Exercise composition: from environment properties to composed problems

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Abstract It is well known that (1) problems have a structure, (2) often this structure comes from its input, output or internal objects, (3) this structure organizes the problem resolution. Decomposition of a problem by its structure is a typical path in problem resolution.
In this article we will do the inverse: composition of simple problems using structure-builder operators.
We will start by (1) discuss problem structures, (2) select a set of important structure-builder operators, (3) study their properties, (4) apply them to a simple math case-study.

1 Introduction

Problems have a structure, usually connected with its input, output or internal objects.

With different names and flavors, problem decomposition was recognized as a crucial step in mathematics, physics and philosophy since the ancient times.

One incredible 4000 years old Egyptian papyrus[1] claimed that (many) problems can be seen as composition of "chip's part problems", "aha problems"[2] or "Pefsu problems"[3]. Greeks discussed problem composition in very different domains. Descartes included this subject in the second principle of *Discourse on the Method*.

Today it continues to be a rather challenging task in a wide number of areas.

The study of the problem structures plays a central role in mathematics and computer science areas [3,7].

Some problems can be described as (1) pairs, triples, tuples of problems – (p1,p2,p3), (2) functional composition – p3(p2(p1())), (3) systems of equation problems – (union of constraints), (4) inductive problems – (sequences, trees), and that internal structure can be used to decompose them in smaller problems. We will discuss some common problem structures in next section.

[1] See Wikipedia, *Moscow Mathematical Papyrus*
[2] Aha stands for a variable – x in modern math notation.
[3] A *pefsu* measures the strength of the beer made from a *heqat* of grain.

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Problem structures and exercise generation –
Studying the problem structure is a very important subject because: (1) it helps in problem resolution, (*a*) we can decompose the problem – divide and conquer approaches, (*b*) we are able to transform the problem into a description where reusable patterns are easier to see; (2) it helps in problem understanding.

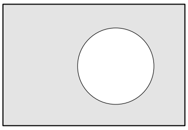
In a complementary way, problem structure can be used to assist in problem composition and exercise generation[1,9]. Beside the main problem text, in the area of exercise generation it is necessary to produce the exercise resolution, the suggestions, the results, the verification functions [5], metadata, etc. Problem decomposition and problem structure may guide in the construction of the exercise components. For example, sub-problems: (1) can be used as sub-questions; (2) may organize the generation of resolutions; (3) are crucial in the generation of suggestions; (4) can be used to organize problem recommendation.

Problem structure and didactic strategy –
Problem structure may also be used to systematically present different didactic choices to apply to different kinds of students or different situations. Example:

- ask for sub-problems before asking for the full problem,
- ask for the full problem and suggest sub-problems if necessary,
- incremental strategy.

A simple example –
The following example is presented to briefly illustrates some of the notions previously presented.

Problem 1: Consider figure bellow. Calculate the area of a rectangle measuring $5\,m$ by $10\,m$ with a circular hole with diameter $3\,m$.



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prop1 :: area = area(rect) - area(circ)
prop2 :: area(rect) = 5 × 10 m
prop3 :: area(circ) = π × (3/2)
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$$p1 = \boxed{prop2 :: a, b \rightarrow area(rect)}$$

$$p2 = \boxed{prop3 :: diameter \rightarrow area(circ)}$$

$$p3 = \boxed{prop1 :: area(rect), area(circ) \rightarrow area(gray)}$$

$$p4 = \boxed{p3 \circ (p1, p2) :: a, b, diameter \rightarrow area(gray)}$$

Problem structure: p1(p2,p3).
Sub-problem order: (p2 before p1); (p3 before p1).
Suggestions that can be provided to students:

1. tips to help in the problem decomposition: "start by calculate area(rect)" or "start by calculate area(circ)";