

COMPUTATIONAL STRATEGIES FOR MASONRY STRUCTURES: MULTI-SCALE MODELLING, DYNAMICS, ENGINEERING APPLICATIONS AND OTHER CHALLENGES

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Abstract *Masonry is a composite material that can be defined as a material incorporating a visible internal structure and having a low strength in tension. The latter characteristic has shaped most civil engineering structures up to the advent of reinforced concrete and iron/steel. Masonry is also present in most of our cultural heritage buildings, which are part of our identity and represent a key attractor for tourism, a major economical asset of Europe with 10% of the GDP. The paper will address different challenges: micro-modelling and homogenization techniques, which represent both a popular and active field on masonry research; dynamics and earthquake engineering, which remain far from being understood and challenge our modelling capacities. The presentation also addresses dynamic identification and inverse problems, or the art of a modelling engineer, as well as the engineering use of sophisticated numerical models, which provide significance to most of the problems addressed before.*

1. INTRODUCTION

In many countries, masonry structures still amount to 30 to 50% of the new housing developments. Also, most structures built before the 19th century and still surviving are built with masonry. Research in the field is essential to understand masonry behaviour, to develop new products, to define reliable approaches to assess the safety level and to design potential retrofitting measures. To achieve these purposes, researchers have been trying to convert the highly indeterminate and non-linear behaviour of masonry buildings into something that can be understood with an acceptable degree of mathematical certainty.

Masonry is usually described as a composite material formed by units and joint, with or without mortar, and different bond arrangements. It is certain that the problems associated

with modelling ancient and modern masonry structures are very different. Physical evidence shows us that ancient masonry is a very complex material with three-dimensional internal arrangement, usually unreinforced, but which can include some form of traditional reinforcement, whereas modern masonry is made usually with a regular arrangement of masonry units, with or without steel reinforcement, see Figure 1. The fact that masonry has so much variability in materials and technology makes computational modelling of masonry structures rather complex.



Figure 1. Examples of different masonry types (from left to right): timber braced “Pombalino” system after the 1755 earthquake in Lisbon; irregular stone wall with a complex transverse cross section; unreinforced masonry using blocks complying with thermal and sound regulations.

2. SELECTED MODELLING APPROACHES

In general, the approach towards the numerical representation of masonry can focus on the micro-modelling of the individual components, units and mortar, or the macro-modelling of masonry as a composite [1]. Depending on the level of accuracy and the simplicity desired, it is possible to use the following modelling strategies, see Figure 2: (a) Detailed micro-modelling, in which units and mortar in the joints are represented by continuum elements whereas the unit-mortar interface is represented by discontinuous elements; (b) Simplified micro-modelling, in which expanded units are represented by continuum elements whereas the behaviour of the mortar joints and unit-mortar interface is lumped in discontinuous elements; (c) Macro-modelling, in which units, mortar and unit-mortar interface are smeared out in a homogeneous continuum. Much effort is made today in the link between the micro- and macro-modelling approaches using homogenization techniques.

2.1. Micro-modelling

Different approaches are possible to represent heterogeneous media, namely, the discrete element method, the discontinuous Finite Element Method (FEM) and limit analysis.

The typical characteristics of discrete element methods are: (a) consideration of rigid or FEM deformable blocks; (b) connection between vertices and sides / faces; (c) interpenetration is usually possible; (d) explicit integration of the equations of motion for the blocks using the

real damping coefficient (dynamics) or artificially large (statics). The main advantages are the formulation for large displacements, including contact update, and an independent mesh for each block, in case of deformable blocks. The main disadvantages are the large number of contact points required for accurate representation of interface stresses and a time consuming analysis, especially for 3D problems. Masonry applications can be found in [2].

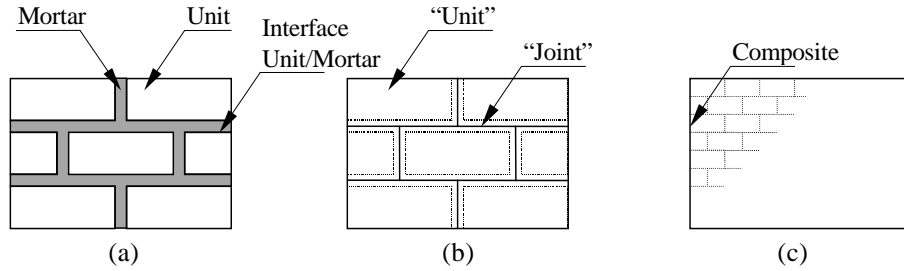


Figure 2. Modelling strategies for masonry structures: (a) detailed macro-modelling; (b) simplified micro-modelling; (c) macro-modelling.

The finite element method remains the most used tool for numerical analysis in solid mechanics and an extension from standard continuum finite elements to represent discrete joints was developed in the early days of non-linear mechanics. Interface elements were initially employed in concrete and rock mechanics, being used since then in a great variety of structural problems. A complete micro-model must include all the failure mechanisms of masonry, namely, cracking of joints, sliding over one head or bed joint, cracking of the units and crushing of masonry, as done in [3] for monotonic loading [4] for cyclic loading.

Computational limit analysis received far less attention from the technical and scientific community for blocky structures. Still, limit analysis has the advantage of being a simple tool, while having the disadvantages that only collapse load and collapse mechanism can be obtained and loading history can hardly be included. A limit analysis constitutive model for masonry that incorporates non-associated flow at the joints, tensile, shear and compressive failure and a novel formulation for torsion is given [5,6].

Next, as an example of the possibilities that can be achieved with micro-modelling, a powerful interface model is detailed and applied to illustrative examples.

2.2. A cyclic interface model

A relation between generalized stress and strain vectors is usually expressed as

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\varepsilon} \quad (1)$$

where \mathbf{D} represents the stiffness matrix. For zero-thickness line interface elements, the constitutive relation defined by Eq. (1) expresses a direct relation between the traction vector and the relative displacement vector along the interface, which reads:

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma \\ \tau \end{Bmatrix} \quad \text{and} \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \Delta u_n \\ \Delta u_t \end{Bmatrix} \quad (2)$$

A constitutive interface model can be defined by a convex composite yield criterion, see Figure 3, composed by three individual yield functions, usually with softening included for all modes so that experimental observations can be replicated, reading

$$\begin{aligned}
 \text{Tensile criterion: } f_t(\boldsymbol{\sigma}, \kappa_t) &= \sigma - \bar{\sigma}_t(\kappa_t) \\
 \text{Shear criterion: } f_s(\boldsymbol{\sigma}, \kappa_s) &= |\tau| + \sigma \tan \phi - \bar{\sigma}_s(\kappa_s) \\
 \text{Compressive criterion: } f_c(\boldsymbol{\sigma}, \kappa_c) &= (\boldsymbol{\sigma}^T \mathbf{P} \boldsymbol{\sigma})^{1/2} - \bar{\sigma}_c(\kappa_c)
 \end{aligned} \tag{3}$$

Here, ϕ represents the friction angle and \mathbf{P} is a projection diagonal matrix, based on material parameters. $\bar{\sigma}_t$, $\bar{\sigma}_s$ and $\bar{\sigma}_c$ are the isotropic effective stresses of each of the adopted yield functions, ruled by the scalar internal variables κ_t , κ_s and κ_c .

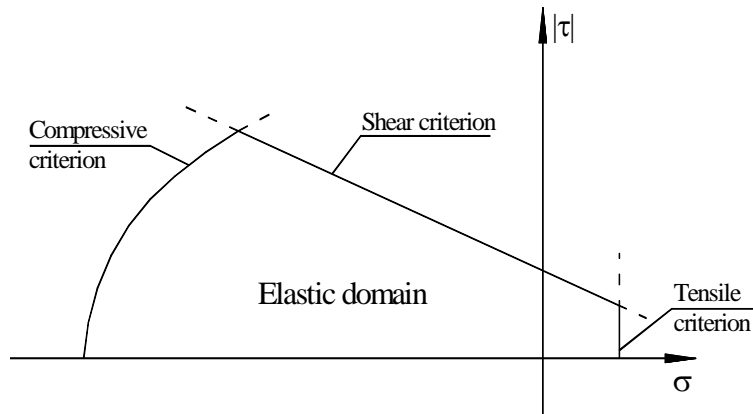


Figure 3. Multisurface interface constitutive model.

In order to include unloading/reloading behaviour in an accurate manner, an extension of the plasticity theory is addressed in [4]. Two new auxiliary yield surfaces (termed unloading surfaces) similar to the monotonic ones were introduced in the monotonic model, so that unloading to tension and to compression could be modelled. Each unloading surface moves inside the admissible stress space towards the similar monotonic yield surface. In a given unloading process, when the stress point reaches the monotonic yield surface, the surface used for unloading becomes inactive and the loading process becomes controlled by the monotonic yield surface. Similarly, if a stress reversal occurs during an unloading process, a new unloading surface is started, subsequently deactivated when it reaches the monotonic envelope or when a new stress reversal occurs. The proposed model comprises six possibilities for unloading/reloading movements.

Unloading/reloading to tension can be started from any allowable stress point, except from points on the monotonic tensile surface, see Figure 4a, ruled according to the yield function

$$f_{Ut}(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \kappa_{Ut}) = \xi^{(1)} - \bar{\sigma}_{i,Ut}(\gamma \kappa_{Ut}) \tag{4}$$

where α is the back-stress vector, $\bar{\sigma}_{i,Ut}$ is the isotropic effective stress and κ_{Ut} is the tensile unloading hardening parameter. The scalar γ provides the proportion of isotropic and kinematic hardening. The relative (or reduced) stress vector ξ is given by

$$\xi = \sigma - \alpha \quad (5)$$

In the same way, unloading/reloading to compression can take place from any acceptable stress point, except from points on the monotonic compressive surface, see Figure 4b, being controlled by the following yield function

$$f_{Uc}(\sigma, \alpha, \kappa_{Uc}) = (\xi^T \mathbf{P} \xi)^{1/2} - \bar{\sigma}_{i,Uc}(\gamma \kappa_{Uc}) \quad (6)$$

where $\bar{\sigma}_{i,Uc}$ is the isotropic effective stress and κ_{Uc} is the compressive unloading hardening parameter.

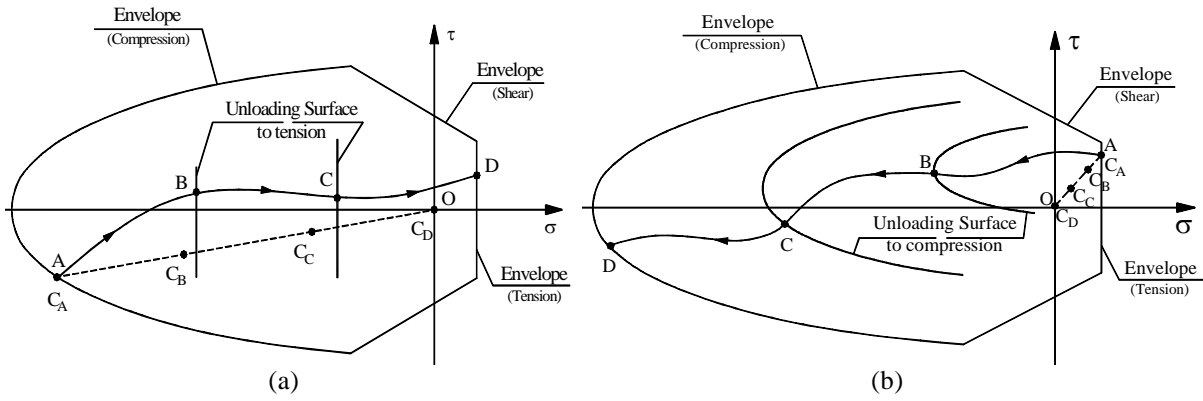


Figure 4. Hypothetic motion of the unloading surface to: (a) tension and to (b) compression.

For each of the six hypotheses considered for unloading movements, a curve that relates the unloading hardening parameter κ_U and the unloading effective stress $\bar{\sigma}_U$ must be defined. Thus, the adoption of appropriate evolution rules makes possible to reproduce non-linear behaviour during unloading. The curves adopted are used in the definition of the isotropic and kinematic hardening laws. Aspects related to the algorithm can be found in [4]. The ability of the model to reproduce the main features of structural masonry elements is shown in Figure 5. It was found that the geometric asymmetry in the micro-structure (arrangement of the units) influenced significantly the structural behaviour of the wall. Note that, depending in the loading direction, the masonry course starts either with a full unit or only with half unit. Figure 6a shows that the monotonic collapse load is 112.0 kN in the LR direction and 90.8 kN in the RL direction, where L indicates left and R indicates right. The cyclic collapse load is 78.7 kN, which represents a loss of ~13% with respect to the minimum monotonic value but a loss of ~30% with respect to the maximum monotonic value. This demonstrates the importance of cyclic loading but also the importance of taking into account the micro-

structure. It is also clear from these analyses that masonry shear walls with diagonal zigzag cracks possess an appropriate seismic behaviour with respect to energy dissipation.

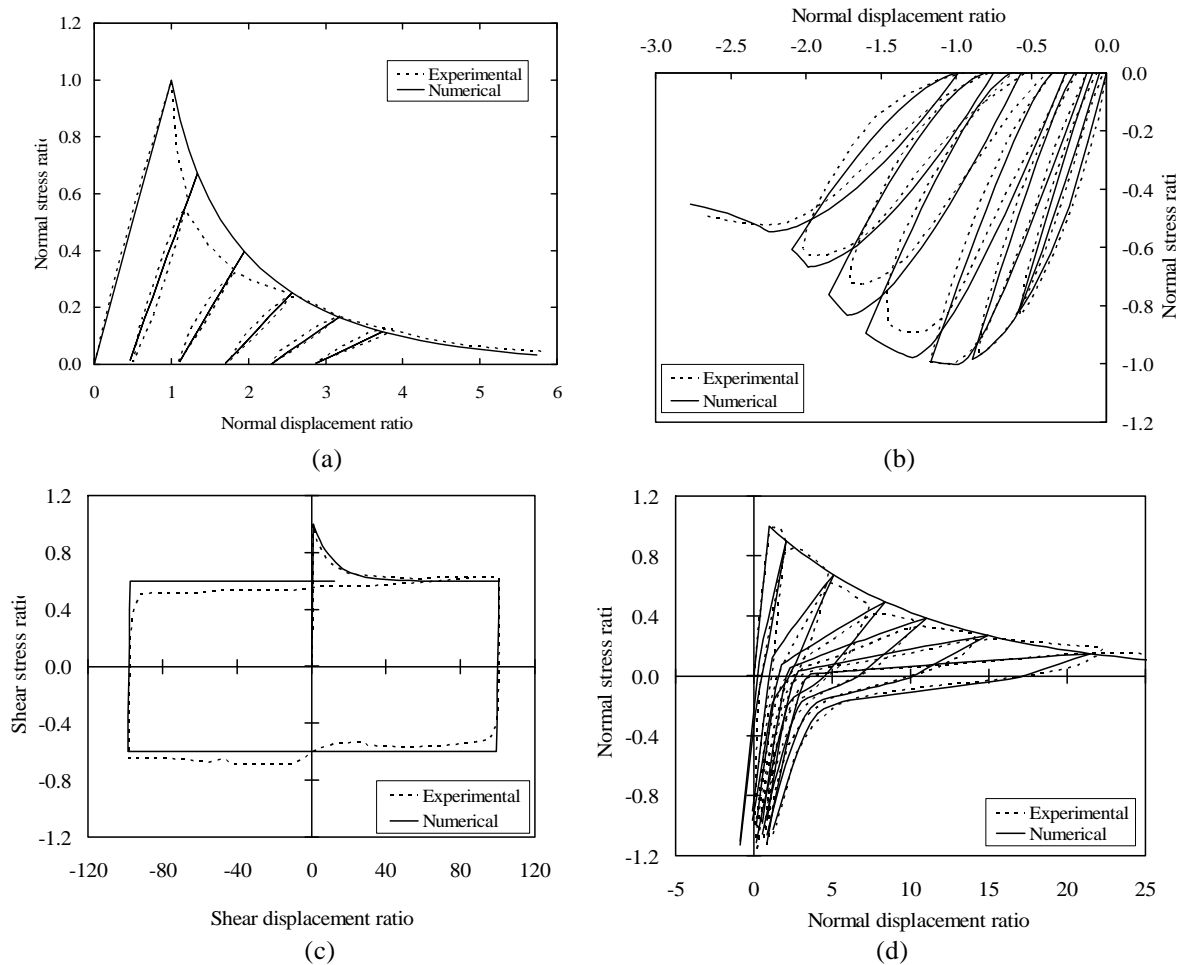


Figure 5. Comparison between experimental and numerical results for uniaxial testing: (a) tension; (b) compression; (c) shear; (d) tension-compression.

Figure 6b presents the results of a high wall, which simply rocks in both ways. The highly non-linear shape of the load-displacement curve is essentially due to the opening and subsequent closing, under load reversal, of the top and bottom bed joints. Similar deformed patterns, involving the opening of extreme bed joints, were observed during the experimental test. Numerical results show that the cyclic behaviour of the wall is controlled by the opening and closing of the extreme bed joints, where damage is mainly concentrated. The model also shows low energy dissipation, which is a consequence of the activated non-linear mechanism (opening-closing of joints). As shown, the failure of this wall is much different from the previous wall, stressing the relevance of the internal structure.

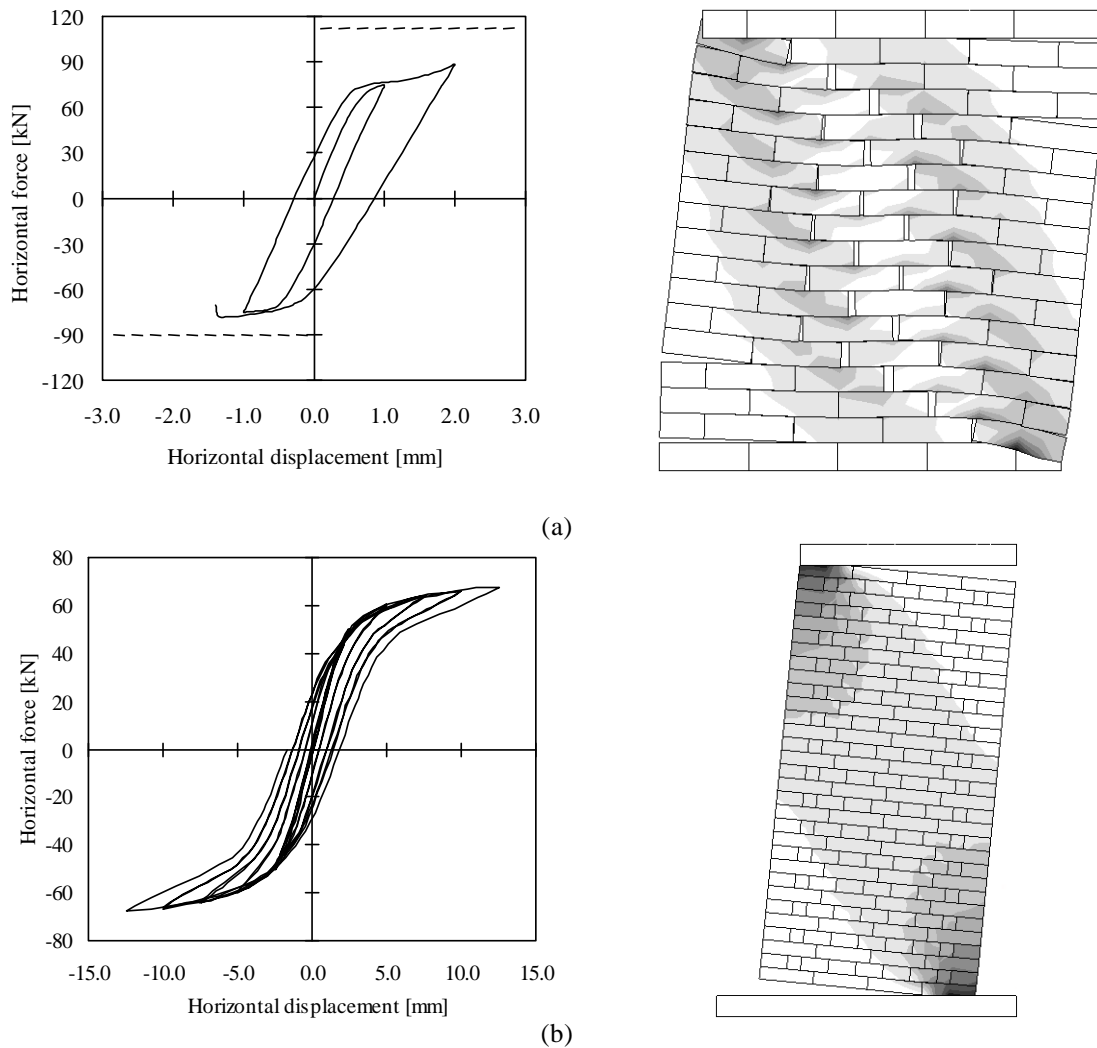


Figure 6. Results obtained with interface cyclic loading model for shear walls, in terms of force-displacement diagram and failure mode: (a) wall failing in shear; (b) wall failing in bending.

2.3. Homogenization techniques

The approach based on the use of averaged constitutive equations [7,8] seems to be the only one suitable to be employed a large scale finite element analyses. Two different approaches are illustrated in Figure 7, one collating experimental data at average level and another from homogenization techniques. A major difference is that homogenization techniques provide continuum average results as a mathematical process that includes the information on the micro-structure.

The complex geometry of the masonry representative volume, i.e. the geometrical pattern that repeats periodically in space, means that no closed-form solution of the problems exists for running bond masonry. One of the first ideas presented was to substitute the complex geometry of the basic cell with a simplified geometry, so that a closed-form solution for the

homogenization problem was possible. This approach, rooted in geotechnical engineering applications, assumed masonry as a layered material [9]. This simplification does not allow including information on the arrangement of the masonry units and provides significant errors in the case of non-linear analysis. Moreover, the results depend on the sequence of homogenization steps. To overcome the limitation, micro-mechanical homogenization approaches that consider additional internal deformation mechanisms have been derived [10,11]. The implementation of these approaches in standard macroscopic finite element non-linear codes is simple and the approaches can compete with macroscopic approaches [12].

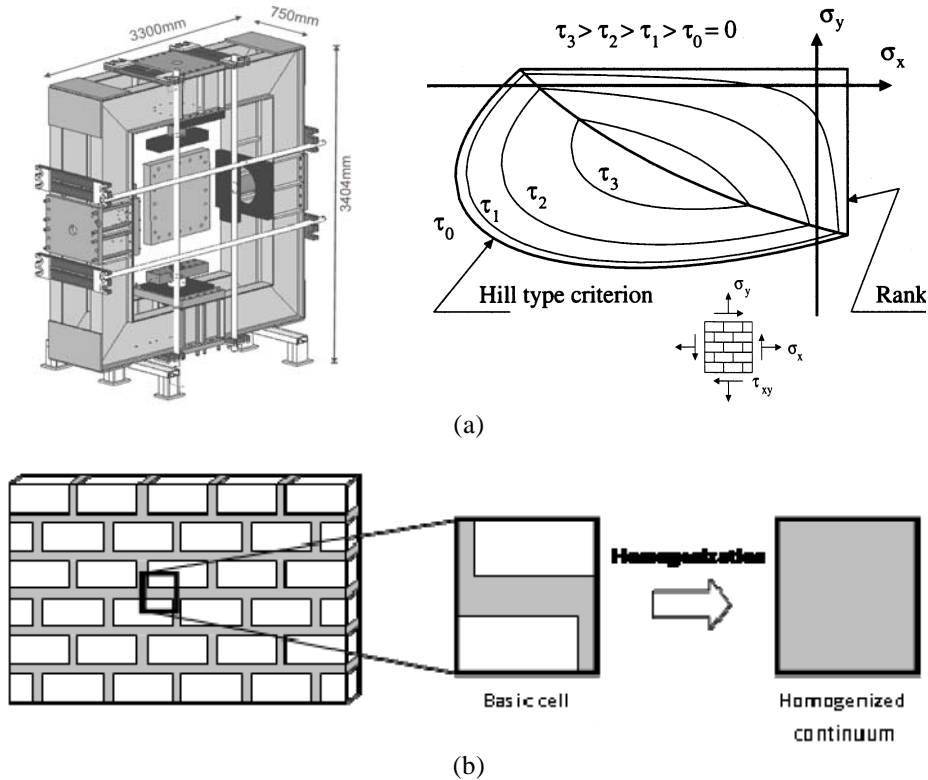


Figure 7. Constitutive behaviour of materials with micro-structure: (a) collating experimental data and failure surfaces; (b) mathematical process using geometry and mechanics of components.

Here, a micro-mechanical model for the limit analysis for masonry is briefly reviewed [13,14]. In the model, the elementary cell is subdivided along its thickness in several layers. For each layer, fully equilibrated stress fields are assumed, adopting polynomial expressions for the stress tensor components in a finite number of sub-domains. The continuity of the stress vector on the interfaces between adjacent sub-domains and suitable anti-periodicity conditions on the boundary surface are further imposed. In this way, linearized homogenized surfaces in six dimensions for masonry in- and out-of-plane loaded are obtained. Such surfaces are then implemented in a finite element limit analysis code for simulation of 3D structures, and including, as recent advances, blast analysis, quasi-periodic masonry internal structure and FRP strengthening.

2.4. A limit analysis approach

Figure 8 shows a masonry wall constituted by a periodic arrangement of bricks and mortar arranged in running bond. For a general rigid-plastic heterogeneous material, homogenization techniques combined with limit analysis can be applied for the evaluation of the homogenized in- and out-of-plane strength domain. In the framework of perfect plasticity and associated flow rule for the constituent materials, and by means of the lower bound limit analysis theorem, can be derived from the following (non-linear) optimization problem:

$$S^{\text{hom}} = \left\{ \max(\mathbf{M}, \mathbf{N}) \mid \left\{ \begin{array}{ll} \mathbf{N} = \frac{1}{|Y|} \int_{Y \times h} \boldsymbol{\sigma} dV & (a) \\ \mathbf{M} = \frac{1}{|Y|} \int_{Y \times h} y_3 \boldsymbol{\sigma} dV & (b) \\ \text{div} \boldsymbol{\sigma} = \mathbf{0} & (c) \\ [[\boldsymbol{\sigma}]] \mathbf{n}^{\text{int}} = \mathbf{0} & (d) \\ \boldsymbol{\sigma} \mathbf{n} \text{ anti-periodic on } \partial Y_l & (e) \\ \boldsymbol{\sigma}(\mathbf{y}) \in S^m \quad \forall \mathbf{y} \in Y^m ; \quad \boldsymbol{\sigma}(\mathbf{y}) \in S^b \quad \forall \mathbf{y} \in Y^b & (f) \end{array} \right. \right\} \quad (7)$$

where:

- \mathbf{N} and \mathbf{M} are the macroscopic in-plane (membrane forces) and out-of-plane (bending moments and torsion) tensors;
- $\boldsymbol{\sigma}$ denotes the microscopic stress tensor;
- \mathbf{n} is the outward versor of ∂Y_l surface;
- $[[\boldsymbol{\sigma}]]$ is the jump of micro-stresses across any discontinuity surface of normal \mathbf{n}^{int} ;
- S^m and S^b denote respectively the strength domains of mortar and bricks;
- Y is the cross section of the 3D elementary cell with $y_3 = 0$, $|Y|$ is its area, V is the elementary cell volume, h represents the wall thickness and $\mathbf{y} = (y_1 \ y_2 \ y_3)$ are the assumed material axes;
- Y^m and Y^b represent mortar joints and bricks respectively.

In order to solve Equations (7) numerically, an admissible and equilibrated micro-mechanical model is adopted [13]. The unit cell is subdivided into a fixed number of layers along its thickness, as shown in Figure 8b. For each layer in the wall thickness direction, one-fourth of the representative volume element is sub-divided into nine geometrical elementary entities (sub-domains), so that the entire elementary cell is sub-divided into 36 sub-domains. For each sub-domain (k) and layer (L), polynomial distributions of degree (m) in the variables (y_1, y_2) are a priori assumed for the stress components. For out-of-plane actions the proposed model requires a subdivision of the wall thickness into several layers, with a fixed constant thickness for each layer.

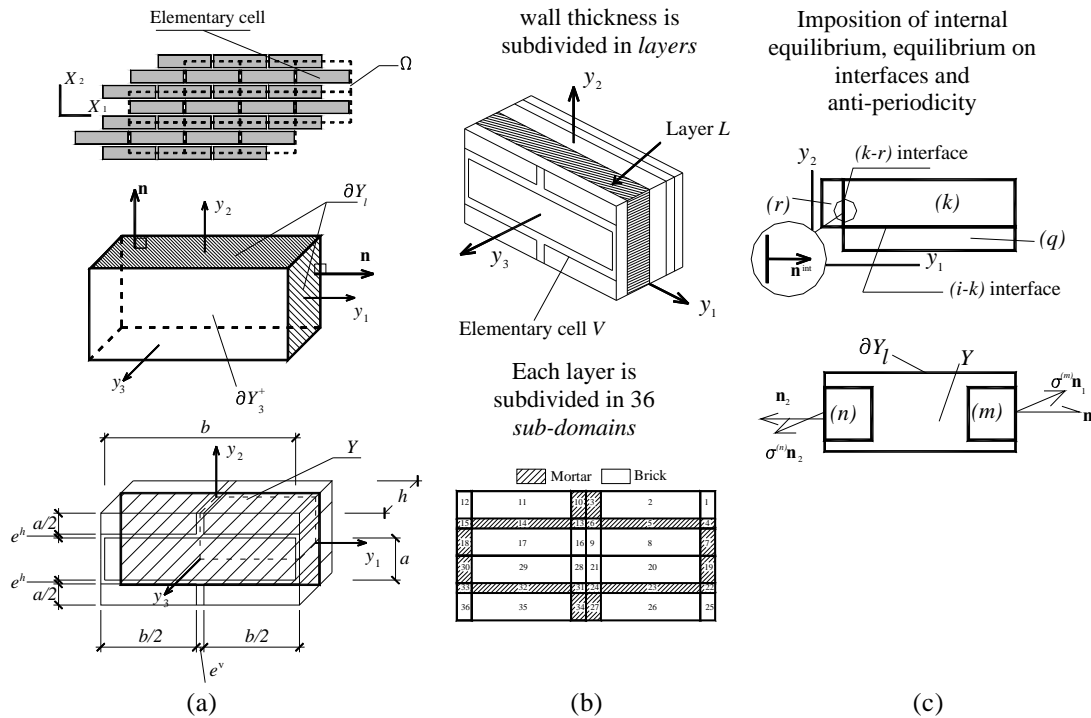


Figure 8. Proposed micro-mechanical model: (a) elementary cell; (b) subdivision in layers along thickness and subdivision of each layer in sub-domains; (c) imposition of internal equilibrium, equilibrium on interfaces and anti-periodicity.

The homogenized failure surface obtained with the above approach has been coupled with finite element limit analysis. Both upper and lower bound approaches have been developed, with the aim to provide a complete set of numerical data for the design and/or the structural assessment of complex structures. The finite element lower bound analysis is based on an equilibrated triangular element, while the upper bound is based on a triangular element with discontinuities of the velocity field in the interfaces. Recent developments include the extension of the model to blast analysis [15], to quasi-periodic masonry [16] and to FRP strengthening [17].

An enclosure running bond masonry wall subjected to a distributed blast pressure is considered first [15]. The wall is supposed simply supported at the base and on vertical edges, and free on top due to the typical imperfect connection between infill wall and RC beam. A full 3D FE heterogeneous elastic-plastic dynamic analysis has been also conducted, in order to have a deep insight into the problem and to collect alternative data to compare with. For the 3D model, a rigid infinitely resistant behaviour for bricks was assumed, whereas for joints a Mohr-Coulomb failure criterion with the same tensile strength and friction angle used in the homogenized approach for joints was adopted. Eight-noded brick elements were utilized both for joints and bricks, with a double row of elements along wall thickness. A comparison between the deformed shapes at $t=400$ msec obtained with the present model and the commercial software is schematically depicted in Figure 9a. As it is possible to notice, the

models give almost the same response in terms of deformed shape for the particular instant time inspected, confirming that reliable results may be obtained with the model proposed. On the other hand, it is worth underlining that the homogenized rigid plastic model required only 101 seconds to be performed on a standard PC Intel Celeron 1.40 GHz equipped with 1Gb RAM, a processing time around 10^{-3} lower than the 3D case. Comparisons of time-maximum displacement diagrams provided by the two models analysed is reported in Figure 9b, together with the evolution of the deformation provided by the homogenized model proposed.

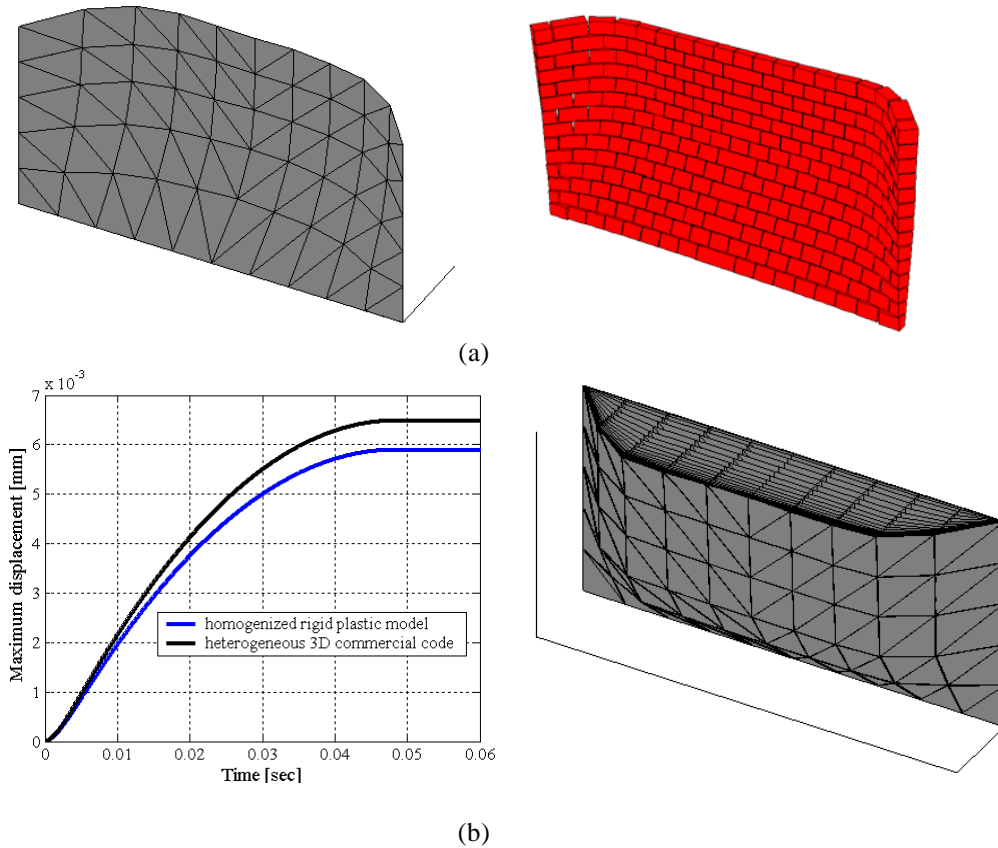


Figure 9. Masonry infill wall subjected to blast pressure: Comparison among deformed shapes at $t = 400$ msec for (a) homogenized limit analysis and heterogeneous 3D elastic-plastic FE approach; (b) comparison between maximum out of plane displacements and limit analysis failure mode.

Recently, two different classes of problems have been investigated [16], the first consisting of full stochastic representative element of volume (REV) assemblages without horizontal and vertical alignment of joints, the second assuming the presence of a horizontal alignment along bed joints, i.e. allowing blocks height variability only row by row. The model is characterized by a few material parameters and it is therefore particularly suited to perform large scale Monte Carlo simulations. Masonry strength domains are obtained equating the power dissipated in the heterogeneous model with the power dissipated by a fictitious homogeneous macroscopic plate. A stochastic estimation of out-of-plane masonry strength domains (both

bending moments and torsion are considered) accounting for the geometrical statistical variability of blocks dimensions is obtained with the proposed model. The case of deterministic block height (quasi-periodic texture) can be obtained as a subclass of this latter case. As an important benchmark, the case in which joints obey a Mohr-Coulomb failure criterion is also tested and compared with results obtained assuming a more complex interfacial behaviour for mortar. In order to show the capabilities of the approach proposed when dealing with large scale structures, the ultimate behaviour prediction of a Romanesque masonry church façade located in Portugal. Comparisons with finite element heterogeneous approaches and “at hand” calculations show that reliable predictions of the load bearing capacity of real large scale structures may be obtained with a very limited computational effort, see Figure 10.

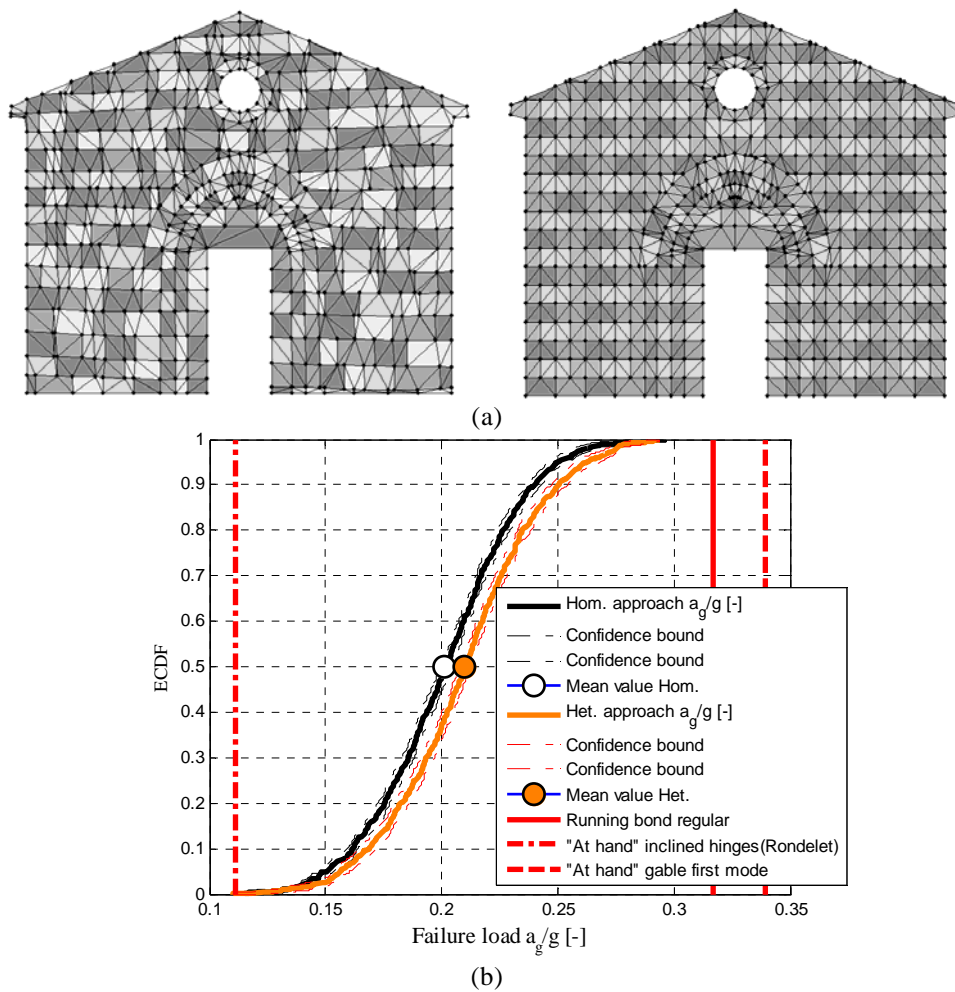


Figure 10. Church of Gondar (Portugal), FE discretization adopted: (a) heterogeneous random mesh vs. mesh for running bond regular heterogeneous and homogenized random analysis; (b) ECDF of the failure load provide through a direct heterogeneous approach and homogenized limit analysis simulations.

Finally, a simple two-step 3D model for the evaluation of the non-linear behaviour of FRP strengthened masonry structures is addressed [17]. Masonry is modelled by means of rigid infinitely resistant wedge elements interconnected by non-linear orthotropic interfaces. FRP strips are modelled by means of triangular rigid elements. To properly take into account the brittle delamination of the strips from the support, it has been supposed that masonry and FRP layers interact by means of interfacial tangential actions between triangles (FRP) and wedges (masonry), following an elastic behaviour with a degradation of the strength until zero in correspondence of a pre-defined slip, in agreement with available codes of practice formulas. Linear piecewise constant approximations of all the stress-strain relationships have been assumed to solve the incremental elastoplastic problem within non-linear programming approaches. In this way, the delamination phenomenon at the FRP/masonry interface and masonry failure may be taken into account suitably. To assess the numerical model proposed, several numerical examples have been analysed, namely a circular arch and a ribbed cross vault, see Figure 11.

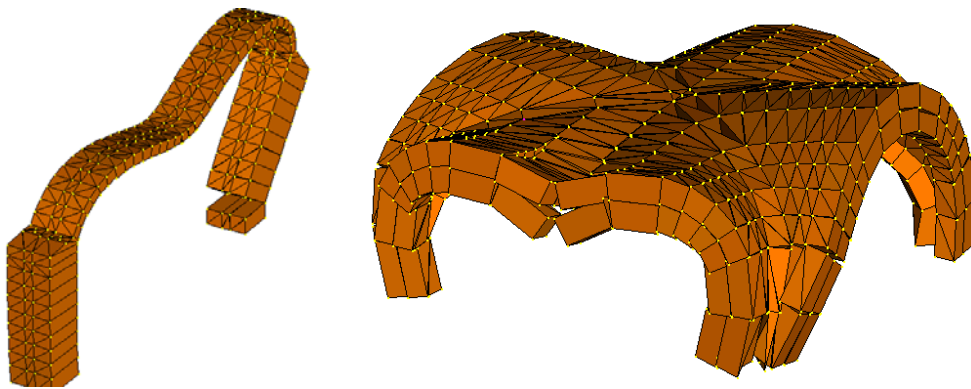


Figure 11. Failure of masonry arches and vaults and the study of FRP strengthening for a circular arch with buttresses and for a ribbed cross vault.

3. SEISMIC RESPONSE

The seismic response of masonry buildings is particularly difficult to characterize due to its nature, the low number of strong events in a given location, site effects, attenuation laws, the non-linear response of the structure, the relevance of execution defects, and many other factors. Next, a brief review is presented on earthquake design and assessment of masonry structures, together with applications of different methodologies. The low tensile strength of masonry is so important that it has determined the shape of ancient and modern masonry constructions. In case of seismic loading, it is certain that non-linear behaviour is triggered at early stages of loading and linear elastic analysis seems not an option. Therefore, the traditional design and assessment method of modal superposition, possibly with a 3-degree-of-freedom system per floor, is not applicable. The alternative options seem to be push-over methods, as recommended in most codes for earthquake safety assessment, or non-linear time integration methods, which is a complex and time consuming tool hardly available for practitioners.

Another much relevant property in case of seismic loading is the presence of floors that provide diaphragmatic action and the so-called “box-behaviour”. This possible feature provides a separation between ancient and modern unreinforced masonry buildings, requiring different models of analysis, addressed next.

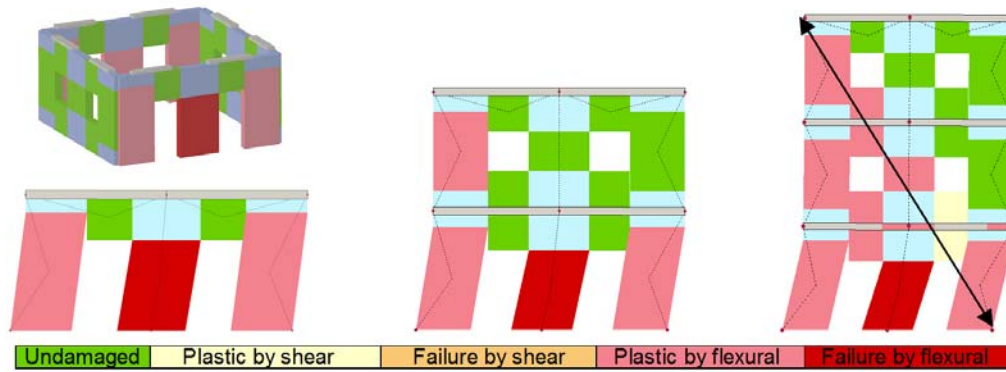
3.1. Masonry structures with box behaviour

Modern masonry buildings usually adopt solutions for the slabs that ensure considerable in-plane stiffness. This is done by using monolithic solutions for the floors, in concrete and steel, and also by establishing an effective connection between slabs and walls. Moreover, many existing buildings originally constructed with timber floors are capable of providing diaphragmatic actions or have been rehabilitated by stiffening the floors and by providing adequate connections. The effect of floor diaphragms combined with the in-plane response of structural walls ensures box behaviour to the building, which usually leads to good performance of the structure when subjected to earthquakes.

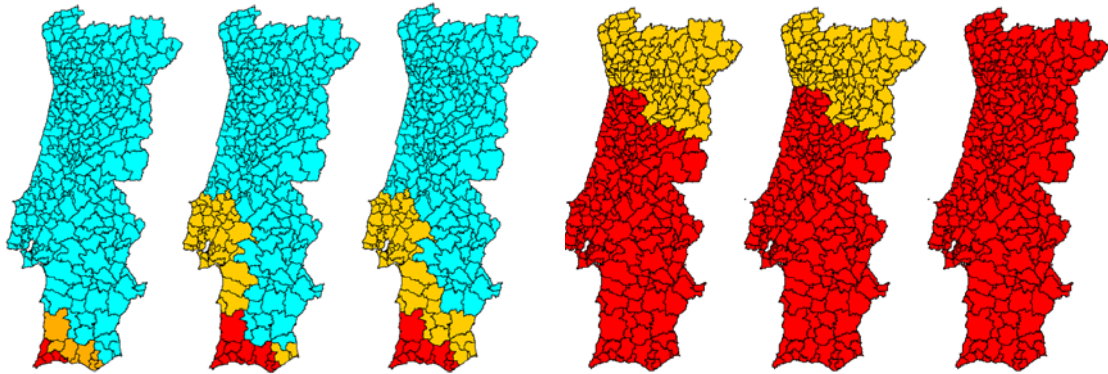
Methods based on macro-elements have been developed, particularly in Italy. These methods seem the most appropriate for design and assessment of masonry buildings, given the simplicity of modelling, the straightforward interpretation of results and the accuracy demonstrated in different validations [18]. To discuss the possibilities of construction with unreinforced masonry in Portugal, the seismic safety of buildings with one up to three storeys, based on a pushover analysis carried out in the 3Muri computer code, is considered. Figure 12a illustrates the ultimate response in terms of deformed configuration and damage of the three buildings, where it can be observed that the collapse mechanisms are essentially induced by flexure, while plastic mechanisms by shear are only found for the three-storey building in spandrels adjacent to the first slab. The possibility to construct the studied buildings in Portugal was evaluated using the non-linear performance based seismic assessment and the European regulations, see Figure 12b, where the latter provides over conservative results.

3.2. Masonry structures without box behaviour

Differently from the structures considered in the previous section, unreinforced masonry structures without box behaviour have shown poor performance in many past earthquakes. The reasons for the poor performance are the inherent brittleness, lack of tensile strength, lack of ductility, the flexible floor diaphragms and the lack of connection between the structural elements. Different methods of seismic assessment exist [19], being the macro-block limit analysis one of the most powerful for engineering application. The strong capabilities of limit analysis and the existence of abacus of possible mechanisms, make this technique particularly helpful for practitioners. An example of seismic assessment is given by S. Torcato church in Portugal. Four mechanisms were defined, based on the inspection and condition of the structure. Figure 13 shows the mechanisms considered in the linear limit kinematic analysis. According to the limit analysis the S. Torcato church is safe and the lowest safety factor is equal to 1.69 (with overturning of the tympanum).



(a)



(b)

Figure 12. Unreinforced masonry buildings: (a) ultimate damage and deformed configurations; (b) safety using a non-linear design method vs. Eurocode 8 (red shows unsafe zones for rock-type soil).

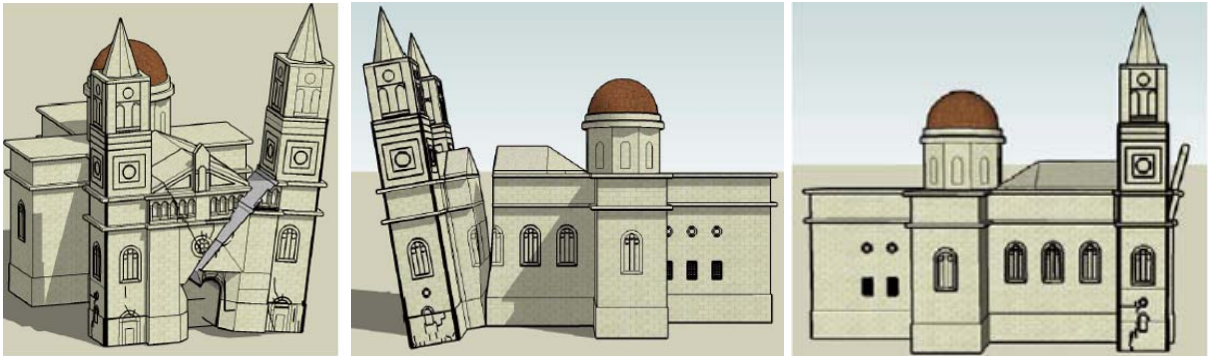


Figure 13. Collapse mechanisms (from left to right): left tower; façade; tympanum.

4. CONCLUSIONS

Constraints to be considered in the use of advanced modelling are the cost, the need of an experienced user / engineer, the level of accuracy required, the availability of input data, the need for validation and the use of the results. As a rule, advanced modelling is a

necessary means for understanding the behaviour and damage of masonry constructions. For large scale applications, average continuum mechanics is usually adopted and homogenization techniques represent a popular and active field in masonry research. The assessment and design of unreinforced masonry structures subjected to seismic loading is particularly challenging. It is advocated that linear elastic analysis can hardly be used, as masonry features low tensile strength and different models must be used in the presence or absence of adequately connected floors, the so-called box behaviour.

In case of box behaviour the available methods have been briefly reviewed. Their performance is good and the knowledge is sound, with some corrections needed in the recent European regulations (Eurocode 8). When box behaviour cannot be guaranteed, the analysis of masonry structures becomes rather complex. The use of macro-models and limit analysis seems the current trend but difficulties arise in its practical use, namely with respect to validation of the hypothesis of the user and the possibility of selecting inadequate failure mechanisms.

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