A SEGURANÇA DAS OBRAS NO BRASIL DISCUTIDA SOB DIFERENTES PONTOS DE VISTA. TRAGA O SEU.

1º CONGRESSO BRASILEIRO DE PATOLOGIA DAS CONSTRUÇÕES

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ANALYTICAL AND EXPERIMENTAL STUDY OF RC SLAB STRIPS STRENGTHENED WITH CFRP LAMINATES

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RESUMO

Um modelo analítico foi desenvolvido para prever a relação força-flecha de estruturas de concreto armado (CA) estaticamente indeterminadas até o seu colapso e o seu desempenho foi avaliado usando os dados obtidos em programas experimentais. Este modelo é baseado no método das forças (flexibilidade), o qual estabelece uma série de equações de compatibilidade de deslocamentos que podem fornecer variáveis desconhecidas. Para se determinar a rigidez à flexão de cada trecho da estrutura, as relações momento-curvatura das seções transversais representativas da estrutura foram obtidas. Este modelo pode ser facilmente implementado e é aplicável a estruturas de CA estaticamente determinadas ou indeterminadas reforçadas segundo a técnica NSM (Near Surface Mounted technique, em língua inglesa) ou pela colagem externa de compósitos segundo a técnica EBR (Externally Bonded Reinforcement technique, em língua externa). Neste trabalho, o desempenho preditivo do modelo foi avaliado por meio da simulação dos ensaios das faixas de laje da série SL30 reforçadas com laminados de carbono aplicados segundo a técnica NSM.

Palavras chave: NSM; CFRP; Reforço à flexão; Lajes; Modelo analítico.

ABSTRACT

To predict the load-deflection response up to the collapse of statically indeterminate reinforced concrete (RC) structures, an analytical model was developed and its predictive performance was appraised by using the data obtained in experimental programs. The proposed approach is based on the force method by establishing a number of displacement compatibility equations that can provide the unknown variables. To determine the tangential flexural stiffness making part of these equations, moment-curvature relationships are determined for the cross sections representative of the structure. This model can be easily implemented and is applicable to statically determinate or indeterminate RC structures strengthened according to the near surface mounted (NSM) or externally bonded reinforcement (EBR) techniques. In this work, the predictive performance of the model was appraised by simulating the SL30 series tests of RC slab strips strengthened with NSM carbon fiber reinforced polymer (CFRP) laminates.

Keywords: NSM; CFRP; Flexural strengthening; Slabs; Analytical model.

1. INTRODUÇÃO

The Externally Bonded Reinforcement, EBR [ACI 440 (2007); FIB - Bulletin 14 (2001)], and the Near Surface Mounted, NSM [Barros and Kotynia (2008), Barros et al. (2007)], are the most used techniques for the strengthening of RC elements. However, when compared to EBR, the NSM technique is especially suitable to increase the negative bending moments (in the intermediate supports, also designated by hogging regions) of continuous RC slabs, since its strengthening process is simpler and faster to apply than other FRP-based techniques [Barros and Kotynia (2008); Bonaldo (2008)]. The effectiveness of the NSM technique for the flexural [Barros and Fortes (2005); De Lorenzis et al. (2000); Carolin (2003); El-Hacha and Rizkalla (2004); Liu el al. (2006); Nordin (2003)] and shear [Barros et al. (2006); Dias and Barros (2008); Dias and Barros (2010)] strengthening of RC members has already been assessed. However, most of the tests were carried out with simply supported members. Although many in situ RC elements are of continuous construction, there is a lack of experimental and theoretical studies in the behavior of statically indeterminate RC members strengthened with FRP materials, and limited information is available dealing with the behavior of continuous structures strengthened according to the NSM technique [Bonaldo (2008); Liu et al. (2006); Liu (2005)]. In this present paper, the effectiveness of an analytical model and its predictive performance was appraised by using the data obtained in experimental programs [Bonaldo (2008) and Dalfré (2013)].

2. MODEL IDEALISATION

Indeterminate structures are being widely used since they can be more economic, safer and develop more ductile behaviour than statically determinate structures. In the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. There are two methods of analysis for statically indeterminate structure depending on the approach selected to establish the system of equations that can derive the unknown variables (Ghali et al., 2003):

1. Force method (also known as flexibility method, method of consistent deformation, flexibility matrix method), where a system of displacement compatibility equations is established, whose number is equal to the unknown redundant supports (extra equations corresponding to selected displacements can also be added).

2. Displacement method (also known as stiffness matrix method), where a system of equilibrium equations is established, whose number is equal to the degrees of freedom of the structure.

In this work, an analytical model based on the force method is proposed. In this method, primary unknowns are forces corresponding to selected redundant supports. To determine simultaneously, not only these forces but also the deflections at the loaded sections, extra displacement compatibility equations are established solving these equations, the redundant forces and the displacement at the loaded sections are determined. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium, as well as the internal forces in the elements forming the structure (Barros, 2004).

2.1 Force method applied to statically indeterminate slab strip of two spans

Figure 1 presents the slab strip used in the experimental program, which is statically indeterminate of one degree, e.g., a displacement compatibility equation corresponding to a reaction support should be established to determine the value of this reaction force. Assuming the principle of superposition of effects can be applied for each relatively small load increment, ΔF , the structure is decomposed into a number of equilibrium configurations. In the present case, three compatibility equations will be established, corresponding to the loaded sections and to the intermediate support, in order to obtain the incremental displacements (Δu_1 and Δu_2) and the incremental reaction (ΔR) due to ΔF (Figure 2).

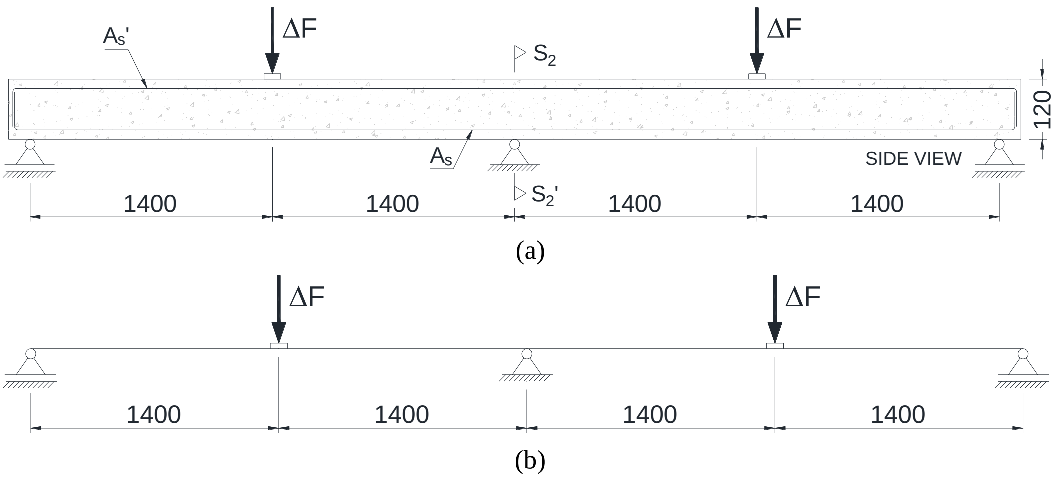


Figure 1: Actual continuous beam (Original)

Figure 2: Basic determinate beam (primary structure) , redundant displacements Δu_1 , Δu_2 and the reaction ΔR .

For each configuration it is determined the deflections corresponding to the applied forces (fictitious ΔF_1 and ΔF_2 forces and unknown reaction ΔR, Figure 3). The terms of the flexibility matrix, $f_{\Delta F_1 \Delta F_1}$, $f_{\Delta R \Delta F_1}$, $f_{\Delta F_2 \Delta F_1}$, $f_{\Delta F_1 \Delta F_2}$, $f_{\Delta R \Delta F_2}$, $f_{\Delta F_2 \Delta F_2}$, $f_{\Delta F_1 \Delta R}$, $f_{\Delta R \Delta R}$ and $f_{\Delta F_2 \Delta R}$, is presented with a generic representation, f_{ij}, that means the displacement in generalized X_i force direction due to the application of an unit load in the X_j direction, is obtained by applying the principle of virtual work to the external and internal forces of the configuration of the X_i forces in the external and internal displacements of the configuration of the X_j forces (Barros, 2004). The diagrams of bending moments (in the present work the work due to axial and shear forces) is neglected for the three configurations of Figure 3 are represented in Figure 4.

From Figure 3 the following three equations of displacements compatibility can be established by applying the principle of superposition effects:

$$\begin{aligned}
\Delta u_1 &= f_{\Delta F_1 \Delta F_1} \times \Delta F_1 + f_{\Delta F_1 \Delta F_2} \times \Delta F_2 + f_{\Delta F_1 \Delta R} \times \Delta R \\
\Delta u_2 &= f_{\Delta F_2 \Delta F_1} \times \Delta F_1 + f_{\Delta F_2 \Delta F_2} \times \Delta F_2 + f_{\Delta F_2 \Delta R} \times \Delta R \\
0 &= f_{\Delta R \Delta F_1} \times \Delta F_1 + f_{\Delta R \Delta F_2} \times \Delta F_2 + f_{\Delta R \Delta R} \times \Delta R
\end{aligned} \tag{2}$$

or

$$\begin{bmatrix} f_{\Delta F_1 \Delta F_1} & f_{\Delta F_1 \Delta F_2} & f_{\Delta F_1 \Delta R} \\ f_{\Delta F_2 \Delta F_1} & f_{\Delta F_2 \Delta F_2} & f_{\Delta F_2 \Delta R} \\ f_{\Delta R \Delta F_1} & f_{\Delta R \Delta F_2} & f_{\Delta R \Delta R} \end{bmatrix} \begin{bmatrix} \Delta F_1 \\ \Delta F_2 \\ \Delta R \end{bmatrix} = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ 0 \end{bmatrix} \tag{3}$$

that can get the following format:

$$[f][F] = [D] \tag{4}$$

where f is the flexibility matrix, F is the vector of applied forces (ΔF_1, ΔF_2 and ΔR are unknown, since the experimental tests were displacement controlled, therefore Δu_1 and Δu_2 are the imposed displacements), and D is the vector of the displacements in the directions of ΔF_1, ΔF_2 and ΔR (where the displacement corresponding to ΔR is null).

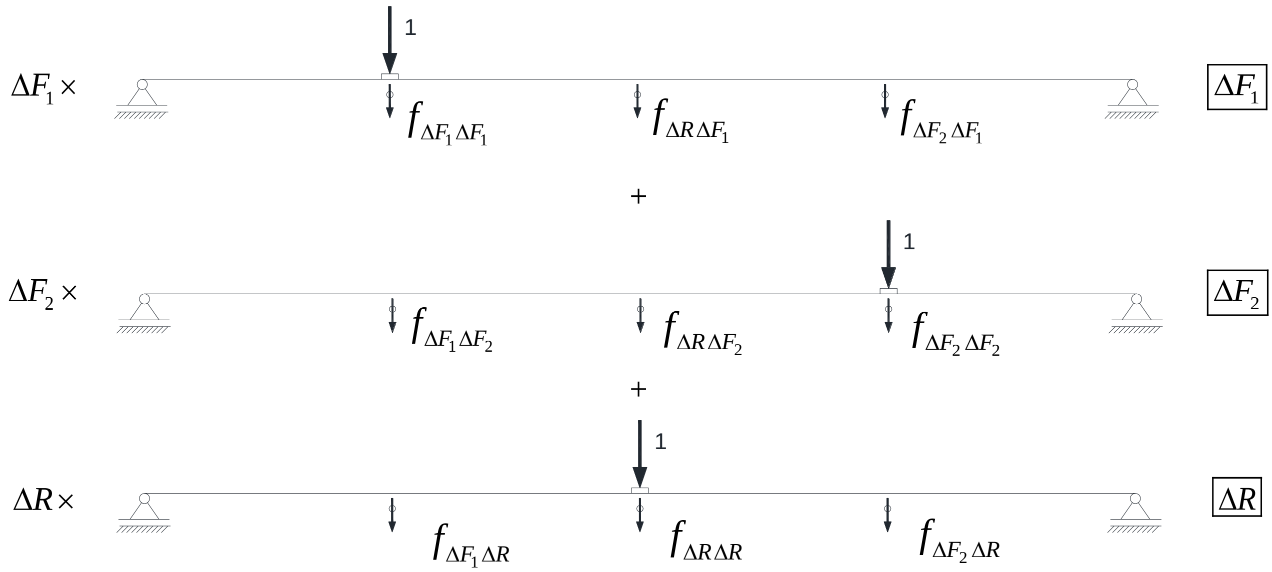


Figure 3: Physical meaning of the terms of the flexibility matrix, based on the displacements for each equilibrium configuration: a) $\Delta F_1 = 1$, b) $\Delta F_2 = 1$, and c) $\Delta R = 1$.

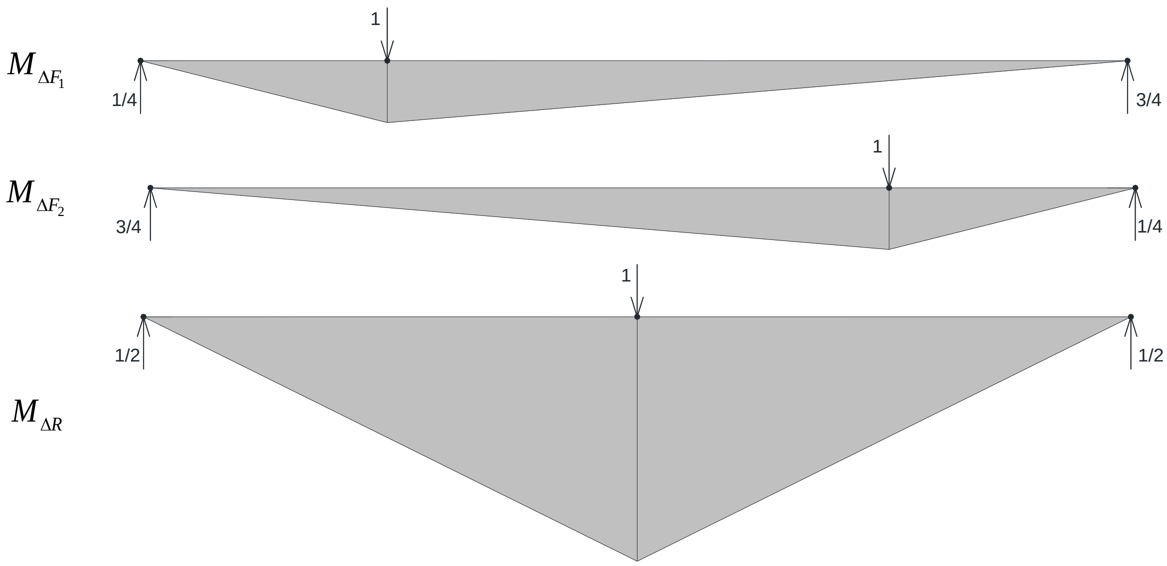
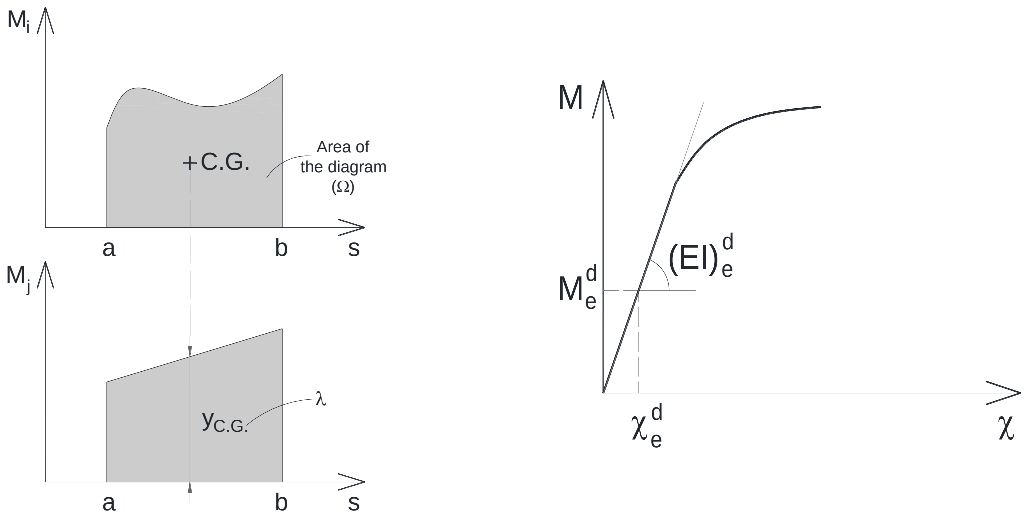


Figure 4: Diagrams of bending moments for the three equilibrium configurations of Figure 3.



(a) (b)

Figure 5: (a) Principle of Bonfim Barreiros's method, and (b) moment-curvature relationship.

3. CASE STUDY – SL30-H

To assess the influence of CFRP NSM flexural strengthening technique, the experimental programs carried out by Bonaldo (2008) and Dalfré (2013), composed of seventeen 120×375×5875 mm^3 RC two-span slabs, was simulated.

3.1 Brief description of the slab strip

The SL30-H is a statically indeterminate RC slab strip designed to assure a moment redistribution percentage, η, of 30% (Bonaldo, 2008). The arrangement of the positive and negative longitudinal steel reinforcement is presented in Figure 6. To evaluate correctly the flexural stiffness of this slab, it is necessary to determine the moment-curvature relationship, $M-\chi$, for each cross section that has distinct reinforcement arrangement. Therefore, each span of the slab strip was discretized in eight different cross-sections, as shown in Figures 7 and 8. The $M-\chi$ of the cross sections was evaluated with the DOCROS computer program [Basto and Barros (2008); Varma (2013)].

According to the model implemented in DOCROS, a cross section is discretized in layers that can have distinct constitutive laws for the characterization of the behaviour of the materials that constitute these layers. It should be noted that the cross section can be composed of plain concrete and can include steel and FRP laminates/bars. A detailed description of DOCROS can be found elsewhere [Varma (2013)].

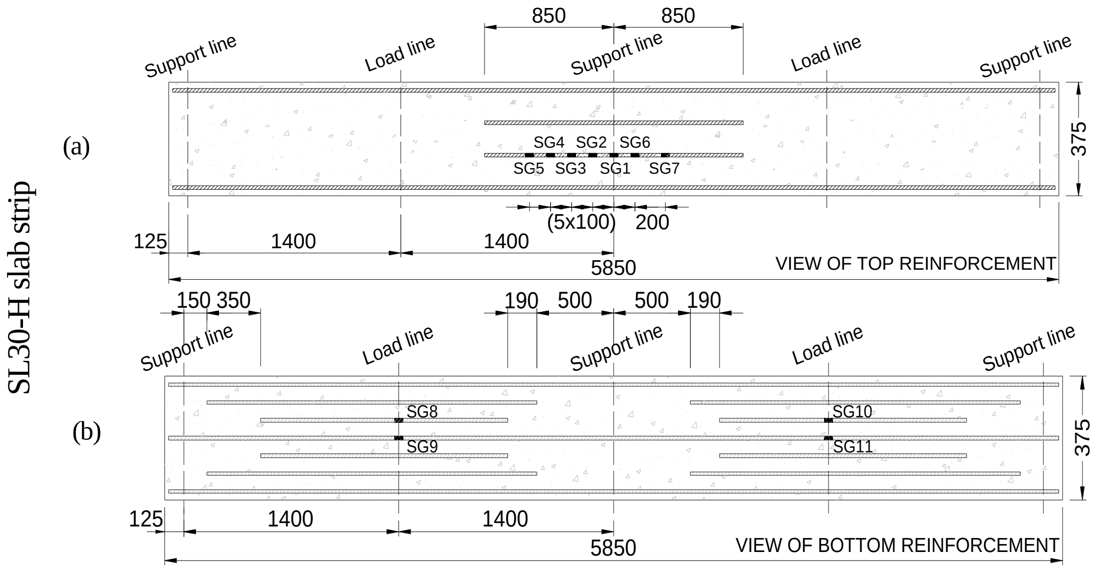


Figure 6: Arrangement of the longitudinal steel reinforcement of the SL30-H slab strip.

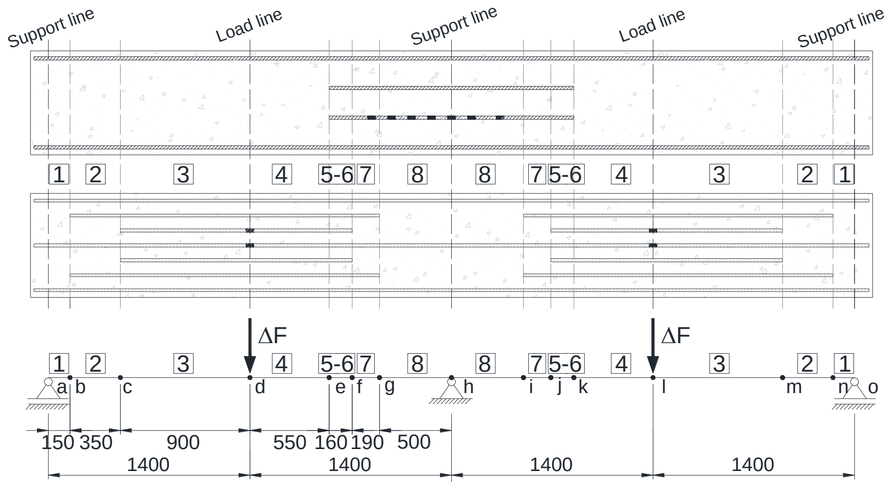


Figure 7: Discretization of the slab strip.

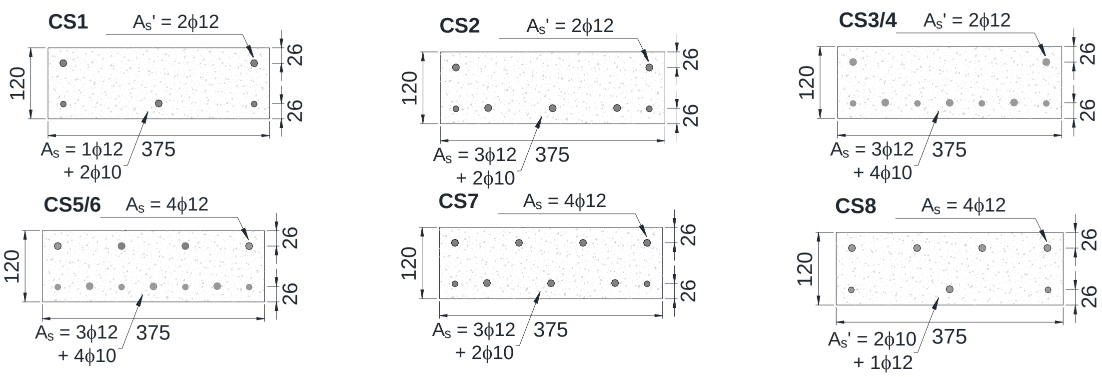


Figure 8: Resume of the cross-section according to the longitudinal steel reinforcement.

Table 1 presents a brief resume with the values of the material parameters adopted for the assessment of the predictive performance of the developed formulation. The ultimate tensile strain, as well as the modulus of elasticity of the CFRP laminates, is included in Table 2. It should be noted that only the description of the slab strip SL30-H is shown in this work, but more details regarding the other slabs can be found in Dalfré (2013). The values that define $M-\chi$ relationship of the cross sections of the SL15-H slab strip are presented in Table 3. The moment diagrams due to the unit loads corresponding to ΔF_1, ΔF_2 and ΔR are represented in Figure 9. Applying the principle of virtual work it is obtained the terms of the flexibility matrix, whose equations are included in Table 4.

Table 1: Mechanical properties of the materials used in the analytical model.

Concrete			Steel reinforcement		
ID	**Compressive strength (f_{cm}) MPa**	**Initial Young's modulus (E_c) GPa**	**Steel bar diameter (ϕs)**	**Modulus of elasticity (kN/mm²)**	**Yield stress (0.2 %)[a] (N/mm²)**
SL15-H	40.07	33.36	8 mm	200.80	421.35
SL15-HS	26.37	29.43			
SL30-H	35.99	32.31	10 mm	178.23	446.95
SL30-HS	28.40	30.09			
SL45-S	41.41	33.69	12 mm	198.36	442.47
SL45-HS	42.38	33.93			

f_{cm} = mean cylinder compressive strength at 28 days; E_c = determined following the recommendations of Eurocode 2 (2010).

Table 2: Mechanical properties of the CFRP laminates.

CFRP laminate cross section height	**Sample ID**	**Ultimate tensile stress (N/mm²)**	**Ultimate tensile strain (‰)**	**Modulus of Elasticity a (kN/mm²)**
10 mm	1	2879.13	18.45	156.100
	2	2739.50	17.00	158.800
	3	2952.00	17.70	166.600
	4	2942.32	17.81	153.620
	5	2825.20	17.40	161.400
	Average	2867.63	17.67	159.304
	Std. Dev.	88.10 (3.07%)	0.54 (3.04%)	5.01 (3.15%)
20 mm	1	2858.799	18.37303	155.5976
	2	2782.862	17.6256	157.8875
	3	2706.926	17.28808	156.5775
	Average	2782.86	17.76	156.69
	Std. Dev.	75.94 (2.73%)	0.56 (3.13%)	1.15 (0.73%)

[a]According to ISO 527-1 and ISO 527-5 (1993)
(value) Coefficient of Variation (COV) = (Standard deviation/Average) x 100

Table 3: Relation M− χ of the cross sections of the SL30-H slab strip

Element 1		Element 2		Element 3		Element 4		Element 5/6		Element 5/6		Element 7		Element 7		Element 8	
M^+	χ	M^+	χ	M^+	χ	M^+	χ	M^+	χ	M^-	χ	M^+	χ	M^-	χ	M^+	χ
0.00	1.83E+12	0.00	1.87E+12	0.00	1.90E+12	0.00	1.90E+12	0.00	1.94E+12	0.00	1.94E+12	0.00	1.92E+12	0.00	1.92E+12	0.00	1.87E+12
1.38	1.73E+12	1.40	1.78E+12	1.41	1.81E+12	1.41	1.81E+12	1.46	1.85E+12	1.49	1.84E+12	1.45	1.82E+12	1.45	1.82E+12	1.40	1.78E+12
3.42	1.52E+12	3.49	1.59E+12	3.53	1.63E+12	3.53	1.63E+12	3.65	1.66E+12	3.68	1.63E+12	3.61	1.62E+12	3.61	1.61E+12	3.49	1.59E+12
4.40	7.01E+11	5.29	9.67E+11	5.64	1.08E+12	5.64	1.08E+12	5.71	1.06E+12	5.25	8.87E+11	5.33	9.37E+11	5.24	9.06E+11	5.22	9.38E+11
4.70	3.47E+11	6.18	5.89E+11	6.84	7.19E+11	6.84	7.19E+11	6.98	7.11E+11	6.19	5.50E+11	6.28	5.81E+11	6.13	5.53E+11	6.04	5.61E+11
6.20	3.15E+11	7.92	5.21E+11	8.67	6.31E+11	8.67	6.31E+11	8.90	6.31E+11	8.00	4.96E+11	8.09	5.20E+11	7.92	4.97E+11	7.77	4.98E+11
7.75	3.05E+11	9.83	5.02E+11	10.71	6.05E+11	10.71	6.05E+11	11.00	6.07E+11	9.94	4.80E+11	10.04	5.02E+11	9.84	4.80E+11	9.64	4.80E+11
9.21	2.98E+11	11.71	4.93E+11	12.74	5.93E+11	12.74	5.93E+11	13.10	5.96E+11	11.86	4.72E+11	11.97	4.94E+11	11.74	4.72E+11	11.50	4.71E+11
10.46	2.83E+11	13.54	4.86E+11	14.72	5.85E+11	14.72	5.85E+11	15.14	5.89E+11	13.71	4.65E+11	13.85	4.87E+11	13.57	4.65E+11	13.31	4.64E+11
11.20	2.47E+11	15.30	4.79E+11	16.63	5.79E+11	16.63	5.79E+11	17.11	5.82E+11	15.41	4.55E+11	15.64	4.79E+11	15.28	4.55E+11	15.00	4.56E+11
11.40	2.05E+11	16.92	4.69E+11	18.45	5.72E+11	18.45	5.72E+11	19.00	5.75E+11	16.79	4.33E+11	17.26	4.67E+11	16.70	4.35E+11	16.48	4.39E+11
11.53	1.76E+11	18.28	4.48E+11	20.17	5.64E+11	20.17	5.64E+11	20.77	5.66E+11	17.61	3.96E+11	18.53	4.42E+11	17.58	3.99E+11	17.49	4.06E+11
11.65	1.54E+11	19.17	4.14E+11	21.74	5.53E+11	21.74	5.53E+11	22.36	5.52E+11	17.98	3.52E+11	19.28	4.05E+11	17.97	3.56E+11	17.95	3.63E+11
11.75	1.37E+11	19.61	3.72E+11	23.10	5.36E+11	23.10	5.36E+11	23.67	5.29E+11	18.15	3.09E+11	19.64	3.64E+11	18.16	3.14E+11	18.18	3.22E+11
11.85	1.23E+11	19.83	3.34E+11	24.18	5.11E+11	24.18	5.11E+11	24.57	4.93E+11	18.24	2.73E+11	19.83	3.27E+11	18.26	2.75E+11	18.29	2.83E+11
11.93	1.12E+11	19.96	3.01E+11	24.89	4.74E+11	24.89	4.74E+11	25.01	4.49E+11	18.33	2.46E+11	19.94	2.92E+11	18.33	2.46E+11	18.34	2.48E+11
12.01	1.04E+11	20.04	2.70E+11	25.19	4.29E+11	25.19	4.29E+11	25.22	4.09E+11	18.42	2.25E+11	20.00	2.62E+11	18.41	2.24E+11	18.40	2.22E+11
12.07	9.65E+10	20.07	2.42E+11	25.33	3.90E+11	25.33	3.90E+11	25.34	3.75E+11	18.49	2.08E+11	20.05	2.38E+11	18.48	2.06E+11	18.45	2.03E+11
12.13	9.05E+10	20.10	2.20E+11	25.42	3.59E+11	25.42	3.59E+11	25.43	3.46E+11	18.55	1.94E+11	20.09	2.20E+11	18.53	1.92E+11	18.50	1.88E+11
12.17	8.54E+10	20.12	2.03E+11	25.48	3.32E+11	25.48	3.32E+11	25.49	3.21E+11	18.60	1.82E+11	20.13	2.05E+11	18.58	1.79E+11	18.54	1.75E+11
12.21	8.10E+10	20.14	1.89E+11	25.52	3.10E+11	25.52	3.10E+11	25.53	3.01E+11	18.63	1.71E+11	20.16	1.92E+11	18.61	1.69E+11	18.56	1.64E+11
12.23	7.71E+10	20.15	1.78E+11	25.54	2.91E+11	25.54	2.91E+11	25.56	2.83E+11	18.65	1.62E+11	20.17	1.81E+11	18.63	1.60E+11	18.58	1.55E+11
12.25	7.37E+10			25.55	2.75E+11	25.55	2.75E+11			18.66	1.54E+11			18.64	1.51E+11	18.58	1.47E+11
12.26	7.06E+10																

M (N.mm); χ (1/mm); M^+ (Positive bending moment); M^- (Negative bending moment)

Using Equation (6) and applying determined displacements (Δu_1 and Δu_2 , where $\Delta u_1 = \Delta u_2$), the ΔF_1, ΔF_2 and ΔR are obtained, and, by equilibrium (or applying the principle of superposition effects), the reactions in the other supports can be determined, as well as the updated diagrams of resultant stresses in the statically indeterminate structure.

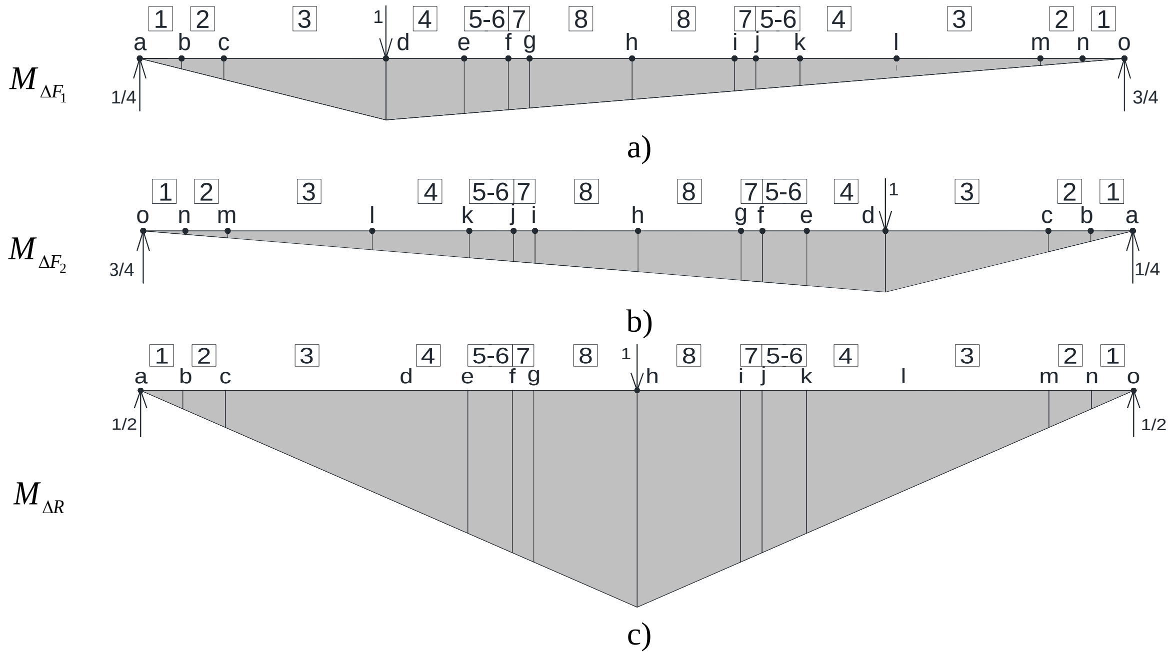


Figure 9: Moment diagrams due to: (a) ΔF_1 =1, (b) ΔF_2 =1, and (c) ΔR =1.

Table 4: Equations for the evaluation of the terms of the flexibility matrix of the structure.

$$f_{\Delta F_1 \Delta F_1} = \sum \left(\begin{aligned} &\int_a^b \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_1} ds + \int_b^c \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_2} ds + \int_c^d \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_3} ds + \int_d^e \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_4} ds + \\ &\int_e^f \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_{5-6}} ds + \int_f^g \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_7} ds + \int_g^h \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_8} ds + \int_h^i \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_8} ds + \\ &\int_i^j \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_7} ds + \int_j^k \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_{5-6}} ds + \int_k^l \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_4} ds + \int_l^m \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_3} ds + \\ &\int_m^n \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_2} ds + \int_n^o \frac{M_{\Delta F_1} M_{\Delta F_1}}{(EI)_1} ds \end{aligned} \right)$$

$$f_{\Delta F_1 \Delta F_2} = \sum \left(\begin{aligned} &\int_a^b \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_1} ds + \int_b^c \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_2} ds + \int_c^d \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_3} ds + \int_d^e \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_4} ds + \\ &\int_e^f \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_{5-6}} ds + \int_f^g \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_7} ds + \int_g^h \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_8} ds + \int_h^i \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_8} ds + \\ &\int_i^j \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_7} ds + \int_j^k \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_{5-6}} ds + \int_k^l \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_4} ds + \int_l^m \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_3} ds + \\ &\int_m^n \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_2} ds + \int_n^o \frac{M_{\Delta F_1} M_{\Delta F_2}}{(EI)_1} ds \end{aligned} \right)$$

$$f_{\Delta F_2 \Delta F_1} = \sum \left(\begin{aligned} &\int_a^b \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_1} ds + \int_b^c \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_2} ds + \int_c^d \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_3} ds + \int_d^e \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_4} ds + \\ &\int_e^f \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_{5-6}} ds + \int_f^g \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_7} ds + \int_g^h \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_8} ds + \int_h^i \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_8} ds + \\ &\int_i^j \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_7} ds + \int_j^k \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_{5-6}} ds + \int_k^l \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_4} ds + \int_l^m \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_3} ds + \\ &\int_m^n \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_2} ds + \int_n^o \frac{M_{\Delta F_2} M_{\Delta F_1}}{(EI)_1} ds \end{aligned} \right)$$

$$f_{\Delta F_2 \Delta F_2} = \sum \left(\begin{aligned} &\int_a^b \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_1} ds + \int_b^c \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_2} ds + \int_c^d \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_3} ds + \int_d^e \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_4} ds + \\ &\int_e^f \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_{5-6}} ds + \int_f^g \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_7} ds + \int_g^h \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_8} ds + \int_h^i \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_8} ds + \\ &\int_i^j \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_7} ds + \int_j^k \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_{5-6}} ds + \int_k^l \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_4} ds + \int_l^m \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_3} ds + \\ &\int_m^n \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_2} ds + \int_n^o \frac{M_{\Delta F_2} M_{\Delta F_2}}{(EI)_1} ds \end{aligned} \right)$$

$$f_{\Delta F_1 \Delta R} = \sum \left(\begin{aligned} &\int_a^b \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_1} ds + \int_b^c \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_2} ds + \int_c^d \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_3} ds + \int_d^e \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_4} ds + \\ &\int_e^f \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_{5-6}} ds + \int_f^g \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_7} ds + \int_g^h \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_8} ds + \int_h^i \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_8} ds + \\ &\int_i^j \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_7} ds + \int_j^k \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_{5-6}} ds + \int_k^l \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_4} ds + \int_l^m \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_3} ds + \\ &\int_m^n \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_2} ds + \int_n^o \frac{M_{\Delta F_1} M_{\Delta R}}{(EI)_1} ds \end{aligned} \right)$$

Table 4 (continued)

$$f_{\Delta R\Delta F_1}=\sum\left(\begin{array}{l}\int_a^b\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_1}ds+\int_b^c\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_2}ds+\int_c^d\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_3}ds+\int_d^e\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_4}ds+\\ \int_e^f\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_{5-6}}ds+\int_f^g\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_7}ds+\int_g^h\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_8}ds+\int_h^i\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_8}ds+\\ \int_i^j\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_7}ds+\int_j^k\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_{5-6}}ds+\int_k^l\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_4}ds+\int_l^m\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_3}ds+\\ \int_m^n\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_2}ds+\int_n^o\frac{M_{\Delta R}M_{\Delta F_1}}{(EI)_1}ds\end{array}\right)$$

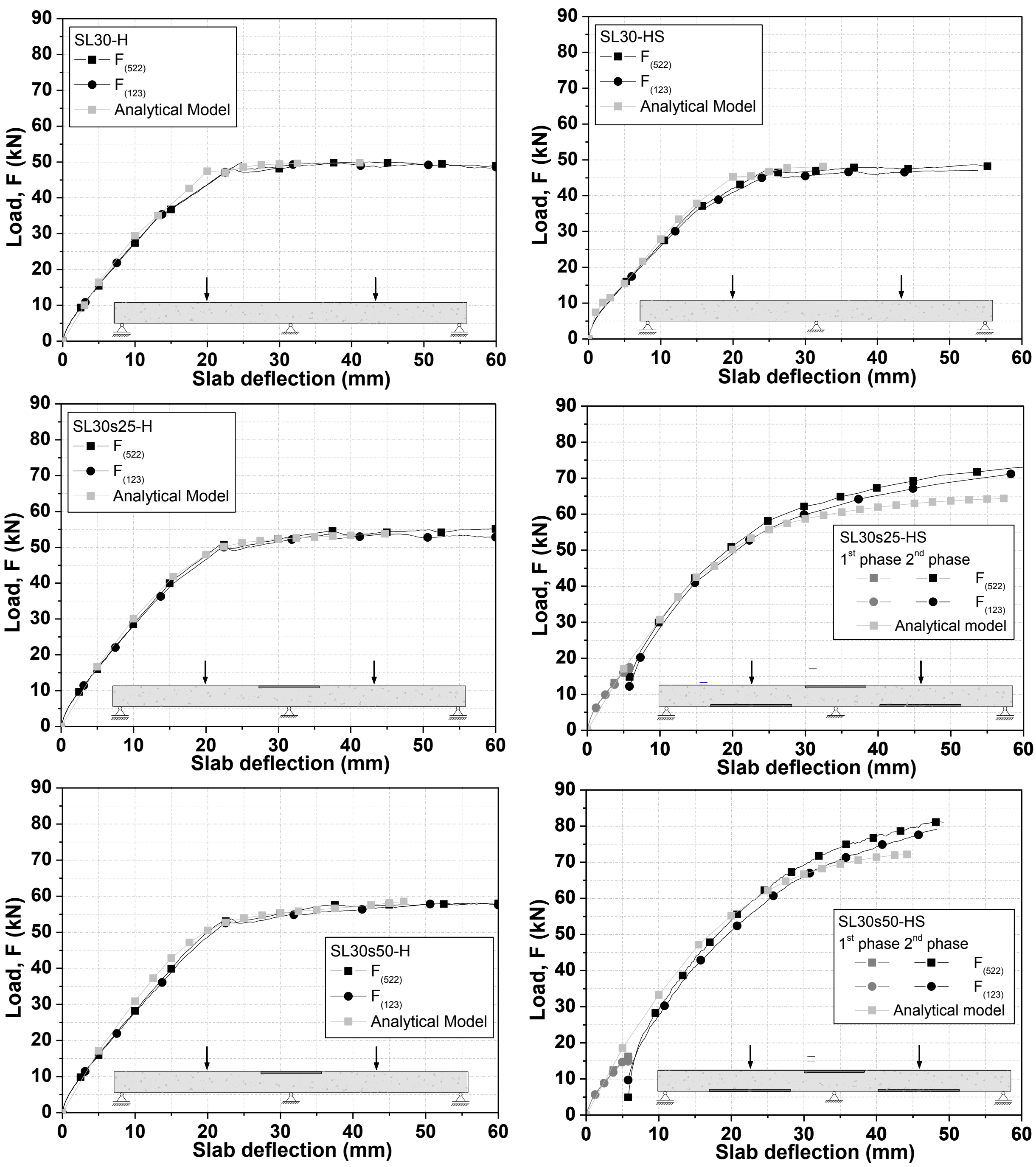
$$f_{\Delta F_2\Delta R}=\sum\left(\begin{array}{l}\int_a^b\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_1}ds+\int_b^c\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_2}ds+\int_c^d\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_3}ds+\int_d^e\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_4}ds+\\ \int_e^f\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_{5-6}}ds+\int_f^g\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_7}ds+\int_g^h\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_8}ds+\int_h^i\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_8}ds+\\ \int_i^j\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_7}ds+\int_j^k\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_{5-6}}ds+\int_k^l\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_4}ds+\int_l^m\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_3}ds+\\ \int_m^n\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_2}ds+\int_n^o\frac{M_{\Delta F_2}M_{\Delta R}}{(EI)_1}ds\end{array}\right)$$

$$f_{\Delta R\Delta F_2}=\sum\left(\begin{array}{l}\int_a^b\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_1}ds+\int_b^c\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_2}ds+\int_c^d\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_3}ds+\int_d^e\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_4}ds+\\ \int_e^f\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_{5-6}}ds+\int_f^g\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_7}ds+\int_g^h\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_8}ds+\int_h^i\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_8}ds+\\ \int_i^j\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_7}ds+\int_j^k\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_{5-6}}ds+\int_k^l\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_4}ds+\int_l^m\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_3}ds+\\ \int_m^n\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_2}ds+\int_n^o\frac{M_{\Delta R}M_{\Delta F_2}}{(EI)_1}ds\end{array}\right)$$

$$f_{\Delta R\Delta R}=\sum\left(\begin{array}{l}\int_a^b\frac{M_{\Delta R}M_{\Delta R}}{(EI)_1}ds+\int_b^c\frac{M_{\Delta R}M_{\Delta R}}{(EI)_2}ds+\int_c^d\frac{M_{\Delta R}M_{\Delta R}}{(EI)_3}ds+\int_d^e\frac{M_{\Delta R}M_{\Delta R}}{(EI)_4}ds+\\ \int_e^f\frac{M_{\Delta R}M_{\Delta R}}{(EI)_{5-6}}ds+\int_f^g\frac{M_{\Delta R}M_{\Delta R}}{(EI)_7}ds+\int_g^h\frac{M_{\Delta R}M_{\Delta R}}{(EI)_8}ds+\int_h^i\frac{M_{\Delta R}M_{\Delta R}}{(EI)_8}ds+\\ \int_i^j\frac{M_{\Delta R}M_{\Delta R}}{(EI)_7}ds+\int_j^k\frac{M_{\Delta R}M_{\Delta R}}{(EI)_{5-6}}ds+\int_k^l\frac{M_{\Delta R}M_{\Delta R}}{(EI)_4}ds+\int_l^m\frac{M_{\Delta R}M_{\Delta R}}{(EI)_3}ds+\\ \int_m^n\frac{M_{\Delta R}M_{\Delta R}}{(EI)_2}ds+\int_n^o\frac{M_{\Delta R}M_{\Delta R}}{(EI)_1}ds\end{array}\right)$$

3.2 Force-Deformation Response

Figures 10 compare the analytical and experimental load-deflection curves for the slabs of SL30-H/HS series. According to the obtained results, a good predictive performance of the adopted formulation is visible. Due to the limited space, only the results concerning to the SL30-H/HS Series are represented, but similar behavior was obtained for all the Series. The entire parametric study is treated in detail elsewhere (Dalfré, 2013).



(a) (b)

Figure 10: Relationship between applied load and deflections at spans of the (a) SL30-H and (b) SL30-HS Series.

4. CONCLUSIONS

In this work an analytical model based on the force method, and using the moment-curvature relationship ($M-\chi$) to determine the actual flexural stiffness, was proposed to evaluate the force deflection relationship of the statically indeterminate RC strips. To correctly evaluate the actual flexural stiffness of a certain slab strip, by taking into account the different arrangements of steel and CFRP reinforcements applied in the simulated slab strips, a slab strip was discretized in several types of cross section according to its reinforcement specificities. The software DOCROS was used to determine the $M-\chi$ of these cross sections. The predictive performance of the developed model was assessed by simulating seventeen slab strips of the experimental programs. The results showed that the developed numerical strategy fits with enough accuracy the registered experimental load-deflection curves of the tested slab strips. Thus, the analytical model presented in this work can be used to predict with high accuracy the behavior of RC continuous slabs strips flexurally strengthened with NSM CFRP laminates.

5. BIBLIOGRAPHY

ACI COMMITTEE 440. **Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures**. American Concrete Institute, 118 p, 2007.

BARROS, J.A.O. **Formulação directa do método das forças.** Technical report 04-DEC/E-25, Department of Civil Engineering, School Engineering, University of Minho, 25 pp., April 2004. http://hdl.handle.net/1822/12986 (in portuguese).

BARROS, J.A.O., DIAS, S.J.E. **Near surface mounted CFRP laminates for shear strengthening of concrete beams**. Cement & Concrete Composites 28, 276–292, 2006.

BARROS, J.A.O., DIAS, S.J.E., LIMA, J.L.T. **Efficacy of CFRP-based techniques for the flexural and shear strengthening of concrete beams**. Journal Cement and Concrete Composites, 29(3), 203-217, March 2007.

BARROS, J.A.O., FORTES, A.S. **Flexural strengthening of concrete beams with CFRP laminates bonded into slits**. Journal Cement and Concrete Composites, 27(4), 471-480, 2005.

BARROS, J.A.O., KOTYNIA, R. **Possibilities and challenges of NSM for the flexural strengthening of RC structures.** Fourth International Conference on FRP Composites in Civil Engineering (CICE2008), Zurich, Switzerland, 22-24July 2008.

BASTO C.A.A, BARROS J.A.O. **Numeric simulation of sections submitted to bending**. Technical report 08-DEC/E-46, Department of Civil Engineering, School Engineering, University of Minho, 73 p., 2008.

BONALDO, E. **Composite materials and discrete steel fibres for the strengthening of thin concrete structures**. PhD Thesis, University of Minho, Guimarães, Portugal, 2008.

CAROLIN, A. **Carbon fibre reinforced polymers for strengthening of structural elements**. Doctoral Thesis, Lulea Univ. of Technology, Lulea, Sweden, 2003.

DE LORENZIS, L., A. NANNI, AND A. LA TEGOLA. **Strengthening of Reinforced Concrete Structures with Near Surface Mounted FRP Rods.** Bibl. International Meeting on Composite Materials, PLAST 2000, Milan, Italy, May 9-11, 2000.

DIAS, S.J.E., BARROS, J.A.O. **Shear strengthening of T cross section reinforced concrete beams by near surface mounted technique**. Journal Composites for Construction, 12(3), 300-311, May/June 2008.

DIAS, S.J.E.; BARROS, J.A.O. **Performance of reinforced concrete T beams strengthened in shear with NSM CFRP laminates**. Engineering Structures, 32(2), 373-384, February 2010

EL-HACHA, R., RIZKALLA, S.H. **Near-surface-mounted fiber-reinforced polymer reinforcements for flexural strengthening of concrete structures**. ACI Structural Journal, 101(5), 717-726, 2004.

EN 1992-1-1: **Eurocode 2: Design of Concrete Structures-Part 1-1: General Rules and Rules for Buildings**. CEN, Brussels, 2010.

FIB - BULLETIN 14. **Externally bonded FRP reinforcement for RC structures.** Technical report by Task Group 9.3 FRP, 130 p, 2001.

GHALI, A., NEVILLE, A. M., BROWN T. G. **Structural Analysis – a unified classical and matrix approach**. Fifth edition, Spon Press, 2003.

ISO 527-1: Plastics - Determination of tensile properties - Part 1: General principles. International Organization for Standardization (ISO), Genèva, Switzerland, 9 pp, 1993.

ISO 527-5: Plastics - Determination of tensile properties - Part 5: Test conditions for unidirectional fibre-reinforced plastic composites. International Organization for Standardization (ISO), Genèva, Switzerland, 9 pp, 1993.

LIU, I.S.T. **Intermediate crack debonding of plated reinforced concrete beams**. PhD Thesis, School of Civil and Environmental Engineering, The University of Adelaide, Adelaide, Australia, November 2005.

LIU, I.S.T., OEHLERS, D.J. AND SERACINO, R. **Tests on the ductility of reinforced concrete beams retrofitted with FRP and steel near-surface mounted plates**. Journal of Composites for Construction, 10(2), 106-114, 2006.

NORDIN, H. **Flexural strengthening of concrete structures with prestressed near surface mounted CFRP rods**. Licentiate Thesis, Lulea Univ. of Technology, Lulea, Sweden, 2003.

VARMA, R. K. **Numerical models for the simulation of the cyclic behaviour of RC structures incorporating new advanced materials**. PhD Thesis, University of Minho, 2013.