NIPE

Documentos de Trabalho

Working Paper Series

"A simple method for testing cointegration subject to regime changes"

Vasco J. Gabriel

NIPE WP 15 / 2001

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS

UNIVERSIDADE DO MINHO

A simple method for testing cointegration subject to regime changes

VASCO J. GABRIEL
Department of Economics, Birkbeck College, UK
University of Minho, Portugal

MARTIN SOLA
Department of Economics, Birkbeck College, UK
Universidad Torcuato Di Tella, Argentina

ZACHARIAS PSARADAKIS
Department of Economics, Birkbeck College, UK

June 2001*

Abstract

In this paper, we propose a simple method for testing cointegration in models that allow for multiple shifts in the long run relationship. The procedure consists of computing conventional residual-based tests with standardized residuals from Markov switching estimation. No new critical values are needed. An empirical application to the present value model of stock prices is presented, complemented by a small Monte Carlo experiment.

Key Words: Cointegration; Markov Switching; Standardized residuals.

JEL Classification: C12; C22; C52.

1 Introduction

Regime changes have always been a major concern when modelling economic time series. Accounting for parameter shifts becomes crucial in cointegration analysis, since it normally involves long spans of data, which, consequently, are more likely to display structural breaks. In recent years, methods have been developed to detect and test for breaks in models with cointegrated variables, see Hansen (1992), Kuo

*We are grateful to Ron Smith and Luis Martins for helpful comments, although retaining responsibility for remaining errors. The first author acknowledges the financial support of the Fundação para a Ciência e Tecnologia, Portugal.

(1998), Seo (1998), Bai, Lumsdaine and Stock (1998) and Hansen and Johansen (1999), for example. A different issue is that of testing cointegration when regime shifts may be present in the data. In fact, conventional procedures to test for cointegration may lead to erroneous inferences, as discussed in Gregory, Nason and Watt (1996) and Gabriel, Sola and Psaradakis (2001). To deal with this problem, Gregory and Hansen (1996) proposed cointegration tests in models that allow for regime changes, while Inoue (1999) developed procedures to test cointegration rank in the presence of breaks in the deterministic trend.

In this paper, we propose a simple method for testing cointegration in single equation models that allow for multiple shifts in the long run relationship. More specifically, we assume that cointegration regimes are governed by an unobserved Markov chain process. Testing for cointegration may be carried out by means of standard residual-based tests, although using the standardized residuals obtained from Markov switching estimation. This approach does not call for the computation of new critical values, as we find that the usual asymptotic distributions seem to offer a good approximation to the actual standardized residuals distribution. We analyse an empirical example and conduct some Monte Carlo experiments to show that these residuals are suitable for cointegration testing, both using tests for the null hypothesis of no cointegration and tests for the null of cointegration.

A major drawback with the above mentioned works is that they either consider a single, deterministic break only, or they assume that the break points are known when cointegration is being tested. A Markov switching cointegration specification is extremely flexible and presents an integrated and very convenient way of dealing with the several steps of cointegration inference. First, it simultaneously allows for an unspecified number of breaks, while guaranteeing their stochastic nature. Secondly, it also permits the possibility of changing variances in the long run relationship. Cointegration thus formulated encompasses a number of empirically plausible and economically meaningful models, including the case of a single permanent regime shift. Furthermore, testing for cointegration arises naturally from the estimation step, since only conventional procedures are to be used. Lastly, information on the timing of the shifts is an immediate by-product of the model estimation.

Since the seminal paper of Hamilton (1989), Markov-switching models have been extensively used to account for regime changes in economic time series (see Kim and Nelson, 1999 for several examples). Concerning cointegration, Hall, Psaradakis and Sola (1997) analyse the Japanese consumption function, finding evidence of Markov-switching changes in the cointegration vector. These authors employ similar methods to ours, although they use simulation-based finite-sample critical values. Also, Krolzig (1997), in an unpublished paper, develops the statistical analysis of cointegrated VAR processes with Markov switching. Moreover, Gabriel *et al.* (2001) study the effects of unaccounted multiple regime shifts of the Markov type in the performance of several cointegration tests. The Markov switching cointegration approach is also related, from a methodological point of view, with the work of Hansen (2000), as this author generalizes Johansen's cointegrated VAR model by allowing for structural breaks.

Therefore, our contribution complements the existing literature on estimating and testing for cointegration in models subject to changes in regime. The paper proceeds as follows. The next section, building upon Driffill and Sola (1998), provides an empirical illustration of the problem using US data on stock prices and dividends. Section 3 analyses a simple Monte Carlo experiment that corroborates our findings and Section 4 concludes.

2 An Empirical Illustration

To motivate the problem of testing for cointegration when several regime shifts have occurred, we look at a simple empirical example, using US data on stock prices and dividends[1]. Several studies have focused on present value models of stock prices and dividends, albeit without providing conclusive evidence, possibly because they fail to account for regime changes. Figure 1 show the series and it is possible to observe the abrupt changes in the path of the variables. Bonomo and Garcia (1990) and Driffill and Sola (1998), for example, explain the deviations from stock prices fundamentals by allowing the dividends process, as well as the present value relationship, to switch between two regimes. Nevertheless, neither address the issue of whether stock prices and dividends are cointegrated or not. Given that the series appear to be non-stationary, we try to answer that question.

Usual testing procedures, such as the ADF-type test or Phillips-Perron-type non-parametric tests, are known to have their power substantially reduced when breaks in the series are present (see Gregory *et al.*, 1996 and Gabriel *et al.*, 2001). This means that the tests do not reject the null of no cointegration in favour of the alternative of an invariant cointegrated relationship. On the other hand, tests for the null of cointegration are badly oversized in the presence of structural breaks, i.e. they tend to reject the hypothesis of cointegration, although one with stable parameters (see Gabriel *et al.*, 2001). The reason is that the residuals will capture unaccounted breaks and, thus, will exhibit a nonstationary behaviour. The researcher may, in this case, resort to the tests of Gregory and Hansen (1996), specifically designed to be robust to regime shifts in the cointegration vector.

Table 1 reports the results from this set of cointegration tests[2], as well as DOLS asymptotically efficient estimates (see Saikkonen, 1991) of the cointegrating relationship[3] $y_t = \beta x_t + u_t$, where y_t and x_t represent real stock prices and dividends, respectively. All tests for the null hypothesis of no cointegration fail to reject, whereas the KPSS-type test (*MLS*) of McCabe, Leybourne and Shin (1997) clearly rejects the existence of

[1] The data is taken from Shiller (1989) and updated by the author. The stock prices are January values for the Standard and Poor Composite Index, from 1900 to 1995, while dividends are year-averages. The series are deflated by January values of the producer price index.

[2] Concerning Gregory-Hansen tests, since we are examining a type of structural break that was not tabulated in the original paper (change in slope, no constant term), we obtained critical values for this case using the response surface technique explained by Gregory and Hansen (1996, p. 110). The critical values at 5% significance level are -4.192 for the *GH-AEG* and *GH-*Z_t tests, and -30.322 for the *GH-*Z_α test, respectively.

[3] The number of leads and lags in the DOLS estimation (corresponding estimates not reported) is 1 and was determined using the BIC criterion.

a long run (stable) relationship between stock prices and real dividends. Note in particular that Gregory-Hansen tests also fail to indicate the presence of cointegration. This is not a surprise, since they are robust only to a single change in the cointegration vector and do not take into account potentially changing variances (see Gabriel *et al.*, 2001).

If more than one shift has occurred, the residuals will reflect this by appearing to be nonstationary, as can be seen in Figure 2. Moreover, we computed the tests of Hansen (1992) for instability of the coefficients, also presented in Table 1, and the null hypothesis of parameter constancy is rejected. Hence, a researcher, using these tools, would find evidence against the existence of cointegration between the variables in this dataset.

What happens if we endogeneize the possibility of having long run parameters switching between different cointegrating regimes? Following Driffill and Sola (1998), we may formulate an explicit cointegration relationship for stock prices and dividends as

$$y_t = \beta_i x_t + \theta_i u_t, \qquad u_t \sim N(0,1) \tag{1}$$

$$\log x_t = \mu_i + \log x_{t-1} + \omega_i v_t, \qquad v_t \sim N(0,1) \tag{2}$$

where $i = 0, 1$ for regime i. The regimes are assumed to follow a first-order homogeneous Markov chain with transition probabilities $p = \Pr(s_t = 1|s_{t-1} = 1)$ and $q = \Pr(s_t = 0|s_{t-1} = 0)$, $s_t \in S = \{0, 1\}$. Note that (1) is specified as a standard cointegrating regression, instead of an implicitly cointegrated, ratio-type formulation (y/x), as in Driffill and Sola (1998, eq. 16).

Table 2 presents the results[4] of fitting a Markov-switching system to the present value relationship and the log of real dividends process, using the procedure described in Driffill and Sola (1998). In the regime 0, we have a low growth/high volatility state in the dividends process, with cointegration vector $[1, -\beta_0]$, $\beta_0 = 19.3636$, while regime 1 corresponds to a high growth/low volatility regime with $[1, -\beta_1]$, $\beta_1 = 30.0884$. The probabilities of staying at each regime are $p = 0.9798$ for regime 0 and $q = 0.9843$ for regime 1. These estimates contrast with the results in Table 1 for the "invariant" model, where $\beta = 25.356$, which is approximately the average of the two regimes. Also notice that the variances are significantly different in the two regimes.

Now, in order to test for cointegration, we employ some of the tests computed in Table 1, but instead we use the standardized residuals obtained from the estimation presented in Table 2. These are computed as

$$e_t = \{y_t - [\beta_0 x_t(\Pr(s_t = 0|I_t)) + \beta_1 x_t(\Pr(s_t = 1|I_t))]\}/\sigma_t, \tag{3}$$

where $\Pr(s_t = i|I_t)$, $i = 0, 1$, are the filter probabilities from the Markov switching estimation and σ_t is the residuals conditional standard deviation. The idea is that, by allowing for an unspecified number of regime

[4] These are maximum likelihood estimates obtained with a numerical optimization procedure using the BFGS algorithm, along with corresponding heteroskedasticity and autocorrelation consistent (HAC) standard errors, computed with the prewhitened quadratic spectral kernel and data-dependent bandwidth, as recommended by Andrews and Monahan (1992).

changes in the estimation step, residuals will be free of unusual observations due to breaks, and therefore will replicate the stationary behaviour of the errors.

From the analysis of the results shown in the second part of Table 2, it is now possible to conclude that there is strong evidence favouring the existence of cointegration between stock prices and dividends. Indeed, all tests with cointegration as the alternative hypothesis clearly reject (at the 1% level of significance) the null hypothesis of no cointegration. By contrast, the KPSS-type test indicates that the residuals are stationary. This is also supported by the observation of Figure 3, in which the standardized residuals appear to be stationary. Thus, the previous conclusion has been reversed.

Obviously, one needs to assure that the tests have good size and power properties. Thus, in order to check the robustness of these results, in the next section we conduct a small Monte Carlo experiment to assess the performance of the approach outlined above. The empirical relevance of our simulation analysis is ensured by plugging the estimates of Table 2 into the DGP. Although artificial DGPs are useful in this context, it is preferable to use more economically meaningful estimated models, even if these only offer a poor approximation to the true DGP.

3 Monte Carlo Analysis

In this section, we present a set of Monte Carlo simulations, where we take model (1)-(2) and the corresponding estimates as our DGP, and evaluate the properties of cointegration tests when standardized residuals are used. The regressor innovation ν_t is generated as $n.i.d.(0,1)$ and independent of u_t. The error term u_t, representing the extent to which the system is out of long-run equilibrium, is simulated as an autoregressive process $u_t = \rho u_{t-1} + \varepsilon_t$, $\varepsilon_t \sim n.i.d.(0,1)$, with $\rho = 0$, $\rho = 0.5$ and $\rho = 0.8$ for the case of cointegration, and $\rho = 1$ corresponding to no cointegration[5]. The idea is to evaluate the tests properties with different error structures, since in an applied work context the disturbances are likely to be, at least, serially correlated. Note that, in our empirical example, the standardized residuals correlation coefficient is $\hat{\rho} = 0.5112$.

We assume that $\log x_t$ is generated by a random walk with switching drift of the Markov type, with transition probabilities $p = 0.9798$ and $q = 0.9843$ (DGP A). We also consider other values for the transition probabilities, namely $(p,q) = (0.95, 0.95)$, $(0.98, 0.9)$, $(0.6, 0.4)$, DGPs (B), (C) and (D), respectively. The last pair of transition probabilities, despite being less empirically plausible, is interesting from a theoretical point of view since it implies that the Markov chain is not autocorrelated and therefore one would expect a great deal of switching. The selected sample dimension is $T = 100$, which is approximately of the same size of the data in our example. We estimate the rejection frequencies using critical values at the 1%, 5% and 10% significance levels. The idea is to assess how the respective quantiles of the sampling distributions are well approximated by the standard asymptotic distributions. In all experiments the number of replications is 5000.

[5] Very seldom in applied work does the researcher allow for possible shifts under the hypothesis of no cointegration. We will consider that case for reasons of "symmetry" with the case of cointegration.

Table 3 reports the results of the simulations. The lines with $\rho = 1$ show the empirical Type-I error probabilities for the tests with null of no cointegration (*AEG*, Z_α and Z_t), while corresponding to empirical power for the *MLS* test. A general conclusion we may draw from the simulations is that the performance of the tests is virtually the same for different values of the transition probabilities, even in case (D), where switching is very frequent. Furthermore, the tests seem to have the correct sizes under their respective nulls.

Concerning the KPSS-type test, we observe that the test performs quite well, attaining a very reasonable power and with little size distortions, except for the case of $\rho = 0.8$. Turning to the other tests, the power attained by these tests is high when the errors are not strongly correlated. Note that for the DGP that more closely resembles that of our illustration ($\rho = 0.5$), all tests perform quite well, in terms of size and finite-sample power. It appears, therefore, that the distributions of the tests statistics using standardized residuals are very close to the standard distributions, thus allowing the researcher to use the method we propose.

4 Summary

In this paper, we have explored a simple, yet effective way of testing for cointegration when multiple regime changes may occur. By specifying the cointegrating relationship to shift between two cointegration regimes, we fit a Markov switching model and use the standardized residuals with conventional cointegration residual-based tests. Although one could expect the asymptotic distributions of the tests statistics (under their respective nulls) to be affected by the fitting of the Markov model, Monte Carlo simulations show that the tests have almost no size distortions and standard critical values may therefore be employed. Using this procedure, we found that the present value model of US stock prices is well described by a Markov switching cointegrated model.

References

[1] Andrews, D. W. K. and Monahan, J. C. (1992), An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator, *Econometrica,* 60, 953-966.

[2] Bai, J., Lumsdaine, R. L. and Stock, J. H. (1998), Testing for and Dating Breaks in Multivariate Time Series, *Review of Economic Studies,* 65, 395-432.

[3] Bonomo, M. and Garcia, R. (1994), Can a Well-Fitted Equilibrium Asset-Pricing Model Produce Mean Reversion?, *Journal of Applied Econometrics*, 9, 19-29.

[4] Driffill, J. and Sola, M. (1998), Intrinsic Bubbles and Regime-Switching, *Journal of Monetary Economics,* 42, 357-373.

[5] Gabriel, V. J., Sola, M. and Psaradakis, Z. (2001), Residual-based Tests for Cointegration and Multiple Regime Shifts, manuscript

[6] Gregory, A. W. and Hansen, B. E. (1996), Residual-based Tests for Cointegration in Models with Regime Shifts, *Journal of Econometrics*, 70, 99-126.

[7] Gregory, A. W., Nason, J. M. and Watt, P. G. (1996), Testing for Structural Breaks in Cointegrated Relationships, *Journal of Econometrics*, 71, 321-341.

[8] Hamilton, J. D. (1989), A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica*, 57, 357-384.

[9] Hall, S. G., Psaradakis, Z., Sola, M. (1997), Cointegration and Changes in Regime: the Japanese Consumption Function., *Journal of Applied Econometrics*, 12, 151-168.

[10] Hansen, B. E. (1992), Tests for Parameter Instability in Regressions with I(1) Processes, *Journal of Business and Economic Statistics*, 10, 321-335.

[11] Hansen, P. R. (2000), Structural Breaks in the Cointegrated Vector Autoregressive Model, Working Paper 00-20, Brown University.

[12] Inoue, A. (1999), Tests of Cointegrating Rank with a Trend Break, *Journal of Econometrics*, 90, 215-237.

[13] Kim, C.-J. and Nelson, C. R. (1999) *State-space Models with Regime Switching*, MIT Press: Cambridge.

[14] Krolzig, H. M. (1997), Statistical Analysis of Cointegrated VAR Processes with Markovian Shifts, manuscript.

[15] Kuo, B. S. (1998), Test for Partial Parameter Instability in Regressions with I(1) processes, *Journal of Econometrics*, 86, 337-368.

[16] McCabe, B. P. M. and Leybourne, S. J. and Shin, Y. (1997), A Parametric Approach to Testing for the Null of Cointegration, *Journal of Time Series Analysis,* 18 (4), 395-413.

[17] Saikkonen, P. (1991), Asymptotically Efficient Estimation of Cointegrating Regressions, *Econometric Theory,* 7, 1-21.

[18] Seo, B. (1998), Tests for Structural Change in Cointegrated Systems, *Econometric Theory*, 14, 222-259.

[19] Shiller, R. (1989), *Market Volatility,* MIT Press, Cambridge, MA.

5 Appendix

Table 1 - Cointegration Analysis

Tests	AEG	Z_α	Z_t	GH-AEG	GH-Z_α	GH-Z_t	MLS
	-2.117	-10.597	-2.022	-3.20	-20.842	-3.217	7.901^{**}
Estimated β (standard error):				25.353	(0.695)		
Regression standard error:				0.1514			
				sup-F	mean-F	L_c	
Instability tests				35.138^{**}	13.954^{**}	1.972^{**}	

Notes: ** means rejection at the 1% significance level. The lag length for the AEG test was chosen by means of a t-test downward selection procedure with initial lag $K = 6$, while for Z_α and Z_t the long run variance is estimated using a prewhitened quadratic spectral kernel with an automatically selected bandwidth estimator.

Table 2 - Markov-switching cointegration results

Eq. (1)	β_0	β_1	θ_0	θ_1	p	q
	19.3636 (0.5795)	30.0884 (0.8339)	0.1466 (0.0192)	0.2995 (0.0635)	0.9798 (0.0376)	0.9843 (0.0422)
Eq. (2)	μ_0	μ_1	ω_0	ω_1		
	-0.0041 (0.0095)	0.0316 (0.0041)	0.1513 (0.0193)	0.0462 (0.0092)		

Cointegration tests using standardized residuals

	AEG	Z_α	Z_t	MLS
	-4.788^{**}	-50.215^{**}	-5.501^{**}	0.143

Note: see notes to Table 1; HAC standard errors in brackets

Table 3 - Monte Carlo results

Tests		*AEG*			Z_α			Z_t			*MLS*		
	ρ	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
	0	0.92	0.958	0.977	1.00	1.00	1.00	1.00	1.00	1.00	0.009	0.047	0.093
(A)	0.5	0.883	0.933	0.959	1.00	1.00	1.00	1.00	1.00	1.00	0.031	0.08	0.124
	0.8	0.438	0.758	0.864	0.507	0.862	0.951	0.503	0.851	0.946	0.254	0.305	0.343
	1	0.011	0.045	0.093	0.009	0.048	0.097	0.01	0.046	0.085	0.837	0.863	0.879
	0	0.922	0.962	0.976	1.00	1.00	1.00	1.00	1.00	1.00	0.008	0.045	0.091
(B)	0.5	0.884	0.933	0.955	1.00	1.00	1.00	1.00	1.00	1.000	0.031	0.074	0.122
	0.8	0.407	0.759	0.868	0.494	0.854	0.952	0.482	0.842	0.941	0.269	0.317	0.354
	1	0.013	0.053	0.10	0.01	0.05	0.103	0.01	0.045	0.096	0.837	0.862	0.875
	0	0.922	0.956	0.974	1.00	1.00	1.00	1.00	1.00	1.00	0.009	0.049	0.097
(C)	0.5	0.878	0.932	0.956	0.999	1.00	1.00	1.00	1.00	1.00	0.028	0.072	0.116
	0.8	0.414	0.744	0.858	0.485	0.847	0.949	0.476	0.839	0.937	0.264	0.317	0.354
	1	0.011	0.054	0.103	0.01	0.049	0.105	0.01	0.045	0.095	0.838	0.862	0.873
	0	0.906	0.945	0.963	0.999	1.00	1.00	1.00	1.00	1.00	0.02	0.063	0.111
(D)	0.5	0.881	0.933	0.955	1.00	1.00	1.00	1.00	1.00	1.00	0.025	0.072	0.121
	0.8	0.43	0.754	0.856	0.506	0.852	0.953	0.493	0.843	0.856	0.264	0.311	0.343
	1	0.01	0.052	0.10	0.01	0.054	0.104	0.01	0.049	0.098	0.836	0.861	0.874

Note: Rejection frequencies at the 1%, 5% and 10% significance levels.

Figure 1: Stock Prices and Dividends

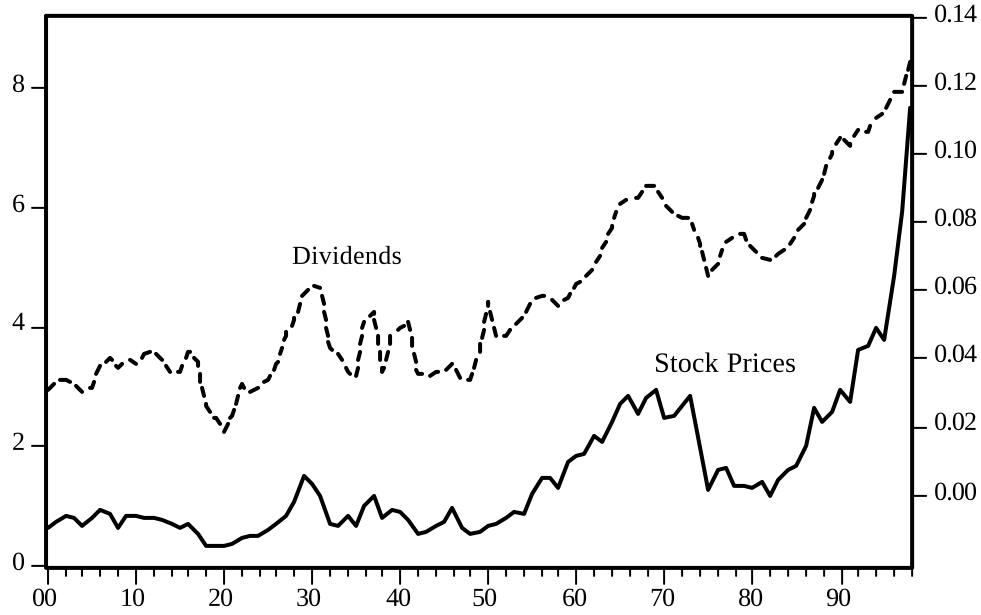

Figure 2: DOLS residuals

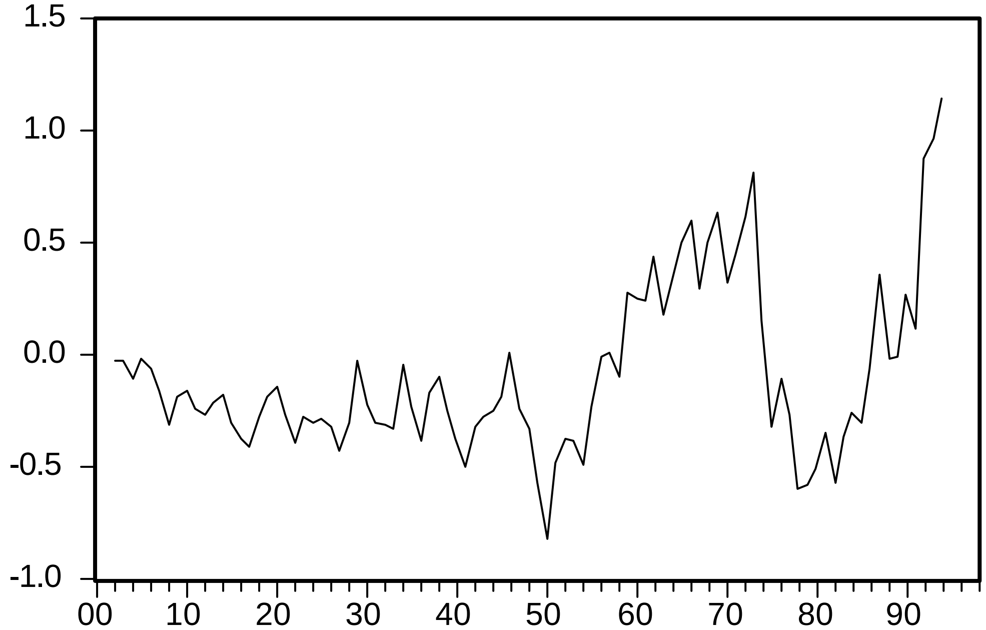

Figure 3: Standardized residuals