Bridge Maintenance, Safety, Management, Life-Cycle Performance and Cost

Editors
Paulo J.S. Cruz, Dan M. Frangopol & Luis C. Neves

International Association for
Bridge Maintenance and Safety

Strength of corroded tapered plate girders under pure shear

P.J.S. Cruz
Civil Engineering Department, University of Minho, Guimarães, Portugal

L. Lourenço
Civil Engineering Department, University of Minho, Guimarães, Portugal

M. Santos
Information Systems Department, University of Minho, Guimarães, Portugal

H. Quintela
School of Technology and Management of the Polytechnic Institute of Viana do Castelo, Viana do Castelo, Portugal

Paulo Cortez
Information Systems Department, University of Minho, Guimarães, Portugal

ABSTRACT: Corrosion is one of the most important causes of deterioration of steel girder bridges. The lack of information concerning the behaviour of corroded web panels makes difficult the civil engineer's task of evaluating the deteriorated member. An in-depth study in front of serviceability and ultimate limit states is necessary to develop efficient techniques to evaluate the structural integrity and safety. By combining information on the rate and location of an eventual corrosion in web and flange panels, it is possible to predict elastic critical shear buckling stress and ultimate shear stress. Nonlinear analyses have been conducted in a three-dimensional finite element model of transversely stiffened corroded tapered plate girders, simulated by thickness reduction, subjected to pure shear. In this paper is presented a finite element model of corroded girder panels (web and flanges), and an application of Data Mining techniques (e.g., Neural Networks) to analyze the data generated in the analytical study to find new and novel knowledge for condition assessment.

1 INTRODUCTION

Steel plate girder structures, composed of nominally flat plates connected together usually by welding, are used in large structures of vital importance for society. Plate girders are usually designed when hot rolled beams are not economic or show insufficient strength for design purposes. Due the postbuckling strength, and depending upon the geometry, the web panel is capable of carrying additional loads considerably in excess of that at which the web starts to buckle. Taking advantage of this postbuckling strength, a plate girder of high strength weight ratio can be designed.

These plate girders can be designed as tapered plate girders, usually done by means of a web panel whose depth varies linearly. Designed in accordance with the distribution of bending moments, along the longitudinal direction of the structural systems, tapered web panels with variable inertia provide the required resistance. In this sense, the use of tapered plate girders is frequently a solution in cases of high moment variation, inducing to a rational and efficient solution (Zárate and Mirambell 2002).

As a result of spectacular bridge failures in Europe, during the 1970s, the design of plated structures attracted great interest. However, the limitations of the favourable behaviour of plates were not known. This lack of knowledge led to large-scale research projects including experiments and theoretical development. The research in the actual area of nonlinear analysis of plated structures is sustained in the power of computers and their availability to researchers.

Since 1930s that had been known that thin plates had a substantial post-critical resistance. Design was based on allowable stresses for plate buckling and the critical stress was the starting point. Design codes started to take the post-critical resistance into account by reducing the safety factors for plate buckling.

Nowadays, the insecurity on modelling in the field of classical mechanics of materials can be considered almost irreducible. In the same sense, mechanical properties of the traditional bridges materials such as steel are readily available, in highly accurate and large volume of statistical data, reducing therefore the uncertainty in the evaluation of its performance. However, when these structures are affected by local corrosion, the estimation of load carrying capacity reduction may contain a relatively high level of uncertainty, and the collection of sufficient field data may take many years (Akgül & Frangopol 2003).

In steel plate construction is unusual that structural members have the exact designed dimensions due to deterioration caused by environment exposure or fabrication tolerances. Corrosion affects the resistance of shear forces. The uncertainty must only be minimized by using models based on sound theoretical principles, validated by extensive and reliable data.

The bridge maintenance costs for bridges will increase quickly, and bridge managers need to use limited budget effectively and plan cost-effective long-term strategies (Sato et al. 2004). There is a need for rational criteria which can be used to determine the actual strength of existing tapered plate girders. For develop efficient techniques to evaluate the structural integrity and safety, due to the large uncertainties related to the deterioration and maintenance of such structures, an in-depth study in front of serviceability and ultimate limit states of corroded tapered plate girders is necessary. Most of the reports are concerned with the causes of corrosion and how to prevent it or how to protect the structures from it, but little is said about the structural safety implications of its presence (Dinno & Birkemoe 1997).

The aim of this study is, therefore, to investigate the effects of a local thickness reduction, due to corrosion, in tapered web plates, on the elastic critical shear buckling load and ultimate shear capacity.

The application of Data Mining (DM) techniques in civil engineering has gained an increasing interest in recent years, due to intrinsic characteristics such as ability to deal with non-linear relationships. Cruz et al. (2004) presented an alternative approach for the prediction of ultimate shear resistance of non-prismatic tapered plate girders studied by Zárate and Mirambell (2000), which used DM techniques and achieved encouraging results. In this study, the same approach is refined and applied using a wide range of experimental cases.

2 NUMERICAL MODEL

2.1 *Shear behaviour of a plate girder*

The considerable slenderness of a tapered web is one of the main distinctive features of tapered plate girders. The web buckling can appear for relatively small values of the shear load with regard to the shear load that yields the web material.

The usual models to evaluate the shear capacity of plate girders are based on the diagonal tension field theory, interpreted by three resistant mechanisms: the first mechanism, the elastic shear buckling strength of the web plate; the second one, the post-buckling strength of the web plate, interpreted by the development of the tension field; the last one, the sway failure mechanism, which implies that the web panel reaches failure when plastic hinges are developed in the flanges.

2.2 *Analyses*

The determination of the elastic critical shear buckling load and ultimate shear capacity should take into account several structures phenomena that are difficult to quantify and differentiate one from another as they occur simultaneously, appearing even at low load levels (web buckling and yielding of the steel). It is, therefore, necessary to study those phenomena using general methods of analysis that provide a realistic approach to the main aspects of their non-linear behaviour (Zárate & Mirambell 1999). These aspects include, first, the influence of the second-order effects on global equilibrium of the structural element (geometric non-linearity) and second, the complexities derived from the real behaviour of the material (material non-linearity) (Zárate & Mirambell 2002).

Nonlinear analyses have been conducted in a three-dimensional finite element model of transversely stiffened corroded tapered plate girders, by thickness reduction, subjected to pure shear. A sensitivity analyses had been made also to verify the influence of the geometric properties in the shear behaviour of the web panel.

The Kirchhoff theory for thin plates is assumed in the present study. In order to reduce running time, the reduced integration algorithm is used. The hypothesis of large displacements and small strains is considered. Large displacements were incorporated, updating repeatedly the geometry of the structure after each load increment to assure global equilibrium over the deformed configuration. Steel properties are defined as a biaxial isotropic material model combined with the von Mises yield surface. The yielding surface may move as stiff solid. The stress-strain relationship is based on the characteristic uniaxial σ-ε diagram of the steel with elasto-plastic behaviour (Zárate & Mirambell, 2002).

2.3 *Geometric and mechanical properties*

The geometric properties of the model are defined in Figure 1, where h_1 is the higher depth of the web panel, and h_0 the lower; a is the transverse stiffener spacing; t_w is the thickness of the web panel; t_f is the thickness of the flange; b_f is the width of the flange; and is the slope of the bottom flange. The standard parameters of a tapered plate girder are: is the aspect ratio of the web, defined by the ratio between the transverse stiffener spacing and the greatest depth of the web ($= a / h_1$); η is the flange width to the maximum web depth ratio ($\eta = b_f / h_1$); λ_f is the flange slenderness ($\lambda_f = b_f / t_f$); and *tg* .

In this study was considered a steel S355, with the next mechanical proprieties: Young's modulus of 210000 N/mm2; Poisson's ratio of 0.3; yield stress of 355 N/mm2; and ultimate stress of 510 N/mm2.

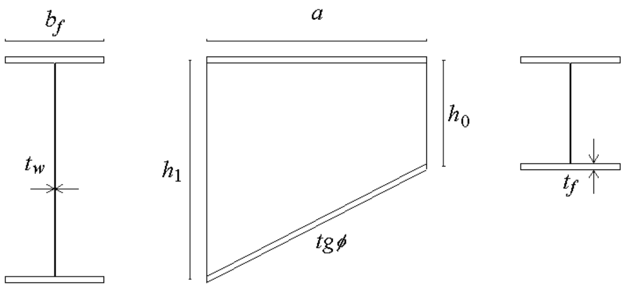


Figure 1. Geometric Properties

2.4 *Boundary conditions*

The presence of the flanges was taken into account in the structural analysis model and therefore also the influence of the stiffness of the flanges of the tapered plate girders, which is manifested as a certain degree of constraint on the rotation of the tapered web panel. The web panel is delimited in the longitudinal direction by two transverse lines of zero strain that reproduce the effects of the transversal stiffeners. This web-stiffener joint is interpreted in the analysis as a simply supported boundary condition.

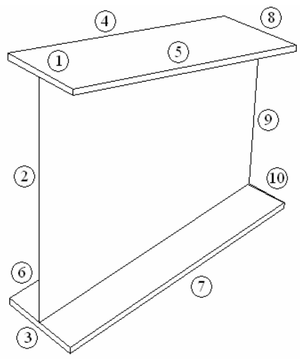
The boundary conditions proposed would match closely the behaviour of a simply supported beam under loading in its middle length. The beam represents, by symmetry, half of the simply supported beam. In the structural model, it is assumed that the cantilever beam is constrained with regard to the perpendicular displacements of the web. The structural analysis model takes into account the bending stresses on the web panel, which are derived from the boundary conditions (Zárate & Mirambell, 2002).

Table 1 show the boundary conditions that were assumed in the structural analyses carried out on tapered plate girders.

Table 1. Boundary conditions.

	u_x	u_y	u_z	θ_x	θ_y	θ_z
①	0	1	1	1	1	0
②	0	1	1	1	0	0
③	0	1	1	1	1	0
④	0	0	0	0	0	0
⑤	0	0	0	0	0	0

⑥	0	0	0	0	0	0
⑦	0	0	0	0	0	0
⑧	1	0	1	1	1	1
⑨	1	0	1	1	0	1
⑩	1	0	1	1	1	1



2.5 *Initial imperfection*

A perfect web panel does not buckle. In this sense, should be introduced in the model a geometric imperfection pattern for a postbuckling load-displacement analysis, turning it into a continuous response problem instead of bifurcation. The referred imperfection is defined as a linear superposition of the previous buckling eigenmodes analysis. The definition of the imperfection is based on the eigenmode data.

The shear behaviour of the web panel depends on the imperfection magnitude. A few numbers of analyses was conducted to investigate the imperfection sensitive. The panel will be easier to analyze if the imperfection is large; however, with large imperfections, the buckling bifurcation point disappears. The results have been obtained for web panels with relatively small initial deformations (h_1/200000), much smaller then allowable initial distortions in ANSI/AASHTO/AWS D1.5-96 Bridge Welding Code (1996).

As shown in Fig. 6 and Table 2, different initial imperfection (0,005mm to 1mm) have different consequences in the presented plot obtained (tangencial load versus displacement). Models with bigger imperfection do not exhibit the same clear cut separation between prebuckling and postbuckling phases. Bifurcation buckling point is not detected for web panels with higher initial deformations, but results for ultimate load appear to be indifferent to the initial deformation.

2.6 *Model validation*

The presented model was validated with various results (Lee & Yoo, 1999), nominated experimental results by Lee & Yoo (1999), design methods by Basler (1959), Cardiff (Porter et al. 1975 & Rockey et al. 1978) and Lee & Yoo (1998 and 1999). In the Table 2 is presented the results obtained. The proposed model presents similar results comparatively to the design methods.

Table 2. Model validation.

Beam	V_u (test)	V_u (model)	Δ	V_u (Lee)	Δ	V_u (Basler)	Δ	V_u (Cardiff)	Δ
G1	282,43	287,09	1,62%	289,10	2,31%	278,31	-1,48%	304,90	7,37%
G2	332,45	359,07	7,41%	317,83	-4,60%	343,63	3,25%	351,73	5,48%
G3	337,35	362,04	6,82%	317,83	-6,14%	343,63	1,83%	386,60	12,74%
G4	268,80	272,27	1,27%	275,57	2,46%	250,68	-7,23%	292,98	8,25%
G5	286,35	313,08	8,54%	296,26	3,35%	285,47	-0,31%	284,43	-0,68%
G6	312,83	320,55	2,41%	296,26	-5,59%	285,47	-9,58%	309,26	-1,15%
G7	258,90	276,87	6,49%	270,17	4,17%	231,14	-12,01%	228,99	-13,06%
G8	276,45	282,81	2,25%	270,17	-2,32%	231,14	-19,60%	251,64	-9,86%

3 SENSITIVITY ANALYSIS

By combining information on the rate and location of an eventual corrosion in web and flange panels, it is possible to predict elastic critical shear buckling stress and ultimate shear stress. In this sense, for a random steel plate girder, with the geometric properties shown in Table 3, nonlinear analyses have been conducted with the objective of detect the critical zones of a web and flange panels, and what strength reduction may we possibly expect.

Table 3. Base model

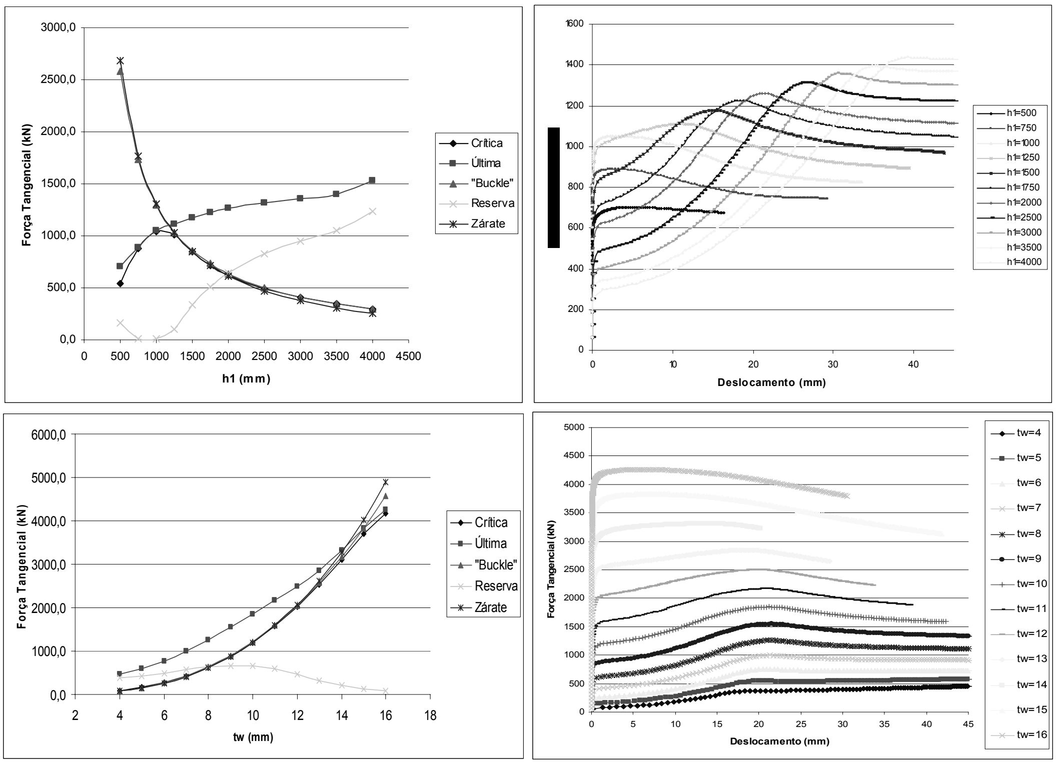
α	tg	h_1 (mm)	t_w (mm)	b_f (mm)	t_f (mm)	Critical Load (kN)	Ultimate Load (kN)
1,00	0,30	2000	8	900	24	635,0	1261,7

In Figure 2 are presented the results obtained for the sensitivity analysis of the geometric proprieties. The single variation was made for h_1, t_w, tg , t_f, b_f and α. There are three predictions of elastic critical shear buckling load: the first, obtained by the BUCKLE procedure (buckling load estimate = “dead loads” + eigenvalue * “live loads”); the second, by the analysis of the plot tangential load versus maximum displacement perpendicular at the web panel; the third, with the numerical formulation proposed by Zárate. In the same figure, there are also presented the ultimate load and postbuckling reserve (ultimate load – elastic critical shear buckling load).

The geometric parameters h1, tw and α are the most influent on the shear behavior of steel panels.

Resist shear force and maintain relative distance between the bottom and top flanges are the primary functions of the web plate in a plate girder. The loss in web material will reduce shear capacity due to both section loss and geometric buckling (Kayser & Nowak 1989).

The results obtained show that there are different consequences upon shear strength for a small thickness reduction of the web panel, depending on the location: for the elastic critical shear buckling load, the central portion of the panel is the most influent; for the ultimate load, the reduction of thickness on the left top corner is the most relevant. The obtained results show also an almost despicable reduction on strength and in the behavior on shear of tapered plate with flanges highly corroded. Generally, corrosion is more prejudicial in the top flange, and in front of elastic critical shear buckling stress. Flanges carry on the bending stresses principally, and the affection of corrosion should be study on this type of loads (Lourenço 2005).



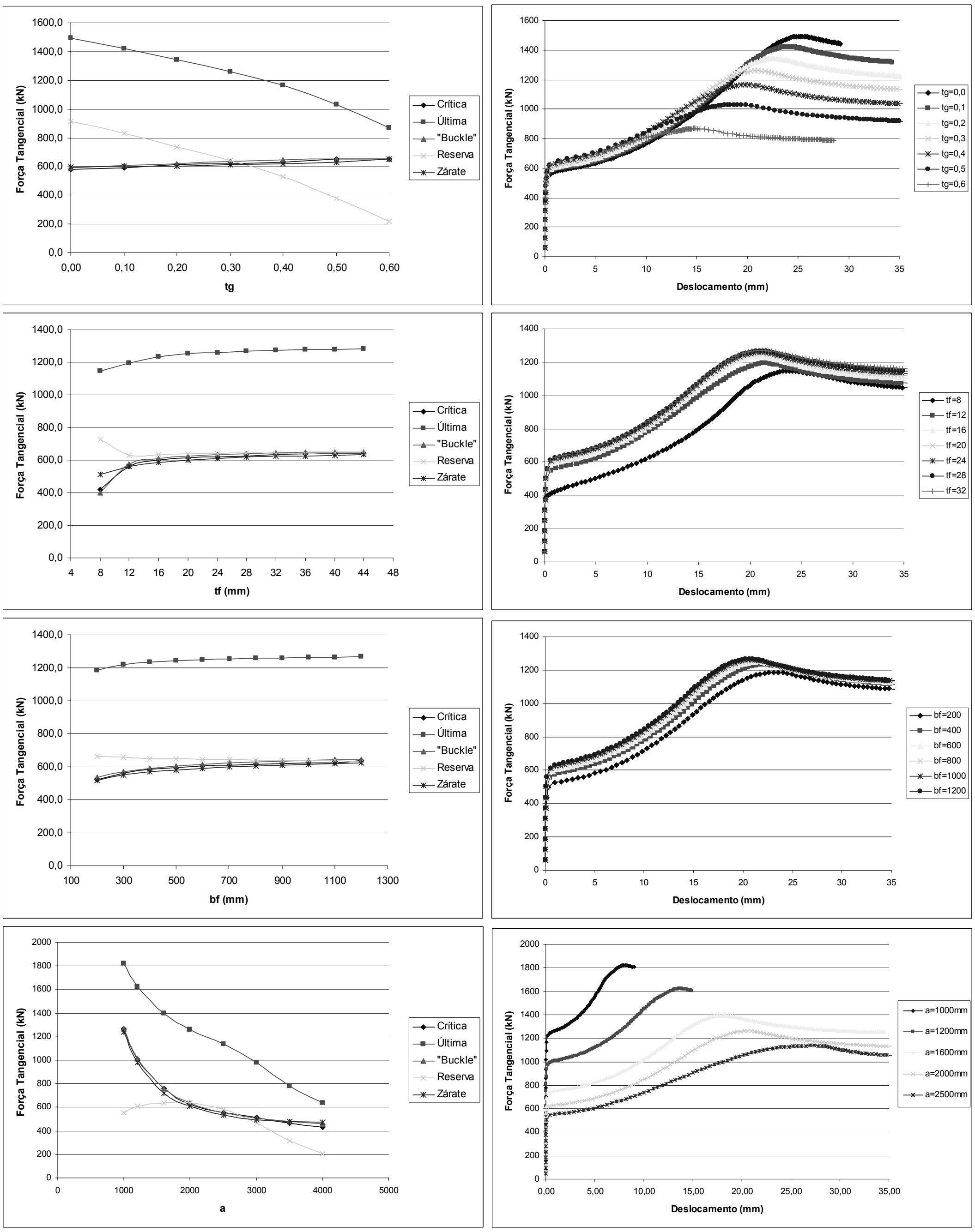


Figure 2. Sensitivity analysis

4 DATA MINING

Data Mining (DM) is often defined as the automated extraction of novel and interesting information from large datasets (Fayyad et al., 1996). DM has it roots in statistics, probability theory and machine learning. One of the underlying principles of knowledge discovery in data is to promote the process of building data-driven expert systems as an extension of the more traditional Artificial Intelligence expert systems approach. The idea is now that experts can learn from new findings in the data as well.

The aim of this study is to produce accurate models to predict the Ultimate Load, using two kinds of DM techniques: Artificial Neural Networks and Decision Trees.

Artificial Neural Networks are connectionist models inspired in the behaviour of the central nervous system of the human brain (Rajkumar et al., 2002), being attractive artefacts for the design of intelligent systems in data mining and control applications. In particular, the multilayer perceptron is the most popular neural architecture for supervised learning, where neurons are grouped in layers and only forward connections exist, providing a powerful base-learner with advantages such as nonlinear learning and noise tolerance. For unsupervised learning such as clustering, the Kohonen's self-organizing map (Kohonen, 1995) (Silipo,2003) is another neural network that is widely applied as an efficient visualization tool for high-dimensional data on a two-dimensional map, while preserving aspects of the underlying topology.

A Decision Tree (Breiman et al., 1984) is a direct and acyclic graph, where each node is either a decision node or a leaf node. To each decision node is associated a test based on attribute values, and a node has two or more successors, depending on the number of possible outcomes of the test. The most commonly used decision tree classifiers are binary trees that use a single feature at each node with two outcomes. This results in decision boundaries that are parallel to the feature axes. Given a learning set *L*, composed on *n* patterns x_i with known classification w_i, the decision tree induction mechanisms follows a top-down strategy for splitting nodes, based on an impurity measure derived from the examples reaching the node. When the tree classifies all learning examples in *L*, the process is stopped.

4.1 *Framework*

The framework used for the experiments involves two steps: (i) a clustering work to search and explore some kind of homogeneity in the data set, and (ii) the generation of prediction models adjusted for each homogeny cluster. This strategy was successful tested in the prediction of the ultimate resistance of steel beams subjected to concentrated loads (Santos et al., 2003) and for prediction of ultimate shear resistance of non-prismatic tapered plate girders (Cruz et al., 2004). After the first step (i), the C5.0 decision tree algorithm (Quinlan, 1997) was applied to each of the 3 clusters (given by the Kohonen network) in order to obtain a set of clustering explanatory rules.

4.2 *Experiments*

The dataset used for the experiments was developed using 5 884 of the Lourenço (2005) study cases. Each row denotes the geometric design parameters of tapered plate girders, the results of critical load and ultimate load and the values of damage in each of the 9 considered sections (Table 4). All experiments were conducted using the Clementine DM package (SPSS).

Table 4: The dataset attributes and range values

Attribute	**Range**
α	[0.5,…,2.00]
λf	[8.33,…,112.50]
n	[0.10,…,1.80]
tg	[0.00,…,0.60]
h1	[500.00,…,4 000.00]
bf	[200.00,…,1 200.00]
tf	[8.00,…,56.00]
fu	[308.40,…,4 262.70]

z1	[4.00,…,16.00]
z2	[4.00,…,16.00]
z3	[4.00,…,16.00]
z4	[4.00,…,16.00]
z5	[4.00,…,16.00]
z6	[4.00,…,16.00]
z7	[4.00,…,16.00]
z8	[4.00,…,16.00]
z9	[4.00,…,16.00]

With a high number of experimental cases in the dataset, it was first carried out a clustering work in order to find homogeneous clusters, producing in this way predictive models more suitable to the data and consequently with a higher degree of accuracy. When accuracy is a critical issue, the predictive models should be well tuned and adjusted. The clustering work was carried out using Kohonen's self organizing maps. In the Kohonen's network several parameters were experimented, being the final topology set to 16 input nodes and 6 output nodes corresponding to a map with a 2x3 grid. Then, the C5.0 algorithm was applied to each of the 3 clusters (Table 5), in order to obtain a set of rules. The training set used a random sample with 2/3 of the available data, while the test set contained the remaining 1/3. The set of classification rules managed to correctly predict the cluster membership with an accuracy of 99%.

Table 5: The frequency values of the 3 clusters obtained by the Kohonen algorithm

Cluster	Frequency	%
1	3 064	52.07
2	1 700	29.89
3	1 120	19.03

As an example, two rules for the cluster 2 membership are presented below:

Rule 1

```
Z5<=7.2
  Z9<=7
```

(n=583, confidence=100%)

Rule 2

```
Z5<=7.2
  Z9>7
    Z3<=7.6
      N>=0.2
        λf >14.8
          Z3<=7
```

(n=128, confidence=98.4%)

In these experiments the most relevant attributes are the damages on the sections 5, 9 and 3.

After the clustering step, a Multilayer Perceptron was used to predict the ultimate load for each cluster, by using a random sample with 2/3 of the data, while the test set contained the remaining 1/3. The results were evaluated using the Mean Absolute Deviation:

$$MAD = \sum_{i=1}^{N} \frac{|y_i - \hat{y}_i|}{N}$$

where N denotes the number of examples, y_i the desired target and $\hat{y}_i$ the value estimated by the neural network.

This study was concentrated in the beams with ultimate load in range [308.40,…, 1 200[kN, taking the assumption that the available data should be representative of the universe in study (56.3% of the beams in the data set have an ultimate load ∈ [308,40, … 1 200] kN). The obtained results appear in Table 6. The accuracy is higher for cluster 1, which contains the majority of the examples (52%), followed by cluster 2 (30%).

Table 6: The best Multilayer Perceptron architectures and results.

Cluster	Multilayer Perceptron architecture (input neurons - neurons in 1st hidden layer - neurons in 2nd hidden layer - output neurons)	MAD
1	14 - 30 - 20 - 1	13.78
2	17 - 30 - 20 - 1	17.70
3	17 - 30 - 20 – 1	24.60

5 CONCLUSIONS

This article presents: (i) the numerical analyses, carry out by using the finite element code ABAQUS, for elastic shear buckling load and ultimate load on tapered web panels with corrosion, and (ii) the use of data mining techniques to generate predictive models for ultimate load.

The carried numerical analyses in this work had allowed to extract the following conclusions:

In steel plate construction is unusual that structural members have the exact designed dimensions due to deterioration caused by environment exposure or fabrication tolerances. Corrosion affects the resistance of shear forces. The uncertainty must only be minimized by using models based on sound theoretical principles, validated by extensive and reliable data. This model can be used to make an extensive database that can be the support to validate a predict model of the shear capacity on web panels in steel plate girders with corrosion or not;

The thickness reduction in all panel origin a reduction on the critical elastic shear buckling load and ultimate load. However, the reduction of strength is not proportional to the area affected but the location of the degradation. For the elastic shear buckling load, the reduction of web thickness in the centre of the panel is the most prejudicial; for ultimate load, the reduction of web thickness in the superior left corner is the most critical;

The thickness reduction on the flanges appears as almost inconsequent in the shear behaviour of web panels. However, the corrosion in these elements should not be ignored. The flanges carried mainly the bending moment, and this type of analyses should be made with this solicitation;

This experimental research contributed to the development of new knowledge about the shear behaviour of corroded web panels in plate girders. Using a finite element method, it is possible to create a large and extensive database, with various scenarios of degradation, which can be an important tool in the inspection and assessment of steel girder bridges.

The availability of large databases enables the use of machine learning techniques for knowledge extraction. In this paper was presented a framework for generation of Artificial Neural Network based models for prediction of the ultimate load that improves the predictive accuracy. In the future should be investigated the embedding of these models in Intelligent Systems for automatic assessment of steel girder bridges.

6 ACKNOWLEDGMENT

The authors acknowledge financial support by the Research Project “Sustainable Bridges” (FP6-PLT-01653), in the aim of the Community’s Sixth Framework Programme. The authors acknowledge too Ángel Víctor Zárate Galván, for sharing his model for further investigations.

7 REFERENCES

Akgül, Ferhat and Frangopol, Dan M., “Rating and Reability of Existing Bridges in a Network”, Journal of Bridge Engineering, Vol.8, No. 6, November/December, pp. 383-393, 2003.

American Association of State Highway and Transportation Officials standard specifications for highway bridges, 16th edition. AASHTO, Washington, D.C., 1996.

Basler, K., and Thürlimann, B. (1959). “Plate Girder Research”, American Institute of Steel Construction, New York, U.S.A.

Breiman, L., Friedman, J., Olshen, R. and Stone, C. (1984), “Classification and Regression Trees”, Wadsworth, CA, USA.

Cruz, P., Quintela, H. and Santos, M.F. (2004), “Prediction of ultimate shear resistance of non-prismatic tapered plate girders using data mining techniques”, Proceedings of IABMAS’04 – Bridge Maintenance, Safety, Management and Cost, Japan 2004, Balkema, ISBN 04 1536 336 X.

Dinno, Deena K. and Birkemoe, Peter C., “Degradation of Strength from Corrosion of Girder Webs”, Proceedings of the 1997 SSRC Annual Technical Session, Toronto, Ontario, June, pp. 295-306, 1997.

Kayser, Jack R. and Nowak, Andrzej S., “Capacity Loss Due to Corrosion in Steel-Girder Bridges”, Journal of Structural Engineering, Vol. 115, n.º 6, June, pp. 1525-1537, 1989.

Lee, S. and Yoo, C. (1998). “Strength of Plate Girder Web Panels under Pure Shear”, Journal of Structural Engineering, Vol. 124, N.º 2, February, 1998, pp. 184-194.

Lee, S. and Yoo, C. (1999). “Experimental Study on Ultimate Shear Strength of Web Panels”, Journal of Structural Engineering, Vol. 125, N.º 8, August, 1999, pp. 838-846.

Lourenço, L. (2005). “Análise do Efeito de Redução de Espessura no Comportamento de Vigas Metálicas de Inércia Variável com Secção em I”, Master Thesis, School Eng., University of Minho.

Porter, D., Rockey, K. and Evans, H. (1975). “The Collapse Behaviour of Plate Girders Loaded in Shear”, Structural Engineer, pp. 313-325.

Quinlan, J. R. (1997), C5.0 Data Mining Tool, http://www.rulequest.com.

Rockey, K., Evans, H. and Porter, D. (1978). “A Design Method for Predicting the Collapse Behaviour of Plate Girders”, Proceedings of the Institution of Civil Engineers, Part 2, pp. 85-112.

Sato, M., Kato, S., Tamba, I., Ikeda, K., Mikami, S. and Oshima, T., “Evaluation of Maintenance Necessity Level (MNL) for aging bridges”, Proceedings of the Second International Conference on Bridge Maintenance, Safety and Management, Kyoto, Japan, pp. 593-595, 2004.

Santos, M. F., Quintela, H. and Cruz, P. (2003), “Forecasting of the ultimate resistance of steel beams subjected to concentrated loads using data mining techniques”, Proceedings of the Fourth International Conference on Data Mining, Rio de Janeiro, Brazil, Wessex Institute – WIT Press.

SPSS Inc., http://www.spss.com.

Zárate, A. V. and Mirambell, E., “Determinación de la tensión crítica de abolladura a cortante de almas esbeltas de vigas metálicas de inercia variable: Utilización del código Abaqus”, IV Reunión Nacional de Usuarios del Programa Abaqus, Spain, 1999.

Zárate, A. V. and Mirambell, E., “Ultimate Shear Capacity of Tapered Plate Girders”, Proceedings of Third European Conference on Steel Structures, September, Coimbra, Portugal, pp. 463-472, 2002.