Assessment of the in-plane shear strength of stone masonry walls by simplified models

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ABSTRACT: The present work intends to represent a further step in the knowledge of ancient stone masonry walls through an experimental approach, from which only few information is available. The main results of an experimental program based on in-plane static cyclic tests conduct on masonry wall panels with distinct masonry bond are presented. It is also revealed that simplified models, often used to predict the lateral shear strength of brick masonry walls submitted to static in-plane lateral loading, are also valid for stone masonry walls. Besides, the prediction of the lateral strength by means of a novel equilibrium model is also pointed out.

1 INTRODUCTION

Unreinforced stone masonry walls were in the past the most relevant structural element used in the construction of monumental and traditional buildings in the northern region of Portugal. These structural elements play the major role on the seismic response of the whole structure since they represent the basic resisting elements to horizontal seismic actions. In the absence of out-of-plane loading, the shear resistance of the walls can be prevailing on the stability conditions of masonry structures and, therefore, contributes to avoid their collapse.

Although the northern region of Portugal has been classified as low to moderate seismicity zone, the assessment of the resisting conditions of the existing typical structural elements comes into play concerning the need of rehabilitation and retrofitting of ancient structures. On the other hand, the understanding of the seismic behavior of masonry stones walls represents an additional advantage in the perspective of new masonry structures design.

In the scope of seismic experimental research, distinct testing approaches have been used for unreinforced masonry structures, namely quasi-static monotonic or cyclic tests, dynamic shaking table tests and pseudo-dynamic tests. By its simplicity, in-plane static cyclic tests are often preferred with respect to dynamic tests due to some important issues that have to be solved in this type of tests when individual masonry walls are considered concerning the high stiffness and the corresponding amplification of the base seismic excitation and the simulation of inertial forces by addition of external masses (Vasconcelos et al. 2006).

In the present work, a brief revision of the in-plane cyclic behavior obtained in a testing program consisting of a set of twenty four static cyclic tests conducted on stone masonry panels is provided. Three distinct masonry bond were considered to be representative of ancient masonry construction. The previous experimental data includes the failure modes and characteristic values of the force-displacement hysteresis diagrams. Subsequently, the experimental results concerning the shear strength are revisited in the light of selected failure criteria. The evaluation of the lateral resistance of the stone masonry walls is additionally assessed by using a strut and tie model. The information of the shear strength properties of masonry joints and the compressive strength of the stone masonry is obtained through a complementary direct shear tests on mortared and dry masonry joints and on masonry prisms.
2 EXPERIMENTAL RESEARCH

2.1 Description of the wall panels

The stone used in the construction of the walls is a two mica, medium-coarsed granite, which is considered to be representative of the material of ancient structures existing in the Northern region of Portugal. As is shown in Fig. 1, three distinct granitic textural typologies were adopted. The dry masonry walls, WS, intend to be representative of historical masonry constructions where no bonding material between stone units is present or where the major part of the bonding material with low strength properties vanished due to weathering effects. Walls WI consist of the assembly of irregular hand-cut units with similar shape but variable dimensions with low strength mortar filling in the head and bed joints. Rubble masonry walls WR are composed by units with variable shape and dimensions randomly assembled with low strength mortar. The masonry wall panels were 1200mm height and 1000mm width, corresponding to a height to length ratio of 1.2. The wall thickness is 200 mm. The adopted dimensions for the walls and stone units are about 1:3 scale for single leaf walls found in the northern region of Portugal.

![Figure 1](image1.png)

Figure 1: Geometry and bond details of the walls; (a) sawn stone units walls, WS; (b) irregular walls, WI; (c) rubble walls, WR.

2.2 Test setup and procedure

The static cyclic tests were carried out for three distinct pre-compression levels of 100kN, 175kN and 250kN, corresponding to normal stresses of $\sigma=0.5\text{N/mm}^2$, $\sigma=0.875\text{N/mm}^2$ and $\sigma=1.25\text{N/mm}^2$ respectively by using the typical testing setup depicted in Fig. 2a. The cantilever wall was fixed to the reaction slab through a couple of steel rods. The pre-compression loading was applied by means of a vertical actuator with reaction in the slab through the cables.

![Figure 2](image2.png)

Figure 2: Experimental details; (a) test setup; (b) displacement time history.
A stiff beam was used for the distribution of the vertical loading and a set of steel rollers were added to allow relative displacement of the wall with respect to the vertical actuator. The deformation of the wall was measured by means of the LVDTs indicated also in Fig. 2a. As has been reported, the selection of the displacement history to simulate the seismic action has some relevance on the results, particularly in the post-peak region, where strength and stiffness degradation occurs (Tomaževič et al. 1996). For this reason, the same displacement history shown in Fig. 2b was used as an input seismic action for walls WI and WR. For dry masonry walls, the displacement increment of 5mm was defined from the prescribed displacement of 10mm. A rate of 100µm/s displacement was adopted.

2.3 Summary of the failure modes

A general overview of the failure modes for the distinct types of walls is given in Table 1. The in-plane behavior under cyclic reversal loading of dry stone masonry walls is governed by flexural response up to the maximum lateral force is reached. This behavior is characterized by the opening of stepped cracks along the joints without any damage on the stones. The opening of diagonal stepped cracks occurs for lateral loads close to the lateral maximum resistance. After the opening of these cracks, the walls tend to rock around the opposite toe. For a normal compressive stress of 1.25N/mm², the collapse of the walls is associated to toe crushing. For low to medium to moderate levels of pre-compression, only residual sliding displacements are visible after lateral loading is ceased.

<table>
<thead>
<tr>
<th>Wall</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS.100 (4)</td>
<td>Rocking mechanism</td>
</tr>
<tr>
<td>WS.175 (3)</td>
<td>Rocking mechanism/ Rocking and toe crushing</td>
</tr>
<tr>
<td>WS.250 (3)</td>
<td>Rocking and toe crushing</td>
</tr>
<tr>
<td>WI.100 (2)</td>
<td>Rocking/flexural</td>
</tr>
<tr>
<td>WI.175 (3)</td>
<td>Flexural/toe crushing</td>
</tr>
<tr>
<td>WI.250 (2)</td>
<td>Flexural/toe crushing – Flexural/shear/toe crushing</td>
</tr>
<tr>
<td>WR.100 (2)</td>
<td>Flexural/shear – Flexural/rocking</td>
</tr>
<tr>
<td>WR.175 (2)</td>
<td>Shear – Flexural/shear</td>
</tr>
<tr>
<td>WR.250 (3)</td>
<td>Shear</td>
</tr>
</tbody>
</table>

The dependence of the failure mode on the normal stress is more evident for irregular and rubble masonry walls. Rocking mechanism prevails for the low normal stress levels, with the opening of considerable bed joint cracks. This behavior is also observed in irregular masonry walls submitted to moderate normal stresses. Typical shear cracks developed for moderate to high level of pre-compression in rubble masonry walls, whereas clear flexural response with toe crushing governs the in-plane behavior of irregular masonry walls in case of vertical stress level of 1.25N/mm². This means that similarly to pre-compression level, masonry bond influences also the failure modes of masonry walls submitted to in-place cyclic loading. Furthermore, it is stressed that the rubble masonry walls are rather sensitive to the workmanship. In fact, for low levels of normal stresses, both rocking and shear failure developed in different wall panels.

2.4 Average values of the lateral strength

The maximum values of the lateral resistance were obtained from the lateral force-lateral displacement diagrams. The average values of the lateral strength are summarized in Table 2 for the different types of walls, being the number of tests indicated inside brackets. It should be noted that in spite of reduced number of specimens could be tested for the same type of walls, a reduced scatter among the different test specimens is observed. Apart from rubble masonry walls, it is clear that the lateral strength increases considerably as the normal stresses increases. The maximum values of the lateral resistance attained by the irregular walls are quite close to the ones obtained for the dry masonry walls for low to intermediate levels of pre-compression,
which is explained by the common failure mechanism associated with flexural rocking response. This tendency is not verified in one specimen submitted to 175kN, whose lateral resistance appears to be influenced by pre-existing microfissures originated by shrinkage. Furthermore, its typical pattern, following the head and bed mortar joints, provides the formation of a flexural mechanism after failure of the upper corners. A tendency for a slight decrease on the values of the lateral strength was observed for walls WI.250 compared to the walls WS.250, which can be attributed to the lower compressive strength of irregular masonry walls. It is noted that in comparison with dry masonry walls, higher scatter in the results is achieved due to variability on the textural arrangement. It should be underlined that stepped shear cracks seem not to control the mechanism of lateral resistance since they coexist with flexural cracks.

In relation to rubble masonry walls, some aspects should be pointed out. For low and intermediate levels of pre-compression, also a similarity of the lateral strength is observed between rubble masonry walls and dry and irregular walls. In spite of the wall WR1.100 fails in shear, the maximum lateral resistance is similar to the one obtained for the wall WR2.100, whose lateral response in governed by rocking. In turn, a noticeable strength decrease is recorded in the walls WR.250. The shear crack resistance is practically coincident with the values of the maximum lateral strength, meaning that the opening of the shear cracks results in the failure of the walls. This result is in agreement with the statements of Magenes and Calvi (1997) indicating diagonal shear cracking can be considered as a limit state since it occurs for lateral loads ranging between 90 and 100% of the maximum resistance.

### Table 2: Average values of the lateral strength for all walls

<table>
<thead>
<tr>
<th>Wall</th>
<th>$H_{\text{max}}$ (kN)</th>
<th>$\delta_{H_{\text{max}}}$ (mm)</th>
<th>$H_{\text{max}}$ (kN)</th>
<th>$\delta_{H_{\text{max}}}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS.100</td>
<td>36.9</td>
<td>12.8</td>
<td>-40.7</td>
<td>-17.8</td>
</tr>
<tr>
<td>WS.175</td>
<td>62.8</td>
<td>22.6</td>
<td>-67.8</td>
<td>-22.5</td>
</tr>
<tr>
<td>WS.250</td>
<td>86.3</td>
<td>24.3</td>
<td>-93.0</td>
<td>-20.9</td>
</tr>
<tr>
<td>WI.100</td>
<td>37.6</td>
<td>14.4</td>
<td>-40.0</td>
<td>-17.9</td>
</tr>
<tr>
<td>WI.175</td>
<td>55.7</td>
<td>19.8</td>
<td>-62.7</td>
<td>-17.8</td>
</tr>
<tr>
<td>WI.250</td>
<td>83.2</td>
<td>17.1</td>
<td>-85.0</td>
<td>-15.8</td>
</tr>
<tr>
<td>WR.100</td>
<td>36.9</td>
<td>12.9</td>
<td>-40.4</td>
<td>-13.7</td>
</tr>
<tr>
<td>WR.175</td>
<td>63.6</td>
<td>13.8</td>
<td>-60.4</td>
<td>-13.4</td>
</tr>
<tr>
<td>WR.250</td>
<td>65.9</td>
<td>7.1</td>
<td>-71.6</td>
<td>-8.3</td>
</tr>
</tbody>
</table>

3 ASSESSMENT OF THE EXPERIMENTAL LATERAL STRENGTH

### 3.1 Simplified models

The prediction of the in-plane behavior of masonry walls by means of complex numerical methods assumed a central role in the research effort in the scope of masonry structures. However, for design purposes, it is often required to use simplified assumptions that enable a simplified analysis. During the last few decades several failure criteria have been proposed to describe the in-plane static strength of masonry walls (Turnsek and Sheppard 1980, Mann and Muller 1982). This section aims at reviewing the most used simplified analytical methods, which are associated to distinct failure patterns consisting of different limit states occurring in masonry walls under in-plane loading.

When masonry walls behave in flexural mode, after flexural cracking and subsequent reduction of the resistant section of the wall, vertical compressive loading migrates towards the compressive toe. Assuming a rectangular stress block at the base of the wall, the lateral strength associated to flexural response is given by the expression:

$$ H_r = \sigma \left( \frac{l^2 t}{2h} \right) \left( 1 - \frac{\sigma}{0.85 f_c} \right) $$

where $f_c$ is the compressive strength of the masonry and a value of 0.85 is assumed for the reduction of the compressive strength, $l$, $t$ and $h$ are respectively the length, thickness and height of the wall. The vertical stress, $\sigma$, is calculated based on the cross section of the wall.
The shear response of masonry walls subjected to combined vertical and horizontal loads can be associated to different cracking typology, which depends essentially on the material properties. Two main simplified criteria have been used to describe the shear failure of panel walls. The first approach consists in a Mohr-Coulomb type criterion, being the ultimate shear strength, $\tau$, given by the expression:

$$\tau = c + \mu \sigma$$

where $c$ and $\mu$ are considered as global strength parameters. This expression is similar to the criteria used to describe the local shear failure at the joint level, but the coefficients $c$ and $\mu$ presents generally lower values since the local stress state of the joint differs from the stress field of the wall, considered as homogeneous material.

The other approach for the shear resistance is based on the Turnšek and Sheppard (1980) criterion, which is based on the assumption that diagonal cracking takes place when the maximum principal stress at the center of the wall reaches the tensile strength of masonry. The stress state is calculated by assuming that masonry is an isotropic and homogeneous material, which is not corresponding to its actual behavior. The tensile principal stress can be calculated as (Turnšek and Sheppard 1980):

$$\sigma_t = \sqrt{\frac{\sigma^2}{2} + \tau_{max}^2} - \frac{\sigma}{2}$$

where the horizontal stress is negligible and $\tau_{max}$ is the maximum shear stress. Considering that the shear stress, $\tau$, assumes a parabolic distribution, the horizontal shear force corresponding to the opening of shear cracks, $H_s$, reads (Turnšek and Sheppard 1980):

$$H_s = f_t \frac{lt}{b} \sqrt{1 + \frac{\sigma}{f_t}}$$

where $f_t$ is taken as the tensile strength of masonry. The variable $b$ assumes the value of 1.5 for walls with height to width ratios larger than 1.5. In case of height to width ratios ($h/l$) ranging between 1.0 and 1.5, the shear strength is calculated by considering a value for $b$ equal to the height to width ratio.

The calculation of the theoretical failure envelopes and further comparison with experimental results are based on the mechanical properties obtained for dry and mortared masonry joints by means of direct shear tests and for masonry prisms through uniaxial compressive tests (Vasconcelos 2005). These properties are summarized in Table 3. The tensile strength of masonry was calculated from (3), based on the shear stress obtained experimentally.

<table>
<thead>
<tr>
<th>Wall</th>
<th>$c$ (N/mm$^2$)</th>
<th>$\mu$</th>
<th>$f_c$ (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walls WS</td>
<td>0.00</td>
<td>0.65</td>
<td>73.0</td>
</tr>
<tr>
<td>Walls WI</td>
<td>0.36</td>
<td>0.63</td>
<td>18.4</td>
</tr>
</tbody>
</table>

The comparison of the analytical flexural and shear strength envelopes with experimental lateral resistance obtained from the shear tests is displayed in Fig. 3a for the walls WS. The lateral resistance obtained in shear tests is remarkably close to the flexural resistance calculated for a compressive strength of $f_c = 73.0$N/mm$^2$. However, flexural response appears to be an upper bound of the lateral resistance, possibly due to the very high compressive strength found in the prism tests, which are hardly representative of the masonry condition and state of stress in the compressed toe. If a statistical regression is fitted to the experimental data, an expressive coefficient of determination ($r^2 = 0.97$) is found to exist for the linear relation between the lateral resistance and the vertical pre-compression: $\tau = 0.382 \sigma$. Fig. 3b shows a good agreement between experimental values corresponding to the opening of the diagonal crack, and the failure envelope of Mann and Müller (1982), obtained with a tensile strength of the units of $f_{tu} = 2.8$N/mm$^2$, corresponding to shear sliding of the bed joints.

Although with acceptable values, it is observed that the experimental lateral resistance of irregular walls is overestimated by flexural resistance envelope corresponding to a compressive strength $f_c = 18.4$N/mm$^2$ for the intermediate and mainly for the highest level of vertical pre-
compression, see Fig. 4a. In terms of average lateral strength, the difference for the flexural envelope is, respectively, 10.8% and 13.8%. This difference is even more remarkable in case of rubble masonry walls. This means that a more detailed characterization of the compressive strength of rubble masonry walls should be made. The flexural resistance would be better adjusted if a lower compressive strength ($f_c = 5.0 \text{N/mm}^2$) was adopted.

Even if flexural typical failure characterizes the in-plane behavior of this type of walls, a significant approximation to the lateral strength of this type of walls is also attained with the Coulomb failure. A linear relation was found to exist between the lateral and the vertical strength, ($\tau = 0.051 + 0.297\sigma$, $r^2 = 0.95$), as the result of the statistical fitting to the experimental data. For the rubble masonry walls, the application of this failure criterion results in a relation between the shear and normal stress as $\tau = 0.112 + 0.193\sigma$, with a coefficient of determination of $r^2 = 0.81$. The higher scatter is, to great extent, influenced by the lower values of the lateral strength obtained for the higher levels of pre-compression. The Turnsek’s envelope based on the assumption that diagonal cracks arises for a certain stress state at the middle of the panel, deviates clearly from the experimental average lateral resistance for irregular masonry walls but approximates considerably well for the rubble masonry walls. For the highest level of pre-compression the difference for the average shear strength in only about 6%. It provides also a reasonable prediction of the average shear strength for walls submitted to intermediate pre-compression levels (difference of about 11%). This is the result of the failure being governed by mixed flexural/shear and shear modes of failure obtained, respectively for moderate and high levels of pre-compression. Note that the experimental results obtained in rubble masonry walls possibly suggest the need of a failure criterion composed by different envelope functions, as
proposed by Mann and Muller (1982) to better describe distinct failure modes occurring for different shear/compression ratios.

### 3.2 Equilibrium model

Although equilibrium models based on struts and ties have been widely accepted by modern concrete codes for design and assessment purposes, almost no information is available on its applicability in the scope of masonry structures. The main reason of this is related to the brittle nature of the masonry material and its reduced ability to conduct tensile stresses. However, some developments arose recently from a simple equilibrium model for assessment of the shear strength of masonry shear walls proposed by Roca (2006). The model is based on the lower bound theorem of plasticity and its applicability can be justified by plastic mechanisms associated to the limit state of the wall: friction in joints and compression failure of masonry. This means that ultimate mechanisms must be governed by the maximum friction in the joints or maximum compression at the nodes. In case of shear walls submitted to uniformly distributed loads, a model composed by smeared struts arranged according to a parallel or fan distribution is shown in Fig. 5. In the fan model indicated in Fig. 5a, the slope of the struts varies gradually to make the stress paths consistent with the geometry, with \( \beta_m \) being defined as the average slope of the struts. The model shown in Fig. 5b consists of a modified fan model that complies with the limitation on the maximum slope of the struts. It becomes equivalent to the fan model in case of very compressed or narrow walls.

![Equilibrium models](image)

Figure 5: Examples of equilibrium models proposed by Roca (2006); (a),(b) walls under uniform load, (c) comparison between the experimental and predicted resistance by the equilibrium model for dry walls

These models were used to predict the horizontal shear resistance of the walls submitted to uniform distributed vertical load resulting from the monotonic and cyclic tests. The usage of these models is validated by the fact the rotation of the upper steel beam is free. In case of dry masonry walls, the model shown in Fig. 5b is possible to develop for any of the pre-compression levels considered. As is shown in Fig. 5c, good agreement was found between the experimental and the predicted values of the maximum horizontal load. Note that for the high compressive strength of masonry considered, \( f_c = 73 \text{N/mm}^2 \), the failure envelope is practically linear, which means that it is practically independent on the compressive strength of the wall. This also means that in the attainment of the ultimate load, sliding failure mechanism takes an important role in the shear response. The evaluation of the usage of the simple models to predict the maximum horizontal load of cohesive walls is performed by means of the results displayed in Fig. 6. Using the value of compressive strength obtained by testing irregular masonry prisms, \( f_c = 18.4 \text{MPa} \), it can be observed that predictions deviate from the experimental strength for the higher level of pre-compression. A much better prediction would be attained for a compressive strength \( f_c = 7.0 \text{N/mm}^2 \). It is noticed that the failure envelope is rather similar to the flexural failure criteria, which results from the fact that both approaches are based on the same plastic mechanism associated to yielding of the masonry in compression. Concerning rubble masonry walls, a good agreement between experimental results and the shear prediction through the simplified model for low and moderate levels of pre-compression would be achieved if a the com-
pressive strength ($f_c = 5.0 \text{N/mm}^2$). Nevertheless, also this model deviates definitely from the experimental lateral strength obtained for the highest level of pre-compression, which is associated to the fact that ultimate state is ruled by diagonal tension cracking that develops before maximum lateral resistance is mobilized. This simplified model also suggests a failure criteria composed by different envelope functions for this type of masonry walls.

![Graph](a) ![Graph](b)

Figure 6: Comparison between the experimental and predicted lateral resistance given by the simple model; (a) irregular masonry walls, WI; (b) rubble masonry walls, WR

4 CONCLUSIONS

Despite the known key role of the geometry of the walls in the shear response of walls, the results obtained in the present work show that vertical normal stresses and masonry bond are also quite relevant. The shear strength is almost insensitive to the arrangement of the stones for low to moderate normal stress states but this observation does not hold for the highest level of pre-compression under consideration. Thus, the shear strength was found to decrease as the randomness of the masonry bond increases, particularly for the highest level of normal stresses, which is associated to typical diagonal shear failure. A good agreement between the experimental lateral resistance and the predicted values based on simplified models was attained. It has been shown that a single simplified failure criterion can be difficult to obtain. However, it is stressed that the lateral strength of all walls can be predicted by a Coulomb type failure criterion with a friction coefficient of approximately 0.4 in dry walls, 0.3 for irregular masonry walls and 0.2 for rubble masonry walls. It should be focused that the latter values is considerably lower than the value pointed out by the EC6 (2005).

REFERENCES