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Study of Friction Force Model Parameters in Multibody Dynamics

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ABSTRACT — *A parametric study on the most relevant friction force models for multibody dynamics is provided in this work. For this, the Dahl, Reset Integrator, LuGre and Gonthier models are revisited, and the model parameters are identified. The classic single degree-of-freedom mass-spring model is utilized to analyze the influence of each of the parameters. The results are presented through the friction-displacement and friction-velocity relations. Although some of the parameters show different localized effects based on the model used, most of them present similar tendencies in terms of the resulting friction forces. This investigation allows for the identification of the most relevant parameters which can affect a system's dynamic response.*

1 Introduction

Friction is a complex phenomenon which is mainly related to the resistance to relative movement of contacting surfaces. Friction properties have been intensively studied, modeled, and used to predict frictional behavior in any mechanical system. Static friction, stick-slip, Stribeck effect, viscous friction, frictional lag, pre-sliding displacement and break-away force are some of the most important phenomena associated with friction modeling. The first major friction model was proposed by Coulomb [1], in which it was stated that friction force is proportional to normal load, and opposes the relative motion of the contacting surfaces. However, several studies indicated that friction force can reach higher values for null relative velocity, which lead to the definition of a higher coefficient of friction for static cases. Furthermore, Stribeck [2] showed experimentally that the transition between the static and kinetic friction is a continuous process; i.e., for low velocities, the friction force decreases with the increasing of relative velocity. This rise of friction force, when there is a drop in the relative velocity, may lead the contact surfaces to stick, and only if the static friction is overcome, the surfaces slide again. This phenomenon is the so-called "stick-slip". Further researches demonstrated that the friction force is not only function of the relative velocity, but also of the displacement [3]. For any applied tangential force, a displacement occurs and, the contact bond presents a similar behavior compared to an elastic spring, often named "pre-sliding displacement". Moreover, another important characteristic is the "frictional lag", which behaves as an inertia towards the change in friction state. Considering the "Stribeck effect", the friction force is lower for decreasing velocities and higher when the velocity is increasing.

In order to evaluate the friction forces generated during the contact, several different friction force models can be employed [4]. They are typically divided into "static" and "dynamic" models. The first group describes the steady-state behavior of friction, which does not allow capturing of the entire friction phenomena. In contrast, the second group uses extra-state variables to increase the complexity and flexibility of the models in order to include the aforementioned phenomena. These approaches present some differences in terms of captured phenomena, number of parameters, and degree of complexity, among others. The choice of a friction model to implement in a dynamic simulation is not an easy task. Nevertheless, in order to have more complex friction models, it is in general necessary to introduce larger number of parameters to fully describe the physics of the friction phenomena. Sometimes, these parameters are obtained experimentally, which could be a significant limitation for modeling approaches that need the determination of many parameters. However, some of them are chosen empirically, since they cannot be physically measured. Some studies have been conducted to examine the friction modelling through the analysis and comparison of different approaches [4,5], although only few of them have been paying attention to the influence of the parameters variation [6]. This work, hence, focuses on the study of the parameters of the most significant friction force models utilized in multibody dynamics. Most of utilized parameters are shared by several friction force models, but they can result in different behaviors depending on the considered approach. It is thus quite important to understand and to quantify the effect of each parameter in the evaluation of the friction force and resulting dynamic behavior of the system.

2 Description of Dynamic Friction Force Models

This section includes the description of some of the most relevant dynamic friction force models used in the context of multibody dynamics. The dynamic friction models generally present a more advanced approach of describing the friction phenomena when compared to the static ones, which have limitations in capturing the pre-sliding displacement or the frictional lag, just to mention a few. In order to have a detailed description of the several friction properties, the dynamic models utilize extra state variables to evaluate the friction force, as well as they utilize a high number of parameters which have different effects in the response of the system.

Dahl [7] established the first dynamic friction model based on the Coulomb's friction, which was the major precursor of the evolution of friction force models. Coulomb [1] stated that the friction force is proportional to normal load, opposes to the relative motion, and is independent of contact area and velocity magnitude. Moreover, the magnitude of Coulomb's friction F_C can be given by

$$F_{\mathrm{C}} = \mu_{\mathrm{k}} F_{\mathrm{N}} \tag{1}$$

where μ_{k} is the kinetic coefficient of friction, and F_{N} denotes the magnitude of the normal contact force. The Dahl friction model [7] was developed with the aim of describing the friction behavior of ball bearings. In that sense, an analogy with the classical stress-strain curve for materials was considered. Dahl observed that in brittle materials, the difference between the stiction and Coulomb friction is difficult to capture. Ductile materials, however, are more probable of having the stiction behavior, and then exhibiting a decrease in the stress until the Coulomb friction level is reached. Moreover, the friction force was shown to be dependent on relative velocity as well as on the displacement. Dahl model states that when the contacting surfaces are subjected to stress, the friction force increases until rupture occurs. The stress-strain curve can be described as

$$\frac{dF}{dx} = \sigma_0 \left| 1 - \frac{F}{F_{\mathrm{C}}} \operatorname{sgn}(v_{\mathrm{T}}) \right|^{\alpha} \operatorname{sgn}\left(1 - \frac{F}{F_{\mathrm{C}}} \operatorname{sgn}(v_{\mathrm{T}}) \right) \tag{2}$$

with

$$\operatorname{sgn}(v_{\mathrm{T}}) = \begin{cases} v_{\mathrm{T}} / |v_{\mathrm{T}}| & \text{if } v_{\mathrm{T}} \neq 0 \\ 0 & \text{if } v_{\mathrm{T}} = 0 \end{cases} \tag{3}$$

where F denotes the friction force, x is the displacement, σ_0 represents the stiffness coefficient, v_{T} is the relative tangential velocity of the contacting surfaces, and α is a parameter that defines the shape of the material curve. This last parameter depends on the material, and usually varies between 0 and 1 for brittle materials, and is higher than 1 for ductile materials. From the analysis of Eq. (2), it can be stated that when F tends to F_C, the derivative tends to zero. Thus, it can be concluded that the magnitude of the friction force does not exceed F_C.

Equation (2) can be transformed into a time derivative. Furthermore, introducing the state variable z, and assuming that $F = \sigma_0 z$, it can be rewritten as

$$\frac{dz}{dt} = \left| 1 - \frac{\sigma_0}{F_{\mathrm{C}}} z \operatorname{sgn}(v_{\mathrm{T}}) \right|^{\alpha} \operatorname{sgn}\left(1 - \frac{\sigma_0}{F_{\mathrm{C}}} z \operatorname{sgn}(v_{\mathrm{T}}) \right) v_{\mathrm{T}} \tag{4}$$

It can be observed from Eq. (4) that when the system reaches the steady state, the friction force is

$$F = F_C \operatorname{sgn}(v_T) \tag{5}$$

which is the mathematical expression for the Coulomb friction model.

Haessig and Friedland [8] proposed an evolution of the Dahl model, which considers that the friction force is originated by the elastic and plastic deformations of the surface asperities. This model takes into account the phenomenon of static friction for the sticking phase, therefore, the magnitude of static friction is evaluated by

$$F_S = \mu_s F_N \tag{6}$$

where μ_s denotes static coefficient of friction.

Each contact is modeled as a bond between two bristles. The reset integrator model does not allow for the bond to break, which means that when the strain of a connection increases until reaching the rupture point, the model ensures that it is kept constant. This model uses the average of bristle deflection, z, to determine the strain in the bond and to account the stiction, as follows

$$\frac{dz}{dt} = \begin{cases} 0 & \text{if } (v_T > 0 \ \wedge \ z \geq z_0) \ \vee \ (v_T < 0 \ \wedge \ z \leq -z_0) \\ v_T & \text{otherwise} \end{cases} \tag{7}$$

Similar to other friction models, the reset integrator model is also composed of two state equations, one for "sticking mode" and another one for "sliding mode". The transition between those two phases occurs when the deflection reaches its maximum value, z_0. This friction force can then be defined as follows

$$F = \begin{cases} \sigma_0 (1+a) z + \sigma_1 \dfrac{dz}{dt} & \text{if } |z| < z_0 \\ \sigma_0 z_0 \operatorname{sgn}(z) & \text{if } |z| \geq z_0 \end{cases} \tag{8}$$

where σ_1 is the damping coefficient that introduces some physical meaning by having damping oscillations and viscous friction effects, a denotes the coefficient pertaining to the stiction, and σ_0 is the contact stiffness. The values of the stiction coefficient, a, and the maximum deflection, z_0, are not independent parameters for this model, since they can be obtained by the following relations, respectively,

$$a = \mu_s / \mu_k - 1 \tag{9}$$

$$z_0 = F_C / \sigma_0 \tag{10}$$

It is worth noting that this friction force model presents a discontinuity when the analysis changes between sticking and sliding situations.

The LuGre model was originally proposed by Canudas de Wit et al. [9], and can be considered as an extension of the Dahl model [7]. This model is capable of capturing the Stribeck and stiction effects. In a simple way, this model considers friction as the result of the interactions of the surfaces bristles. When a force is applied, the bristles start to deform with spring behavior during the sticking phase. Then if the force is sufficiently large, the bodies start to slip. The model is described as

$$\frac{dz}{dt} = \left(1 - \frac{\sigma_0}{g(v_T)} z \cdot \operatorname{sgn}(v_T) \right) v_T \tag{11}$$

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + f(v_T) \tag{12}$$

where σ_0 is the stiffness of the bristles, σ_1 denotes the damping of the bristles, $f(v_T)$ is an arbitrary function that describes the viscous effect, and $g(v_T)$ is an arbitrary function that accounts for the Stribeck effect as

$$g(v_T) = F_C + (F_S - F_C) e^{-(v_T/v_S)^2} \tag{13}$$

where F_C is as before the Coulomb friction, F_S denotes the static friction, and v_S represents the characteristic velocity of the Stribeck friction [2]. For $f(v_T)$, typically a linear viscous friction is considered, that is

$$f\left(v_{\mathrm{T}}\right)=\sigma_2 v_{\mathrm{T}} \tag{14}$$

For a constant velocity, that is, when the system reaches the steady state (dz/dt=0), the expression to the friction force can be reduced to

$$F=g\left(v_{\mathrm{T}}\right)\operatorname{sgn}\left(v_{\mathrm{T}}\right)+f\left(v_{\mathrm{T}}\right) \tag{15}$$

Gonthier et al. [10] introduced a dynamic friction model based on LuGre's approach [9]. This model considers a force from the bending of the bristles, given by

$$F_{\mathrm{br}}=\sigma_0 z+\sigma_1\frac{dz}{dt} \tag{16}$$

where σ_0 is the stiffness coefficient, and σ_1 denotes the damping coefficient. To ensure a smooth transition between the stick-slip friction regimes, an auxiliary parameter is defined as,

$$s=e^{-\left(v_{\mathrm{T}}/v_{\mathrm{S}}\right)^2} \tag{17}$$

where v_{S} is the Stribeck velocity. When the bodies are sticking $(s=0)$, the deformation rate will be equal to the relative velocity, while for the sliding regime $(s=1)$, the resultant friction force approaches the Coulomb regularized friction force, F_{CR},

$$\frac{dz}{dt}=sv_{\mathrm{T}}+\left(1-s\right)\left(\frac{1}{\sigma_1}F_{\mathrm{CR}}-\frac{\sigma_0}{\sigma_1}z\right) \tag{18}$$

The Coulomb regularized friction is always along the relative velocity direction, and can be evaluated as

$$F_{\mathrm{CR}}=F_{\mathrm{C}}\mathrm{dir}_{\epsilon}\left(v_{\mathrm{T}},v_{\epsilon}\right) \tag{19}$$

where $\mathrm{dir}_{\epsilon}(v_{\mathrm{T}},v_{\epsilon})$ returns the sign of the relative velocity, and it smooths out the velocity oscillations for values under a certain tolerance, v_{ϵ}, in order to diminish the discontinuities in the vicinity of the null velocity. The tolerance velocity is commonly approximated by $v_{\epsilon}=0.01v_{\mathrm{S}}$, and hence

$$\mathrm{dir}_{\epsilon}\left(v_{\mathrm{T}},v_{\epsilon}\right)=\begin{cases}\dfrac{v_{\mathrm{T}}}{\left|v_{\mathrm{T}}\right|} & \text{if } \left|v_{\mathrm{T}}\right|\geq v_{\epsilon}\\[2ex] \dfrac{v_{\mathrm{T}}}{v_{\epsilon}}\left(\dfrac{3}{2}\dfrac{\left|v_{\mathrm{T}}\right|}{v_{\epsilon}}-\dfrac{1}{2}\left(\dfrac{\left|v_{\mathrm{T}}\right|}{v_{\epsilon}}\right)^3\right) & \text{if } \left|v_{\mathrm{T}}\right|<v_{\epsilon}\end{cases} \tag{20}$$

This approach includes a temporal lag associated with the dwell-time dependence. To capture that phenomenon, a new state variable is defined as

$$\dot{s}_{dw}=\begin{cases}\left(s-s_{dw}\right)/\tau_{dw} & \text{if } s-s_{dw}\geq 0\\ \left(s-s_{dw}\right)/\tau_{\mathrm{br}} & \text{if } s-s_{dw}<0\end{cases} \tag{21}$$

where τ_{dw} is the dwell-time dynamics time constant, and $\tau_{\mathrm{br}}=\sigma_1/\sigma_0$ is the bristle dynamics time constant. The time constants should be set according to the desired time delay, a large one for sticking, and a small time delay for sliding. Thus, the maximum friction force can be defined as

$$F_{\max}=F_{\mathrm{C}}+\left(F_{\mathrm{S}}-F_{\mathrm{C}}\right)s_{dw} \tag{22}$$

where F_{C} and F_{S} are the Coulomb and static friction, respectively. Thus, this friction force can be expressed as

$$F=\begin{cases}F_{\mathrm{br}}+\sigma_2 v_{\mathrm{T}} & \text{if } \left|F_{\mathrm{br}}\right|\leq F_{\max}\\ F_{\max}\operatorname{sgn}\left(F_{\mathrm{br}}\right)+\sigma_2 v_{\mathrm{T}} & \text{if } \left|F_{\mathrm{br}}\right|>F_{\max}\end{cases} \tag{23}$$

where σ_2 is the viscous damping coefficient. The use of this model results in a set of ordinary differential equations that are quite stiff at low relative velocities and cannot be solved using explicit ODE solvers.

3 Single Degree-of-Freedom Spring-Mass Model with Friction

In this section, the classical 1-DoF mass-spring system shown in Fig. 1 is utilized as an illustrative example of application to evaluate the effect of the friction parameters. Since the dynamic response of this system is highly dependent of the evaluation of frictional forces, it is commonly used as a benchmark for validation of friction force models [4, 5, 8, 10]. The system is constituted by a block with mass m, which is located on a conveyor belt with constant velocity v_b. The block is connected by a spring element with stiffness k_s. The dimensions of the block are neglected, since the model only considers one degree of freedom.

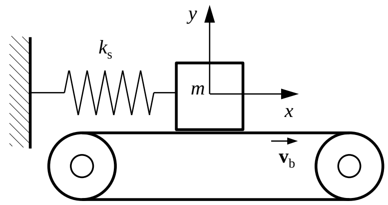

Fig. 1: Representation of the 1-DOF spring-mass model with frictional contact

Before performing the sensitivity analysis on model parameters, a standard simulation was carried out in order to be utilized as a reference. The parameters of the spring-mass system are listed in Tab. 1. The initial conditions of the simulation consider that the block is located at the origin of the coordinate system, and it is moving with equal velocity to the belt. Furthermore, the standard parameters for the described friction force models are displayed in Tab. 2. The utilized values have been chosen according to most of the analyses presented on the literature.

Parameter	**Value**	**Parameter**	**Value**
Mass of the block (m)	1 kg	Time step (Δt)	0.00005 s
Velocity of the belt (v_b)	0.1 m/s	Simulation time	20 s
Spring stiffness (k_s)	2 N/m	Integrator algorithm	Runge-Kutta 4th order

Tab. 1: Simulation parameters for the spring-mass model

Parameter	**Symbol**	**Value**	**Parameter**	**Symbol**	**Value**
Kinetic coefficient of friction	μ_k	0.1	Stiffness coefficient	σ_0	10^5 N/m
Static coefficient of friction	μ_s	0.15	Damping coefficient	σ_1	$10^{5/2}$ Ns/m
Curve shape parameter	α	1	Viscosity coefficient	σ_2	0.1 Ns/m
Stribeck velocity	v_s	0.001 m/s	Dwell-time constant	τ_{dw}	2 s

Tab. 2: Standard parameters for the friction force models

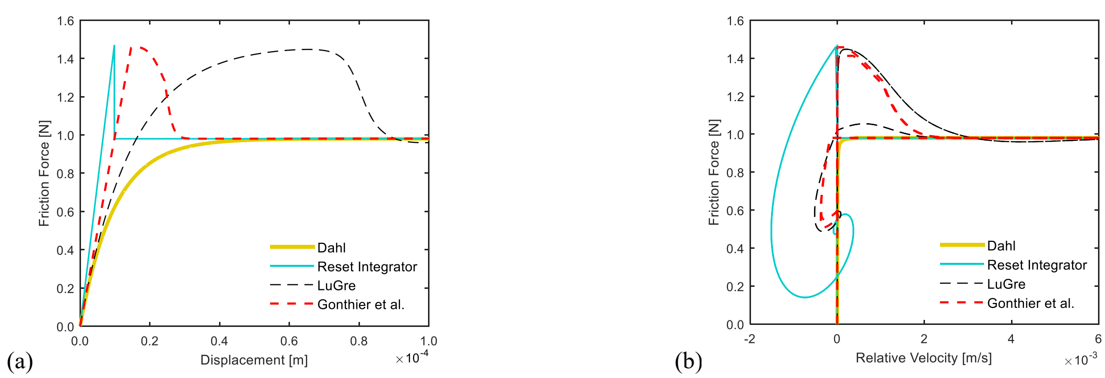

Fig. 2: Friction models behavior: (a) Friction force vs displacement; (b) Friction force vs relative velocity

The results of the dynamic simulations are analyzed through the plots of the friction force versus displacement and friction force versus relative velocity, as depicted in Fig. 2. These plots allow the identification of some frictional phenomena, such as the pre-sliding displacement or frictional lag, which cannot be so easily recognized by the kinematic outputs as the position or velocity of the block. As it was expected, the Dahl model is the only approach that does not capture the static friction behavior. Although the same stiffness coefficient is utilized for all the models, the slope of the pre-sliding displacement is not exactly the same, as shown in Fig. 2a. The reset integrator model has an abrupt transition for sliding phase, while the Gonthier, and mainly the LuGre approaches present a smoother behavior. From Fig. 2b, it can be observed that the reset integrator model has a larger oscillation of the relative velocity and friction force during the sticking phase when compared with remaining ones. The frictional lag can be identified in the LuGre and Gonthier models, since they show higher friction force when the relative velocity is increasing compared to when it is decreasing.

In the following, an analysis of the influence of the several parameters of each model is presented. For the sake of simplicity, the coefficients of friction, either kinetic or static, are not in the scope of this study, since their influences in the friction force have been examined extensively in the literature. Each parameter has been set to half and twice the original value in order to study how they affect the output motion of the system.

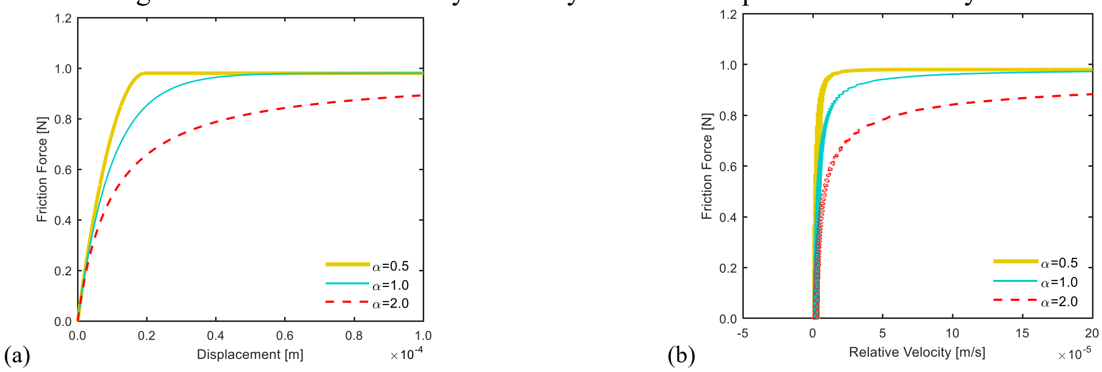

Fig. 3: Influence of curve shape parameter (α) in Dahl model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

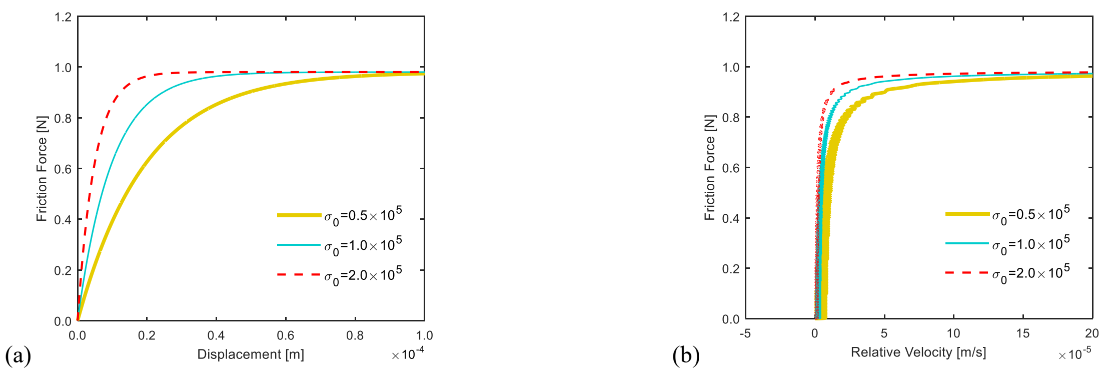

Fig. 4: Influence of stiffness coefficient (σ_0) in Dahl model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

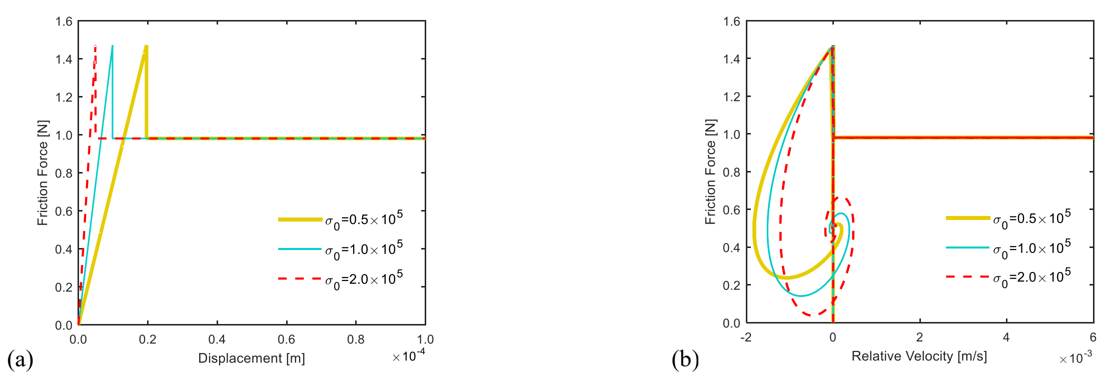

Fig. 5: Influence of stiffness coefficient (σ_0) in Reset Integrator model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

As described in Section 2, the Dahl model utilizes three parameters to be fully defined. Since one them is the kinetic coefficient of friction, only the curve shape parameter and the stiffness coefficient will be addressed. The analysis of the curve shape parameter based on the materials in contact is represented in Figs. 3a and 3b. For a low value of α, it should have properties of a brittle material, while for the larger one, a more ductile behavior is expected. This can be corroborated by Fig. 3a, which shows a slower transition between the elastic and plastic phases of the contact when this parameter increases. Figures 4a and 4b show the influence of the stiffness coefficient σ_0 which is associated to the slope of the pre-sliding displacement. Moreover, the increase of this coefficient diminishes the velocity oscillations in the sticking phase, as is observed from Fig. 4b.

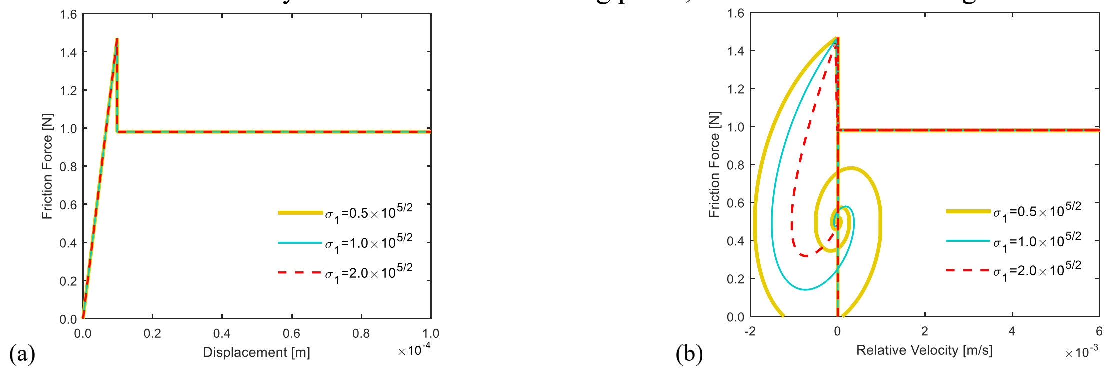

Fig. 6: Influence of damping coefficient (σ_1) in Reset Integrator model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

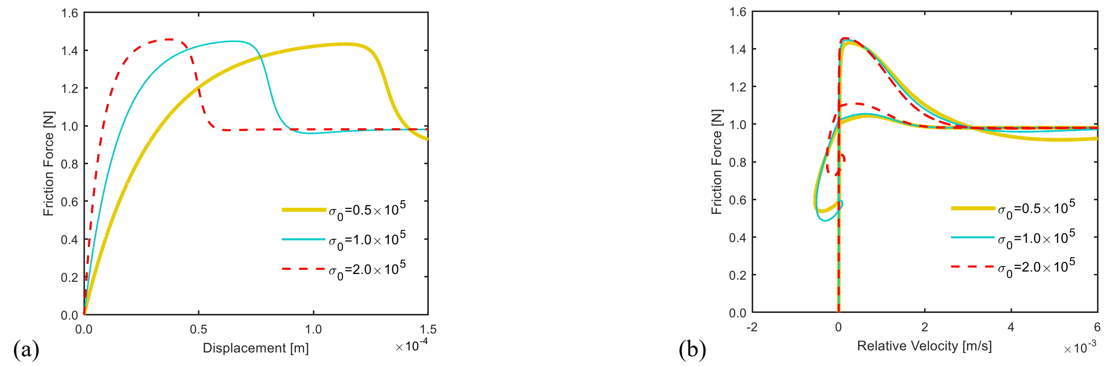

Fig. 7: Influence of stiffness coefficient (σ_0) in LuGre model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

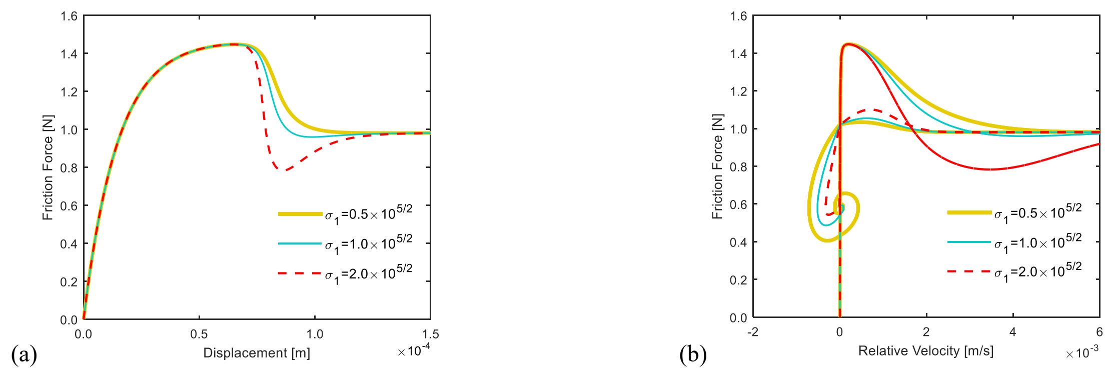

Fig. 8: Influence of damping coefficient (σ_1) in LuGre model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

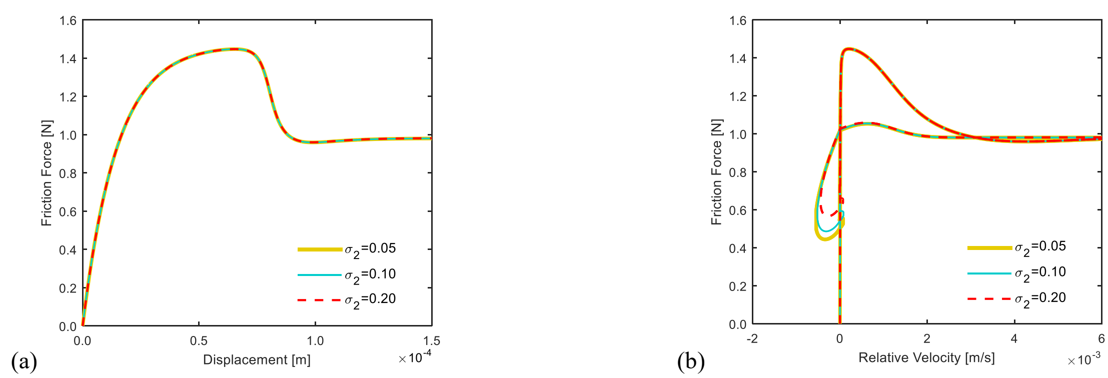

Fig. 9: Influence of viscosity coefficient (σ_2) in LuGre model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

The stiffness and damping coefficients (σ_0 and σ_1) are two of the four parameters utilized by the reset integrator model, and their influence is shown in Figs. 5 and 6, respectively. Similar to the previous model, the stiffness coefficient affects the slope of the pre-sliding displacement curve. In the friction-velocity relation, with the increase of the stiffness, the oscillation of the velocity decreases, but the variation of friction force is higher. In what concerns the damping coefficient, it does not affect the friction-displacement relation, although the reduction of this parameter largely increases the size of the sticking oscillation cycle, as seen in Fig. 6b.

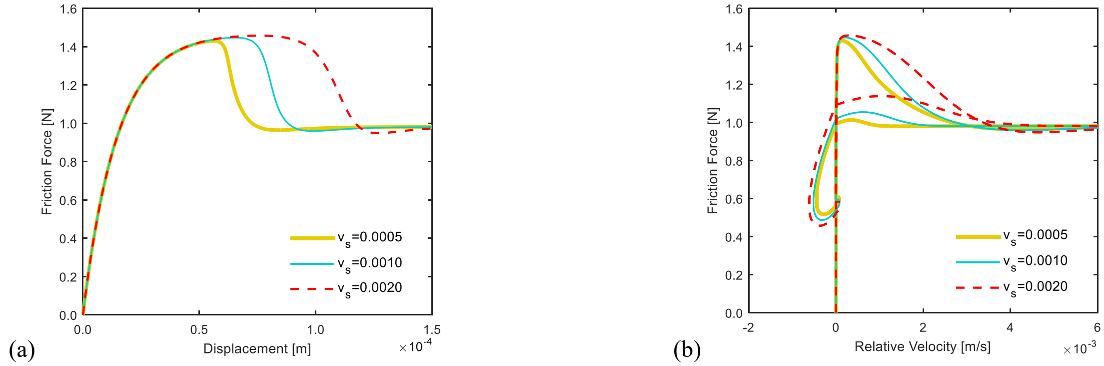

Fig. 10: Influence of Stribeck velocity (v_S) in LuGre model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

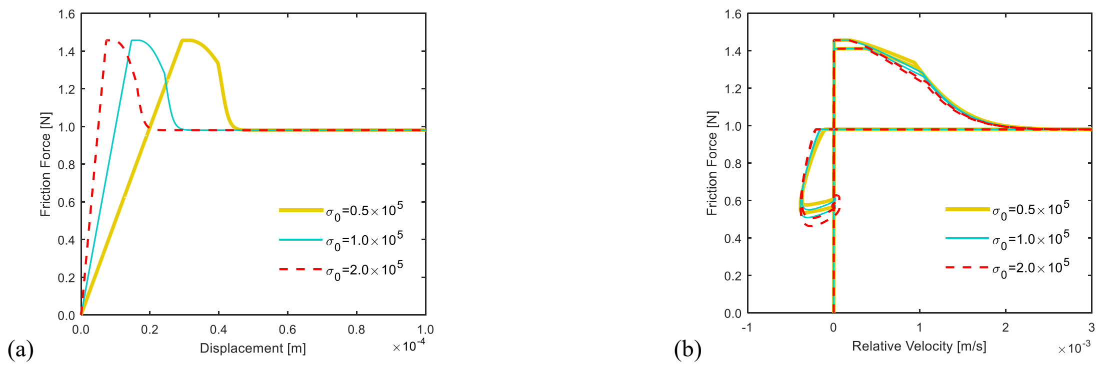

Fig. 11: Influence of stiffness coefficient (σ_0) in Gonthier model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

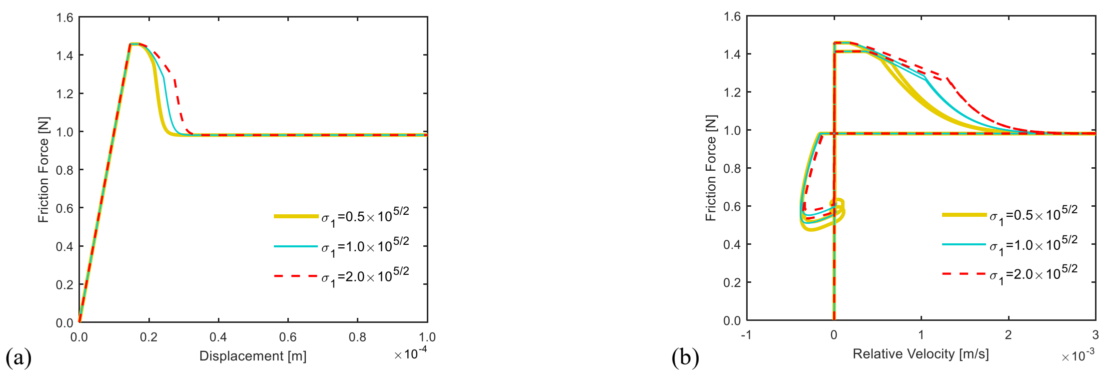

Fig. 12: Influence of damping coefficient (σ_1) in Gonthier model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

As presented previously, the LuGre model has six different parameters. The stiffness, damping and viscosity coefficients, as well as the Stribeck velocity, will be analyzed. Figure 7a shows that the stiffness coefficient σ_0 produces the same result as in the Dahl and reset integrator models, by controlling the slope of the pre-sliding displacement and therefore, increasing the displacement range of the sticking phase. The increase of this parameter diminishes the effect of frictional lag by having higher friction force when the velocity approaches zero, as shown in Fig. 7b. The damping coefficient σ_1 does not change the pre-sliding displacement, although the lower the parameter the smoother the transition from sticking to sliding regime, as depicted in Fig. 8a. With a high damping coefficient, the friction force can even reach a value lower than the Coulomb friction before starting the sliding phase. Similarly to reset integrator model, as observed in Fig. 8b, this parameter has a direct influence in the size of the friction-velocity oscillation in the sticking mode. The viscosity coefficient σ_2 does not influence the friction-displacement relation, but it mitigates the friction force oscillation in the sticking mode, as represented in Fig. 9b. As it was expected, the increase of the Stribeck velocity increases the dimension of the sticking phase. Figure 10b shows that, for a high Stribeck velocity v_S, it is easier to detect the growth on the friction force when the velocity approaches zero.

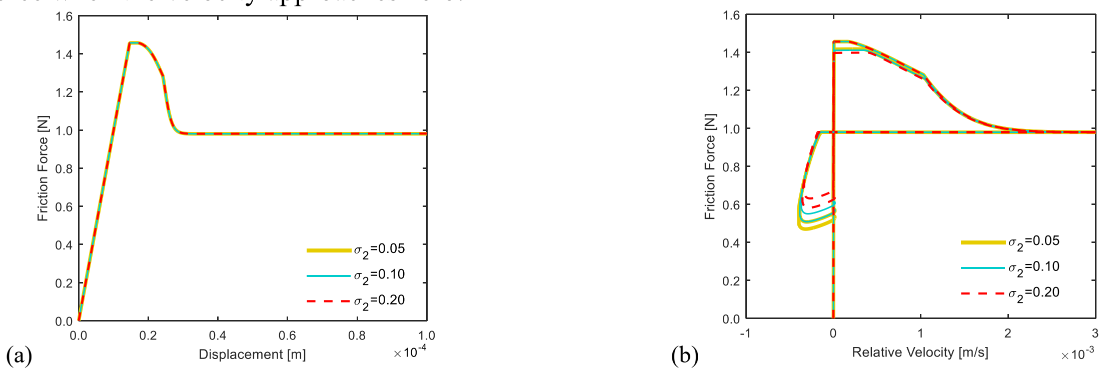

Fig. 13: Influence of viscosity coefficient (σ_2) in Gonthier model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

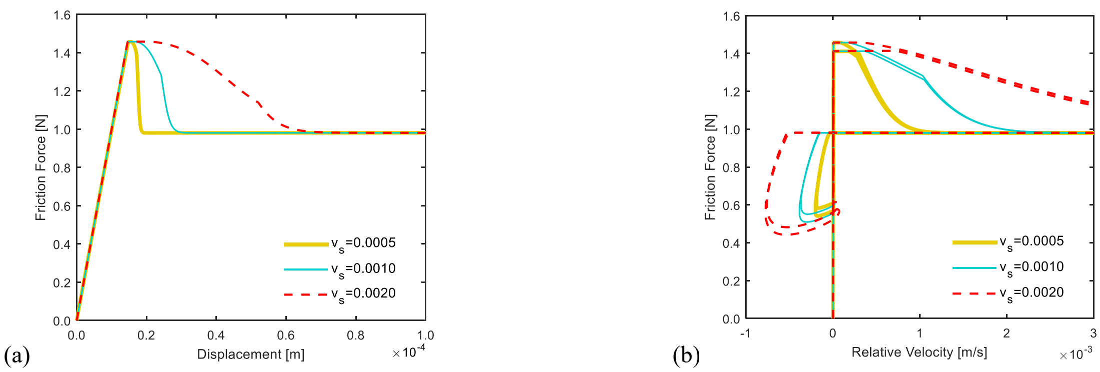

Fig. 14: Influence of Stribeck velocity (v_S) in Gonthier model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

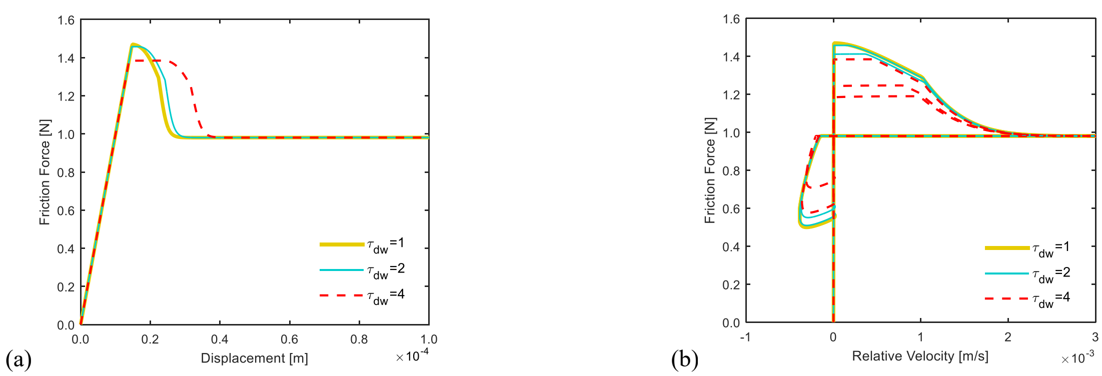

Fig. 15: Influence of dwell-time constant (τ_{dw}) in Gonthier model: (a) Fric. Force vs Displacement; (b) Fric. Force vs Rel. Velocity

The Gonthier model utilizes seven parameters to be fully implemented. The influence of five of them is assessed through the plots of Figs. 11-15. The effects of stiffness coefficient σ_0 corroborates the analogous parameter in the previous models, as it affects the slope of the pre-sliding displacement. In contrast with LuGre model, the increase of the damping coefficient σ_1 makes the transition from sticking to sliding smoother, as shown in Fig. 12a. Moreover, as in LuGre model, the viscosity coefficient σ_2 does not affect the friction-displacement relation, but it reduces the oscillation of the friction force during the sticking phase. The Stribeck velocity v_S enlarges the duration of the sticking phase, as depicted in Fig. 14 This parameter also increases the oscillation of the relative velocity during the sticking regime. Finally, the dwell-time constant τ_{dw} affects the maximum value of friction force, as the increase of this parameter diminishes the maximum friction force, but it enlarges the duration of sticking phase, as illustrated in Fig. 15.

4 Conclusions

A parametric study on some of the most relevant dynamic friction force models used in the context of multibody dynamics has been presented in this work. The models under investigation were first described in detail and their parameters were identified in order to enable their correct implementation. The study of the parameters was performed based on the analysis of the dynamic response of the single degree-of-freedom spring-mass system with frictional contact. From the obtained results, it was observed that some parameters maintain their influence independently of the friction model. However, few of them, such as the damping coefficient, have different effects based on the utilized friction force model. In general, the stiffness coefficient is related with the slope of the pre-sliding displacement curve. The damping coefficient diminishes the friction-velocity oscillations during the sticking phase, although when the magnitude of velocity is increasing, it can extend or decrease the static friction range. Furthermore, the viscosity coefficient does not produce a significant influence in the presented results. The increase of Stribeck velocity typically enlarges the extent of static friction. The parameters in general produce mainly localized effects on the resulting friction forces not their steady-state values. Overall, this study shows that to correctly model all frictional phenomena in a multibody system, an appropriate friction force model must be adopted for which a suitable set of parameters must be selected.

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