Kinematics and Dynamics of Mechanical Systems with Lubricated Revolute Joints: the Infinitely-Short Journal-Bearing Approach

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Abstract – This paper deals with a methodology to assess the influence of the lubricated revolute joints in the kinematics and dynamics of mechanical systems. The existence of the clearance at revolute joints is inevitable in all mechanical systems and most of them are designed to operate with a lubricant fluid. In the present work, the infinitely-short journal-bearing approach for dynamic loads is used to evaluate the resulting hydrodynamic forces of the pressure distribution in the lubricated revolute joints. These hydrodynamic forces are then introduced into the system's equations of motion as external applied forces. A numerical example is presented in order to demonstrate the efficiency and accuracy of the methodology and procedures adopted in this work. The numerical results point out that the existence of lubricated joint can affect the kinematics and dynamics of system. Copyright © 2007 Praise Worthy Prize S.r.l. - All rights reserved.

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I. Introduction

Kinematics and dynamics of mechanical systems is often conducted under the assumption that bodies are rigid. Material deformation and damping during motion are routinely ignored as well as clearance and lubrication effects [1]. Over the last few decades, some authors have studied planar and spatial mechanisms with joint clearances [2-4]. Haines [5] derived equations of motion that describe the contributions at an idealized revolute joint with clearance but with no lubrication present.

Roger and Andrews [6] developed mathematical models for the journal-bearing elements, which take into account the effect of clearance, surface compliance and lubrication. However, their lubrication model only accounts for the squeeze-film effect. Later, Liu and Lin [7] extended Roger and Andrews' work to include both squeeze-film and wedge-film actions. Schwab [8], based on the Moes et al. [9] work, applied the impedance method to model lubricated revolute joints in a slider-crank mechanism. Flores et al. [10] proposed a hybrid model for revolute clearance joints in which the dry contact and the pure squeeze-film effect are combined. Flores et al. [11], based upon the Pinkus and Sternlicht [12] formulation, presented a model for dynamic analysis of a planar slider-crank mechanism with a dynamically loaded journal-bearing. An overview literature on the topic of modeling mechanical systems with imperfect kinematic joints is given by Flores [13].

The main aim of this work is to present a general methodology for modeling lubricated revolute joints in constrained mechanical systems.

Both squeeze-film and wedge-film hydrodynamic effects are included in the dynamically loaded journal-bearings model. Lubricated revolute joints in mechanical systems deal with force constraints rather than kinematical constraints (position limitation). Therefore, in mechanical systems, a lubricated revolute joint does not produce any kinematic constraint like the ideal joint. Instead, it acts in a similar way to a force element producing time dependent forces.

For dynamically loaded journal-bearings the classic analysis consists of predicting the motion of the journal centre under arbitrary and known loading, using, for instance, the well-known mobility method [14]. Conversely, in the present analysis the time variable parameters are known from the dynamic analysis and the instantaneous forces on the journal bearing are calculated. In a simple way, the forces built up by the lubricant fluid are evaluated from the state of variable of the system and included into the equations of motion of the mechanical system.

The great contribution of this paper deals with a general methodology developed for mechanical systems with lubricated revolute joints, in which the infinitely-short journal-bearing theory is employed. In fact, most of the available works on the issue are not very realistic, in the measure of most of them only use joints without lubricant [1]-[4]. Other works, when including lubrication effect, the method to account for this phenomenon can not be considered appropriate. For instance, Ravn et al. [21] use the infinitely-long journal-bearing approach to model lubricated revolute joints. However, it is well known that almost all journal-bearings that exist in mechanical systems are short or finite. So this simplification or approximation can be

erroneous, being necessary to develop new methods to model joints in a realistic way, as it is the case of the present work.

An elementary slider-crank mechanism, in which a lubricated revolute joint in the gudgeon-pin exists, is selected as a demonstrative example, in order to provide the necessary results for the comprehensive discussion of the model presented throughout this work

II. Reynolds' Equation

When the journal and bearing have relative rotational velocities with respect to each other, the amount of eccentricity adjusts itself until the pressure generated in the converging lubricating film balances the external loads. The pressure generated, and, consequently, the load capacity of the journal-bearing depends on the journal eccentricity, the relative angular velocity, the effective viscosity of the fluid lubricant and the journal bearing geometry and clearance. Usually, the effect of hydrodynamic pressure is looked as the contribution of two different actions; wedge and squeezing. The squeeze action relates the radial journal motion with the generation of load carrying capacity in the lubricant film, whilst the wedge action deals with the relation between relative rotational velocity of the journal and bearing ability to produce such pressure.

Fig. 1. Cross section of a journal-bearing dynamically loaded

The tangential velocity component on the journal surface, due to the journal rotation, is equal to ωR , while the translational velocity components of the journal with respect to the bearing centre are $\dot{e}$ and $e\dot{\phi}$ as illustrated in Fig. 1. The tangential velocity component at point P , due to the journal rotation, is $\omega R \cos \alpha$. Since the angle α is very small, $\omega R \cos \alpha$ is approximately equal to ωR . In addition, the tangential velocity component at point P caused by the translational motion is $\dot{e} \sin \theta - e\dot{\phi} \cos \theta$.

Thus, the tangential velocity at point P can be written as:

$$U = \omega R + \dot{e} \sin \theta - e\dot{\phi} \cos \theta \quad (1)$$

In a similar way, the normal velocity, V , is equal to sum of the normal components of the rotational and translational velocities. The normal velocity component due to rotational velocity is $\omega R \sin \alpha$. Again, noting that α is very small, then $\omega R \sin \alpha$ is approximately equal to $\omega R \tan \alpha = \omega \frac{\partial h}{\partial \theta}$ [12]. The translational motion of the journal also causes normal velocity component as $\dot{e} \cos \theta + e\dot{\phi} \sin \theta$.

Thus, the global normal velocity at point P is expressed by:

$$V = \omega \frac{\partial h}{\partial \theta} + \dot{e} \cos \theta + e\dot{\phi} \sin \theta \quad (2)$$

The Reynolds' equation for a journal-bearing subjected to dynamic loads can be written as:

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6h \frac{\partial U}{\partial x} - 6U \frac{\partial h}{\partial x} + 12V \quad (3)$$

where h is the fluid film thickness, μ is the absolute fluid viscosity, p represents the film pressure, U is the relative tangential velocity and V is the relative normal velocity between journal and bearing. The radial and axial directions are denoted by x and z .

The three terms on the right-hand side of equation (3) contribute to the hydrodynamic forces in the journal bearing. The term $6h \partial U / \partial x$ implies a variation of tangential velocity along the bearing surface. The second term, $6U \partial h / \partial x$, represents the action of the journal rotation over a wedge-shaped fluid film. Finally, the $12V$ is the expression for the radial velocity of the journal centre and is responsible for the squeeze film effect.

The Reynolds' equation (3) should be expressed in term of the journal bearing variables e , ϕ and θ [13].

Thus, neglecting the curvature of the fluid film, the x coordinate in the unwrapped film is written as:

$$x = R\theta \quad (4)$$

and, hence:

$$\frac{\partial}{\partial x} = R \frac{\partial}{\partial \theta} \quad (5)$$

Now, using equation (5) together with the tangential and normal velocities given by equations (1) and (2), the terms of the right-hand side of equation (3) can be expressed as:

$$6h \frac{\partial U}{\partial x} = 6 \left(\frac{c\dot{e}}{R} \cos \theta + \frac{ce\dot{\phi}}{R} \sin \theta + \frac{e\dot{e}}{R} \cos^2 \theta + \frac{e^2\dot{\phi}}{R} \cos \theta \sin \theta \right) \quad (6)$$

$$-6U \frac{\partial h}{\partial x} = 6 \left(\omega e \sin \theta + \frac{e\dot{e}}{R} \sin^2 \theta - \frac{e^2\dot{\phi}}{R} \cos \theta \sin \theta \right) \quad (7)$$

$$12V = 12(-\omega e \sin \theta + \dot{e} \cos \theta + e\dot{\phi} \sin \theta) \quad (8)$$

The detailed derivation of these equations can be found in work presented by Flores et al. [15]. Using equations (6) up to (8), the Reynolds' equation for finite length journal bearings is:

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 12c \left[\dot{e} \cos \theta + \varepsilon (\dot{\phi} - \bar{\omega}) \sin \theta \right] \quad (9)$$

III. Hydrodynamic Forces

It is well known that equation (9) is one nonhomogeneous partial differential of elliptical type. The exact solution of the Reynolds' equation is difficult to obtain and requires considerable numerical effort. However, it is possible to solve this equation approximately where the first or second term on the left-hand side is treated as zero. These solutions correspond to those for an infinitely-short and infinitely-long journal-bearing.

Dubois and Ocvirk [16] considered a journal-bearing where the pressure gradient around the circumference was very small compared with those along the length. Practically, this assumption is in general valid for length-to-diameter (L/D) ratios up to 0.5. Consequently, the Reynolds' equation for an infinitely-short journal-bearing can be written as:

$$\frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 12c \left[\dot{e} \cos \theta + \varepsilon (\dot{\phi} - \bar{\omega}) \sin \theta \right] \quad (10)$$

If the relative pressure is zero at journal-bearing ends, the pressure in the fluid film is given by [17]:

$$p(\theta, z) = -\frac{3\mu}{h^3} \left(\frac{L^2}{4} - z^2 \right) \left((\omega - 2\dot{\gamma}) \frac{\partial h}{\partial \theta} + 2\dot{e} \cos \theta \right) \quad (11)$$

Equation (11) enables the calculation of the pressure distributions in hydrodynamic infinitely-short loaded journal-bearings as functions of the dynamic parameters and geometries. The force components of the resultant pressure in directions tangent and perpendicular to the

line of centers have to be calculated. These force components can be obtained by integrating the pressure field either in the entire domain 2π or half domain π . These boundary conditions, associated with the pressure field, correspond to Sommerfeld's and Gumbel's boundary conditions. The Sommerfeld's boundary conditions, complete or full film, do not take into account the cavitation phenomenon and, consequently, contemplate the existence of negative pressures for the region $\pi < \theta < 2\pi$. This case is not realistic in many applications due to the fluid incapacity to sustain significant sub-ambient pressures. The Gumbel's conditions, which account for the rupture film, preconize the existence of a zero pressure zone between π and 2π . Though the Gumbel's, or half Sommerfeld's solution, results in more realistic predictions of the load capacity, it leads to a violation of the continuity of flow at the outlet end of the pressure curve.

Thus, for the Sommerfeld's conditions, the force components of the fluid film for infinitely-short journal-bearing are written as:

$$F_r = -\frac{\pi\mu L^3 R}{c^2} \frac{\dot{e}(1+2\varepsilon^2)}{(1-\varepsilon^2)^{3/2}} \quad (12)$$

$$F_t = \frac{\pi\mu L^3 R}{c^2} \frac{\varepsilon(\omega-2\dot{\gamma})}{2(1-\varepsilon^2)^{3/2}} \quad (13)$$

where F_r is the radial component of the force, while F_t is the tangential component.

For the Gumbel's conditions, the force components of the fluid film for infinitely-short journal-bearing can be expressed as:

$$F_r = -\frac{\mu L^3 R}{2c^2 (1-\varepsilon^2)^2} \left(\frac{\pi\dot{e}(1+2\varepsilon^2)}{(1-\varepsilon^2)^{3/2}} + 2\varepsilon^2 (\omega-2\dot{\gamma}) \right) \quad (14)$$

$$F_t = \frac{\mu L^3 R \varepsilon}{2c^2 (1-\varepsilon^2)^2} \left(4\dot{e} + \frac{\pi}{2} (\omega-2\dot{\gamma}) \sqrt{1-\varepsilon^2} \right) \quad (15)$$

Finally, the force components of the resulting pressure distribution for the directions tangent and perpendicular to the line of centres have to be projected onto the X and Y directions. From Fig. 1 it is clear that:

$$F_x = F_r \cos \gamma - F_t \sin \gamma \quad (16)$$

$$F_y = F_r \sin \gamma + F_t \cos \gamma \quad (17)$$

where γ is the angle between the eccentricity vector and X -axis, as it is illustrated in Fig. 2.

IV. Modeling Lubricated Revolute Joints

In order to evaluate the forces produced by the fluid lubricant on the journal-bearings, the different dynamic parameters on which these forces depend need to be evaluated. These hydrodynamic forces are nonlinear functions of the time parameters, ω , ε , $\dot{\varepsilon}$, γ , and $\dot{\gamma}$, which can be evaluated at any instant of time from the dynamic system configuration. Thus, in this section the basic kinematic aspects of a lubricated revolute joint are presented [11].

In regard to Fig. 2, the eccentricity vector $\mathbf{e}$, which connects the centers of the bearing and journal, is calculated as:

$$\mathbf{e} = \mathbf{r}_j^p - \mathbf{r}_i^p \quad (18)$$

where both $\mathbf{r}_j^p$ and $\mathbf{r}_i^p$ are described in global coordinates with respect to the inertial reference frame [18]. The magnitude of the eccentricity vector is evaluated as:

$$e = \sqrt{\mathbf{e}^T \mathbf{e}} \quad (19)$$

An unit vector, $\mathbf{r}$, along the eccentricity direction can be defined as:

$$\mathbf{r} = \frac{\mathbf{e}}{e} \quad (20)$$

The unit vector has the same direction as the line of centres of the bearing and the journal, here denoted as radial direction, while tangential direction is obtained by rotating vector $\mathbf{r}$ 90° in counter clockwise direction.

Fig. 2. Generic configuration of a lubricated revolute joint

The parameter ε which defines the eccentricity ratio is obtained from distance between the bearing and journal centres divided by radial clearance, that is:

$$\varepsilon = \frac{e}{c} \quad (21)$$

The parameter $\dot{\varepsilon}$ can be obtained by differentiating equation (18) and divide the result by radial clearance:

$$\dot{\varepsilon} = \frac{\dot{e}}{c} \quad (22)$$

The line of centres between the bearing and journal makes an angle γ with X -axis as shown in Fig. 2. Since the unit radial vector $\mathbf{r}$ has the same direction as the line of centres, the angle γ can be defined as:

$$\begin{bmatrix} \cos \gamma \\ \sin \gamma \end{bmatrix} = \begin{bmatrix} r_x \\ r_y \end{bmatrix} \quad (23)$$

Thus:

$$\gamma = \tan^{-1} \frac{r_y}{r_x} \quad (24)$$

Finally, the parameter $\dot{\gamma}$ can be obtained by differentiating equation (24) with respect to the time, yielding:

$$\dot{\gamma} = \frac{e_x \dot{e}_y - \dot{e}_x e_y}{e^2} \quad (25)$$

In the present work, the relative journal-bearing motion characteristics are known from the dynamic configuration of the system. Afterwards the fluid force, from the pressure distribution in the lubricant, is calculated. Thus, since all the state of variables are known from dynamic analysis of the mechanical system, the hydrodynamic forces given by equations (16) and (17) can be evaluated and introduced as external generalized forces into the system's equations of motion.

V. Equations of Motion

The equations of motion for a mechanical system subjected to holonomic constraints can be stated in the form [18]:

$$\begin{bmatrix} \mathbf{M} & \Phi_q^T \\ \Phi_q & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \gamma \end{Bmatrix} \quad (26)$$

with the reference frame placed at the centre of mass for each body, $\mathbf{M}$ is the system mass matrix, Φ_q is the Jacobian matrix of constraint equations, the vector $\ddot{\mathbf{q}}$ contains the generalized state accelerations, λ is the vector that contains the Lagrange multipliers, $\mathbf{g}$ is the vector of generalized forces and γ is the vector of quadratic velocity terms. Equation (26) is formed as a combination of the equations of motion and kinematic constraint equations, often referred to as a mixed set of differential and algebraic equations.

A set of initial conditions (positions and velocities) is required to start the dynamic simulation. The selection of the appropriate initial conditions plays a key role in the prediction of the dynamic performance of mechanical system. In the present work, the initial conditions are based on the results of kinematic simulation of mechanisms in which all the joints are assumed to be ideal, that is, without clearance. The subsequent initial conditions for each time step in the simulation are obtained in the usual manner from the final conditions of the previous time step. In order to stabilize or keep under control the constraints violation, equation (26) is solved using the Baumgarte stabilization technique [19]. In turn, the integration process is performed using a predictor-corrector algorithm with both variable step size and order [20].

VI. Application Example

In order to illustrate the efficiency and accuracy of the methodology presented in this work, a slider-crank mechanism is used as an application example (see Fig. 3).

The slider-crank mechanism consists of four rigid bodies, including ground, two ideal revolute joints and one ideal translational joint. A lubricated revolute joint connects the connecting-rod and sliding block. This is an example of a dynamically loaded journal-bearing in so far as load varies in both direction and magnitude. This revolute joint is modeled with the hydrodynamic model presented in sections II and III of this work.

Fig. 3. Slider-crank mechanism with a lubricated revolute joint between the connecting and slider

The crank is driven with a constant angular velocity equal to 5000 rpm.

The dimensions and inertia bodies' properties are listed in Table I, and Table II lists the dynamic properties for the simulation.

TABLE I
GOVERNING PROPERTIES FOR THE SLIDER-CRANK MECHANISM

Body Nr.	Length [m]	Mass [Kg]	Moment of inertia [Kg·m ²]
2	0.05	0.30	1.0×10^{-4}
3	0.12	0.21	2.5×10^{-4}
4	-	0.14	-

TABLE II
PARAMETERS USED IN THE DYNAMIC SIMULATION

Bearing radius	10.0 mm
Journal radius	9.8 mm
Dynamic fluid viscosity	400 cP

The performance of the slider-crank mechanism is quantified by plotting the following dynamic parameters, the slider acceleration, the reaction force produced by the fluid on the lubricated joint and driving crank moment. Additionally, the journal centre orbit inside the bearing boundaries is also plotted. The time interval used corresponds to two complete crank rotations.

The results are compared to those obtained when the system is modeled with all ideal joints, as illustrated in Figs. 4 to 7.

From Figs. 4-7 it is clear that the results concerning to the slider acceleration, joint reaction and crank moment are of the same order as those obtained with ideal joint simulation.

The smooth curve obtained for reaction force is propagated throughout the mechanical system until the crank moment.

Furthermore, it can be observed that, the first and the second crank rotation show the same results which indicate that the system has reached a good operation conditions. This can be confirmed by the orbit of the journal centre relative to the bearing centre, in which the journal moves far away from the bearing wall, meaning that there is always a minimum film lubricant in between the two bodies avoiding the metal-to-metal contact (see Fig. 7). In fact, a hydrodynamic fluid film journal bearing exhibits damping effect that plays a very important role in the stability of this kind of journal-bearing.

Fig. 4. Slider acceleration

Fig. 5. Joint reaction force

Fig. 6. Crank reaction moment

Fig. 7. Journal centre trajectory inside of the bearing

VII. Concluding Remarks

A general methodology has been presented in this research work for modeling and evaluating the forces produced by a dynamic journal-bearing in mechanical systems. In an application for slider-crank mechanism, the reaction moment necessary to drive the crank with a constant angular velocity is of the same order as the case of an ideal joint, meaning that the use lubricant at the machine joints is an effective way to ensuring better performance. The crank moment required to maintain constant the crank angular velocity is very smooth, meaning that the global motion of the slider-crank mechanism with a lubricated joint is periodic or regular. The lubricant acts like a nonlinear spring-damper in so far as lubricated journal-bearing absorbs some of the

energy produced by the slider when it accelerates or decelerates, which results in lower reaction moment compared to ideal joints. Indeed, the lubricant introduces effective stiffness and damping to the slider-crank mechanism.

The quantitative solutions presented in this work are quite useful in so far as they are given in an analytical form with all the inherent advantages over numerical solutions. The hydrodynamic model for lubricated revolute joints in mechanical systems is numerically efficient and fast because the pressure distribution does not need to be evaluated. Further, the methodology is easy and straightforward to implement in a computational code because resultant forces due to the fluid action are in explicit form. Some numerical difficulties can be observed if either the fluid viscosity is very low or the radial clearance of the journal-bearing is too large, which leads to large eccentricities and consequently the system becomes stiff. In general, the present work is expected to be of great use in the analysis and design of mechanical systems containing lubricated revolute joints. Further, this formulation is conducted in a general way, which makes the model easy to implement in any mechanical system analysis.

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