A PROPOSAL FOR MODELING DEBONDING OF NSM FRP STRIPS FOR SHEAR STRENGTHENING OF RC BEAMS

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Abstract

Gluing of FRP strips within thin shallow slits cut in the concrete cover of RC beams is a strengthening technique that is gaining increasing attention in the FRP community. It has proven to be effective for both shear and flexural strengthening of existing RC structures that need a retrofitting intervention. Recent findings have highlighted that such near surface mounted (NSM) strips may fail due to: debonding, concrete semi-conical tensile fracture and strip tensile rupture. The necessity to improve an analytical model for predicting the NSM FRP strips shear strength contribution led to further address issues related to bond mechanism through which force in the strip is transferred to the surrounding concrete. A new local bond stress-slip relationship is proposed and closed-form equations to be implemented in that analytical model are derived and appraised on the basis of some of the most recent experimental results.

Keywords: bond, debonding, elastic phase, free slipping, softening, softening friction.

1. Introduction

In the framework of a study aiming at developing a comprehensive analytical model for predicting the NSM FRP strips shear strength contribution to RC beams [1], it was demonstrated that the failure modes affecting the behaviour, at ultimate, of the NSM FRP strips include: strip tensile rupture, concrete semi-conical tensile fracture and debonding (Fig. 1). Concrete semi-conical fracture occurs when the force transferred by bond to the surrounding concrete is such as to induce principal tensile stresses in the concrete exceeding its tensile strength. Concrete fractures along a surface, envelope of the compression isostatics, whose shape can be conveniently assumed as semi-conical. The term “debonding” is adopted to designate failure occurring within the adhesive or just a few millimetres inside the surrounding concrete. In that occasion, the modelling strategy adopted to simulate the concrete fracture failure mode was developed in closed form, resulting robust and rational. However, the strategy adopted to simulate the possibility of debonding was based on analytical expressions obtained by regression of experimental data. That part of the analytical model resulted, this way, not scientifically rigorous and therefore, susceptible of improvements. Moreover, among the possible failure modes affecting the behaviour, at ultimate, of an NSM CFRP strip, a mixed failure mode composed of a shallow semi-cone plus debonding should be also taken into consideration (Fig. 1d). The possibility of this latter failure mode can be easily explained considering the interaction between the progressive value of force transferred by bond to the concrete, due to the imposed end slip consequent to the shear crack widening process and originating at the end of the strip available bond length lying on the critical diagonal crack (CDC), and the progressive concrete fracture capacity. The details of that interaction are herein omitted for the sake of brevity but can be found elsewhere [2,3].

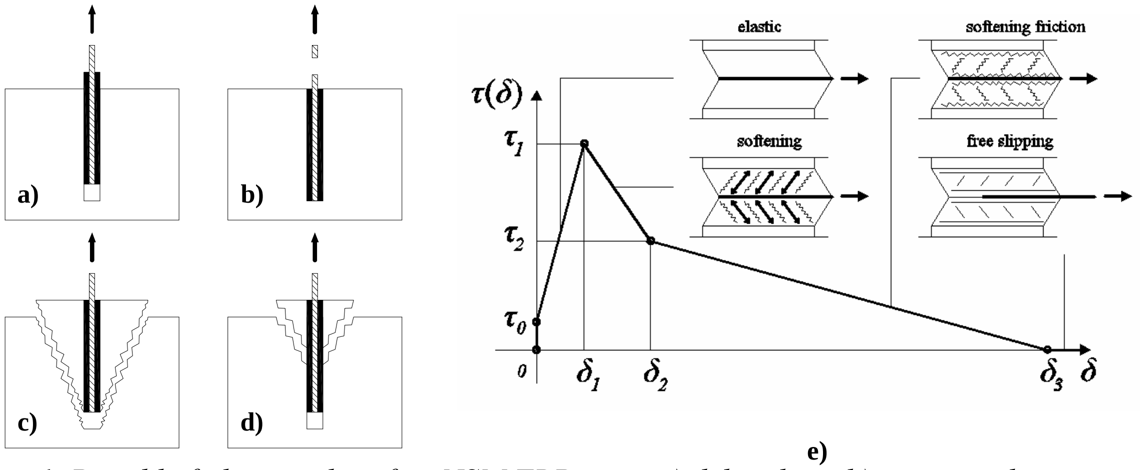

Fig. 1. Possible failure modes of an NSM FRP strip: a) debonding; b) strip tensile rupture; c) concrete semi-conical tensile fracture; d) mixed shallow semi-cone plus debonding; and e) proposed local bond stress-slip relationship.

2. Debonding Model

The understanding and analytical modelling of debonding, affecting the behaviour of externally bonded FRPs, has reached to date a high level of accuracy [4,5]. As regards more specifically the case of NSM strips it arises, from the most recent works [6-8], that debonding is more complicated, mainly due to the higher number of parameters it depends on. Moreover, several aspects still need to be clarified. In the present work a new interpretation of the phenomenon is provided. By analyzing the most recent specialized publications, regarding the newest findings in terms of the use of high performance adhesives in engineering applications [9,10], it arises that, due to the relative novelty of those materials, a lot of aspects still need to be clarified, even from a micro-structural and chemical standpoint. In that scenario, a new interpretation of the phenomenon of loss of bond for an NSM FRP strip is provided by putting forward some hypotheses that should be further confirmed, *a posteriori*, also by means of more specialized contributions.

2.1 Local Bond Stress-Slip Relationship

The local bond stress-slip relationship herein proposed to simulate the subsequent phases undergone by bond during the loading process is composed of four different linear branches (Fig. 1e). Those phases, representing the physical phenomena occurring in sequence within the adhesive layer by increasing the imposed end slip, are: “elastic”, “softening”, “softening friction” and “free slipping”. The first rigid branch $(0-\tau_0)$ represents the overall initial shear strength of the joint, independent of the deformability of the adhesive layer and attributable to the micro-mechanical and chemical properties of the involved materials and interfaces. In fact, the parameter τ_0 is the average of the following physical entities encountered in sequence by forces flowing from the strip to the surrounding concrete, *i.e.*: adhesion at the strip-adhesive interface, cohesion within the adhesive itself, and adhesion at the adhesive-concrete interface. From τ_0 up to the peak strength τ_1, a macro-mechanical strength due to the adhesive layer elastic stiffness adds to the constant adhesive-cohesive strength. That macro-mechanical strength due to the elastic stiffness of the intact adhesive layer can be conveniently modelled by a linear elastic behaviour. Approaching the peak strength, the adhesive fractures along diagonal planes orthogonal to the traction isostatics as was outlined by means of post-test optical microscope photos [8] and finite element materially nonlinear analysis [7]. During the subsequent *softening* phase, force is transferred from the strip to the surrounding concrete by the resulting diagonal micro-struts. Throughout the softening phase, by increasing the imposed slip, the adhesive at the extremities of those struts progressively deteriorates, so that, by increasing the imposed slip, micro-cracks parallel to the strip start to appear at both the strip-adhesive and adhesive-concrete interfaces. Approaching the *softening friction* phase, the softening resisting mechanism is gradually replaced by friction and micro-mechanical interlock along those micro-cracks. Nonetheless, even those mechanisms undergo softening due to progressive degradation. When the resisting force provided by friction is exhausted, those micro-cracks result in smooth discontinuities. The *free slipping* phase follows, during which the strip keeps being pulled out without having to overcome any opposing restraint left. For computational ease, both softening and softening-frictional behaviours are modelled as linear. The resulting analytical relationship is the following:

$$\tau(\delta)=\begin{cases}\tau_0+\dfrac{\tau_1-\tau_0}{\delta_1}\cdot\delta & 0\le\delta\le\delta_1\\[2ex] \tau_1-\dfrac{\tau_1-\tau_2}{\delta_2-\delta_1}\cdot(\delta-\delta_1) & \delta_1<\delta\le\delta_2\\[2ex] \tau_2-\dfrac{\tau_2}{\delta_3-\delta_2}\cdot(\delta-\delta_2) & \delta_2<\delta\le\delta_3\\[2ex] 0 & \delta>\delta_3\end{cases} \tag{1}$$

2.2 Governing Equations

The analytical expressions are derived hereafter with reference to a typical push-pull test of an FRP strip near surface mounted on a concrete prism. The strip width and thickness are denoted by b_f and a_f, respectively, and those of the concrete prism by b_c and a_c, respectively. The strip and concrete Young's modulus is E_f and E_c, respectively. Based on equilibrium considerations, the following fundamental equations can be written:

$$\frac{d\sigma_f(x)}{dx}-\tau(x)\cdot\frac{L_p}{A_f}=0 \quad \text{and} \quad \sigma_f(x)\cdot A_f+\sigma_c(x)\cdot A_c=0 \tag{2}$$

where $\tau(x)$ is the shear stress acting on the surface of the strip, $\sigma_f(x)$ is the axial stress in the strip, $\sigma_c(x)$ is the axial stress in the concrete prism, A_f ($a_f\cdot b_f$) and A_c ($a_c\cdot b_c$) are the area of the cross-section of the strip and concrete prism, respectively, and L_p is the effective bond perimeter of the strip cross-section, *i.e.*:

$$L_p=2\cdot b_f+a_f \tag{3}$$

The constitutive equations for the adhesive layer and the two adhering materials can be written as:

$$\tau=\tau(\delta) \quad \text{and} \quad \sigma_f=E_f\cdot\frac{du_f}{dx} \quad \text{and} \quad \sigma_c=E_c\cdot\frac{du_c}{dx} \tag{4}$$

The interfacial slip δ is defined as the punctual relative displacement between the two adhering materials, that is:

$$\delta(x) = u_f(x) - u_c(x) \tag{5}$$

The governing differential equation can be obtained by introducing Eqs. (3)-(5) into Eq. (2), resulting in:

$$\frac{d^2\delta}{dx^2} - \tau\left[\delta(x)\right] \cdot J_1 = 0 \quad \text{with} \quad J_1 = \frac{L_p}{A_f} \cdot \left(\frac{1}{E_f} + \frac{A_f}{A_c \cdot E_c} \right) \tag{6}$$

Eq. (6) can be solved once the local bond stress-slip model of Eq. (1) is reliably known. Once the relationship $\delta(x)$ has been obtained by solving Eq. (6) with the convenient boundary conditions, the expressions for the stress in the strip and the tangential stress along this latter can be deduced as follows [2]:

$$\sigma_f(x) = J_2 \cdot \frac{d\delta}{dx} \quad \text{with} \quad J_2 = \frac{E_f \cdot E_c \cdot A_c}{E_c \cdot A_c + E_f \cdot A_f} \tag{7}$$

and:

$$\tau(x) = J_3 \cdot \frac{d^2\delta}{dx^2} \quad \text{with} \quad J_3 = \frac{E_f \cdot A_f \cdot E_c \cdot A_c}{L_p \cdot \left(E_c \cdot A_c + E_f \cdot A_f\right)} \tag{8}$$

2.3 Debonding process for an infinite bond length

The entire debonding process affecting the behaviour of an NSM strip is herein described by distinguishing the several subsequent phases and by referring to an infinite bond length. Those phases are singled out according to the value of the imposed slip and with respect to the assumed local 'bond stress-slip' relationship. Further details can be found elsewhere [2].

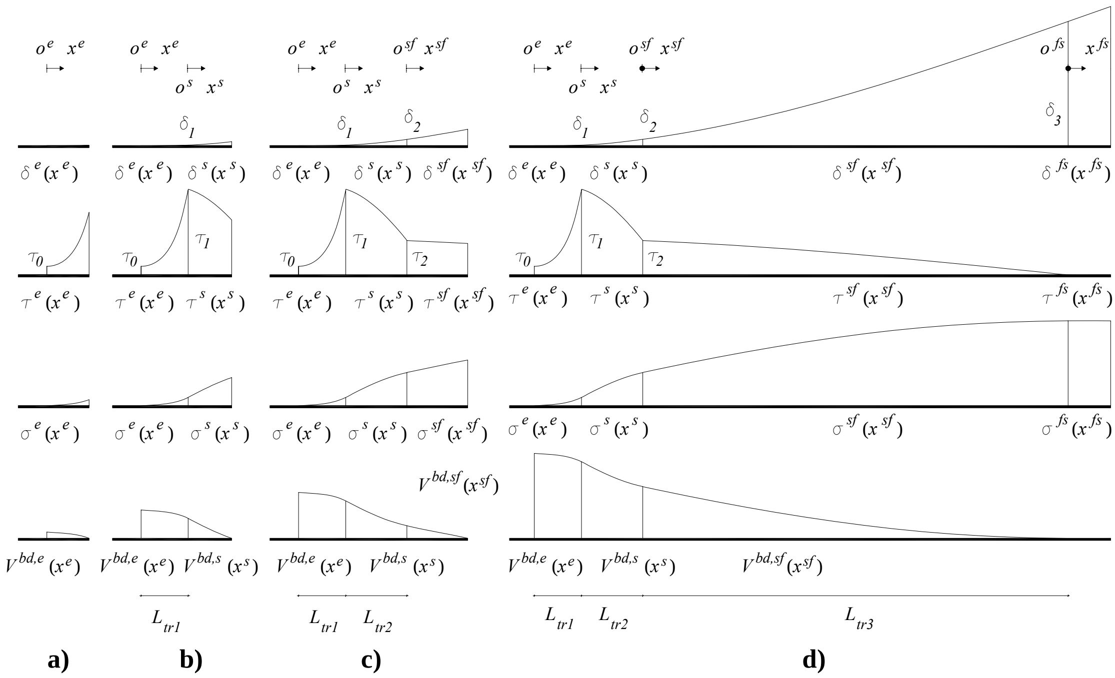

Fig. 2. Differentt phases of the debonding failure occurring within the adhesive layer for an infinite bond length: a) elastic; b) softening; c) softening friction; d) free slipping.

2.3.1 Elastic phase

When the imposed end slip is $\delta_{Li} \le \delta_1$, the governing differential equation solved in the local reference system ox^e originating in the leftward unloaded extremity of the transfer length, becomes (see Fig. 2a):

$$\frac{1}{\lambda^2}\cdot\frac{d^2\delta^e}{dx^{e2}} - \delta^e = \left(J_1\cdot\tau_0\right)\cdot\frac{1}{\lambda^2} \quad \text{with} \quad \frac{1}{\lambda^2} = \frac{\delta_1}{\left(\tau_1-\tau_0\right)\cdot J_1} \tag{9}$$

and the boundary conditions are:

$$\delta^e = 0 \;\text{ at }\; x^e = 0 \;\text{ and }\; \delta^e = \delta_{Li} \;\text{ at }\; x^e = L_{tr}^e\left(\delta_{Li}\right) \tag{10}$$

By solving the resulting equations, the following expressions of the integration constants are obtained:

$$C_1^e = \left[\delta_{Li} + \frac{\tau_0\cdot J_1}{\lambda^2}\cdot\left(1-e^{-\lambda\cdot L_{tr}^e}\right)\right]\cdot\frac{1}{e^{\lambda\cdot L_{tr}^e} - e^{-\lambda\cdot L_{tr}^e}} \quad \text{and} \quad C_2^e = \frac{\tau_0\cdot J_1}{\lambda^2} - C_1^e \tag{11}$$

And the expression for the interfacial slip:

$$\delta^e\left(x^e\right) = C_1^e\cdot e^{\lambda\cdot x^e} + C_2^e\cdot e^{-\lambda\cdot x^e} - \frac{\tau_0\cdot J_1}{\lambda^2} \tag{12}$$

The corresponding expressions for the axial stress in the laminate and the bond stress are determined by introducing Eq. (12) into Eqs. (7) and (8), respectively. By imposing the equilibrium condition:

$$L_p\cdot\int_0^{L_{tr}^e}\tau^e\left(x^e\right)\cdot dx^e = A_f\cdot\sigma_f\left(L_{tr}^e\right) \tag{13}$$

the expression of the transfer length for the first phase can be obtained as function of the imposed slip:

$$L_{tr}\left(\delta_{Li}\right) = L_{tr}^e\left(\delta_{Li}\right) = \frac{1}{\lambda}\cdot\operatorname{arcosh}\frac{B^e}{2\cdot A^e} \;\text{ with }\; A^e = \frac{\tau_0\cdot J_1}{2\cdot\lambda^2} \;\text{ and }\; B^e = \delta_{Li} + \frac{\tau_0\cdot J_1}{\lambda^2} \tag{14}$$

The force transferred by bond to the surrounding concrete, along the transfer length $L_{tr}^e\left(\delta_{Li}\right)$, can be determined as follows:

$$V^{bd,e}\left(\delta_{Li}\right) = L_p\cdot\int_0^{L_{tr}^e}\tau^e\left(x^e\right)\cdot dx^e \tag{15}$$

The transfer length at the end of the elastic phase L_{tr1} and the corresponding value of force transferred to concrete V_1^{bd}, both invariants for given input parameters, are obtained respectively by Eq. (14) and (15) imposing $\delta_{Li} = \delta_1$:

$$V_1^{bd} = V^{bd,e}\left(\delta_1\right) \quad \text{and} \quad L_{tr1} = L_{tr}^e\left(\delta_1\right) \tag{16}$$

2.3.2 Softening phase

When the imposed slip is $\delta_1 < \delta_{Li} \le \delta_2$, the solving differential equation, limited to the amount of the total transfer length in the softening phase, becomes (see Fig. 2b):

$$\frac{1}{\beta^2}\cdot\frac{d^2\delta^s}{dx^{s2}}+\delta^s=\frac{\tau_1\cdot J_1}{\beta^2}+\delta_1 \quad \text{with} \quad \frac{1}{\beta^2}=\frac{\left(\delta_2-\delta_1\right)}{\left(\tau_1-\tau_2\right)\cdot J_1} \tag{17}$$

and the boundary conditions, for a reference system ox^s that originates at the point of the bond length where slip is equal to δ_1, are:

$$\delta^s=\delta_1 \text{ at } x^s=0 \quad \text{and} \quad \delta^s=\delta_{Li} \text{ at } x^s=L_{tr}^s \tag{18}$$

By solving the resulting equations, the following expressions of the integration constants are obtained:

$$C_1^s=\frac{1}{\sin\left(\beta\cdot L_{tr}^s\right)}\cdot\left\{\delta_{Li}-\delta_1+\frac{\tau_1\cdot J_1}{\beta^2}\cdot\left[\cos\left(\beta\cdot L_{tr}^s\right)-1\right]\right\} \text{ and } C_2^s=-\frac{\tau_1\cdot J_1}{\beta^2} \tag{19}$$

The expression for the interfacial slip is:

$$\delta^s\left(x^s\right)=C_1^s\cdot\sin\left(\beta\cdot x^s\right)+C_2^s\cdot\cos\left(\beta\cdot x^s\right)+\frac{\tau_1\cdot J_1}{\beta^2}+\delta_1 \tag{20}$$

The expression of the transfer length $L_{tr}^s\left(\delta_{Li}\right)$ corresponding to the amount of the infinite bond length undergoing softening is:

$$L_{tr}^s\left(\delta_{Li}\right)=\frac{1}{\beta}\cdot\phi+\frac{1}{\beta}\cdot\arcsin\frac{C^s}{\sqrt{\left(A^s\right)^2+\left(B^s\right)^2}} \tag{21}$$

with:

$$A^s=V_1^{bd};B^s=J_3\cdot L_p\cdot\frac{\tau_1\cdot J_1}{\beta};C^s=J_3\cdot L_p\cdot\beta\cdot\left(\delta_{Li}-\delta_1-\frac{\tau_1\cdot J_1}{\beta^2}\right);\phi=\arcsin\frac{B^s}{\sqrt{\left(A^s\right)^2+\left(B^s\right)^2}} \tag{22}$$

The overall transfer length, for $\delta_1<\delta_{Li}\le\delta_2$, is then:

$$L_{tr}\left(\delta_{Li}\right)=L_{tr1}+L_{tr}^s\left(\delta_{Li}\right) \tag{23}$$

and the force transferred by bond to the surrounding concrete is:

$$V^{bd}\left(\delta_{Li}\right)=V_1^{bd}+V^{bd,s}\left(\delta_{Li}\right)=V_1^{bd}+L_p\cdot\int_0^{L_{tr}^s}\tau^s\left(x^s\right)\cdot dx^s \tag{24}$$

The maximum value of the transfer length that can undergo softening and the relevant value of the force transferred to the surrounding concrete are the following invariants:

$$L_{tr2}=L_{tr}^s\left(\delta_2\right) \quad \text{and} \quad V_2^{bd}=V^{bd,s}\left(\delta_2\right) \tag{25}$$

2.3.3 Softening friction phase

When the imposed slip is larger than the value at which *softening friction* begins, $\delta_2<\delta_{Li}\le\delta_3$, the solving differential equation becomes (see Fig. 2c):

$$\frac{1}{\gamma^2}\cdot\frac{d^2\delta^{sf}}{dx^{sf2}}+\delta^{sf}\left(x^{sf}\right)=\delta_3 \quad \text{with} \quad \frac{1}{\gamma^2}=\frac{\left(\delta_3-\delta_2\right)}{\tau_2\cdot J_1} \tag{26}$$

and the boundary conditions, for a reference system ox^{sf} that originates at the point of the infinite bond length where slip is equal to δ_2, are:

$$\delta^{sf} = \delta_2 \ \text{ at } \ x^{sf} = 0 \ \text{ and } \ \delta^{sf} = \delta_{Li} \ \text{ at } \ x^{sf} = L_{tr}^{sf} \tag{27}$$

The integration constants are:

$$C_1^{sf} = \frac{1}{\sin\left(\gamma \cdot L_{tr}^{sf}\right)} \cdot \left[\delta_{Li} - \delta_3 + \left(\delta_3 - \delta_2\right) \cdot \cos\left(\gamma \cdot L_{tr}^{sf}\right) \right] \text{ and } C_2^{sf} = \delta_2 - \delta_3 \tag{28}$$

The expression for the interfacial slip is:

$$\delta^{sf}\left(x^{sf}\right) = C_1^{sf} \cdot \sin\left(\gamma \cdot x^{sf}\right) + C_2^{sf} \cdot \cos\left(\gamma \cdot x^{sf}\right) + \delta_3 \tag{29}$$

The expression of the transfer length $L_{tr}^{sf}\left(\delta_{Li}\right)$ corresponding to the amount of length undergoing *softening friction* is:

$$L_{tr}^{sf}\left(\delta_{Li}\right) = \frac{1}{\gamma} \cdot \psi + \frac{1}{\gamma} \cdot \arcsin \frac{C^{sf}}{\sqrt{\left(A^{sf}\right)^2 + \left(B^{sf}\right)^2}} \tag{30}$$

with:

$$A^{sf} = V_1^{bd} + V_2^{bd}; \ B^{sf} = J_3 \cdot L_p \cdot \gamma \cdot \left(\delta_3 - \delta_2\right); \ C^{sf} = J_3 \cdot L_p \cdot \gamma \cdot \left(\delta_{Li} - \delta_3\right) \tag{31}$$

and:

$$\psi = \arcsin \frac{B^{sf}}{\sqrt{\left(A^{sf}\right)^2 + \left(B^{sf}\right)^2}} \tag{32}$$

The overall transfer length, for $\delta_2 < \delta_{Li} \le \delta_3$, is:

$$L_{tr}\left(\delta_{Li}\right) = L_{tr1} + L_{tr2} + L_{tr}^{sf}\left(\delta_{Li}\right) \tag{33}$$

and the value of force transferred by bond to the surrounding concrete:

$$V^{bd}\left(\delta_{Li}\right) = V_1^{bd} + V_2^{bd} + V^{bd,sf}\left(\delta_{Li}\right) = V_1^{bd} + V_2^{bd} + L_p \cdot \int_0^{L_{tr}^{sf}} \tau^{sf}\left(x^{sf}\right) \cdot dx^{sf} \tag{34}$$

The maximum value of the infinite bond length that can undergo *softening friction* and the relevant value of the force transferred to the surrounding concrete are:

$$L_{tr3} = L_{tr}^{sf}\left(\delta_3\right) \quad \text{and} \quad V_3^{bd} = V^{bd,sf}\left(\delta_3\right) \tag{35}$$

2.3.4 Free slipping phase

When the imposed slip is larger than the value at which *free slipping* begins, $\delta_{Li} > \delta_3$, the solving differential equation becomes (see Fig. 2d):

$$\frac{d^2\delta^{fs}}{\left(dx^{fs}\right)^2} = 0 \tag{36}$$

and the boundary conditions, for a reference system ox^{fs} that originates at the point of the bond length where slip is equal to δ_3, are:

$$\delta^{fs} = \delta_3 \ \text{ at } \ x^{fs} = 0 \text{ and } \delta^{fs} = \delta_{Li} \ \text{ at } \ x^{fs} = L_{tr}^{fs} \tag{37}$$

The integration constants are:

$$C_1^{fs} = \frac{\delta_{Li} - \delta_3}{L_{tr}^{fs}} \quad \text{and} \quad C_2^{fs} = \delta_3 \tag{38}$$

The expression for the interfacial slip is:

$$\delta^{fs}\left(x^{fs}\right) = C_1^{fs} \cdot x^{fs} + C_2^{fs} \tag{39}$$

The expression of the transfer length $L_{tr}^{fs}\left(\delta_{Li}\right)$ corresponding to the amount of length undergoing *free slipping* is:

$$L_{tr}^{fs}\left(\delta_{Li}\right) = J_3 \cdot L_p \cdot \frac{\delta_{Li} - \delta_3}{V_1^{bd} + V_2^{bd} + V_3^{bd}} \tag{40}$$

The overall transfer length, for $\delta_{Li} > \delta_3$, is:

$$L_{tr}\left(\delta_{Li}\right) = L_{tr1} + L_{tr2} + L_{tr3} + L_{tr}^{fs}\left(\delta_{Li}\right) \tag{41}$$

and the force transferred by bond to the surrounding concrete:

$$V^{bd}\left(\delta_{Li}\right) = V_1^{bd} + V_2^{bd} + V_3^{bd} \tag{42}$$

2.4 Debonding process for a finite bond length

Once the constants defining the debonding process have been determined as above specified, based on the input parameters, the values of the transfer length $L_{tr,i}\left(L_{Rfi};\delta_{Li}\right)$, the corresponding force $V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right)$ transferred by bond, and the free end slip $\delta_{Fi}\left(L_{Rfi};\delta_{Li}\right)$ for an imposed slip δ_{Li} can be determined for whatever value of the resisting bond length L_{Rfi} as hereafter specified. Debonding propagation can be thought of as a constant "wave" progressing from the loaded end inwards, towards the free extremity of the NSM strip (Fig. 3). For the sake of brevity, the expressions for the evaluation of the progressive value of force transferred to the surrounding concrete are herein omitted but can be found elsewhere [2].

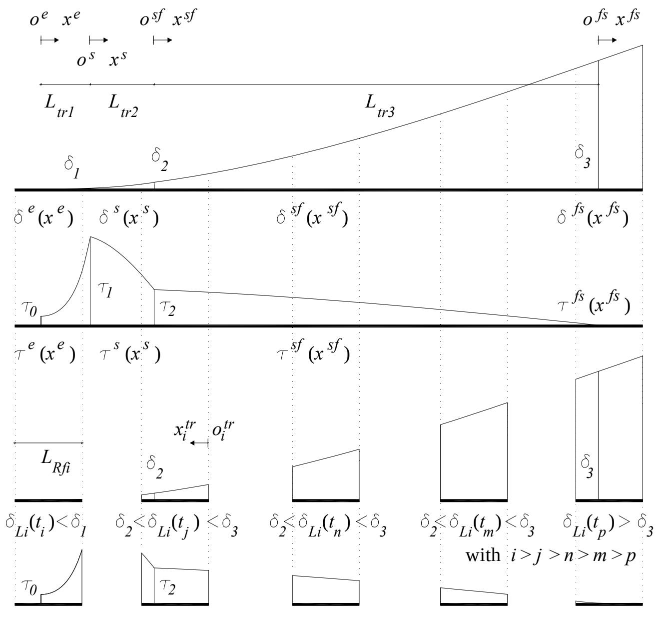

Fig. 3. Debonding process for a finite bond length: "bond wave" progressing towards the free end.

2.4.1 Elastic phase

If the imposed slip is $0 \le \delta_{Li} \le \delta_1$, the corresponding value of $L_{tr}\left(\delta_{Li}\right)$ is calculated by Eq. (14) and then, if $L_{Rfi} \ge L_{tr}\left(\delta_{Li}\right)$, it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{tr}\left(\delta_{Li}\right);\; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = L_p \cdot \int_0^{x_2^e} \tau^e\left(x^e\right)\cdot dx^e;\; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = 0 \tag{43}$$

with $x_2^e = L_{tr}\left(\delta_{Li}\right)$. If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$, it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = L_p \cdot \int_{x_1^e}^{x_2^e} \tau^e\left(x^e\right)\cdot dx^e; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^e\left(x_1^e\right) \tag{44}$$

with $x_1^e = L_{tr}\left(\delta_{Li}\right) - L_{Rfi}$ and $x_2^e = L_{tr}\left(\delta_{Li}\right)$

2.4.2 Softening phase

If the imposed slip is $\delta_1 < \delta_{Li} \le \delta_2$, the corresponding value of $L_{tr}\left(\delta_{Li}\right)$ is calculated by Eq. (23) and then, if $L_{Rfi} \ge L_{tr}\left(\delta_{Li}\right)$, it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{tr}\left(\delta_{Li}\right); V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = V_1^{bd} + L_p \cdot \int_0^{x_2^s} \tau^s\left(x^s\right)\cdot dx^s; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = 0 \tag{45}$$

with $x_2^s = L_{tr}\left(\delta_{Li}\right) - L_{tr1}$. If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $L_{tr}\left(\delta_{Li}\right) - L_{Rfi} < L_{tr1}$ it is:

$$\begin{gathered} L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = L_p \cdot \left[\int_{x_1^e}^{L_{tr1}} \tau^e\left(x^e\right)\cdot dx^e + \int_0^{x_2^s} \tau^s\left(x^s\right)\cdot dx^s\right]; \\ \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^e\left(x_1^e\right) \end{gathered} \tag{46}$$

with $x_1^e = L_{tr}\left(\delta_{Li}\right) - L_{Rfi}$ and $x_2^s = L_{tr}\left(\delta_{Li}\right) - L_{tr1}$. If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $L_{tr}\left(\delta_{Li}\right) - L_{Rfi} \ge L_{tr1}$:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = L_p \cdot \int_{x_1^s}^{x_2^s} \tau^s\left(x^s\right)\cdot dx^s; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^s\left(x_1^s\right) \tag{47}$$

with $x_1^s = L_{tr}\left(\delta_{Li}\right) - \left(L_{Rfi} + L_{tr1}\right)$ and $x_2^s = L_{tr}\left(\delta_{Li}\right) - L_{tr1}$.

2.4.3 Softening friction phase

If the imposed slip is $\delta_2 < \delta_{Li} \le \delta_3$, the corresponding value of $L_{tr}\left(\delta_{Li}\right)$ is calculated by Eq. (33) and then, if $L_{Rfi} \ge L_{tr}\left(\delta_{Li}\right)$ it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{tr}\left(\delta_{Li}\right); V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = V_1^{bd} + V_2^{bd} + L_p \cdot \int_0^{x_2^{sf}} \tau^{sf}\left(x^{sf}\right)\cdot dx^{sf}; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = 0 \tag{48}$$

with $x_2^{sf} = L_{tr}\left(\delta_{Li}\right) - \left(L_{tr1} + L_{tr2}\right)$. If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $L_{tr}\left(\delta_{Li}\right) - L_{Rfi} < L_{tr1}$ it is:

$$\begin{gathered} L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = V_2^{bd} + L_p \cdot \left[\int_{x_1^e}^{L_{tr1}} \tau^e\left(x^e\right)\cdot dx^e + \int_0^{x_2^{sf}} \tau^{sf}\left(x^{sf}\right)\cdot dx^{sf}\right]; \\ \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^e\left(x_1^e\right) \end{gathered} \tag{49}$$

with $x_1^e = L_{tr}\left(\delta_{Li}\right) - L_{Rfi}$ and $x_2^{sf} = L_{tr}\left(\delta_{Li}\right) - \left(L_{tr1} + L_{tr2}\right)$. If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $L_{tr1} \le L_{tr}\left(\delta_{Li}\right) - L_{Rfi} \le \left(L_{tr1} + L_{tr2}\right)$ it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = L_p \cdot \left[\int_{x_1^s}^{L_{tr2}} \tau^s\left(x^s\right)\cdot dx^s + \int_0^{x_2^{sf}} \tau^{sf}\left(x^{sf}\right)\cdot dx^{sf}\right]; \quad (50)$$

$$\delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^s\left(x_1^s\right)$$

with $x_1^s = L_{tr}\left(\delta_{Li}\right) - \left(L_{Rfi} + L_{tr1}\right)$ and $x_2^{sf} = L_{tr}\left(\delta_{Li}\right) - \left(L_{tr1} + L_{tr2}\right)$.

If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $\left(L_{tr1} + L_{tr2}\right) < L_{tr}\left(\delta_{Li}\right) - L_{Rfi} \le \left(L_{tr1} + L_{tr2} + L_{tr3}\right)$ it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = L_p \cdot \int_{x_1^{sf}}^{x_2^{sf}} \tau^{sf}\left(x^{sf}\right)\cdot dx^{sf}; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^{sf}\left(x_1^{sf}\right) \quad (51)$$

with $x_1^{sf} = L_{tr}\left(\delta_{Li}\right) - \left(L_{tr1} + L_{tr2} + L_{Rfi}\right)$ and $x_2^{sf} = L_{tr}\left(\delta_{Li}\right) - \left(L_{tr1} + L_{tr2}\right)$.

2.4.4 Free slipping phase

If the imposed slip is $\delta_{Li} > \delta_3$, the corresponding value of $L_{tr}\left(\delta_{Li}\right)$ is calculated by Eq. (41) and then, if $L_{Rfi} \ge L_{tr}\left(\delta_{Li}\right)$ it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{tr}\left(\delta_{Li}\right); V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = V_1^{bd} + V_2^{bd} + V_3^{bd}; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = 0 \quad (52)$$

If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $L_{tr}\left(\delta_{Li}\right) - L_{Rfi} < L_{tr1}$ it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = V_2^{bd} + V_3^{bd} + L_p \cdot \int_{x_1^e}^{L_{tr1}} \tau^e\left(x^e\right)\cdot dx^e; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^e\left(x_1^e\right) \quad (53)$$

with $x_1^e = L_{tr}\left(\delta_{Li}\right) - L_{Rfi}$. If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $L_{tr1} \le L_{tr}\left(\delta_{Li}\right) - L_{Rfi} \le \left(L_{tr1} + L_{tr2}\right)$ it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = V_3^{bd} + L_p \cdot \int_{x_1^s}^{L_{tr2}} \tau^s\left(x^s\right)\cdot dx^s; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^s\left(x_1^s\right) \quad (54)$$

with $x_1^s = L_{tr}\left(\delta_{Li}\right) - \left(L_{Rfi} + L_{tr1}\right)$.

If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $\left(L_{tr1} + L_{tr2}\right) < L_{tr}\left(\delta_{Li}\right) - L_{Rfi} \le \left(L_{tr1} + L_{tr2} + L_{tr3}\right)$ it is:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = L_p \cdot \int_{x_1^{sf}}^{L_{tr3}} \tau^{sf}\left(x^{sf}\right)\cdot dx^{sf}; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^{sf}\left(x_1^{sf}\right) \quad (55)$$

with $x_1^{sf} = L_{tr}\left(\delta_{Li}\right) - \left(L_{tr1} + L_{tr2} + L_{Rfi}\right)$. If $L_{Rfi} < L_{tr}\left(\delta_{Li}\right)$ and $L_{tr}\left(\delta_{Li}\right) - L_{Rfi} > \left(L_{tr1} + L_{tr2} + L_{tr3}\right)$:

$$L_{tr,i}\left(L_{Rfi};\delta_{Li}\right) = L_{Rfi}; V_{fi}^{bd}\left(L_{Rfi};\delta_{Li}\right) = 0; \delta_{Fi}\left(L_{Rfi};\delta_{Li}\right) = \delta^{fs}\left(x_1^{fs}\right) \quad (56)$$

with $x_1^{fs} = L_{tr}\left(\delta_{Li}\right) - \left(L_{tr1} + L_{tr2} + L_{tr3} + L_{Rfi}\right)$

3. Model Appraisal

The modelling strategy outlined above was appraised on the basis of the experimental results of the pull-out bending tests carried out by Sena-Cruz and Barros [8]. In that experimental program, those authors intentionally employed fibre reinforced concrete to avoid concrete fracture and force debonding to occur. Comparisons between analytical and experimental results for some of the specimens tested are plotted in Fig. 4. The generic label adopted for each series, composed of three specimens, was fcmXX_LbYY where XX is the concrete compressive strength in *MPa* and YY is the bond length in *mm*. Herein, the analytical results are compared with the experimental recordings regarding the series characterized by concrete compressive strength of 45 *MPa* and with bond lengths 40 and 60 *mm*, but further comparisons can be found elsewhere [2]. The comparison regards both the relationship between force and imposed slip and force versus free end slip. From those comparisons, a satisfactory data-fitting performance of the proposed model arises. The assumed values of the parameters entering the adopted bond stress-slip relationship were: τ_0 equal to 2.0 *MPa*, τ_1 ranging from 18.0 to 25.0 *MPa*, τ_2 from 7.5 to 11.5 *MPa*, δ_1 from 0.06 to 0.2 *mm*, δ_2 from 0.6 to 0.9 *mm* and δ_3 from 12.0 to 15.0 *mm*. Variations in some of those parameters are due to inevitable disturbance affecting the experimental results and attributable, for the cases herein examined, to the dissymmetrical position of the strip cross section with respect to the groove's and/or some irregularities in the adhesive layer or on either the concrete surface or the composite surface.

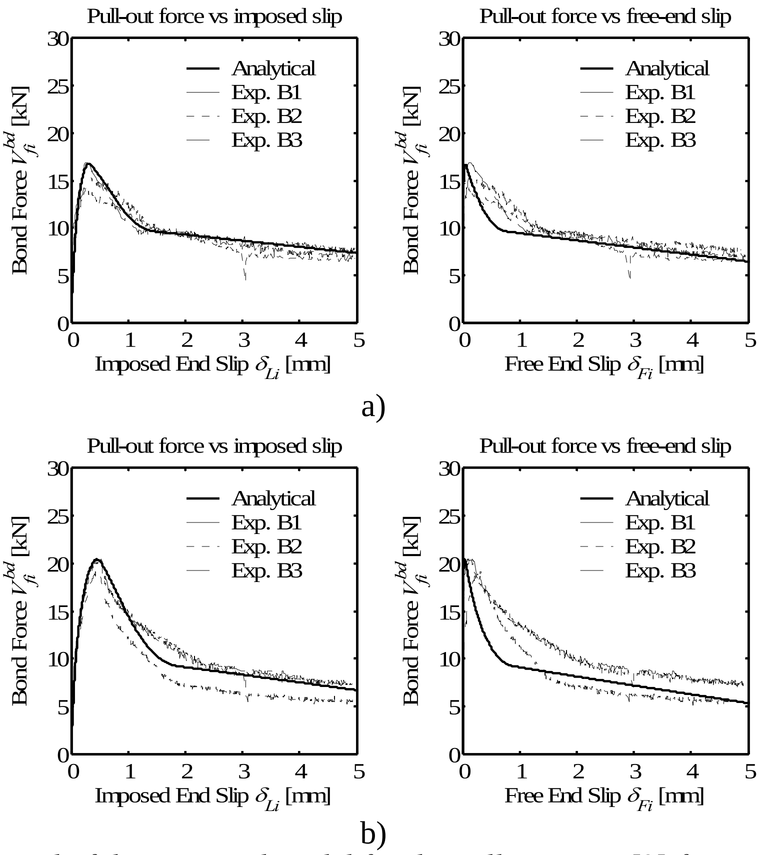

Fig. 4. Appraisal of the proposed model for the pull-out tests [8] for specimens: (a) fcm45_Lb40; (b) fcm45_Lb60.

4. Conclusions

The need to improve an already existing analytical model developed to predict the NSM shear strength contribution for RC beams, led to further address some issues related to debonding failure mode. Due to the relative novelty of powerful adhesives in the field of structural rehabilitation using FRP composite materials, the necessity arises to further investigate the micromechanical and chemical properties of the materials involved. A new physical-mechanical interpretation of the debonding process and a corresponding simplified analytical model of the local 'bond stress-slip' relationship were proposed. The entire debonding process was described and closed-form analytical expressions to be implemented in the model for shear were derived. The comparison between the analytical predictions and some of the most accredited experimental results available to date, showed the high level of accuracy of the proposed model.

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