STATISTICAL ANALYSIS OF COMPRESSIVE STRENGTH OF CONCRETE SPECIMENS

B. Aguiar*, V. Veiga and P. Oliveira
University of Minho, Portugal
Fax: +351 253 510 217
E-mail: aguiar@civil.uminho.pt

ABSTRACT

This paper presents the statistical analysis of compressive strength of concrete specimens delivered in a laboratory of Northern of Portugal. Three types of concretes were analyzed defined as C20, C25 and C30. In the study we used the results of three years, 1995 to 1997. The one-sample Kolmogorov-Smirnov Test was used to verify the normal distribution of the strength results. The results presented in this paper are important for predicting the performance of structural elements and for calculating resistance factors for limit state design codes. As concrete is made in majority on site, conformity is more difficult than for other construction materials produced on factories. For the first analysis the concrete applied on the different sites were separated in groups. A total of 82 groups of concrete produced in 20 sites, were analyzed. We verified that in about 32% of the groups the characteristic compressive strength established for the design was not achieved. For the second analysis the total results were considered, without separation by site or by group. When the exigency was classes C20 or C25, the characteristic compressive strength was below the exigency. On the contrary, for the concrete mentioned as C30, the characteristic compressive strength was above the exigency.

KEYWORDS: Concrete, Quality, Statistical analysis, Compressive strength.

* Author to whom correspondence to be addressed

1. INTRODUCTION

Reliability based methods have been utilized successfully in the development of design codes for reinforced concrete [1-4]. To determine safety factors for limit state design codes it is necessary the knowledge about the statistical characteristics of load and strength parameters and their probability distributions is necessary.

The statistical analysis of the compressive strength of three types of concretes, C20, C25 and C30, is presented. The normal distribution of the compressive strength is verified by the Kolmogorov-Smirnov test. The analysis developed is important for evaluating the variation in the performance of structural elements and determining their reliability levels. It is also important for calculating the safety factors for various limiting states.

2. CONCRETE SAMPLING

For this study the results of compression tests of 150x150x150 mm made at 28 days were used. Since 1995 the determination of the conformity of the concrete in Portugal follows the European standard ENV 206 [5]. First, the data was divided into groups. It is known that the sites were small or medium. Thus, was not possible to produce in a week more than 450 m^3 of concrete. In that case the group is the concrete produced in a week [5].

For the three years period of study, 1995 to 1997, data from 20 sites were collected. At least 6 samples for each group were needed, except for the concretes not higher than C25. For these concretes at least 3 samples were needed. The verification of the conformity of the groups followed the criteria defined by ENV 206 [5]. The first criterion is applied when 6 or more samples exist. The resistance, in MPa, should respect the following conditions:

$$\overline{X}_n \geq f_{ck} + \lambda \; S_n \qquad [1]$$

$$X_{min} \geq f_{ck} - k \qquad [2]$$

where:

X_{min} is the smaller individual strength value of the samples;
$\overline{X}_n$ is the average of the strength of the samples;
S_n is the standard deviation of the strength of the samples;
f_{ck} is the characteristic strength specified for concrete;
λ and k are values listed in Table [1] in accordance with the number of samples (n).

Table [1]: Values of λ and k

N	λ	K
6	1.87	3
7	1.77	3
8	1.72	3
9	1.67	3
10	1.62	4
11	1.58	4
12	1.55	4
13	1.52	4
14	1.50	4
15	1.48	4

The second criterion is applied when the conformity is verified with three samples with the strength X_1, X_2 and X_3. The strength, in MPa, should respect the following conditions:

$$\overline{X}_3 \geq f_{ck} + 5 \qquad [3]$$

$$X_{min} \geq f_{ck} - 1 \qquad [4]$$

where:

$\overline{X}_3$ is the average strength of the three samples.

The European standards do not provide a procedure when 4 or 5 samples exist. In that case the second criterion was considered.

3. RESULTS

In 20 sites, data collected during 69 weeks were analyzed, where different types of concretes were produced. The number of groups where the characteristic strength of concrete specified for the site was not achieved were determined. 26 groups in these conditions were found. So, we have about 32 % of groups where the exigency was not achieved. The situation for the different types of concretes is described in Table [2].

Table 2: Non conformable groups for different concrete types.

Concrete Type	Total of groups	Non conformable groups	
		Number	%
C20	2	1	50
C25	51	23	45
C30	29	2	7

The following observations can be made:

a) the percentage of non conformable groups decreases when the concrete strength increase;

b) the number of groups analyzed for the C20 was not enough to draw conclusions.

When the characteristic strength of concrete specified for the site was not achieved the safety of the concrete structure is not guaranteed. The lack of safety increase with the increase of the difference between the specified and the achieved characteristic strength of concrete. The analysis presented in Table [2] does not clarify this aspect. In order to evaluate the lack of safety a statistical analysis of the data was made. The results of 37 specimens mentioned as C20, 430 mentioned as C25 and 333 mentioned as C30 were used.

The validity of the model was checked using the Kolmogorov-Smirnov goodness of fit test at the 5 % significance level [6]. The statistical parameter D was calculated using a software. D is the largest of the absolute values of the n differences between the hypothesized function $F_X(X^{(i)})$ and the observed cumulative histogram i/n:

$$D = \max{}_{i=1}^{n}\left[\left|\frac{i}{n} - F_X(X^{(i)})\right|\right] \qquad [5]$$

For the three types of concretes listed in Table [3], the following observations can be made:

a) the concretes types C20 and C25 show a normal distribution, whereas that of C30 has a log-normal distribution. From the results of Kolmogorov-Smirnov tests D is less than the critical value D_c, which indicates that the models of the variation in strength listed in Table

[3] are acceptable at the 95% confidence level. We also test the normal distribution for the C30 and D higher than 0.074 was found.

Table [3]: Statistical analysis of three types of concretes

Concrete type	$P_{5\%}$	V_C(%)	KS test		Distribution type
			D	D_c	
C20	17	24	0.123	0.224	Normal
C25	17	27	0.043	0.066	Normal
C30	31	18	0.049	0.074	Log-normal

b) the coefficients of variation V_c for concretes types C20 and C25 are higher than 20 %. For the concrete type C30 the coefficient of variation is about 18 %.
c) the fifth percentiles of the distribution functions are smaller than the wanted strengths for the concretes types C20 and C25. For the type C30 the five percentile is higher than the wanted strength which indicates that this concrete is of good quality.

Concrete strength is considered to follow a normal distribution [7,8]. The results obtained in the present study follows a normal distribution except for the concrete type C30, where a log-normal distribution was observed. This could be explained by the fact that specimens from different sites were analyzed, the execution of the cubes was not controlled and the majority of the concrete was produced on the sites. This could also explain the high values of the coefficients of variation. The lowest value of the coefficient of variation was for the concrete type C30, about 18 %.

4. CONCLUSIONS

This study presents the statistical analysis of the results in two ways. First, the analysis was made to each site considering different groups. The percentage of non conformable groups decreases when the characteristic strength increases. For the concrete type C20 we have 50 % of non conformable groups and for the concrete type C30 we have only 7 %. The second analysis was made considering the total results for each type of concrete. Only for the concrete type C30 was five percentile higher than the exigency. For concretes types C20 and C25 we have strengths 17 % and 32% below the exigency were observed, respectively.

The global analysis could dissimulate some problems in the concrete structures. For example for the concrete C30, the global analysis showed that the concrete had good quality, but the group analysis showed 7 % of non conformable groups. The change of concrete quality within the structure should be considered.

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