ADVANCED MASTERS IN STRUCTURAL ANALYSIS
OF MONUMENTS AND HISTORICAL CONSTRUCTIONS

Master's Thesis

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A Bayesian approach to NDT Data Fusion for St. Torcato Church

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DECLARATION

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

I hereby declare that the MSc Consortium responsible for the Advanced Masters in Structural Analysis of Monuments and Historical Constructions is allowed to store and make available electronically the present MSc Dissertation.

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Dedicated to the memory of my Late Grandmother, Priyamvada Mishra.

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ABSTRACT

The main objective of this thesis is to combine information gathered from different Non Destructive tests (NDT) (direct and indirect) and fuse it by using Bayesian approach. Many time practitioners working with NDT data want to choose parameters based on results of different NDT tests with different levels of reliability and uncertainty quantification. As suggested by literature the use of a single technique might not suffice to gain information and the combination of different techniques is recommended. Also for the case of masonry structures it might not be possible to perform destructive tests but since the parameter has to be estimated based on information provided by various NDT data sources coupled with literature information.

NDT data from San Torcato Church was used in this thesis to test a Methodology to transform the data into a single and uniform format by the help of Bayesian approach. A simple Matlab Toolbox NDT_FUSION was developed and tested with different models available and modified later by using a Trust Factor which takes into account the weightage of different NDT tests. The developed toolbox is very easy to use since it has Graphical user interface (GUI) and does not required practitioner to learn the complex mathematics involved in calculation behind the Bayesian black box. The data fusion was done at different levels and steps so every time an updating takes place we arrive to a more realistic value of parameter.

Two geomechanical parameters namely the Elastic modulus (E) and compressive strength (f_c) of granite blocks from St. Torcato Church were studied in this thesis. The normal probability distribution function for the parameter of interest was calculated by using Jeffrey's Prior and Conjugate Prior, considering different levels of initial knowledge. The Elastic modulus (E) was updated by using data from Literature knowledge, sonic, ultrasonic and direct compressive strength tests to arrive to a more certain value in form of a posterior distribution. In both the cases the raw data from direct and indirect sources was processed and combined with data fusion toolbox to transform values into statistical distribution. The reliability confidence intervals of parameters were updated every time a new data becomes available providing more broad information. Different levels of uncertainty are present in data fusion system proposed in this report starting from the literature knowledge to direct compression test core data which were quantified and addressed in this thesis.

The tests of different reliability levels were weighed by circulating a survey form among professors and graduate students experts in the field to take their opinion. The results of the surveys come was the calculation of Trust Factor to update the spread of the parameters and incorporate in the model to obtain better predication of the parameters. The application developed comes with a Matlab compiler runtime (MCR) installer which allows the application to run on computers without the prerequisite of having Matlab installed.

Keywords: *NDT Data fusion, Bayesian updating, Uncertainty, Mechanical parameter*.

RESUMO

Quando na inspeção e diagnóstico de estruturas se utilizam diferentes métodos de ensaios destrutivos e não destrutivos, pela natureza dos seus resultados (qualitativos e/ou quantitativos), existem muitas incertezas associadas na quantificação de parâmetros mecânicos essenciais para as análises estruturais. Tal como a literatura da especialidade sugere, o uso de uma técnica de inspeção isolada pode não ser suficiente para obter-se a informação desejada, tornando-se recomendável a utilização de diferentes técnicas ou métodos para corroborar os resultados. No caso de construções históricas muitas vezes não é possível realizar ensaios destrutivos, sendo apenas realizados ensaios não destrutivos que, muitas vezes, oferecem apenas resultados qualitativos, sendo usual combinar métodos e ensaios com valores de referência existentes na literatura mas caracterizados por terem grande dispersão de resultados.

O objetivo principal da presente dissertação é combinar os resultados obtidos por via de diferentes ensaios não-destrutivos (diretos e indiretos) e fundi-los usando uma metodologia com recurso á inferência bayesiana e a técnicas de fusão de dados. Os ensaios não destrutivos realizados na igreja de S. Torcato em Guimarães foram usados para validar a metodologia. Como resultado foi elaborada uma toolbox no software Matlab que permite a fusão de diferentes dados para a estimativa de parâmetros mecânicos, tais como o módulo de elasticidade ou a resistência à compressão do granito utilizado na construção da igreja. A toolbox tem uma interface gráfica simples de utilizar e permite uma análise incremental, obtendo-se no final valores médios, desvios padrão e intervalos de confiança para cada parâmetro em estudo.

Tendo em consideração as incertezas e as diferenças entre os diferentes métodos de ensaio, foi também adicionado à metodologia um fator de confiança aplicável a cada método de ensaio. Para tal foi realizado um inquérito a um conjunto de especialista e utilizadores de ensaios não-destrutivos. Esse inquérito permitiu aferir a confiança do utilizador perante a utilização de um método na quantificação de um parãmetro estrutural, quando comparado com outros diferentes métodos. À metodologia adotada inicialmente foi então aplicado o fator de confiança majorando ou minorando as incertezas associadas a cada método.

Palavras Chave: *Fusão de dados, análise Bayesiana, incertezas, estimativa de parâmetros mecânicos*

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1 INTRODUCTION

1.1 General considerations

In many cases it is not possible to carry on destructive tests on historical constructions and one has to rely on data got from non-destructive tests and convert them into meaningful information. Data sources might also vary from qualitative to quantitative, so it becomes difficult for practitioner to arrive at some reasonable value of parameter due to so many complexities in data. This thesis takes Saint Torcato church (Figure 1.1) which is located in a small village near Guimãraes as a case study to fuse data using Bayesian inference.

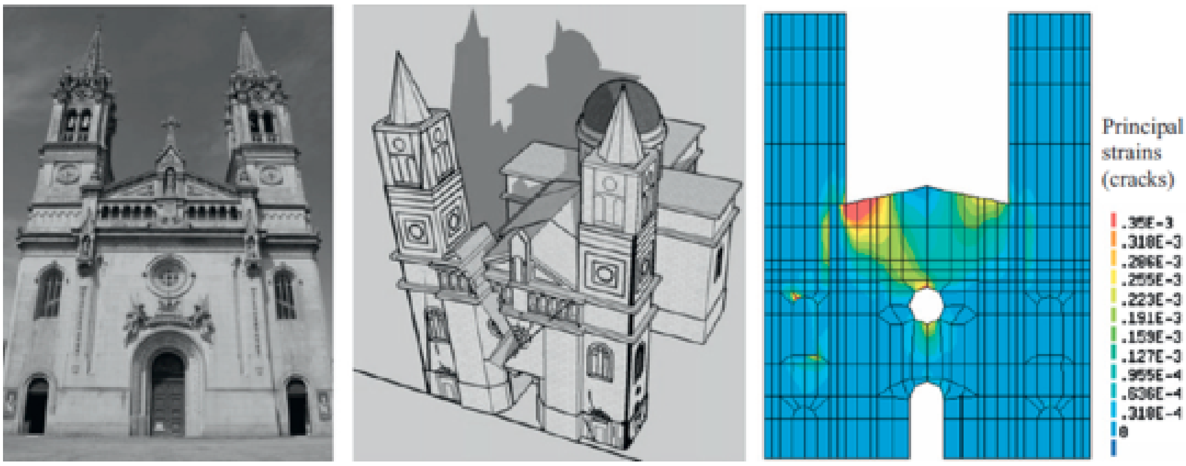


Figure 1.1: San Torcato Church [1]

Several ND tests have already been carried out in the church and a monitoring system has been installed to control the current condition and to assess the success of the future intervention. This thesis aims at analyzing the recent ND test carried out on Saint Torcato church. The main objective of this thesis is to combine the information gained from different ND techniques using Bayesian approach to arrive to a more certain value of parameter for helping the practitioners in their decision making. Instead of choosing some random value for the modelling purposes for example for a parameter the model incorporates some criteria to select a more certain value with less uncertainty in it.

1.2 Objectives

The main aim is the development of methodologies for merging the information gained with different NDT methods, by means of data fusion and sensitivity analysis. As indicated in the literature review, most researchers confirm that the use of single and isolated non-destructive techniques might not be sufficient to reliably detect or confirm a particular feature (object, feature, damage, etc.) with the exception of simple cases, which is not the case of historic masonry constructions. Therefore, it is necessary to apply different techniques within the same area or object. Secondly, different techniques use different theories and different results, which imply a large knowledge of all these techniques by the practitioner. Therefore, the objective of this thesis is to combine data produced by all different techniques, or a suitable combination of techniques, convert them into a single and uniform format and process them as a whole (or in steps) using data fusion methodologies (bayesian approach in this case).

1.3 Chapter-wise breakup

The thesis has been divided into several chapters:

Chapter 1: Introduction and Objectives of the thesis are presented.

Chapter 2: Literature review on different ND testing and monitoring techniques that can be used for monuments and historical constructions mentioning if they are direct/indirect and what parameter they measure. Also, how different NDT techniques can be used to correlate with geomechanical parameters like Elastic modulus (E) and compressive strength (f_c) of granite blocks is studied.

Chapter 3: Bayesian interface to combine data from different tests is presented and explained. The formulation to model uncertainties on uncertainties is presented along with its modified form including trust factor.

Chapter 4: Collecting data form different ND testing, monitoring reports of S. Torcato Church related to its structural condition and conservation. The Chapter will includes the identification of the level of information available about the construction safety and conservation. Also this chapter proposes a method to combine information in different stages of data collection and finally update the posterior to its final form.

Subsequently, the raw data is fused in an attempt to have in a single result the contributions from different techniques. The choice of the techniques to fuse will depend on their ability to contribute positively to a feature the other techniques will fail or whose contribution is not relevant, or, on the possibility to confirm with high reliability a particular feature. The aim of this task is to look for a combination of analytical and in situ techniques capable providing broad information of the structure/construction with reliable confidence intervals.

Chapter 5: This chapter mentioned the calculation of Trust Factor. Before this proposed factor weightage of each NDT test with respect to each other is calculated by using AHP- Analytic Hierarchy Process. Results got from Chapter 5 are again recalculated to include the result of trust factor and see how it affects the results.

A simple Graphical User Interface (GUI) was also presented in Matlab to explain and carry out this fusion process of combining data. The data is considered from St. Torcato church used as a case study and finally the Elastic modulus (E) and compressive strength of granite blocks (f_c) is estimated by this method.

Chapter 6: The previous task will lead to recommendations for designers and practitioners. Results will be regularly analysed and re-implemented during Chapters 1 to 7 and they will be reported. Also future research work is presented in this chapter which can be done after calculating the parameters compressive strength (f_c) and elastic modulus of stone (E). A need for use of Trust factor is emphasised in this approach which can be used to scale the importance of different tests and carry out the update process.

2 TESTING AND MONITORING TECHNIQUES FOR MASONRY CONSTRUCTIONS

The structural monuments and Historical constructions needs to be inspected since they posses risk owing to their old construction for ensuring safety of the people. The testing methods come into two classes- Destructive in which there is some damage in the building tested and Non-destructive testing (NDT) which as such poses no damage to component being inspected. In this chapter, a list of NDT tests (See figure 2.1) will be presented and how they can be correlated to get geomechanical properties of the material.

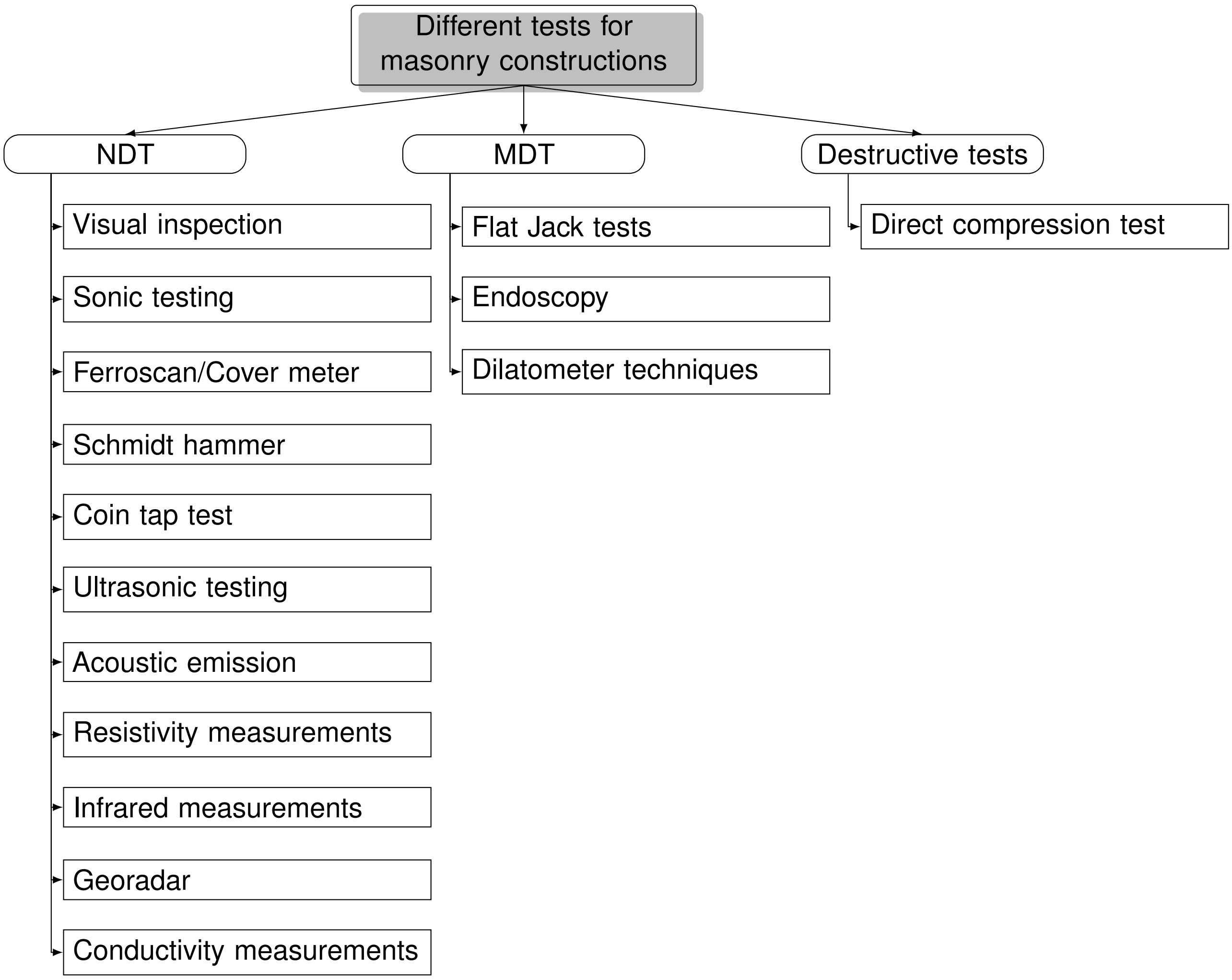


Figure 2.1: Classification of different tests used in masonry and historical constructions

Since in monuments and historic constructions many things needs to be conserved like art work, graffiti and painting, NDT is a good option as far as inspection is concerned. Many tests are defined for investigating the quality of masonry some of which are explained later in this chapter.The most common NDT techniques for monuments are used in tandem to provide information about hidden characteristics and state of degradation of masonry structures.These tests can be used to evaluate many important information like detection of voids and discontinuities inside masonry, location of reinforcement, determination of physical properties of a material like compressive strength, elastic modulus, width of cracks, corrosion etc. The defects mentioned above start with minor flaws at early stage and develop into more severe flaws if not detected in time and intervened.

These tests can vary from the type of data they provide i.e. qualitative and/or quantitative and often the combination of these tests is needed to reach a conclusion. For example sonic tests can give qualitative information about a void present inside masonry or the sonic velocities can be correlated to some other property like elastic modulus or compressive strength. For example in case of Schmidt hammer results a correlation like is used to convert rebound number into equivalent compressive strength (f_c). This list mentioned in this chapter is not exhaustive but contains most of them which are frequently used in case of masonry structures.

2.1 Non-Destructive Techniques (NDT)

2.1.1 Visual Inspection

This is most widely used of all the nondestructive tests for preliminary survey as its simple and easy to apply, quickly carried out with minimal equipments and usually lowest in cost. This is one of the most basic methods to get a rough idea depending upon what scale we want to look in. For example visual inspection can be used to detect wide prominent cracks by naked eye or with help of some supplemental aids like magnifying glass. Tools like image processing can be used in tandem to improve the quality of visual inspection. Also the images obtained can also be enhanced by using smoothing and filtering facilities.The test is very qualitative in nature and doesn't provide any value to a parameter of interest.

2.1.2 Sonic Testing

The is done by constructing a square grid 80 cm x 80 cm (see Figure 2.3) spacing 20 cm in horizontal direction and 20 cm in vertical on a masonry wall in both inside and outside faces of the wall. A hammer (Figure 2.2) is used to generate pulse which is received by the receiver at the other end of the wall of known thickness. After this test sonic velocity contours (Figure 2.4) and sonic velocity histograms (Figure 2.5) are plotted to get an idea of masonry wall characteristics. Sonic testing can be interpreted that if the distribution of velocities is not homogeneous which is indicative of a masonry with remarkable voids and defects. Further research needs to be done on correlation like this to get a better understanding of the knowledge of level of the building. This method has been used to evaluate homogeneity of the material, depth of surface cracks, presence of voids and to get an estimate of average compressive strength and elastic modulus of the material. The basis for determining the sonic velocity goes by measuring the time difference between the signal between the transmitter and receiver. The three set up of the tests can be seen from Figures 2.6, 2.7 and 2.8 depending upon location of transmitter and receiver.

Figure 2.2: Hammer to generate pulse for sonic test [2]

Figure 2.3: Installation of Grid for sonic test [2]

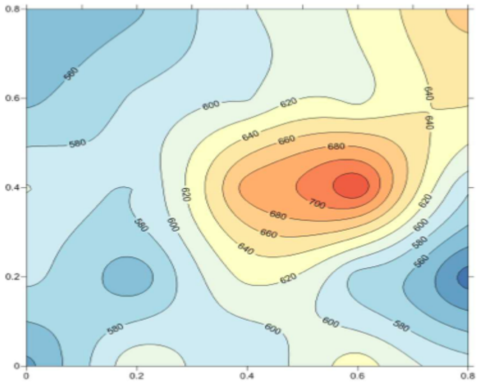


Figure 2.4: Distribution of sonic velocities [2]

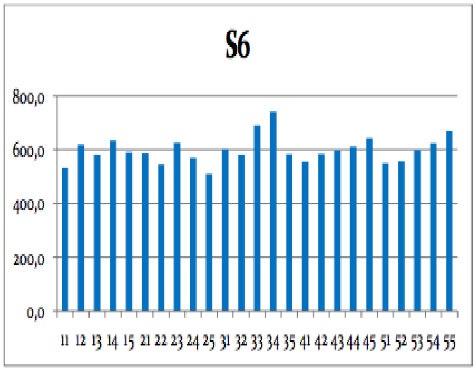


Figure 2.5: Histogram of velocities [2]

The test can be done in several transmission modes namely direct, indirect and semidirect tests shown in Figure 2.6, 2.7, and 2.8. The test data can be related to compressive strength and elastic modulus to get some quantitative information about the parameter of interest.

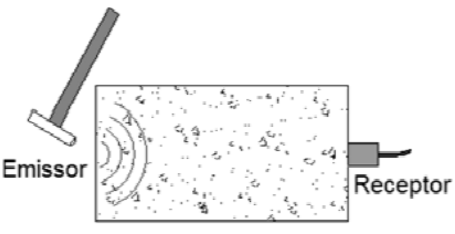


Figure 2.6: Direct Test [3]

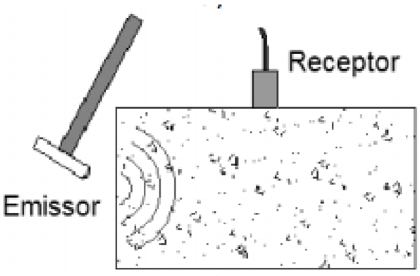


Figure 2.7: Semi-Direct test [3]

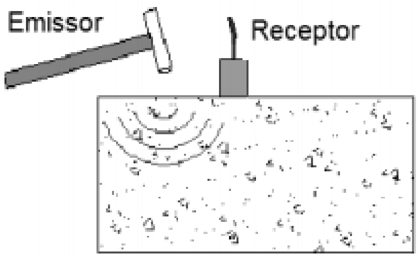


Figure 2.8: Indirect Test [3]

2.1.3 Cover meter / Ferroscan Tests

The instrument commercially known as cover meter uses electromagnetic methods to determine the location and thickness of concrete above the reinforcement bars. The principle is based on the fact that steel rods embedded in concrete change electromagnetic field around the coils positioned in iron-core inducted in covermeter. It is a battery power equipment which determines the position of reinforcement, measures depth of the concrete cover and estimates the diameter of the rebar [4] in a structure in a non-destructive manner (Figure 2.9) . Its principle of operation is based on generation and detection of electromagnetic fields by conductive material. The intensity of the field generated depends on the depth and diameter of the rebar. Figure 2.10 demonstrates how the scanner is moved along the grid in horizontal and vertical directions to obtain a scan image (Figure 2.12) which in turn can be viewed in the monitor (Figure 2.11) and then later on computer screen.

Figure 2.9: Hilti Ferroscan PS200 - Scanner [2]

Figure 2.10: Testing of wall by ferroscan [2]

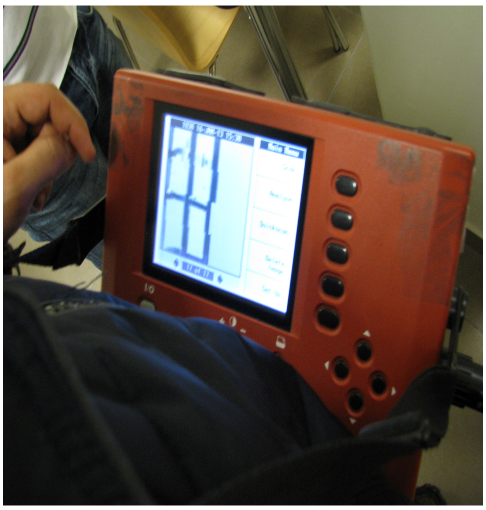


Figure 2.11: Display on Ferroscan screen [2]

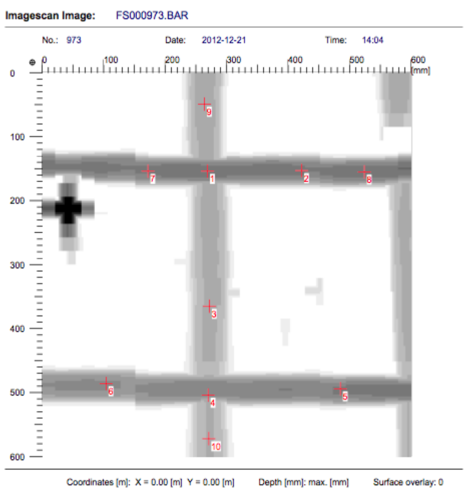


Figure 2.12: Details of imported Ferroscan survey file on computer [2]

It is a very useful equipment to find out problems when the depth of the concrete cover is inadequate and where rebar is corroded. Also when engineering defects are present from the beginning it can be used to test the distance of rebars without use of traditional drilling techniques which are destructive in nature. This is a very good test for building inspection and quality control. This scan helps in cases where construction drawings get lost or we need to find the reinforcement positions and sizes since load carrying capacity depends solely on them. There might be some error in cases when concrete is penetrated with saline water since it may effect the electrical conductivity of concrete.

2.1.4 Schmidt Hammer

Schmidt hammer test shown in Figure 2.13 is a non-destructive test which measures hardness of a material (Rebound value R) which can be correlated to the Compressive strength (f_c) by help of conversion charts . The test is an indirect test since it doesn't give compressive strength value directly. The rebound reading (10-100) is affected by the orientation of hammer, when used in a vertical position (on the underside of a suspended slab for example) gravity will increase the rebound distance of the mass and vice versa for a test conducted on a floor slab. The test is more useful when comparison is made between samples. For example in San Torcato church test was performed on granite which came from similar quarry and compared with original sample to prove that they are similar in characteristics. Attention must be paid to BS 1881 Point 202 [5] which states that the use of universal calibrations, such as those produced by the manufacturers of rebound hammers, can lead to serious errors and should be avoided. The conversion charts are mostly available for concrete only and for new material they need to be calibrated by doing tests.

Figure 2.13: Scmidt Hammer [6]

2.1.5 Coin tap test

This is a simple variation of impact echo method to detect defect or cavities behind linings of tunnels or areas of rendered wall where rendering has separated from stonework. The procedure is very simple in which the wall is tapped with a lightweight hammer and the ringing or echo change in frequency is observed in the defected area. The method is very effective since human ear is much sensitive to resonant frequencies. One application of this test was to identify debonding of metal plates [7] glued to underside of concrete deck on a bridge in scotland.

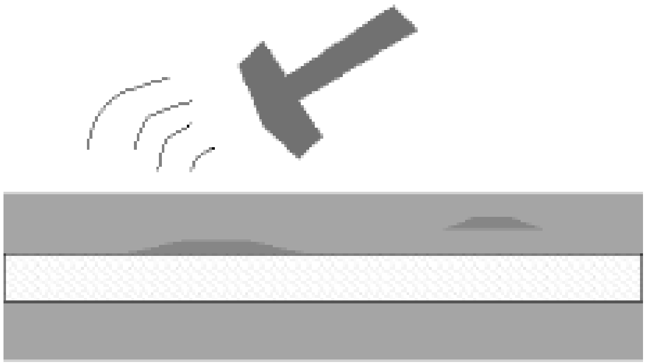


Figure 2.14: Sonic resonance method/coin tap test [8]

2.1.6 Ultrasonic Testing

This method uses ultrasonic waves ($f > 20$ KHz) for material examination and detection of internal flaws. As shown in Figure 2.15 by measuring the time difference betwenn the two waves the thickness or the location of the defect can be easily measured.

$$d_o = vt_1/2 \tag{2.1}$$

where
d_o= distance of the defect from specimen,
v= Speed of ultrasonic wave in the medium,
t_1= Time measured between the two peaks.

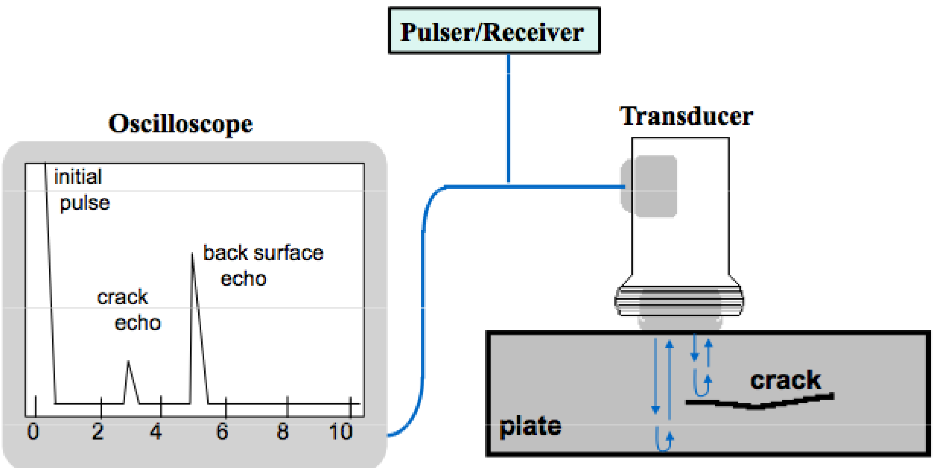


Figure 2.15: a) A Scan plot b) Typical Ultrasonic pulse echo system [9]

Also by measuring the velocity it can be correlated to other properties of the material like Elastic modulus (E) etc. The technique can range from pulse echo in which single probe is used to measure and transmit the signal (pulse echo) and pitch catch in which two transducers are used (through transmission) and both have their advantages and disadvantages. The velocity results can be related to geomechanical parameters to obtain some estimate of Elastic modulus. The Figures 2.16, 2.17 and 2.18 show the steps to carry out a Ultrasonic test on a crack and interpret the result. As seen from Figure 2.18 it can be interpreted that crack can go up to 40 cm at some location of wall being inspected.

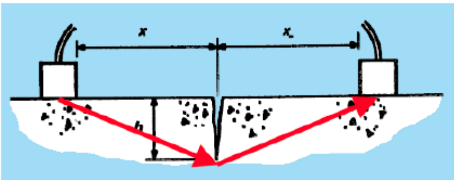


Figure 2.16: Ultrasonic Test setup [10]

Figure 2.17: Performing Ultrasonic Test [10]

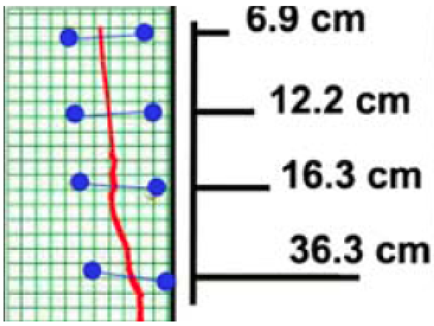


Figure 2.18: Ultrasonic Test crack values [10]

2.1.7 Acoustic Emission

Acoustic emission works in the principle that when a crack opens, the energy released in form of acoustic emission and high frequency stress waves can be recorded and analysed. Its main application comes in the area of crack monitoring and defect localisation [11]. Sources of AE vary from natural events like earthquakes and rockbursts to the initiation and growth of cracks, slip and dislocation movements, melting, twinning, and phase transformations in metals. It has to be supplemented with techniques like signal processing and filtering to obtain good optimum results. The methods finds its applicability in laboratory much better than on site monitoring since it can be time consuming. Unfortunately, AE systems can only qualitatively gauge extent of damage is contained in a structure. In order to obtain quantitative results about size, depth, and overall acceptability of a part, other NDT methods (often ultrasonic testing) are necessary. They allow us to estimate the depth of all important cracks observed through crack pattern survey (See Figure 2.19). However it is difficult to interpret the results of acoustic emission tests with the geomechanical parameters.

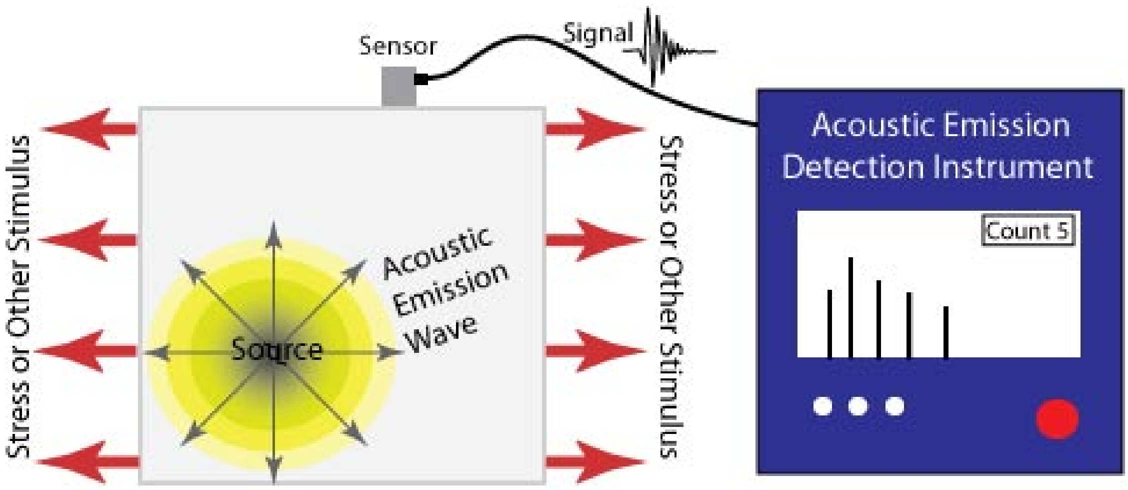


Figure 2.19: Acoustic Emission system [12]

2.1.8 Resistivity measurements

This method is a version of electrical resistivity method (See figure 2.20) used to find corrosion rates within a reinforced concrete structure. In this method the electrode is used to map the electrical resistivity through out the length of the beam. The changes of resistivity can be related to the ability of corrosion currents to flow though the reinforced concrete beam which can be function of water cement (w/c) ratio, moisture and salt content. Some precautions should be used like the contact must be very good to use this method which can be accomplished by drilling of small holes.

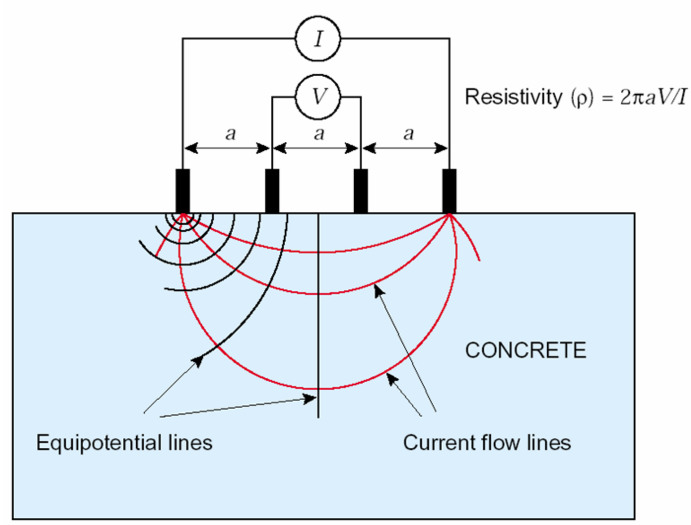


Figure 2.20: Set up for resistivity measurement [13]

2.1.9 Infrared measurements

In Infra-red thermography the heat at any temperature is converted into a thermal image using Infra-red cameras. The buildings with defects (cavities, moisture presence, change of material etc) differ in amounts of infra-red radiation. For example the building free with defects and concrete surface even colour will appear uniform when viewed from infra-red camera. If the concrete surface has cracks then they will heat up faster under solar radiation and hot spots will appear on thermal scan. These areas which appear as hot spots can be examined more closely for further investigation. This methods has gained much popularity in assessment of large buildings with high rise apartment blocks [14] . Figure 2.21 shows the detection of hidden tie rods using thermovision. The method can be active of passive depending upon if forced heating is applied to structure or not.

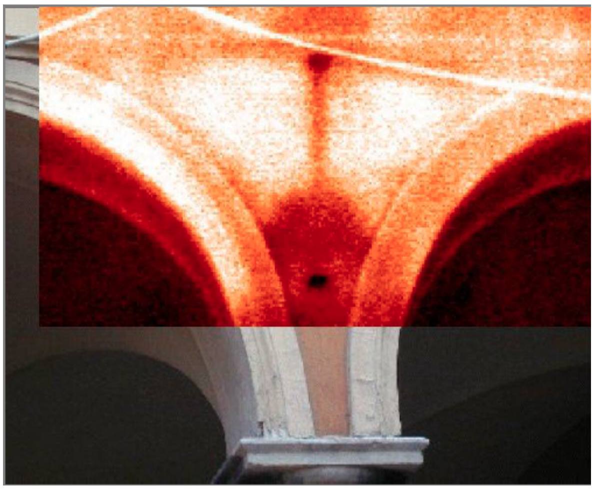


Figure 2.21: Detection of hidden tie rods using thermal vision [10]

2.1.10 Georadar

This is also a NDT method used for masonry and to locate presence of large voids, inclusion of different materials, presence of moisture levels and morphology of the wall section in multiple leaf masonry. Figure 2.22 and Figure 2.23 shown below shows the interpretation of radar gram test results on a target wall for one value of depth slice showing presence of a local void due to the energetic reflection.

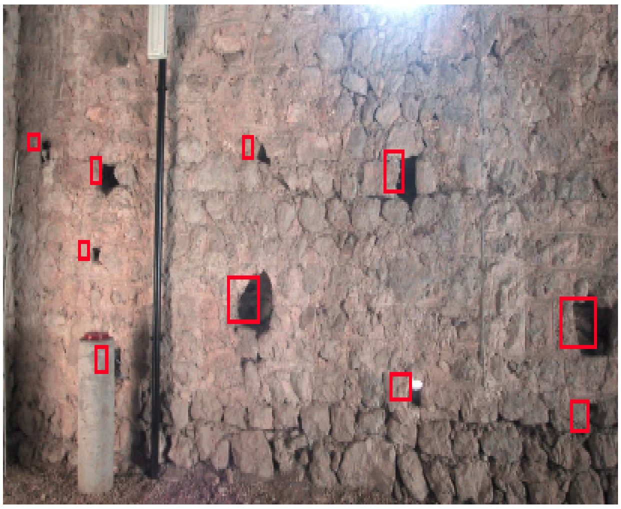


Figure 2.22: Target wall [10]

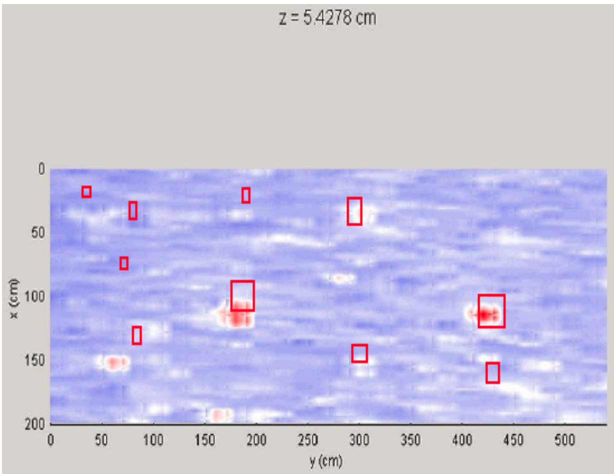


Figure 2.23: Radargram profile showing several voids [10]

2.1.11 Conductivity Measurements

As we know electrical conductivity depends on degree of water saturation and their electrical properties. Electromagnetic waves propagated inside structure can give information on materials investigated. The equipment can be non-contacting but some problems might be caused when reinforcing rods are inside. As ingress of water inside masonry is of great engineering importance it needs to be monitored. They can be used to give various measurements like moisture content, salt content and presence of metal reinforcements, pipes, etc in the wall.

Figure 2.24: Detection of hidden tie rods using thermal vision [10]

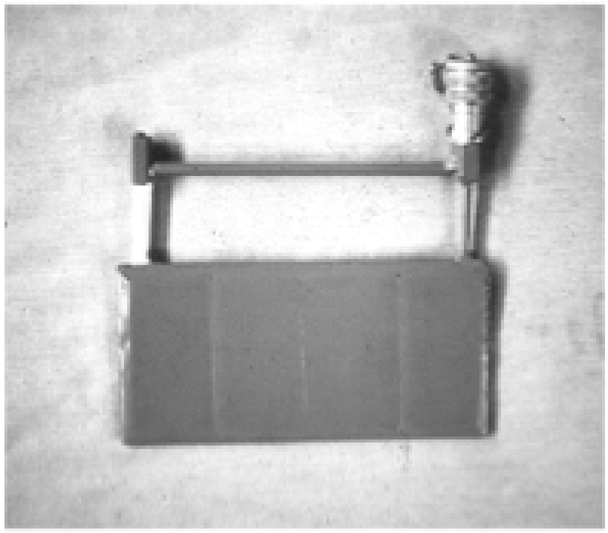
2.2 Minor-Destructive Testing (MDT)

2.2.1 Single and Double Flat jack tests

Strength of masonry is an important consideration in finding out the condition of the building. In case of heritage building removing parts of the masonry is unacceptable and requires some tests which do not alter building. Testing of masonry strength by flatjacks is a minor destructive testing (since some portions of mortar have to be removed for testing) plus instead of loading the whole wall only a small portion of wall is loaded by a small hydraulic jack instead of whole wall. The things that we can measure using flatjack are: (1). compressive strength of masonry if it is allowed to test until masonry fails. (2). Elastic modulus (in linear part) since the stress (σ) vs strain relation (ε) is given. and (3). in plane shear strength.

To start the test first a layer of mortar is cut from the masonry wall and then a thin flat jack is inserted into the mortar layer which has been cut. Since there is some cut so it comes into minor destructive testing. Because of the cut there is stress relaxation since distance will be less than before. The flat jack will slowly try to increase the pressure so as to restore it into old settings. The extensometer can be used to measure displacement after cutting and then during the testing process.

(a) Rectangular flat jack



(b) Drilling of the mortar joint

Figure 2.25: Procedure flat jack test [10]

The Figure 2.25 shows how a cut is made in brick masonry since its easy to make rectangular cut for this type (as opposed to irregular stone masonry where it is difficult to find joints). Then after this step a second cut is made from 40-50 cm from first one and 2^{nd} jack is inserted in that cut. Then since the masonry sample is delimited by these two jacks so it can perform axial compression test on this part of sample sandwiched between two jacks. The LVDTs can give axial and transverse strains attached to this masonry sample. Various loading and unloading cycles have to be performed to give a better idea of elastic modulus and if test allows we can continue it to find ultimate strength of sample until it collapses.

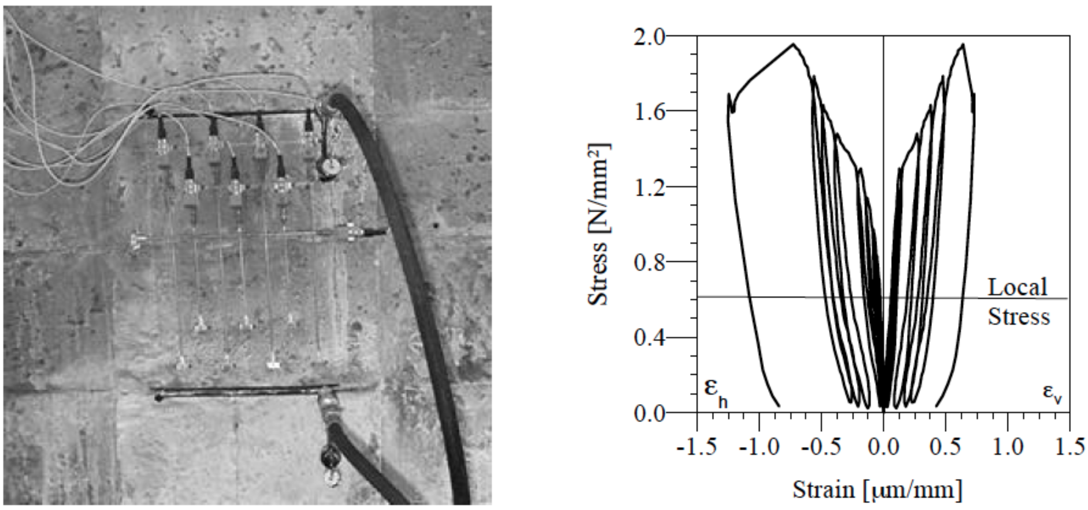


Figure 2.26: a) Double flat jack test on regular stone masonry b) Stress strain curve [10]

The state of stress in a flat jack test is given by : [10]

$$S_f = K_j K_a P_f \tag{2.2}$$

where:
S_f = calculated stress value,
K_j = jack calibration constant (≤ 1),
K_a = jack/slot area constant (≤ 1),
P_f = flat-jack pressure.

Interpretation of Data of Flatjack tests can be in terms of Elastic modulus values. The double flat-jack test allows to measure the modulus of elasticity which can be used to classify different kinds of masonries. For example, Levels of masonry according to elasticity modulus are Masonry of rural buildings E $<$ 900 N/mm^2, Masonry of Civil buildings 900 N/mm^2 $<$ E $<$ 1500 N/mm^2, and Masonry of Monumental buildings E $>$ 2000 N/mm^2 [10].

2.2.2 Dilatometer techniques

In this test shown in Figure 2.27 the perforation is made by the help of drill and then a cylindrical tube conforming with the dimensions of the specimen is introduced in it so it can expand inside the hole. After getting test data of the curve pressure given by the tube and increase of volume obtained can be used to estimate the module of deformation of the masonry. This is a quantitative test as it gives us directly the value of deformability modulus of masonry.

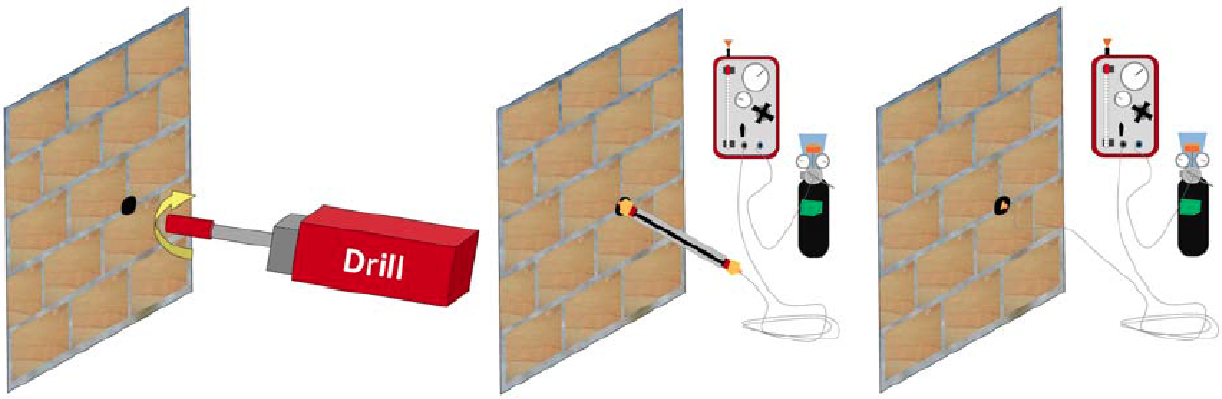


Figure 2.27: Phases of the dilatometer test [15]

2.2.3 Endoscopy

Endoscopy is simply an extension of the essential visual survey (Section 2.1.1) into areas inaccessible to the naked eye. The equipment ranges from relatively simple borescopes consisting of a light source, a small diameter rigid tube with built-in optics and an eye-piece to complex controllable systems with numerous specialised attachments. By drilling a hole (normally less than 12 mm) and inserting the tube, it is possible to inspect voids under floors or behind panelling for example. Any hidden problems such as fungal growth can, in theory, be identified. The more sophisticated and expensive equipment is fully flexible and can be steered by wires built into the casing. Systems are available down to 6 mm diameter, and more specialised systems down to less than 2 mm. It is possible to attach still or closed-circuit television camera (CCTV) to the eye-piece to record the findings on a videotape (Video boroscopy). The theory is fairly simple, but in practice it can be very difficult to retain a sense of scale of the image observed, and keep track of the location and orientation of the tip. The focal range, depth of field and strength of light is greatly reduced in the smaller diameter systems.

Figure 2.28: Execution of endoscopic investigation (left) and it's endoscopic picture (right) [16]

Endoscopic technique allows one to observe, inspect and document masonry panels in their section and generally hidden portions of structures. Endoscopy can be applied for a lot of different uses: documentation of structural elements (walls, floors, vaults) in order to investigate their materials, techniques and construction phases; analysis of degradation and instability (moisture, cracks); evaluation of effectiveness of intervention in progress. However, endoscopy requires execution of a small hole, it is therefore a micro-invasive test, but sometimes it provides detailed and reliable information difficulty to obtain using other techniques, especially if non-invasive.

2.3 Destructive tests

2.3.1 Compression tests

This tests are Destructive in nature and requires samples to be take out from core or casted and then tested with Universal Testing Machine (UTM). They are direct tests since they give directly the compressive strength of the material. These tests can also be used to find Young's modulus of the material by placing LVDTs at approximately one third and two thirds of the specimen's height. The Figure 2.29 shows a test performed on a concrete sample to measure its strength against compression. This test for concrete specimen is most widely used test to measure its compressive strength with concrete specimen ranging from cubes & cylinders.

Figure 2.29: Compression testing on a concrete specimen [17]

2.4 Crack Monitoring

2.4.1 Glass Crack Meters

This is a very qualitative way to monitor cracks. It consists of pieces of glass installed (See Figure 2.30) where the crack propagates and if the glass cracks or shows any damage then it means that the crack has opened.

Figure 2.30: Glass pieces inserted into the wall to monitor cracks [2]

2.4.2 Crack Meters

The displacement transducers (LVDT) are available with different operating ranges and the square of the frequency signal is directly proportional to the amount of displacement. These units are used in crackmeters and rod extensometers. The crackmeters allows us to measure displacement of a crack in different axes up-to accuracy of 0.1 to 1.00 mm. These crackmeters (Figure 2.31) are quick and easy to read, adaptable to data loggers or data acquisition system.

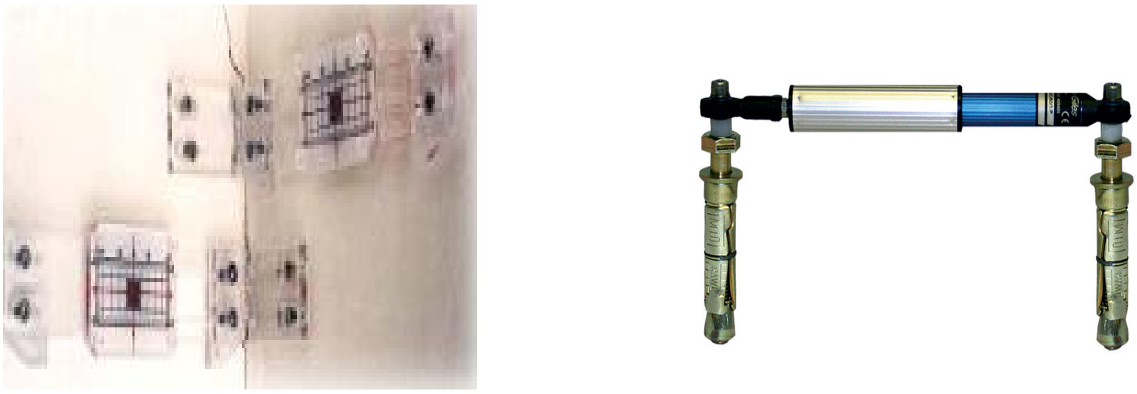


Figure 2.31: Different types of Crack Meters a) Manual Crack Meters b) Electronic Crack Meter (LVDT) [18]

2.5 Computers in NDT

Visualisation of NDT data can be very tedious job and time consuming. But with advent of softwares for data analysis the task is becoming more simpler and less prone to misinterpretation. Many visualisation tools available for different NDT tests: for example Surfer for sonic tests, FerroScan software for Ferroscan data to mention a few. Graphic possibilities of computers enable us to get us a overall view in different NDT tests. There has been a continuos development in visualisation of NDT techniques and display of information with enhance data analysis tools. Quality images are produced to ease the communication of data and interpretation with other scientists working in this area. The use of colours have been a very efficient development for data visualisation and readability of an image. The coloured images (Example see Figure 2.4) can be seen much easier in identifying stress points, locating extremes and defective regions of material.

2.6 Discussion of NDT methods used for St Torcato case study

Certainly the advent of computers (See Section 2.5) and data loggers have decreased the time of performing NDT survey but the results must be interpreted with great care. Also there is a urgent need in scientific community to develop some standards for NDT surveys for interpretation of results in relation to structures. One of the challenges can be to combine the results of different NDT techniques with different reliability techniques and fuse them together to obtain value for engineers and scientists for decision making. So, to summarise a wide range of NDT methods can be used depending on what kind of information is required, on which scale, economy, ease of use etc.However in the case study data from St. Torcato was used to calculate the parameter of interest (Elastic modulus and compressive strength of granite blocks). There are different possibilities of using NDT/MDT tests according to parameter desired. The list is summarized in table 2.1. In this table many tests correlate with the geomechanical properties of the material and each test can vary in their complexity to reliability of data one gets from these tests.

Table 2.1: Different possibilities to use NDT or MDT according to parameter of interest

Correlation with Geomechanical parameters			
Parameter	List of methods	Application	Direct measurement
Elastic Modulus (E)	Sonic velocity	In-situ	No
	Ultrasonic testing	In-situ	No
	Schmidt Hammer	In-situ	No
	Dilatometer	In-situ	Yes
	Flat jack test	In-situ	Yes
	Direct compression test	Lab	Yes
Compressive strength (f_c)	Schmidt hammer	In-situ	No
	Flat jack test	In-situ	Yes
	Direct compression test	Lab	Yes
Diameter of rebar (d)	Cover meter	In-situ	Yes
	Thermography	In-situ	No
Concrete cover (cc)	Cover meter	In-situ	Yes
State of stress of masonry(σ, ε)	Flat jack test	In-situ	Yes
Crack depths (d)	Ultrasonic testing	In-situ	Yes
Width of crack (w mm)	Glass crack meters	In-situ	No
	Displacement transducers	In-situ	Yes

In most of cases the data with low reliability will show a large standard deviation than with data which has more reliability. Series of data coming at different times from St. Torcato church was used in to get a reliable estimate of the parameter. For this thesis we have only used literature knowledge, schmidt hammer with direct compression test to evaluate the compressive strength (f_c) of the granite block. And for second parameter literature knowledge, sonic, ultrasonic tests and direct compression tests were used to calculate the value of elastic modulus (E) for the granite block. Summarised Table 2.2 discusses the advantages and disadvantages of NDT methods and how we can use these methods to calculate the desired parameter.

2.7 Conclusions

The ND tests and the laboratory tests should be used in sequence to characterize the masonry typology and mechanical behaviour of masonry material. In some cases in monuments when its not possible to remove material so the parameters have to be deduced by carrying only NDT tests and the data coming from these tests needs to be combined in a logical way to arise to a conclusion. In some cases when no direct relation is available to characterize masonry then some range can be used on previous studies carried out on other masonry samples. Also some minimum number of tests should be carried out to get data with some reliability keeping in mind the budget allocated to the testing scheme. Still a great deal of research is necessary for the interpretation of the NDT results and for their correlation to masonry characteristics.

Table 2.2: Summary of NDT Methods adapted from [19]

Inspection method	Parameter measured	Advantage	Disadvantage	Cost
Visual	Surface condition	Quick; modest skills required	Superficial	Low
Sonics	Wave velocity; tomographic cross-sections	Moderately slow; gives useful information on major elements	Requires skill to interpret data	Moderate High
Cover meter	Concrete cover and diameter of rebars	Relatively quick	Error when concrete is penetrated with saline water	Moderate High
Schmidt hammer	Rebound number	Simple to use, equipment inexpensive and readily available	No direct relationship to strength or deformation properties	Low
Coin tap test	Change in frequency	Procedure to perform test very simple	Superficial	Low
Ultrasonics	Wave velocity, location of defect	High penetrating power,Greater accuracy, portable equipment	technical knowledge is required for the development of inspection procedures, Couplants needed	Moderate High
Acoustic emission	Energy released in form of stress waves	High sensitivity, localisation of failure zone by time of arrival measurement	Only estimate qualitatively how much damage is in the material	High
Resistivity	Changes in resistivity	Shallow investigations are rapid	Deep investigations require long cables and much time, data interpretation difficult	Moderate
Infrared measurement	Amounts of infrared radiation	Visual image easy to interpret, located hidden tie bars	Qualitative image, not much information gained about other things	High
Radar	Electromagnetic wave velocity	Quick; can give good penetration; can give good image of internal structure	Poor penetration through clay infill and salt contaminated fill; requires skill to understand data	Moderately high
Conductivity	Relative conductivity	Quick; gives relative conductivities over a large area to a maximum depth of 1.5 m	Limited depth penetration of 1.5 m; complements radar	Low
Dilatometer	Modulus of deformation of masonry	Quantitative test, gives direct estimate of parameter	Requires skill to interpret data	Moderate

3 DATA COLLECTION, DATA FUSION AND INTRODUCTION OF BAYESIAN STATISTICS

The objective of this chapter is to explain how Bayesian approach can be applied to combine data from different ND tests. St Torcato church was used as a case study to see how the bayesian model can be adapted to different data sets. Mainly in this study two granite stone blocks were studied and different tests were performed on it to get an estimate of geomechanical parameters of granite used in the church. Many minor details like the places where data was collected is not mentioned since this thesis emphasises more on data values then on locations where data was fetched.

3.1 Data Collection for S. Torcato church

Data collection was done by reading and going through several testing reports of San Torcato church [20], and also though several excel data sheets. Two types of data were collected in this case : one was experimental data (Direct and Indirect) obtained from testing and another type was monitoring data obtained from continuous monitoring of San Torcato church. But only the experimental data was used in this thesis and the monitoring data count not be used. In data collection, two blocks were taken from the site to the lab and tested. We had the information that the blocks came from the same quarry, but they uses two quarries to build the church. So, there is an uncertainty related with this aspect.

3.1.1 Schmidt Hammer Tests

The data from blocks was obtained to make sure that the granite has come from same quarry. However schmidt hammer provides surface hardness it is poorly be related to Elastic modulus of the stone block. It can be related to compressive strength of granite but the value obtained were so close and needs some empirical relationship as suggested by Ang [21] to be used as a prior for combining it with initial level of Knowledge. The relationships developed to relate compressive strength of granite with schmidt hardness value in paper of Graca Vasconcelos $f_c = 12.24N - 739.94$ [22] were not suited for this data because of lower range as can be seen by comparison of Figure 3.1 and Table 3.1. Further information about conducting this test can be found in Chapter 2, Section 2.1.4. The average value of the rebound number for in-situ tests is 63.0 with a standard deviation of 2.8 and a coefficient of variation of 4.4. The value obtained by this correlation was around 31.2 MPa and was discarded as its too low for granite compressive strength.

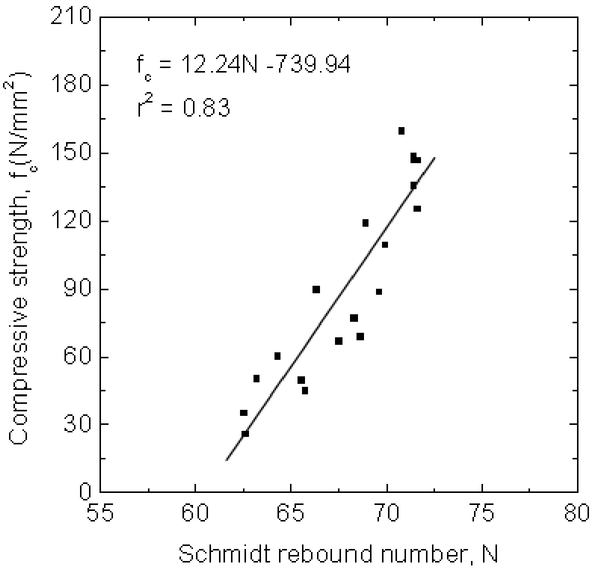


Figure 3.1: Figure showing correlation of Schimdt Hammer rebound number and compressive strength of granite [22]

Stone No	Rebound No.
Stone no 1.	62.6
Stone no 2.	62.5
Stone no 3.	63.3
Stone no 4.	63.7

Table 3.1: Table showing Schmidt hammer test results for San Torcato Church [20]

3.1.2 Granite Sonic Tests

The sonic tests were done on 5 points on northwest wall of west tower and the same comparison was made with the granite blocks. Some values shown in table 3.5 are a bit low which shows discontinuity in rubble masonry with voids. The same sample taken from the quarry was used to perform three types of tests: direct, semi-direct and indirect. The velocity can be used as a indicative to get an idea about the voids present inside the wall. The table 3.2 gives an idea about what test was performed on granite blocks and what was done on masonry wall. For carrying out the data fusion process, results of direct sonic tests of granite stone blocks from Table 3.3 and 3.4 were taken into account since they show less coefficient of variation than the other indirect and semi-direct tests. The velocities obtained from Table 3.8 and 3.9 were lower since there were difficulties in distinguishing P-wave from R-wave. The table 3.2 shows which tests were available for sonic tests for blocks and in-situ for comparison in both cases. However only direct test data was used in the model. Other data from the table can also be used with some additional uncertainty.

Table 3.2: Granite sonic test results (Block and in-situ) [20]

Results from sonic testing			
Serial number	Test Type	Lab	In-situ
1	Direct sonic	✓	✓
2	Semi-direct sonic	✓	✗
3	Indirect sonic	✓	✗

Table 3.3: Granite Stone Block 1 Direct sonic test results [20]

Results from sonic testing Stone Block 1		
Surfaces	velocity [m/s]	COV
A-C	4751	15.3
D-E	5332	16.7
ALL	4945	15.8

Table 3.4: Granite Stone Block 2 Direct sonic test results [20]

Results from sonic testing Stone Block 2		
Surfaces	velocity [m/s]	COV
A-C	4542	16.1
D-E	4508	18.6
ALL	4530	17.0

Table 3.5: Torcato church granite Direct sonic test results In-situ [20]

Results from sonic testing S.Torcato In-situ Direct			
S.No	Location	Avg. Velocity [m/s]	COV
1	P1	2073	1.84
2	P2	4220	4.54
3	P3	3244	2.39
4	P4	3821	3.38
5	P5	3322	2.39
	Mean	3336	2.91

Table 3.6: Granite Stone Block 1 Semi-Direct sonic test results [20]

Results from sonic testing Stone Block 1			
Surfaces	Distance b/w points (m)	velocity [m/s]	COV
B2-C2	0.14	3737	57.7
B2-C4	0.22	4523	37.8
B4-C2	0.22	6016	33.9
B4-C4	0.28	4589	21.8
ALL		4716	20.1

Table 3.7: Granite Stone Block 2 Semi-Direct sonic test results [20]

Results from sonic testing Stone Block 2			
Surfaces	Distance b/w points (m)	velocity [m/s]	COV
B5-B1	0.14	5875	28.3
B5-B2	0.22	4779	16.9
B5-B3	0.22	3872	39.5
B4-B3	0.283	4854	38.1
ALL		4845	30.7

Table 3.8: Granite Stone Block 1 Indirect sonic test results [20]

Results from sonic testing Stone Block 1			
Surfaces	Distance b/w points (m)	velocity [m/s]	COV
B6-B1	0.25	2714	29.5
B6-B2	0.20	4133	42.8
B5-B2	0.15	4279	48.7
B5-B3	0.10	3083	46.2
ALL		3552	21.7

Table 3.9: Granite Stone Block 2 Indirect sonic test results [20]

Results from sonic testing Stone Block 2			
Surfaces	Distance b/w points (m)	velocity [m/s]	COV
B5-B1	0.20	3116	26.9
B5-B2	0.15	2066	15.2
B5-B3	0.10	3167	50.8
B4-B3	0.05	4500	24.8
ALL		3212	31.1

3.1.3 Granite Ultrasonic tests

The same kind of tests which were performed on sonic were repeated for ultrasonic range details of which are explained in Report no 6 [20]. Also these tests were not good since the analysis of signal was done internally making it impossible to know which wave form was used in determining the results. The table 3.10 shows which tests were available ultrasonic for blocks and in-situ for comparison in both cases.

Table 3.10: Granite Ultrasonic test results (Block and in-situ) [20]

Results from Ultrasonic tests			
Serial number	Test Type	Lab	In-situ
1	Direct Ultrasonic	✓	✗
2	Semi-direct Ultrasonic	✓	✓
3	Indirect Ultrasonic	✓	✗

Table 3.11: Granite Stone Block 1 Direct Ultrasonic test results [20]

Results from Ultrasonic testing Stone Block 1	
Surfaces	velocity [m/s]
A-C	3836
D-E	3885
ALL	3852

Table 3.12: Granite Stone Block 2 Direct Ultrasonic test results [20]

Results from Ultrasonic testing Stone Block 2	
Surfaces	velocity [m/s]
A-C	3694
D-E	3849
ALL	3750

Table 3.13: Granite Stone Blocks Semi-Direct Ultrasonic test results [20]

Semi-Direct Ultrasonic testing Stone Blocks		
Surfaces	velocity [m/s]	COV
B2-C2	3720	4.3
B2-C4	3589	4.0
B4-C2	3862	0.7
B4-C4	3742	2.2
ALL	3728	3.6

Table 3.14: Granite Stone S. Torcato Semi-Direct Ultrasonic test [20]

Ultrasonic testing Insitu San Torcato		
Surfaces	velocity [m/s]	COV
A2-B2	4729	5.6
A2-B4	4422	10.1
A4-B2	4671	9.8
A4-B4	4438	11.7
ALL	4565	8.7

The other test data with high coefficient of variation (COV) was discarded due to more variability and less reliability. The same argument can be used as the tests can also be incorporated with additional uncertainty making the model more complex.

Table 3.15: Granite Stone Block 1 InDirect Ultrasonic test results [20]

Results from Ultrasonic testing Stone Block 2		
Surfaces	Distance b/w points (m)	velocity [m/s]
C1-C2	0.05	4630
C1-C3	0.10	3922
C1-C4	0.15	2650
C1-C5	0.20	2747
C1-C6	0.25	2283
C1-C7	0.30	2627
C1-C8	0.35	2513
ALL		3053

Table 3.16: Granite Stone Block 2 InDirect Ultrasonic test results [20]

Results from Ultrasonic testing Stone Block 1		
Surfaces	Distance b/w points (m)	velocity [m/s]
C1-C2	0.05	4630
C1-C3	0.10	3257
C1-C4	0.15	3119
C1-C5	0.20	3106
C1-C6	0.25	3153
C1-C7	0.30	2290
ALL		3259

3.1.4 Granite cylinder compression tests

Since in Section 3.1.1 it was presented that granite blocks from same quarry have similar hardness values. It is reasonable to assume that the compression tests and young's modulus values on cylinders should be almost similar [20]. The six cores were drilled from Stone block 2 with a diameter of 75 mm and a height equal to 155 mm. Also 3 LVDT's were placed at approximately one third of specimen height to measure the Elastic Modulus. This test is a direct test to measure the compressive strength and elastic modulus.

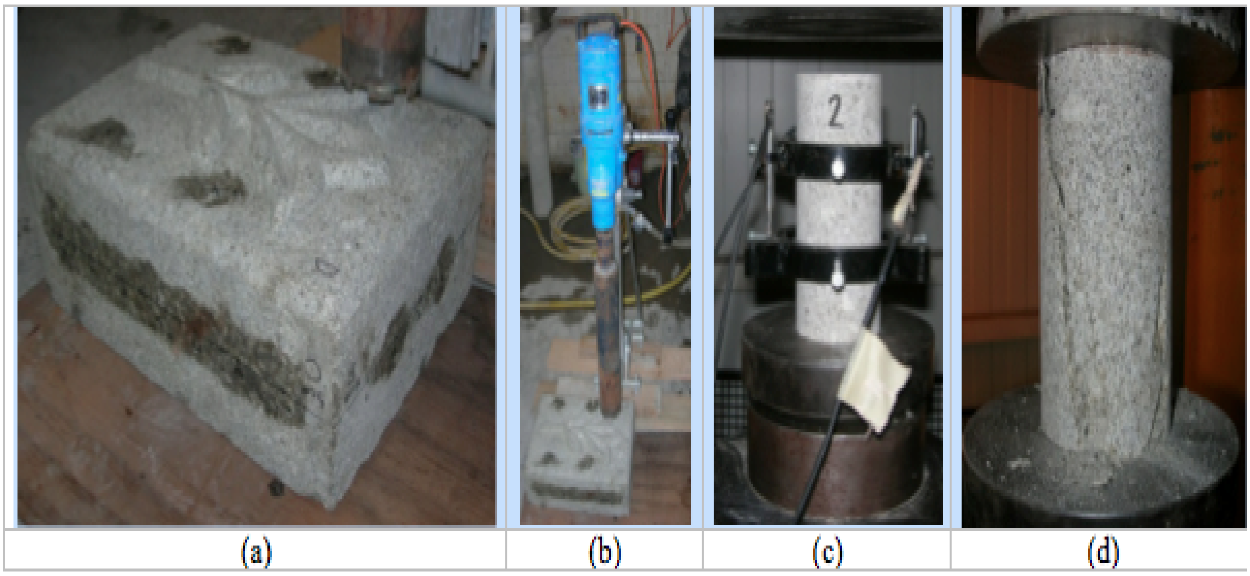


Figure 3.2: Figure showing different stages of Compression test on a granite cylinder a)Stone 2 b)Drilling of sample c) Setup to measure elastic modulus d)Failure during compression test [20]

Table 3.17: Granite compression test results [20]

Results from core testing			
Core number	Compressive strength	Young's Modulus 2^{nd} Cycle (GPa)	Young's Modulus 3^{rd} Cycle (GPa)
1	79.6	–	–
2	87.2	31.4	32.5
3	85.2	29.8	33.3
4	66.9	32.1	33.2
5	77.8	32.8	33.3
6	73.8	31.4	33.3
Average	78.4	31.5	33.1
Std Deviation	7.5	1.1	0.3
COV%	9.5	3.5	1.1

3.2 Data Fusion

3.2.1 Definition and General overview

Data fusion (data integration) sometimes known as information fusion is the process to combine different sources and different points in time into a representation that provides effective support for human or automated decision making. Moving from Biology to technology the most simple example of data fusion system can be given by the human brain (Figure 3.3). It integrates information from different senses (sensors) for e.g. eyes, ear, nose etc to arrive to a conclusion. For example while watching television sound of a voice combined with visual information helps in identifying a person. For sensor level data fusion any type of sensor data can be fused with the condition they should represent the same measurement and if not then it has to be processed to obtain an identical format. Most of the data fusion is done to integrate information from multiple sensors. Chair and Varshney [23] produced an optimised data fusion algorithm to weight each signal coming from sensors according to their reliability. Same kind of technique is applied in this report in which different NDT tests are weighed according to survey from experts and then fused using Bayesian methodology.

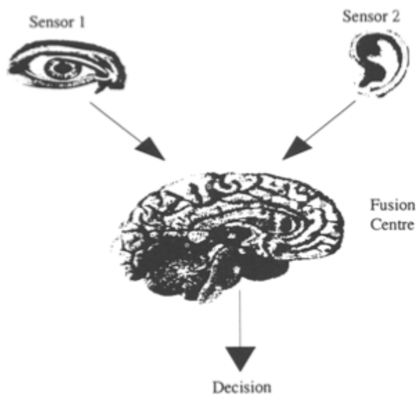


Figure 3.3: Illustration of human data fusion system [24]

The information coming from different sources can be conflicting, incomplete or vague and needs to be combined from different sources to help practitioners in decision making. Data fusion is defined as synergistic use of information coming from different sources to understand the phenomenon [24]. The Flowchart shown in Figure 3.4 shows the concept of data fusion when the information is combined using Bayesian techniques to reach to a higher confidence level. The whole methodology behind Data Fusion process is to combine information from Literature and multiple NDT tests of different reliability levels to get a more accurate picture and assessment of parameters of interest than possible with a single NDT method. A Bayesian approach is used to combine data from different indirect and direct tests for San Torcato in this case study. The role of data fusion is its use to manage uncertainty and improve accuracy and provide a rational and mathematically valid approach to fuse the data from different test to help in the decision making process of selecting proper parameters. There is some difference between data fusion and data integration which is used to describe the combining of data, whereas data fusion is integration followed by reduction or replacement. Data integration might be viewed as set combination wherein the larger set is retained, whereas fusion is a set reduction technique with improved confidence.

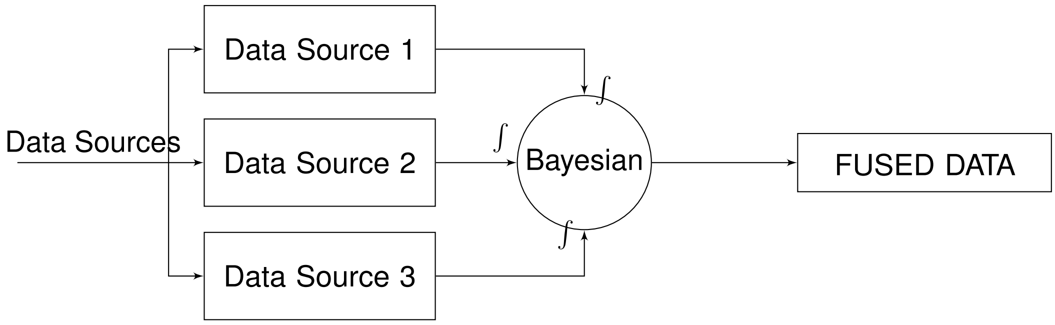


Figure 3.4: Illustration of Data Fusion system combining diverse data sets into a unified (fused) data set

The different NDT techniques used for monuments were described in Chapter 2 with their advantages and disadvantages. In addition the efficiency of different models to incorporate data is also presented in this thesis. In brief different NDT techniques were studied with different probability approaches to NDT data fusion and their efficiency in combining information was assessed using different statistical models. The main role of Data fusion through several publications is its ability to manage uncertainties and improve the accuracy of the system. In this study for fusion center we can get, for e.g.. information from sonic tests that can be supplemented from core testing data to obtain the missing information.

Data fusion in the field of NDT is still a new concept and it still needs to be understood and practised by engineers. Some research papers using NDT data fusion using Bayesian techniques related to geomechanical parameters include:

1. Assessment of bridges using Bayesian updating in which yield strength of reinforcement bars and concrete cover was updated using NDT tests [25].
2. Bayesian assessment of the characteristic concrete compressive strength using combined vague informative piers for updating strength distribution of concrete [26].
3. Bayesian methodology for updating geomechanical parameters and uncertainty quantification [27]

One of the most important examples of data fusion is to combine information coming from different NDT sensors and improve the performance of inspection. The data fusion technologies are used in many fields like target tracking, robotics, traffic control, image processing depending upon the popularity.

Bayes theorem is basically a plan of changing beliefs in face of evidence (NDT data in our case). The more the evidence is related to beliefs, the stronger the hypothesis becomes after each Bayesian updating. In this thesis we have calculated Elastic modulus of granite (E) and compressive strength (f_c) as more and more test data become available and starting with only vague knowledge about the parameter and then arriving to the posterior as our evidence correlates to our beliefs.

3.2.2 Levels of Data fusion

Data fusion can occur basically at three levels

1. **First Level:Raw data**: This involves fusion of Raw data. Most of the applications of Multi-sensor data fusion are focused on this area.
2. **Second Level:Decision**: Fusion at Decision level. In this format data may be different but needs to be converted to the format desired as it has to be in identical format to qualify for fusion.
3. **Third Level:Fusion**: This is the last level in which the data has to be fed in Data fusion center and processed by a mathematical algorithm to produce a coherent global result.

We have addressed levels 2 and 3 in this thesis. There are many methodologies of data fusion but we have focussed only on Bayesian approach in this case.

3.2.3 Different techniques of data fusion

3.2.3.1 Probabilistic Fusion

Probabilistic methods rely on the probabilistic distribution functions (PDF) to express data uncertainty. The most commonly used is Bayesian Fusion which fuses pieces of data from various sources. One can apply Bayes estimator each time and update the PDF by fusing it with new information. The data fusion algorithm implemented in this case took the form of probabilistic inference processes such as the Bayesian inference theory.

3.2.3.2 Data fusion using Artificial Intelligence (AI)

In these systems data association takes the place by emulating the decision making ability of human brain. The efficiency of these systems depend on amount of knowledge and training they undergo before using them for new data. One of the most common used (AI) systems are artificial neural networks. They come with processing units or nodes and are trained in order to solve problems. The training is done by the help of historical data and known outcomes and then they can be tested. The weight of each node (See Equation 3.1) is adjusted after each trained data set by some algorithm specified to the neural network system. They find their applicability in areas where its difficult to specify a algorithm and come with processing units known as neurons (Figure 3.5). Each node can act as an unit to process the input data and an output signal expressed as:

$$y = f(\sum w_i x_i) \tag{3.1}$$

where y is the output associated to input node i, w_i is the weight associated to the input node and x_i is the input at node i. Detailed information has not been discussed since it is beyond scope of this thesis.

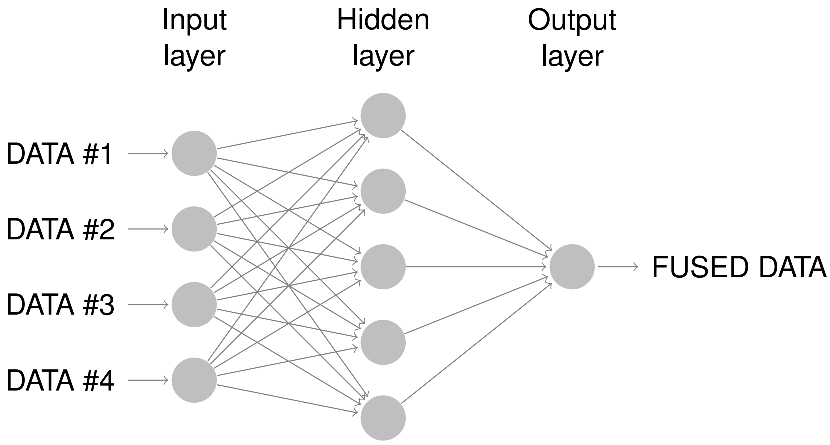


Figure 3.5: Figure Illustrating a two layer Perceptron neural network

3.2.4 Performed Data fusion using S. Torcato as case study

After collecting data from S Torcato church, it was analysed and used for the fusion process. The tests were of two types: Direct and Indirect tests. The metho dology to combine data from these two different types of sources is presented in section 3.2.4.1 and explained with the help of flowchart in Figure 3.6. However acoustic emission data could not be incorporated in this approach since no correlation was found to relate it with desired parameters of interest. However, the data fusion was done stepwise i.e dealing with two data sources at a time and then combining them into a single data source. The single data source was then combined with the next data to get another single data in an uniform format. Most trustworthy data i.e data from direct test was combined in last steps of data fusion process.

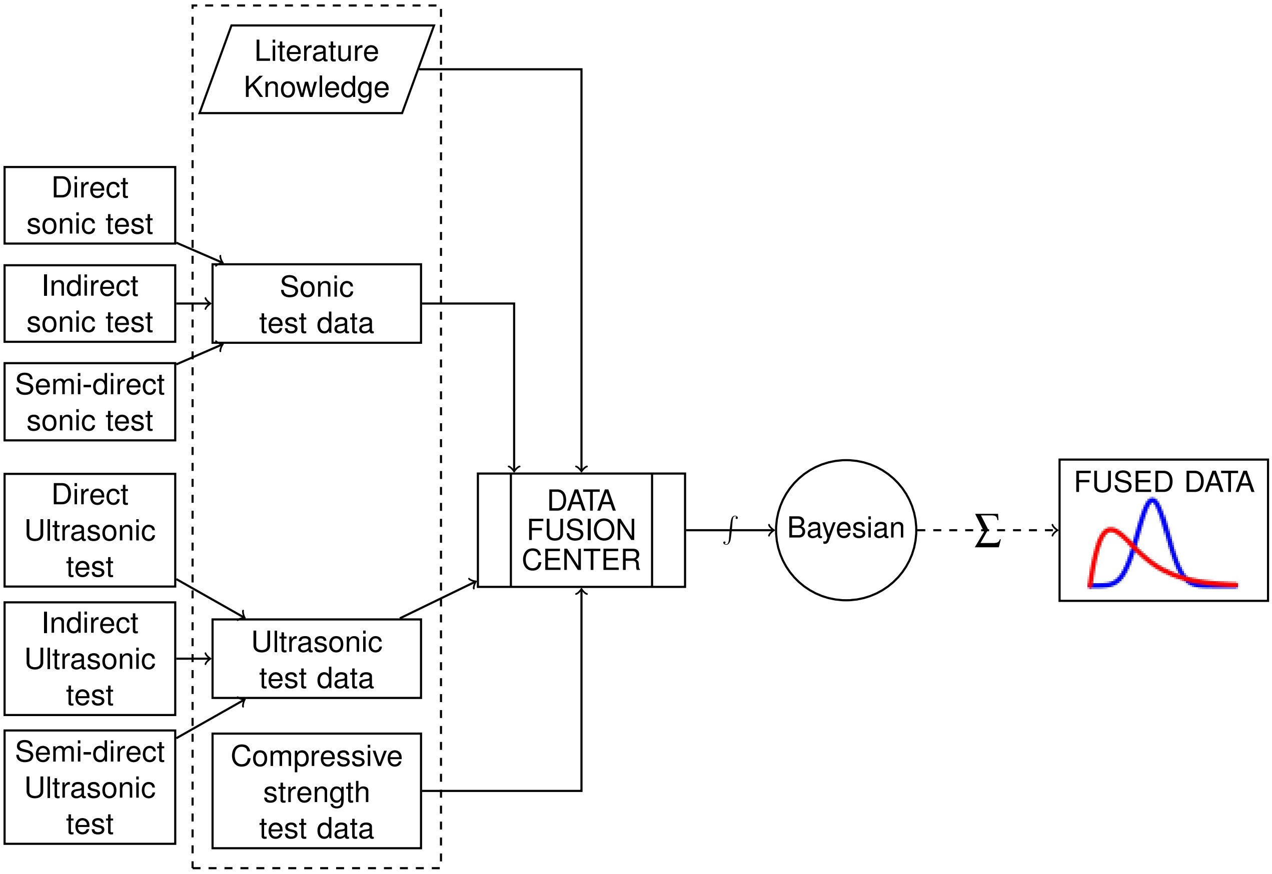


Figure 3.6: Illustration of Data Fusion system using Bayesian approach for San Torcato Church

3.2.4.1 Data Fusion from Direct and Indirect data sources

For using the Bayes theorem to update the prior to posterior it is mandatory to have the new data in the same format as prior distribution. So the indirect data has to be converted into equivalent E data to use Bayes theorem. The first thing suggested is that the data type should be same i.e if the data from source is telling some other parameter must be converted into equivalent data [21] by developing some regression relationship or by using some empirical formulas with some uncertainty. The three step procedure described by Tang is shown by the help of flowchart in Figure 3.7. Tang [28] suggested a three step procedure. The initial prior is obtained by combining the indirect test data with prior pdf to obtain the posterior PDF. The posterior PDF is considered as a prior for the second set of data and an another updated posterior is obtained. This posterior got from two indirect test data is considered as prior when we combine this with the direct test data to obtain the final Posterior distribution taken into account all types and sets of data.

So to illustrate this first, lets say for example data from Ultrasonic tests i.e velocity was converted into dynamic elastic modulus (based on vibration and wave propagation) E_d values by using Equation 3.2

$$\boxed{E_d = v_p^2 \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \rho} \tag{3.2}$$

where ν= Poisson ratio of material, ρ= density of the material and v_p= P-wave velocity. Since in this case $E = f(v, \nu, \rho)$ and all the input parameters are associated with some uncertainty, the values of E were generated using Monte Carlo simulations and the mean and variance from the generated values was found and assigned suitable reliability values depending upon the type of data in the fusion system. The Table 3.18 shows the range of uncertainty parameters on the three input parameters depending upon the characteristic of granite.

Table 3.18: Range of parameter values for granite [Engineering Toolbox]

Parameter	Range
$v_{ultrasonic}$(m/s) (Stone Block)	3694–3885
v_{sonic} (m/s)(Stone Block)	4508–5332
ν	0.2–0.3
ρ (kg/m^3)	2600–2800

As we obtain the range of values of Elastic modulus for rock E to be 20-50 Gpa can be used as a prior for combining it with indirect data source1.

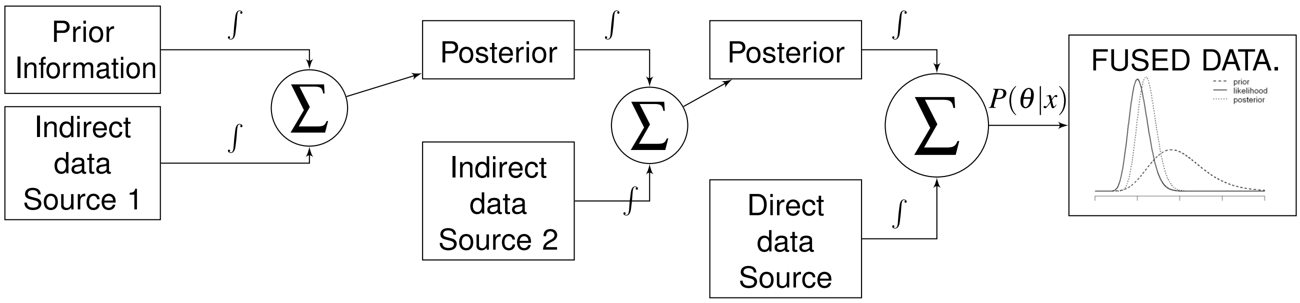


Figure 3.7: Illustration of Data Fusion system using Bayesian approach in case of indirect and direct sources of data [28]

Flowchart shown in Figure 3.7 represents the different stages of data fusion process. At stage 1 we combine the information from literature which was subjective in nature with indirect source of data to arrive to a posterior. This posterior 1 acts as a prior for the second stage of fusion process and is combined with indirect data source 2 to get posterior. Again the posterior acts as a prior and is combined with direct source of data to arrive to a final posterior distribution of interest. The number to fusion processes can be limited by the amount of data sources and also if one wants to use initial knowledge present in the literature into the model of data fusion using Bayesian methodology. Bayesian statistics combines prior knowledge with observed data. This can be done by updating previous knowledge that is prior with new knowledge gathered over time and obtaining the posterior. The Bayesian approach enables us to update our beliefs as probabilities in this case as new data is available and arrive to more accurate estimations of the parameters under study. The main objectives of this thesis are:

- To explain how Bayesian analysis can be used to combine data to arrive from prior to posterior.
- To present different data fusion methodologies used on San torcato church.
- To study and compare how different models can arrive to various intervals of parameters under study.

The Framework presented and formulated is applied to two cases in San Torcato church one is the update of the Elastic Modulus (E) of the granite blocks and another one is the update of Compressive strength (f_c) of the same blocks.

3.3 Introduction to Bayesian Statistical theory

For two events shown in Figure 3.8 and using the Venn diagram the intersection region in white colour is the conditional probability $P(A|B)$ of event A given B and is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{3.3}$$

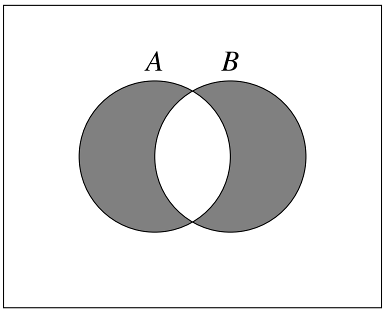


Figure 3.8: Ven Diagram representing the Probability of two events

Since all events are mutually exclusive B_j, Bayes rule is given by:

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{\sum_{j=1}^{n} P(B_j)P(A \cap B_j)} \tag{3.4}$$

where:
$P(B_i|A)$: Posterior probability of B_i;
$P(A|B_i)$: The conditional term is the likelihood;
$P(B_i)$: Prior probability of event B_i.

The Bayesian updating is done by use of relationships 3.3 and 3.4 [29]. The basic Bayesian interface can be summarised as it starts with a prior probability which is updated to a posterior probability as new information is conveyed to the system. All the A events associate conditional probability based only on new data. In monitoring the data is continuously measured and sometimes the strength and deformability parameters also experience some change in value due to several factors like aging of structure, atmospheric conditions, decay, etc. The Flowchart of Figure 3.10 represents the general decision engineering cycle [30] which can be adapted to our case of updating parameters of San Torcato church. It starts from determining the parameters under some uncertainty and included in engineering models. Then as more and more data is gathered related to the parameters they are updated with less uncertainty and used in engineering calculations. The process is complex since different types of data are available from different tests like

Schmidt hammer, sonic, ultrasonic and compressive strength data. Also data present is of different reliability levels and even the parameter of interest is not measured directly in some cases. The Bayesian approach to deal with this data makes the process easier and more rational as data is coming from different sources and different stages of the project ([31], [27]).

Bayesian methods deal with uncertainties into a mathematical form by using probability functions for the random variables and these functions are updated as more and more information is available. The whole process is composed of 3 steps [32] : (1) Setting up of a joint probability distribution function; (2) update the knowledge as more data is obtained to compute the posterior; (3) Evaluate the model to see if conclusions from it make sense and analysing how results change considering several modelling assumptions.

The flowchart shown in Figure 3.9 explains the overall updating process which takes place. First the prior is chosen with the help of some professional judgement or test results from previous knowledge including some uncertainties (rather than randomness). As the data is gathered concerning the parameters it is updated to a posterior. The main key for this whole process is to the prior as the new evidence is presented. Choice of prior can vary from very accurate prior with less standard deviation to even no knowledge which is Jeffrey's prior presented here. It is reasonable to assume prior as a normal distribution (for mathematical considerations) for modelling of many mechanical parameters [33] . When the posterior (p(θ|x)) and the prior (p(θ)) has the same parametric form is called conjugacy. For example, the Gaussian family is conjugate to itself (or self-conjugate) with respect to a Gaussian likelihood function: if the likelihood function is Gaussian, choosing a Gaussian prior over the mean will ensure that the posterior distribution is also Gaussian. A conjugate prior is an algebraic convenience, giving a closed-form expression for the posterior: otherwise a difficult numerical integration may be necessary. All members of the exponential family have conjugate priors.

3.3.1 Bayesian inference

The whole process allows to arrive from prior distribution $p(\theta)$ to a posterior distribution $p(\theta|x)$ using the likelihood of data. Since we arrive at the posterior by integrating more knowledge we expect it to be less variable than the former (however some exceptions can occur arise which are explained later for compressive strength data). The Bayesian inference in this case used two types of priors : Jeffreys and conjugate prior to arrive to a posterior distribution. The parameters of interest are considered random variable with variable moments which means that mean (μ) and variance (σ^2) are random variable following some distribution (Refer Equations 3.13 and 3.14). In this report, both mean (μ) and variance (σ^2) of E are considered to be random variables and are updated as new data is obtained. The concept of variable moments rather than fixed ones intends to incorporate several levels of uncertainty in the model. In other words, it tends to integrate the innovative concept of uncertainty on uncertainty. Both the formulations are presented in this Chapter. The figure 3.9 shows the general updating scheme which can also be adapted from data from S. torcato church.

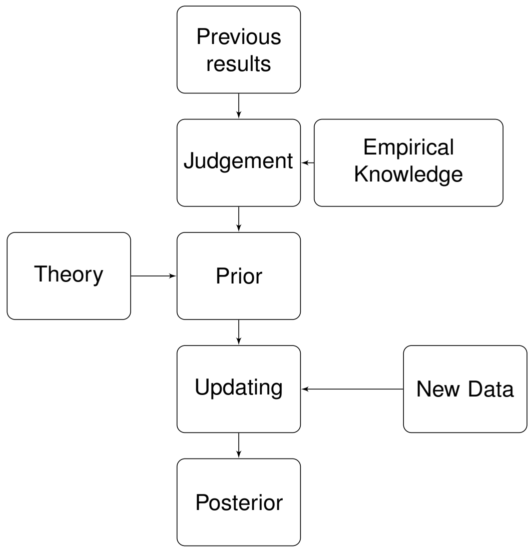


Figure 3.9: Scheme of the updating process [34]

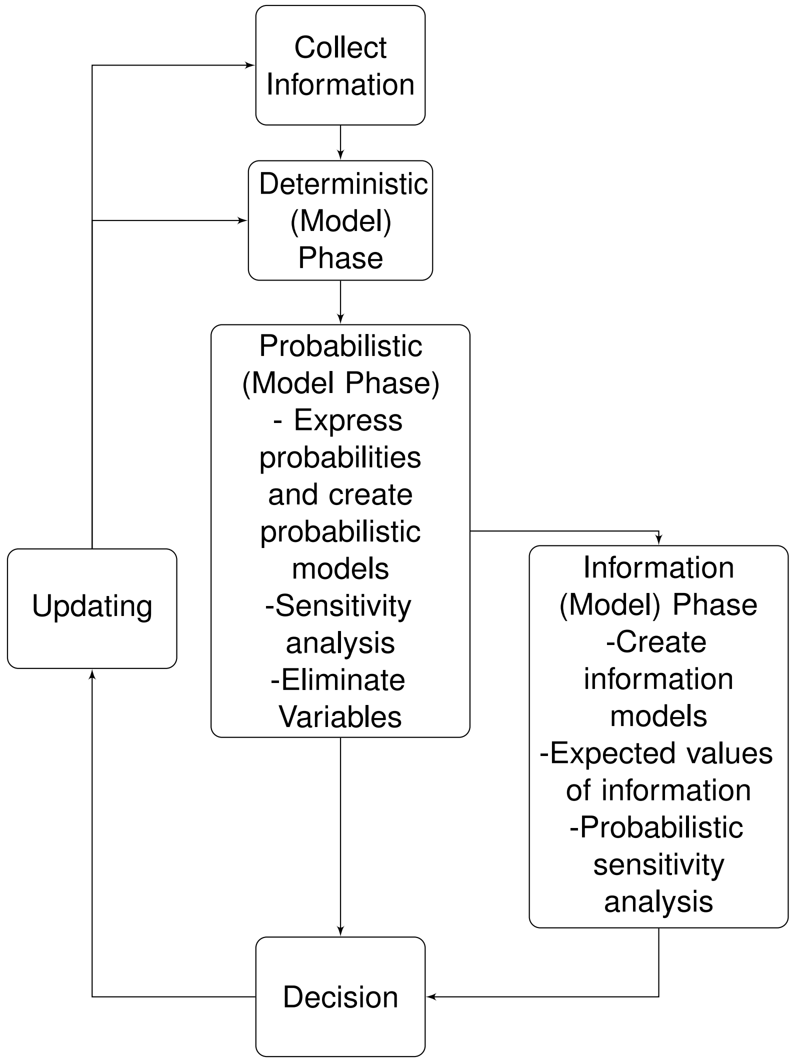


Figure 3.10: The decision cycle [30]

3.3.2 Bayesian inference using Jeffreys prior

In this case it will be assumed that mean and variance are independent of each other and assume a vague prior distributions for these two parameters. For the normal model they are given below:

$$p(\mu) \propto c, \quad -\infty < \mu < \infty \tag{3.5}$$

$$p(\sigma) \propto \frac{1}{\sigma^2}, \quad \sigma^2 > 0 \tag{3.6}$$

This is our equivalent Jeffreys prior for (μ, σ^2).

$$p(\mu, \sigma^2) \propto \frac{1}{\sigma^2}, \quad -\infty < \mu < \infty, \quad \sigma^2 > 0 \tag{3.7}$$

It is necessary to draw some inference from this improper prior to reach to a posterior distribution as more observations X=$(x_1, x_2, ...x_n,)$ are obtained.

$$p(\mu,\sigma^2|X) \propto p(\mu,\sigma^2)p(x|\mu,\sigma^2) \propto \sigma^{-n-2}exp\left\{-\frac{1}{2\sigma^2}[(n-1)s^2+n(\bar{x}-\mu)^2]\right\} \tag{3.8}$$

where

$$s=\frac{1}{n-1}\sum(x_i-\bar{x})^2 \tag{3.9}$$

This Equation 3.8 shows that conditional posterior is a normal distribution with mean $\bar{x}$ and variance σ^2/n, and marginal posterior for σ^2 is a inverse χ^2 distribution of the form:

$$\mu,\sigma^2|X \to N\left(\bar{x},\frac{\sigma^2}{n}\right) \tag{3.10}$$

$$\frac{(n-1)s^2}{\sigma^2} \to \chi^2_{n-1} \tag{3.11}$$

3.3.3 Bayesian inference using conjugate prior

For conjugate prior distribution, the joint distribution of μ and σ^2 has the form:

$$p(\mu,\sigma^2) \propto \sqrt{\frac{n_o}{\sigma^2}}exp\left\{-\frac{n_o}{2\sigma_o}(\mu-\mu_o)^2]\right\} \times \left(\frac{1}{\sigma_o^2}\right)^{\{\frac{v_o}{2}+1\}} \times exp\left(-\frac{S_o}{2\sigma_o^2}\right) \tag{3.12}$$

n_o= Initial size of sample;
S_o= Initial sum of the squared differences between the values and their mean;
μ_o= Initial mean;
σ_o= Standard deviation Initial sample.
It can be stated that the prior is a product of the density of a inverted gamma distribution with argument σ^2 and degrees of freedom v_o and density of normal distribution (μ) with variance proportional to σ^2. So finally to conclude this conjugate prior is a normal-gamma distribution with four parameters: μ_o, σ_o, n_o and v_o. Therefore, the prior on μ conditional on σ^2 is a normal with mean μ_o and variance σ^2/n_o.

$$\mu|\sigma^2 \rightsquigarrow N\left(\mu_o,\frac{\sigma^2}{n_o}\right) \tag{3.13}$$

$$\frac{1}{\sigma_o^2} \rightsquigarrow \Gamma\left(\frac{v_o}{2},\frac{S_o}{2}\right) \tag{3.14}$$

The two equations 3.13 and 3.14 are related to each other since one equations gives value of another. The appearance of σ^2 in the conditional distribution of $\mu|\sigma^2$ means that μ and σ^2 are interdependent. For instance, if σ^2 is large, then prior distribution with high variance is induced on μ. Since this formulation considers conjugate distributions, the posterior distributions for the parameters will follow the same form as the priors. The equations for posterior mean and marginal posterior density of $1/\sigma^2$ will follow the same parameters but a bit modified and are explained below:

$$\mu|\sigma^2,x \rightsquigarrow N\left(\mu_1,\frac{\sigma^2}{n_1}\right) \tag{3.15}$$

$$\frac{1}{\sigma^2}|x \rightsquigarrow \Gamma\left(\frac{\nu_1}{2}, \frac{S_1}{2}\right) \tag{3.16}$$

where:
n_1= Total Final size of sample;
S_1= Posterior sum of the squares;
μ_1= Final weighted mean;

Also seen $n_1 = n_0 + n, \nu_1 = \nu_o + n, \mu_1 = \frac{n_o}{n_o+n}\mu_o + \frac{n}{n_o+n}\bar{x}$ and $S_1 = S_o + (n-1)s^2 + \frac{n_o n}{n_o+n}(x-\mu_o)^2$.

$$\boxed{S_1 = \overbrace{\underbrace{S_o}_{\text{prior deviation}} + \underbrace{(n-1)s^2}_{\text{posterior deviation}} + \underbrace{\frac{n_o n}{n_o+n}(x-\mu_o)^2}_{\text{additional uncertainity}}}^{\text{Total deviation of posterior}}} \tag{3.17}$$

As it can be seen the parameters from the posterior distribution will combine the prior information and the information contained in the data. As seen from equation for μ_1, which is the weighted average of the prior and sample mean, with weights determined by the relative precision of the two pieces of information.The posterior sum of squares has information combined from different sources as explained above. Obtaining the posterior distribution is the fundamental objective of Bayesian analysis. To obtain the complete posterior distribution simulation methods like Markov Chain Monte Carlo (MCMC) algorithm with the Gibbs sampler was implemented to proceed with the simulation of distributions details of which are presented in Appendix A.

3.4 Dealing with different uncertainties

Risk and reliability analysis are gaining increasing importance in decision support for civil engineering problems. Risk management includes the consideration of different types of uncertainties present in a given problem and its effect. The proposed model deals with many kinds of uncertainties at different levels of fusion and from different types of data sources (See Figure 3.11) and how they can be managed into the proposed reliability model of data fusion. Uncertainties can be represented in terms of mathematical concepts based on probability theory [32] and in many cases its enough to model them using random variable with a given distribution. The parameters of these distribution functions can be estimated based on statistical and/or subjective information ([34], [35]).

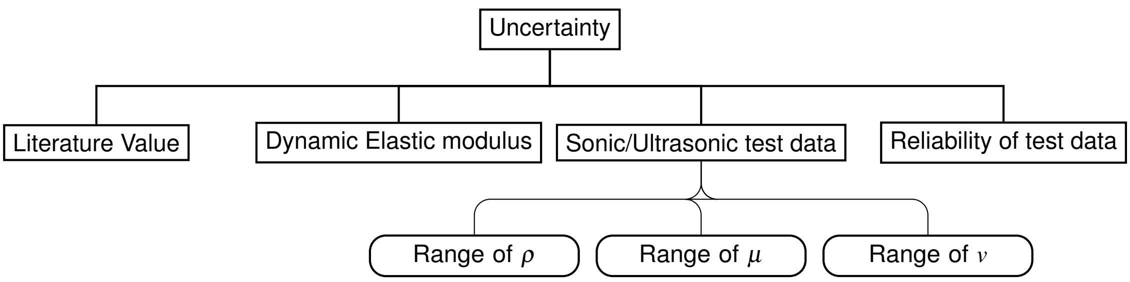


Figure 3.11: Figure showing various types of uncertainties addressed

At the step I, the data from literature sources from granite comes with uncertainty since type of granite, age, etc is not known so a range was taken to start with the model. Also while dealing with sonic/ultrasonic data the values of velocity (v), density (ρ) of granite and poisons ratio (μ) were not known so again some range was taken (For intervals adopted refer Table 3.18). The elastic modulus population was generated by using Monte Carlo simulation for different values of the above three parameters mentioned above. Also the elastic modulus computed using these tests is dynamic elastic modulus which is not exactly the Young's modulus which we are looking for from compression tests. So this needs to be multiplied by some conversion factor which can be modelled as an additional uncertainty. Currently in the scope of this thesis no factor was used. The last and most important uncertainty dealt in this thesis is the framework to modify the standard deviation considering the difference in the quality of data. (For example data coming from high quality *in situ* tests will be much more trustworthy than schmidt hammer test). All these uncertainties mentioned are included in the model in the form of standard deviation and can be reduced as more data is obtained.

4

BAYESIAN RELATIONSHIPS FOR ESTIMATING COMPRESSIVE STRENGTH AND ELASTIC MODULUS OF STONE GRANITE BLOCKS

This chapter presents two case studies in which the in situ strength of granite (f_c) and Elastic modulus (E) is found out by combining results from Literature and NDT tests (Direct and Indirect). Besides core tests (Direct), NDT tests like pulse velocity and rebound hammer (Indirect) can also be used to give some estimate regarding strength of granite and elastic modulus of rock. In this case we are testing everything on granite blocks so we assume all the Bayesian models for concrete holds for granite also with few minor modifications. Due to economic reasons it is preferred to obtain some data from NDT testing on samples before doing core tests and to develop regression relationship between them. Also it is not possible in many cases to take sample out of cultural heritage buildings as mentioned in guidelines of ICOMOS [36] due to policy of minimum intrusion.

Vague prior information is available for assessment of structures and needs to be combined with other experimental data to arrive at a better estimate. To include more information as the structure tests are carried out Bayesian methods [21] provide a more rational and consistent approach to encompass this new information. Many classical relationships to arrive at a fixed value of strength are available but they are valid when the amount of data is higher so the statistical procedures is needed to convert data in an uniform format. In this first study test data was used to predict in-situ compressive strength of granite. Since core specimens cannot be taken out for testing from church similar tests were done from the stone which was supposed to come from same quarry and results were similar [20]. Bayesian relationships are used in this case which can combine small or large amounts of data with prior knowledge which can be qualitative or quantitative in nature. It is very beneficial to use when the data supply is intermittent and frequent update of random variable is necessary. In this chapter various Bayesian approaches are used to combine data and comparison is made between different approaches.

The following timeline shows the different research carried out in Bayesian updating of mechanical parameters. The same have been applied to the data of San Torcato church to arise to some definitive value and confidence interval for the parameters. The timelines shown in Figure 4.1 cite the different researches done in geomechanical parameters using Bayesian approach. For the Fusion process the model used considers both mean (μ) and standard deviation (σ) to have moments [27] enabling model uncertainties to a great extent. Along with this uncertainties arising from literature and sonic/ultrasonic velocities are also put into this model to arrive to the posterior estimates of parameter and characterstic values (5% lower fractile).

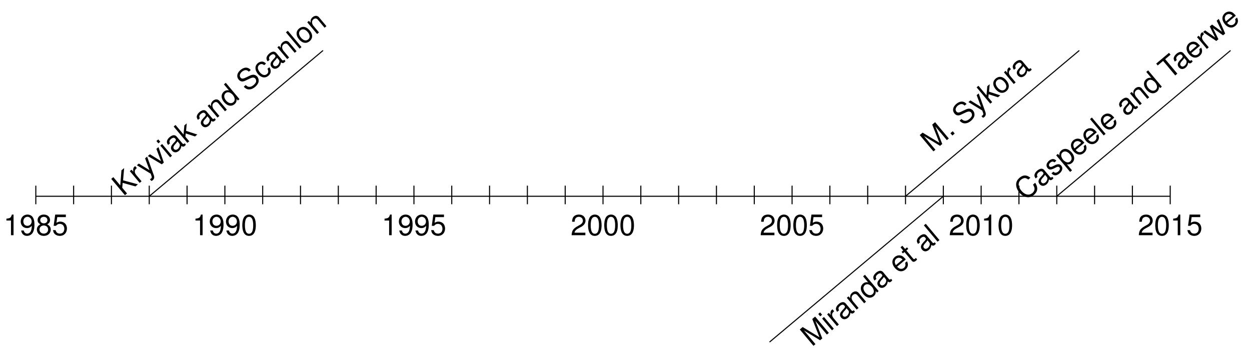


Figure 4.1: Different researches done in geomechanical parameters using Bayesian approach

4.1 Estimation of mean granite strength (f_c) when only core compressive strength testing data is available using approach by Kryviak et al.

4.1.1 Prior information

It is common to use a normal distribution for the compressive strength of concrete [37] which we are using in case of granite also. Therefore, the probability density function for the strength of the granite block can be expressed as

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} \tag{4.1}$$

where:
x: random variable usually concrete strength, granite in this case;
σ^2: variance of random variable;
μ: mean or expected value of the random variable.

In order to use Bayes theorem in this case it is pertinent to predict mean μ and σ^2 based on some prior information (construction documents) or by some experience. If they cannot be predicted the starting point can be a prior with a large variance. Since a prior distribution for mean strength is required and as such no information was available for granite compressive strength so mean value (μ)=100 and a value of variance (σ)=20 based on literature was chosen. So the prior pdf was given by [31]:

$$f(\mu) = \frac{1}{\sqrt{2\pi}\sigma_{prior}} e^{\left(-\frac{\left(x-\mu_{prior}\right)^2}{2\sigma^2_{prior}}\right)} \tag{4.2}$$

where:-
μ_{prior}: Assumed mean of granite compressive strength, 100 MPa
σ^2: assumed value of prior variance of mean compressive strength, (20 MPa)^2
μ: random variable, i.e mean granite compressive strength.

After this step of choosing the prior the next step was the determination of the likelihood function. The likelihood will give the conditional probability of obtaining test data assuming a mean of μ and since all the population comes from same population variance σ_o^2 can be assumed same for all datas. The likelihood function is given by:

$$f(x_i|\mu) = \frac{1}{\sqrt{2\pi}\sigma_o} e^{\left(-\frac{(x-\mu)^2}{2\sigma_o^2}\right)} \tag{4.3}$$

where x_i: Granite compressive strength of ith core.

4.1.2 Bayesian updating and test results

The test results for granite compressive strength can be taken from Table 3.17 in chapter 3. The table shows a total of 6 tests for compressive strength. The posterior PDF can be determined using the test results i=1 to 6 (Table 3.17) (f_c=[79.6 87.2 85.2 66.9 77.8 73.8]) using the following given by [31] :

$$f(x_i|\mu) = \frac{1}{\sqrt{2\pi}\sigma_{prior}} e^{\left(-\frac{(x-\mu_{post})^2}{2\sigma_{prior}^2}\right)} \tag{4.4}$$

where $\boxed{\sigma_{post} = \frac{1}{\frac{n}{\sigma_o^2} + \frac{1}{\sigma_{prior}^2}}}$ and $\boxed{\mu_{post} = \frac{\frac{\sigma_o^2}{n}\mu_{prior} + \sigma_{prior}^2 \bar{x}}{\frac{\sigma_o^2}{n} + \sigma_{prior}^2}}$

The plots for the granite concrete strength using this method considering only core data given (See Table 3.17) and the comparison for prior and posterior is shown in Figure 4.2.

Table 4.1: Prior and posterior values for granite strength f_c in MPa using constant mean and variance

Parameter	Prior	Posterior
μ	100	78.9
σ	20	3.03
95% confidence for mean	67.1-132.9	73.9-83.9

This approach is simple and does not has much mathematical formulation however it fails to deal with many uncertainties. The approach presented in the next section bit complex but treats different kinds of uncertainties explained in Section 3.4. However the data from Schmidt hammer could not be integrated with the data from literature and core compressive strength tests since there was no proper correlation to convert the indirect test data into equivalent compressive strength data.

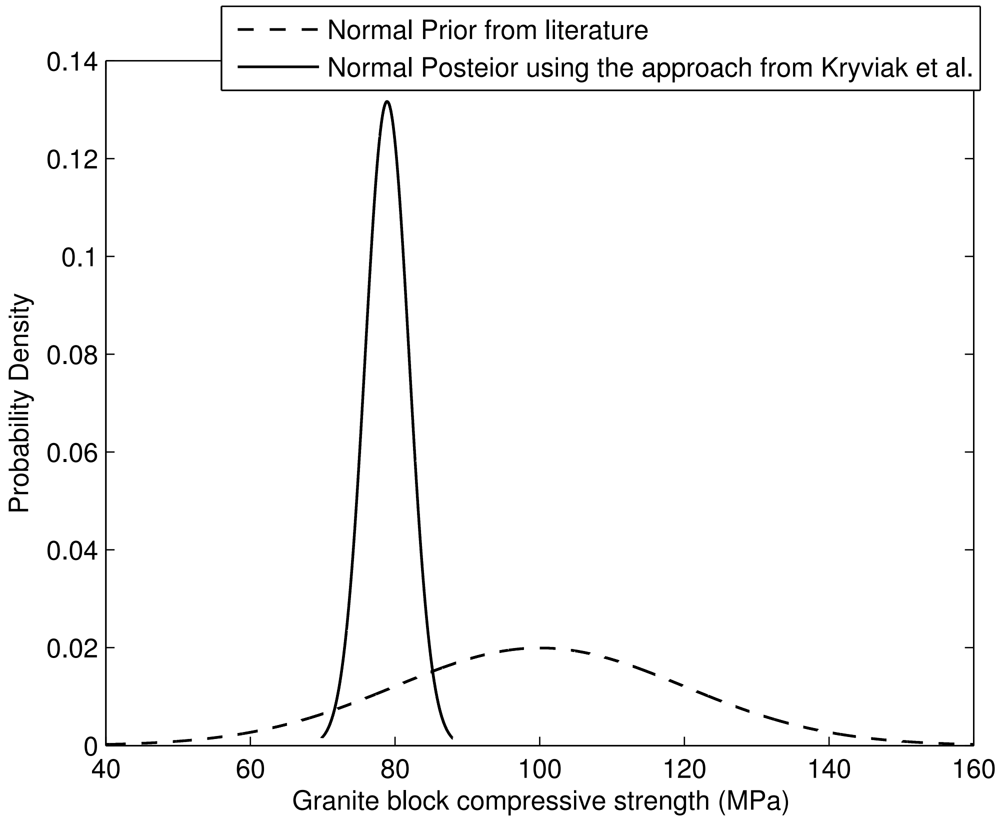


Figure 4.2: Prior and Posterior density functions for Granite compressive strength f_c in MPa using core data only [31]

4.2 Estimation of mean granite strength when only core compressive strength testing data is available using Jeffreys prior

The results obtained by using Jeffreys prior [27] were not suitable in this case (See Table 4.2) since they give a very high value of mean and variance. Reason may be the use of χ^2 distribution function with low degrees of freedom is more skewed and does not provide good results. If a higher number of test results were available then this problem might be reduced and give better accurate results.

Table 4.2: Posterior values for granite strength f_c in MPa using Jeffreys prior

Parameter	Prior	Posterior (n=6)	Posterior (n=25)	Posterior (n=50)	Posterior (n=100)
μ	100	92.21	80.47	79.34	78.94
σ	20	71.64	10.99	9.17	8.23
95% confidence for mean	67.1–132.9	-22.6–216.4	62.4–98.6	64.25–94.44	65.14–92.50

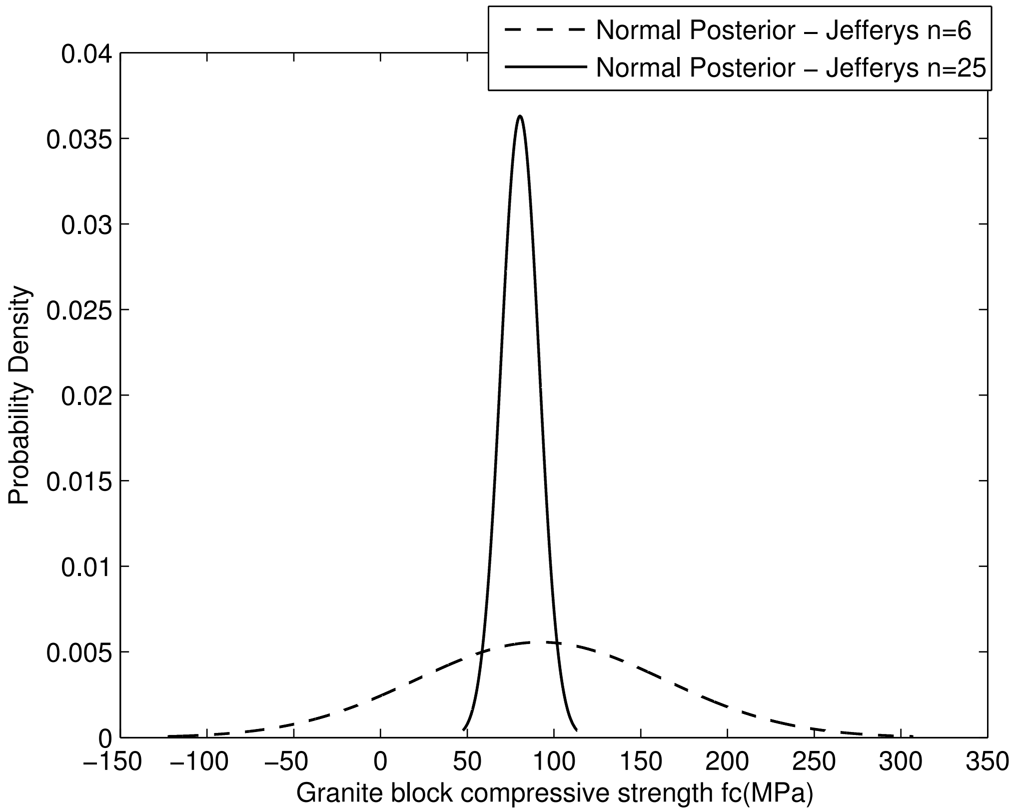


Figure 4.3: Posterior density functions for Granite compressive strength f_c in MPa using Jeffreys Prior for sample size n=6 & n=25

The results of Jeffreys prior may be used in case when sample size is large. The sample size which we used n=6 shows higher standard deviation and a large confidence interval which might not be suited in this case. Further a high number of tests will decrease the standard deviation from current value of 7.5 to lower value which will further improve the model accuracy. This factor was not taken into account while doing the simulations.

4.3 Estimation of mean granite strength when only core compressive strength testing data is available using conjugate prior

In this framework the Bayesian approach is used to update the parameter when new data is available. Regarding the initial level of knowledge considered two approaches were used: no knowledge was present about the parameter and second prior distribution was available based on some data about the parameter. The Bayesian approach is used to reduce uncertainty related to parameters actual value. In the approach [27] they Bayesian technique is used to update geomechanical parameter E by considering the mean and standard deviation as random variable and arriving at a more reliable value of E. As seen the uncertainty is represented by the standard deviation the random variable and our interest is to develop a Bayesian framework to reduce uncertainty and increase reliability of the parameter.
The same thing was done in this case but since this approach requires a initial value of n_o to start with

so 6 samples were considered and was make sure that the values of these samples satisfy the criteria of sample mean (μ_{sample}=100) and standard deviation (σ_{sample}=20) for comparison purposes with previous results. The results were also studied parametrically by varying the prior information, the variance by using trust factor and amount of data obtained. The study was performed using data obtained from real investigation.

Table 4.3: Prior and Posterior estimates of compressive strength f_c in MPa using Conjugate prior

Parameter	Prior	Posterior (n=6)	Posterior (n=50)	Posterior (n=100)
μ	–	89.21	89.20	89.20
$\sigma(\mu)$	–	4.21	1.49	1.06
σ	–	14.43	15.02	15.06
$\sigma(\sigma)$	–	3.10	1.07	0.75
95% confidence for mean	–	82.21–96.21	86.21–91.67	87.4–90.95
μ_{pop}	100	89.25	89.24	89.19
$\sigma_{population}$	20	18.98	16.47	16.18
95% confidence for population mean	67.1–132.9	58.02–120.48	62.18–116.13	62.5-115.82

where:
μ = mean value of the mean; $\sigma(\mu)$ = standard deviation of mean value; σ = mean value of the standard deviation; $\sigma(\sigma)$ = standard deviation of mean value of standard deviation; μ_{pop} = mean value of the population mean; $\sigma_{population}$ = standard deviation of the population mean.

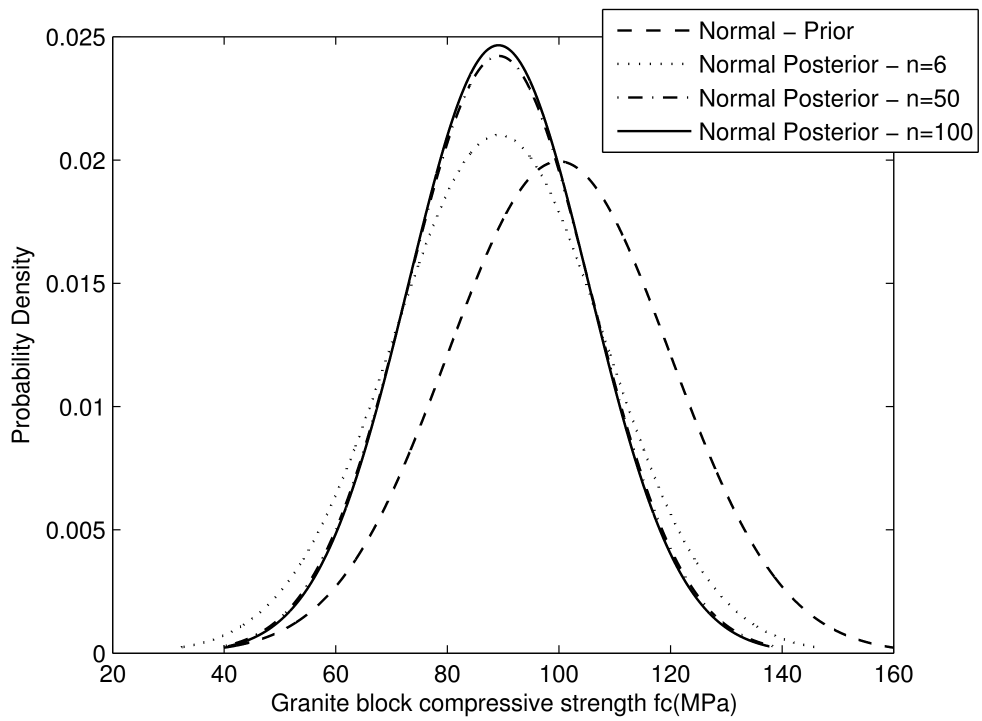


Figure 4.4: Prior and Posterior density functions for Granite compressive strength f_c in Mpa using conjugate distribution

4.3.1 Conclusions and Results

Since the number of observation data are so small to improve the prior so it can be only said that it gives a conservative value of mean = 89.25 which means the updating did take place but was not good enough to drag the mean further down.

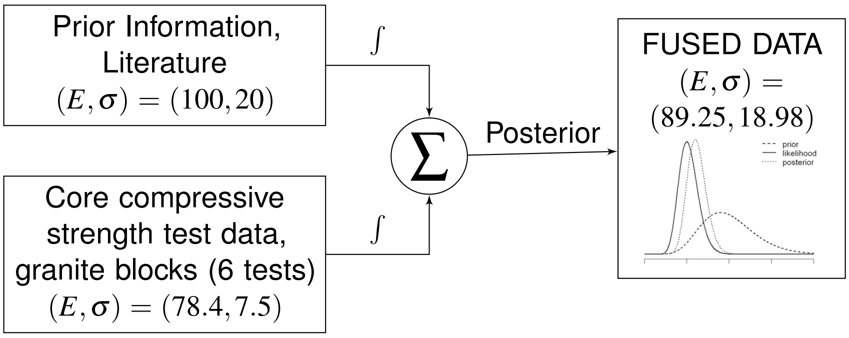


Figure 4.5: Data Fusion system using Bayesian approach for updating Compressive strength f_c in case of data from Literature and core compressive strength data for granite

In these cases, Bayesian updating does not decrease uncertainty by greater extend since the initial mean value of f_c and the value of new data were considerably different. As it can be seen from Figure 4.4, as number of samples increases there is a increase in reliability interval of the parameter. One of the most important thing while increasing number of tests is as they increase sigma will decrease which was not taken account in this case. Had this factor taken into account the graphs will be clearly separated.

4.4 Update of Elastic Modulus of Granite E using Bayesian Technique

From the database of elastic modulus (See Table 4.4) the granite modulus interval was estimated to be 20-50 GPa (Prior Information). This interval was used to establish to establish prior parameters namely, mean (μ) = 35 and standard deviation (σ) = 7.5 to cover the range given in Table 4.4 on choosing elastic modulus for rocks. Choosing these interval range allows us to cover 95 % of confidence interval ($\mu - 2\sigma \leq$ E $\leq \mu + 2\sigma$) as shown in Figure 4.6. Same calculations were performed to obtain the posterior distribution as done in Chapter 4 , Section 4.3. The normal distribution was used to model the geomechanical parameter E. The matrix for E value [31.4 29.8 32.1 32.8 31.4 32.5 33.3 33.2 33.3 33.3] was used as got from Table 3.17 from doing compressive test on granite blocks. The new values from compressive strength direct test data show mean (μ) of 32.3 and standard deviation (σ) of 1.1551.

Rock Type	Modulus of Elasticity (GPa)
Limestone	3–27
Dolomite	7–15
Limestone(very hard)	70
Sandstone	10–20
Quartz-sandstone	60–120
Greywacke	10–14
Siltstone	3–14
Granite - slightly altered	10–20
Granite - good	20–50
Quartzite - Micaceous	28
Quartzite - sound	50–80
Dolerite	70–100
Basalt	50
Andesite	20–50
Amphibolite	90

Table 4.4: Guideline table for choosing Elastic Modulus for Rocks [38]

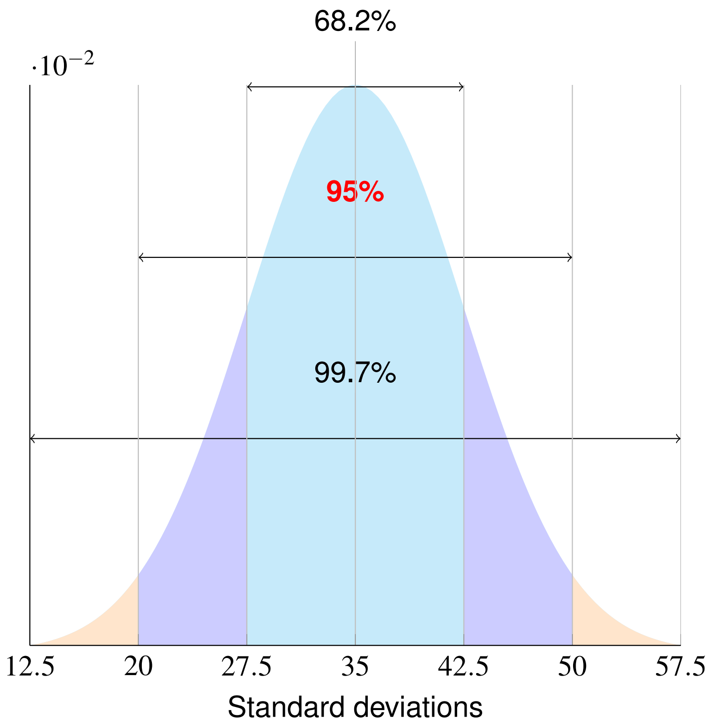


Figure 4.6: The Normal distribution for Literature values of Elastic Modulus (E) of granite

4.4.1 Determination of Posterior using Jeffreys prior for Elastic Modulus of granite

The results obtained by using an non-informative Jeffreys prior for estimating the elastic modulus for granite blocks are shown in Table 4.5 and the distribution is plotted in figure 4.7. The results obtained by jeffreys prior mainly depend on the variance of direct compression test results, since no initial knowledge was considered.

Table 4.5: Posterior values for granite Elastic modulus(E) using Jeffreys prior

Parameter	Posterior
μ	32.64
σ	1.64
95% confidence for mean	29.93–35.35

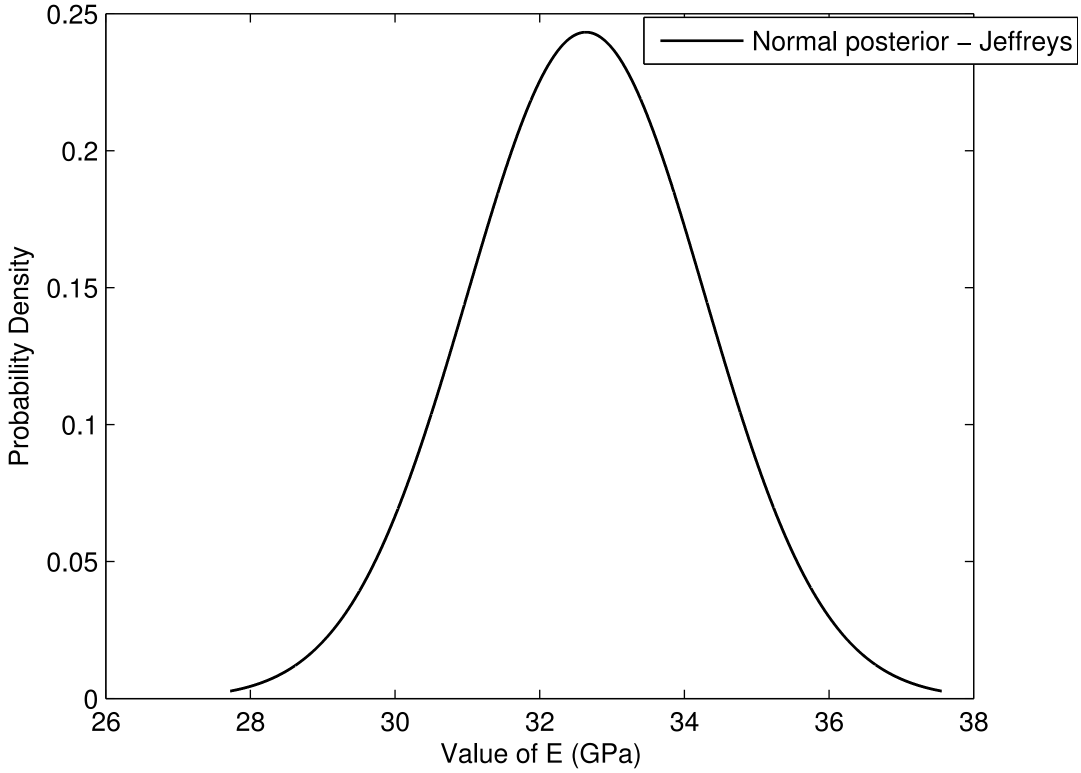


Figure 4.7: Posterior density functions for Granite Elastic Modulus for normal case using Jeffreys Prior

4.4.2 Bayesian Model Incorporating data from sonic tests using conjugate prior

The model proposed in the Section 3.2.4.1 on data fusion Chapter 3.2 Flowchart [3.2.4.1] was used in which the values of $E = f(v_s, \nu, \rho)$ [Table 3.18] were generated using Monte Carlo simulation methods. A total of 10000 values were generated by writing a simple Matlab code using equation 3.2 with velocity, Poisson's ratio and density varying randomly over their specified range. Mean and standard deviation of them was found which was combined with the initial knowledge of Elastic modulus from the literature and posterior was obtained. The used Matlab code is presented in appendix B.
Since no weight or preference of the values available from literature is known so 50% weightage was given to both of them to arrive to a posterior which acts as a prior when combining it with data from the direct tests to get to a Final posterior. The flowchart of Figure 4.9 summarises all the results with respective values.

Table 4.6: Values for granite Elastic modulus (E) for sonic velocity data (Block) generated using Monte Carlo simulation with uncertainty in Poisson ratio ν and Density ρ

Parameter	Value (GPa)
E	31.51
σ	5.06

Table 4.7: Prior and Posterior estimates of E (Normal Distribution) in GPa

Parameter	Literature	1^{st} Update	2^{nd} Update
μ	–	33.20	32.54
$\sigma(\mu)$	–	0.66	0.53
σ	–	6.59	5.29
$\sigma(\sigma)$	–	0.47	0.38
95% confidence interval for mean	–	32.12-34.29	31.67-33.40
μ_{pop}	35	33.21	32.52
$\sigma_{population}$	7.5	7.25	5.82
95% confidence interval for population mean	20.3-49.7	21.28-45.14	22.94-42.10

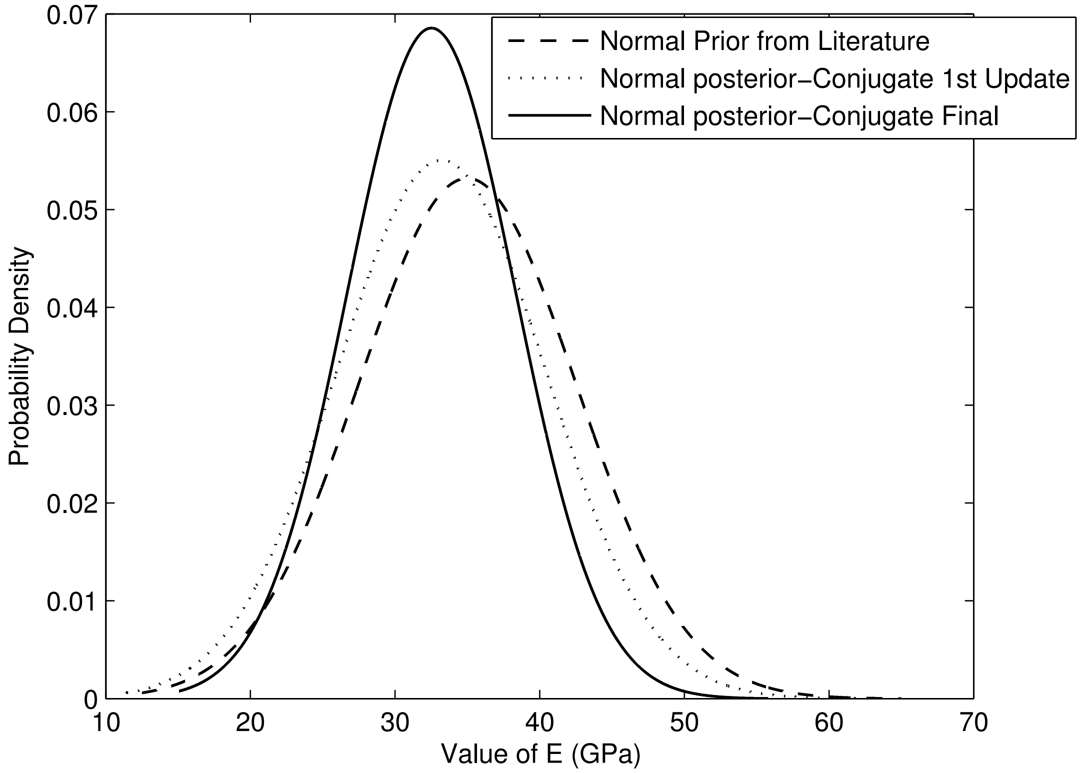


Figure 4.8: Posterior density functions for Granite Elastic Modulus E before and after updating considering data from Literature, sonic and direct compressive strength data

The updated mean of E was closer to the mean obtained from direct compression tests and a significant impact on reducing uncertainties was obtained. However, since the initial deviation for literature data was a bit more so the mean of final posterior didn't decrease by a large amount. The standard deviation value was reduced to 22.8 % with direct impact on a substantial narrowing of the 95 % CI for the mean. The uncertainty in the E was clearly reduced by using this methodology. When comparing it with Jeffreys prior (Table 4.5), the uncertainty is higher for the conjugate distributions. The fact is due to consideration of the prior information uncertainty which does not exist using Jeffreys prior.

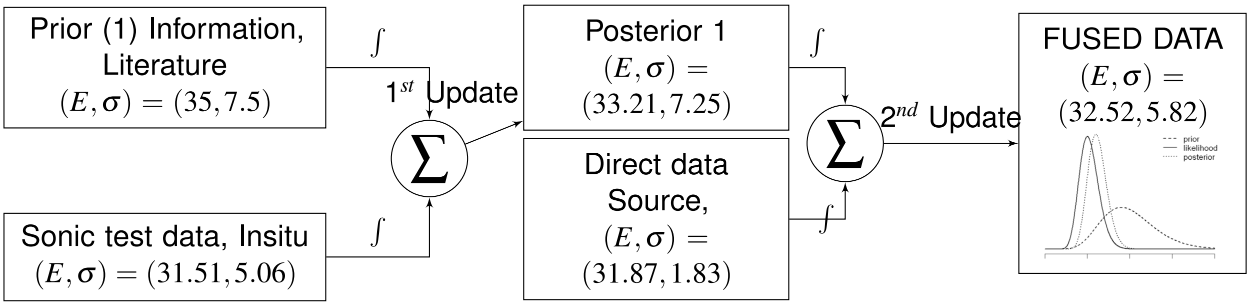


Figure 4.9: Data Fusion system using Bayesian approach for updating Elastic modulus E in case of data from sonic tests and compressive strength data

4.4.3 Bayesian Model Incorporating data from sonic and ultrasonic tests

Same steps as stated in Section 4.4.2 for generation of values were repeated and the two tables 4.8 and 4.9 for sonic and ultrasonic data were obtained.

Table 4.8: Values for granite elastic modulus (E) for sonic velocity data (block) generated using Monte Carlo simulation with uncertainty in Poisson ratio ν and density ρ

Parameter	Value
E	31.51
σ	5.06

Table 4.9: Values for granite elastic modulus (E) for ultrasonic velocity data (block) generated using Monte Carlo simulation with uncertainty in Poisson ratio ν and density ρ

Parameter	Value
E	32.20
σ	2.16

Table 4.10: Prior and Posterior estimates of E (Normal Distribution) in GPa considering sonic and ultrasonic data

Parameter	Literature	1^{st} Update	2^{nd} Update	3^{rd} Update
μ	–	33.20	32.70	32.28
$\sigma(\mu)$	–	0.66	0.54	0.44
σ	–	6.59	5.33	4.35
$\sigma(\sigma)$	–	0.47	0.38	0.31
95% confidence interval for mean	–	32.12-34.29	31.82-33.59	31.57-33.00
μ_{pop}	35	33.21	32.70	32.26
$\sigma_{population}$	7.5	7.25	5.89	4.80
95% confidence interval for population mean	20.3-49.7	21.28-45.14	23.00-42.39	24.37-40.15

As in this case, it can be seen that the mean doesn't change significantly but the impact on uncertainty is important.

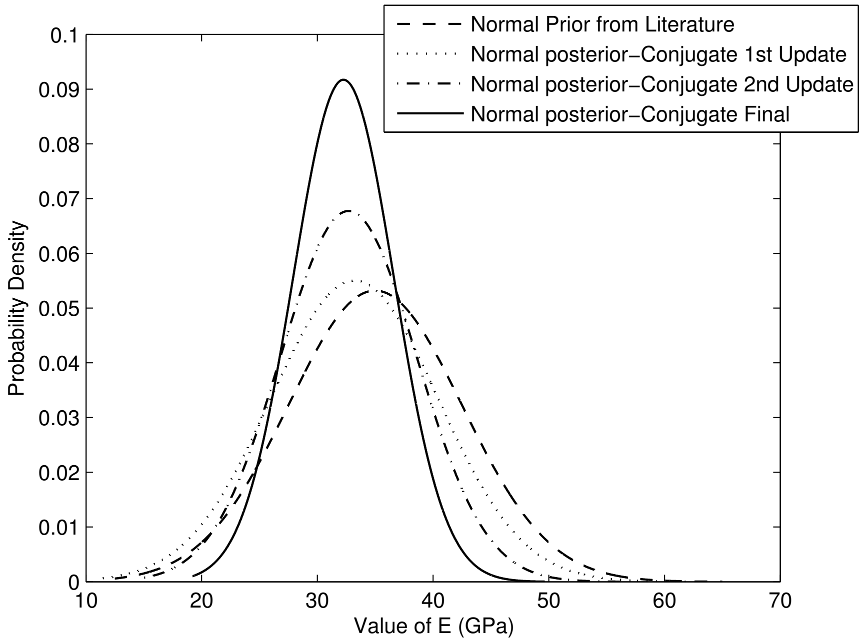


Figure 4.10: Posterior density functions for Granite Elastic Modulus E before and after updating considering data from Literature, Sonic, Ultrasonic and direct compressive strength data

The flowchart of Figure 4.9 below summarises all the results with respective values. As seen the final value is closer to the value obtained from direct compression test results of elastic modulus E. However, same logic applied when comparing the values obtained from conjugate and Jeffreys prior since the values obtained from conjugate prior show more deviation than Jeffreys prior. The standard deviation values was reduced to 36 % in this case as compared to 22.8 % with direct impact on a substantial narrowing of the 95 % CI for the mean. This model takes into account all the data from different NDT and Destructive tests from S. Torcato case study and presents good results after fusion of data from many steps.

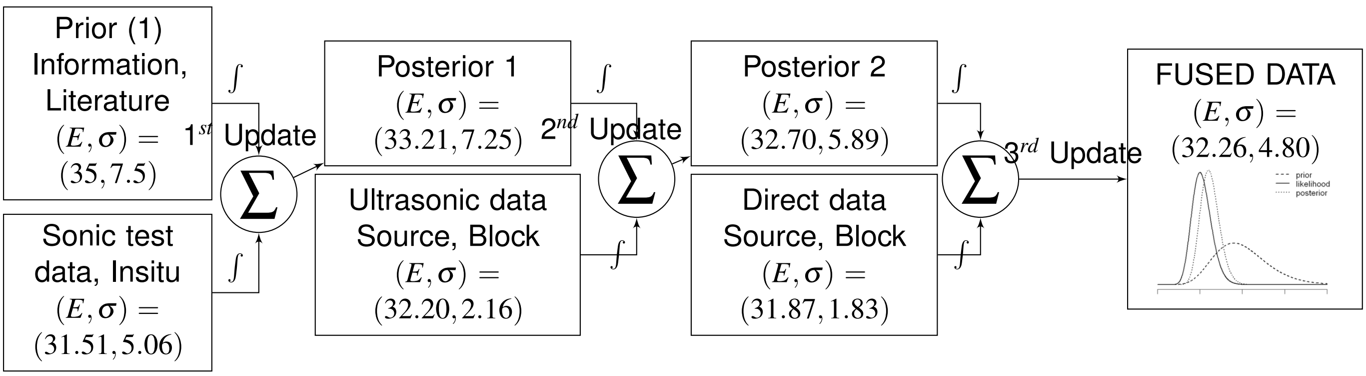


Figure 4.11: Data Fusion system using Bayesian approach for updating Elastic modulus E in case of data from sonic, Ultrasonic and compressive strength data

4.4.4 Conclusions from Bayesian model to calculate Elastic modulus (E)

The Figure 4.12 summarises the change in elastic modulus mean values and standard deviation. The error bars on the y-axis represent the standard deviation of the parameter/ CI intervals and can be seen there is reduction from 7.5 to 4.8 in their values. The numbers on x axis represent different stages of data fetching and are mentioned below:

1. Literature database
2. Sonic test data
3. Ultrasonic test data
4. Direct compression strength test data

As it can be seen from 1 to 4 there is uncertainty reduction for all levels. For the normal distribution case the standard deviation of the mean ($\sigma(\mu)$) was reduced from 0.66 GPa to 0.44 Gpa, i.e 33% decrease from the value of prior 2. The mean of the standard deviation (σ) underwent a 34 % decrease from 6.59 GPa to 4.35 GPa. Also for the population values the updating process allowed a significant reduction on the dispersion measures which means the uncertainty of the parameter.

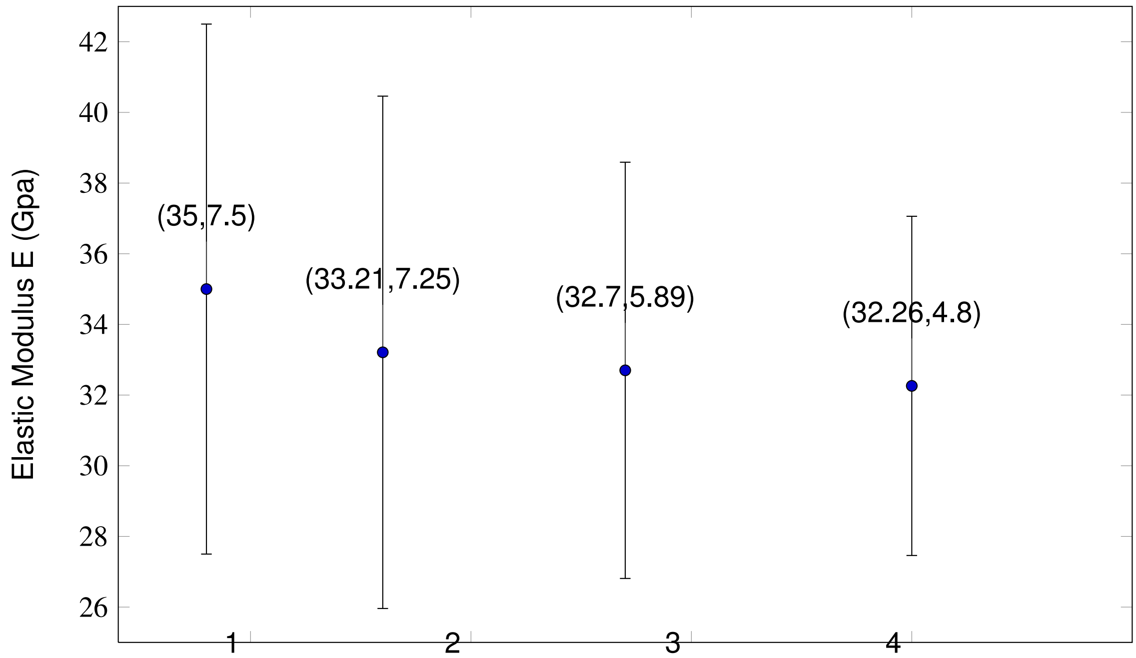


Figure 4.12: Elastic modulus updating after different steps without trust factor

5 DESCRIPTION OF MATLAB TOOLBOX MADE FOR DATA FUSION AND CALCULATIONS OF TRUST FACTOR (T)

A simple Matlab [39] toolbox (NDT_FUSION) was created to fuse data and arrive to a more certain value of parameter of interest using Bayesian methodology. A screenshot of the GUI is shown in the Figure 4.10 and how to use manual is in Appendix E. The GUI was tested for values from the paper of [27] and the same values and graph for elastic modulus (E) was obtained same as mentioned in the paper (Table 6). However this GUI needs some modifications to include Trust factor (see Section 5.2) to give more logical results for cases in which the reliability of data from different tests are different. We can apply this Bayes estimator updating (Figure 4.10) each time and update the probability distribution function and confidence interval of the parameter by fusing it with new piece of data.
In the second part of this chapter the calculations for trust factors are explained and the results including this trust factor in the Bayesian model are shown. Since, no guideline for using trust factor was stated previously so some values are proposed and parametric study needs to be done for different choice of trust factors. Some of the values proposed for choosing trust factor are mentioned in the conclusions in section 6.2.

5.1 Description of Matlab toolbox NDT_FUSION and NDT_FUSION_TRUST

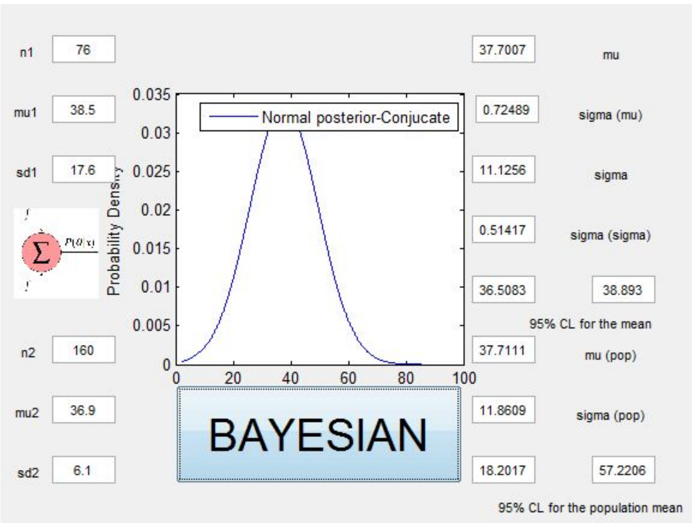


Figure 5.1: Graphical User Interface (GUI) for updating elastic modulus

The GUI (NDT_FUSION) (See Figure 5.1) was replaced my new modified GUI (NDT_FUSION _TRUST) including trust factor T_1 and T_2. Screenshot of which is shown in the Figure 5.2. The calculation of Trust factor are explained later in this section.

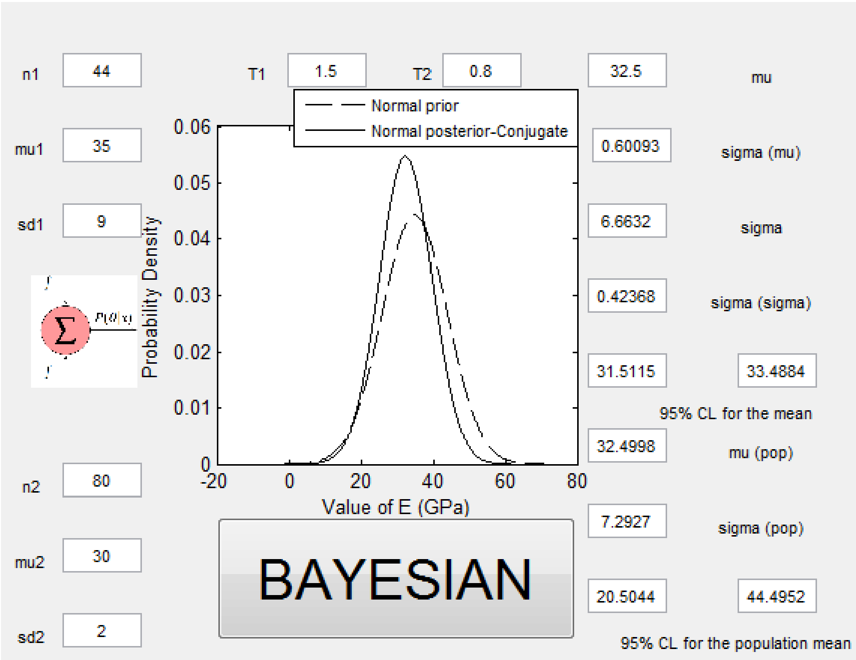


Figure 5.2: Graphical User Interface (GUI) for updating elastic modulus using trust factors T_1 and T_2

5.2 Need for trust factor for NDT tests

The Trust Factor (TF) is intended to introduce the subjective concept of having more confidence in the results of some tests than others. As some tests are more important than others and this fact should be inputted in the Bayesian approach. For all the NDT tests above there is a need for some factor to scale our responses mainly the standard deviation (σ) by dividing it with factor less than 1 or in case of less trustworthy results multiplying it by factor less than 1. Since in some cases the data from indirect tests has less spread than that from direct tests so it needs to have some correction to give correct results otherwise the data from indirect tests will have more contribution in final result. To scale the tests a survey was carried out by asking Professors and P.hd students expert in NDT field to fill their preference form by giving them rating among different tests using AHP- Analytic Hierarchy Process [40]. Their ratings along with some mathematical formulations applied and explained later in this chapter helps us to find the trust factor to scale our responses.

5.2.1 Description of the survey form used for calculation of weights for different tests

The survey is meant for people expert in area of NDT testing. The survey form is to find the importance given to each NDT method relative to another for evaluating a certain parameter of interest for example Elastic Modulus (E) in survey 1 and Compressive strength (f_c) in survey 2. The survey uses AHP- Analytic

Hierarchy Process scale shown in Table 5.2 to rate each test as compared with another. The table is divided into 9 scales from 1-9.
For example if method A is 2 times more important than method B, then it also implies method B is 1/2 times more important than method A to be consistent in filling the form.
Example: How is knowledge from Literature more important in relation to other tests for evaluating value of elastic modulus of stone?

Table 5.1: Sample question of the form

Sonic	Ultrasonic	Direct Compression test
1/5	1/2	1/8

Table 5.2: Sample question of the form [40]

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
2	Weak	–
3	Moderate Importance	Experience and Judgment slightly favor one activity over another
4	Moderate plus	–
5	Strong Importance	Experience and Judgment strongly favor one activity over another
6	Strong plus	–
7	Very strong or demonstrated importance	The evidence of favoring one activity over another is of highest possible order, its dominance demonstrated in practice
8	Very, very strong	–
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

5.2.2 Methodology for calculating weightage of each NDT test

To calculate the relative weights a fuzzy approach based in three step is needed as mentioned below:
1. First the three triangular fuzzy numbers (TFNs) [See Equation 5.1] are computed and can be seen from Figure 5.3 as they represent pessimistic, moderate and optimistic estimate of opinions given by experts for each survey.

$$\tilde{a}_{ij} = (\alpha_{ij}, \delta_{ij}, \gamma_{ij}) \tag{5.1}$$

$$\alpha_{ij} = Min(\beta_{ijk}), \quad k = 1, ..., n \tag{5.2}$$

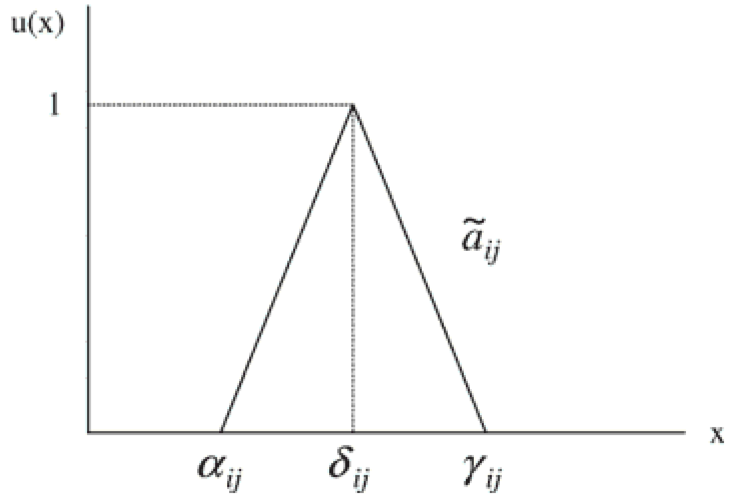


Figure 5.3: Membership function of fuzzy delphi method [40]

$$\delta_{ij} = \left(\prod_{k=1}^{n} \beta_{ijk}\right)^{1/n}, \quad k = 1, ..., n \tag{5.3}$$

$$\gamma_{ij} = Max(\beta_{ijk}), \quad k = 1, ..., n \tag{5.4}$$

As we can see the fuzzy numbers $\alpha_{ij} \leq \delta_{ij} \leq \gamma_{ij}$, $\alpha_{ij}, \delta_{ij}, \gamma_{ij} \in [1/9, 1] \bigcup [1, 9]$ and $\alpha_{ij}, \delta_{ij}, \gamma_{ij}$ are calculated by using equations 5.2, 5.3 and 5.4 respectively where :
α_{ij}- Indicates the lower bound.
β_{ijk}- Relative intensity of importance of expert k between activities i and j.
γ_{ij}- Indicates the upper bound.
n- number of people in survey.

2. After step 2 the fuzzy positive reciprocal matrix is obtained as given below:

$$\tilde{A}_{ij} = [\tilde{a}_{ij}], \quad \tilde{a}_{ij} \times \tilde{a}_{ji} \approx 1, \forall \quad i, j = 1, 2, ..., n \tag{5.5}$$

Or

$$\tilde{A} = \begin{bmatrix} (1,1,1) & (\alpha_{12}, \delta_{12}, \gamma_{12}) & (\alpha_{13}, \delta_{13}, \gamma_{13}) \\ (1/\gamma_{12}, 1/\delta_{12}, 1/\alpha_{12}) & (1,1,1) & (\alpha_{23}, \delta_{23}, \gamma_{23}) \\ (1/\gamma_{13}, 1/\delta_{13}, 1/\alpha_{13}) & (1/\gamma_{23}, 1/\delta_{23}, 1/\alpha_{23}) & (1,1,1) \end{bmatrix}$$

3. Last step will be calculation of relative fuzzy weights of evaluation factors.

$$\tilde{Z}_i = [\tilde{a}_{ij} \otimes .... \otimes \tilde{a}_{in}]^{1/n} \tag{5.6}$$

$$\tilde{W}_i = \tilde{Z}_i \otimes (\tilde{Z}_i \oplus ... \oplus \tilde{Z}_n)^{-1} \tag{5.7}$$

where $\tilde{a}_1 \otimes \tilde{a}_2 = (\alpha_1 \times \alpha_2, \beta_1 \times \beta_2, \gamma_1 \times \gamma_2)$; Explanation of symbols is given below:
$\otimes$: Multiplication of fuzzy numbers
$\oplus$: Addition of fuzzy numbers
$\tilde{W}_i = (\omega_1, \omega_2, ...\omega_n)$: Row vector consists of fuzzy weight of ith factor.
The defuzzification is explained later in calculations done for calculating the different weight factors explained in this thesis.

5.2.3 Calculations Explained- The weighting factor determination

Among the 11 people who filled the survey, 8 were university professors and 3 were Phd students who had some experience about NDT testing methods. Based on their responses, the method was applied as explained in Section 5.2.2. Only one response was not filled with proper factors and was rejected. It does not satisfy the criteria $\alpha_{ij}, \delta_{ij}, \gamma_{ij} \in [1/9, 1] \bigcup [1, 9]$. All the individual responses which we got are attached in appendix C. The response of survey 1 in which weightage factors for calculating Elastic modulus E were found to be mainly biased on direct test results. Direct compression results were ranked much higher than other results. However the literature knowledge, sonic and ultrasonic results were put in same category since there was not much difference in responses. Some people preferred one result while some another so the weights came out to be almost same for these three tests. Similar trends were observed for the case of NDT tests for calculating granite block compressive strength test (f_c). The respondents preferred direct compression test much more than literature values and schmidt hammer tests. All the conclusions deduced in this section about the weights can be easily seen by checking the results obtained later in this section.

5.2.4 Weightage factor for for NDT tests used in calculating elastic modulus of granite block

1. The matrix is presented below which we have got from the survey responses.

Table 5.3: All acceptable responses for survey: Part 1

Test method	Literature Knowledge	Sonic testing	Ultrasonic Testing	Direct compressive test
Literature Knowledge	1	(1/2,2,1/5,1/2,3, 7,3,1/3,1/5,1)	(1/3,1/2,1/5,1/5,5, 9,4,1/3,1/3,1/2)	(1/7,1/7,1/8,1/9,1/8, 1/3,1/8,1/5,1/3,1/8)
Sonic testing	Positive reciprocal	1	(1/2,1/3,1,1/2,3, 5,1/4,1,1/5,1/2)	(1/6,1/8,1/5,1/8,1/8, 1/9,1/5,1/9,1/3,1/7)
Ultrasonic Testing	Positive reciprocal	Positive reciprocal	1	(1/5,1/5,1/5,1/5,1/8, 1/9,1/6,1/9,1/3,1/6)
Direct compressive test	Positive reciprocal	Positive reciprocal	Positive reciprocal	1

2. Following outline above from Table 5.3 , we obtain a fuzzy positive reciprocal matrix $\tilde{A}$.

$$\tilde{A} = \begin{bmatrix} (1,1,1) & (1/5,0.91691,7) & (1/5,0.7628,9) & (1/9,0.1618,1/3) \\ (1/7,1.0906,5) & (1,1,1) & (1/5,0.70711,5) & (1/9,0.15431,1/3) \\ (1/9,1.3110,5) & (1/5,1.41421,5) & (1,1,1) & (1/9,0.17215,1/3) \\ (3,6.1804,9) & (3,6.48046,9) & (3,5.809,9) & (1,1,1) \end{bmatrix}$$

3. Calculation of relative fuzzy weights of the evaluation factors.

$$\tilde{Z}_1 = [\tilde{a}_{11} \otimes \tilde{a}_{12} \otimes \tilde{a}_{13} \otimes \tilde{a}_{14}]^{1/4} = [0.2582, 0.5800, 2.1407]$$

$$\tilde{Z}_2 = [\tilde{a}_{21} \otimes \tilde{a}_{22} \otimes \tilde{a}_{23} \otimes \tilde{a}_{24}]^{1/4} = [0.2374, 0.5873, 1.6990]$$

$$\tilde{Z}_3 = [\tilde{a}_{31} \otimes \tilde{a}_{32} \otimes \tilde{a}_{33} \otimes \tilde{a}_{34}]^{1/4} = [0.2229, 0.7516, 1.6990]$$

$$\tilde{Z}_4 = [\tilde{a}_{41} \otimes \tilde{a}_{42} \otimes \tilde{a}_{43} \otimes \tilde{a}_{44}]^{1/4} = [2.2795, 3.9192, 5.1961]$$

$$\sum \tilde{Z} = [2.9980, 5.8381, 10.7348]$$

$$\tilde{W}_1 = \tilde{Z}_1 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3 \oplus \tilde{Z}_4)^{-1} = [0.0241, 0.0993, 0.7140]$$

$$\tilde{W}_2 = \tilde{Z}_2 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3 \oplus \tilde{Z}_4)^{-1} = [0.0221, 0.1006, 0.5667],$$

$$\tilde{W}_3 = \tilde{Z}_3 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3 \oplus \tilde{Z}_4)^{-1} = [0.0208, 0.1287, 0.5667],$$

$$\tilde{W}_4 = \tilde{Z}_4 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3 \oplus \tilde{Z}_4)^{-1} = [0.2123, 0.6713, 1.7332]$$

Therefore, $W_1 = (\prod_{i=1}^{3} \omega_i)^{1/3}$ =0.1195, W_2 = 0.1080 ,W_3 = 0.1149 ,W_4 = 0.6275 .
The weighing factors for three NDT tests for finding the Elastic modulus are Literature values **(0.1195)**, sonic testing **(0.1080)**, ultrasonic testing **(0.1149)** and Direct compression test **(0.6275)**. So the survey results we arrive to the following weight-age factors given in form of tree (Figure 5.4).

Total Rating (1.0)

Literature Knowledge (0.1195) | **Sonic testing** (0.1080) | **UltraSonic testing** (0.1149) | **Direct compression test** (0.6275)

Figure 5.4: Weighing factors for NDT test used to find Elastic modulus

5.3 Weightage factor for NDT tests used in calculating granite compressive strength

Same steps were repeated as done in Section 5.2.2.

5.3.1 The Weighting factor determination for calculating compressive strength

1. The matrix is presented below which we have got from the survey responses.

Table 5.4: All acceptable responses for survey

Test method	Literature Knowledge	Schmidt Hammer	Direct compressive test
Literature Knowledge	1	(1/2,1,1/3,1/5,7, 5,6,3,1/5,1/2)	(1/8,1/8,1/8,1/9,1/7, 1/7,1/8,1/6,1/3,1/7)
Schmidt Hammer	Positive reciprocal	1	(1/7,1/8,1/5,1/6,1/8, 1/9,1/8,1/8,1/3,1/5)
Direct compressive test	Positive reciprocal	Positive reciprocal	1

From this matrix compute the triangular fuzzy numbers. Details of which are presented in Appendix D

2. Following outline above, we obtain a fuzzy positive reciprocal matrix $\tilde{A}$

$$\tilde{A} = \begin{bmatrix} (1,1,1) & (1/5,1.07702,7) & (1/9,0.146,1/3) \\ (0.14285,0.92848,5) & (1,1,1) & (1/9,0.15614,1/5) \\ (6,6.8504,9) & (5,6.4045,9) & (1,1,1) \end{bmatrix}$$

3. Calculation of relative fuzzy weights of the evaluation factors.

$$\tilde{Z}_1 = [\tilde{a}_{11} \otimes \tilde{a}_{12} \otimes \tilde{a}_{13}]^{1/3} = [0.2811, 0.5397, 1.3264]$$

$$\tilde{Z}_2 = [\tilde{a}_{21} \otimes \tilde{a}_{22} \otimes \tilde{a}_{23}]^{1/3} = [0.2513, 0.5247, 1]$$

$$\tilde{Z}_3 = [\tilde{a}_{31} \otimes \tilde{a}_{32} \otimes \tilde{a}_{33}]^{1/3} = [3.1072, 3.53, 4.3267]$$

$$\sum \tilde{Z}_i = [3.6396, 4.5944, 6.6531]$$

$$\tilde{W}_1 = \tilde{Z}_1 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0423, 0.1175, 0.3644]$$

$$\tilde{W}_2 = \tilde{Z}_2 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0378, 0.1142, 0.2748],$$

$$\tilde{W}_3 = \tilde{Z}_3 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.4670, 0.7683, 1.1888]$$

Therefore, $W_1 = (\prod_{i=1}^{3} \omega_i)^{1/3}$ =0.1147, W_2=0.1059 ,W_3=0.7672 .
The weighing factors for three NDT tests for finding the compressive strength are Literature values **(0.1218)**, Schmidt hammer **(0.1059)** and Direct compression test **(0.7528)**. So the survey results we arrive to the following weight-age factors given in form of tree (Figure 5.5).

Total Rating (1.0)

Literature Knowledge (0.1218) | **Schmidt Hammer** (0.1059) | **Direct compression test** (0.7528)

Figure 5.5: Weighing factors for NDT test used to find compressive strength of granite block

5.4 Proposal for Trust Factors

The trust factors proposed for this thesis depend on weighting factors calculated in Section 5.2.4 and 5.3. These factors will tend to increase the standard deviation of sample in case when the weightage is small and will decrease the deviation for cases in which weightage is higher so it has more contribution. The maximum to minimum values of these factors were set to vary from 1.3 to 0.7. Since this trust factor is implemented for the first time we are proposing more conservative values of T i.e 1.3 to 0.7 allowing maximum 30 % impact on uncertainity. More high of low values will increase the spread so much or vice versa and will result in much increase in uncertainty values. They will vary linearly from 1.3 to 0.7 depending on the weights obtained of different NDT tests. The trust factor proposed can be multiplied by the standard deviation to increase or decreased its spread i.e degree of confidence in that testing technique.

5.5 Calculated trust factors for two NDT tests

5.5.1 Compressive strength of granite blocks including proposed trust factor

The same calculations that are described in Section 4.3 were performed on the model using trust factors which we got from Table 5.5. However the Schmidt hammer trust factor could not be used so the weight-age was divided among Literature knowledge and direct compression tests according to their weights. So, the trust factor for Literature knowledge was calculated as 1.22 and for direct compression test as 0.81 as seen from Tables 5.5 & 5.6. Repeating the similar steps and data fusion Matlab toolbox the results obtained are mentioned in Table 5.7, Figure 5.8 and 5.9. As we are increasing standard deviation by a greater value of 1.3 as compared to 0.7 for direct test data the standard deviation of posterior was found to be more than of prior. The model can be improved if the test results are more with less standard deviation. As seen from Figure 5.8 on increasing the number of tests the posterior graph has less standard deviation which also implies less spread. One more fact was not taken into account which will increase the accuracy of results

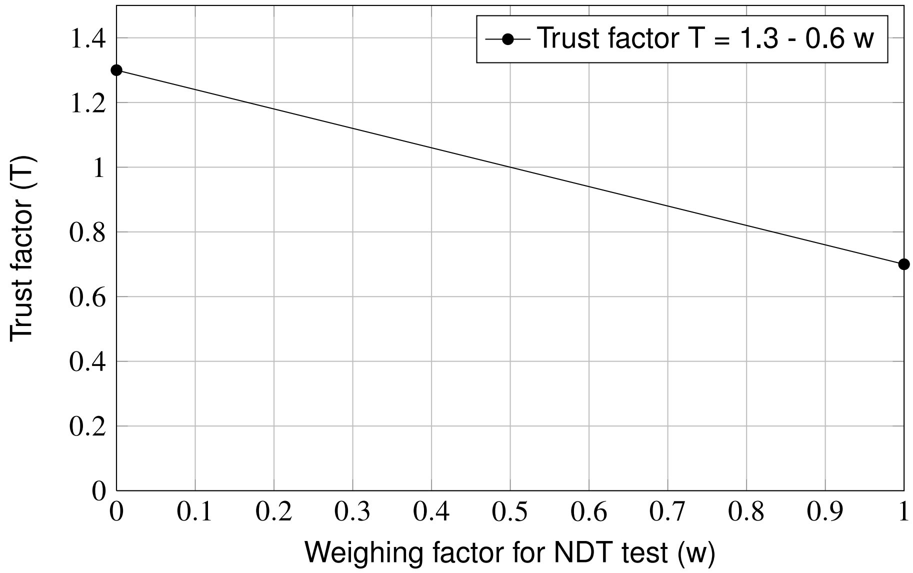


Figure 5.6: Proposed trust factor for NDT tests

by a greater extend as the test samples increase the test becomes more reliable and standard deviation is lower. If this fact is taken into account the model will show much better results and the graphs of proir and posterior can be easily distinguishable.

5.5.2 Modified Algorithm for data fusion

The equation 3.17 was modified to Eq. 5.8 to take into account proposed trust factors in the data fusion algorithm.

$$S_{1modified} = \overbrace{\underbrace{T_1 S_o}_{\text{prior deviation with } T_1} + \underbrace{T_2(n-1)s^2}_{\text{posterior deviation with } T_2} + \underbrace{\frac{n_o n}{n_o + n}(x-\mu_o)^2}_{\text{additional uncertainity}}}^{\text{Total deviation of posterior including trust factors}} \quad and \quad \boxed{S_{omodified} = T_1 S_o} \tag{5.8}$$

5.5.3 Results including trust factor for compressive strength (f_c) of granite block

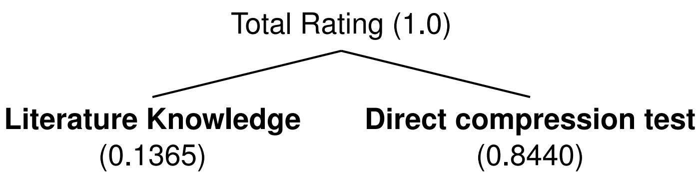


Figure 5.7: Weighing factors for NDT test used to find compressive strength of granite block without schmidt hammer

Table 5.5: Proposed Trust Factor for NDT tests used to find Compressive strength (f_c)

NDT Test	Weightage (w)	Value of Trust Factor (T)
Literature Knowledge	0.1218	1.23
Schmidt Hammer	0.1059	1.24
Direct Compression test	0.7528	0.85

Table 5.6: Proposed Trust Factor for NDT tests used to find Compressive strength (f_c) without Schmidt Hammer test

NDT Test	Weightage (w)	Value of Trust Factor (T)
Literature Knowledge	0.1365	1.22
Direct Compression test	0.8440	0.81

Table 5.7: Prior and Posterior estimates of compressive strength f_c in MPa using Conjugate prior and including trust factors

Parameter	Prior	Posterior (n=6)	Posterior (n=25)	Posterior (n=50)	Posterior (n=100)
μ	–	89.20	89.20	89.20	89.20
$\sigma(\mu)$	–	4.59	2.29	1.63	1.14
σ	–	15.54	16.14	16.22	16.27
$\sigma(\sigma)$	–	3.35	1.64	1.15	0.81
95% confidence for mean	–	81.66–96.75	85.43–92.97	86.52–91.9	87.31–91.09
μ_{pop}	100	89.22	89.21	89.20	89.23
$\sigma_{population}$	20	20.49	18.50	17.94	17.40
95% confidence for population mean	55.52–122.92	56.85–121.67	58.79–119.64	59.70–118.70	60.64-117.88

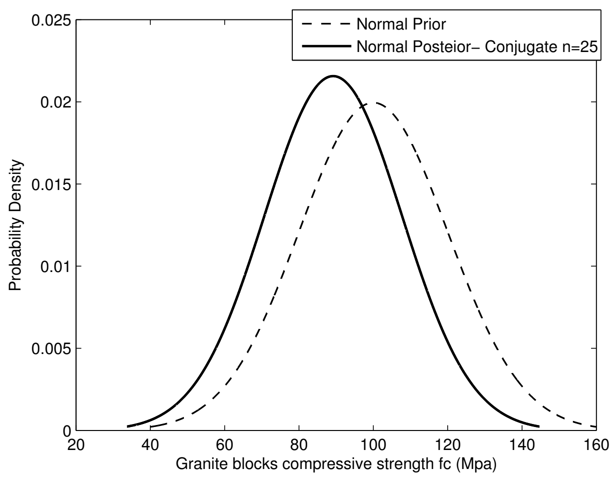


Figure 5.8: Prior and Posterior density functions for Granite compressive strength f_c in MPa using conjugate distribution including trust factors

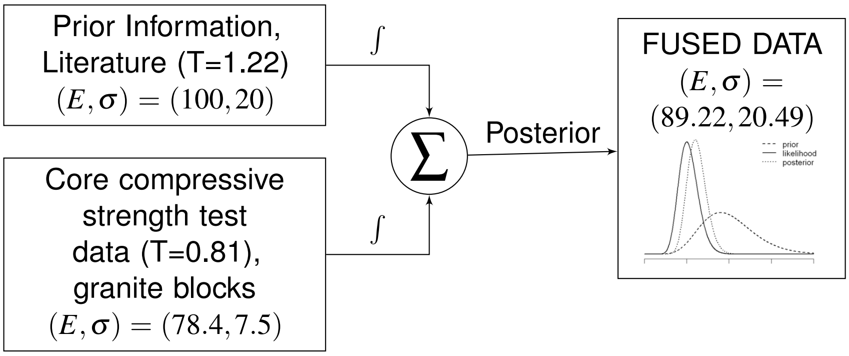


Figure 5.9: Data Fusion system using Bayesian approach for updating Compressive strength f_c in case of data from Literature and core compressive strength data for granite including trust factors for n=6 samples

5.5.4 Elastic modulus of granite blocks including proposed trust factor

Table 5.8: Proposed Trust Factors for NDT tests used to find Elastic Modulus (E)

NDT Test	Weightage (w)	Value of Trust Factor (T)
Literature Knowledge	0.1195	1.23
Sonic Testing	0.1080	1.24
Ultrasonic Testing	0.1149	1.23
Direct Compression test	0.6275	0.92

In this case, since all the results are available so no distribution of weights was done for missing tests and the trust factors calculated in Table 5.6 were used. The flowchart of Figure 5.11 summarizes the results with inclusion of trust factor.
The results shown in flowchart of 5.11 similar trend as the results for compressive strength with increase in standard deviation as the trust factor increase the uncertainty of the sample.

Table 5.9: Prior and Posterior estimates of E (Normal Distribution) in GPa considering sonic and ultrasonic data including trust factors

Parameter	Literature	Prior 2	Prior 3	Posterior 3
μ	–	33.25	32.72	32.30
$\sigma(\mu)$	–	1.42	1.36	1.27
σ	–	6.89	6.58	6.19
$\sigma(\sigma)$	–	1.03	0.98	0.91
95% confidence interval for mean	–	30.91-35.59	30.49-34.95	30.20-34.40
μ_{pop}	35	33.24	32.73	32.32
$\sigma_{population}$	7.5	8.38	8.00	7.52
95% confidence interval for population mean	20.3-49.7	19.44-47.04	19.56-45.89	19.95-44.68

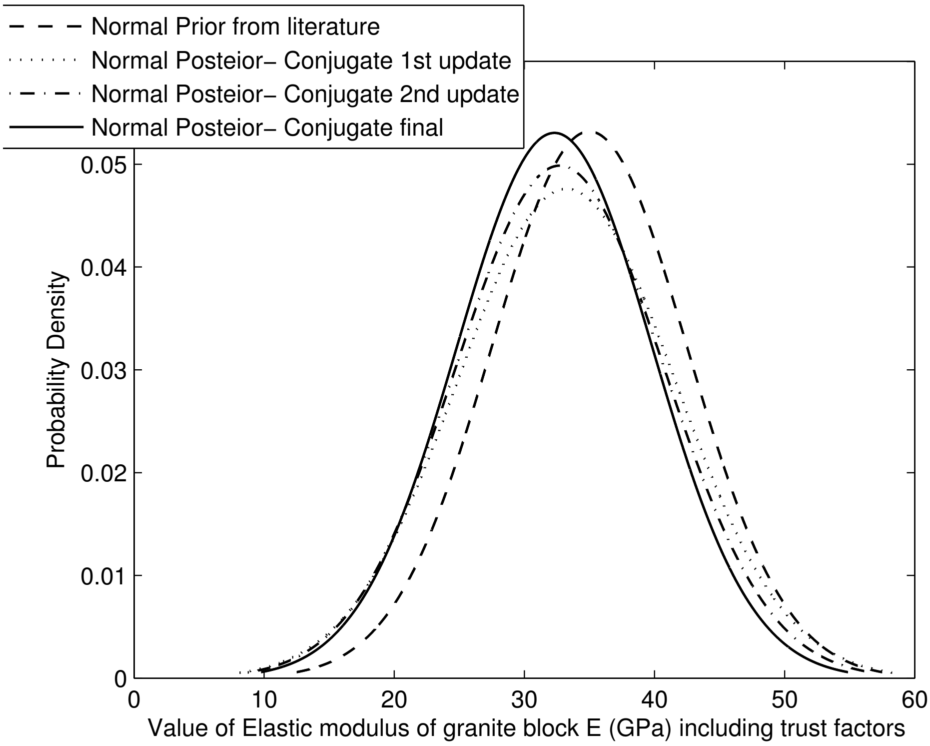


Figure 5.10: Posterior density functions for Granite Elastic Modulus E before and after updating considering data from Literature, Sonic, Ultrasonic and direct compressive strength data including trust factors

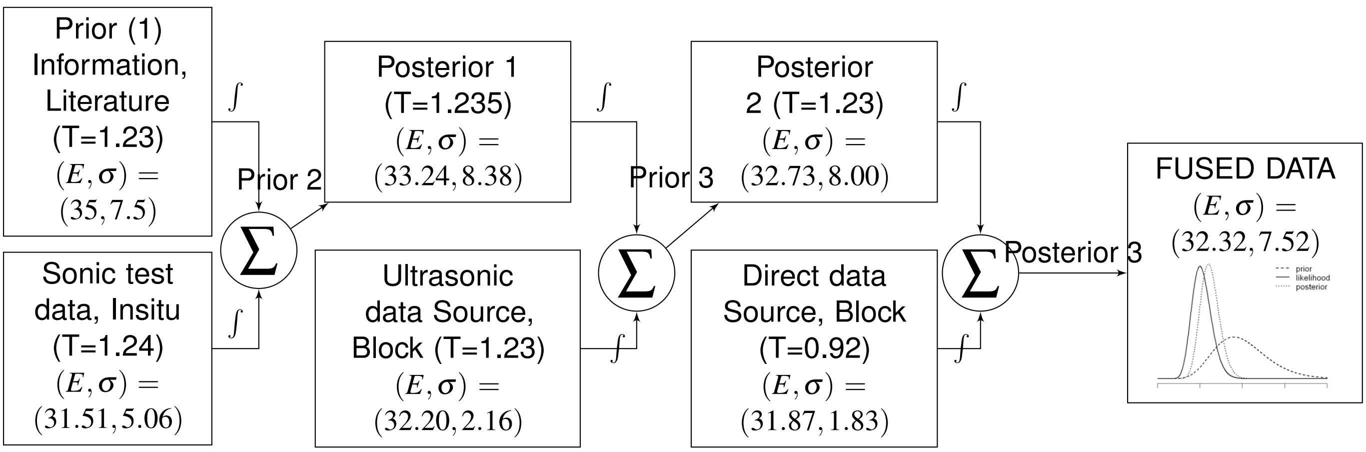


Figure 5.11: Data Fusion system using Bayesian approach for updating Elastic modulus E in case of data from sonic, Ultrasonic and compressive strength data including trust factors

5.6 Conclusion including trust factor

As seen from the figure 5.12 standard deviation got a bit increased since for trust factor the fusion of data is done at may steps. Since, the data fusion in flowchart of figure 5.11 occurred in many time steps and every time the trust factor was included it was multiplied by some factor. This approach will be useful in data fusion applications where it is done in one step with less introduction of uncertainty in data. The main concept behind the proposal of trust factor is to take more contribution from the more reliable data by decreasing its standard deviation by multiplying with a factor less than 1 and vice versa for less reliable data i.e multiplication by a factor greater than 1. So, the trust factor acts as a penalty factor before data is fused to get a more reliable confidence interval. The results should be better calibrated with parametric tests to obtain to a more reasonable conclusion.

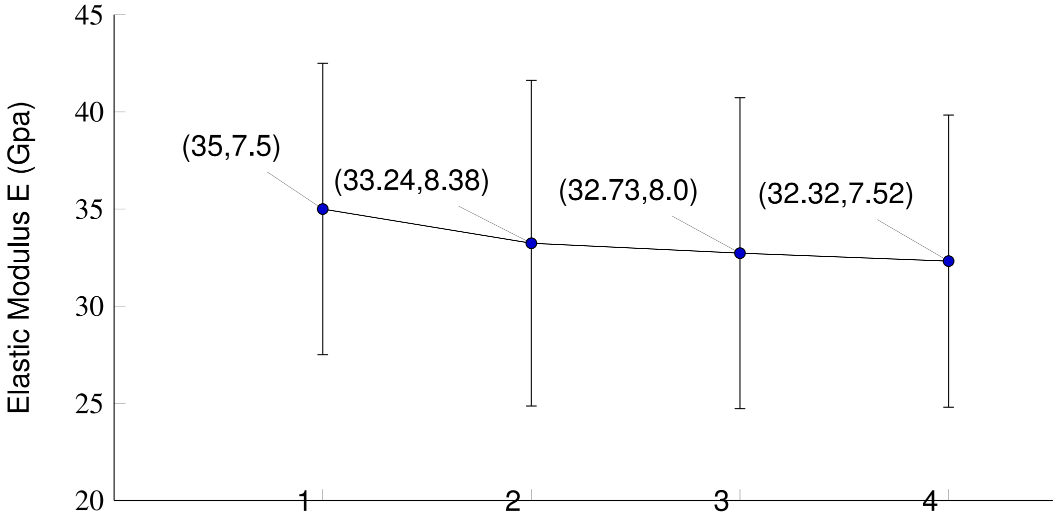


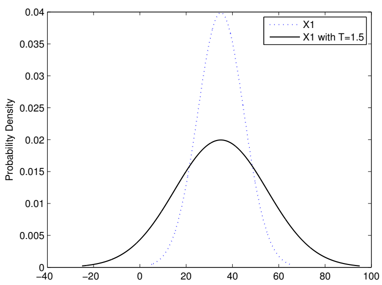
Figure 5.12: Plot showing update for elastic modulus after each data step with trust factor

6 CONCLUSIONS AND FUTURE RESEARCH

6.1 Conclusions

The methodology developed involves data fusion at different levels and from subjective, indirect and direct sources of data which is a new concept. Using Bayesian approach to combine information and dealing with certain levels of uncertainty at different levels to formulate the deformability modulus and compressive strength can be considered as main contribution of these thesis. The developed NDT Data fusion Matlab toolbox has the following advantages:

- Combines information from different NDT tests (direct and indirect) to fuse it into single uniform format;
- Includes trust factor which weights the importance of each test on the basis of user trust in the testing procedure;
- To present the final parameter in a numerical format easy to interpret by the practitioners;
- Draws the distribution of both prior and posterior for comparison of the parameter;
- Comes with a MCR installer which makes nonobligatory to install Matlab for running this program.



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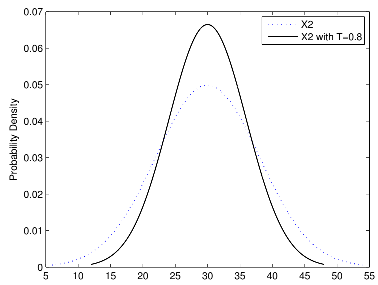

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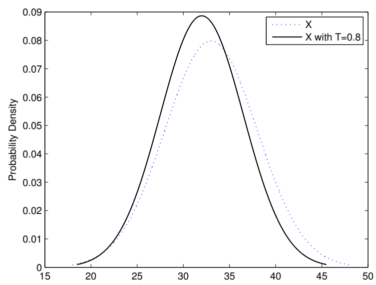


Figure 6.1: Prior 1 (X1) with and without trust factor

Figure 6.2: Prior 2 (X2) with and without trust factor

Figure 6.3: Posterior distribution (X) with and without trust factor

For fusion system to work properly, we would recommend collection of large amount of experimental data prior to fusion, on different types of NDT tests to built a database to assign prior probabilities to each NDT tests. The Figure 6.1 shows prior 1 (X1) dotted line and Figure 6.2 prior 2 (X2) dotted line combine to give posterior distribution (X = X1 + X2) also by dotted line shown in Figure 6.3 without including any trust factors for different NDT tests. After the survey for example the prior 1 was modified to prior 1 with trust factor shown in Figure 6.1 and the prior 2 by applying a factor of 1.3 and prior 2 to prior 2 with trust factor by applying trust of 0.7 as shown in Figure 6.2. By doing this the posterior in Figure 6.3 got a bit changed with mean closer to prior 2 mean but shows a bit more spread since we are increasing the prior deviation by a higher value and mean is closer to mean of prior 2.

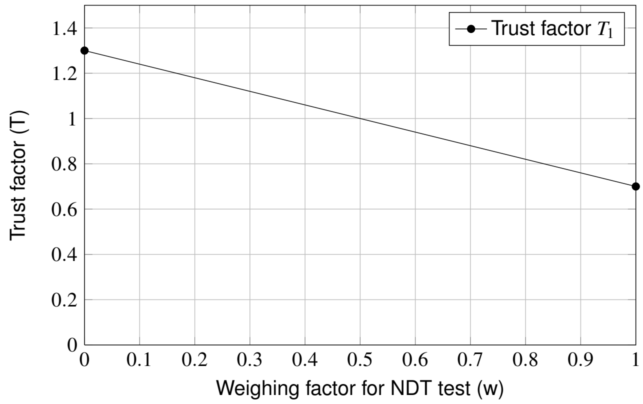
The model works well when the amount of data available is considerable. It was also seen that the results are presented with associated probability intervals and favour the input with maximum degree of confidence. The fusion from NDT test results would not gain much importance in the posterior if the results are poor from the input data i.e with larger standard deviation and less weightage associated with them. In the advanced NDT fusion system proposed many uncertainties could be coupled with information provided from different data sources to make inferences.

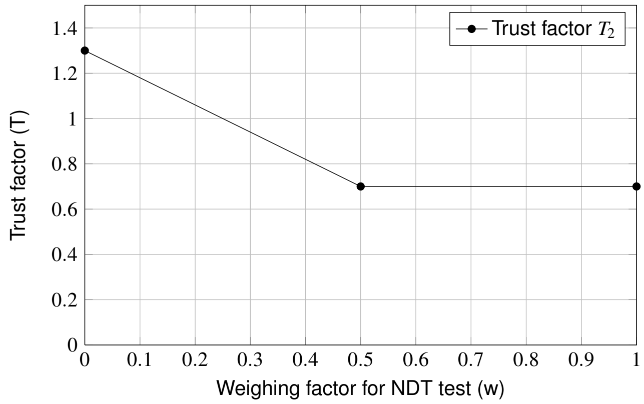
6.2 Future Research

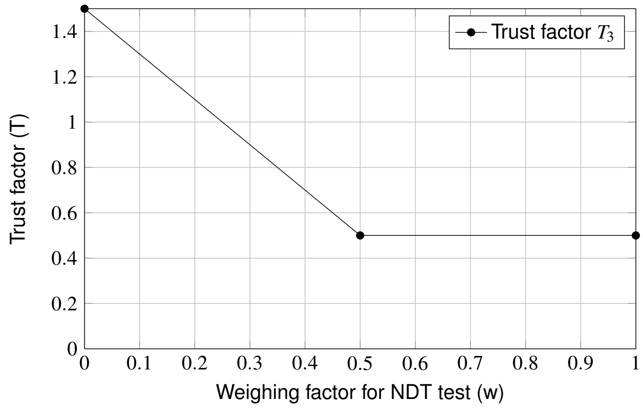
The project aims in proposing the development of a prototype system which can help practitioners to fuse data from different NDT tests and arrive to a more certain value of parameter. The developed Standalone application can be developed for iPhones so it must be free from the constraints of having computer all the time. At present we have made two Matlab toolboxes NDT_FUSION & NDT_FUSION_TRUST that uses Bayesian algorithm to compute reliable intervals levels of parameters. These toolboxes comes with a MCR installer which has no need to have Matlab preinstalled in computer. Similar toolboxes can be developed to compute E using Monte Carlo methodology. The same algorithm can be used to develop an applet for Handheld devices like iPhone and other devices like personal digital assistants (PDA) which can be used by engineers working in field. These devices already have an operating system installed which supports these applications and are easy to carry, portable and can be used even at remote locations.

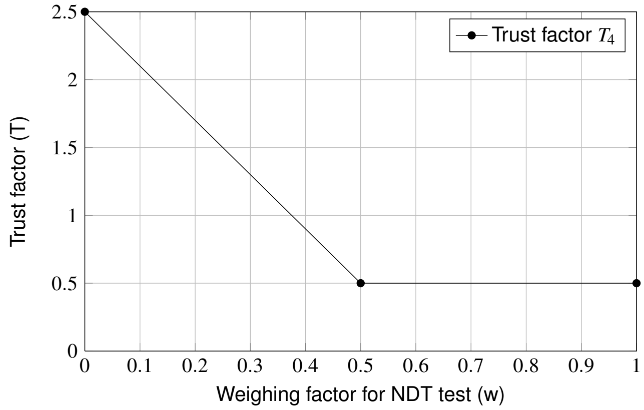
One of the main issues is the bias in the trust factor which are handled before in this report. Many people have different opinions about the reliability of different NDT tests and they assign different importance to different tests. A parametric study needs to done for determining trust factors since the factors which we proposed are on conservative side. The study can also be used to optimise the number of tests from NDT and core when a predetermined amount of money is allotted for a project. Variance can be minimised by using a balance between the two tests keeping in mind the cost factor.

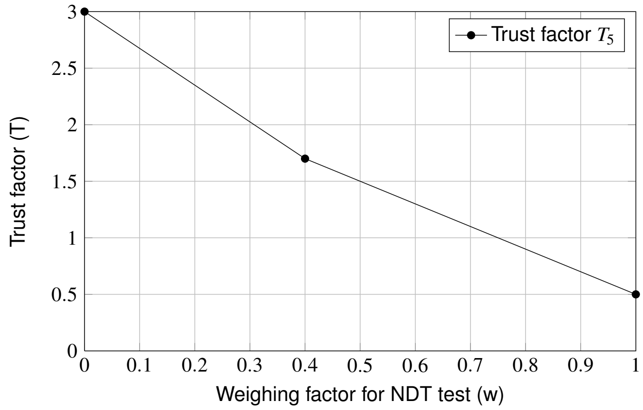
Different combinations shown in the next page can be tried for calculating the trust factor and doing a parametric study on which combination yield more reliable results. Also different case studies can be studies like for example data from flat jack tests of elastic modulus can be fused with literature knowledge. It is a growing field and many colleges have separate departments on Information fusion to carry out the desired task.

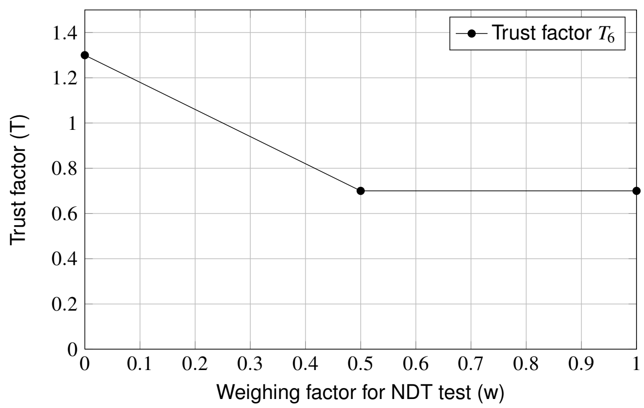

Trust factor T_1
Trust factor (T)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)

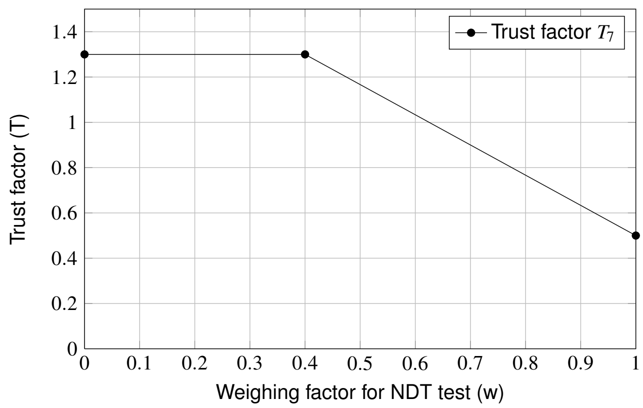

Trust factor T_2
Trust factor (T)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)

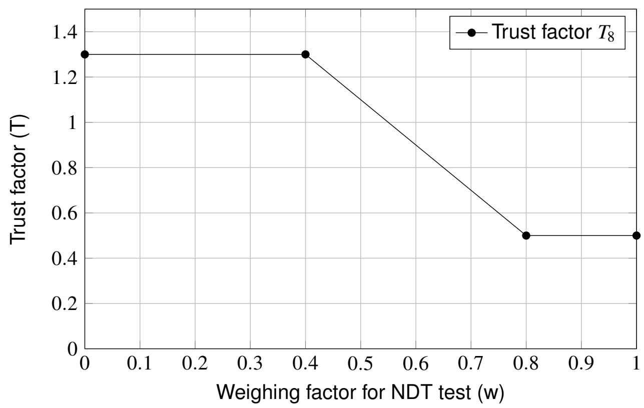

Trust factor T_3
Trust factor (T)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)


Trust factor T_4
Trust factor (T)
0 0.5 1 1.5 2 2.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)


Trust factor T_5
Trust factor (T)
0 0.5 1 1.5 2 2.5 3
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)


Trust factor T_6
Trust factor (T)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)


Trust factor T_7
Trust factor (T)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)


Trust factor T_8
Trust factor (T)
0 0.2 0.4 0.6 0.8 1 1.2 1.4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Weighing factor for NDT test (w)

List of Symbols, Abbreviations, and Notation

Symbol	Meaning	Page
E	Elastic modulus	2
f_c	Compressive strength	2
GUI	Graphical User Interface	3
R	Rebound value	9
d_o	Distance of the defect from specimen	10
v	Speed of ultrasonic wave in the medium	10
t_1	Time measured between the two peaks	10
AE	Acoustic Emission	11
S_f	Calculated stress value	16
K_j	Jack calibration constant (≤ 1)	16
K_a	Jack/slot area constant (≤ 1)	16
P_f	Flat-jack pressure	16
UTM	Universal testing machine	17
COV	Coefficient of variation	24
LVDT	Linear variable displacement transducer	27
E_d	Dynamic Elastic Modulus	32
v_p	Velocity of P-wave	32
ν	Poisons ratio of material	32
ρ	Density of the material	32
LNEC	Laboratório Nacional de Engenharia	35
S_o	Initial sum of the squared differences between the values and their mean	37
Γ	Gamma distribution	37
S_1	Posterior sum of the squares	38
MCMC	Markov Chain Monte Carlo	38
x	Random variable, usually concrete strength, granite in this case	42
σ^2	Variance of random variable	42
μ	mean of expected value of random variable	42
PDF	Probability density function	43
σ_{post}	Posterior deviation	43
μ_{post}	Posterior mean	43
CI	Confidence interval	52
$\sigma(\mu)$	Standrd deviation of the mean	53
T	Trust Factor	55
AHP	Analytic Hierarchy Process	56
TFNs	Triangular fuzzy numbers	57
α_{ij}	Indicates the lower bound	58
β_{ijk}	Relative intensity of importance of expert k between activities i and j	58
γ_{ij}	Indicates the upper bound	58
n	number of people in survey	58
$\tilde{A}$	Fuzzy positive reciprocal matrix	58
$\otimes$	Multiplication of fuzzy numbers	59
$\oplus$	Addition of fuzzy numbers	59
$\tilde{W}_i$	Row vector consists of fuzzy weight of ith factor	59
$S_{1modified}$	Modified posterior sum of squares	63
MCR	Matlab compiler runtime	70
PDA	Personal digital assistants	70

A

Supplementary Material for Simulation Algorithm and MCMC

Simulation Algorithm and MCMC

Simulation Algorithms:-

For generating populations when parameters are variable many simulation algorithms exist. In this report we have used Markov Chain Monte Carlo (MCMC) which consists of series of draws in which sample values of distribution only depend on the last value. In probability theory, a Markov chain for random variable $x_1, x_2, x_3, ..., x_n$ for any time t, x_t depends only on most recent value x_{t-1}. Gibbs sampler is used for Markov chain algorithm in this report.

To explain the Gibbs sampler let us consider a problem with two parameters x_1 and x_2 in which the conditional distributions p(x_1| x_2) and p(x_2| x_1) are known, and it is necessary to compute one or both marginal distribution p(x_1) and p(x_2). The Gibbs sampler starts with an initial value x_2^0 for x_2 and obtains x_1^0 from the conditional distribution p(x_1| x_2= x_2^0). Then the sampler uses x_1^0 to generate a new value x_2^1 drawing from the conditional distribution based on the value x_1^1, p(x_2| x_1= x_1^0). In mathematical terms the samples are taken from the two conditional distributions in the following sequence:

$$x_1^t = p(x_1|x_2 = x_2^{t-1}) \tag{A.1}$$

$$x_2^t = p(x_2|x_1 = x_1^t) \tag{A.2}$$

This sequence of draws is a Markov chain because the values at step t only depend on the value at step t-1. If the sequence is run long enough the distribution of the current draws converges to the simulated distribution.

More specifically, to implement the Gibbs sampler, for instance in the case of the normal model with conjugate priors for unknown mean and variance, it is first necessary to obtain draws from the marginal posterior distribution of the variance and then simulate the mean value from the conditional posterior distribution on the variance and data. The mathematical form of this procedure is the following:

$$\boxed{\frac{1}{\sigma^{2(1)}}|x \rightsquigarrow \Gamma\left(\frac{\nu_1}{2}, \frac{S_1}{2}\right)} \tag{A.3}$$

$$\boxed{\mu^1|\sigma^2, x \rightsquigarrow N\left(\mu_1, \frac{\sigma^{2(1)}}{n_1}\right)} \tag{A.4}$$

....

$$\boxed{\frac{1}{\sigma^{2(t)}}|x \rightsquigarrow \Gamma\left(\frac{\nu_1}{2}, \frac{S_1}{2}\right)} \tag{A.5}$$

$$\boxed{\mu^t|\sigma^2, x \rightsquigarrow N\left(\mu_1, \frac{\sigma^{2(t)}}{n_1}\right)} \tag{A.6}$$

B

MATLAB CODES FOR DIFFERENT BAYESIAN MODELS

MATLAB CODE FOR DATA FUSION

MATLABCODE FOR Fc; WITH CONSTANT MEAN AND VARIANCE

```
%␣Gaussian␣Distribution␣Basian␣updating␣only␣core␣strength␣is␣given
STD_prior␣=␣20;
MEAN_prior␣=␣100;
x␣=␣35:0.1:150;
f␣=␣(1/(STD_prior*sqrt(2*pi)))*exp(-0.5*((x-MEAN_prior)/STD_prior).^2);
hold␣on;
p=plot␣(x,f);
set(p,'Color','red','LineWidth',1.)
legend('prior')
confidence_5_prior=norminv(0.05,100,20)
confidence_95_prior=norminv(0.95,100,20)
n=6;
x_bar=78.4;
STD_o=7.5;
STD_post=sqrt((STD_o^2␣*␣STD_prior^2)/(STD_o^2+n*STD_prior^2));
MEAN_post=(STD_prior^2*x_bar+STD_o^2*MEAN_prior/n)/
(STD_prior^2+(STD_o^2/n));
g␣=␣(1/(STD_post*sqrt(2*pi)))*exp(-0.5*((x-MEAN_post)/STD_post).^2);
plot␣(x,g);
legend('prior','posterior')
title('Mean␣Compressive␣Strength␣of␣granite␣from␣only␣core␣data');
xlabel('Granite␣compressive␣Strength');
ylabel('Probability␣Density');
confidence_5_post=norminv(0.05,MEAN_post,STD_post)
confidence_95_post=norminv(0.95,MEAN_post,STD_post)
```

MATLABCODE FOR Fc; JEFFREYS PRIOR

```
for␣i=1:10000
p1=rand();
chiinv␣=␣chi2inv(p1,9);
c2(i)=12.0083/chiinv;
d2(i)=sqrt(c2(i));
prior(i)␣=␣norminv(p1,32.3,c2(i)/10);
priorsim(i)␣=␣norminv(p1,prior(i),c2(i)^0.5)
end
average_c2=mean(c2)
average_d2=mean(d2)
average_prior=mean(prior)
average_priorsim=mean(priorsim)␣%mean␣of␣posterioR
```

```
std_c2=std(c2)
std_d2=std(d2) %STD of posterior
std_prior=std(prior)
std_priorsim=std(priorsim) %STD of posterior
confidence_5_mean=norminv(0.05,average_prior,std_prior)
confidence_5_population=norminv(0.05,average_priorsim,std_priorsim)
confidence_95_mean=norminv(0.95,average_prior,std_prior)
confidence_95_population=norminv(0.95,average_priorsim,std_priorsim)
STD_post = std_priorsim;
MEAN_post = average_priorsim;
mu = average_priorsim;
sd = std_priorsim;
ix = mu-3*sd:1e-3:mu+4*sd; %covers more than 99% of the curve
iy = pdf('normal', ix, mu, sd);
plot(ix,iy);
hold on;
STD_prior = 15;
MEAN_prior = 35;
x = 10:0.1:60;
f = (1/(STD_prior*sqrt(2*pi))) * exp(-0.5*((x-MEAN_prior)/STD_prior).^2);
hold on;
p=plot (x,f);
xlabel('Granite Elastic Modulus E (GPa)')
ylabel('Probability Density')
set(p,'Color','red','LineWidth',1.)
legend('Posterior-Jeffreys E','prior for E')
confidence_5_prior=norminv(0.05,35,15)
confidence_95_prior=norminv(0.95,35,15)
```

MATLABCODE FOR Fc; CONJUGATE PRIOR AND POSTERIOR DISTRIBUTION

```
for i=1:10000
p1=rand();
p2=rand();
invsigmao2(i) = gaminv(p1,7.5,1/684);
sigmao(i)=(1/invsigmao2(i))^0.5;
prior(i) = norminv(p2,32.3,sigmao(i)/(14^0.5));
priorsim(i) = norminv(p2,prior(i),sigmao(i));
end
average_sigmao=mean(sigmao)
average_prior=mean(prior)
average_priorsim=mean(priorsim) %mean of posterior
```

```
std_sigmao=std(sigmao)
std_prior=std(prior)
std_priorsim=std(priorsim) %STD of posterior
confidence_5_mean=norminv(0.05,average_prior,std_prior)
confidence_5_population=norminv(0.05,average_priorsim,std_priorsim)
confidence_95_mean=norminv(0.95,average_prior,std_prior)
confidence_95_population=norminv(0.95,average_priorsim,std_priorsim)
mu = average_priorsim;
sd = std_priorsim;
ix = mu-3*sd:1e-3:mu+4*sd; %covers more than 99% of the curve
iy = pdf('normal', ix, mu, sd);
plot(ix,iy,'--');
hold on;
mu1 = 35;
sd1 = 15;
ix1 = 0:1e-3:100; %covers more than 99% of the curve
iy1 = pdf('normal', ix1, mu1, sd1);
p=plot(ix1,iy1);
xlabel('Value of E (GPa)')
ylabel('Probability Density')
title('Prior Posterior probability density functions of E
using conjucate prior distributions')
set(p,'Color','red','LineWidth',1.)
legend('Normal Posterior-Conjucate','Prior mean-Normal')
```

MATLABCODE FOR generation values of E using Monte Carlo Similulation

```
for i=1:1000
p1=rand(1);
p2=rand(1);
p3=rand(1);
v_lb= 3244;
v_ub= 4220;
poissons_lb=0.2;
poissons_ub=0.3;
density_lb=2600;
density_ub=2800;
poissons(i)=0.2+(0.1).*p1;
density(i) = density_lb +((density_ub-density_lb).*p2);
v(i)=v_lb+(976*p3);
r(i)=(1+poissons(i)).*(1-(2*poissons(i)))/(1-poissons(i));
E(i)=(v(i).^2).*density(i).*r(i)
```

```
end
avg_v= mean(v)
avg_density= mean(density)
avg_poissons= mean(poissons)
avg_E= mean(E)
std_E=std(E)



% avg_v =  3732.4
% avg_density =  2700.5
% avg_poissons =  0.25008
% avg_E =  3.1372e+10
% std_E =  5.0638e+09
```

C

FORM for Survey for preference of NDT tests

SURVEY FORMS FILLED FOR NDT RATINGS AMONG DIFFERENT TESTS

Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/2	1/3	1/7

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	2	1/2	1/6

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	3	2	1/5

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	7	5	6

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	1/2	1/8

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	2	1/7

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	8	7

PART A: Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	2	1/2	1/7

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/2	1/3	1/8

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	2	3	1/5

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	7	5	8

PART B: Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	1	1/8

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	1	1/8

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	8	8

remark1: ultrasonic testing gives a dynamic modulus, which is not always comparable to the values obtained from compression tests, so it is rather difficult to make the above comparison.

remark2: this survey questions the 'importance' and I assumed that importance means 'how well does the test give you the actual, standardized, correct value'. If I also have to take into account the impact of the method, the importance of the NDT's of course becomes larger.---

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/5	1/5	1/8

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	5	1	1/5

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	5	1	1/5

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	8	5	1

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	1/3	1/8

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	3	1/5

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	8	5

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/2	1/5	1/9

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	2	1/2	1/8

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	5	2	1/5

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	9	5	8

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	1/5	1/9

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	5	1/6

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	9	6

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	3	5	1/8

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/3	3	1/8

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	1/5	1/3	1/8

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	8	9	9

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	7	1/7

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	1/7	1/8

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	7	9

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	7	9	1/3

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/7	5	1/9

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	1/9	1/5	1/9

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	3	9	7

PART B:

Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	5	1/7

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	1/5	1/9

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	7	9

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	6	5	1/9

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/6	5/6	1/54

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	1/5	6/5	1/45

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	9	45	54

Comment [h1]: Sei que estes valores estão for a da escala, mas mantive coerência entre as diferenças relativas na 1ª questão

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	9	1/9

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	1/9	1/81

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	9	81

Comment [h2]: O mesmo que na part A

PART A: Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	3	4	1/8

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/3	1/4	1/5

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	1/4	1/3	1/6

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	8	8	9

PART B: Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	6	1/8

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	1/6	1/8

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	8	9

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	1/3	1/3	1/5

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	3	1	1/9

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	3	1	1/9

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	5	9	9

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	3	1/6

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	1/3	1/8

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	6	8

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	1	1	1

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	5	3	1

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	3	5	1

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	3	3	3

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	1	1

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	5	1

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	3	3

PART A:
Survey for preference of ND or MD test in finding Elastic Modulus of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of elastic modulus of stones?

Literature	Sonic testing	Ultrasonic testing	Direct compression tests (samples in Lab)
	1	1/2	1/8

2. How is knowledge from Sonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Sonic testing	Literature values	Ultrasonic testing	Direct compression tests (samples in Lab)
	1	1/2	1/7

3. How is knowledge from Ultrasonic testing more important in relation to other tests for evaluating value of elastic modulus of stones?

Ultrasonic testing	Literature values	Sonic testing	Direct compression tests (samples in Lab)
	2	2	1/6

4. How is knowledge from **Direct compression tests** more important in relation to other tests for evaluating value of elastic modulus of stones?

Direct compression tests (samples in Lab)	Literature values	Ultrasonic testing	Sonic testing
	8	6	7

PART B:
Survey for preference of NDT test in finding Compressive Strength of stones

1. How is knowledge from Literature values more important in relation to other tests for evaluating value of compressive strength of stone?

Literature values	Schmidt Hammer	Direct compression tests (samples in lab)
	1/2	1/7

2. How is knowledge from Schmidt Hammer more important in relation to other tests for evaluating value of compressive strength of stone?

Schmidt Hammer	Literature values	Direct compression tests (samples in lab)
	2	1/5

3. How is knowledge from Direct compression test more important in relation to other tests for evaluating value of compressive strength of stone?

Direct compression test (samples in lab)	Literature values	Schmidt Hammer
	7	5

D

Calculation to calculate weightage of each NDT test

CALCULATIONS EXPLAINED TO FIND WEIGHTS FOR DIFFERENT NDT TESTS

Sample Calculations:

$\tilde{a}_{ij} = (\alpha_{ij}, \delta_{ij}, \gamma_{ij}), \alpha_{ij} = Min(\beta_{ijk}), \delta_{ij} = \left(\prod_{k=1}^{n} \beta_{ijk}\right)^{1/n}, \gamma_{ij} = Max(\beta_{ijk})$

In all the responses n=8, and subscript ij represents that i and j vary from 1 to 3. For example B_{121}=1/2, B_{124}=1/5, B_{134}=1/9, B_{235}=1/8, B_{211}=2, B_{214}=5, B_{314}=9, B_{325}=7, ... etc.

Similarly the triangular fuzzy numbers given by Equations 5.2, 5.3 and 5.4.

For example $\alpha_{12} = Min(B_{12k}) = 1/5, \alpha_{13} = Min(B_{13k}) = 1/9, \alpha_{23} = Min(B_{23k}) = 1/9$

$\delta_{12} = \left(\prod_{k=1}^{8} \beta_{12k}\right) = 1.46311, \delta_{23} = \left(\prod_{k=1}^{8} \beta_{23k}\right) = 0.13769, \delta_{13} = \left(\prod_{k=1}^{8} \beta_{13k}\right) = 0.1320$

$\gamma_{12} = Max(\beta_{12k}) = 7, \gamma_{13} = Max(\beta_{13k}) = 1/6, \gamma_{23} = Max(\beta_{23k}) = 1/5$

According to the positive reciprocal rule $\alpha_{ij} \leq \delta_{ij} \leq \gamma_{ij}$, $\alpha_{ij}, \delta_{ij}, \gamma_{ij} \in [1/9, 1] \bigcup [1, 9]$

$\alpha_{31} = (1/\gamma_{13}, 1/\delta_{13}, 1/\alpha_{13}) = (6, 1/0.1320, 9) = (6, 7.57575, 9)$

Similarly $\alpha_{32} = (1/\gamma_{32}, 1/\delta_{32}, 1/\alpha_{32}) = (5, 7.26269, 9)$

E

MANUAL TO USE MATLAB TOOLBOX FOR DATA FUSION

MATLAB DATA FUSION TOOLBOX
NDT_FUSION _TRUST

The toolbox based on principle of bayes estimator (Figure E.2) consists on the left side with number of samples for two tests n_1 and n_2, mean μ_1 and μ_2 of these data samples along with the standard deviation σ_1 and σ_2 of these two tests. The trust on these two tests can be filled in the boxes of T_1 and T_2.
Output also comes in graphical form so one can compare the prior and posterior in two plots with different lines. In case of no knowledge of trust factor, both values can be put equal to 1. The results from the toolbox are summarised in Table E.1.

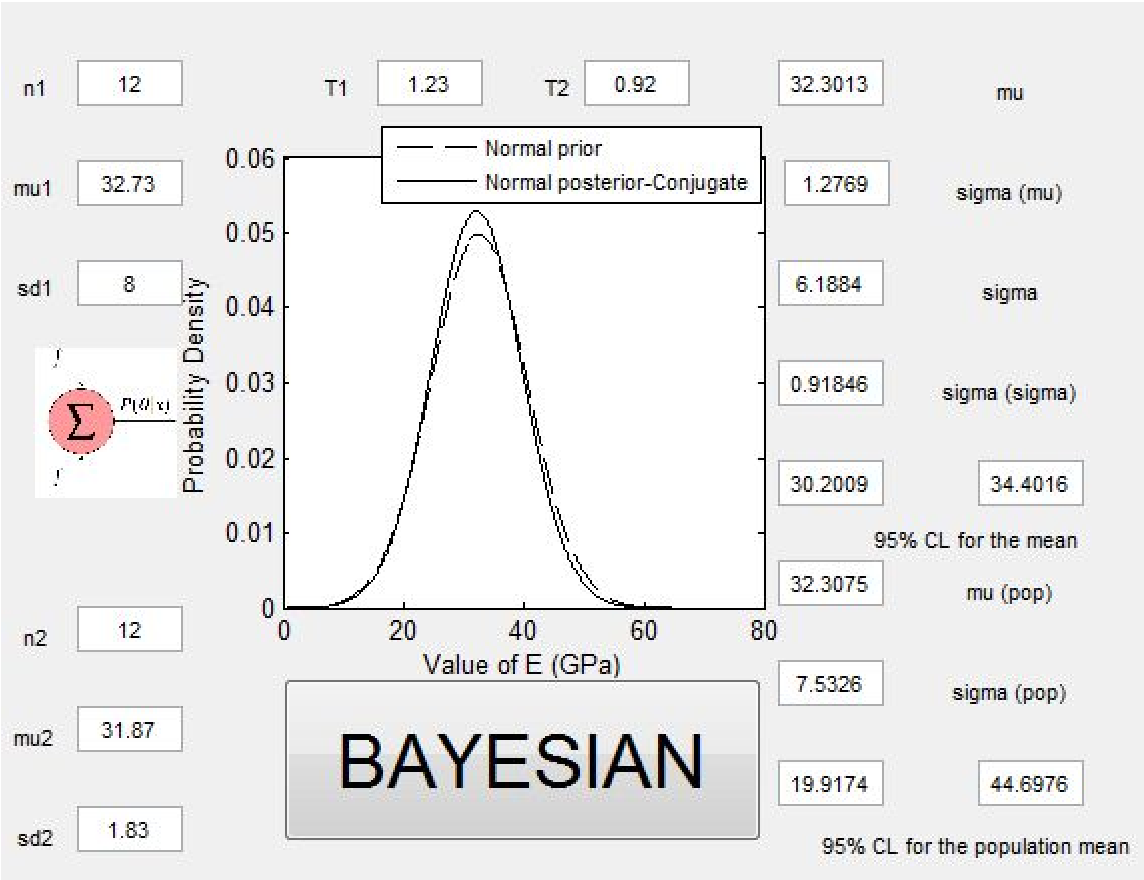


Figure E.1: Screenshot showing data fusion results of Figure 5.11 for last step including trust factors

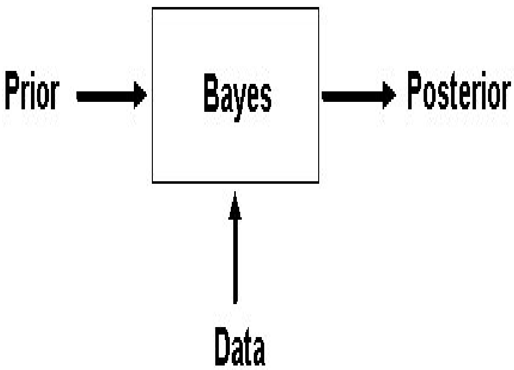


Figure E.2: Bayes estimator

Parameter	Posterior
μ	32.30
$\sigma(\mu)$	1.28
σ	6.12
$\sigma(\sigma)$	0.92
95% confidence interval for mean	30.20-34.40
μ_{pop}	32.31
$\sigma_{population}$	7.53
95% confidence interval for population mean	19.92-44.70

Table E.1: Posterior estimates of E (normal distribution) in GPa

Bibliography

[1] L. F. Ramos, R. Aguilar, P. B. Lourenco, and S. Moreira, "Dynamic structural health monitoring of saint torcato church," *Mechanical systems and Signal processing, Elsevier*, vol. 35, pp. 1–15, February 2013.

[2] A. Agzamova, C. Grande, and M. Mishra, "Seismic verification and inspection of donghi building; a case study," Master's thesis, University of Padova, March 2013.

[3] L. F. B. Miranda, "Ensaios acústicos e de macacos planos em alvenarias resistentes," Ph.D. dissertation, Universidade do Porto, Junho 2011.

[4] J. Alldred, Ed., *Improvements to the orthogonal method for determining reinforcing bar diameter using a cover meter*, vol. 2, no. 11-5, Proceedings of Six international conference on international faults and repair. Engineering Technics Press, 1995.

[5] *Recommendations for Surface Hardness Testing by Rebound Hammer*, pt. 202 ed., BS 1881, 1986.

[6] Wikipedia. (2013, May) Notes on schmidt hammer. [Online]. Available: http://en.wikipedia.org/wiki/Schmidt

[7] M. Force and R. Mackie, "Non-destructive evaluation of a bonded externally reinforced concrete bridge using the frequency response function method," *ICCI, University of Arizona*, vol. 8, no. 1031, January 1996.

[8] N. composites, "Interactive knowledge base on nde of composites, introduction coin and tap testing."

[9] S. Salamone, "Non-destructive testing special topic," *Class notes*, October 2010.

[10] L. Binda, "Insitu general purpose ndt and mdt-investigation for the diagnosis of historic buildings: Application at different scales," *Department of Structural Engineering, Politechnico of Milan, Italy*, 2003.

[11] B. Raj and B. Jha, "Fundamentals of acoustic emission," *British Journal of Non Destructive Testing, Springer Verlag*, 1993.

[12] "Introduction to acoustic emission testing," June 2013. [Online]. Available: www.ndt-ed.org

[13] Proceq. (2013, June) Concrete resistivity method. [Online]. Available: http://www.canin-concrete-corrosion.com/analyzing-methods.html

[14] C. Stanley and R. Balendran, "Developments in assessing the structural integrity of applied surfaces to concrete buildings and structures using infra-red thermography," vol. 3, pp. 39–44, July 1995.

[15] I. Lombillo, L. Villegas, and J. Elices, "Minor destructive techniques applied to the mechanical characterization of historical rubble stone masonry structures," *Structural Survey*, vol. 28, no. 1, pp. 53–70, 2010.

[16] A. Bianco, "Endoscopic analysis supporting issues of historic stratigraphic investigations: the case history of saint domenico monastery in naples-italy." 18th world conference on Nondestructive testing, 2012.

[17] Wikipedia. (2013, June) Concrete compressive testing. [Online]. Available: http://en.wikipedia.org/wiki/Compressive_strength

[18] "Rst instruments innovation in geotechnical instruments," June 2013. [Online]. Available: http://www.rstinstruments.com/

[19] D. McCann and M. Forde, "Review of ndt methods in the assessment of concrete and masonry structures," *NDT and E International*, vol. 34, pp. 71–84, 2001.

[20] "Assessment of san torcato church: Non destructive survey and proposed measures, improved and innovative techniques for the diagnosis and monitoring of historical masonry." University of Minho, Tech. Rep. 6, 2013.

[21] A. H. Ang. and W. Tang, "Probabilistic concepts in engineering planning and design," *John Wiley and Sons Ltd., Chickhester, England*, vol. I, 1975.

[22] G. Vasconcelos, P. B. Lourenco, C. Alves, and J. Pamplona, "Prediction of the mechanical properties of granites by ultrasonic pulse velocity and schmidt hammer hardness," June 2007.

[23] Z. Chair and P. Varshney, "Optimal data fusion in multiple sensors detection system," *Institute of Electrical and Electronics Engineers Transactions on Aerospace and Electronic Systems*, vol. 22, no. 1, pp. 98–101, Jan 1986.

[24] X. E. Gros, *NDT Data fusion*. John Wiley and Sons, NY, 1997, no. ISBN 0 470 23724 4.

[25] M. Sykora, "Assessment of bridges using bayesian updating," *Third international conference on Reliability, safety and diagnosis of transport structures and means, University of Pardubice, Czech Republic*, 2008.

[26] R. Caspeele and L. Taerwe, "Bayesian assessment of the characteristic concrete compressive strength using combined vague-informative piers," *Elsevier*, vol. 28, pp. 342–350, 2012.

[27] T. Miranda, A. G. Correia, and L. R. Sousa, "Bayesian methodology for updating geomechanical parameters and uncertainty quantification," *International Journal of Rock Mechanics and Mining sciences*, no. 1144-1153, 2009.

[28] W. Tang, “A bayesian evaluation of information for foundation engineering design,” *First International Conference for applications of Statistics and Probability to Soil and Structural Engineering*, pp. 174–185, 1971.

[29] *Bases for design of structures- Assesment of existing structures*, ISO 13822, 2003.

[30] E. Haas C, “Updating the decision aids for tunnelling,” *SCE J Constr Eng Management*, no. 128, 2002.

[31] G. J. Kryviak and A. Scanton, “Bayesian analysis of in-situ data for estimating the compressive strength of existing structures,” Master’s thesis, University of Alberta, Canada, July 1986.

[32] M. O Ditlevson, *Structural reliability methods*. New York: Wiley, 1996.

[33] L. Report, “Lnec collaboration in the geotechnics and geotechnical studies concerning the hydraulic circuit of venda nova ii,” Lisboa, Portugal, Tech. Rep. 47/1/7084, 1983.

[34] M. Faber, “Risk and safety in civil, surveying and environmental engineering,” 2005.

[35] A. Gelman, J. Carlin, H. Stern, and D. Rubin, *Bayesian data analysis*. London: Chapman and Hall, 2004.

[36] I.-I. scientific committee for analysis, restroration of structures, and architechtural heritage, “Recommendations for the analysis, conservation and structural restoration of architectural heritage,” November 2005.

[37] M. Mirza S.A., Hatzinikolas and J. MacGregor, “Statistical descriptions of strength of concrete,” *ASCE Journal of the Structural Division*, vol. 105, no. ST6, pp. 1021–1037, 1979.

[38] S. Jade and T. G. Sitharam, “Characterization of strength and deformation of jointed rock mass based on statistical analysis,” vol. Inernational journal of Geomechanics, ASCE, 2003.

[39] MATLAB, *version 7.10.0 (R2010a)*. Natick, Massachusetts: The MathWorks Inc., 2010.

[40] Y. C. Liu and C. S. Chen, “A new approach for application of rock mass classification on rock slope stability assessment,” *Engineering Geology, Elsevier*, vol. 89, pp. 129–143, November 2006.