

A DISCRETE MACRO-MODEL USING HOMOGENIZATION WITH STRAIN-RATE DEPENDENCY FOR THE OUT-OF-PLANE STUDY OF MASONRY PANELS SUBJECTED TO IMPACT LOADING

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Abstract. *In recent decades, a great deal of effort has been made to develop solutions to reduce destructive damage and casualties due to blast loads and impacts, also in light of a major protection of the built heritage against terrorist attacks. In the present study, a simple and reliable Homogenization approach coupled with a Rigid Body and Spring Model (HRBSM) accounting for high strain rate effects is utilized to analyse masonry panels subjected to impact load.*

The homogenization approach adopted relies into a coarse FE discretization where bricks are meshed with a few elastic constant stress triangular elements and joints are reduced to interfaces with elastic-plastic softening behaviour including friction, a tension cut-off and a cap in compression. Strain rate effects are accounted for assuming the most meaningful mechanical properties in the unit cell variable through the so-called Dynamic Increase Factors (DIFs), with values from literature data. The HRBS model, which has been implemented at structural level in the commercial code ABAQUS resorts on a discretization into rigid quadrilateral elements with homogenized bending/torque non-linear springs on adjoining edges.

The model is tested on a masonry parapet subjected to a standardized impact. A number of previous results obtained by literature models are available for comparison, as well as experimental data. Satisfactory agreement is found between the present results and existing literature in the field, both experimental and numerical.

1 MESO-SCALE: HOMOGENIZED MODEL

1.1 Overview

A multi-scale approach is assumed for the study of masonry panels subjected to different load types. Such strategy lies on the periodicity feature of a given media and it is therefore a suitable strategy for masonry [17,18]. First, a micro-scale mechanical characterization on a representative volume element (hereafter, RVE) is achieved by solving a boundary value problem (BVP). Then, the macroscopic constitutive response is accomplished through the assemblage of these RVE units. The main features will be explained in what follows and, for further information of the quasi-static approach, the reader is referred to [19] and [20]. Please note that the description will be made for a running bond texture, see Fig. 1.

In brief, homogenization consists in deriving the upper-scale properties by introducing averaged quantities for macroscopic strain and stress tensors (\mathbf{E} and $\mathbf{\Sigma}$, respectively) obtained at a micro-scale on the RVE (Y , elementary cell). The main concept of the homogenization process implies that the macroscopic stress and strain tensors are calculated as Eq. (1).

$$\mathbf{E} = \langle \boldsymbol{\varepsilon} \rangle = \frac{1}{V} \int_Y \boldsymbol{\varepsilon}(\mathbf{u}) dY \quad ; \quad \mathbf{\Sigma} = \langle \boldsymbol{\sigma} \rangle = \frac{1}{V} \int_Y \boldsymbol{\sigma} dY \quad (1)$$

where $\langle * \rangle$ is the average operator, $\boldsymbol{\varepsilon}$ is the local strain value, which is directly dependent of the displacements field \mathbf{u} , $\boldsymbol{\sigma}$ is the local stress value and V is the volume of each elementary cell. The latter is governed by the Hill-Mandel principle [21,22] that establishes the energy equivalence between the macroscopic stress power with the micro-scale stress power over the volume of the RVE. All the mechanical quantities are considered as additive functions and periodicity conditions (local periodicity) are imposed on the stress field $\boldsymbol{\sigma}$ and the displacement field \mathbf{u} [23] so that:

$$\boldsymbol{\sigma} \text{ periodic on } \partial Y \text{ and } \boldsymbol{\sigma} \mathbf{n} \text{ antiperiodic on } \partial Y_1 \quad (2)$$

$$\mathbf{u}^{per} \text{ periodic on } \partial Y_1 \quad (3)$$

In the present model, the RVE is modelled as a continuum FE model, whereas joints are reduced to interfaces with zero thickness and bricks are discretized by means of a mesh constituted by plane-stress elastic triangles, Fig. 1. The formulation assumes that cracking and all non-linearity of each RVE are concentrated exclusively on interfaces between adjoining elements, both on brick and joints. The elastic domain of joints is bound by a composite yield surface that includes tension, shear and compression failure with softening (see Fig. 1). A multi-surface plasticity model is adopted, with softening, both in tension and compression. The parameters f_t and f_c are, respectively, the tensile and compressive Mode-I strength of the masonry or mortar-brick interfaces, c is the cohesion, F is the friction angle, and Y is the angle which defines the linear compression cap.

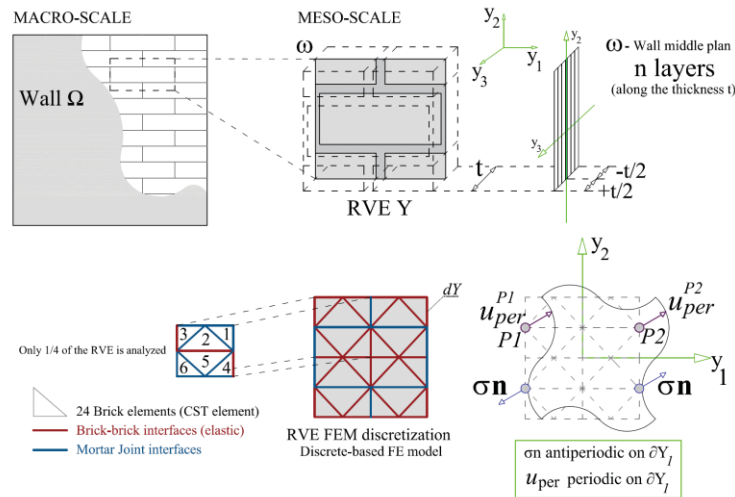


Fig. 1. Micro-mechanical model adopted for the present homogenized model and strength domain for mortar joints reduced to interfaces.

The response of the RVE under out-of-plane actions is obtained subdividing along the thickness the unit cell into several layers (40 subdivisions). A displacement driven approach is adopted, meaning that macroscopic curvature increments $\Delta\chi_{11}$, $\Delta\chi_{22}$, $\Delta\chi_{33}$ are applied through suitable periodic boundary displacement increments.

In this way, homogenized curves are approximated to define the nonlinear behaviour of the interfaces. For each interface at a structural scale, the cross-section equilibrium is iteratively calculated aiming to obtain the M- θ homogenized curves. The strategy to derive these curves is based on the macroscopic mode-I and mode-II stresses. Such assumption is plausible because masonry failure mechanisms tend to be mainly governed by joints failure due to its low tensile strength. The interface orientations accounted at a meso-scale are guided by the discrete mesh representation at a structural scale.

Quasi-static range	
Young's modulus of the brickwork composite (MPa)	20000
Young's modulus of the mortar (MPa)	1500
Young's modulus of the brick (MPa)	20000
Poisson coefficient (-)	0.30
Density of the brickwork composite (kg/m ³)	2295
Shear modulus (MPa)	600
Cohesion, c (MPa)	0.63
Tensile strength f_t (MPa)	0.45
Compressive strength f_c (MPa)	30.0
Friction angle (ϕ) (degrees)	30.0
Linearized compressive cap angle (ψ) (degrees)	50.0
Mode I fracture energy, G_f^I (N/mm)	0.012
Mode II fracture energy, G_f^{II} (N/mm)	0.019
K_n - axial truss (MPa)	174.43
K_n - torque truss (MPa)	130449

Table 1. Static Mechanical properties adopted for the homogenization step.

1.2 Strain-rate effect

Existing research proves that the use of static strength properties can lead to unreliable results for the masonry behaviour under fast dynamic actions, see [13,24]. Static strength material properties may exhibit an enhancement according to the strain rate level of the applied load. A useful and practical way to numerically represent the material properties change is to define dynamic increase factors (DIFs). The use of DIF laws is found suitable to study masonry structures subjected to fast loads application [24,25]. In this way, a homogenized model that may account the latter is relevant.

Focusing on the present homogenized approach, the material model reflects the dynamic characteristics of mortar and brick, and is derived from the static in-plane homogenized model (see also [20]). The values that define the elastic behaviour and the strength envelope of the unit cell, i.e. the parameters that directly rule the plasticity model, are strain-rate dependent. Specifically, the Young's modulus of the brick E_b , Young's modulus of the mortar E_m , tensile strength of the mortar f_{tm} , shear modulus of the mortar G_m and cohesion c . Compressive behaviour is in practice scarcely active in out-of-plane loaded periodic masonry and therefore, excluded from the present considerations on DIF for the sake of simplicity. According to the strain rate level, the values of the material properties are obtained through the product between the quasi-static property value and DIF, Eq. (4).

$$\begin{cases} E_b = DIF_{E_b} \times E_b \\ f_{tm} = DIF_{f_{tm}} \times f_{tm} \\ E_m = DIF_{E_m} \times E_m \\ c = DIF_c \times c \end{cases} \quad (4)$$

Therefore, it is required to define such DIFs by: (i) introducing strain-rate laws, typically logarithmic curves, for each selected parameter; or (ii) using a discrete DIF value, independent from the strain rate level, which is a priori assumed and adopted as a constant value. If the former yields more realistic values, it is also true that the latter is a straightforward, simple and more aligned with normative proposals.

For the present study, the information proposed by [24] is used to obtain rate dependent homogenized relations and so, the former strategy is adopted. The bending and torsional moment curves may be integrated along the thickness for each strain rate level. The nonlinear curvature-bending moment flexural and torsional behaviours of the interfaces are approximated using 5-node simplified curves, see for instance the curves defined in [20]. The implementation of this information in a finite element package at a macro-scale will allow to represent and study three-dimensional structures due to out-of-plane dynamic actions.

2 MACRO-SCALE: DISCRETE FE-MODEL

2.1 Overview

The adopted two-fold strategy relies into a homogenization approach at a meso-scale and on a discrete FE model at a macro-scale level. The dynamic out-of-plane analyses of masonry walls are performed using a novel discrete mechanical system at a macro-scale. Fig. 2 presents the model for a clear understanding and was already validated for quasi-static purposes, see [20]. The work by Kawai [26,27] serves as background for its formulation.

Briefly, the system is composed by quadrilateral rigid plates. On the interfaces and connecting these rigid elements, deformable truss and rigid beams are placed on each node. These truss-

beam system mimics the presence of flexural and torsional springs, governing the deformation and damage of the equivalent continuum. Additionally, mid-span hinges placed on interfaces allow to fix the axis of rotation for torsion movements without compromising the deformed shape. It should be noted that a decoupled characterization of flexural and torsional actions is adopted and such behaviour is ruled by the mechanical and material information derived beforehand at a meso-scale level. Nodal mass elements are lumped on the centre of each rigid plate. These elements concentrate the mass of the equivalent basic cell of the system, see Fig. 2.

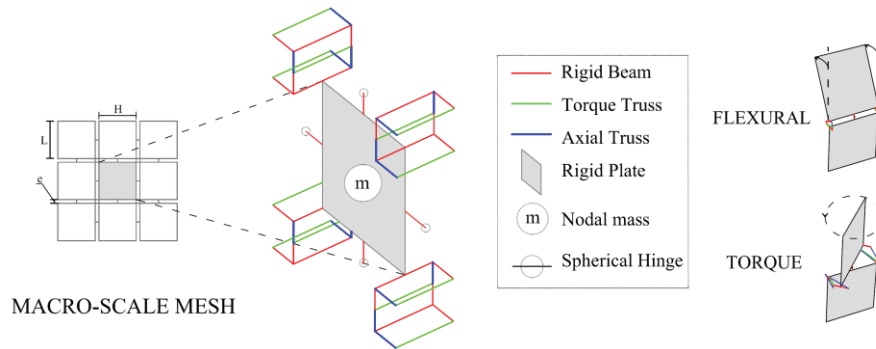


Fig. 2. Description of the novel discrete element system proposed.

3 NUMERICAL MODEL VALIDATION

The interest of researchers regarding the dynamic analysis of masonry structures subjected to impacts has been growing, but experimental studies on the behaviour of these structures is still scarce [11]. Recognizing the importance to develop further studies on the field, numerical models play an important role. In this framework, a discrete homogenized strain-rate sensitive model is proposed here and the main features of the discrete model were discussed in previous Sections.

To assess the ability of the approach proposed for the study of impact loading, a ‘stretcher’ bond masonry parapet tested by Gilbert et al. [31] is used. The wall is subjected to a low velocity impact load that tries to represent the impact of a vehicle. The comparisons will be performed by means of the experimental determined data [31], but also with numerical results collected from the studies of [25,32]. The selected parapets (from 21 tested) are designated as C6 and C7 and are replicates, see Fig. 3. Their assemblage was executed with strong concrete blocks and weak mortar (class iii mortar according to BS-5628). The walls and brick dimensions are $9150 \times 1130 \times 215 \text{ mm}^3$ (length x height x thickness) and $440 \times 215 \times 215 \text{ mm}^3$ (length x height x thickness), respectively.

At the wall base, the surface was coated with epoxy sand to reproduce the roughness of a given street floor. For the lateral supports, two abutment blocks connected to the walls through epoxy mortar were used. Numerically, the boundary conditions are considered to be fixed for each lateral edge and simple supported at the base of the walls. Aiming to model a car-like impact at both mid-height and length of the walls, a triangular out-of-plane load was applied through a steel plate. The load is idealized as a triangular time history distribution, in which the peak value is equal to 110 kN [31]. The deformation of the studied parapets was recorded in a node located 580 mm above the base and deviated 250 mm from the centre.

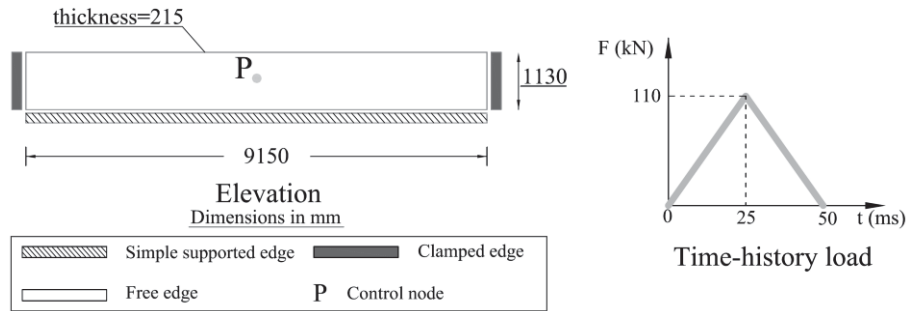


Fig. 3. Geometry of the running bond masonry parapets C6 and C7 studied by [31].

In the present model, the numerical simulation of the parapet walls will be accomplished through the aforementioned discrete element homogenized methodology. Firstly, the homogenization step at a meso-scale is performed to mechanically characterize the running bond masonry. The static material properties required are gathered in Table 1 and the rate-dependency issue is addressed. Hao and Tarasov [24] studied the experimental dynamic behaviour of a series of brick and mortar specimens under uniaxial compressive tests through a tri-axial static-dynamic apparatus. The analytical expressions required to describe the value of the DIFs derive from the latter research. It may be noted that the study by Rafsanjani et. al. in [32] that covers the current masonry parapets, use the same laws hereby presented to define the strain-rate dependency of masonry interfaces using a micro-modelling strategy.

The implementation of such laws in the homogenized model allows deriving stress-strain rate-dependent homogenized curves. Additionally, moment-curvature relationships are simply derived by the integration of the latter stress-strain curves through the thickness of the wall.

Fig. 4a reports the displacement magnitude with respect to time. A numerical model with a mesh size of 200 mm was adopted at a macro-scale. Curves resultant from (i) the experimental results [31], (ii) the numerical model by Burnett et. al [25], (iii) the numerical model by Rafsanjani et. al [32] and, the simulation results of the discrete homogenized-based model are depicted. The curve by Burnett et. al [25] leads to excessive displacements (and understiff response) because it considers quasi-static values for the material parameters. Conversely, both the present and the micro-model by Rafsanjani et. al [32] are accurate in predicting the peak displacement, with a relative error of around 10%.

Fig. 4b indicates the observed damage pattern for the present model. Vertical cracks are clear around both the central area and the two supported edges. Also, horizontal cracks spread from the centre along the height of the masonry parapet. As expected, it is evident that damage tends to concentrate on the impact zone. This was implicitly concluded in the experiments tests [31].

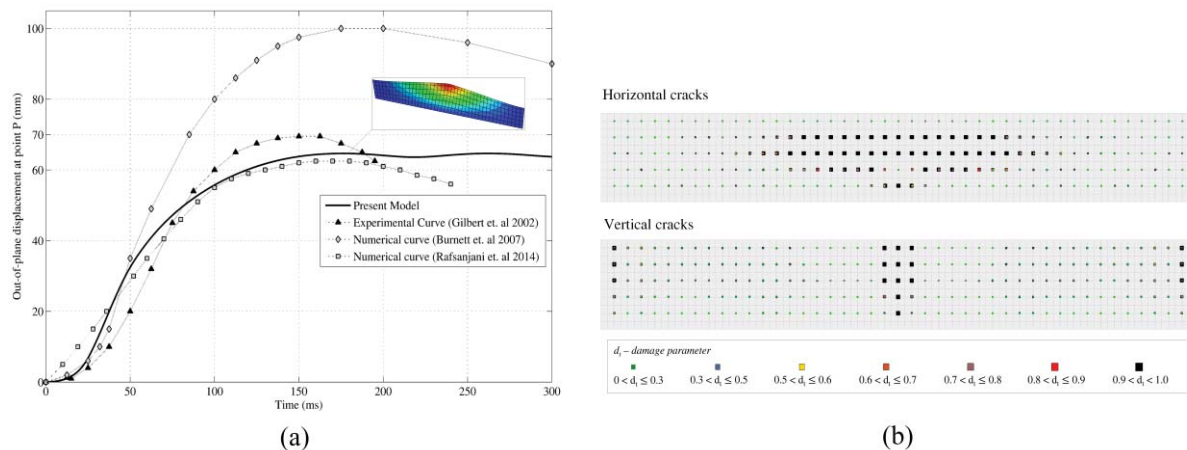


Fig. 4. (a) Time history of the out-of-plane displacement for the control node of parapets C6 and C7, with deformed shape for the instant 180 ms; (b) Damage pattern obtained for the applied load wall side: horizontal cracks and vertical cracks.

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