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Seismic assessment of Christchurch Catholic Basilica, New Zealand

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ABSTRACT: The paper presents a numerical study on the seismic assessment of the Christchurch Catholic Basilica. The Basilica, also known as Cathedral of the Blessed Sacramento, is located in Christchurch, New Zealand and is listed as category I heritage – International or National significance. The four strong earthquakes events (September 2010 to June 2011) led to a progressive damage and to local collapses. A seismic analysis was carried to study its structural behavior. The seismic assessment involved the preparation and calibration of a three-dimensional numerical model based on the Finite Element Method. The dynamic behavior of the Cathedral was evaluated through pushover analyses with distribution of forces proportional to its mass in each direction. The capacity curves allowed collecting the damage patterns, the most vulnerable elements and the maximum load capacity. Taking into account this information, a comparison between the numerical and real behavior was carried out and an acceptable correlation was obtained.

Keywords: Cathedral, masonry, earthquake, seismic performance, pushover analysis.

1 INTRODUCTION

Cultural heritage buildings outline a set of particular values which symbolically allow the structure to become part of a given cultural identity and continuity [1]. Nowadays, the conservation principles on these particular structures (according to ICOMOS/ISCARSAH) seek to "retain the historic integrity of heritage places by doing as much as necessary to preserve them" [2]. These guidelines take into account centuries of ideological evolution, related to aesthetic and technical issues [3][4], aiming at maintaining the identity and symbolism of these structures. However, the lack of intervention has been pointed out as the main cause of important cultural heritage masonry buildings' collapse, for instance the St. Marco's Campanile at Venice (Italy) in 1902, Civic Tower of Pavia (Italy) in 1989, St. Martinus Church at Kerksken (Belgium) in 1990, among others [5].

It is known that existing masonry buildings present, in general, high seismic vulnerability, which is related with its (i) low resistance to horizontal forces, (ii) lack of capacity to dissipate energy [6][7] and (iii) absence of seismic requirements in the time of its construction [7]. Furthermore, the accumulated damage due to long-term masonry behavior, load conditions changes such as fire, floods, foundation settlements or accidental overloads had also stability implications [8]. The intervention process on these constructions is complex, not only due to lack of structural information but also due to its intangible importance. An intervention based on scientific knowledge is less exposed to inadequate actions. In this context and in order to minimize misunderstandings, the methodology should be flexible and iterative, including several studies, such as a historical one, inspections, monitoring

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actions and structural analysis [9][4]. The present paper describes the analysis of the seismic performance of the Cathedral of the Blessed Sacrament, also known as Christchurch Catholic Basilica. The Basilica is located in New Zealand (NZ) and is listed as category I heritage (International or National significance) [10]. The four relevant earthquakes occurred in a period of nine months, namely: (i) September 4, 2010 earthquake; (ii) December 26, 2010 earthquake; (iii) February 22, 2011 earthquake; and (iv) June 13, 2011 aftershocks. They led to a progressive damage and several local collapses.

A brief description related to the location and main structural features of the Cathedral will be presented. A FEM (Finite Element Method) numerical model was prepared. The material properties, the constitutive law (nonlinear analysis) and material deterioration were collected and inserted in DIANA software [11]. The FEM model was calibrated taking into account the experimental results obtained from the dynamic identification tests, namely the frequencies of the structure. The nonlinear static analysis was used to study the seismic response of the structure. The goal was to analyze the damage patterns, the vulnerable elements and the maximum load capacity, in order to enable a comparison between the numerical and real behavior.

2 HISTORICAL OVERVIEW

The Cathedral of the Blessed Sacrament, Figure 1, was designed by Architect Francis William Petre. The Cathedral is based on Roman style, where the typical arrangement features, i.e. a nave, a transept, chapels, apse, a dome and bell towers are observed. It was built in a time period of only four years, between 1901 and 1905 and Oamaru limestone was used. According to a very common practice of the 19th and early 20th centuries, the walls were made with two-leaf stone masonry, externally, and a concrete core in the middle. The stones are linked through grout-filled cavities and have about 500 mm of thickness. The internal and external claddings have 125 mm thickness of limestone [6]. The thickness of the concrete core is equal to 20 mm. The domes are copper lined and the main one is supported on four large arches, made from no-fines concrete, that spring from the first floor level.



Figure 1. Cathedral of the Blessed Sacramento: North and West façades (left); South façade (right).

The arches are supported by four large columns with an internal spiral above [6]. The main dome is placed above the sanctuary, which is not common in this type of structures. In general, the main dome is located at the crossing of transepts and nave. The nave contains colonnades with diverse capitals and spacious arcades. The Basilica has, in plan, a length of 63 meters (North and South façades) and a width of 33 meters (West and East façades). The highest element is the dome with 36 meters followed by the two bell towers with 32 meters, see Figure 2 [12].



Figure 2. Basilica of the Blessed Sacramento: West elevation (left); plan (right).

In order to assure an appropriated behavior for seismic events, a structural study was performed in 2002, adopting basic preliminary analyses and engineering practical knowledge for the conclusion [6]. The strengthening process introduced: (i) a new reinforced concrete slab (RC) with 100 mm of thickness in the first floor and roof level, aiming at obtaining the diaphragm action; (ii) a steel brace with 32 mm of thickness and RC ring-beams with a section 200 x 600 mm² in the front of the two bell towers, above and below the window openings; (iii) RC ring beams with a section of 200 x 600 mm² at the top section of the main dome, above and below the windows; (iv) the post-tensioning at alternate columns of the nave colonnade and the grouting of external parapets and ornaments; (v) the attachment of the gable ends of the roof, the transepts and ornaments [6].

The strengthening operation had as main objective the increasing of the building strength capacity. The adopted techniques are focused on the weakest elements of the buildings. This decision allowed to improve the building response to the required standard values of lateral load capacity, resisting to the legal minimum of 0.05 g (g is the gravitational acceleration) and 20% of the code value [6].

3 DAMAGE OBSERVED

3.1. Earthquake of September 4, 2010

The first earthquake occurred in the early morning of 4th September, with a magnitude of M7.1. The epicenter was located near Darfield town, a region of Canterbury located 40 km west of Christchurch [13]. The earthquake caused moderate damage to Christchurch Catholic Basilica. The walls were affected, with minor cracking as well as significant displacements at the stones of the West sacristy wall and evidence of an out-of-plane movement at the middle column of the North bell tower (eastern elevation). Despite that, cracks were also observed in retrofitted portions, as the underside of the first floor diaphragm all around the nave and the main dome, with shear cracks at the joints [12][14]. After the visual survey, the building was tagged with a red placard, which prohibits its usage [15].

3.2. Earthquake of February 22, 2011

The earthquake of February 22 of 2011 was, quantitatively, less severe than the September 4 (2010) earthquake, with a magnitude of 6.3 instead of 7.1 (ML). However, qualitatively, the 2011 earthquake caused more damage in Christchurch, possibly due to the vulnerability of many buildings after the damage induced by the first earthquake. The higher damage can also be related, according to [16] [17], to the proximity of the site, to the shallowness of the rupture and to the weak underlying soils. The main damage induced by the February 22, 2011 earthquake in the Basilica are described next [12] [15][16][17][18], see Figure 3 and Figure 4: (i) North and South tower belfries collapsed. Failure occurred in a North-West and South-West direction, respectively; (ii) Similarity to the North and South elevation, severe shear cracking behind the bell towers is visible. Spalling of the ground piers (due to in-plane rocking), large displacements and cracking around the windows and doors also occurred; (iii) Cracking of the outward wall of the North and South transepts. The North wall had a shear crack

around one window on the ground level, which is not-significant comparatively to the South one; (iv) Significant damage at the main dome drum occurs, with a permanent lateral displacement of approximately 75 mm and a vertical settlement of 100-200 mm, together with collapse of the supporting North arch and severe damage to the South arch; (v) For the two levels that integrate the sanctuary, below the Dome and Rotunda, some concrete flat roof areas of the upper level (retrofitted in 2004), collapsed and present large shear cracks.



Figure 3. West façade of the Christchurch Catholic Basilica: damage at the two bell towers (left); Detail of the West wall collapse of the South bell tower (middle); Collapse occurred in the North arch of the sanctuary (right).



Figure 4. Severe damage to the South arch spandrel (left); Collapse of a part of the RC sanctuary flat (middle); Shear cracks around the upper arcade of the North elevation (right).

3.3. Aftershocks of June 13, 2011

The aftershocks occurred on June 13 of 2011 and induced further damage to the buildings, already vulnerable after the main earthquakes of September 2010 and February 2011 [13]. In the case of the Christchurch Catholic Basilica, the two aftershocks had several repercussions. Besides of damage being aggravated, these events led to the necessity of changing the prior strategies for the main dome removal [17].



Figure 5. Collapse of a North of the adjacent RC flat roof and interior arch (left); Damage along the West arch (middle); Developing of a diagonal block failure in the South transept façade (right).

The main structural consequences of the aftershocks are described (see Figure 5) as: (i) A portion of the North Rotunda wall, parts of the adjacent reinforced concrete flat roof and the North interior arch, already with severe damaged, collapsed; (ii) The East and West arches suffered significant damage around the keystone; (iii) The South transept façade suffered additional damage, exhibiting cracking and developing a diagonal block failure. A crack across the transept roof level, with 25 mm of width, is now present.

4 NUMERICAL MODEL PREPARATION

The numerical model of Christchurch Catholic Basilica could be prepared according to different approaches, as there are several strategies to idealize masonry structures. The differences are related to material and structural behavior assumptions, number of input parameters, modelling efforts, computational time required, post-processing results, among others aspects. Simplified approaches, specially focused on collapse mechanisms, as the kinematic analysis (limit analysis) [19] and the analyses based on macro-elements [19], can hardly replace the more advanced tools in the analysis of complex study cases, as the present case.

Finite element methods (FEM) [20][21] are nowadays a very common tool, mainly due to the computational evolution [22]. Masonry structures could be assumed in FEM as continuous isotropic/orthotropic models, which represent a homogeneous material, or micro-models, where the masonry units and mortar are considered separately [22]. In the present case of study, a FEM numerical model was prepared using the software DIANA [11], assuming masonry as an ensemble of continuous and homogeneous elements (macro-modeling). To represent the Basilica's geometry beam, shell and solid elements were used. For beam elements, a two nodes and three-dimensional beam was chosen, named as L12BE (6 DOF), based on Bernoulli theory. For shell elements, threedimensional degenerated curved shell elements according to Mindlin-Reissner theory were adopted. The adopted triangular (T15SH) and guadrilateral (Q20SH) shell elements are isoparametric elements with linear interpolation. For solid elements, brick (hexahedral HX24L) and wedge (tetrahedral TP18L) elements were used. Elements with more nodes, i.e. quadratic instead of linear, are more computational time demanding but offer a more accurate solution [23]. Nevertheless, owing the considerable dimensions of the Basilica, the use of these elements would greatly increase the number of nodes. Thus, it is expected that the linear elements allow to achieve a solution with an acceptable number of degrees of freedom and avoid a huge numerical model. The model has a total of 36,758 elements and the total number of degrees of freedom is 178,719.

After the earthquake of February 22th, 2011, dynamic identification tests were carried out to estimate the mode shapes and frequencies of the Basilica [12]. It would be expected that this information could be used to calibrate the numerical model. However, the Basilica was already damaged when the experimental tests were performed and, consequently, the calibration process must be carried out for a numerical model with damage.

Motorial	Material Properties			
Material	γ (kN/m³)	E (GPa)	ν	
Two leaf limestone masonry	20	2	0.15	
Reinforced concrete	25	30	0.20	
Steel	78	200	0.25	
Timber	7	11	0.30	

Table 1. Elastic material parameters of the model.

The damage was inserted in the original FEM model according to two methodologies that create discontinuities and weak boundaries in the structure. The damage assessment was achieved by gathering the information from OPUS, an engineering consultant designated by CERA (Canterbury Earthquake Recovery Authority), with images collected during the dynamic identification tests. For the calibration process, several attempts were done in order to approximate the frequency of the 1st numerical with the experimental one. Taking this into account and the information provided by the NZ authorities, the adopted linear material properties are described in **Table** 1.

5 NON-LINEAR STATIC ANALYSIS

The performed non-linear static analysis (pushover analysis) allowed to evaluate the seismic behavior of the Basilica. The seismic forces were simulated by applying horizontal loads, monotonically increased and with a distribution proportional to the mass. These lateral loads were applied to the structure considering its nonlinear properties.

For representing the physical nonlinear behavior, the total strain fixed crack model [11] was adopted. This approach allows describing the tensile, compression and shear nonlinear behavior for a continuum model. In this material model, it is assumed that the cracks are orthogonal and fixed, according to the principal strain directions at the onset of cracking, and this direction remains unchanged during the load increase. Furthermore, a shear retention factor, which defines the shear behavior after cracking, was defined [24]. The models that represent each material in terms of stress-strain relationship were defined too. In the present case study, both masonry and concrete behavior are given by a parabolic stress-strain relationship for compression and an exponential stress-strain relationship for tension.

The load direction taken into account is in agreement with the global coordinate system of the numerical model and followed the positive and negative axis in the longitudinal direction of the Basilica (\pm X). In the transversal direction, only the positive direction was studied (+Y) (Figure 7). A total of five control nodes were defined for the analysis. The nodes are positioned in elements with higher relative horizontal displacements. These control nodes are identified in Figure 7. The damage assessment takes into account the maximum principal tensile strains, which is a good indicator of tensile damage (cracking).



Figure 6. Numerical model: Applied load directions (longitudinal – X; transversal – Y) (left); Nodes of control (right).

5.1. Non-linear material properties

The used nonlinear material parameters are defined in Table 2 (the non-linear behavior was only assumed for masonry and concrete). The information collected from the New Zealand authorities studying the Basilica were adopted, specifically the compressive and tensile strength for masonry and

concrete. For the fracture energy, the recommendations presented in the literature were adopted. The values for concrete are estimated through Model Code 90 [22]. The Modified Newton-Raphson iterative method was adopted for the pushover analysis.

Material	Compressive strength (f _c) [kPa]	Tensile strength (f _t) [kPa]	Compressive fracture energy (G _{fc}) [kN/m]	Tensile fracture energy (G _f) [kN/m]
Masonry	8000	100	12.8	0.120
Concrete	30000	2000	25.0	0.054

Table	2.	Non-linear	material	parameters	for t	the	model
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5.2. Direction +X

The adopted control nodes for the pushover analysis in the direction +X are located at the West tympanum wall and at the top of the main dome. The capacity curves are shown in Figure 7. The capacity curves of the control nodes present nonlinear behavior, in which the linear limit (initial cracking) occurs for a load factor of about 16%. The top of the tympanum West wall (Node 1) has a slightly larger horizontal displacement than the main dome node (Node 2). A horizontal load of about 30% of the structure self-weight was applied, and a maximum displacement of 30 mm for Node 1 and 26.3 mm for node 2 were obtained. It was also observed that the capacity curve of Node 1 does not start from the origin position. This aspect is due to the out-of-plane movement (negative displacement) caused by the vertical loads (self-weight analysis).



Figure 7. Capacity curve in +X direction for nodes 1 and 2 (left and middle); Bell tower stories' displacements for a horizontal load of 0.30g (+X; st.: storey; N3: node 3) (right).

In Figure 7 it is observed that the horizontal in-plane displacement is higher at the top (27 mm). However, the maximum inter-storey displacement (drift) occurs at the first floor, which justifies that the shear-cracks are only visible in bell towers' walls of the first and second storeys. On the other hand, the West tympanum wall develops horizontal cracks at the top of the RC slab reinforcement. This damage is in agreement with the expected failure mode, i.e. the out-of-plane movement of a slender element (bending) leads to high stresses, which cause a rotation line, with out-of-plane rocking of the tympanum.

Figure 8 presents the deformed shape of the pushover analysis in the positive longitudinal direction with the principal tensile strains of the South façade walls. It is clear that the larger displacements are located on the top of the elements. However, the horizontal relative displacements do not have a linear increase through the structure height. The first floor walls present the most important in-plane

displacement (and damage). This aspect is particularly noted in the deformed shape of the bell tower, where the parabolic shape of the first floor is replaced in height by a linear one. This is due to the added elements in the 2004 strengthening works, i.e. the RC slabs in the bell towers and nave, as well as the steel braces and ring beams of the third bell tower floor which improves the seismic behavior of the structure. The principal tensile strains distribution leads to the same conclusion, in which it is observed that the damage concentrates at the first floor walls, i.e. in openings, walls connection (shell elements) and rotunda walls (solid elements).



Figure 8. Principal tensile strains of the South façade walls for a horizontal load of 0.30g in +X direction (the nave roof frame is not represented).

5.3. Direction -X

The maximum horizontal displacement of the pushover analysis in the direction –X occurs at the top of the bell tower and is equal to 30 mm. The horizontal displacements decrease to about the half for elements closer of the sacristy. Significant differences in the capacity curve of Node 1 are observed, in comparison to the +X direction. The node presents the same maximum displacement, but for a load coefficient approximately 5% lower. The same occurs for the initial cracking load, i.e with a coefficient of 13% rather than 16%, see Figure 9. These differences show that the West gable wall has less strength capacity in the -X direction and, consequently, more damage. This can be explained by the confinement action caused by the rotunda walls, which occur in the +X direction (the confinement is only possible due to the modeled roof trusses). This action does not occurred for the –X direction, because the out-of-plane movement of the West façade tympanum led to tensile stresses that cannot be withstand by the rotunda walls, due to the low tensile strength of masonry elements.

Similarly to the +X direction, the relative displacements between the storeys of the bell tower are higher between the first and second ones, as observed by the inter-storey displacements curve of Figure 9. However, even if the horizontal displacement that occurred at the top is 5 mm greater (30mm, in comparison to \approx 25mm), the differences are smaller for this direction and consequently the cracks due to in-plane shear stresses occur, but are not as clear as before. The crack pattern is similar to the one obtained for the +X direction, i.e. a high concentration of cracks in the North, South walls (nave and sacristy) near the openings. It is clear that the damage on the shell elements oriented in the load direction (façade walls) is caused by in-plane shear stresses. Thus, the difference in the – X direction corresponds to the way that cracks propagate (diagonal shear cracks) (Figure 10). Another difference in the damage pattern is highlighted in Figure 10 by a dashed circle located in the connection between the South nave and South stairwell wall.



Figure 9. Capacity curve in -X direction for nodes 1 and 2 (left and middle); Bell tower stories' displacements for a horizontal load of 0.27g (–X; st.: storey; N3: node 3) (right).

Thus, if the +X load direction is vulnerable for the bell tower walls, by creating a crack pattern prone to an out-of-plane mechanism, the -X load direction makes it more likely to happen. In fact, for the performed study, a horizontal load of 0.20 g is the value that initiates cracking at these connections. This is not related to a shear action but it is instead caused by the push-over action in these elements (tensile stresses).



Figure 10. Principal tensile strains of the South façade walls for a horizontal load of 0.27g in -X direction (the nave roof frame is not represented).

5.4. Direction +Y

Concerning the pushover analysis in the direction +Y, different control nodes were used from the previous analysis. Node 1 was not assumed, because the gable West wall is transversally positioned with respect to the Y direction. This wall has a high stiffness and a maximum displacement of 12 mm was obtained for this action, which are much smaller that the displacements obtained in the X direction. In Figure 11 it is possible to observe that for the considered capacity curves the initial cracking load ranges from 10% to 13% of the self-weight. For both nodes a maximum load coefficient of 22% was obtained and it is possible to conclude that the main dome node has, for the same load, a less significant non-linear behavior.



Figure 11. Capacity curve in +Y direction: (a) Node 4; (b) Node 5 (left and middle). Bell tower storeys' displacements (+Y); st: storey; N4: node 4).

The main damage obtained from the +Y direction analysis is presented next. Figure 12 indicates with a dashed line the initial cracking occurrence, caused by the concentration of tensile stresses due to the difference of behavior between structural elements, i.e. the North bell tower has larger displacements than the main West façade wall (if the load had the -Y direction, it would be symmetric). This is valid to the North end of the West entablature and also for the finite elements at the connection between the West façade and North bell tower, where a vertical crack is prone to be active.



Figure 12. Principal tensile strains due to a horizontal load in +Y direction: West façade (left) and East façade (right).

Nevertheless, for a load coefficient of 22% of the self-weight the demand for tensile strength is small, in which the cracking do not have further repercussions on a macro-elements failure collapse. A set of diagonal cracks is also presented in Figure 12, due to in-plane shear stresses near the openings of the West and East elevations and near the walls connections, mainly in the transversal walls of the transepts and in the second floor of the East wall (dashed circles).

On the other hand, interior damage was also observed, for instance the damage in the first floor at the transversal bell tower wall, near the arch passage and near the aisle passage of the second floor. Furthermore, diagonal shear cracks are very clear at the transversal walls of the South and North façades. There is no occurrence of toe crushing and, similarly to what occurs in direction X, the difference between displacements of the first and second storeys is considerable, being the main cause of the concentration of shear cracks in both bell tower walls at the first floor level Figure 12.

6 CONCLUSIONS

The nonlinear static analysis allowed the authors to make remarks about the structural behavior of the Christchurch Catholic Basilica when subject to horizontal loads. A good connection between elements must be guaranteed to improve the distribution of horizontal loads. The so-called box behavior, which is presented in the Basilica (at least in the numerical model) has also a decisive role. The diaphragm effect is not common in historical constructions [25][26][27], but has an important effect in the maintenance of the façades stability. The in-plane stiffness avoids flexible movements of the façades in the in-plan and out-of-plan direction. The most vulnerable direction of the structure is the transversal one, since the maximum applied gravity load coefficient is equal to 20% and 30% for the transversal and longitudinal directions, respectively. The analyses allowed obtaining the cracks patterns and failure mechanisms. Several in-plane and out-of-plane mechanisms were observed, caused by in-plane shear stresses and orthogonal loads, respectively.

Finally, the elements of the main dome wall do not present any relevant damage which leads to the conclusion that the installed RC ring beams in 2002 strengthening have an effective action. In general, it is possible to conclude that the damage obtained from the numerical model is in reasonable agreement with the observed crack pattern after the series of earthquakes. The model also indicates that the structure is unsafe for an earthquake such as the one experienced, in which significant damage would be expected.

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