Collaborative Dynamic Decision Making: a Case Study from B2B Supplier Selection

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Summary. The problem of supplier selection can be easily modeled as a multiple-criteria decision making (MCDM) problem: businesses express their preferences with respect to suppliers, which can then be ranked and selected. This approach has two major pitfalls: first, it does not consider a dynamic scenario, in which suppliers and their ratings are constantly changing; second, it only addressed the problem from the point of view of a single business, and cannot be easily applied when considering more than one business. To overcome these problems, we introduce a method for supplier selection that builds upon the dynamic MCDM framework of Campanella and Ribeiro [1] and, by means of a linear programming model, can be used in the case of multiple collaborating businesses planning their next batch of orders together.

1 Introduction

Complex decision making is a process extended in time: real-world decision problems are dynamic in the sense that they are the outcome of a sequence of decisions or of an exploratory process, during which both alternatives and criteria may vary. Dynamic decision making problems are characterized by three key properties:

1. the temporal profile of an alternative matters for comparison with other alternatives;
2. alternatives are not fixed, since they might be deemed nonviable and discarded, and likewise new options might be taken into consideration and added;
3. criteria are not fixed, not only because corresponding values might change over time, but also because new criteria might be considered, or existing ones removed.

Multiple-criteria decision making (MCDM) methods deal with selection of alternatives from evaluations over criteria. They model two fundamental aspects of decision problems:

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1. they capture the relevant quantitative and qualitative criteria – such as cost, efficiency, and performance – for a specific decision problem;
2. they support the intricate trade-off process when considering different alternatives.

They are, in general, methods for eliciting evaluations and aggregating criteria measurements in order to compose a ranked list of alternatives. However, they are unable to deal with dynamic situations: they usually assume a fixed set of alternatives and criteria, and thus cannot be used to adequately model the characteristics of a dynamic decision making process in a changeable environment.

Recently, Campanella and Ribeiro proposed a framework for *dynamic* MCDM [1] to address this gap in the decision making literature [2]. This framework was developed in the context of a specific spatial-temporal decision problem [3, 4]; it is, however, domain-independent and thus useful for any domain that includes dynamic decision making. In the present paper, we extend the dynamic MCDM framework of Campanella and Ribeiro [1] to the problem of supplier selection. This problem cuts across many different industries for which suppliers play a key role in the supply chain [5], and in its standard form can be easily stated as a MCDM problem.

This chapter is organized as follows. In Section 2, we describe the basic MCDM operations and the dynamic MCDM framework. In Section 3, we discuss how the problem of supplier selection can be tackled using the proposed framework, in the case of a single business as well as in the case of multiple collaborating businesses served by the same set of suppliers. In Section 4, we work through an illustrative example to facilitate the understanding and demonstrate the effectiveness of the proposed method. Finally, in Section 5, we conclude by suggesting other possible application areas, and by proposing directions for future research.

2 Dynamic multiple-criteria decision making

In general, the aim of MCDM methods is to identify the best compromise solution from a set of feasible alternatives assessed with respect to a predefined set of (usually conflicting) criteria.

The decision making literature contains many approaches to the *static* version of this problem [6, 7]. It makes sense that current techniques are geared towards a one-shot decision; complex decision problems – the kind that benefits from decision support systems – are often about making one important decision. These methods, however, are not easily adapted to *dynamic* decision making, which, quoting Brehmer [8], can be defined as

> . . . decision making under conditions which require a series of decisions, where the decisions are not independent, where the state of the world changes, both autonomously and as a consequence of the decision maker's actions.

Dynamic decision making methods must thus be able to support interdependent decisions in an evolving environment, in which both criteria and alternatives may change, and later decisions need to take into account *feedback* from previous ones.

2.1 Structure of static MCDM methods

Before we introduce the dynamic MCDM framework of Campanella and Ribeiro [1], it is important to briefly describe the high-level structure of most static MCDM methods, since it constitutes the foundation upon which the framework is built. Broadly speaking, we can identify two phases in the decision making process, namely preference elicitation and aggregation.

Preference elicitation The first step of static MCDM methods is known as *preference elicitation* and consists in identifying the available alternatives, fixing the criteria that will be used in the evaluation, and deriving the preference structure, which may be expressed using different types of scales [9]. As noted by Aloysius et al. [10], this process is of paramount importance and may even directly affect user acceptance of MCDM methods; many different techniques have thus been proposed in the literature (see, for example, [11] and references therein).

Once the preference structure is obtained, it is necessary to translate it into numerical values that express the relative performance of each alternative with respect to each criterion; these values are usually assumed to belong to the unit interval $\mathbb{I} = [0, 1]$. Mathematically, a typical MCDM problem with m alternatives and n criteria is modeled by the matrix

$$\begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \dots & c_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \end{array} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_m \end{bmatrix}, \qquad (1)$$

where $x_{ij} \in \mathbb{I}$ represents the level of achievement of alternative a_i, $i = 1, \dots, m$ with respect to criterion c_j, $j = 1, \dots, n$, with 0 interpreted as "no satisfaction" and 1 corresponding to "complete satisfaction".

In the case of imprecise or uncertain data, fuzzy logic can be used to guarantee normalization and comparability of input variables, which would be represented by means of fuzzy membership functions [12].

Aggregation After numerical values for each alternative have been elicited, they can aggregated into another numerical value, also belonging to the unit interval, that is understood to represent the preferableness of that alternative relative to all others. Given these values, alternatives may then be ordered, thus producing a ranking, and the best one can be selected.

Aggregation is achieved by means of an *aggregation function* (sometimes improperly called aggregation operator), formally defined as follows [13].

Definition 1 (Aggregation function). *An* aggregation function $f : \mathbb{I}^n \to \mathbb{I}$ *is a function of $n > 1$ variables that maps points $\mathbf{x} = (x_1, \ldots, x_n)$ in the unit hypercube $\mathbb{I}^n$ to single values in the unit interval $\mathbb{I}$ and that satisfies, for all $\mathbf{x}, \mathbf{y} \in \mathbb{I}^n$,*

$$\begin{cases} f(\underbrace{0, 0, \ldots, 0}_{n \ times}) = 0 \\ f(\underbrace{1, 1, \ldots, 1}_{n \ times}) = 1 \end{cases} \qquad \textit{(preservation of bounds)}, \tag{2}$$

$$\mathbf{x} \leq \mathbf{y} \Rightarrow f(\mathbf{x}) \leq f(\mathbf{y}) \qquad \textit{(monotonicity)}. \tag{3}$$

Remark 1 (Weight vector). In this context, it is common to introduce a weight vector $\mathbf{w} \in [0, 1]^n$ whose generic element w_j, $j = 1, \ldots, m$ is the weight associated to criterion c_j expressing its importance relative to all others. These weights must satisfy the normalization condition $\sum_j w_j = 1$.

Intuitively, mathematical properties of the function chosen for aggregation will directly affect output values and, therefore, the final ranking of alternatives. An important property that can be required of aggregation functions is *associativity*, defined as follows.

Definition 2 (Associativity). *A bivariate aggregation function $f : \mathbb{I}^2 \to \mathbb{I}$ is said to be* associative *if, for all $x_1, x_2, x_3 \in \mathbb{I}$, it holds that*

$$f(f(x_1, x_2), x_3) = f(x_1, f(x_2, x_3)). \tag{4}$$

Remark 2. By iterative application, any bivariate aggregation function unambiguously defines a family of n-ary aggregation functions for $n \geq 2$.

It is also common to classify aggregation functions according to their behavior; we have the following definitions [13].

Definition 3 (Conjunctive aggregation function). *An aggregation function is said to be conjunctive if, for every $\mathbf{x} \in \mathbb{I}^n$, it holds that $f(\mathbf{x}) \leq \min(\mathbf{x})$.*

Definition 4 (Averaging aggregation function). *An aggregation function is said to be averaging if, for every $\mathbf{x} \in \mathbb{I}^n$, it holds that $\min(\mathbf{x}) \leq f(\mathbf{x}) \leq \max(\mathbf{x})$.*

Definition 5 (Disjunctive aggregation function). *An aggregation function is said to be disjunctive if, for every $\mathbf{x} \in \mathbb{I}^n$, it holds that $f(\mathbf{x}) \geq \max(\mathbf{x})$.*

Definition 6 (Mixed aggregation function). *An aggregation function is said to be mixed if it behaves differently on different parts of its domain, and thus does not belong to any of the classes hitherto presented.*

The literature on aggregation functions is extremely rich, and entire books have been written on the subject. As a general introduction, we suggest the books by Beliakov et al. [13] and Torra and Narukawa [14]; other important references can be found in the paper by Campanella and Ribeiro [1].

2.2 Dynamic MCDM framework

Having briefly presented the structure of static MCDM methods, we shall now introduce the dynamic MCDM framework proposed by Campanella and Ribeiro [1]. Its most prominent feature is the addition of *feedback* to the decision process, a critical aspect of how humans reach a decision, even in situation where the problem is fully specified [15]. The operations performed at each decision moment are schematically depicted in Figure 1, and will be fully described in the rest of this section.

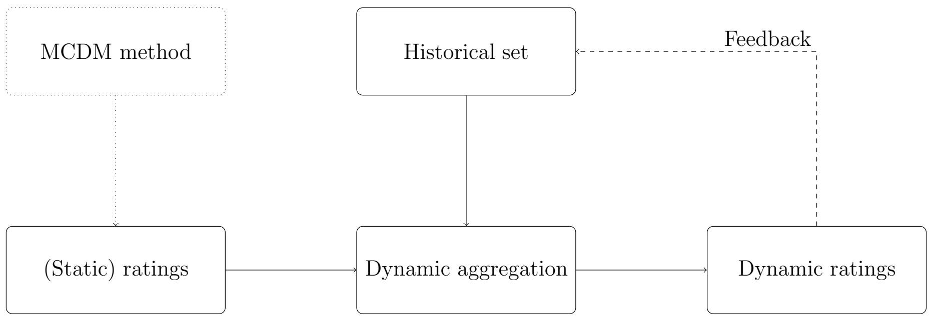

Fig. 1. Operations performed at each decision moment $t \in \mathcal{T}$ in the dynamic MCDM framework of Campanella and Ribeiro [1]: (static) ratings, computed using some MCDM method, are aggregated with information stored in the historical set to produce dynamic ratings, which are then used to update the historical set.

To present the framework, let us introduce the following notation. We shall denote by $\mathcal{T} = \{1, 2, \ldots\}$ the (possibly infinite) set of discrete decision moments, and by $\mathcal{A}_t$ the set of alternatives that are available at each decision moment $t \in \mathcal{T}$. Furthermore, we assume that a static MCDM method is being used at each decision moment $t \in \mathcal{T}$ to compute ratings for each available alternative in $\mathcal{A}_t$, and that these (static) ratings are represented by the function $r_t : \mathcal{A}_t \to \mathbb{I}$. Note that we are only interested in the final aggregated value associated to each available alternative, meaning that criteria and, possibly, weights may vary among decision moments. Moreover, since we are dealing with a constantly changing set of alternatives, we have replaced the more common matrix notation presented earlier in the text with an appropriate set notation; when convenient, it is of course possible to carry out computations for a single decision moment in matrix form.

The dynamic nature of the decision process is dealt with by means of a feedback mechanism, controlled by an aggregation function f that makes use of an *historical set* of alternatives – its "memory" – defined as follows.

Definition 7 (Historical set). *The* historical set *of alternatives at decision moment $t \in \mathcal{T}$ is a subset of all alternatives that have ever been available up to and including that decision moment,*

$$\mathcal{H}_t \subseteq \bigcup_{s \leq t} \mathcal{A}_s, \quad s, t \in \mathcal{T}. \tag{5}$$

Remark 3 (Retention policy). In practical applications, the historical set is updated incrementally. Let us define $\mathcal{H}_0 = \emptyset$ by convention; at each decision moment $t \in \mathcal{T}$, the historical set can thus be defined as follows,

$$\mathcal{H}_t \subseteq \mathcal{A}_t \cup \mathcal{H}_{t-1}, \quad t \in \mathcal{T}. \tag{6}$$

It is therefore necessary to define a *retention policy* that can be used to select alternatives that will be included in the historical set and carried over to the next decision moment.

Let us now define the dynamic rating function $\tilde{r}_t : \mathcal{A}_t \cup \mathcal{H}_{t-1} \to \mathbb{I}$ that, for each decision moment $t \in \mathcal{T}$, gives the rating of alternatives that belong to the current set of alternatives, or that have been carried over from a previous decision moment. We can distinguish three cases:

1. if the alternative belongs only to the current set of alternatives, meaning that no historical information is available, its dynamic rating corresponds to its (static) rating;
2. if the alternative belongs to both the current and historical set of alternatives, its dynamic rating is obtained by aggregating its (static) rating with its dynamic rating at the previous decision moment;
3. finally, if the alternative belongs only to the historical set of alternatives, meaning that no updated information is available, its dynamic rating corresponds to the one it had at the previous decision moment.

More formally, we have the following definition.

Definition 8 (Dynamic rating function). *For any alternative $a \in \mathcal{A}_t \cup \mathcal{H}_{t-1}$, the dynamic rating function is defined as follows,*

$$\tilde{r}_t(a) = \begin{cases} r_t(a) & a \in \mathcal{A}_t \setminus \mathcal{H}_{t-1} \\ f(r_t(a), \tilde{r}_{t-1}(a)) & a \in \mathcal{A}_t \cap \mathcal{H}_{t-1} \\ \tilde{r}_{t-1}(a) & a \in \mathcal{H}_{t-1} \setminus \mathcal{A}_t \end{cases}, \tag{7}$$

where f is some associative *aggregation function.*

Remark 4. The associativity requirement ensures that repeated pairwise application of the aggregation function f will yield, at decision moment $t \in \mathcal{T}$, the same result as application over the whole set of past (static) ratings $\{r_s(a), s = 1, \ldots, t\}$; this also means that this computation can be performed incrementally.

Apart from the associativity requirement, any aggregation function can be used for dynamic aggregation, which in this way takes into account satisfaction of criteria not only at the time of decision, but also at previous decision moments. The dynamic MCDM framework, in fact, provides a way of capturing an intrinsic part of dynamic decision making problems – the *temporal profile* of ratings. Choosing appropriate aggregation functions, it is thus possible to reward alternatives that were consistently rated highly in the past, even if their most recent rating is somewhat lower than average, or conversely to favor large and recent increases in rating, ignoring poorer past performance. We will present an example of this mixed behavior in Section 4.

3 Dynamic MCDM for supplier selection

Supplier selection is a typical decision problem that goes beyond simple optimization. Due to its criticality, many authors have focused on the problem of identifying and analyzing supplier selection criteria. Already in 1966, Dickson [16] examined different supplier selection strategies by means of questionnaires that were distributed among selected managers from the United States and Canada. Clearly, as companies become more and more dependent on suppliers, outcomes of wrong decisions become more and more severe: for example, on-time delivery and material costs are both affected by careful selection of suppliers, especially in industries where raw material accounts for a significant part of the total cost [17].

The problem of supplier selection can be naturally modeled as a multiple-criteria decision making problem: businesses express their preferences on suppliers, which are then ranked and selected. In fact, numerous (static) MCDM methods, ranging from simple weighted averaging to complex mathematical programming models, have been applied to the supplier selection problem; among them, we note the work of Chan [18] (see also [19]), which is based on the well-known Analytic Hierarchy Process (AHP) of Saaty [20, 21], and the Data Envelopment Analysis (DEA) method that was originally developed by Charnes et al. [22] and that is now widely used. Regarding the collaborative aspect of supplier selection, however, as noted by Shi et al. [23],

> ...only a few studies have explored the multiple-participant characteristic of the supplier selecting process.

Many authors have also considered integrated approaches that combine two or more techniques, usually in a multiple-step process (for a detailed overview, see [24]).

While these methods are able to deal with criteria as diverse and competing as quality, service, reliability, organization, and other technical issues, they are not as effective in coping with varying supplier performances, and also do not take into account the possibility that the set of available suppliers might be altered, for example because some of them went out of business, while others emerged in the market. Moreover, they do not consider the possibility of a network of collaborating businesses planning their next batch of orders together. The first limitation can be easily addressed by understanding that the problem at hand is a prime example of a dynamic MCDM problem, since businesses *periodically* interact with suppliers and express their preferences. As regards the possibility of planning for more than one business, in the rest of this section we shall present a further extension to the dynamic MCDM framework of Campanella and Ribeiro [1] that, by means of a linear programming model, makes it possible to handle situations in which a number of collaborating businesses face several suppliers. As before, we shall begin by first considering the simpler case of a single business, and then extend it to more complex one of multiple businesses.

3.1 Single business

Let us first consider the case in which a single business has to periodically select one or more suppliers to fulfill its needs for a certain period of time.

As in Section 2, we consider a discrete set of decision moments $\mathcal{T} = \{1, 2, \ldots\}$, and denote by $\mathcal{A}_t$ the set of n alternatives – i.e., suppliers – that are being considered at decision moment $t \in \mathcal{T}$. Note that the number of suppliers needs not be constant along time, as they can be both removed (for example, because they went out of business) and added (for example, because new business opportunities opened up). At each decision moment $t \in \mathcal{T}$, each supplier is also assumed to be assessed by the business according to some set of criteria (such as reliability, speed, and cost) that may also change over time; these assessments are then distilled down to single (static) ratings using some MCDM method, and further aggregated with information stored in the historical set to produce dynamic ratings from which a final ranking can be produced.

The dynamic MCDM framework of Campanella and Ribeiro [1] can thus be applied straightforwardly to the problem of supplier selection, bringing about the improvements over static MCDM methods that were discussed in the previous section.

3.2 Multiple businesses

Let us now consider the more complex case of m businesses that are *collaboratively* planning their orders to a set of n suppliers; the situation is depicted in Figure 2. Note that we require *complete collaboration* among businesses, but not among suppliers; weaker collaborations among businesses could be studied in the context of game theory, though we do not do so here.

At a fixed decision moment $t \in \mathcal{T}$, we assume that each business b_j, $j = 1, \ldots, m$, has rated each supplier s_i, $i = 1, \ldots, n$, using some MCDM method, and that these ratings have been aggregated with historical information into dynamic ratings, as described before; in order to avoid a cumbersome notation, these ratings will simply be denoted by $\tilde{r}_t(i, j)$. Furthermore, we assume that each business has a certain demand $d_t(j)$, $j = 1, \ldots, m$, and that each supplier has a maximum capacity $c_t(i)$, $i = 1, \ldots, n$. The variables of the problem are represented by the quantities $x_t(i, j)$ that business b_j, $j = 1, \ldots, m$, shall order from supplier s_i, $i = 1, \ldots, n$ at decision moment $t \in \mathcal{T}$, as summarized in Figure 3. Clearly, the allocation of orders to suppliers (encoded by the variables $x_t(i, j)$) will change over time as a result of varying ratings, demands and capacities.

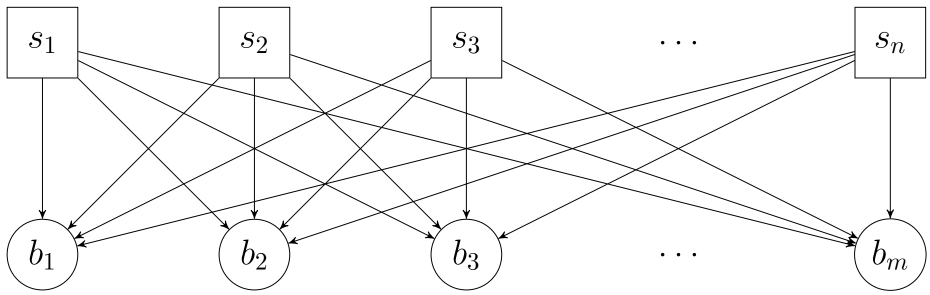

Fig. 2. Network of businesses and suppliers: at decision moment $t \in \mathcal{T}$, each business b_j, $j = 1, \dots, m$, depicted here as a circle, orders a certain quantity $x_t(i, j)$ from supplier s_i, $i = 1, \dots, n$, depicted here as a square.

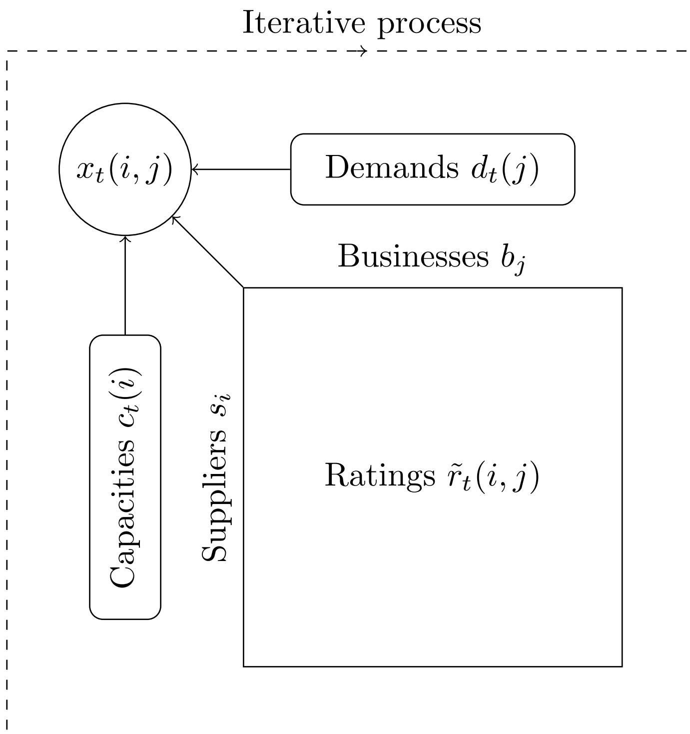

Fig. 3. Collaborative model: different variables enter into the construction of the optimal order quantities $x_t(i, j)$.

Before proceeding, let us introduce the following definitions.

Definition 9 (Satisfaction of a business). *The* satisfaction of a business b_j, *$j = 1, \dots, m$, at time $t \in \mathcal{T}$ with respect to a certain allocation of orders to suppliers is defined as follows,*

$$\sigma_t(j) = \sum_i \tilde{r}_t(i,j)\, x_t(i,j). \tag{8}$$

Definition 10 (Total satisfaction). *The* total satisfaction *of all businesses at time $t \in \mathcal{T}$ with respect to a certain allocation of orders to suppliers is defined as follows,*

$$\sigma_t = \sum_j \sigma_t(j). \tag{9}$$

We are now in a position to define the following linear program that maximizes the total satisfaction, making sure all demands are met and no capacity is exceeded,

$$\begin{aligned} \max \;\; & \sigma_t = \sum_j \sigma_t(j) & & \\ \text{s.t.} \;\; & \sum_i x_t(i,j) = d_t(j) & & j = 1,\ldots,m, \\ & \sum_j x_t(i,j) \le c_t\,(i) & & i = 1,\ldots,n. \end{aligned} \tag{10}$$

This linear program would then be solved at each decision moment $t \in \mathcal{T}$ to determine the optimal order quantities. Note that similar linear programming models usually consider costs instead of satisfactions, and consequently seek to minimize the objective function. Using the proposed approach, on the other hand, each business can consider many more criteria that are then condensed into a single (static) rating; this rating is then further aggregated with historical information to yield a dynamic rating that is finally used in the linear program presented above.

4 Illustrative example

To better understand how the dynamic MCDM framework of Campanella and Ribeiro [1] can be applied to the supplier selection problem with multiple businesses, we now work through a small illustrative example.

For simplicity, we consider a fixed set of four suppliers, named s_1 through s_4, and three businesses, named b_1 through b_3. Moreover, we also fix values for capacities and demands as follows,

$$\begin{aligned} c_1 &= 20, & d_1 &= 30, \\ c_2 &= 20, & d_2 &= 20, \\ c_3 &= 15, & d_3 &= 25, \\ c_4 &= 25. & & \end{aligned}$$

It is easy to verify that the total capacity exceeds the total demand, so that the linear program of Equation (10) will always have a solution. To keep the example small, we consider only three decision moments (which could correspond to monthly supplier evaluations, for example), and present three matrices each time, as shown in Figure 4:

1. the first matrix shows the (static) ratings for each pair of business and supplier;
2. the second matrix gives the dynamic ratings obtained using the dynamic MCDM framework;
3. finally, the third matrix represents the solution to the linear program of Equation (10).

Fig. 4. Layout of matrices presented for each decision moment.

Changes in these matrices are highlighted in bold and indicated by a small arrow; for example, a value that rose from 0.5 to 0.6 would be written as $\mathbf{0.6}^{\uparrow}$, whereas a change in the opposite direction (i.e., from 0.6 to 0.5) would be written as $\mathbf{0.5}^{\downarrow}$.

The aggregation function used for dynamic aggregation is as follows,

$$f(x, y) = \frac{x\,y\,(1-e)}{x\,y + e\,(1-x-y)}, \quad x, y \in \mathbb{I}, \tag{11}$$

where $e \in (0, 1)$ is the so-called *neutral element* that was chosen here equal to 1/2. This aggregation function belongs to the class of *uninorms* [25, 26], which are associative aggregation function that exhibit an interesting kind of mixed behavior known as full reinforcement [27]: they are conjunctive when presented with input values below a given neutral element $e \in (0, 1)$), disjunctive for input values above e, and averaging otherwise (i.e., when one input value is below e and the other one is above it). This particular behavior is exemplified in Figure 5 for different values of e.

At the first decision moment, we have the following situation,

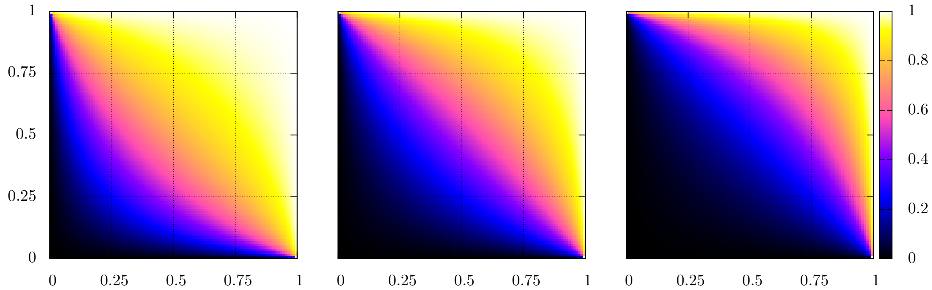

Fig. 5. Contour map of the aggregation function of Equation (11) for three different values of the neutral element e: from left to right, we have $e = 1/4$, $e = 1/2$ and $e = 3/4$.

$$
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 0.40 & 0.50 & 0.70 \\ 0.60 & 0.20 & 0.60 \\ 0.70 & 0.90 & 0.80 \\ 0.90 & 0.70 & 0.90 \end{bmatrix} \end{array}
\quad
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 0.40 & 0.50 & 0.70 \\ 0.60 & 0.20 & 0.60 \\ 0.70 & 0.90 & 0.80 \\ 0.90 & 0.70 & 0.90 \end{bmatrix} \end{array}
\quad
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 0 & 5 & 15 \\ 5 & 0 & 10 \\ 0 & 15 & 0 \\ 25 & 0 & 0 \end{bmatrix} \end{array}
$$

Since no historical information is available, the first and second matrices are of course equal.

At the second decision moment, we have the following situation,

$$
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 0.40 & \mathbf{0.60}^{\uparrow} & 0.70 \\ 0.60 & 0.20 & \mathbf{0.50}^{\downarrow} \\ \mathbf{0.80}^{\uparrow} & 0.90 & 0.80 \\ \mathbf{0.80}^{\downarrow} & \mathbf{0.60}^{\downarrow} & \mathbf{0.80}^{\downarrow} \end{bmatrix} \end{array}
\quad
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} \mathbf{0.31}^{\downarrow} & \mathbf{0.60}^{\uparrow} & \mathbf{0.84}^{\uparrow} \\ \mathbf{0.69}^{\uparrow} & \mathbf{0.06}^{\downarrow} & 0.60 \\ \mathbf{0.90}^{\uparrow} & \mathbf{0.99}^{\uparrow} & \mathbf{0.94}^{\uparrow} \\ \mathbf{0.97}^{\uparrow} & \mathbf{0.78}^{\uparrow} & \mathbf{0.97}^{\uparrow} \end{bmatrix} \end{array}
\quad
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 0 & \mathbf{0}^{\downarrow} & \mathbf{20}^{\uparrow} \\ \mathbf{15}^{\uparrow} & 0 & \mathbf{0}^{\downarrow} \\ 0 & 15 & 0 \\ \mathbf{15}^{\downarrow} & \mathbf{5}^{\uparrow} & \mathbf{5}^{\uparrow} \end{bmatrix} \end{array}
$$

We can clearly recognize the mixed behavior of the dynamic aggregation function in the dynamic ratings it produces: for example, since the value associated with the pair (s_1, b_1) was consistently lower than the neutral element, the corresponding dynamic rating was brought down; conversely, since the value associated with the pair (s_2, b_1) was consistently higher than the neutral element, the corresponding dynamic rating was pushed up. This was also reflected in the amount that business b_1 should order from supplier s_2, which tripled from the previous iteration.

Finally, at the third decision moment, we have the following situation,

$$
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \left[\begin{array}{ccc} \mathbf{0.50}^{\uparrow} & 0.60 & 0.70 \\ \mathbf{0.70}^{\uparrow} & \mathbf{0.10}^{\downarrow} & \mathbf{0.40}^{\downarrow} \\ \mathbf{0.90}^{\uparrow} & 0.90 & 0.80 \\ 0.80 & \mathbf{0.40}^{\downarrow} & \mathbf{0.70}^{\downarrow} \end{array}\right] \end{array}
\quad
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \left[\begin{array}{ccc} 0.31 & \mathbf{0.69}^{\uparrow} & \mathbf{0.93}^{\uparrow} \\ \mathbf{0.84}^{\uparrow} & \mathbf{0.01}^{\downarrow} & \mathbf{0.50}^{\downarrow} \\ \mathbf{0.99}^{\uparrow} & \mathbf{1.00}^{\uparrow} & \mathbf{0.98}^{\uparrow} \\ \mathbf{0.99}^{\uparrow} & \mathbf{0.70}^{\uparrow} & \mathbf{0.99}^{\uparrow} \end{array}\right] \end{array}
\quad
\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array}
\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \left[\begin{array}{ccc} 0 & \mathbf{5}^{\uparrow} & \mathbf{15}^{\downarrow} \\ 15 & 0 & 0 \\ 0 & 15 & 0 \\ 15 & \mathbf{0}^{\downarrow} & \mathbf{10}^{\uparrow} \end{array}\right] \end{array}
$$

The mixed behavior of the chosen dynamic aggregation function is again evident, even though it is now also being smoothed by the availability of more data: understandably, as more information becomes available, dynamically aggregated values tend to reflect the underlying trend in (static) ratings, though modulated in a non-linear fashion that could, for example, make the aggregation very sensitive to abrupt changes.

Even in our small example, it is interesting to observe how the quantities to order vary between iterations: for example, the quantity ordered from supplier s_4 by business b_3 consistently increases throughout the three iterations. Two effects are behind this change: firstly, b_3 rates s_4 better than s_2 in the second iteration, and thus is more willing to place orders there; secondly, b_1 collaborates with the two other businesses to maximize total satisfaction, and is thus willing to share some of the capacity of s_4 with them.

5 Conclusions

In this paper we have proposed an extension of the dynamic MCDM framework of Campanella and Ribeiro [1] to the problem of supplier selection for multiple collaborating businesses. The proposed method uses the dynamic MCDM framework as the dynamic component for individual decision makers, and a linear programming model for the collaborative component. It provides a unified method to assess supplier performances in a context in which businesses share information about suppliers among themselves, suppliers can appear and disappear, and supplier performances change over time.

As directions for future research, it would be important to understand the effect of missing or imprecise data and how it could be handled effectively. Regarding the supplier selection problem, the linear program could be reformulated to include more constraints, such as thresholds that would veto suppliers with consistently low ratings. It would also be interesting to relax the assumption of complete collaboration among businesses, as well as to apply the dynamic MCDM framework to the situation in which a number of businesses must jointly select some suppliers. This problem can actually be restated more broadly as a consensus problem, which was already identified as closely related to the dynamic MCDM framework [1].

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