A Multi-objective Approach to the Optimization of Home Care Visits Scheduling

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Abstract: Due to the increasing of life expectancy in the developed countries, the demand for home health care services is growing dramatically. Usually, home services are planned manually and lead to various optimization problems in their activities. In this sense, health units are confronted with appropriate scheduling which may contain multiple, often conflicting, objectives such as minimizing the costs related to the traveling distance while minimizing the traveling time. In order to analyze and discuss different trade-offs between these objectives, it is proposed a multi-objective approach to home health care scheduling in which the problem is solved using the Tchebycheff method and a Genetic algorithm. Different alternative solutions are presented to the decision maker that taking into account his/her preferences chooses the appropriate solution. A problem with real data from a home health care service is solved. The results highlight the importance of a multi-objective approach to optimize and support decision making in home health care services. Moreover, this approach provides efficient and good solutions in a reasonable time.

1 INTRODUCTION

Over the last decade, the increased life expectancy resulted in increased demand for Home Health Care (HHC) (Fikar and Hirsch, 2017). In this sense, the number of people who needs home care services are consequently growing worldwide (Nickel et al., 2012). Therefore, health units that provide home care services need to optimize their activities in order to meet the constantly increasing demand for home care services (Koeleman et al., 2012). This is usually due to the manual planning of home care visits, making it a time and effort consuming task that often leads to inefficient solutions. Thus, the operation management within the HHC turns to be very hard because of the high number and diversity of human and material resources that participate in the process of home care and the criteria to be optimized (Benzarti et al., 2013). Regarding the literature in this field, there are some issues relevant to the home care visits schedule. Some authors present an overview of works related to HHC (Benzarti et al., 2010). The work of Yalçındag et al. (2011) besides presenting the particular scheduling of visits also lists the well-known routing problems in HHC. The problems that address static multi-vehicle routing and assignment are a very well-known and researched topic in Operations Research (Baker and Ayechew, 2003; Toth and Vigo, 2014). In the context of HHC, these problems consist in scheduling vehicles with nurses to patients (Bredström and Rönqvist, 2008), in order to determine the best order and time that visits should be performed. In this subject, some works propose an approach to find the best schedule, minimizing the costs (Nguyen and Montemanni, 2013). On the other hand, some problems were modeled to be solved by the “branch and price” algorithm (Rasmussen et al., 2012). Recently, Alves et al. (2018) proposed an approach using the genetic algorithm to solve the scheduling of home visits as a single objective optimization problem. The work of Allaoua et al. (2013) also presents the home care scheduling problem as a single objective by minimizing the travel cost or the travel time.

In this context, it is important to mention that new research in HHC is being targeted with multi-objective optimization approaches (Ombuki et al., 2006; Pasia et al., 2007). For example, Braekers et al. (2016) presents an approach that analyzes the trade-off between costs and client inconvenience, using a meta-heuristic algorithm based on multi-directional
local search. Thus, multi-objective solution procedures derive a set of Pareto optimal solutions that are not yet common in HHC scheduling (Braekers et al., 2016; Gayraud et al., 2013).

In this work, the home health care scheduling problem is formulated as a multi-objective problem and an instance with real data is solved. Conflicting objectives, such as costs, distances or unexpected and inconvenient events are important to be considered. The developed approach and model try to handle the objectives of home care services and the patient’s objectives. To achieve this goal, additional costs related to the travelling time and the travelling distance as well as the waiting time are minimized. The obtained solutions assist the decision making process in terms of these objectives. As a result of using the Tchebycheff method and a Genetic algorithm, multiple alternatives are presented to decision makers that can inspect benefits and trade-offs to derive daily schedules based on individual or mutual preferences.

The paper is organized as follows. In Section 2, the assumptions and the mathematical formulation of the problem are presented. The multi-objective optimization approach is described in Section 3. Section 4 presents the results for a case study based on real data. In this section, the results are also analyzed and discussed. Finally, some conclusions and future work are drawn in Section 5.

2 PROBLEM DEFINITION

The problem arises when it is necessary to overcome the difficulties that HHC services entail, such as the vehicles delay that consequently cause bad schedules with no viable replacement options and consequently increase the expenses of Health Unit.

However, it is important to define the general characteristics of the problem, such as the number and characterization of health professionals, the number of vehicles available, the number of patients and treatments they need, locations that can be traveled and their distances. This information and data allow to create a mathematical formulation of the problem, as an attempt to reduce the time spent on visits and consequently the reduction of costs.

2.1 Assumptions

The vehicle scheduling carries out home care visits in order to perform the necessary treatments for patients belonging to a Health Unit. This problem was modeled, considering the number of vehicles involved in the teams and the patients who request this type of health care. Thus, considering a Health Unit in Bragança, with a domiciliary team that provides home care to patients that require different types of treatments, all the entities involved in the problem were identified.

Consider the following information:

- the trips duration and distances between the different locations;
- the time of travel, in the same location, to visit different patients;
- the distance, in the same location, to visit different patients;
- the list and duration of treatments are known for each patient (defined and provided by the Health Unit);
- the number of patients who need health care, and who are assigned to days of home visits, are known in advance;
- the number of vehicles available;
- all visits begin and end at the Health Unit.

The mathematical formulation considers the following sets:

- \( P \) is the set of patients, \( P = \{p_1, \ldots, p_{np}\} \);
- \( V \) is the set of vehicles, \( V = \{v_1, \ldots, v_{nv}\} \);
- \( L \) is the set of locations, \( L = \{l_1, \ldots, l_{nl}\} \);
- \( T \) is the set of treatments, \( T = \{t_1, \ldots, t_{nt}\} \);
- \( R_i \), where \( i \in \{1, \ldots, nt\} \), is the set of patients that receive the treatment \( i \), \( R_i \subset P \);
- \( Q_i \), where \( i \in \{1, \ldots, nl\} \), is the set of locations for home care visits, \( Q_i \subset P \);

The parameters of the model are:

- \( D_{nl \times nl} \) is the matrix of distances between the locations (the diagonal indicate the distance required to visit different patients in the same location);
- \( H_{nl \times nl} \) is the matrix of time required to travel between the locations (the diagonal indicate the time required to visit different patients in the same location);
- \( Q_{nl \times 1} \) is the vector of times required to apply each of the treatments to patients.

Taking into account the information regarding these sets and parameters, a vehicle scheduling optimization problem can be formulated to reduce the time spent on visits and the costs.
2.2 Mathematical Formulation

The goal is to find the vehicle scheduling solution that minimizes, simultaneously, the time spent on visits and costs. A solution for this problem can be expressed by the vector \( x \) of dimension \( n = 2 \times np \) with the following structure:

\[
x = (y, z) = (y_1, \ldots, y_{np}, z_1, \ldots, z_{np})
\]

where the patient \( y_i \in P \) will be visited by the vehicle \( z_i \in V \), for \( i = 1, \ldots, np \). In this vector, \( y_i \neq y_j \) for \( \forall i \neq j \) with \( i = 1, \ldots, np, j = 1, \ldots, np \). Therefore, for a given \( x \) it is possible to define the vehicles scheduling taking into account the order of the components of \( x \). For instance, consider a scheduling problem with \( np = 5 \) patients to be visited by \( m = 3 \) vehicles. For \( P = \{p_1, p_2, p_3, p_4, p_5\} \) and \( V = \{v_1, v_2, v_3\} \), the vector \( x = (4, 3, 1, 2, 5, 2, 3, 2, 1, 1) \) means that patients \( p_3, p_5, p_1, p_2 \) and \( p_4 \) will be visited by vehicles \( v_2, v_3, v_2, v_1 \), respectively. Thus, vehicle \( v_1 \) will visit patients \( p_2 \) and \( p_5 \); vehicle \( v_2 \) will visit patients \( p_4 \) and \( p_1 \); and vehicle \( v_3 \) will visit patient \( p_3 \).

For a vehicle schedule \( x \), the functions \( t^l(x) \) and \( d^l(x) \), for \( l = 1, \ldots, m \), give the total time and total distance required to perform all visits of the vehicle \( l \), respectively. The two objective functions are defined as

\[
f_1(x) = \max_{l=1,..,m} t^l(x) \tag{1}
\]

\[
f_2(x) = \max_{l=1,..,m} d^l(x) \tag{2}
\]

which represent, respectively, the maximum time and maximum distance spent by all the vehicles to carry out all the visits. Then, the optimization problem can be defined as:

\[
\min_{x \in \Omega} \{f_1(x), f_2(x)\} \tag{3}
\]

where \( x \in \Omega \) is the decision variable space and \( \Omega = \{ (y, z) : y \in P^p, z \in V^p \} \) is the feasible set.

3 MULTI-OBJECTIVE APPROACH

The vehicle scheduling problem formulated previously is a multi-objective problem, since the objectives conflict each other. The goal is to find the set of feasible trade-off solutions \( x \) that simultaneously minimize \( f_1(x) \) and \( f_2(x) \) in order to facilitate the decision maker select the solution according to his/her preferences.

3.1 Multi-objective Optimization

When several conflicting objectives are optimized at the same time, the search space becomes partially ordered. In such scenario, solutions are compared on the basis of the Pareto dominance. Without loss of generality, consider a multi-objective optimization problem where \( m \) objectives are to be minimized, for two solutions \( a \) and \( b \) from the feasible set \( \Omega \), a solution \( a \) is said to dominate a solution \( b \) (denoted by \( a < b \)) if:

\[
\forall i \in \{1, \ldots, m\} : f_i(a) \leq f_i(b) \wedge \exists j \in \{1, \ldots, m\} : f_j(a) < f_j(b). \tag{4}
\]

Since solutions are compared against different objectives, there is no longer a single optimal solution but a set of optimal solutions, generally known as the Pareto optimal set. This set contains equally important solutions representing different trade-offs between the given objectives and can be defined as:

\[
\mathcal{PS} = \{ x \in \Omega : \nexists y \in \Omega : y < x \}. \tag{5}
\]

The images of the solutions of the Pareto optimal set define a Pareto front in the objective space. This Pareto front allow to identify the trade-offs between solutions and therefore facilitate the decision making.

3.2 Tchebycheff Scalarization Method

Approximating the Pareto optimal set is the main goal of a multi-objective optimization algorithm. The scalarization methods can be used to obtain an approximation to the Pareto-optimal set (Miettinen, 2012) such as:

- the weighted sum method, where the weighted linear combination of the objectives is minimized;
- the \( \varepsilon \)-constraint method, in which one of the objectives is minimized and the others are included as constraints;
- the minimization of a weighted distance to a reference point, e.g., the weighted Tchebycheff method.

When using these scalarization methods, the multi-objective problem is transformed into a single objective optimization problem that have to be solved using single objective optimization algorithms. There must be some care in choosing the scalarization method since the weighted sum method does not allow to achieve solutions belonging to non-convex regions of the Pareto front. On the other hand, in the \( \varepsilon \)-constraint method is difficult to properly fix the limits of the constrained objectives. Since the vehicle scheduling problem is a non-continuous and
non-convex problem, then the weighted Tchebycheff method was adopted to solve this problem. The main goal is to approximate the optimal Pareto solutions that represent different trade-offs between the objectives. In the weighted Tchebycheff method the $L_\infty$ norm (i.e., minimization of the maximum difference) is used. This distance is computed to a reference point (or aspiration levels). The weighting coefficients allow to obtain different trade-offs and lead with different scaling factors of each objective. In this method, the scalarized function is a weighted function based on the Tchebycheff metric, to the reference point. The weighted problem of Tchebycheff is given as follows (Miettinen, 2012):

$$\min \max \{w_i | f_i(x) - z_i^\star\}$$  \hfill (6)$$

where $w_i$ are the weighting coefficients for objective $i$, $z_i^\star$ are the components of a reference point. An approximation to the ideal vector can be used as reference point $z^\star = (z_1^\star, \ldots, z_m^\star) = (\min f_1, \ldots, \min f_m)$. This approach can be implemented as an a priori technique in which the reference point and the weights are defined by the decision maker before the search. There are some drawbacks in this approach, namely, the difficulty to define properly weights and aspiration levels that reflect the decision maker preferences. Other perspective is to implement an a posteriori technique in which the decision making process takes place after the search. In this case, the reference point is defined as the ideal solution and the weights are uniformly varied. In this manner, after the search, a set of Pareto optimal solutions is presented as alternatives and the decision maker can identify the compromises and choose according to his/her preferences. Thus, due to its ability to solve this kind of problems, a Genetic algorithm was assumed.

### 3.3 Genetic Algorithm

A Genetic algorithm (GA) (Holland, 1992) is used to solve each single objective optimization problem that results of the weighted Tchebycheff method. GA is inspired by the natural biological evolution, uses a population of individuals and new individuals are generated by applying the genetic operators of crossover and mutation.

Genetic algorithm are particularly well suited to solve vehicle scheduling problem that is non-convex with discrete decision variable and couple with the combinatorial nature of the search space. The flexibility of representation of the solutions and genetic operators allow to handle hard constraints.

The GA used in this work is summarized in Algorithm 1. Initially, a population of $N_{\text{pop}}$ individuals is randomly generated. Each individual in the population is a vector of decision variables $x$. Afterwards, each generation, crossover and mutation operators are applied to generate new solutions. These genetic operators were designed in order to guarantee that new solutions are feasible.

The best individuals in the population have a high probability of being selected to generate new ones by crossover and mutation. Therefore, the good features of the individuals tend to be inherited by the offspring. In this manner, population converges towards better solutions (Ghaheri et al., 2015). The iterative procedure terminates after a maximum number of iterations ($NI$) or a maximum number of function evaluations ($NFE$).

![Algorithm 1: Genetic Algorithm](image)

#### 3.4 Multi-objective Optimization Framework

Figure 1 describes and illustrates the entire framework of this multi-objective approach.  

![Figure 1: Multi-objective optimization framework](image)
of weights allow to define an approximation to the Pareto optimal front. The dominance relation is used to filter the non-dominated solutions.

Based on the dominance concept, it is possible to determine the non-dominated solution of a given set of solutions. Among a set of solutions \( S \), the non-dominated set of solutions are those that are not dominated by any members of the set \( S \). This non-dominated set is the best approximation to the Pareto optimal set.

Therefore, after obtaining all the solutions, it is necessary to create a routine, which with efficient computational procedures, allows obtaining the set of non-dominated solutions. In the literature, there are different procedures, such as “Naive and Slow”, or “Continuously Updated”, which allow to compare and identify the set of solutions (Deb, 2014). Regarding the “Naive and Slow” approach, each solution \( i \) is compared to all population solutions in order to test the dominance by any of the other solutions. If the solution \( i \) is dominated by any solution, it means that there is at least one solution in the population that is better than \( i \) in all objectives. Hence the solution \( i \) can not belong to the non-dominated set. In this way, a “flag” is marked against the solution \( i \) to denote that it does not belong to the non-dominated set. However, if no solution is found to dominate the solution \( i \), it becomes a member of the non-dominated set. Thus, with this procedure, any other solution in the population can be checked for future analysis.

This approach can be presented step-by-step to find the set not dominated in a given set \( S \) of size \( N \) (Deb, 2014):

- **Step 1** - Define the solution counter \( i = 1 \) and create an empty non-dominated set \( S' \).
- **Step 2** - For solution \( j \in S \) (but \( j \neq i \)), check that solution \( j \) dominates solution \( i \). If yes, proceed to step 4.
- **Step 3** - If more solutions are left in \( S \), increment \( S' \) by one and proceed to step 2; Otherwise, set \( S' = S' \cup i \).
- **Step 4** - Increase \( i \) by one. If \( i \leq N \), go to step 2; Otherwise, stop and declare \( S' \) as the set non-dominated.

Thus, a routine similar to the previous one was implemented, but with a small improvement in the classification scheme. That is, each solution is compared to all members of the set \( S' \), one by one. If the solution \( i \) dominates any member of \( S' \), then this solution is removed from \( S' \). In this way, non-members of non-dominated solutions are deleted from \( S' \). Otherwise, if solution \( i \) is dominated by any member of \( S' \), solution \( i \) is ignored. However, if the solution \( i \) is not dominated by any member of \( S' \), it is inserted in \( S' \) (similar to the continuously updated procedure). It is in this way that the set \( S' \) grows with non-dominated solutions, where the other members of \( S' \) are thus the non-dominated set.

This approach has made possible to improve the efficiency and performance of the implemented routine as it allows the dominated solutions to be removed quickly.

## 4 NUMERICAL RESULTS

In this section, the multi-objective optimization framework is applied to a case study.

### 4.1 Case Study

The case under study is related to a day of the month of April in the year 2017. On that day the Health Unit of Bragança had five cars available (\( nv = 5 \)) to perform the home care visits to 22 patients (\( np = 22 \)). Each patient required a certain treatment of a total of five treatments (\( nt = 5 \)). Table 1 identifies which treatments each patient needs (\( R_i \) for \( i = 1, \ldots, nt \)).

<table>
<thead>
<tr>
<th>Patients</th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
<th>Treatment 4</th>
<th>Treatment 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1, 2, 3, 6, 7, 8, 9, 17, 21, 22</td>
<td>4, 5</td>
<td>13, 14</td>
<td>11, 12, 15, 16, 18, 19, 20</td>
<td>10</td>
</tr>
</tbody>
</table>

The treatments needed by the patients have different care and average times (\( O_i \) for \( i = 1, \ldots, nt \)), as can be seen in the Table 2.

<table>
<thead>
<tr>
<th>Patients</th>
<th>Average Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment 1 (Curative)</td>
<td>30</td>
</tr>
<tr>
<td>Treatment 2 (Surveillance and Rehabilitation)</td>
<td>60</td>
</tr>
<tr>
<td>Treatment 3 (Curative and Surveillance)</td>
<td>75</td>
</tr>
<tr>
<td>Treatment 4 (Surveillance)</td>
<td>60</td>
</tr>
<tr>
<td>Treatment 5 (General)</td>
<td>60</td>
</tr>
</tbody>
</table>

On the other hand, the patients are dispersed in nine different locations (\( nl = 9 \)) in the region of Bragança. Regarding the places of domiciliary visit, Table 3 shows the locations and cities (\( Q_i \) for \( i = 1, \ldots, nl \)) (inserted with abbreviation for confidential data protection) of each of the patients.
For this case study, simulations were carried out on a PC Intel(R) Core(TM) i7 CPU 2.2 GHz with 6.0GB of RAM. The optimization framework was implemented in MatLab® (MATLAB, 2015).

The values of the control parameters used in GA for this problem were tuned after preliminary experiments. A population size of 30 individuals (N\text{pop} = 30) and a probability rate of 50% for crossover and mutation procedures was used. The stopping criterion was based on the maximum number of iterations defined as 100 (N\text{I} = 100) or the maximum number of function evaluations of 5000 (N\text{FE} = 5000). Since GA is a stochastic algorithm, 10 runs were carried out with random initial populations.

The value of the reference point was an approximation to the ideal vector and the weights for the objectives varied in the set \((w_1, w_2) \in \{(1, 0), (0.75, 0.25), (0.5, 0.5), (0.25, 0.75), (0, 1)\}\). For each of the weight vectors, the execution in MatLab® (MATLAB, 2015) was performed.

The multi-objective optimization framework allowed to identify a set of non-dominated solutions, crucial to provide to the decision maker through the visualization of solutions in the objective space. Figure 2 shows the the set of Pareto optimal solutions, i.e. the non-dominated solutions (represented by a red circle), as well as the dominated solutions (represented by a blue circle). The position of these solutions express the trade-offs between the objectives (time in minutes in the x axis and distance in kilometers in the y axis). It can be seen from Figure 2 that five non-dominated solutions were found, representing the set of most interesting solutions. Therefore, these are the solutions that must be taken into account, because they are optimal in terms of the objectives, and each solution exhibits the trade-off that
should be explored. However, it is also important to analyze the solution in the decision variable space. In order to illustrate the solutions found in the decision space, Gantt charts are used. Thus, the Gantt chart aims to represent the vehicle schedule of a solution. In this sense, three different vehicle schedules are presented for home visits, referring to three non-dominated solutions. The three selected non-dominated solutions refer to the extreme points in terms of each of the objectives and the closest solution to the ideal vector. First, the non-dominated solution related to the best solution in terms of minimizing the distance traveled in kilometers (one extreme of Pareto front) is presented in Figure 3.

In conclusion, the analysis of viable alternatives gives new valuable information to the decision maker. Moreover, the multi-objective approach provides alternative optimal solutions that are essential to support the decision maker to choose adequate schedules for home visits in a Health Unit.

5 CONCLUSIONS

Home visits at the Health Unit are usually planned manually and without any computational support, which implies that, in addition to being a complex and time-consuming process, the solution obtained may not be the best. Thus, in an attempt to optimize the process it is necessary to use strategies that allow to minimize certain objectives, without, however, worsening the quality of services provided and, always, looking for the best optimization of the scheduling. Thus, a multi-objective optimization model was developed to simultaneously minimize two conflicting objectives: the time spent and the distance traveled (which consequently affects the costs). The multi-objective optimization problem is scalarized...
applying the Tchebycheff method and solved by a Genetic algorithm. Different compromise solutions are obtained. An efficient and fast routine to compute the non-dominated solutions is implemented. This decision support system was applied to a case study with real data.

The optimal alternatives found were analyzed both in terms of objective functions and decision variables values. For the decision maker the extreme and “elbow” solutions can be particularly interesting and therefore may be carefully investigated. In addition, the approach allows the possibility of replicating the problem with different instances, without incurring additional costs or deficiencies in the service, thus continuing to be able to serve as a support system, which today does not yet exist.

As future work perspectives, it is intended to improve the efficiency of the the optimization algorithm, as well as the model by the inclusion of new objectives and constraints.

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