A simple homogenized micro mechanical model for the analysis at the collapse of out-of-plane loaded masonry walls

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SUMMARY: The paper presents a simple micro-mechanical model for the limit analysis of out-of-plane loaded masonry walls by means of homogenization techniques. In this framework, masonry thickness is subdivided into several layers and for each layer polynomial distributions for the stress fields are a-priori assumed inside a fixed number of sub-domains. In this way, a simple linear programming problem is derived with the aim of obtaining out-of-plane homogenized failure surfaces of masonry. Finally, such strength domains are implemented in FE limit analysis codes for upper and lower bound analyses on entire masonry panels out-of-plane loaded. One of these numerical analyses is reported in order to show the reliability (in terms both of collapse loads and failure mechanisms) of the model at hand in comparison with experimental data.

1. INTRODUCTION

The prediction of the ultimate load bearing capacity of masonry walls out-of-plane loaded is of great technical relevance. As a matter of fact, out-of-plane failures are mostly related to seismic and wind loads. Furthermore, earthquake surveys have shown that the lack of out-of-plane strength is a primary cause of failure in many traditional forms of masonry and this is confirmed in the case of historical buildings, where the façades are often characterized by a relative small thickness (Spence and Coburn 1992). Many damages suffered by historical masonry buildings during the Friuli (1976), Umbria-Marche (1997-1998) and Molise (2002) earthquakes might be ascribed to out-of-plane collapses. Another important aspect to underline is that masonry structures are usually subjected simultaneously to in-plane compressive vertical loads and out-of-plane actions. As shown by experimentations, vertical loads increase not only the ultimate out-of-plane strength but also the ductility of masonry.

On the other hand, laboratory tests conducted on brick masonry walls subjected to lateral loads, have demonstrated that failure takes place along a definite pattern of lines, so inspiring approximate analytical solutions based on the yield line theory (Sinha 1978). Up to now, the yield line method seems the only suitable to be applied in practice for the evaluation of the ultimate load bearing capacity of masonry out-of-plane loaded. Furthermore, probably for its theoretical simplicity, it has been adopted by many codes of practice, as for instance BS 5628 and EC 6. Nevertheless, all codes employ only horizontal and vertical out-of-plane masonry strengths (which are experimentally available directly), leading unavoidably to an approximate estimation of the collapse load, which does not take into account brickwork torsion contribute. A limit analysis approach has been recently adopted also by OPCM 3274, where masonry is modeled as a no tension material.

For this reason, limit analysis combined with a homogenization approach seems a powerful tool able to predict masonry behavior at collapse. Furthermore, this approach both requires only a reduced number of material parameters and allows to avoid an independent modeling of units and mortar. On the other hand, it is able to provide limit load multipliers, failure mechanisms and, at least on critical sections, the stress distribution at collapse. Nevertheless, an evident drawback of homogenization is that it requires to solve (usually by means of FE techniques) a field problem on the elementary cell and different loading conditions require different expensive simulations.

The simple micro-mechanical model presented in this paper allows to avoid a FE discretization at a cell level; the elementary cell is subdivided along the thickness in several layers, for each layer fully equilibrated stress fields are assumed, a-priori fixing polynomial expressions for the stress tensor components in a finite number of sub-domains, imposing the continuity of the stress vector on the interfaces and anti-periodicity conditions on the boundary surface. As the lower bound theorem of limit analysis states, such stress distribution represents a statically admissible micro-stress field, provided that admissibility conditions for the constituent materials are imposed on the unit cell. A simple linear optimization problem with few variables is obtained in order to recover out-of-plane failure surfaces of masonry. Finally, such homogenized strength domains are implemented in FE limit analysis codes (both upper and lower bound) for a limit analysis of entire panels out-of-plane loaded.

In Section 2, after a brief review of the homogenization theory combined with limit analysis, the fully equilibrated micro-mechanical model is presented in detail.

In Section 3 a comparison between the results obtained by means of the micro-mechanical model at hand and experimental data by Gazzola and Drysdale (1986) is presented. The comparison refers to the out-of-plane strength of specimens in four point bending at different angles ϑ of the ultimate moment with respect to bed joints orientation. A further comparison at a cell level between the equilibrated micro-mechanical model proposed and a recently presented kinematic approach by Sab (2003) in the case of joints reduced to interfaces with a Mohr-Coulomb failure criterion and bricks infinitely resistant is reported. The comparison shows that the equilibrated model offers reliable results even for a relatively coarse subdivision of masonry thickness. Finally, an adding numerical simulation is carried on for a technically meaningful case, varying progressively vertical in-plane compressive load with the aim of testing the ability of the model to reproduce the influence of membrane actions on out-of-plane masonry strength.

In Section 4 a numerical example on a masonry panel out-of-plane loaded and simply supported at three edges is reported. Both lower and upper bound FE limit analyses are dealt with in detail. The lower bound approach is based on the equilibrated triangular element by Hellan (1967) and Herrmann (1967), whereas the upper bound is based on the triangular element by Munro and Da Fonseca (1978). The results, in terms of collapse load and failure mechanism, show the reliability of the simple model presented.

2. A SIMPLE MICRO-MECHANICAL MODEL

Let us consider a masonry wall Ω constituted by a periodic arrangement of bricks and mortar disposed in stretcher bond texture. As shown in a classical paper by Suquet (1983), homogenization techniques combined with limit analysis can be applied for an extimation of the homogenized out-of-plane strength domain S^{hom} of masonry.

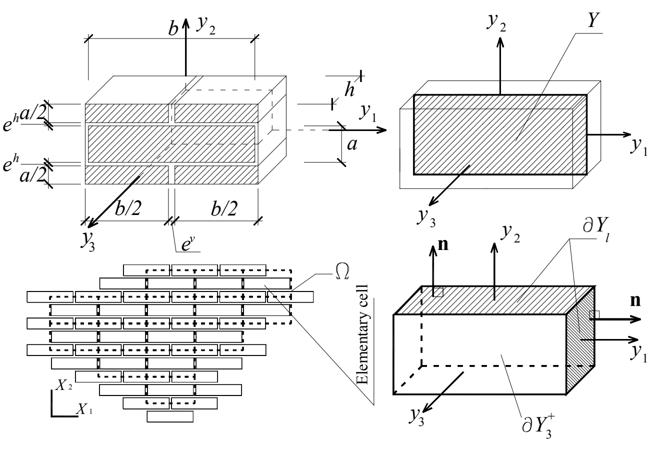


Figure 1: Periodic structure ($X_1 - X_2$ macroscopic frame of reference) and elementary cell ($y_1 - y_2 - y_3$ local frame of reference)

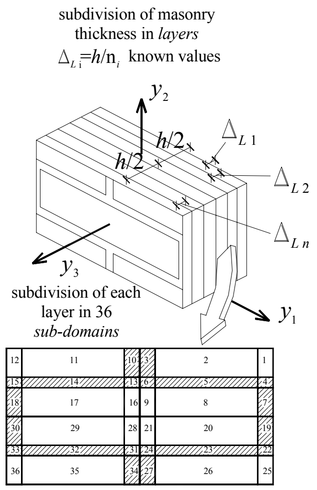


Figure 2: The micro-mechanical model proposed. Subdivision in layers along the thickness and subdivision of each layer in sub-domains.

In this framework, bricks and mortar are assumed rigid-perfectly plastic materials with associated flow rule. As the lower bound theorem of limit analysis states and under the hypotheses of homogenization, S^{hom} can be derived by means of the following (non-linear) optimization problem (see also Figure 1).

$$
S^{\text{hom}} = \left\{ (\mathbf{M}, \mathbf{N}) \,|\, \left\{ \begin{array}{lr}
\mathbf{N} = \dfrac{1}{|Y|} \displaystyle\int_{Y \times h} \boldsymbol{\sigma} dV & (a) \\
\mathbf{M} = \dfrac{1}{|Y|} \displaystyle\int_{Y \times h} y_3 \boldsymbol{\sigma} dV & (b) \\
div \boldsymbol{\sigma} = \mathbf{0} & (c) \\
[\![\boldsymbol{\sigma}]\!] \mathbf{n}^{\text{int}} = \mathbf{0} & (d) \\
\boldsymbol{\sigma}\mathbf{n} \quad \text{anti-periodic} \quad \text{on} \quad \partial Y_l & (e) \\
\boldsymbol{\sigma}(\mathbf{y}) \in S^m \quad \forall \mathbf{y} \in Y^m \;;\; \boldsymbol{\sigma}(\mathbf{y}) \in S^b \quad \forall \mathbf{y} \in Y^b & (f)
\end{array} \right. \right\} \tag{1}
$$

where:

- $\mathbf{N}$ and $\mathbf{M}$ are the macroscopic in-plane (membrane forces) and out-of-plane (bending moments) tensors;

- $\boldsymbol{\sigma}$ denotes the microscopic stress tensor and $\mathbf{n}$ is the outward versor of ∂Y_l surface;

∂Y_l is defined in Figure 1;

- $[[\boldsymbol{\sigma}]]$ is the jump of micro-stresses across any discontinuity surface of normal $\mathbf{n}^{\text{int}}$;

- S^{m} and S^{b} denote respectively the strength domains of mortar and bricks;

- Y is the cross section of the 3D elementary cell with $y_3 = 0$ (Figure 1), $|Y|$ is its area, V is the elementary cell, h represents the wall thickness and $y = (y_1 \quad y_2 \quad y_3)$.

In order to solve problem (1) in a simple manner, the unit cell is subdivided into a fixed number of layers along its thickness, as shown in Figure 2. According to classical limit analysis plate models (Capurso 1971), for each layer out-of-plane components σ_{i3} ($i = 1, 2, 3$) of the micro-stress tensor $\boldsymbol{\sigma}$ are set to zero, so that only in-plane components σ_{ij} ($i, j = 1, 2$) are considered in the optimization.

Then, σ_{ij} ($i, j = 1, 2$) are kept constant along the Δ_{i_L} thickness of each layer. As proposed by the authors for in-plane actions (Milani et al. 2005a), for each layer one-fourth of the REV is sub-divided into nine geometrical elementary entities (*sub-domains*), so that all the cell is sub-divided into 36 sub-domains (Figure 2).

Inside each sub-domain (k) and layer (i_L), polynomial distributions of degree *(m)* are assumed for the stress components. Being stress fields polynomial expressions, the generic *ij*th component of the stress tensor can be written as follows:

$$\sigma_{ij}^{(k,i_L)} = \mathbf{X}(\mathbf{y})\mathbf{S}_{ij}^{(k,i_L)T} \qquad \mathbf{y} \in Y^{(k,i_L)} \tag{2}$$

where:

- $\mathbf{X}(\mathbf{y}) = \lfloor 1 \quad y_1 \quad y_2 \quad y_1^2 \quad y_1 y_2 \quad y_2^2 \quad \ldots \rfloor$;

- $\mathbf{S}_{ij}^{(k,i_L)} = \lfloor S_{ij}^{(k,i_L)(1)} \quad S_{ij}^{(k,i_L)(2)} \quad S_{ij}^{(k,i_L)(3)} \quad S_{ij}^{(k,i_L)(4)} \quad S_{ij}^{(k,i_L)(5)} \quad S_{ij}^{(k,i_L)(6)} \quad \ldots \rfloor$ is a vector representing the unknown stress parameters of sub-domain (k) of layer (i_L);

- $Y^{(k,i_L)}$ represents the *k*th sub-domain of layer (i_L).

The imposition of equilibrium inside each sub-domain, the continuity of the stress vector on interfaces and the anti-periodicity of $\boldsymbol{\sigma}\mathbf{n}$ permit a strong reduction of the total number of independent stress parameters.

For instance, the imposition of micro-stress equilibrium ($\sigma_{ij,j} = 0 \quad i = 1,2$) in each sub-domain yields:

$$\sum_{j=1}^{2} \mathbf{X}(\mathbf{y})_{,j} \mathbf{S}_{ij}^{(k,i_L)T} = \mathbf{0} \tag{3}$$

If p is the degree of the polynomial expansion, $p(p+1)$ equations can be written.

A further reduction of the total unknowns is obtained imposing the continuity of the (micro)-stress vector on internal interfaces ($\sigma^{(k,i_L)}_{ij} n_j^{\text{int}} + \sigma^{(r,i_L)}_{ij} n_j^{\text{int}} = 0 \quad i = 1,2$) for every (k, i_L) and (r, i_L) contiguous sub-domains with a common interface of normal $\mathbf{n}^{\text{int}}$. Other $2(p+1)$ equations in the stress coefficients can be written for each interface as follows:

$$\left(\hat{\mathbf{X}}_{ij}^{(k,i_L)}(\mathbf{y}) \hat{\mathbf{S}}^{(k,i_L)} + \hat{\mathbf{X}}_{ij}^{(r,i_L)}(\mathbf{y}) \hat{\mathbf{S}}^{(r,i_L)T} \right) n_j^{\text{int}} = 0 \quad i = 1,2 \tag{4}$$

Furthermore, anti-periodicity of $\boldsymbol{\sigma}\mathbf{n}$ on ∂V requires other $2(p+1)$ equations per pair of external faces (m, i_L) and (n, i_L), i.e. it should be imposed that stress vectors $\boldsymbol{\sigma}\mathbf{n}$ are opposite on opposite sides of ∂V :

$$\hat{\mathbf{X}}_{ij}^{(m,i_L)}(\mathbf{y}) \hat{\mathbf{S}}^{(m,i_L)} \mathbf{n}_{1,j} = -\hat{\mathbf{X}}_{ij}^{(n,i_L)}(\mathbf{y}) \hat{\mathbf{S}}^{(n,i_L)} \mathbf{n}_{2,j} \tag{5}$$

Where $\mathbf{n}_1$ and $\mathbf{n}_2$ are oriented versors of the external faces of the paired sub-domains (m, i_L) and (n, i_L).

After some trivial elementary assemblage operations on the local variables, stress vector of layer i_L inside sub-domain (k) can be written as follows:

$$\tilde{\boldsymbol{\sigma}}^{(k,i_L)} = \tilde{\mathbf{X}}^{(k,i_L)}(\mathbf{y}) \tilde{\mathbf{S}}^{(i_L)} \tag{6}$$

Where $\tilde{\mathbf{S}}^{(i_L)}$ is the vector of unknown stress parameters of layer i_L.

As it has been show for the in-plane case by the authors (Milani et al. 2005a), reliable results can be obtained if a fourth order polynomial expansion is chosen for the stress field. For this reason, in what follows, expansions of degree four are adopted.

Once fixed the polynomial degree, the out-of-plane model presented requires a subdivision (n_L)

of the wall thickness into several layers (Figure 2-a), with an a-priori fixed constant thickness $\Delta_{L_i} = t / n_L$ for each layer. In this way, the following simple (non) linear optimization problem is derived:

$$
\left\{
\begin{array}{l}
\max\{\lambda\} \\
\text{such that}
\left\{
\begin{array}{lr}
\mathbf{N} = \int_{k,i_L} \tilde{\boldsymbol{\sigma}}^{(k,i_L)} dV & (a) \\
\mathbf{M} = \int_{k,i_L} y_3 \tilde{\boldsymbol{\sigma}}^{(k,i_L)} dV & (b) \\
\mathbf{M} = \begin{bmatrix} M_{xx} & M_{xy} \\ M_{xy} & M_{yy} \end{bmatrix} = \lambda \begin{bmatrix} \cos(\psi)\cos(\vartheta) & \sin(\vartheta) \\ \sin(\vartheta) & \sin(\psi)\cos(\vartheta) \end{bmatrix} & (c) \\
\psi = [0;\, 2\pi] \quad \theta = [0;\, \pi/2] & (d) \\
\tilde{\boldsymbol{\sigma}}^{(k,i_L)} = \tilde{\mathbf{X}}^{(k,i_L)}(\mathbf{y})\tilde{\mathbf{S}} & (e) \\
\tilde{\boldsymbol{\sigma}}^{(k,i_L)} \in S^{(k,i_L)} & (f) \\
k = 1,\ldots, \text{ number of } \textit{sub-domains};\ i_L = 1,\ldots, \text{number of } \textit{layers} & (g)
\end{array}
\right.
\end{array}
\right. \tag{7}
$$

where:
- λ is the direction of the ultimate bending moment in the $M_{xx} - M_{yy} - M_{xy}$ space;
- ψ and ϑ are spherical cooridinates in $M_{xx} - M_{yy} - M_{xy}$, given by $\tan(\vartheta) = \dfrac{M_{xy}}{\sqrt{(M_{xx}^2 + M_{yy}^2)}}$,
- $\tan(\psi) = \dfrac{M_{yy}}{M_{xx}}$;
- $S^{(k,i_L)}$ denotes the (non-linear) strength domain of the constituent material (mortar or brick) corresponding to the k^{th} sub-domain and i_L^{th} layer;
- $\tilde{\mathbf{S}}$ collects all the unknown polynomial coefficients (of each sub-domain of each layer).

For the sake of simplicity, membrane actions are kept constant and independent from load multiplier. In this way, in-plane actions effect optimization only in the evaluation of M_{xx}, M_{yy}, M_{xy} strength domains. This assumption is technically acceptable for the experimental tests analyzed next, since in these cases a fixed in-plane compressive load (if present) $N_{yy} = -N_0$ is applied before out-of-plane actions and kept constant until failure, whereas $N_{xx} = N_{xy} = 0$.

Finally, we refer the reader to classical papers (Anderheggen and Knopfel 1972, Maier 1977) for a critical discussion both on the procedures adopted to reduce (7) to a linear programming problem and on the algorithms used (based on the revised simplex method) to solve efficiently the linearized problem derived from (7).

3. OUT-OF-PLANE STRENGTH FOR DIFFERENT ORIENTATIONS ϑ OF THE LOADING WITH RESPECT TO THE BED JOINT

In this section, the ability of the homogenization procedure proposed to reproduce the strength of different masonry walls subjected to out-of-plane loads is tested for different orientations ϑ of the bending moment with respect to the bed joint direction. A further comparison with a kinematic approach recently presented in the technical literature (Sab 2003) is reported when the bricks are supposed infinitely resistant and joints are reduced to interfaces with a pure Mohr-Coulomb failure criterion.

It is worth mentioning that experimental data available from different authors are reported in terms of maximum bending moments or flexural tensile strengths along horizontal and vertical directions. Usually, flexural tensile strengths f_t are quantities derived from experimental failure moments M_u by means of the elastic relation $f_t = M_u / W_{el} = 6M_u /(bh^2)$, see also Figure 3-b, where h is the wall thickness and b is a unitary length. Of course, these values of f_t are not the real uniaxial tensile strengths. A more realistic stress distribution along the thickness of the wall at failure (under the assumption of perfect plasticity for the constituent materials) is depicted in Figure 3-a. This implies that mechanical properties to adopt for mortar and units in the homogenization model has to be chosen in order to fit horizontal and vertical uniaxial tensile strengths of Figure 3-a, i.e. experimental values divided roughly by 3 (see also stress/strain diagrams reported in EC6 code).

The most complete set of experimental strength data for specimens subjected to out-of-plane loading in four-point bending seems to be given by Gazzola et al. (1985) and Gazzola and Drysdale (1986), who tested 25 wallettes of hollow concrete block masonry, with different dimensions and with the bed joints making a variable angle ϑ with the direction of loading.

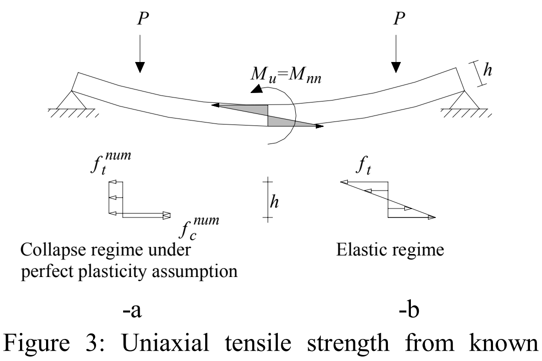


Figure 3: Uniaxial tensile strength from known values of failure moment M_u in four point bending. –a: collapse stress distribution, perfect plasticity (present model). –b: experimental procedure (elastic properties of section)

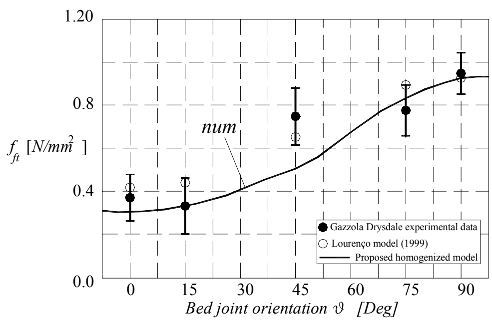


Figure 4: Comparison among experimental results by Gazzola and Drysdale (1986), plasticity model by Lourenço (1999) and proposed model for the evaluation of flexural strength at different values of ϑ angle.

In order to compare experimental data with the model at hand mechanical properties of mortar and bricks are taken in order to reproduce exactly the experimental value of f_{ft} reported by Gazzola Drysdale (1986) for $\vartheta = 90°$. Mechanical properties of mortar and bricks are reported in Table 1, whereas bricks dimensions and joints thickness are assumed $390x190x150\ mm^3$ and $10\ mm$ respectively.

Mortar	Brick
Mohr Coulomb plane strain with tension cut-off	Compression cut-off
$f_{tm} = M_{uh} / (h^2/6)$ $\left[\frac{N}{mm^2}\right]$ (tension cut-off) $c_m = 2 f_{tm}$ (cohesion) $\Phi_m = 36°$ (friction angle)	$f_{cb} = 22.7\ \frac{N}{mm^2}$

Table 1: Comparison with experimental data by Gazzola et al. (1985) for masonry specimens in four-point bending

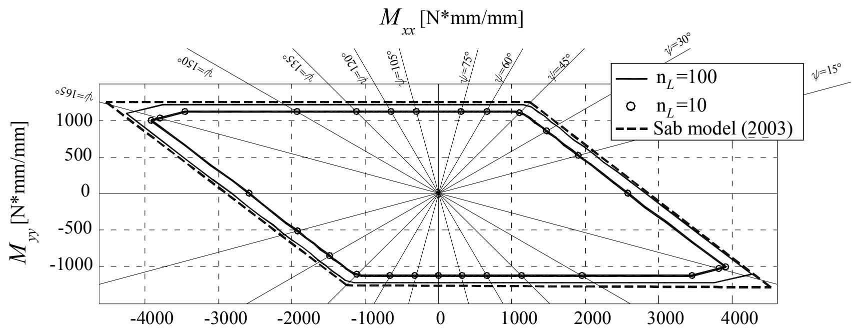


Figure 5: Failure surfaces in the plane M_{xx}-M_{yy} obtained using the micro-mechanical model proposed (number of layers $n_L = 10$ and $n_L = 100$) and a kinematic approach recently presented in literature.

A comparison between experimental values and results from the numerical model for different orientation of ϑ angle is given in Figure 4, which shows the average and standard deviation of the tests for each orientation of loading. In general, f_{ft}^{num} values, depend on both geometry and mechanical properties of mortar and bricks, and are obtained solving the following optimization problem:

$$\max\left\{ f_{ft} \mid f_{ft}\frac{h^2}{2} = M_{nn} = M_{xx}\sin^2\vartheta + M_{yy}\cos^2\vartheta - M_{xy}\sin(2\vartheta),\ \mathbf{A}^{\mathbf{in}}\mathbf{M} \le \mathbf{b}^{\mathbf{in}} \right\} \quad (8)$$

Where $\mathbf{A}^{\mathbf{in}}\mathbf{M} \le \mathbf{b}^{\mathbf{in}}$ represents the linearized out-of-plane strength domain and h is the wall thickness (see Figure 3).

It is worth noting both that for the evaluation of M_{nn}, also the torsion moment M_{xy} is taken into account and that stress distribution along the thickness of the wall is assumed as in Figure 3-a.

In this section, a further comparison with a kinematic approach recently presented by Sab (2003) is presented in order to evaluate the capability of the equilibrated micro-mechanical model proposed to approximate the homogenized failure surfaces obtained by means of kinematic limit analyses. In the kinematic approach proposed by Sab (2003), bricks are supposed infinitely resistant and a Mohr-Coulomb failure criterion is chosen for joints reduced to interfaces. In this way, a "closed form" solution is obtained only when torsion M_{xy} is set to zero.

Here, a masonry wall with joints reduced to interfaces and bricks of dimensions $bxhxa = 250x120x55\ mm^3$ (width x thickness x height) and $m = 2a/b = 0.44$ is considered.

Mechanical characteristics of joints are $c = 0.1\left[N/mm^2\right]$ (cohesion) and $\Phi = 37°$ (friction angle). In Figure 5 a comparison between failure surfaces (in the plane M_{xx} and M_{yy}) obtained using the micro-mechanical model proposed and the kinematic approach by Sab (2003) is reported. As it is possible to note, the model proposed both approximates accurately results from the kinematic procedure and is able to reproduce the orthotropic masonry behavior at failure. On the other hand, different mechanical characteristics for bricks and mortar can be taken into account, as well as the actual thickness of joints.

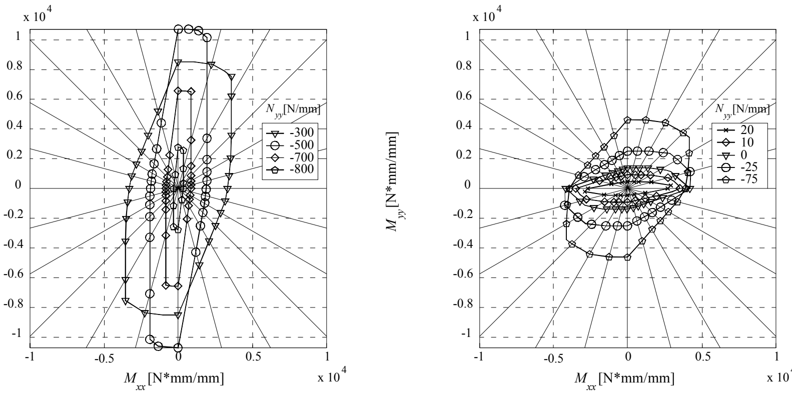


Figure 6: Some cross sections of the failure surface with $M_{xy} = 0$ and $N_{yy} = \alpha$ for different values of α. Brickwork by Raijmakers and Vermeltfoort (1992), numerical results obtained using two layers of unknown thickness and non-linear programming.

Finally, it is worth noting that the influence of a vertical compressive in-plane load can be easily taken into account with the model at hand. As already mentioned, a vertical compressive load can increase out-of-plane strength. In order to study this effect on a meaningful technical case, the brickwork considered by Raijmakers and Vermeltfoort (1992) for performing some experimental tests on shear walls is considered. The units dimensions are $210x52x100\ mm^3$, whereas the thickness of the mortar joints is $10\ mm$. For the sake of simplicity, joints are reduced to interfaces assuming for them a frictional type failure criterion ($\Phi_m = 37°$, $c_m = 1.4 f_{tm}$) with a tension cut-off ($f_{tm} = 0.16\ N/mm^2$) and a linearized cap in compression ($f_{cm} = 11.5\ N/mm^2$, $\Phi_{cm} = 30°$), see Milani 2004 for further details, whereas bricks are assumed infinitely resistant.

In Figure 6 some cross sections of the failure surface in the space $M_{xy} = 0$ with $N_{yy} = \alpha$ are reported for different values of α in-plane vertical load.

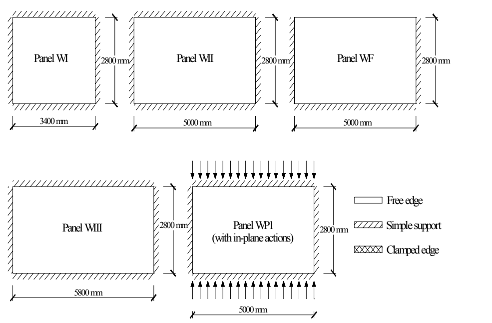


Figure 7: Gazzola et al. (1985) experimental tests on out of-plane loaded masonry walls. Panels dimensions and boundary conditions

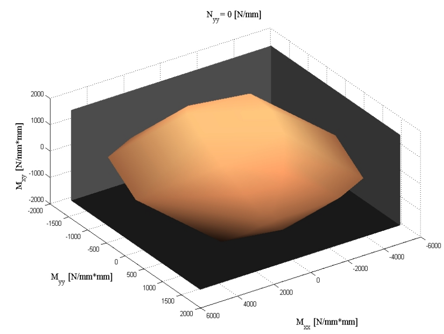


Figure 8: Homogenized failure surface for Gazzola et al. 1985 tests.

4. STRUCTURAL EXAMPLE

In this section, the homogenized model previously presented is validated by means of some comparisons with experimental data on entire masonry panels out-of-plane loaded. In order to make the comparison, both upper and lower bound FE limit analysis codes have been implemented (Matlab 6.5TM). The lower bound approach is based on the triangular elements by Hellan (1967) and Herrmann (1967), whereas the upper bound is based on the finite element presented by Munro and Da Fonseca (1978). For the sake of conciseness, a detailed description of the elements used is not reported here and the reader is referred to Milani et al 2005b.

The panels here analyzed consist of hollow concrete block masonry. The tests were carried out by Gazzola et al. 1985 and are denoted by W. Five panels were tested by the authors (WI, WII, WIII, WP1 and WF), as shown in Figure 7. The panels were loaded until failure with increasing out-of-plane uniform pressure p. For each configuration, three different tests were carried out and the results reported by the authors represent the average of the tests. The only panel with in-plane action was WP1, which was loaded, previously to the application of the out-of-plane loading, with an in-plane confining vertical pressure of $0.2\ N/mm^2$.

In this paper, for the sake of conciseness, only panel WF is analyzed with the homogenized model at hand. Referring to the incremental non-linear analysis conducted by Lourenço (1997 and 1999), these panels have a relatively ductile behavior and therefore are suitable for a homogenized limit analysis.

Mortar	Brick
Mohr Coulomb plane strain with tension cut-off	Compression cut-off
$f_{tm} = 0.157 \dfrac{N}{mm^2}$ (tension cut-off) $c_m = 3.8 f_{tm}$ (cohesion), $\Phi_m = 36°$ (friction angle)	$f_{cb} = 22.7 \dfrac{N}{mm^2}$

Table 2: Comparison with experimental data by Gazzola et al. (1985) on out-of-plane loaded panels

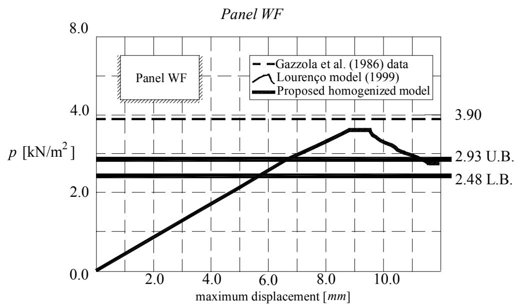
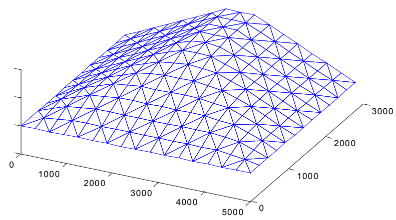


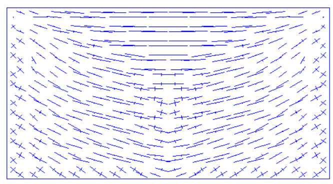
Figure 9: Comparison between experimental and numerical results, Gazzola et al. 1985 tests, panels WII and WF



-a

Figure 10: Gazzola et al. 1985 experimental tests, lower and upper bound FE limit analysis results. – a: Principal moments at collapse, panel WF, -b: failure mechanism from the upper bound FE limit analysis and mesh used, panel WF

-b



Inelastic properties of mortar and bricks are reported in Table 2 and are chosen in order to fit experimental vertical/horizontal masonry strengths reported by Gazzola et al. (1985). The homogenized failure surface obtained solving problem (7) for several directions of λ is reported in Figure 8.

Figure 9 shows a comparison among the failure loads obtained numerically (both upper and lower bound methods), the load-displacement diagrams obtained by Lourenço (1997 and 1999) and experimental failure loads. It is worth noting that no information is available from Gazzola et al. 1995 regarding experimental load-displacement diagrams, as well as about the scatter of their tests.

Finally, in Figure 10 principal moments distribution at collapse from the lower bound analysis for panel WF and failure mechanism (with the relative mesh used) from the upper bound analysis are reported. The comparison shows that reliable predictions can be obtained using the homogenized model proposed.

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