_{\min}}^{p_A - \gamma} f\left(p_B\right)dp_B\right) f\left(p_A\right)dp_A \tag{48}$$

When price discrimination is not permitted or when firms are myopic (i.e. $\delta = 0$),

$$\Pr\left(\text{Demand Sharing},\ \delta = 0\right) = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}}\left(1 - \frac{p_{\min}+\gamma}{p_A}\right)\frac{p_{\min}+\gamma}{\left(p_A - \gamma\right)^2}dp_A$$

Thus, for $p_{\min} = 1.414\gamma$ and $p_{\max} = 3.414\gamma$ it follows that $\Pr\left(\text{Demand Sharing},\ \delta = 0\right) \simeq 0.81$. When firms are forward looking and $\delta = 1$,

$$\Pr\left(\text{Demand Sharing},\ \delta = 1\right) = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}}\left(1 - \frac{p_{\min} - 0.414\gamma}{p_A - 1.414\gamma}\right)\frac{p_{\min} + 2.414\gamma}{\left(p_A + 0.414\gamma\right)^2}dp_A$$

In this case $p_{\min} = 1.082\gamma$ and $p_{\max} = 3.082\gamma$. Thus, $\Pr\left(\text{Demand Sharing},\ \delta = 1\right) = 0.71$. *Q.E.D.*

Proof of Corollary 3. The average first period price consumers expect to pay in the discrimination game when $\delta = 1$ is given by

$$\begin{aligned} E\left(p^1\right)_D &= \int_{\sup} p^1 g(p^1)dp^1 = \left(\int_{1.082\gamma}^{2.082\gamma}\frac{0.668\gamma p^1}{\left(p^1 - 0.414\gamma\right)^2}dp^1 + \int_{(2.082)\gamma}^{(3.082)\gamma}\frac{3.496\gamma p^1}{\left(p^1 + 0.414\gamma\right)^2}dp^1\right) \\ &\cong 1.87\gamma. \end{aligned}$$

The variance of prices is given by $Var(p) = E(p^2) - [E(p)]^2$, where

$$E\left(\left(p^1\right)^2\right)_D = \int_{\sup} \left(p^1\right)^2 g(p^1)dp^1 \cong 3.87\gamma^2.$$

It follows that

$$Var\left(p^1\right)_D = 3.87\gamma^2 - [1.87\gamma]^2 \cong 0.373\gamma^2. \; Q.E.D.$$

Proof of Proposition 7. Suppose there is an asymmetric pure strategy equilibrium with prices $\left(p_i^1, p_j^1\right)$ such that firm i serves the entire market in period 1 while firm j has no demand. For a given p_i^1, it must be the case that $p_j^1 > p_i^1 + \gamma$. Because the market is entirely served by the same firm in the initial period, both firms learn nothing by the second period. In this situation, both firms set their prices randomly as in the static case with anonymous consumers. Thus, overall equilibrium profits per firm are given by,

$$\pi_i = p_i^1 + \delta(1.207\gamma)$$

and

$$\pi_j = \delta(1.207\gamma).$$

This can only be an equilibrium if firm j has no incentive to deviate. When firm j deviates two possible situations may occur. First, when firm j sets a price such that $p_j^1 = p_i^1 + \gamma$ it shares the first-period market equally with firm i and its profit from deviation, say π_j^d, is

$$\pi_j^d = \frac{1}{2}\left(p_i^1 + \gamma\right) + \delta\left(\frac{1}{2}\gamma\right).$$

Second, when firm j sets a price such that $p_j^1 = p_i^1 - \gamma - \varepsilon$ it captures the entire market. In this case its profit from deviation is

$$\pi_j^d = p_i^1 - \gamma - \varepsilon + \delta(1.207\gamma).$$

Summing up, firm j has no incentive to deviate as long as $\pi_j = \delta(1.207\gamma) \geq \pi_j^d$, i.e. if the two following conditions hold:

$$\delta(1.207\gamma) \geq \frac{1}{2}\left(p_i^1 + \gamma\right) + \delta\left(\frac{1}{2}\gamma\right)$$

and

$$\delta(1.207\gamma) \geq p_i^1 - \gamma - \varepsilon + \delta(1.207\gamma).$$

Or, equivalently if,

$$p_i^1 \leq 1.414\delta\gamma - \gamma \tag{49}$$

and

$$p_i^1 \leq \gamma + \varepsilon \lessapprox \gamma \text{ for } \varepsilon \text{ sufficiently small.} \tag{50}$$

Since the equilibrium price must be equal or above the marginal cost, $p_i^1 \geq 0$, this implies that if the asymmetric equilibrium exists it must be the case that $\delta \geq 0.707$.

From (49) and (50) it follows that $1.414\delta\gamma - \gamma \leq \gamma$,if and only if $\delta \leq 1.414$. In sum, when $0.707 < \delta \leqq 1.414$ firm i's first period price is given by $p_i^1 = 1.414\delta\gamma - \gamma$ whilst when $\delta > 1.414$ firm i's first period price is $p_i^1 = \gamma$. Thus, when $0.707 < \delta \leqq 1.414$ overall equilibrium profits are:

$$\begin{aligned} \pi_i &= 2.621\delta\gamma - \gamma \\ \pi_j &= 1.207\delta\gamma. \end{aligned}$$

When $\delta > 1.414$, overall equilibrium profits are:

$$\begin{aligned} \pi_i &= (1 + 1.207\delta)\,\gamma \\ \pi_j &= (1.207\delta)\,\gamma. \end{aligned}$$

Finally, to finish the proof one needs to verify that firm i has also no incentive to increase its price and share the market with firm j. Given that p_i^1 is such that allows firm i to serve the entire market, firm i could increase its price by 2γ and share the market with firm j. Its profit from deviation would be equal to:

$$\pi_i^d = \frac{p_i^1 + 2\gamma}{2} + \delta\left(\frac{1}{2}\gamma\right).$$

This deviation would not be profitable if

$$\frac{p_i^1 + 2\gamma}{2} + \delta\left(\frac{1}{2}\gamma\right) \leq p_i^1 + \delta\,(1.207\gamma)$$

or, if

$$p_i^1 \geq 2\gamma - 1.414\delta\gamma.$$

When $p_i^1 = 1.414\delta\gamma - \gamma$ the previous condition is satisfied if and only if

$$1.414\delta\gamma - \gamma \geq 2\gamma - 1.414\delta\gamma \Rightarrow \delta \geqq 1.06.$$

Otherwise, when $p_i^1 = \gamma$, the above condition is satisfied if and only if

$$\gamma > 2\gamma - 1.414\delta\gamma \Rightarrow \delta > 0.707$$

which for $\delta > 1.414$ is always true. *Q.E.D.*

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