IT'S MATHEMATICS!: A MODELLING DAY TO DEVELOP REAL-WORLD PROBLEM SOLVING

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Abstract

Mathematics is a basic and structural discipline of several Higher Education courses. Despite its comprehensive application to numerous real-world situations, students frequently demonstrate low levels of engagement and negative attitudes towards mathematics, which are also translated in poor achievement. In this context, teaching approaches might make the difference to engage learners' attention and to drive them into a process of active and significant learning. The challenge here is to develop learning environments that makes mathematics an enjoyable subject, providing simultaneously a meaningful learning through the stimulation on thinking skills. One of the innovative teaching approaches that emerged under the premise of develop students' problem-solving skills, today required in most professional areas, is the Problem-Based Learning (PBL).

The School of Management and Technology (ESTG), from Polytechnic of Porto (P.PORTO), Portugal, has numerous courses with subjects in different areas of Mathematics. The applications of Mathematics was a common students' question present on classes. Indeed, the students related their low motivation level to the difficulty in understanding how they could use the mathematical knowledge on their professional future.

The workshop "It's Mathematics!" consisted in a modelling day, inspired in the PBL methodology, designed to address this challenge. This event joined students from different program courses, degree levels and future professional profiles into contact with real industrial problems (or their simplifications) and also to work in groups. In small teams, and using methods, concepts and techniques from different areas of Mathematics, students worked together in order to answer these problems with the support of a teacher. This initiative was firstly motivated by the need to develop students' awareness of the application of Mathematics in solving industrial problems, as a way to increase their motivation and engagement in their study fields. At the same time, the program aimed to train and develop problem solving, teamwork, oral and written communication. These are relevant skills for professionals in the industrial and business fields that have usually deserved less attention in traditional pedagogic methods.

Since the first edition, the students showed a good receptivity to this workshop, which was demonstrated by a great enthusiasm in the closing of the initiative. They considered the event very challenging and expressed the experience of public presentation at the end of the day as very positive. As negative aspects, they reported the feeling of pressure and the short time to solve the problems. The organization is currently discussing the extension of the experience over time, moving from a one-day experience to several days throughout the semester. To determine which problems are more appropriate for different students' levels, in order to be sufficiently stimulating but simultaneously not overwhelming, continues to be a challenge for the organizers. In addition, there is the need to develop adequate instruments that could help on identifying and evaluating the impact of this event in a more systematic and formal way, in order to stimulate the continuous improvement of this experience.

Keywords: Mathematics, teaching and learning experience, industrial problems, problem-based learning.

1 INTRODUCTION

Mathematics is a basic and structural discipline of several Higher Education courses. Despite its comprehensive application to numerous real-world situations, students frequently demonstrate low levels of engagement and negative attitudes towards mathematics, which are also translated in poor achievement [1]. In this context, teaching approaches might make the difference to engage learners'

attention and to drive them into a process of active and significant learning. The challenge here is to develop learning environments that makes mathematics as an enjoyable subject, providing simultaneously a meaningful learning through the stimulation on thinking skills [2].

One of the innovative teaching approaches that emerged under the premise of develop students' problem-solving skills, today required in most professional areas, is the Problem-Based Learning (PBL). This method is characterized by being student-centred, where learning usually occurs in small groups and under the guidance of a tutor, in confronting authentic problems [3]. Because problems are usually presented in unstructured and complex ways (as they are in real-world), it is expected that new information needs to be acquired to solve the problems, and so self-directed learning is required [3]. A typical PBL cycle is initiated with a problem scenario presented to the students. Then, students formulate and analyse the problem by identifying relevant facts. After, students generate hypotheses about possible solutions. Once students have reached this stage, they should be able to identify knowledge deficiencies, which will drive the search for new knowledge, that is, self-directed learning. Following that, students apply new knowledge and evaluate the formulated hypothesis. The final stage implies the reflection about their learning process [4]. Therefore, this method contrasts with traditional lectures, where problems are usually presented in a structured way, with a single possible solution, and appealing to specific and identified content, of the curricular unit. With PBL, students are challenged with the opportunity to combine aspects of their learning to deal with complex and unfamiliar problems with ill-defined boundaries and solutions, managing their learning goals and strategies.

This approach to learning allows students to acquire skills needed for lifelong learning, and therefore allowing a better preparation for their future professional practices [4], [5]. At the same time, and because students work in small groups in search of a problem solution, they need to establish common ground, resolve discrepancies actions to take and arrive to agreement [6], being team-players. Because students are involved in meaningful tasks and the learning is under their control, PBL is commonly highlighted because of its potential to increase motivation for learning practices [4] and higher learning outcomes [7].

2 SHORT OVERVIEW OF THE PROGRAM "IT'S MATHEMATICS!: A MODELLING DAY TO DEVELOP REAL-WORLD PROBLEM SOLVING"

The School of Management and Technology (ESTG), from Polytechnic of Porto (P.PORTO), Portugal, is an higher education institution that began its activity in 1985 and offers degree, master and professional higher technical (TeSP) courses in different areas from Informatics to Management Sciences and Law (see http://estg.ipp.pt, for further information). ESTG, is located at campus 3 of P.PORTO, in the city of Felgueiras, and plays a significant role in the economic development of the region of Tâmega and Sousa, in the North of Portugal, which is a highly industrial area, characterized mostly by SMEs (small and medium-sized enterprises). All courses, with the exception of Legal Practice and Services, have subjects in the different areas of Mathematics such as, Linear Algebra, Calculus, Numerical Methods, Statistics and Optimization. The applications of the different areas of Mathematics was a common students' question present on almost every class. Indeed, the students related their low motivation level to the difficulty in understanding how they could use the mathematical knowledge on their professional future.

In April 2016, the workshop "It's Mathematics!" was organized for the first time. This event was inspired in the concept of modelling week, that ECMI (European Consortium for Mathematics in Industry) has been running annually since 1988, essentially for master and PhD students. The idea was to adapt this event to a format that could work on a single day, therefore having a modelling day which brought students from different courses, degree levels and future professional profiles into contact with real industrial challenges (or their simplifications) and also to work in groups. In small teams, and using methods, concepts and techniques from different areas of Mathematics, students worked together in order to answer these challenges with the support of a teacher. Since 2016, this event has been held one time per curricular year. Indeed, the second edition took place in December 2017 and the third edition in January 2019. A total of 230 students participated in the event through the three editions.

The day starts with the presentation of the real problems by the respective coordinator. The coordinators are teachers of mathematical subjects, researchers, invited academic or industrial professionals. The problem is formulated in non-mathematical terms. That is, the coordinator does not identifies the mathematical contents or techniques that should be used, and also there is not a single

possible solution or methodology to use. In a second moment, after all problems have been exposed, the students choose the groups. These groups may include elements of different courses or not. Right after, starts the work in group. The day ends with presentation to all participants of the work developed by the different groups of students. It is not expected that the problems are completely solved in the end of the day but that the students can show in what way they have contributed to a specific part (large or small) of the problem's resolution.

The event is prepared at the beginning of the semester and the teachers are invited to propose a problem for this day event. They can also invite a researcher, academic or industrial professional to do the problem presentation. The teachers are encouraged to think of a way to integrate the student's work of this day on the evaluation of the subject that they teach. The organization gathers the contribution of all and tries that problems to be worked are aligned with the syllabus of the different subjects and can be solved using the contents that are being taught in the different classes.

The first edition of "It's Mathematics!" took place from 11a.m. to 4p.m. on April 27th, 2016. A total of 51 students from the degrees of Computer and Networks Security, Informatics Engineering, Management and Business of SMEs and five teachers, participated (see Table 1). The mathematical topics that were covered were Statistics, Linear Algebra and Graph Theory. The following four problems were proposed: Prob1.1 - Characterization of noise exposure of Portuguese industry workers; Prob1.2 - Reservation programming in an Hotel establishment; Prob1.3 - Image processing; Prob1.4 - What is the best route?

The informal feedback from the students was very positive, but no satisfaction questionnaire was applied at this time. However, this feedback justified and motivated a second edition. In this second edition, the workshop started at 11a.m. and finished at 23p.m., enabling that the evening students could also participated. In addition to the extension of the workshop to evening students, the workshop included a seminar of the representative of PT-MATHS-IN (Portuguese Network for Mathematics in Industry and Innovation) that took place at 6p.m. The students presentations took place at 5p.m. and 23p.m., for day and evening classes, respectively. Numerical Methods, Linear Algebra, Statistics, and Linear and Nonlinear Optimization, were the areas that could be used to solve the five problems: Prob2.1 - Fitting Carreau-Yasuda fits model in experimental data; Prob2.2 - How to load pallets of different sizes in trucks; Prob2.3 - Analysis of Tâmega and Sousa indicators for the attribution Governmental incentives; Prob2.4 - Diagnosis on a firm's occupational safety and health; Prob2.5 - Improvement of the grape reception process (see Table 1).

Table 1: Number of students and group coordinators, courses involved and problems proposed

Courses	***Problem***	***Math Area***	***Evaluation***
1st Edition - April 27th 2016 - 51 students; 4 coordinators			
TeSP in Management and Business of SMEs; Degree in Occupational Safety and Environment; Informatics Engineering; Computer and Networks Security	Prob1.1 - Characterization of noise exposure of Portuguese industry workers Prob1.2 - Reservation programming in an Hotel Prob1.3 - Image processing Prob1.4 - What is the best route?	Statistics, Linear Algebra, Graph Theory	Prob1.1 Prob1.4
2nd Edition - December 7th 2017 - 79 students; 5 coordinators			
Degree in Informatics Engineering; Computer and Networks Security; Degree in Business Sciences; Degree in Occupational Safety and Environment; Master in Methods for Business Decision Making	Prob2.1 - Fitting Carreau-Yasuda fits model in experimental data Prob2.2 - How to load pallets of different sizes in trucks Prob2.3 - Analysis of Tâmega and Sousa indicators for the attribution Governmental incentives Prob2.4 - Diagnosis on a firm's occupational safety and health Prob2.5 - Improvement of the grape reception process	Numerical Methods, Linear Algebra, Statistics, Linear and Nonlinear Optimization	Prob2.1 Prob2.2 Prob2.5
3rd Edition – January 3rd 2019 – 100 students; 5 coordinators			

TeSP in Development for Web and Mobile Devices; Degree in Business Sciences (day and evening classes); Degree in Informatics Engineering; Degree in Industrial Management and Logistics	Prob3.1 - Use of computer security windows in municipal facilities in Portugal Prob3.2 - Unemployment rate and inflation, Portugal versus USA Prob3.3 - Extrusion process modeling Prob3.4 - A transportation problem Prob3.5 - Socially influential: the impact of social networks	Statistics, Numerical Methods, Linear Algebra	All problems

In the third edition, that took place in January 2019, the event started with a seminar by an academic researcher with a wide experience with academy-industry synergies. Several cases from different industries were presented and highlighted the mathematical concepts used. After that, five problems were presented to be solved: Prob3.1 - Use of computer security windows in municipal facilities in Portugal; Prob3.2 - Unemployment rate and inflation, Portugal versus USA; Prob3.3 - Extrusion process modeling; Prob3.4 - A transportation problem; Prob3.5 - Socially influential: the impact of social networks (see Table 1). Both, day and evening students presented their work.

In the first edition, only for two problems the students' work could be used in the evaluation of the maths subject, while in the second edition this number increased to three. In the third edition, for all problems the students' work on the workshop could be used for evaluation of the respective math subject. Each subject's teacher defined specifically the weight of the workshop for the final evaluation. However, it was mandatory that the students identified the mathematical concepts and methods that were being used, and justify it. Next, some examples of the problems that were addressed are presented.

Prob1.1 - Characterization of noise exposure of Portuguese industry workers – For a professional in Occupational Safety noise exposure at the local of work is an important issue, [8]. The students were invited to study the impact of noise in different sector in industry. For this purpose, they used data from 280 firms across the main industrial sectors collected within the "PREVENIR – Prevenção como Solução" (PREVENT – Prevention as a Solution) that took place from 2005 to 2011 and was developed by AEP – Associação Empresarial de Portugal (Business Association of Portugal), in partnership with ACT – Autoridade para as Condições do Trabalho (Authority for Working Conditions) and Support of POAT – Programa Operacional de Assistência Técnica (Operational Program of Technical Assistance). After the students identify the problem at hand, which is one of the topics studied in the degree in Occupational Safety and Environment, they discussed with each other the possibilities to respond to this problem. In discussion with the problem's coordinator it was decided to use statistical techniques, more concretely, ANOVA Test, followed by the Tukey Test. Some of the students continued afterwards the work, which resulted in a publication in a Portuguese journal on the occupational health.

Prob1.4 - What is the best route? – There are multiple industrial scenarios where it is necessary a collaborator to collect items/products from different locations in an warehouse, to meet customer orders. An example is the challenge "Supermarket Order Picking" proposed at the 92nd European Study Group with Industry, and consisted in determining the best routes for a person to collect items from a supermarket bought by the client on an on-line service [9]. The students' challenge was to consider the problem of collecting items in a warehouse, or store, such that the workers took the smaller time as possible. To answer the problem, the students used concepts and technics from Graph Theory, making reference to the well-known Travelling Salesman Problem and used the Scilab software, used in the Discrete Mathematics (DM) subject, as well as C programming language, used in other subjects of the degree. The different groups developed and adapted algorithms in order to find a solution.

Prob2.1 - Fitting Carreau-Yasuda fits model in experimental data - Extrusion technology allows the production of a variety of geometry components widely used worldwide in traffic transportation, civil architecture, aerospace and/or automobile [10]. These industries use materials to manufacture different products, in various shapes, sizes and colors. Aluminum extrusion is a very complex process which requires proper process parameters to be used to get optimum settings to produce a high quality product [10], [11]. A dataset containing the dynamics results $|\eta^*|$ - complex viscosity of the material extruded, under different operating conditions was given to the students. The challenge was to find mathematical models for that to describe the behavior of material and interpret the results. The Carreau-Yasuda was presented as a case but the students should investigate further models and

present results for two or three different models. The students who were attending Computational Mathematics II could complete their work with a report and this could be used in the evaluation. However, this was not mandatory. From the 52 students enrolled on this subject, 38 delivered the report concerning the workshop. The mean value of the report evaluation was approximately 13.8 values (over 20) with a standard deviation of approximately 4.3 values, while the results of the continuous evaluation tests' mean value was 8.8.

For the second edition, from the 79 participants, only 27% answered a questionnaire concerning the global appreciation of the workshop. The items were composed by a lickert scale, ranging from 1 to 4 where 1 was "totally disagree" and 4 was "totally agree". Sixty two percent of the participants classified as 3 or 4 the question: "The event corresponded to my expectations", while 67% classify as 3 or 4 the question: "The event added value to my professional future?"

3 CHALLENGES AND OPPORTUNITIES OF THE IMPLEMENTATION OF THE PROGRAM

This initiative was firstly motivated by the need to develop students' awareness of the application of Mathematics in solving industrial problems, as a way to increase their motivation and engagement in their study fields. At the same time, the program aimed to train and develop problem solving, teamwork, oral and written communication. These are relevant skills for professionals in the industrial and business fields, which have usually deserved less attention in traditional pedagogic methods. The students were free to select the work groups' elements (from three to six elements each) as well as the problem to solve. In several cases the team-work developed during the workshop could be used as part of the evaluation Mathematics subjects, through report elaborated by students where they described the work that was developed by the group, exposed the used methodologies and the results obtained. This allowed to stimulate the reflection process about their own learning during the modelling day. Also, students reported to have increased their motivation levels regarding the subjects.

Since the first edition, the students showed a good receptivity to this workshop, which was demonstrated by a great enthusiasm in the closing of the initiative. They considered the event very challenging and expressed the experience of public presentation at the end of the day as very positive. As negative aspects, they reported the feeling of pressure and the short time to solve the problems. The organization is currently discussing the extension of the experience over time, moving from a one-day experience to several days throughout the semester.

To determine which problems are more appropriate for different students' levels, in order to be sufficiently challenging but simultaneously not overwhelming, continues to be a challenge for the organizers. Also, there is the need to develop adequate instruments that could help on identifying and evaluating the impact of this event in a more systematic and formal way, in order to stimulate the continuous improvement of this experience.

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