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NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS

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Using Wavelets to decompose time-frequency economic relations[*]

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Abstract

Economic agents simultaneously operate at different horizons. Many economic processes are the result of the actions of several agents with different term objectives. Therefore, economic time-series is a combination of components operating on different frequencies. Several questions about the data are connected to the understanding of the time-series behavior at different frequencies. While Fourier analysis is not appropriate to study the cyclical nature of economic time-series, because these are rarely stationary, wavelet analysis performs the estimation of the spectral characteristics of a time-series as a function of time.

In spite of all its advantages, wavelets are hardly ever used in economics. The purpose of this paper is to show that cross wavelet analysis can be used to directly study the interactions different time-series in the time-frequency domain. We use wavelets to analyze the impact of interest rate price changes on some macroeconomic variables: Industrial Production, Inflation and the monetary aggregates M1 and M2. Specifically, three tools are utilized: the wavelet power spectrum, wavelet coherency and wavelet

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phase-difference. These instruments illustrate how the use of wavelets may help to unravel economic time-frequency relations that would otherwise remain hidden.

Keywords: Monetary policy, time-frequency analysis, non-stationary time series, wavelets, cross wavelets, wavelet coherency.

1 Introduction

Economic agents simultaneously operate at different horizons. For example, central banks have different objectives in the short and long run, and operate simultaneously at different timescales (see Ramsey and Lampart 1998a). More than that, many economic processes are the result of the actions of several agents, who have different term objectives. Therefore, a macroeconomic time-series is a combination of components operating on different frequencies. Several questions about the data are connected to the understanding of the time-series behavior at different frequencies.

Fourier analysis allows us to study the cyclical nature of a time-series in the frequency domain. For example, spectral techniques can be used to identify seasonal components, such as a Christmas effect (see Wen 2002), or to highlight different relations among economic variables at distinct frequencies, such as the paradoxical relations between production and inventories (see Wen 2005). In spite of its utility, however, under the Fourier transform, the time information of a time series is completely lost. Because of this loss of information it is hard to distinguish transient relations or to identify when structural changes do occur. Moreover, these techniques are only appropriate for time-series with stable statistical properties, i.e. stationary time-series. Unfortunately, typical economic time-series are noisy, complex and strongly non-stationary, hence, these methods have never become popular in economics and most of the economic data analysis is done exclusively in the time domain.

However, some interesting relations may exist at different frequencies. For example, it is possible that monetary policies have different impacts in the short or long-run (high or low frequencies), therefore affecting the economy in different ways at different frequencies. On another perspective, it is possible that monetary authorities react to inflation news in the short-run, while, in the long-run, the price level is essentially determined by the money supply. Or, finally, it is possible that the effects of a certain policy change and evolve with time. These types of relations are difficult to uncover using pure time-domain or pure frequency domain methods.

As an alternative, wavelet analysis has been proposed. Wavelet analysis performs the estimation of the spectral characteristics of a time-series as a function of time revealing how the different periodic components of the time-series change over time. One major advantage afforded by the wavelet transform is the ability to perform natural local analysis of a time series. It stretches into a long wavelet function to measure the low frequency movements; and it compresses into a short wavelet function to measure the high frequency movements.

In this paper, we use wavelets to analyze the impact of interest rate price changes on some macroeconomic variables: Industrial Production, Inflation and the monetary aggregates M1 and M2. Specifically, three tools are utilized: the wavelet power spectrum, wavelet coherency and wavelet phase-difference. These instruments illustrate how the use of wavelets may help to unravel economic time-frequency relations that would otherwise remain hidden.

This paper proceeds as follows. In section 2, we discuss the Continuous Wavelet Transform (CWT), its localization properties and discusses in some detail the optimal characteristics of the Morlet wavelet. Section 3 describes the Cross Wavelet Transform (XWT), the Cross Wavelet Coherence (WTC), and the phase difference and discusses how to assess their statistical significance. Section 4 applies CWT, XWT, WTC and the phase difference to macroeconomic data and discusses its insights. Section 5 concludes.

2 Wavelet analysis

2.1 The Morlet wavelet

The minimum requirements imposed on a function $\psi(t)$ to qualify for being a *mother (admissible or analyzing) wavelet* are that $\psi(t)$ is a square integrable function, and also fulfills a technical condition, known as the *admissibility condition*: $\int_{-\infty}^{\infty} \frac{|\Psi(f)|}{|f|} df < \infty$. The square integrability of ψ is a very mild decay condition; wavelets used in practice have much faster decay or even compact support. For functions with sufficient decay, it turns out that the admissibility condition is equivalent to requiring $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$. This means that the function

ψ has to wiggle up and down the t-axis, i.e. it must behave like a wave; this, together with the decaying property, justifies the choice of the term wavelet (small wave) to designate ψ. The wavelet ψ is usually normalized to have unit energy: $\|\psi\|^2 = \int_{-\infty}^{\infty} |\psi(t)|^2 \, dt = 1$.

There are several types of wavelet functions available with different characteristics, such as Morlet, Mexican hat, Haar, Daubechies, etc. The choice of each one depends on the particular application one has in mind. In this paper we choose a complex wavelet as it yields a complex transform, with information on both the amplitude and phase, which is essential to study synchronism between different time-series. To be more precise, we will use the Morlet wavelet, which has optimal joint time-frequency concentration, in the sense that it reaches the Heisenberg uncertainty lower bound. The Morlet wavelet:

$$\psi(t) = \pi^{-\frac{1}{4}} \exp(i\omega_0 t) \exp\left(-\frac{1}{2}t^2\right). \tag{1}$$

We will consider, $\omega_0 = 6$. For this particular choice the wavelet scale, s, is inversely related to the frequency, $f \approx \frac{1}{s}$, simplifying the interpretation of the wavelet analysis.

2.2 Continuous Wavelet Transform

The continuous wavelet transform, with respect to the wavelet ψ, is a function $W_x(s,\tau)$ defined as:

$$W_x(s,\tau) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s}\right) dt, \tag{2}$$

where * denotes the complex conjugate form. The parameter s is a scaling or dilation factor that controls the length of the wavelet (the factor $1/\sqrt{|s|}$ being introduced to guarantee that wavelets have unit variance) and τ is a location parameter that indicates where the wavelet is centered. Scaling a wavelet simply means stretching it (if $|s| > 1$), or compressing it (if $|s| < 1$)

If wavelet function $\psi(t)$ is complex, the wavelet transform W_x will also be complex. The transform can then be divided into the real part ($\mathcal{R}\{W_x\}$) and imaginary part ($I\{W_x\}$), or amplitude, $|W_x|$, and phase, $\tan^{-1}\left(\frac{\mathcal{I}\{W_x\}}{\mathcal{R}\{W_x\}}\right)$. The phase of a given time-series $x(t)$ can be viewed as the position in the pseudo-cycle of the series and it is parameterized in radians ranging from $-\pi$ to π. For real-valued wavelet functions the imaginary part is zero and the phase is undefined. Therefore, in order to separate the phase and amplitude information of a time series it is important to make use of complex wavelets, which is the case of the Morlet wavelet.

Under some regularity conditions, we can reconstruct $x(t)$ from its continuous wavelet transform:

$$x(t) = \frac{2}{C_\psi} \int_0^\infty \left[\int_{-\infty}^{\infty} \mathcal{R}\left(W_x(s,\tau)\, \psi\left(\frac{t-\tau}{s}\right)\right) d\tau \right] \frac{ds}{s^2}, \tag{3}$$

where $C_\psi = \int_{-\infty}^{\infty} \frac{|\Psi(f)|}{|f|} df$, and $\Psi(f)$ is the Fourier transform of $\psi(t)$.

Since we can go from $x(t)$ to its wavelet transform, and from the wavelet transform we can obtain $x(t)$, both are representations of the same mathematical entity.[1] They just present information in a new manner, allowing us to gain some insights that would, otherwise, remain hidden.

3 Wavelet Tools [2]

3.1 Wavelet Power Spectrum

We simply define the wavelet power as $|W_n^x|^2$. Following Torrence and Compo (1998), the statistical significance of wavelet power can be assessed relative to the null hypotheses that the signal is generated by a stationary process with a given background power spectrum (P_f). Torrence and Compo assumed a first order auto-regressive model and, using Monte Carlo simulations, showed that on average, the local wavelet power spectrum is indistinguishable from the Fourier power spectrum. They then derive, under the null, the corresponding distribution

[1] Note, however, that one can also the integration over a range of scales, performing a band-pass filtering of the original series.

[2] A MatLab software package for performing and displaying the XWT and WTC, which also computes the levels of significance as described above, was developed by A. Grinsted, J. C. Moore, and S. Jevrejeva and can be found at http://www.pol.ac.uk/home/ research/waveletcoherence/.

for the local wavelet power spectrum,

$$D\left(\frac{|W_n^x(s)|^2}{\sigma_x^2} < p\right) = \frac{1}{2}P_f\chi_v^2, \tag{4}$$

at each time n and scale s. The value of P_f in (4) is the mean spectrum at the Fourier frequency f that corresponds to the wavelet scale s (in our case $s \approx \frac{1}{f}$, see equation (**??**)) and v is equal to 1 for real and 2 for complex wavelets.

In Figure 1, we can see the continuous wavelet power spectrum of the several variables.[3] The thick black contour designates the 5% significance level against a red noise and the cone of influence is shown as a lighter shade. The color code for power range from blue (low power) to red (high power).

Looking at the time-scale decomposition of the several variables some interesting facts are revealed:

1. Interest rates: it is clear that most of the action, especially at high scales (low frequencies) appears after the 1960s, suggesting a structural change in that decade.

2. Inflation: until 1950 inflation rate variance was quite high both at low and high scales. Again in The 1970s and 1980s, probably as a consequence of very active oil shocks, the variance of the inflation rate became higher, but in this case, the effect is clearer at medium and high scales, suggesting that we were facing permanent shocks to inflation.

3. Industrial Production: the variance, at all scales of the industrial production was quite high until 1950s. After that it has been steadily decreasing, with an exception between mid 1970s and mid 1980s, when the variance at the business cycle frequency (2 to 8 years) was quite high. It has become common in the literature to argue that we have been

[3]We use monthly data. We have a measure for interest rates (Moody's Seasoned Aaa Corporate Bond Yield) running from 1921:01 to 2007:04, and a measure for the inflation rate (based on the Consumer Price Index) running from 1921:02 to 2007:4. To measure Economic Activity we use the Industrial Production Index, available from 1921:1 to 2007:4. We also have data for money stocks. We have data for M1 (since 1947:01) and M2 (since 1948:1). All data is available at the Federal Reserve Bank of St. Louis (data for M1 and M2 were complemented with the estimations provided by Rasche, 1987). Data for industrial production and the money stocks were transformed in logarithms. The trend was removed using a wavelet based filter, which has properties similar to a band pass filter.

observing, in the last decade, a decrease in the volatility of GDP in the United States (e.g. see Blanchard and Simon 2001). Some authors call it the "Great Moderation". In reality, we can observe that this is a secular, and not decadal, trend. Before the 2nd World War, the volatility was quite high at all scales (at least scales above 6 months). In the 1960s, the volatility decreased at all scales. It then increased, probably due to the oil shocks, at the business cycle frequency in the 1970s, however this increase was temporary.

4. M1 and M2: we observe a distinct evolution of the behavior of two different monetary aggregates. The volatility of M1 is very high at very low scales (high frequencies), which is something we do not observe for M2. It is also very clear the different behavior in the 1970s, with M2 with a very high power in the $3 \sim 6$ year scale, while M1 only became more active after 1980, suggesting a structural change in the monetary policy. This coincides with the presidential election of Ronald Reagan and the beginning of the appointment of Paul Volcker as a chairman of the Federal Reserve. Volcker implemented a very restrictive monetary policy as a reaction to the inflationary pressures of the second oil shock.

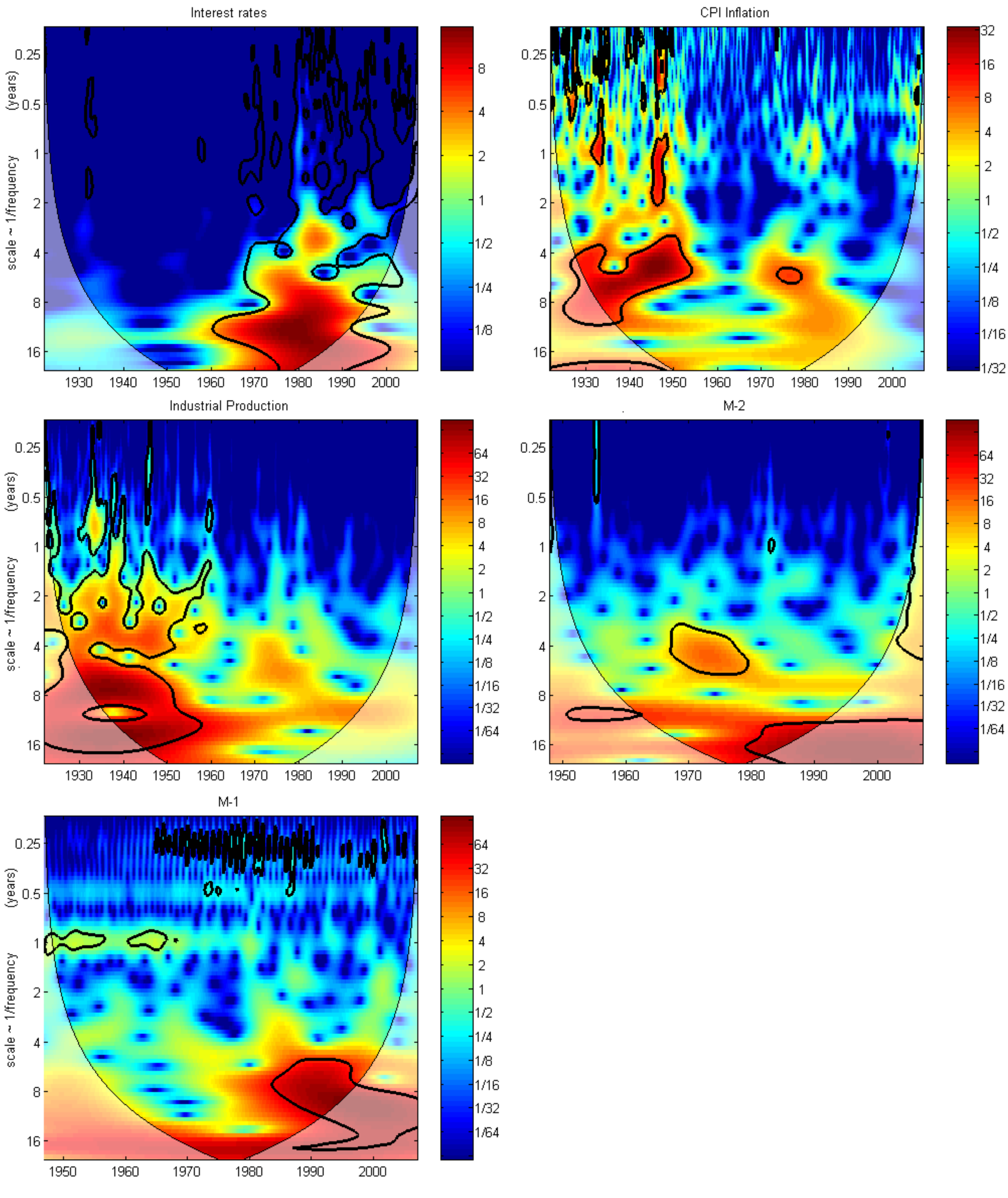

Figure 1: The continuous wavelet power spectrum. The thick black contour designates the 5% significance level against an AR1 and the cone of influence is shown as a lighter shade.

3.2 The Cross Wavelets and the Phase Difference

The cross wavelet transform of two time series, $x = \{x_n\}$ and $y = \{y_n\}$, first introduced by Hudgins et al. (1993) is simply defined as

$$W_n^{xy} = W_n^x W_n^{y*}, \tag{5}$$

where W_n^x and W_n^y are the wavelet transforms of x and y, respectively. The cross wavelet power is given by $|W_n^{xy}|$. While the wavelet power spectrum depicts the variance of a time series, with times of large variance showing large power, the cross–wavelet power of two time series depicts the covariance between these time series at each scale or frequency. Therefore, cross–wavelet power gives us a quantified indication of the similarity of power between two time series.

We follow Torrence and Compo (1998) to assess the statistical significance of the cross wavelet power. If two time-series have Fourier Spectra P_f^x and P_f^y, then the cross wavelet distribution is given by

$$D\left(\frac{\left|W_x W_y^*\right|}{\sigma_x \sigma_y} < p\right) = \frac{Z_v\left(p\right)}{v}\sqrt{P_f^x\ P_f^y}, \tag{6}$$

where $Z_v\left(p\right)$ is the confidence level associated with the probability p for a pdf defined by the square root of the product of two χ^2 distributions (see Torrence and Combo 1998, for details).

As in the Fourier spectral approaches, wavelet coherence can be defined as ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local correlation, both in time and frequency, between two time-series. Here, again, we follow Jevrejeva et al. (2003) and define the wavelet coherence between two time series $x = \{x_n\}$ and $y = \{y_n\}$ as follows:

$$R_n^2\left(s\right) = \frac{\left|S\left(s^{-1}W_n^{xy}\left(s\right)\right)\right|}{S\left(s^{-1}\left|W_n^x\right|\right)^{\frac{1}{2}} S\left(s^{-1}\left|W_n^y\right|\right)^{\frac{1}{2}}}, \tag{7}$$

where S denotes a smoothing operator in both time and scale.[4] Theoretical distributions for wavelet coherence have not been derived yet. So to assess the statistical significance of the estimated wavelet coherence we follow Grinsted et al. (2004) and use Monte Carlo methods. Again, see Grinsted et al. (2004) for details.

Phase differences are useful to characterize phase relationships between two time series, $x = \{x_n\}$ and $y = \{y_n\}$. As we said before, the phase of a given time-series, ϕ_x, can be viewed as the position in the pseudo-cycle of the series.The phase difference, $\phi_{x,y}$, characterizes phase relationships between the two time-series. The phase difference is defined as

$$\phi_{x,y} = \tan^{-1}\left(\frac{\mathcal{I}\left\{W_n^{xy}\right\}}{\mathcal{R}\left\{W_n^{xy}\right\}}\right), \qquad \text{with } \phi_{x,y} \in [-\pi, \pi]\,. \tag{8}$$

A phase difference of zero indicates that the time series move together (analogous to positive covariance). If $\phi_{x,y} \in \left(0, \frac{\pi}{2}\right)$ then the series move in phase, but the time-series y leads x. If $\phi_{x,y} \in \left(-\frac{\pi}{2}, 0\right)$ then it is x that is leading. A phase difference of π (or $-\pi$) indicates an anti-phase relation (analogous to negative covariance). If $\phi_{x,y} \in \left(\frac{\pi}{2}, \pi\right)$ then x is leading. Time-series y is leading if $\phi_{x,y} \in \left(-\pi, -\frac{\pi}{2}\right)$. In Figures 2 and 3, the phase difference is represented by arrows.

[4]Smoothing is a necessary step, because, without that step, coherence is identically one at all scales and times. Smoothing is achieved by a convolution in time and scale. The time convolution is done with a Gaussian and the scale convolution is performed by a rectangular window; see Grinsted et al. (2004) for details.

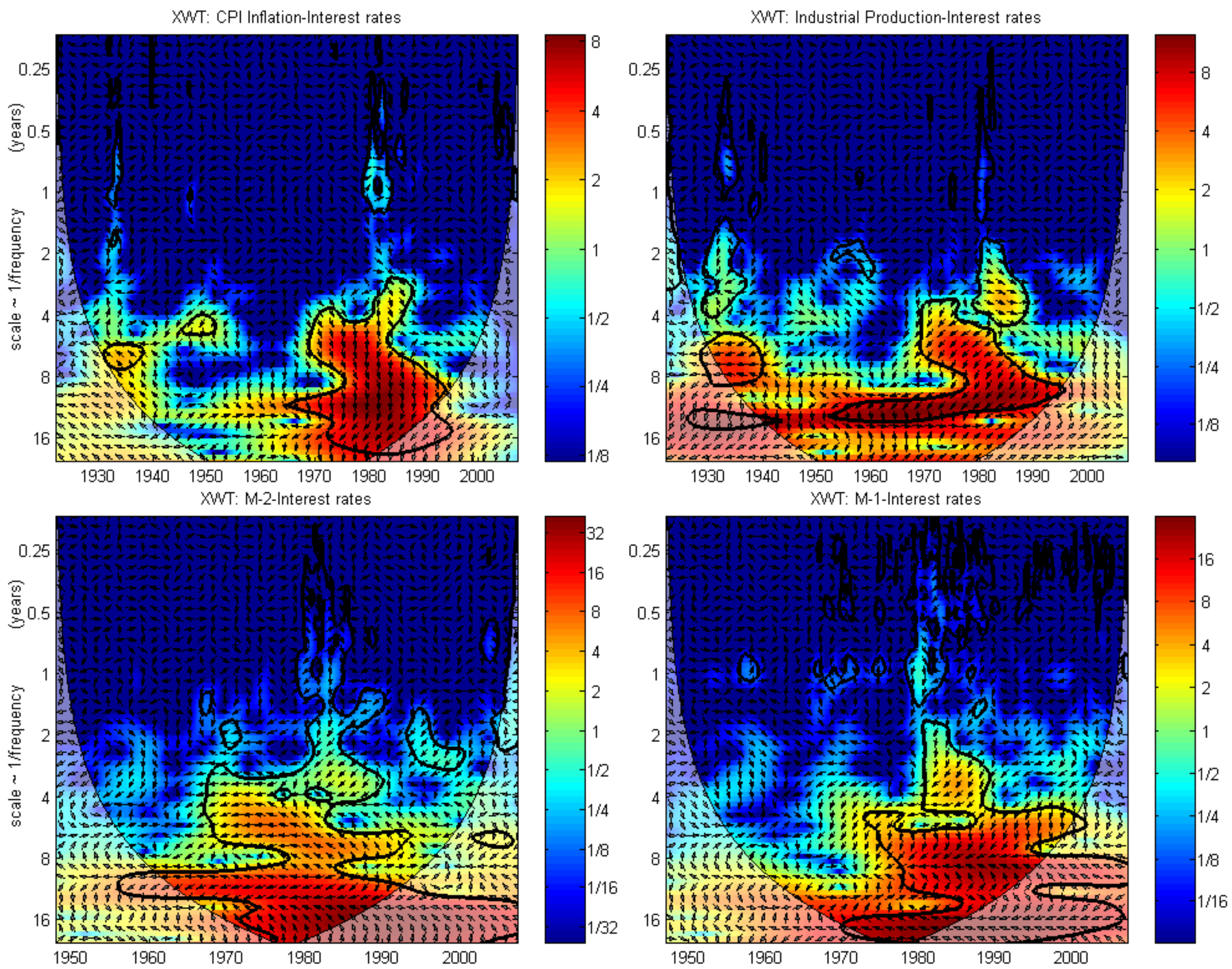

Figure 2: Cross Wavelet Transform between several variables and Interest rates. The phase difference is shown as arrows. In-phase pointing right, anti-phase pointing left. In-phase relations with interest rates leading (lagging) pointing up (down) and to the right. Anti-phase realations with interest rates leading (lagging) pointing down (up) and to the left.

In Figure 2, we can observe the estimated cross wavelet between the interest rates and several other variables. We focus on the interest rates because according to Sims (1980, 1992) the role of money in output determination is very minor, when interest rates are included in the system. According to his results interest rates play a leading role both in determining output and inflation. The 5% significance level is shown with a thick contour and the cone of influence is shown as a lighter shade. The phase difference is shown as arrows. In-phase pointing right, anti-phase pointing left. In-phase relations with interest rates leading (lagging) pointing up (down) and to the right. Anti-phase relations with interest rates leading (lagging) pointing down (up) and to the left. The color code for power range from blue (low power) to red (high power).

1. Inflation and Interest rates: in the 1970s and 1980s their covariance was quite high in the $3 \sim 20$ year scale. Note that the causality is not the same at the different scales. Arrows pointing down and to the right (in the $3 \sim 8$ year scales) suggest that these variables are in-phase, with the inflation rates leading (therefore, an increase in the inflation rate is followed by an increase in the interest rate). In the $12 \sim 20$ year scales, arrows point down and to the left, suggesting that the variables are anti-phase, with the interest rates leading (therefore, and increase in the interest rates will precede a decrease in inflation). This suggests that, at the business cycle frequency, interest rate increases follow inflation increases, which is consistent with a Central bank that follows some kind of Taylor rule[5], but in the longer run an increase in the interest rate will have a negative effect on inflation. If we were restricted to classical time-series we would have no information about the frequency differences. Therefore, this type of conclusion is not easy to get if we are restricted to classical methods.

2. Industrial Production and Interest rates: during the 1920s and 1930s, increases in the interest rates preceded decreases in the industrial production suggesting that Friedman was right when blaming contractionary monetary policy for aggravating the effects of the big recession. In late 1950s and in the decade of 1960, long run changes (at the $10 \sim 14$ year scales) in the interest rates caused anti-cyclical movements in the industrial production. In the 1970s and 1980s this effect was extended to the business cycle frequency ($4 \sim 10$ years). Interestingly, and starting in 1980, coinciding with Paul Volcker as the chairman of the Federal Reserve, one can see that interest rates, in the 2-4

[5] According to the Taylor rule the Federal Reserve should change the interest rates in response to real divergences of real GDP from potential GDP and divergences of actual rates of inflation from a target rate of inflation. See Taylor (1993).

year band, reacted pro-cyclically with industrial production, again suggesting that the Fed was following some kind of Taylor rule, having contractionary effects in the longer run.

3. Monetary aggregates (M1 and M2) and Interest rates: a structural change in this relation has clearly happened in the 1970s and 1980s. Generally, arrows point to the left, suggesting an obvious anti-phase relation, higher interest rates are the results of contractionary monetary policies. After 1980, we observe that interest rates become the leading variable. This change in behavior suggests a structural change in monetary policy and coincides with the end of the monetary targeting.

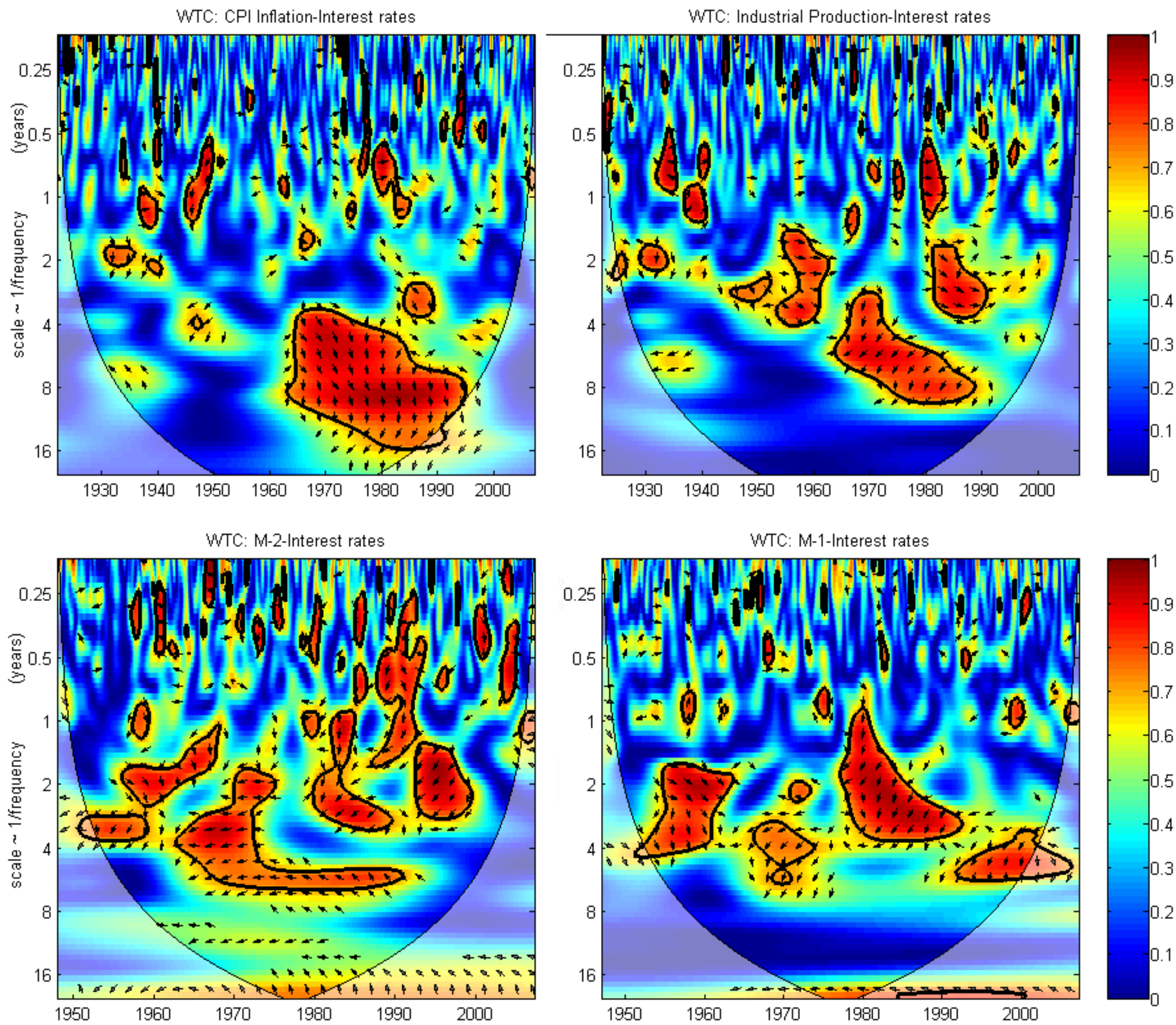

Figure 3: Squared wavelet coherence between several variables and Interest Rates. The 5% significance level against red noise is shown as a thick contour.

While the cross wavelet transform gives us something that can be interpreted as the covariance between two variables at different time-scales, the wavelet coherency (Figure 3) can be interpreted as the local correlation; therefore complementing, and correcting the previous analysis, highlighting relations that could, otherwise, remain hidden.

In Figure 3, we can observe that:

1. The relation between interest rates and inflation has changed a lot. In the 1930s the relation is not very strong, except for an island in the $6 \sim 9$ year band, where arrows pointing upwards and to the left suggest that inflation was the leading variable. After the

1960s, strong medium and long run relations are uncovered confirming the conclusions we reached with the XWT analysis.

2. A negative relation between interest rates and industrial production is uncovered in the 1930s at the business cycle frequency. Again we observe that monetary policy exacerbated the great depression. The restrictive monetary policy led to an increase in interest rates, which hurt investment and industrial production. In the 1950s, interest rates react pro-cyclically to changes in the industrial production ($1 \sim 5$ year band). In the 1970s and 1980s, in the $3 \sim 10$ year band, arrows suggest that increases in the interest rates had contractionary effects, supporting the conclusions of some authors (Barsky and Kilian, 2001, and Leduc and Sill, 2004) who argued that monetary policy reinforced the recessionary effects of the oil shocks. Still in the 1980s, but at lower scales ($0.5 \sim 1.5$ and $2 \sim 5$ year band), interest rates seem to follow industrial production, with the negative long run effects already noted. After 2000, this pattern seems to have moved to higher scales (but since this island is under the cone of influence it is still too early for decent inference).

3. The relation between money stocks and interest rates has changed quite often since 1950. For example, if we focus on M2, we can see that in the 1950s the interest rate was the leading variable, with M2 reacting in the opposite direction. In the decade of 1960, in the $1 \sim 3$ year band, M2 became the leading variable, with the interest rate following M2. In the 1970s, at the business cycle frequencies, $2 \sim 8$ year band, interest rates and M2 were in an anti-phase relation, and it is not clear which variable was the leading one. But in the 1980s, at higher scales, $6 \sim 8$ year M2 became the leading variable, while at lower scales, $1 \sim 3$ year, interest rates were leading. A similar pattern continued to be observed in the 1990s.

4 Conclusions

In this paper, we claimed that wavelet analysis can very useful to analyze economic data. We illustrated how wavelet analysis can naturally be applied to the study of business cycles (given its periodic nature), or to any field of economics, or finance, especially when there is a distinction between short and long-run relations. Wavelet analysis can help us to interpret multi-frequency, non-stationary time-series data, revealing features we could not see otherwise. We have argued that the wavelet transform is much better suited for economic data than the Fourier transform. The main advantage of the wavelet approach is the ability to analyze transient dynamics, both for single time-series or for the association between two time-series.

We showed that some of the shortcomings that economists have found when applying wavelet techniques to study two or more time series disappears once the concept of cross wavelet is introduced. We used three tools that, to our knowledge, have not been used yet by economists: the Cross Wavelet Transform, the Cross Wavelet Coherence and the phase difference. While the wavelet power spectra quantifies the main periodic component of a given time-series and its time evolution, the Cross Wavelet Transform and the Cross Wavelet Coherence Wavelet are used to quantify the degree of linear relation between two non-stationary time-series in the time–frequency domain. Phase analysis is a nonlinear technique that makes possible to study the phase synchronization of two time-series.

This paper's main contribution to the literature is to clearly demonstrate the utility of wavelets for the analysis of economic time series and to illustrate how relationships between macroeconomic variables change over time and across different frequencies. In fact, wavelets allowed us to detect transient effects which would be very difficult to detect using classical econometric techniques. For example, we were able to see that the reduction in the US output volatility decreased in the 1960s at all frequencies (and not in the 1980s as it is usually claimed), but that it was temporarily revived in the 1970s (especially at the business cycle frequency) probably because of the oil price shocks. We were also able to disentangle different short run and medium run relations. For example, we saw that after 1980, coinciding with

Paul Volcker as a chairman of the Federal Reserve, interest rates, in the 2 $\sim$ 4 year band, reacted pro-cyclically with industrial production, having contractionary effects in the longer run.

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