Exercise composition: from environment properties to composed problems

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Abstract It is well known that (1) problems have a structure, (2) often this structure comes from its input, output or internal objects, (3) this structure organizes the problem resolution. Decomposition of a problem by its structure is a typical path in problem resolution.
In this article we will do the inverse: composition of simple problems using structure-builder operators.
We will start by (1) discuss problem structures, (2) select a set of important structure-builder operators, (3) study their properties, (4) apply them to a simple math case-study.

1 Introduction

Problems have a structure, usually connected with its input, output or internal objects.

With different names and flavors, problem decomposition was recognized as a crucial step in mathematics, physics and philosophy since the ancient times.

One incredible 4000 years old Egyptian papyrus¹ claimed that (many) problems can be seen as composition of “chip’s part problems”, “aha problems”² or “Pefsu problems”³. Greeks discussed problem composition in very different domains. Descartes included this subject in the second principle of Discourse on the Method.

Today it continues to be a rather challenging task in a wide number of areas.

The study of the problem structures plays a central role in mathematics and computer science areas [3,7].

Some problems can be described as (1) pairs, triples, tuples of problems – (p1,p2,p3), (2) functional composition – p3(p2(p1())), (3) systems of equation problems – (union of constraints), (4) inductive problems – (sequences, trees), and that internal structure can be used to decompose them in smaller problems. We will discuss some common problem structures in next section.

¹ See Wikipedia, Moscow Mathematical Papyrus
² Aha stands for a variable – x in modern math notation.
³ A pefsu measures the strength of the beer made from a heqat of grain.
Problem structures and exercise generation –

Studying the problem structure is a very important subject because: (1) it helps in problem resolution, (a) we can decompose the problem – divide and conquer approaches, (b) we are able to transform the problem into a description where reusable patterns are easier to see; (2) it helps in problem understanding.

In a complementary way, problem structure can be used to assist in problem composition and exercise generation[1,9]. Beside the main problem text, in the area of exercise generation it is necessary to produce the exercise resolution, the suggestions, the results, the verification functions [5], metadata, etc. Problem decomposition and problem structure may guide in the construction of the exercise components. For example, sub-problems: (1) can be used as sub-questions; (2) may organize the generation of resolutions; (3) are crucial in the generation of suggestions; (4) can be used to organize problem recommendation.

Problem structure and didactic strategy –

Problem structure may also be used to systematically present different didactic choices to apply to different kinds of students or different situations. Example:

– ask for sub-problems before asking for the full problem,
– ask for the full problem and suggest sub-problems if necessary,
– incremental strategy.

A simple example –

The following example is presented to briefly illustrates some of the notions previously presented.

Problem 1: Consider figure bellow. Calculate the area of a rectangle measuring 5 m by 10 m with a circular hole with diameter 3 m.

\[
\begin{align*}
\text{prop1} & \:: \text{area} = \text{area(rect)} - \text{area(circ)} \\
\text{prop2} & \:: \text{area(rect)} = 5 \times 10 \text{ m} \\
\text{prop3} & \:: \text{area(circ)} = \pi \times (3/2)
\end{align*}
\]

\[
\begin{align*}
p1 = & \quad \text{prop2} :: a, b \rightarrow \text{area(rect)} \\
p2 = & \quad \text{prop3} :: \text{diameter} \rightarrow \text{area(circ)} \\
p3 = & \quad \text{prop1} :: \text{area(rect)}, \text{area(circ)} \rightarrow \text{area(gray)} \\
p4 = & \quad p3 \circ (p1, p2) :: a, b, \text{diameter} \rightarrow \text{area(gray)}
\end{align*}
\]

Problem structure: p1(p2,p3).
Sub-problem order: (p2 before p1); (p3 before p1).
Suggestions that can be provided to students:

1. tips to help in the problem decomposition: “start by calculate \text{area(rect)}” or “start by calculate \text{area(circ)}”;