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Macro vs Micro Limit Analysis models for the seismic assessment of in-plane masonry walls made with quasi-periodic bond types

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Abstract

Masonry bond patterns can considerably affect the seismic performance of in-plane walls. Although several numerical and experimental works addressed this topic, few attempts tried to investigate such an issue using analytical formulations. This paper aims to compare macro and micro limit analysis models investigating masonry walls arranged with different bond types, namely Running, Flemish and English. A dataset involving 81 combinations is generated by varying geometrical (panel aspect ratio, block aspect ratio, bond type) and mechanical (friction coefficient) parameters. Finally, one-way and two-way factor interactions are used to evaluate how each parameter affects the horizontal load multiplier and assess matching among the two adopted formulations.

Keywords: Bond Pattern; Limit Analysis; Parameter Influence, In-Plane Masonry Wall.

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1. Introduction

Historical Masonry Structures (HMS) are often constituted by the assemblage of blocks having various bond patterns. While the influence of the component strength on masonry is relatively well studied, few studies have been focused on the influence of masonry texture properties (Sharma et al. 2021; Shaqfa and Beyer 2022; Zhang and Beyer 2019; Stepinac et al. 2020). To fully understand the influence of different bond types, investigations through parametric

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analyses using advanced numerical or analytical strategies are required. Structural simulations are typically performed with Finite Element Method (FEM) (Szabó et al. 2021; da Silva and Milani 2022; Dauda et al. 2021) or Discrete Element Method (DEM) (Gonen and Soyoz 2021; Bui et al. 2017; Gonen et al. 2021; Kim et al. 2021; Funari et al. 2021). FEM allows a more versatile application as masonry can be represented either through a homogeneous equivalent media (Fortunato et al. 2017) (designated as macro-modelling) or by a discrete representation of units and joints (Funari et al. 2022) (designated as micro-modelling). Typically, in a DEM-based discontinuum analysis, masonry constructions are represented via a system of rigid or deformable distinct polyhedral blocks that can interact based on the point contact hypothesis (Pulatsu et al. 2019). In this context, Malomo et al. (Malomo et al. 2019) conducted parametric DEM analyses on masonry walls with different bond patterns (Flemish, English, Dutch cross-bond, header and running bonds). As an alternative to sophisticated numerical approaches, analytical approaches based on limit analysis (LA) theorems at both micro (Portioli et al. 2014; Gilbert et al. 2006; Cascini et al. 2018) and macro scales (Funari et al. 2020; Casapulla et al. 2021; Funari et al. 2020) are extensively adopted, even though few studies investigated the influence of the masonry bond type.

In particular, being a motivation for this work, it is worth mentioning the research developed by Rios et al. (2022). The authors investigated the effects of different geometrical (panel ratio, block ratio, and bond type) and mechanical (friction ratio) parameters on the in-plane structural response of dry-stack masonry panels. The analytical simulation was performed using a kinematic upper bound micro LA model with an associative flow rule.

This brief literature survey underlines the limited investigation of brickwork bonds using LA formulations. In particular, studies comparing micro and macro strategies and assessing their pros and cons remain scarce. To this end, adopting micro and macro LA formulations, the influence of typically employed periodic and quasi-periodic bond types (i.e. Running, Flemish, English) on the in-plane seismic response of masonry walls is investigated. The paper is divided as follows. Section 2 briefly describes both macro and micro LA formulations. Section 3 presents the design of the parametric analysis. Section 4 discusses the simulations' outcomes. Finally, relevant conclusions are drawn in Section 5.

2. Limit analysis formulations

In this section, macro and micro LA formulations are described. Macro LA is formulated according to the model proposed in (Casapulla et al. 2021), where the frictional resistance definition proposed in (M.F. Funari et al. 2022) for non-periodic masonry is generalised for quasi-periodic bond types. Micro LA formulation is formulated according to the pioneering work developed in (Gilbert et al. 2006). Both approaches are implemented in a customised code in the Java programming language, linked with a masonry pattern generator.

2.1. Macro Limit Analysis formulation

The in-plane sliding-rocking failure mechanism of unreinforced masonry structures, through macroblock LA, has been extensively investigated in the literature (Funari et al. 2021; Casapulla et al. 2021; Casapulla et al. 2014; Colombo et al. 2022). As shown in Fig. 1, the sliding-rocking mechanism is pre-defined, and the equation of equilibrium can be formulated employing the virtual work principle in which the only unknown is the horizontal load multiplier. The external virtual work contains both the overturning as well as the stabilising works performed by the inertial forces, whereas the internal work is derived from the friction force at contact interfaces:

$$\begin{aligned} \delta W_{ext} &= \lambda \cdot W_{OBC} \cdot \delta_{O,OBC} - W_{OBC} \cdot \delta_{S,OBC} \\ \delta W_{\text{int}} &= F_{real} \cdot \delta_{S,f} \end{aligned} \tag{1}$$

where W_{OBC} is the inertial force arising from the self-weight of the macro-block OBC, $\delta_{O,OBC}$ and $\delta_{S,OBC}$ are the virtual overturning and stabilising displacements of the centre of gravity of the macro-block, and F_{real} is the frictional resistance generated by the wall. Regarding the internal work, it is worth remarking that the failure mechanisms often involve mix-mode sliding-rocking with consequently uplifting of the blocks that reduce the number of the bed joints in full contact. In order to consider this phenomenon and compute the actual frictional resistance, the solution proposed in (Casapulla et al. 2021) is adopted, where the frictional force is defined as a function of the crack inclination angle α_c. This is given by:

$$F_{real} = W_{OAB} \cdot \mu \cdot \left(1 - \frac{\alpha_c}{\alpha_b}\right) = \frac{\left(\bar{H} - Z_o\right)^2}{2} \cdot \tan\left(\alpha_b\right) \cdot t_w \cdot \gamma \cdot \mu \cdot \left(1 - \frac{\alpha_c}{\alpha_b}\right) \tag{2}$$

where t_w is the thickness of the in-plane wall, μ is the frictional coefficient, γ is the specific weight of the masonry, α_c is the actual crack inclination and α_b is the crack inclination upper threshold, which in the case of Running bond type is the function of the block aspect ratio:

$$\tan\left(\alpha_b\right) = \frac{v}{h} \tag{3}$$

Here, v and h are half-width and height of the unit blocks, respectively.

Since Eq. (3) may only be adopted for Running patterns, an alternative solution to define α_b for different quasi-periodic bond types is introduced next (Fig. 1).

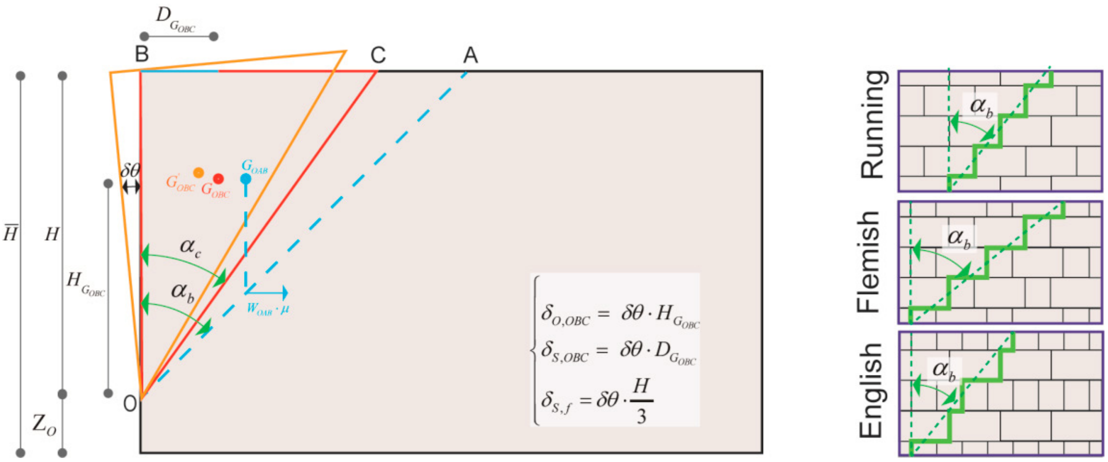

Fig. 1. Kinematic description of the sliding-rocking mechanism for an in-plane shear wall and definition of α_b for quasi-periodic bond patterns

As proposed in (M.F. Funari et al. 2022), in order to compute the upper thresholds of crack inclination, one can refer to a representative masonry pattern window (RMPW) and calculate α_b accordingly (Fig. 1):

$$\tan\left(\alpha_b\right) = \frac{\sum_{i=1}^{n_c} v_i}{\left(n_c + 1\right) \cdot h} \tag{4}$$

It is worth remarking that, in this case, n_c refers to the number of courses inside the RMPW, and v_i is the i-th horizontal segments of the structured path UP-RIGHT-UP-RIGHT.

According to the analytical formulation proposed in (Casapulla et al. 2021), once the crack inclination upper threshold is defined, it is possible to apply the macroblock formulation, which allows computing horizontal load multiplier and geometry of the failure mechanisms. In particular, the horizontal load multiplier can be evaluated by equating external and internal virtual work and solving for λ. According to the upper-bound theorem of the LA, the computation of the horizontal load multiplier requires the solution of a constrained minimisation problem in which the parameters defining the failure mechanism's geometry, i.e. α_c and Z_O, are adopted as variables to explore all the panorama of possible solutions:

$$\begin{aligned} \text{minimise:} \quad & \lambda \\ \text{subject to:} \quad & Z_O \le \bar{H} \\ & \alpha_c \le \alpha_b \end{aligned} \tag{5}$$

where Z_O is the height position of the pivot point and $\bar{H}$ is the total height of the wall.

2.2. Micro Limit Analysis formulation

In the micro LA formulation, dry-stack assemblage is represented by rigid blocks connected by frictional contact interfaces with a non-associative flow rule, with zero dilation (Fig. 2).

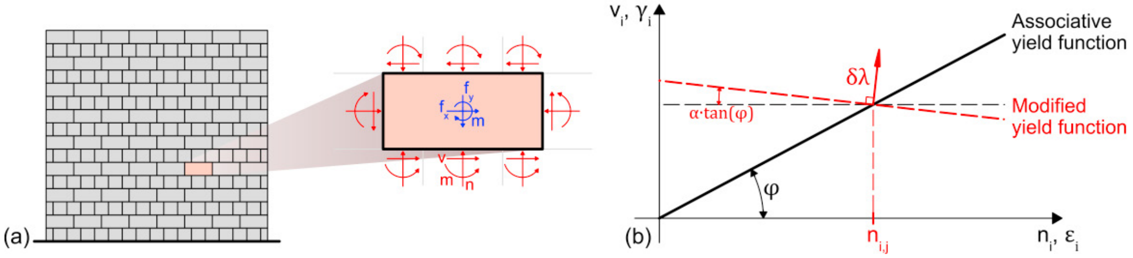

Fig. 2. (a) Dry-stack masonry wall; (b) Modification of yield function for the non-associative solution

The solution scheme proposed by Gilbert et al. (2006), involving a non-associative frictional flow rule consisting of sequential solutions of linear programs, is adopted (Fig. 2b). At each iteration a linear program is defined as follows:

$$\begin{aligned} \text{Maximize} \quad & \lambda \\ \text{Subject to} \quad & \mathbf{Bq} - \lambda \mathbf{f}_L = \mathbf{f}_D \\ & \mathbf{C}^T [\mathbf{q} - \mathbf{c}] \le 0 \end{aligned} \tag{6}$$

where λ is the load multiplier and $\mathbf{q}$ the vector of unknown contact forces, $\mathbf{f}_L$ and $\mathbf{f}_D$ are the live and dead loads, $\mathbf{c}$ is the cohesion vector, $\mathbf{B}$ and $\mathbf{C}$ are the equilibrium and yield constraints matrices. The first constraint represents the equilibrium of forces, whereas the second is the condition for yielding (failure) of the interfaces.

The yield conditions are updated at each iteration based on the normal forces at the previous iterations:

$$\begin{aligned} & v_{i,j} \le c_i + \alpha \cdot \mu_i \cdot n_{i,j} \\ & c_{i,j+1} = c_i^0 + (1+\alpha) \cdot \left(\beta \cdot n_{i,j} + (1-\beta) \cdot n_{i,j-1} \right) \cdot \tan(\varphi_i) \end{aligned} \tag{7}$$

Here $v_{i,j}$ and $n_{i,j}$ are the shear and normal forces of the *i-th* interface at the *j-th* iteration. α and β are algorithm parameters set to 0.01 and 0.6, respectively. The reader can refer to Gilbert at al. (2006) for more details about the iterative solution algorithm.

3. Parametric analysis design

As stated before, this work aims to understand the in-plane seismic behaviour of masonry walls arranged with different bond types, namely Running, Flemish and English. The discussed wall specimens are constrained at the base with simple support, while the horizontal load is assumed proportional to the mass (Rios et al. 2022). According to Rios et al. (2022), a full fractional dataset involving all the 81 combinations of the input parameters is generated by varying geometrical (panel aspect ratio, block aspect ratio, bond type) and mechanical parameters (friction coefficient). One can note that the geometrical parameters are assumed to be consistent with Rios et al. (2022), whereas more reasonable values of the frictional parameters have been assumed. Tab. 1 summarises the range of values adopted for each parameter.

Table 1. 3. Parametric analysis design: parameters' values

Panel aspect ratio (H/B)	$[0.72/1.44;\ 1.44/1.44;\ 2.88/1.44]$
Block aspect ratio (b/h)	$[0.24/0.06;\ 0.12/0.06;\ 0.06/0.06]$
Bond type	[*Running*; *Flemish*; *English*]
Friction	$[0.50;\ 0.65;\ 0.80]$

4. Results

The 81 simulations have been performed in both micro and macro limit analysis and the resulting horizontal load factors have been collected. The effect of input parameters on the results and the relation of the two approaches have been investigated with the analysis of variance (ANOVA) approach, where the average effect and its standard deviation are calculated for one-parameter (linear factor) or the joint effect of two or more parameters (two- or multiple-way factor) as:

$$
\begin{aligned}
\bar{\lambda}_{i...} &= \sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\lambda_{ijkl} \qquad & s_{\bar{\lambda}_{i...}} &= \sqrt{\frac{1}{n}\cdot\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{d}\left(\lambda_{ijkl}-\bar{\lambda}_{i...}\right)^2} \\
\bar{\lambda}_{ij..} &= \sum_{k=1}^{c}\sum_{l=1}^{d}\lambda_{ijkl} \qquad & s_{\bar{\lambda}_{ij..}} &= \sqrt{\frac{1}{n}\cdot\sum_{k=1}^{c}\sum_{l=1}^{d}\left(\lambda_{ijkl}-\bar{\lambda}_{ij..}\right)^2}
\end{aligned}
\tag{8}
$$

where $\bar{\lambda}_{i...}$ is the mean value of the load factor for all the cases, with the first input parameter having the value of i. $\bar{\lambda}_{ij..}$ is the mean value, where the first two input parameters have the values of i and j, respectively.

In Fig. 3, parameters' individual effects are shown for both micro and macro LA models. The two formulations show good agreement in terms of both mean and standard deviation values.

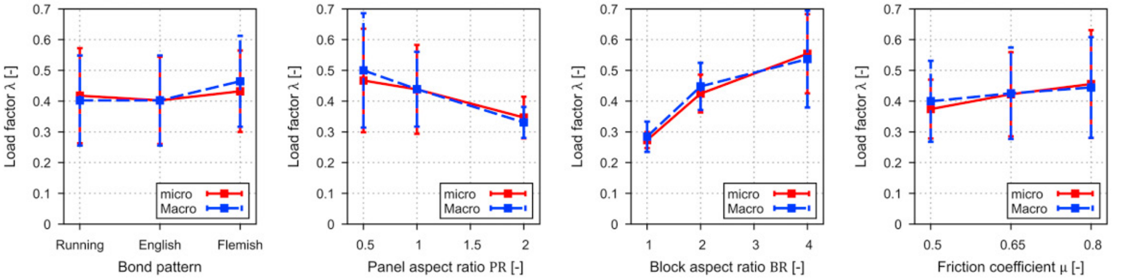

Fig. 3. Linear interaction: micro and macro LA

Fig. 3 remarks small differences in terms of horizontal multiplier for the three analysed bond types. As expected, the higher friction coefficient tends to increase the horizontal capacity of the in-plane walls but generates more scattered results. Panel aspect ratio (PR=H/B) and the block aspect ratio (BR=b/h) strongly influence the horizontal load multiplier. In both cases, the standard deviation is very sensitive to PR and BR.

The macro and micro LA two-way factor interactions are reported in Fig. 4 and Fig. 5, respectively. The good agreement among the LA formulations is again confirmed. It is worth remarking that in macro LA, α_b was set equal for the Running and English bonds, so the corresponding mean and standard deviation values are the same. Micro LA underlines slight differences among the bond types that could not be caught with macro LA, where the Flemish bond shows significantly higher mean values than the other bond types. The difference decreases with the increase of PR and BR but stays constant with the change in friction coefficient. Macro LA presents a small scatter for the BR-PR interaction plots, while in micro LA the scattering significantly grows with the increase of BR and decrease of PR. For the friction-PR interaction, macro LA result's trend is not influenced by the value of PR, meaning the two parameters are not correlated. On the contrary, micro LA visualises an evident two-way friction-PR interaction. The friction-BR interaction shows excellent agreement between macro and micro LA models, where the mean values tend to increase within the BR. Finally, standard deviation values tend to be variable with PR and BR.

5. Final Remarks

Parametric analysis was performed on in-plane masonry walls by varying geometrical (panel aspect ratio, block aspect ratio, bond type) and mechanical (friction coefficient) parameters. The resulting horizontal load factors have been compared by adopting the ANOVA approach. A good agreement between macro and micro LA is observed. The panel and block aspect ratios significantly influenced the horizontal load factors, while the friction coefficient has a slight effect.

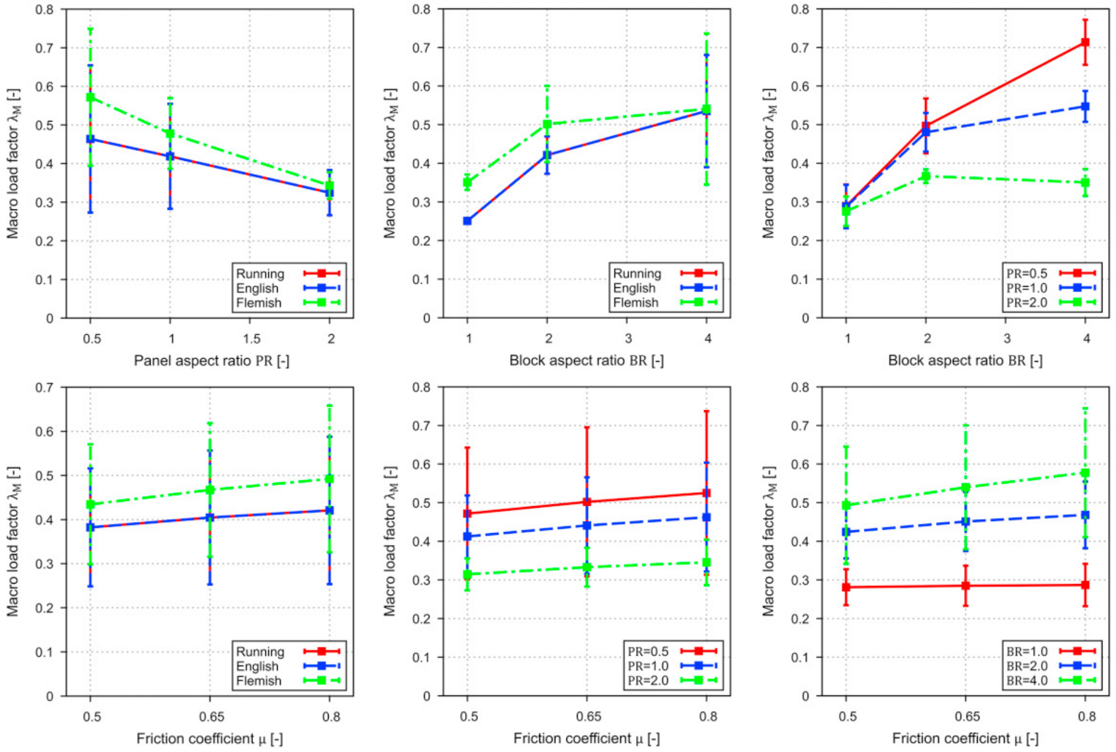

Fig. 4. Two-way interaction plots: macro LA

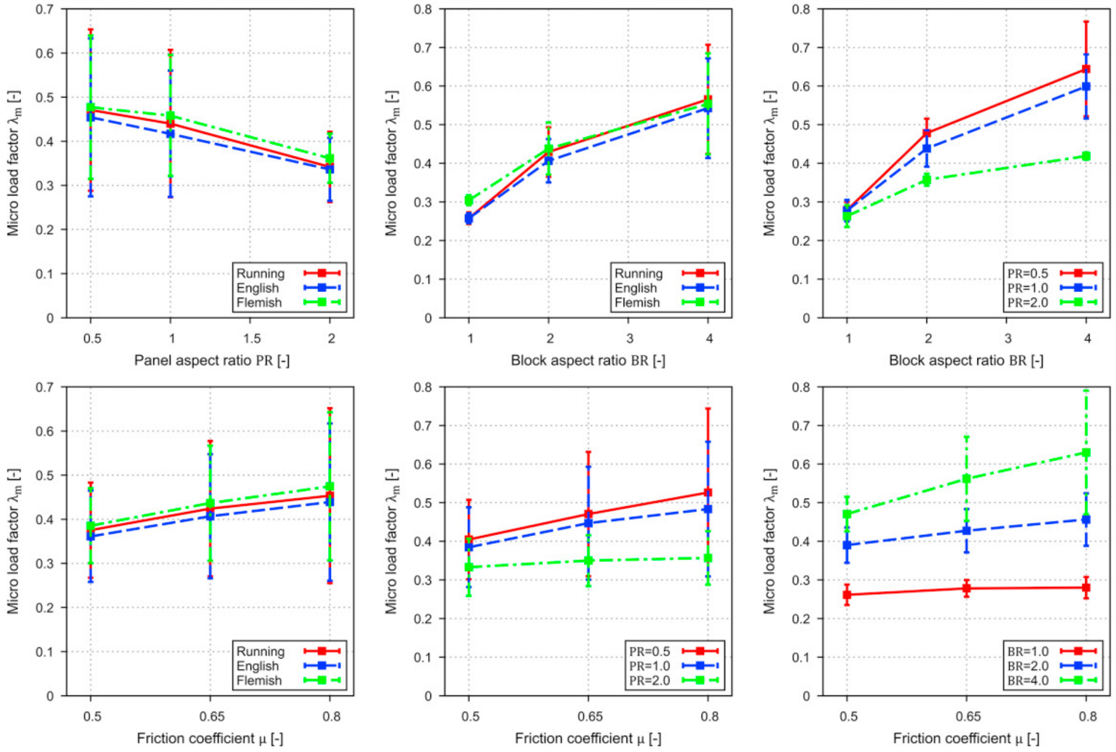

Fig. 5. Two-way interaction plots: micro LA

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