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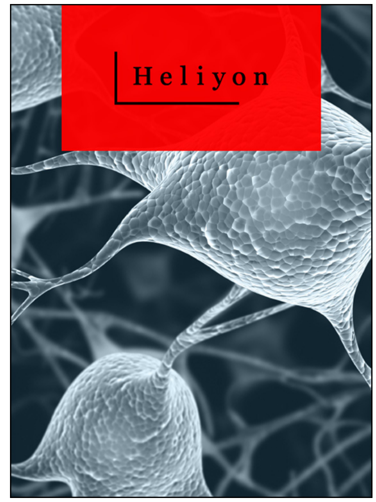

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PII: S2405-8440(24)00022-7
DOI: https://doi.org/10.1016/j.heliyon.2024.e23991
Reference: HLY e23991

To appear in: *Heliyon*

Received date: 20 May 2023
Revised date: 25 December 2023
Accepted date: 2 January 2024

Please cite this article as: V. Kuppusamy, M. Shanmugasundaram, P.B. Dhandapani et al., Addressing a decision problem through a bipolar Pythagorean fuzzy approach: A novel methodology with application in digital marketing, *Heliyon*, 9, e23991, doi: https://doi.org/10.1016/j.heliyon.2024.e23991.

Highlights

- We integrate bipolarity and Pythagorean fuzzy theory in marketing as a novel methodology.
- The integration of bipolarity redefines decision-making in digital marketing.
- We enhance the precision of digital marketing strategies by employing bipolarity.
- Our advantage lies in the reshaping of uncertain decisions through a bipolarity-infused approach.
- We apply our methodology to enhance digital marketing strategies on the Facebook platform.

Addressing a decision problem through a bipolar Pythagorean fuzzy approach: A novel methodology with application in digital marketing

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ARTICLE INFO

Keywords:
Bipolar Pythagorean matrices
COPRA, CRITIC, MAIRCA, MARCOS methods
Fuzzy theory
Open soft matrices
Score function

ABSTRACT

Decision-making in real-world scenarios often faces the challenge of uncertainty. Traditionally, fuzzy theory has been a means to represent and navigate such uncertainty. In this study, we propose a pioneering approach that incorporates a bipolarity analysis into multi-criteria decision-making processes, with a focus on its application in digital marketing. The proposal allows the analysis to be more encompassing by considering both the positive and negative dimensions of data, leading to better-informed decisions. A cornerstone of our contribution is integrating bipolarity into Pythagorean fuzzy soft matrices, a fresh mathematical framework that broadens the utility of bipolar fuzzy theory. Through rigorous computational experimentation, we determine the prioritization of alternatives, ultimately identifying the most effective strategy for digital marketing platforms. In our study, Facebook emerges as the foremost platform for implementing digital marketing strategies. When compared to existing techniques, our approach showcases significant advantages, underlining its potential to improve decision-making in uncertain scenarios. Our research offers profound insights for businesses aiming to refine their digital marketing strategies in an ever-evolving digital landscape.

1. Introduction

In a world characterized by pervasive uncertainty, the usage of fuzzy sets provides a framework to handle this uncertainty, as initially presented in [1]. The subsequent exploration of the role of fuzzy sets in topology, crucial for grasping their functionality, was tackled later [2]. A groundbreaking advance was the evolution of bipolar fuzzy sets, a mechanism to differentiate between positive and negative data, as highlighted in [3]. Moreover, the nuanced role of fuzzy sets has been explored within broader system dynamics, particularly emphasizing their impact in bio-economic and industrial sectors through control methodologies [4].

Building upon the foundational understanding of bipolar fuzzy sets, research directions have diversified. For instance, certain studies have considered into numerical solutions of differential systems by considering a pure hybrid fuzzy neutral delay theory, thus highlighting the broad applications of fuzzy theories in complex mathematical computations [5]. Furthermore, an extension of bipolar fuzzy sets emphasizes their extended range, allowing membership degrees to span the interval [-1, 1] from the traditional interval [0,1].

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The dynamic field of digital marketing is intrinsically laden with uncertainties and bipolar tendencies. The task of pinpointing the ideal social media platform (SMP) often encounters hurdles due to the uncertainties surrounding aspects such as advertising expenses, audience demographics, engagement metrics, return on investment (ROI), and user behavior. These complexities necessitate methodologies capable of integrating fuzzy and crisp solutions, particularly in evolving scenarios such as those modeled by fractional pandemic models [6, 7].

Traditional methodologies, anchored in precise data, occasionally fall short in navigating these aspects, as outlined in [8]. One can champion a methodology that seamlessly integrates the mentioned uncertainties, leveraging the bipolar valued fuzzy sets discussed in [9]. The methodology can broaden such uncertainties to encapsulate intuitionistic fuzzy soft matrices and introduce innovative operations in the soft set paradigm [10–14]. An intuitionistic fuzzy matrix is a pair of fuzzy matrices, namely, membership and non-membership functions capturing the nuances of universal elements in relation to certain attributes (characteristics).

The pioneering work presented in [15] applied bipolar intuitionistic fuzzy soft sets to decision-making problems. The emergence of the complex proportional assessment (COPRAS) method, proposed in [16], provided a new strategy for handling decision-making frameworks. The COPRAS method was later adapted to intuitionistic fuzzy soft sets in multi-criteria decision-making problems, as discussed in [17, 18]. Similarly, the introduction of the intercriteria correlation (CRITIC) method in [19] offered a novel strategy for determining attribute weights in bipolar Pythagorean fuzzy (BPF) environments, explored further in [20–22]. A Pythagorean fuzzy set is a generalization of an intuitionistic fuzzy set.

In the evaluation of alternative preferences, the measurement and ranking based on the compromise solution (MARCOS) method, developed in [23], provides a valuable tool. Additionally, the multi-attribute ideal-real comparative analysis (MAIRCA) method, introduced in [24], serves as an effective approach for estimating the gap between ideal and empirical detections.

With the preceding backdrop, our study is anchored on four objectives as follows: (i) to introduce and articulate the BPF soft regular generalized matrices; (ii) to present an allied topology, expanding the horizons of the existing techniques; (iii) to delineate operations specifically designed for these matrices, priming them for tangible real-world applications; and (iv) to craft and expose algorithms deeply rooted based on the proposed approach with a spotlight on digital marketing deployments. A distinctive feature of our research domain is the detailed exploration of BPF soft regular generalized matrices and their operations – a domain unexplored in prior academic endeavors. The significance of our novel approach becomes even more pronounced within the context of digital marketing, an arena constantly navigating nuances and uncertainties. Our proposed approach and the resultant algorithms hold the potential to redefine the decision-making trajectory in digital marketing, bestowing it with fortified resilience in the face of uncertain data. Also, the growing exploration of topological structures in information landscapes reveals new pathways and insights, as demonstrated in studies such as [25, 26].

Our new approach offers a decision-making paradigm in digital marketing, integrating fuzzy theory, bipolarity, and soft settings. This approach allows for the handling of uncertain or imprecise data and the accommodation of multiple criteria simultaneously through matrix operations, providing a promising contribution to decision-making methodologies. Therefore, our research addresses a crucial gap in the literature and advances the current understanding of decision-making under uncertainty.

The article is planned as follows. In Section 2, we introduce the preliminaries and notations ensuring clarity in our exploration. In Section 3, four algorithms are described tailored for digital marketing, as applications of the bipolar Pythagorean fuzzy soft matrix (BPFSM) topology. Section 4 presents a multi-criteria decision-making application utilizing the bipolar Pythagorean fuzzy soft regular generalized matrices, underscored by various methods infused with BPF data. Lastly, Section 5 offers a comprehensive discussion on our exploration of bipolarity in Pythagorean fuzzy soft regular generalized matrices, drawing conclusions, limitations, and potential avenues for future research.

2. Preliminaries

The present article draws upon topological concepts, particularly from fuzzy set theory and its bipolar extensions. To ensure clarity, this section begins by introducing a selection of notations, as detailed in Table 1, which are fundamental to our work. Subsequently, we provide foundational concepts that are consistently employed throughout the document.

Table 1
Acronyms and notations used in the present article.

Acronym/notation	Definition
AAI	Anti-ideal solution
AI	Ideal solution
BPF	Bipolar Pythagorean fuzzy
BPFSM	Bipolar Pythagorean fuzzy soft matrix
$\text{BPFSM}_{m\times n}$	Set of all $m \times n$ BPF soft matrices
$\text{BFSM}_{m\times n}$	Set of all $m \times n$ bipolar fuzzy soft matrices
BPFSCM	BPF soft closed matrix
BPFSMTS	BPF soft matrix topological space
BPFSRCM	BPF soft regular closed matrix
BPFSRGCM	BPF soft regular generalized closed matrix
BPFSRGOM	BPF soft regular generalized open matrix
BPFSOM	BPF soft open matrix
BPFSROM	BPF soft regular open matrix
$\text{FSM}_{m\times n}$	Set of all $m \times n$ fuzzy soft matrices
NDM	Normalized decision matrix
ROI	Return on investment
SMP	Social media platform
WNDM	Weighted normalized decision matrix

2.1. Notations

Prior to delving deeper, we lay out the notations pivotal to our discussions. These notations provide the scaffolding for the terminologies inherent in our proposed algorithms. Table 1 offers a concise guide, introducing concepts like the BPFSM, soft regular closed and open matrices, and the normalized decision matrix (NDM). Each notation is imperative for the effective conceptualization of the algorithms and their applications.

2.2. Topological concepts

Topology, which is intrinsically concerned with continuity and boundaries, is fundamental for understanding various mathematical and information science structures. The advancements of recent literature highlight the escalating significance of topological concepts in information systems. Within this work, understanding topology is essential to fully grasp the intricacies of bipolar fuzzy matrices. To further assist the reader, in the next subsection, we present all requisite definitions, particularly emphasizing the topological concepts at the heart of our exploration. We recommend that readers familiarize themselves with this foundational content to enhance their understanding of the subsequent sections.

2.3. Definitions

Next, some definitions are provided to facilitate the reading. Let $U = \{u_1, \dots, u_m\}$ be the universal set, $E = \{e_1, \dots, e_n\}$ a set of characteristics (attributes or criteria), A a subset of E ($A \subseteq E$), $\emptyset$ the empty (null) set, τ a topology on $\{U, E\}$, and the trinity $\{X, \tau, E\}$ be a topological space over the space set X where the attributes E are defined. For example, X can be five SMPs as $X = \{\text{Facebook, Instagram; LinkedIn; WhatsApp; YouTube}\}$ with attributes $E = \{\text{advertisement cost; demography; marketing goal; monthly active users; product cost}\}$.

Definition 2.1 (Fuzzy soft set and matrix form). *The pair $\{F, A\}$ is called a fuzzy soft set over U whenever F is a mapping given by $F\colon A \mapsto I^U$, with I^U denoting the collection of all fuzzy subsets of U. If $\{F, A\}$ is a fuzzy soft set in the fuzzy soft class $\{U, E\}$, then $\{F, A\}$ can be represented in a matrix form with $A = [a_{ij}]$, where $a_{ij} = \mu_j(u_i)$, if $e_j \in A$; otherwise, that is, if $e_j \notin A$, $a_{ij} = 0$, with $\mu_j(u_i)$ being the membership of u_i in the fuzzy set $F(e_j)$, for $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, n\}$.*

Definition 2.2 (Intuitionistic fuzzy soft set and its matrix form). *Let $\{F, A\}$ be an intuitionistic fuzzy soft set in the intuitionistic fuzzy soft class $\{U, E\}$, that is, $\{F, A\}$ is a pair of fuzzy matrices with membership and non-membership functions. Then, F is an intuitionistic fuzzy matrix that maps elements in A to membership and non-membership values μ and ν, capturing the nuances of elements in U in relation to attributes in A. Thus, $\{F, A\}$ can be represented in a matrix form with $A = [a_{ij}]$, where $a_{ij} = \{\mu_j(u_i), \nu_j(u_i)\}$ if $e_j \in A$; otherwise, $a_{ij} = \{0, 1\}$. In this context, for any u_i in U and e_j in A, $\mu_j(u_i)$ is the membership of u_i in the intuitionistic fuzzy set $F(e_j)$, while $\nu_j(u_i)$ is its non-membership, with $0 \leq \mu_j(u_i) + \nu_j(u_i) \leq 1$, for $i \in \{1, \ldots, m\}$ and $j \in \{1, \ldots, n\}$.*

Definition 2.3 (Pythagorean fuzzy soft set and its matrix form). *Let $\{F, A\}$ be a Pythagorean fuzzy soft set in the Pythagorean class $\{U, E\}$, that is, $\{F, A\}$ is a generalization of an intuitionistic fuzzy set. Then, $\{F, A\}$ can be represented in a matrix form with $A = [a_{ij}]$, where $a_{ij} = \{\mu_j(u_i), \nu_j(u_i)\}$, if $e_j \in A$; otherwise, $a_{ij} = \{0, 1\}$, with $\mu_j(u_i)$ being the membership of u_i in the Pythagorean fuzzy set $F(e_j)$ and $\nu_j(u_i)$ the associated non-membership, holding $0 \leq (\mu_j(u_i))^2 + (\nu_j(u_i))^2 \leq 1$, for $i \in \{1, \ldots, m\}$ and $j \in \{1, \ldots, n\}$.*

Definition 2.4 (Bipolar fuzzy soft set and its matrix form). *Let $\{F, A\}$ be a bipolar fuzzy soft set in the bipolar fuzzy soft class $\{U, E\}$, that is, $\{F, A\}$ possesses membership degrees that span the interval [-1, 1] departing from the traditional interval [0,1]. Thus, $\{F, A\}$ can be represented in a matrix form with $A = [a_{ij}]$, where $a_{ij} = \{\mu_j^-(u_i), \mu_j^+(u_i)\}$, if $e_j \in A$; otherwise, $a_{ij} = \{0, 0\}$. Here, $\mu_j^-(u_i)$ denotes the negative membership of u_i in the bipolar fuzzy set $F(e_j)$, and $\mu_j^+(u_i)$ the positive membership, considering $-1 \leq \mu_j^-(u_i) \leq 0, 0 \leq \mu_j^+(u_i) \leq 1$, for $i \in \{1, \ldots, m\}$ and $j \in \{1, \ldots, n\}$.*

Definition 2.5 (BPF set in matrix form). *Let $\{F, A\}$ be a BPF soft set in the fuzzy soft class $\{U, E\}$. Then, $\{F, A\}$ may be formulated as a matrix with $A = [a_{ij}]$, where $a_{ij} = \{\mu_j^+(u_i), \nu_j^+(u_i), \mu_j^-(u_i), \nu_j^-(u_i)\}$, if $e_j \in A$; otherwise, $a_{ij} = \{0, 1, 0, -1\}$, with $\mu_j^+(u_i)$ being the positive membership of u_i in the BPF set $F(e_i)$ and $\mu_j^-(u_i)$ its negative membership, whereas $\nu_j^+(u_i)$ is the positive non-membership of $u_i \in F(e_i)$ and $\nu_j^-(u_i)$ its negative non-membership, for $0 \leq (\mu_j^-(u_i))^2 + (\nu_j^-(u_i))^2 \leq 1$, $0 \leq (\mu_j^+(u_i))^2 + (\nu_j^+(u_i))^2 \leq 1$, $i \in \{1, \ldots, m\}$, and $j \in \{1, \ldots, n\}$.*

Definition 2.6 (BPF soft null and universal matrices). *Given matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ in $BPFSM_{m\times n}$, matrix A is termed a BPF soft submatrix of B, denoted by $A \subseteq B$, if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$ for all $i \in \{1, \ldots, m\}$ and $j \in \{1, \ldots, n\}$. Within $BPFSM_{m\times n}$: (i) a BPF soft empty (null) matrix, denoted as $\emptyset_{m\times n}$, is when all elements are $\{0, 1, 0, -1\}$, indicating no positive membership and full negative non-membership; and (ii) a BPF soft universal matrix, denoted by $U_{m\times n}$, has its elements as $\{1, 0, -1, 0\}$, implying full positive membership and no negative non-membership.*

Definition 2.7 (Operations on the BPFSM set). *Let $A = [a_{ij}]$ and $B = [b_{ij}] \in BPFSM_{m\times n}$, with μ_A, μ_B be their corresponding membership functions, for $i \in \{1, \ldots, m\}, j \in \{1, \ldots, n\}$. Then, we define:*

(i) [Addition] $A + B = C = [c_{ij}]$, with $c_{ij} = \left\{ \max\{\mu_A^+, \mu_A^-, \mu_B^+, \mu_B^-\}, \min\{\nu_A^+, \nu_A^-, \nu_B^+, \nu_B^-\}\right\}$;

(ii) [Subtraction] $A - B = C = [c_{ij}]$, with $c_{ij} = \left\{ \min\{\mu_A^+, \mu_A^-, \mu_B^+, \mu_B^-\}, \max\{\nu_A^+, \nu_A^-, \nu_B^+, \nu_B^-\}\right\}$;

(iii) [Product] $A \times B = C = [c_{ij}]$, where

$$c_{ij} = \left\{ \max_{i,j} \min\{\mu_A^+, \mu_B^+\}, \min_{i,j} \max\{\nu_A^+, \nu_B^+\}, \min_{i,j} \max\{\mu_A^-, \mu_B^-\}, \max_{i,j} \min\{\nu_A^-, \nu_B^-\}\right\}.$$

Here, for each element c_{ij}, the indices i and j run through the entire set of row and column indices of the matrices A and B. This means that each c_{ij} considers the combination of all elements from both matrices;

(iv) [Complement] For $A = [a_{ij}]$, with $a_{ij} = \{\mu_j^+, \nu_j^+, \mu_j^-, \nu_j^-\}$, its complement is $A^c = [a_{ij}^c]$, with $a_{ij}^c = \{\nu_{ij}^+, \mu_{ij}^+, \nu_{ij}^-, \mu_{ij}^-\}$;

(v) [Union] $A \cup B = C = [c_{ij}]$, where $c_{ij} = \{\mu_{ij}^{+C}, \nu_{ij}^{+C}, \mu_{ij}^{-C}, \nu_{ij}^{-C}\}$, with $\mu_{ij}^{+C} = \max\{\mu_{ij}^{+A}, \mu_{ij}^{+B}\}$, $\nu_{ij}^{+C} = \min\{\nu_{ij}^{+A}, \nu_{ij}^{+B}\}$, $\mu_{ij}^{-C} = \min\{\mu_{ij}^{-A}, \mu_{ij}^{-B}\}$, $\nu_{ij}^{-C} = \max\{\nu_{ij}^{-A}, \nu_{ij}^{-C}\}$;

(vi) [Intersection] $A \cap B = C = [c_{ij}]$, with $c_{ij} = \{\mu_{ij}^{+C}, \nu_{ij}^{+C}, \mu_{ij}^{-C}, \nu_{ij}^{-C}\}$, where $\mu_{ij}^{+C} = \min\{\mu_{ij}^{+A}, \mu_{ij}^{+B}\}$, $\nu_{ij}^{+C} = \max\{\nu_{ij}^{+A}, \nu_{ij}^{+B}\}$, $\mu_{ij}^{-C} = \max\{\mu_{ij}^{-A}, \mu_{ij}^{-B}\}$ and $\nu_{ij}^{-C} = \min\{\nu_{ij}^{-A}, \nu_{ij}^{-B}\}$.

Definition 2.8 (BPFSMTS). *A topology τ on $\{U, E\}$ is the family of $BPFSM_{m\times n}$ over $\{U, E\}$, $\tau_{m\times n}$ say, satisfying the following properties: (i) $\emptyset$, $U \in \tau_{m\times n}$; (ii) If $A, B \in \tau_{m\times n}$, then $A \cup B \in \tau_{m\times n}$; and (iii) if $A, B \in \tau_{m\times n}$, then $A \cap B \in \tau_{m\times n}$. The trinity $\{X, \tau_{m\times n}, E\}$ is said to be a BPFSMTS over the space set X.*

As topological structures evolve, we introduce sophisticated constructs like the bipolar Pythagorean fuzzy soft open matrix (BPFSOM), fundamental to advanced topological domain. This introduction sets the stage for nuanced discussions essential in this domain.

The roles of certain sets in such a domain, harmonious with traditional topology, are:

- [Set V] Analogous to subsets in traditional topology, this is a foundational set subject to interior and closure operations.
- [Set G] It represents the open conditions within the topology and defines the interior of set V by forming unions that adhere to specific criteria, reminiscent of open sets in traditional topology.
- [Set K] It has functions similar to closed sets in traditional topology, establishing the boundaries or closure for sets like V. The closure is discerned by intersections of sets within K under defined closed conditions.
- [Set W] This is a distinctive set without a direct analog in traditional topology and serves as a reference, ensuring sets remain compliant with certain topological properties.

These concise roles, extended from usual concepts, navigate through more intricate as non-binary topological relationships.

Definition 2.9 (BPFSOM). *Let $A_{m\times n}$ be a matrix wherein each element signifies a bipolar fuzzy state within a topological space. This matrix, crucial for our subsequent operations and relations, is identified as a BPFSOM.*

Definition 2.10 (Pythagorean Fuzzy soft interior and closure). *In a BPFSMTS $\{X, \tau_{m\times n}, E\}$, for a set $\{V, E\}$: (i) the "interior" is defined as $int\{V, E\} = \bigcup\{\{G, E\} \mid \{G, E\} \subseteq \{V, E\}\}$, where each $\{G, E\}$ is a BPFSRGOM; and (ii) the "closure" is states as $cl\{V, E\} = \bigcap\{\{K, E\} \mid \{V\} \subseteq \{K\}\}$, where each $\{K, E\}$ is a BPFSRGCM. In this context, "int" and "cl" refer to the interior and closure operations, respectively.*

Definition 2.11 (BPF soft regular open matrix —BPFSRGOM). *In a BPFSMTS $\{X, \tau_{m\times n}, E\}$, a set $BPFS_{m\times n}$ defined by $\{K, E\}$ is called a BPFSRGOM if it satisfies the condition $\{K, E\} = int\{cl\{K, E\}\}$.*

Definition 2.12 (BPF soft regular closed matrix —BPFSRGCM). *In a BPFSMTS $\{X, \tau_{m\times n}, E\}$, a set $\{K, E\}$ is called a BPFSRGCM if it satisfies the condition $\{K, E\} = cl\{int\{K, E\}\}$.*

Definition 2.13 (BPFSCM soft closed matrices —BPFSCM). *In a BPFSMTS $\{X, \tau_{m\times n}, E\}$, a set $\{K, E\}$ is considered a BPFSCM if $cl\{K, E\} \subseteq W_{m\times n}$ under the condition that $\{K, E\} \subseteq W_{m\times n}$, where $W_{m\times n}$ is a BPFSROM within the space $\{X, \tau_{m\times n}, E\}$.*

2.4. Properties and operations on BPF soft matrices

Theorem 2.14. *Let $\{V, E\}$ and $\{W, E\}$ be $BPFSRGCM_{m\times n}$. Then, the disjunction of $\{V, E\}$ and $\{W, E\}$ is also a BPFSRGCM in the space $\{X, \tau_{m\times n}, E\}$.*

Remark 2.15. *The conjunction of two $BPFSRGCM_{m\times n}$ is not guaranteed to be a BPFSRGCM in the space $\{X, \tau_{m\times n}, E\}$.*

Theorem 2.16. *Let $\{V, E\}$ be a BPFSRGCM and $\{V, E\} \subseteq \{W, E\} \subseteq cl_{\mathrm{BPFS}}\{V, E\}$. Then, $\{W, E\}$ is a BPFSRGCM in the space $\{X, \tau_{m\times n}, E\}$.*

After understanding the dynamics of the closed matrices, it is equally pivotal to explore the landscape of the open matrices. In essence, their definitions provide a contrasting perspective that enhances our topological elements.

Table 2
Proposed algorithms and their associated method

Algorithm	Method
1	BPF CRITIC
2	BPF CRITIC COPRA
3	BPF CRITIC MARCOS
4	BPF CRITIC MAIRCA

Definition 2.17 (BPFSRGOM). *Consider a BPFSMTS* $\{X, \tau_{m\times n}, E\}$. *A set* $\{K, E\}$ *within this space is termed a BPFSRGOM if its complement* $\{K, E\}^c$ *is a BPFSRGCM, and for any matrix* $G_{m\times n}$ *that is a BPFSRCM satisfying* $\{K, E\} \supseteq G_{m\times n}$, *we have* $int\{K, E\} \supseteq G_{m\times n}$, *indicating that the interior of* $\{K, E\}$ *contains* $G_{m\times n}$.

The properties of the BPFSRGOM bring forward intriguing dynamics, especially when combined with other similar matrices.

Theorem 2.18. *Let* $\{V, E\}$ *and* $\{W, E\}$ *be two BPFSRGOM in the space* $\{X, \tau_{m\times n}, E\}$. *Then, the conjunction of* $\{V, E\}$ *and* $\{W, E\}$ *is also a BPFSRGOM in the space* $\{X, \tau_{m\times n}, E\}$.

Remark 2.19. *The disjunction of any two* $BPFSRGOM_{m\times n}$ *is not guaranteed to be a BPFSRGOM in the space* $\{X, \tau_{m\times n}, E\}$.

Beyond the inherent properties and operations on the presented matrices, it is also essential to understand how we can quantitatively assess them, providing a tangible measure of their characteristics.

Definition 2.20 (Score function). *For any BPFSM, the score function of* $A_{m\times n}$ *is defined as*

$$S(A_{m\times n}) = S([a_{ij}]) = 1 - \left| \frac{(\mu_A^+)^2 - (\nu_A^+)^2 + (\mu_A^-)^2 - (\nu_A^-)^2}{2} \right|,$$

where $S(A_{m\times n}) \in [-1, 1]$.

To summarize, this section has introduced a robust foundation with definitions, theorems, and remarks around the BPF soft matrices. The introduced concepts form the core theoretical background for the algorithms discussed in the next section, demonstrating the practical application of the topological concepts used.

3. Algorithms for BPF soft matrices topology

To prepare the discussions and analyses that follow, we incorporated a comprehensive summary and background in Section 2. The summary and background defined and elaborated upon recurring terms and concepts throughout the article, ensuring a fluid and accessible reading experience. This section begins by articulating four algorithms, which are proposed as applications of a BPFSMTS [29, 30] within the domain of digital marketing. Specific methods embedded within each algorithm are illustrated in Table 2.

3.1. Context

Next, we introduce algorithms rooted in advanced topological concepts, which have been emphasized for their growing significance in information systems [25, 26]. Within the scope of the present work, these abstract mathematical structures are applied to digital marketing. The algorithms offer innovative solutions for tackling real-world challenges such as customer segmentation, advertising optimization, and ROI calculations.

Consider the complex environments of a digital marketing ecosystem, populated by numerous touchpoints – ranging from social media engagement and web analytics to customer journey mapping. Traditional data analysis methods [27] often falter under the weight of the high volume, velocity, and variety of data (often named as big data [28]) generated in such environments. In contrast, the algorithms discussed in this section provide a robust, data-driven framework to manage this complexity effectively. They allow for more informed decision-making processes in the field of digital marketing.

In the subsequent sections, we detail these algorithms. Each algorithm utilizes topological structures in unique ways to optimize various facets of digital marketing. As summarized in Table 2, we outline four specific algorithms: BPF CRITIC, BPF CRITIC COPRA, BPF CRITIC MARCOS, and BPF CRITIC MAIRCA, namely. To cater to a diverse audience, each algorithm is provided into three detailed formats: either as a textual algorithm with an accompanying breakdown, as a flowchart, and an accompanying breakdown to assist in understanding their underlying mechanics. These tailored formats enable readers to understand the complexities of each algorithm in the manner that they find most accessible.

Remark 3.1. *Throughout the following algorithms, whenever we mention the computation of the score values $S([a_{ij}])$, refer to the score function previously established.*

3.2. BPF CRITIC algorithm

Presented in Algorithm 1, the BPF CRITIC method offers a valuable framework for analyzing data in digital marketing. Specifically designed for user-friendliness and efficiency, it excels in managing complex datasets. One of its core functionalities is the normalization of the decision matrix and the estimation of attribute weights, a frequently encountered aspect in the domain of marketing analytics, thereby providing a structured approach to handle BPFSMs. This aspect enables a more sophisticated interpretation of customer behavior and engagement metrics. Consequently, it empowers marketers to make strategically informed decisions based on nuanced data insights.

Algorithm 1 BPF CRITIC method [19]

procedure (BEGIN)

Input BPFSOM $A_{m\times n} = [a_{ij}]$, where each a_{ij} is a four-tuple $\{\mu_A^+, \nu_A^+, \mu_A^-, \nu_A^-\}$ indicating positive and negative feedback ratios.

Construct a BPFSMTS $\{X, \tau_{m\times n}, E\}$ such that $\{E, A\}$ is a BPFSRGOM in the space $\{X, \tau_{m\times n}, E\}$.

Compute the score values, $S([a_{ij}])$ say, for each element in the matrix $A_{m\times n}$.

Normalize the decision matrix using

$$\text{NDM} = a_{ij}^* = \frac{a_{ij} - a_j^{\text{worst}}}{a_j^{\text{best}} - a_j^{\text{worst}}},$$

where $a_j^{\text{worst}} = \min_i\{a_{ij}\}$ and $a_j^{\text{best}} = \max_i\{a_{ij}\}$.

Calculate the correlation coefficient between the attributes e_i to e_k utilizing

$$r_{ik} = \frac{\sum_{i=1}^m (a_{ij}^* - \overline{a}_j^*)(a_{ik}^* - \overline{a}_k^*)}{\sqrt{\left(\sum_{i=1}^m (a_{ij}^* - \overline{a}_j^*)^2\right)\left(\sum_{i=1}^m (a_{ik}^* - \overline{a}_k^*)^2\right)}},$$

where

$$\overline{a}_j^* = \frac{1}{m}\sum_{i=1}^m a_{ij}^*, \quad \overline{a}_k^* = \frac{1}{m}\sum_{i=1}^m a_{ik}^*.$$

Determine the standard deviation $s_i = \sqrt{(1/m)\sum_{i=1}^m (a_{ij}^* - \overline{a_j})^2}$, for $j \in \{1, \dots, n\}$.

State the deviation degree ϕ of criterion e_j from the other criteria by using $\varphi_j = s_j \sum_{k=1}^n (1 - r_{jk})$, for $j \in \{1, \dots, n\}$.

Estimate the weights of the attributes by considering $w_j = \varphi_j / \sum_{j=1}^n \varphi_j$, for $j \in \{1, \dots, n\}$.

Output Weights w_j for $j \in \{1, \dots, n\}$.

END

To assist in understanding the underlying mechanics of the BPF CRITIC method, a breakdown of Algorithm 1 is provided as:

- [Input definition] The algorithm starts by receiving the BPFSOM, which incorporates both positive and negative feedback ratios and serves as the foundational data from which the rest of the algorithm operates.

- [Topology construction] Here, a BFS matrix topology is fashioned, ensuring compatibility with the BPFSRGOM.
- [Score valuation] This step calculates the score values for each element in the matrix, allowing us to gauge variances.
- [Data normalization] A fundamental step where the decision matrix is standardized, providing uniformity and ensuring a balanced data treatment.
- [Correlation computation] This measures the inter-dependencies and relationships within the matrix, giving an understanding of how different attributes interact with each other.
- [Standard deviation] This step provides insights into the data's variability.
- [Deviation degree] Here, it is evaluated how each criterion differs from the others, providing a measure of distinctiveness.
- [Attribute weighting] This determines the importance of each attribute, indicating a clear hierarchy.
- [Output presentation] Lastly, the computed attribute weights are presented as the algorithm's output, providing a concise summary of the attributes' relative importance.

Our methodology ensures comprehensive data processing, highlighting its significance in making informed digital marketing decisions. For further clarity and visualization, refer to the schematic representation of the BPF CRITIC's workflow in Figure 1.

3.3. BPF CRITIC COPRA algorithm

Expanding on the BPF CRITIC method, the BPF CRITIC COPRA method in Algorithm 2 integrates optimization indexes for positive and negative criteria, essential for nuanced decision-making in digital marketing. This method is particularly helpful in scenarios where decisions need to be made considering various attributes' importance.

Algorithm 2 BPF CRITIC COPRA method

procedure (BEGIN)
  Input BPFSOM $K_{m\times n}$ with $A_{m\times n} = [a_{ij}]_{m\times n}$ where a_{ij} comprises a four-tuple $\{\mu_A^+, \nu_A^+, \mu_A^-, \nu_A^-\}$.
  Construct BPFSMTS $\{X, \tau_{m\times n}, E\}$ ensuring $\{K_{m\times n}, A\}$ forms a BPFSRGOM in $\{X, \tau_{m\times n}, E\}$.
  Obtain score values $S([a_{ij}])$ for each a_{ij} in $A_{m\times n}$.
  Invoke Algorithm 1 to determine the attribute weight w_j for each criterion j.
  Normalize the matrix, where WNDM $= [\hat{a}_{ij}]_{m\times n}[w_j]^\top$, implying that $[\hat{a}_{ij}]$ are the normalized values.
  Compute optimization indexes using the weights stated as $\gamma_i^+ = \sum_{j=1}^{r} w_j \hat{a}_{ij}$ for the positive impact criteria and $\gamma_i^- = \sum_{j=r+1}^{m} w_j \hat{a}_{ij}$ for the negative ones, where $\hat{a}_{ij}$ represents the normalized score of each criterion.
  Estimate the priority value B_i for each alternative, considering both positive and negative criteria impacts by means of $B_i = \mathbb{R}^*(\gamma_i^*) + (\min_i\{\mathbb{R}^*(\gamma_i^-)\}/\mathbb{R}^*(\gamma_i^-)) \sum_{i=1}^{n} \mathbb{R}^*(\gamma_i^-)$, where $\mathbb{R}^*(\gamma_i^*)$ and $\mathbb{R}^*(\gamma_i^-)$ are the aggregated scores from positive and negative criteria, respectively.
  Calculate utility degree ψ_i for each a_i as $\psi_i = (B_i/B_{\max})\,100\%$, indicating the performance of each alternative in comparison to the best performer $B_{\max}$.
  Rank values ψ_i, forming ranking $\mathcal{R}$, and make decisions based on this ranking, considering the strategic implications of each ψ_i.
  Output Values ψ_i for each alternative, providing nuanced insights for decision-making.
  END

To elucidate the BPF CRITIC COPRA method's workflow, a concise breakdown of Algorithm 2 is presented as:

- [Input definition] The method starts by receiving the BPFSOM $K_{m\times n}$ containing the foundational data for subsequent processing.
- [Topology construction] At this stage, the BPFSMTS is constructed, ensuring that the pair $\{K_{m\times n}, A\}$ forms a BPFSRGOM in the prescribed space.
- [Score valuation] Score values are computed for each entry a_{ij} in the matrix $A_{m\times n}$.
- [Attribute weighting via BPF CRITIC] This method invokes Algorithm 1 to determine the weights of attributes w_j for each criterion j.

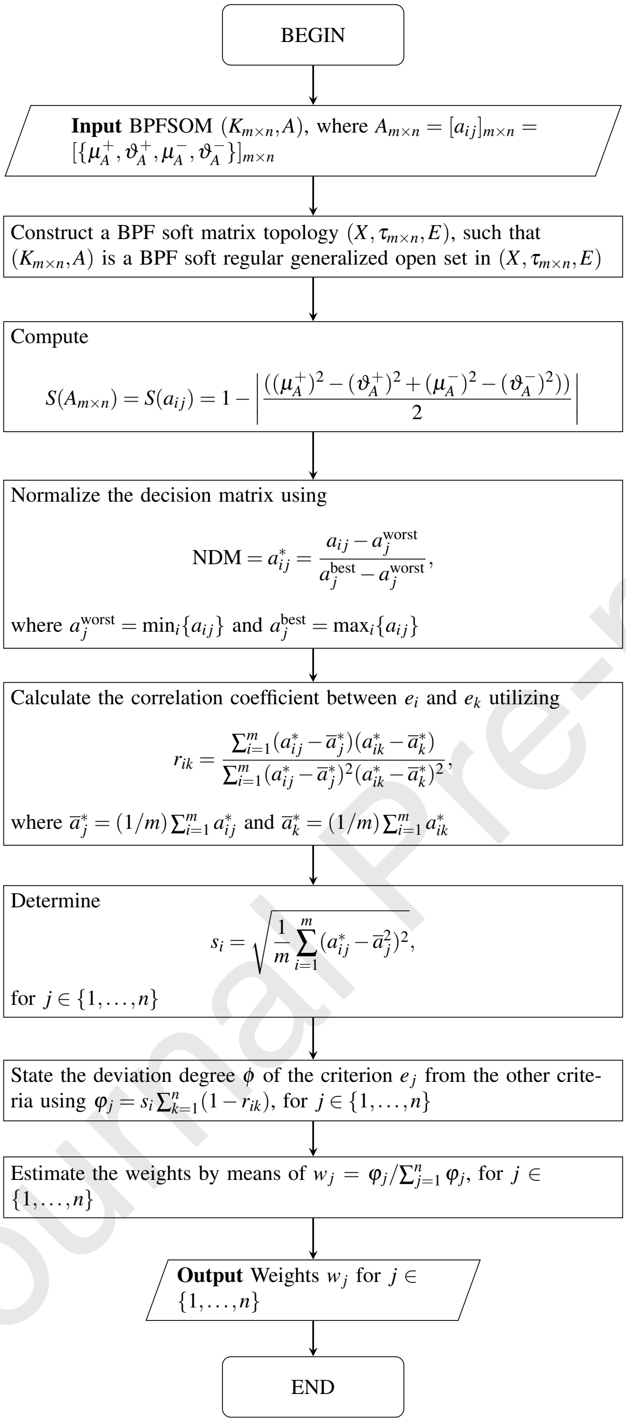

Figure 1: BPF CRITIC method.

- [Matrix normalization] Here, the data matrix is normalized using the determined weights to get the weighted normalized decision matrix (WNDM).
- [Optimization index computation] Optimization indexes for both positive and negative impacts of criteria are calculated using the derived weights.
- [Priority value estimation] The method estimates a priority value B_i for each alternative, taking into account both the positive and negative impacts of the criteria.

- [Utility degree calculation] The method computes the utility degree ψ_i for each entry a_i, which measures each alternative's performance compared to the best performer $B_{\max}$.
- [Ranking and decision-making] Based on the computed values ψ_i, the alternatives are ranked, forming $\mathcal{R}$ and decisions are made based on this ranking, keeping in mind the strategic implications of each ψ_i.
- [Output presentation] The algorithm concludes by outputting the values ψ_i for each alternative, providing a comprehensive perspective for informed decision-making.

The BPF CRITIC COPRA method offers a nuanced approach to decision-making by not only taking into account the weights of the criteria but also by providing a comprehensive ranking system based on both positive and negative impacts of the criteria.

3.4. BPF CRITIC MARCOS algorithm

Building on the BPF CRITIC method, Algorithm 3 provides the BPF CRITIC MARCOS approach, designed specifically for the intricacies of digital marketing challenges. By integrating both the optimal and non-optimal solutions in a matrix representation, it offers a comprehensive view of potential decision outcomes. This is especially vital in digital marketing where professionals must navigate through complex key performance indicators and market dynamics. Evaluating both the best and worst-case scenarios facilitates efficient strategic decision-making.

Algorithm 3 BPF CRITIC MARCOS method

procedure (BEGIN)

Input Matrix $K_{m\times n}$ containing entries a_{ij}, where each entry represents the feedback (positive or negative) for the corresponding criterion, facilitating a comprehensive evaluation.

Construct a topological structure $\{X, \tau_{m\times n}, E\}$ compatible with the matrix dimensions of $K_{m\times n}$ and aligned with the criteria set A, ensuring each criterion's relevance is reflected.

Compute the significance scores $S([a_{ij}])$ for each element in $A_{m\times n}$ to capture the relative importance of each feedback within the decision matrix, enhancing the accuracy of subsequent analysis.

Apply Algorithm 1 to ascertain the attribute weight w_j for each criterion j, thereby differentiating the influence of each criterion on the final outcome.

Expand matrix x_{ij} to include ideal (AI) and anti-ideal (AAI) solutions, extracted from the extremities of feedback scores, providing reference points that signify the best and worst possible scenarios under each criterion.

Normalize x_{ij} by applying $N_{ij} = a_{ij}/\max\{a_{ij}\}$, if j pertains to non-benefit criteria; otherwise, $N_{ij} = \min\{a_{ij}\}/a_{ij}$, if j pertains to benefit criteria; ensuring scores are dimensionless and comparable, considering the nature of each criterion for $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, n\}$.

Derive $R_{ij} = N_{ij}w_j$, incorporating the weight w_j of criterion j, to obtain a weighted matrix that shows the compounded influence of criteria preferences and performance scores.

Evaluate $e_i^+ = S_i/S_{\text{AAI}}$ and $e_i^- = S_i/S_{\text{AI}}$ to ascertain each alternative's proximity to the anti-ideal and ideal states, respectively, so indicating their overall desirability or undesirability.

Establish utility functions $f(e_i^+)$ and $f(e_i^-)$ reflecting the desirability (for $f(e_i^+)$) and undesirability (for $f(e_i^-)$) of each option through the equations stated as $f(e_i^+) = e_i^-/(e_i^+ + e_i^-)$ and $f(e_i^-) = e_i^+/(e_i^+ + e_i^-)$, highlighting the trade-off between optimal and suboptimal performance.

Compute the aggregated utility score $f(e_i)$ considering both desirability and undesirability, facilitating a balanced evaluation as $f(e_i) = (e_i^+ + e_i^-)/1 + (1 - f(e_i^+))/f(e_i^+) + (1 - f(e_i^-))/f(e_i^-))$, and organize the alternatives based on $f(e_i)$, with a higher score indicating a more favorable option.

Output Utility scores $f(e_i)$ for each alternative, providing a ranked list that signals their suitability grounded on the cumulative assessment from multiple criteria.

END

Breaking down the workings of the BPF CRITIC MARCOS method, Algorithm 3 consists of:

- [Input definition] The process begins by acquiring the matrix $K_{m\times n}$, with each entry a_{ij} in this matrix representing feedback for a respective criterion, offering a comprehensive evaluation platform.
- [Topology construction] At this juncture, a topological structure $\{X, \tau_{m\times n}, E\}$ is constructed, ensuring it aligns with the matrix dimensions of $K_{m\times n}$ and syncs with the criteria set A, guaranteeing that the relevance of each criterion is appropriately captured.

- [Significance score computation] Each element in $A_{m\times n}$ is assigned a significance score $S([a_{ij}])$, whose score captures the relative importance of each feedback within the decision matrix, which improves the accuracy of the subsequent evaluations.
- [Attribute weighting] Algorithm 1 is employed to determine the weight w_j of each criterion j, helping in differentiating the influence of each criterion on the eventual outcome.
- [Inclusion of ideal solutions] The matrix X_{ij} is expanded to integrate both the ideal (AI) and anti-ideal (AAI) solutions, whose solutions, derived from the feedback score extremities, act as reference points that exemplify the best and worst potential outcomes for each criterion.
- [Matrix normalization] Normalizing the matrix X_{ij} ensures that the scores are dimensionless and can be compared, whose normalization also takes into account the nature of each criterion for $i \in \{1,\dots,m\}$ and $j \in \{1,\dots,n\}$.
- [Weighted matrix derivation] The matrix is weighted by deriving $R_{ij} = N_{ij}w_j$, emphasizing the compounded influence of criteria preferences and performance scores.
- [Desirability computation] The proximity of each alternative to the ideal and anti-ideal metrics is ascertained through e_i^+ and e_i^-, respectively, whose metrics shed light on the overall attractiveness or unattractiveness of each choice.
- [Utility function establishment] The utility functions $f(e_i^+)$ and $f(e_i^-)$ portray the desirability and undesirability of each option, whose functions underscore the trade-off between optimal and suboptimal performance.
- [Aggregated utility score calculation] The aggregated utility score $f(e_i)$ is computed, considering both desirability and undesirability, whose balanced score helps in organizing the alternatives, with higher scores indicating better options.
- [Output presentation] Concluding the algorithm, the utility scores $f(e_i)$ for each alternative are presented, whose output provides a ranked list based on the combined evaluation from multiple criteria, indicating the suitability of each alternative.

The BPF CRITIC MARCOS method offers an intricate approach for decision-making by holistically assessing both the ideal and anti-ideal scenarios. This in-depth evaluation aids in a nuanced differentiation of the options, presenting a ranked list of alternatives based on a multiplicity of criteria.

3.5. BPF CRITIC MAIRCA algorithm

Building upon the foundation of the BPF CRITIC approach, Algorithm 4 introduces the BPF CRITIC MAIRCA method, specifically designed for the multifaceted environment of digital marketing. Recognizing that not all decision criteria hold equal importance, this algorithm emphasizes the priority of certain indicators, adapting to the dynamic needs of digital marketing where priorities shift based on metrics such as customer engagement, ROI, or brand visibility.

A breakdown of the BPF CRITIC MAIRCA method, as described in Algorithm 4, is provided as:

- [Input definition] The algorithm initiates by accepting the BPFSOM $\{K_{m\times n}, A\}$. Each a_{ij} in A depicts a specific feedback parameter characterized by the tuple $\{\mu_A^+, \nu_A^+, \mu_A^-, \nu_A^-\}$.
- [Topology formation] At this juncture, a BPFSMTS $\{X, \tau_{m\times n}, E\}$ is formulated, certifying that $\{K_{m\times n}, A\}$ operates under the BPFSRGOM regime.
- [Relevance determination] Here, $S([a_{ij}])$ is evaluated, signifying the relevance of every feedback parameter within $A_{m\times n}$.
- [Criterion significance extraction] The method employs Algorithm 1 to distill w_j, a weighting metric that conveys the importance of each criterion j.
- [Indicator priority assessment] This step evaluates J_{e_j} for every indicator j, underpinned by a uniform distribution for equity.
- [Merging weight with priority] Here, the algorithm integrates the determined priority and weight for each indicator to obtain $G_{p_{ij}}$.
- [Criterion-based adjustments] Every $G_{r_{ij}}$ undergoes modifications as per the type of criterion j, where these criteria can be classified as either beneficial or non-beneficial; for beneficial ones, adjustments elevate $G_{r_{ij}}$; and in contrast, non-beneficial ones lead to reductions in $G_{r_{ij}}$.
- [Discrepancy measurement] This stage caculates ρ_{ij}, showcasing the variance between the desired and true outcomes.

Algorithm 4 BPF CRITIC MAIRCA method [24]

procedure (BEGIN)
Input BPFSOM $\{K_{m\times n}, A\}$, where each a_{ij} in A represents a specific feedback parameter, defined by the tuple $\{\mu_A^+, \nu_A^+, \mu_A^-, \nu_A^-\}$.
Establish a BPFSMTS $\{X, \tau_{m\times n}, E\}$ as a structural configuration ensuring that $\{K_{m\times n}, A\}$ operates under the BPFSRGOM framework.
Compute $S([a_{ij}])$, reflecting the relevance of each feedback parameter in $A_{m\times n}$.
Invoke Algorithm 1 to derive w_j, the weighting factor expressing the importance of each criterion j.
Evaluate J_{e_j} as the priority of each indicator j, established using a uniform distribution for equity given by $J_{e_j} = 1/m$; where j ranges from 1 to n.
Determine $G_{p_{ij}}$ by integrating the priority and weight of each indicator, calculated as $G_{p_{ij}} = J_{e_j}\, w_j$; applicable for i and j within the range 1 to n.
Adjust each $G_{r_{ij}}$ based on the type of criterion j, where the criteria are considered either beneficial or non-beneficial to the outcome stated as:

- For a beneficial criterion, where higher values are preferable, adjust $G_{r_{ij}}$ using the formula defined as $G_{r_{ij}} = G_{p_{ij}}(a_{ij} - a_i^{\text{worst}})/(a_i^{\text{best}} - a_i^{\text{worst}})$;
- Conversely, for a non-beneficial criterion, where lower values are preferable, use the alternative formulation stated as $G_{r_{ij}} = G_{p_{ij}}(a_{ij} - a_i^{\text{best}})/(a_i^{\text{worst}} - a_i^{\text{best}})$;

and then apply the respective formula for each criterion, iterating through all i from 1 to m and j from 1 to n.
Calculate ρ_{ij} representing the disparity between the ideal and true performance, expressed as $\rho_{ij} = G_{p_{ij}} - G_{r_{ij}}$; with each i and j spanning from 1 to m and 1 to n, respectively.
Obtain θ_i as the consolidated metric reflecting overall performance or suitability, derived from $\theta_i = \sum_{j=1}^{n} \rho_{ij}$.
Rank the alternatives via θ_i, identifying the most suitable option as the one with the lowest θ_i, succeeded by others as per ascending values θ_i; applied for i within 1 to m.
Output A definitive ranking of alternatives through scores θ_i, providing a basis for selection or prioritization in subsequent decision-making processes.
END

- [Comprehensive metric formation] The algorithm formulates θ_i, a metric encapsulating the overall performance.
- [Hierarchy establishment] Utilizing θ_i, alternatives are organized in a sequence.
- [Output generation] The method culminates by delivering a ranking of alternatives via θ_i scores, guiding subsequent decision-making processes.

Figure 2 states a visual representation of the BPF CRITIC COPRA, MARCOS, and MAIRCA methods.

The BPF CRITIC MAIRCA method offers a systematic approach to multi-criteria decision analysis. By integrating different evaluation dimensions like feedback parameters, criteria weights, and indicator priorities, the method outputs a ranked list of alternatives that can guide stakeholders in their decision-making processes.

The algorithms discussed share a common goal: to provide efficient solutions for decision-making in complex scenarios, with a special focus on digital marketing. They are grounded in the BPFSMTS framework, which enables systematic interpretation and data analysis, catering to the varied demands of this rapidly changing field.

While these methods vary in their specific methodologies, they are designed to offer comprehensive views across digital marketing indicators. Each introduces unique considerations and approaches within its structure, collectively shaping a versatile toolbox for decision-makers in digital marketing. This ensemble of algorithms aids in key processes such as normalization, weighting, and ranking, ultimately enhancing the decision-making efficiency in a dynamic landscape.

4. Application of the BPF approach to marketing

In this section, we present a multi-criteria decision-making application of the BPF soft regular generalized matrix using the BPF CRITIC COPRA, BPF CRITIC MAIRCA and BPF CRITIC MARCOS methods that are equipped with BPF data. We illustrate our proposed methods with a numerical example.

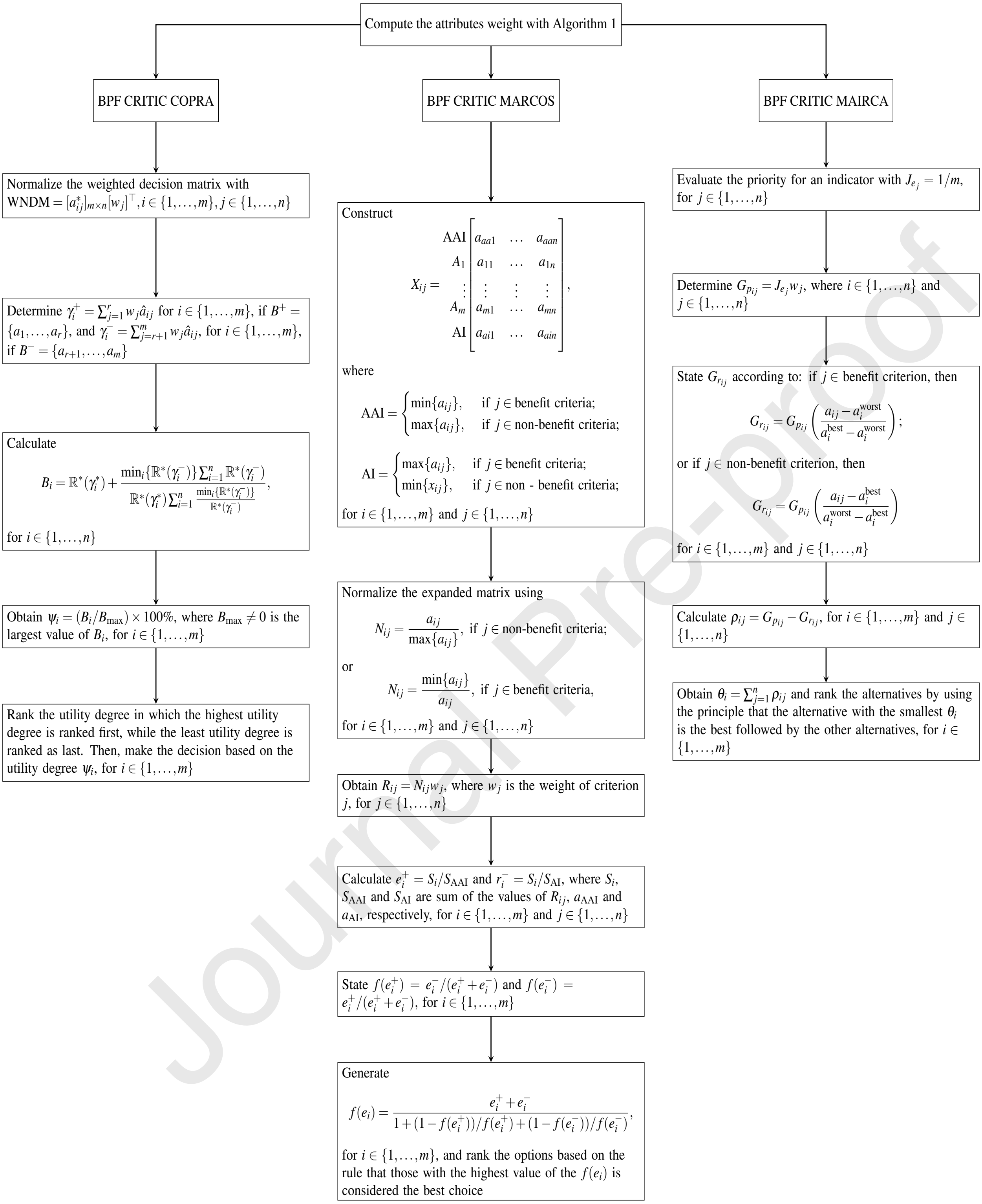

Figure 2: Visual representation of the BPF CRITIC COPRA, MARCOS, and MAIRCA methods

4.1. Social media platform

An SMP is an internet-based medium of communication that encompasses applications or websites where users can create and share content and connect with others. SMPs include Facebook, Instagram, Linkedin, Pinterest, Twitter, WhatsApp, and YouTube. Today, the majority of individuals engage with some form of social media.

An SMP not only allows us to communicate with friends and family but also enables corporations to interact with their audience, gather customer feedback, and enhance their brands. There are various types of social media sites available that enrich the digital marketing.

4.2. Empirical example

Next, we apply the BPFSRGOM to address an empirical decision-making scenario, focusing on a company's objective to identify the optimal SMP to maximize its ROI. We consider five prominent SMPs as alternatives: $[x_1]$ Instagram; $[x_2]$ YouTube; $[x_3]$ LinkedIn; $[x_4]$ WhatsApp; and $[x_5]$ Facebook. The goal is to pinpoint the most effective platform for business development that also offers substantial reach within the target audience. The process begins by evaluating each platform based on certain criteria. Think of this as rating each social media on various aspects that matter to a business, like how many users they can reach or how much it costs to advertise.

The decision-making process involves five evaluation criteria (attributes): $[e_1]$ marketing goals; $[e_2]$ demographics; $[e_3]$ cost of advertisement; $[e_4]$ monthly active users; and $[e_5]$ cost of product. Of these, e_1, e_2, and e_4 are considered benefit criteria, while e_3 and e_5 are viewed as cost criteria.

A point worth noting here is that not all criteria are equal: some are beneficial (the higher, the better), while others, like costs, are the opposite (the lower, the better).

Given these evaluation criteria, the BPFSMTS approach offers a distinct advantage: it maintains the integrity of the original data during the decision-making process. Unlike traditional methods where the union or intersection of various opinions may distort the original dataset, BPFSMTS ensures that the resulting set remains within the initial data space. This is crucial for making a sound and accurate decision. In simpler terms, the BPFSMTS method respects the original data given and do not let them to get "watered down" or distort as we process the data.

In the following, we outline the steps for constructing a BPFSMTS and achieving this purpose:

- [Step 1] Arrange the given BPFSOM organizing all the ratings and feedback to have about each SMP into a neat table (or matrix) which is the foundation of our analysis and given by

$$S_{5\times 5} = \begin{bmatrix} (0.3,0.7,-0.4,-0.8) & (0.9,0.1,-0.9,-0.2) & (0.5,0.4,-0.7,-0.7) & (0.6,0.6,-0.5,-0.7) & (0.4,0.8,-0.4,-0.7) \\ (0.4,0.8,-0.3,-0.7) & (0.8,0.1,-0.8,-0.1) & (0.4,0.4,-0.6,-0.6) & (0.5,0.5,-0.6,-0.6) & (0.5,0.9,-0.4,-0.5) \\ (0.5,0.3,-0.7,-0.5) & (0.3,0.7,-0.5,-0.7) & (0.6,0.3,-0.3,-0.3) & (0.3,0.4,-0.3,-0.8) & (0.4,0.3,-0.2,-0.3) \\ (0.3,0.3,-0.2,-0.7) & (0.1,0.4,-0.6,-0.4) & (0.2,0.7,-0.6,-0.6) & (0.5,0.4,-0.9,-0.3) & (0.3,0.4,-0.5,-0.5) \\ (0.3,0.5,-0.5,-0.3) & (0.4,0.3,-0.7,-0.8) & (0.7,0.6,-0.3,-0.6) & (0.4,0.3,-0.8,-0.6) & (0.4,0.4,-0.3,-0.4) \end{bmatrix}.$$

- [Step 2] Express the score values $S([a_{ij}])$, giving each platform a score based on our criteria, with higher scores being better and stated as:

$$S([a_{ij}]) = \begin{bmatrix} 0.560 & 0.215 & 0.955 & 0.880 & 0.595 \\ 0.800 & 0.680 & 0.865 & 0.690 & 0.990 \\ 0.560 & 0.370 & 1.000 & 1.000 & 0.675 \\ 0.775 & 0.975 & 0.775 & 0.595 & 0.965 \\ 1.000 & 0.960 & 0.930 & 0.825 & 0.965 \end{bmatrix},$$

  where Best $= \{1.000, 0.975, 0.775, 1.000, 0.595\}$ and Worst $= \{0.560, 0.215, 1.000, 0.595, 0.990\}$.

- [Step 3(i)] Normalize the decision matrix converting scores from different tests to a common scale so they can be compared easily (it is like changing all currencies to dollars for easy comparison) reaching:

$$\text{NDM} = [a^*_{ij}]_{5\times 5} = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \begin{bmatrix} 0.0000 & 0.0000 & 0.2000 & 0.7037 & 1.0000 \\ 0.5455 & 0.6118 & 0.6000 & 0.2346 & 0.0000 \\ 0.0000 & 0.2039 & 0.0000 & 1.0000 & 0.7975 \\ 0.4886 & 1.0000 & 1.0000 & 0.0000 & 0.0633 \\ 1.0000 & 0.9803 & 0.3111 & 0.5679 & 0.0633 \end{bmatrix}.$$

Table 3
Indexes γ_i^+ and γ_i^- obtained with the BPF CRITIC COPRA method for the indicated SMP.

SMP	γ_i^+	γ_i^-
e_1	0.1470	0.3029
e_2	0.2506	0.1054
e_3	0.2454	0.2136
e_4	0.2617	0.1925
e_5	0.4628	0.0716

- [Step 3(ii)] Obtain the correlation coefficient of the attributes e_1 to e_5 to understand how each criterion relates to the others, where if two criteria often move together (for example, cost and quality), they have a high correlation, that in our case is stated as:

$$[r_{ik}] = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix}\begin{bmatrix} 1.0000 & 0.8761 & 0.4094 & -0.4897 & -0.8604 \\ 0.8761 & 1.0000 & 0.7005 & -0.7023 & -0.9192 \\ 0.4094 & 0.7005 & 1.0000 & -0.9820 & -0.7158 \\ -0.4897 & -0.7023 & -0.9820 & 1.0000 & 0.7673 \\ -0.8604 & -0.9192 & -0.7158 & -0.7673 & 1.0000 \end{bmatrix}.$$

- [Step 3(iii)] Calculate the standard deviation for each attribute that in our case is given by:

$$s_i = \begin{bmatrix} 0.4209 & 0.4509 & 0.3890 & 0.3926 & 0.4753 \end{bmatrix}.$$

- [Step 3(iv)] Determine the deviation degree ϕ from one criterion to the other criteria, whose step is about understanding how different each criterion is from the others, it being another way to measure their uniqueness, and in our case established as:

$$\phi_j = \begin{bmatrix} 1.7110 & 1.8239 & 1.7849 & 2.1227 & 2.7224 \end{bmatrix}.$$

- [Step 3(v)] State the weights of attributes, after understanding the importance and uniqueness of each criterion, assigning them, where the weights tell us which criteria are more important in our decision-making process, that in our case are presented as:

$$w_j = \begin{bmatrix} 0.1683 & 0.1794 & 0.1756 & 0.2088 & 0.2678 \end{bmatrix}.$$

  Thus, the weightage of each criterion has been evaluated using the BPF CRITIC method. Next, for the BPF CRITIC COPRA method, follow Step 1, Step 2 and Step 3 as in BPF CRITIC method and then:

- [Step 4] Compute the WNDM, adjusting our scores based on the weights of each criterion, giving extra importance to criteria that matter more to us and is established as WNDM $= [a^*_{ij}]_{m\times n} = [w_j]^\top$ and reported as:

$$\begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix}\begin{bmatrix} 0.0000 & 0.0000 & 0.2000 & 0.7037 & 1.0000 \\ 0.5455 & 0.6118 & 0.6000 & 0.2346 & 0.0000 \\ 0.0000 & 0.2039 & 0.0000 & 1.0000 & 0.7975 \\ 0.4886 & 1.0000 & 1.0000 & 0.0000 & 0.0633 \\ 1.0000 & 0.9803 & 0.3111 & 0.5679 & 0.0633 \end{bmatrix}.$$

- [Step 5] Establish the optimization indexes γ_i^+ and γ_i^- for B^+ and B^- as in Table 3, where these indexes help us to understand the performance of each platform compared to the best and worst possible scenarios.

- [Steps 6 and 7] Compute the priority values B_i, utility degrees ψ_i, and ranks, where now we rank our SMP based on all the previous steps. The platform with the highest rank is our best option. The rank of the alternatives, SMPs in our case, are evaluated using the BPF CRITIC COPRA method. The results are presented in Table 4.

Table 4
Priority values B_i, utility degrees ψ_i, and ranks obtained with the BPF CRITIC COPRA method for the indicated SMP.

SMP	B_i	ψ_i	ψ_i 100%	Rank
e_1	0.4848	0.6055	60.55	5
e_2	0.5884	0.7349	73.49	3
e_3	0.5833	0.7285	72.85	4
e_4	0.5995	0.7488	74.88	2
e_5	0.8006	1.0000	100.00	1

Proceeding with our analysis, we apply the BPF CRITIC MARCOS method. Notably, the initial stages (Steps 1, 2, and 3) of the BPF CRITIC MARCOS method are consistent with those of the BPF CRITIC method, which was designed to ensure the resulting set remained within the initial data space. This approach assists decision-makers in identifying the optimal SMP for marketing. The steps associated with the BPF CRITIC MARCOS method are introduced below:

- [Step 4] Represent an expanded matrix by adding an ideal solution, AI, and its anti-ideal solution, AAI, as:

$$\begin{matrix} \text{AAI} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ \text{AI} \end{matrix}\begin{bmatrix} 0.560 & 0.215 & 1.000 & 0.595 & 0.990 \\ 0.560 & 0.215 & 0.955 & 0.880 & 0.595 \\ 0.800 & 0.680 & 0.865 & 0.690 & 0.990 \\ 0.560 & 0.370 & 1.000 & 1.000 & 0.675 \\ 0.775 & 0.975 & 0.775 & 0.595 & 0.965 \\ 1.000 & 0.960 & 0.930 & 0.825 & 0.965 \\ 1.000 & 0.975 & 0.775 & 1.000 & 0.595 \end{bmatrix}.$$

  This matrix extends the existing data to consider an additional set of criteria, enabling more comprehensive decision analysis.

- [Step 5] Calculate a matrix that extends the existing data to consider an additional set of criteria, enabling more comprehensive decision analysis, as given by:

$$N_{ij} = \begin{matrix} \text{AAI} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ \text{AI} \end{matrix}\begin{bmatrix} 0.5685 & 0.2205 & 0.7750 & 0.5950 & 0.6010 \\ 0.5685 & 0.2205 & 0.8115 & 0.8800 & 1.0000 \\ 0.8122 & 0.6974 & 0.8960 & 0.6900 & 0.6010 \\ 0.5685 & 0.3795 & 0.7750 & 1.000 & 0.8815 \\ 0.7868 & 1.0000 & 1.0000 & 0.5950 & 0.6166 \\ 1.000 & 0.9846 & 0.8333 & 0.8250 & 0.6166 \\ 1.000 & 1.000 & 1.000 & 1.000 & 1.000 \end{bmatrix}.$$

- [Step 5] Move forward by obtaining the weighted normalized expanded matrix, assigning weights to our criteria to emphasize the importance of certain factors over others, whose expanded matrix is established as:

$$R_{ij} = \begin{matrix} \text{AAI} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ \text{AI} \end{matrix}\begin{bmatrix} 0.0975 & 0.0396 & 0.1361 & 0.1243 & 0.1610 \\ 0.0957 & 0.0396 & 0.1425 & 0.1838 & 0.2678 \\ 0.1367 & 0.1251 & 0.1573 & 0.1441 & 0.1610 \\ 0.0957 & 0.0681 & 0.1361 & 0.2088 & 0.2361 \\ 0.1324 & 0.1794 & 0.1463 & 0.1723 & 0.1651 \\ 0.1569 & 0.3261 & 0.0946 & 0.1749 & 0.1148 \\ 0.1683 & 0.1794 & 0.1756 & 0.2088 & 0.2678 \end{bmatrix}.$$

- [Steps 7, 8, 9] Rank the SMPs so that the decision-maker can immediately recognize which platform best suits the company's needs, with the rank of the SMP being computed using the BPF CRITIC MARCOS method, and the results are summarized in Table 5.

Table 5
Results obtained with the BPF CRITIC MARCOS method for the indicated SMP.

SMP	Sum	e_i^+	e_i^-	$f(e_i^+)$	$f(e_i^-)$	$f(e_i)$	Rank
AAI	0.5566	-	-	-	-	-	-
e_1	0.7294	0.7294	1.3105	0.6424	0.3275	0.6083	4
e_2	0.7242	0.7242	1.3012	0.6424	0.3275	0.6040	5
e_3	0.7448	0.7448	1.3382	0.6424	0.3275	0.6212	3
e_4	0.7769	0.7769	1.3958	0.6424	0.3275	0.6479	2
e_5	0.8287	0.8287	1.4890	0.6424	0.3275	0.6912	1
AI	1.000	-	-	-	-	-	-

Continuing our analysis, we now consider into the BPF CRITIC MAIRCA method. Similar to the previous procedures, the initial stages (Steps 1, 2, and 3) of this method align with those of the BPF CRITIC method. The additional stages of this method are as follows:

- [Step 4] Evaluate the priority for an indicator using a relation $J_{e_j} = 1/5 = 0.2$, for $j \in \{1, \dots, n\}$.
- [Step 5] Determine the ranking matrix as:

$$G_{p_{ij}} = J_{e_j} w_j = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \begin{bmatrix} 0.0337 & 0.0359 & 0.0351 & 0.0418 & 0.0536 \\ 0.0337 & 0.0359 & 0.0351 & 0.0418 & 0.0536 \\ 0.0337 & 0.0359 & 0.0351 & 0.0418 & 0.0536 \\ 0.0337 & 0.0359 & 0.0351 & 0.0418 & 0.0536 \\ 0.0337 & 0.0359 & 0.0351 & 0.0418 & 0.0536 \end{bmatrix}.$$

- [Step 6] Normalize the decision matrix as:

$$\text{NDM} = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \begin{bmatrix} 0.0000 & 0.0000 & 0.2000 & 0.7037 & 1.0000 \\ 0.5455 & 0.6118 & 0.6000 & 0.2346 & 0.0000 \\ 0.0000 & 0.2039 & 0.0000 & 1.0000 & 0.7975 \\ 0.4886 & 1.0000 & 1.0000 & 0.0000 & 0.0633 \\ 1.0000 & 0.9803 & 0.3111 & 0.5679 & 0.0633 \end{bmatrix}.$$

- [Step 7] Compute the quantity matrix as:

$$G_{r_{ij}} G_{r_{ij}} = \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \begin{bmatrix} 0.0000 & 0.0000 & 0.0070 & 0.0294 & 0.0536 \\ 0.0184 & 0.0220 & 0.0211 & 0.0098 & 0.0000 \\ 0.0000 & 0.0073 & 0.0000 & 0.0418 & 0.0427 \\ 0.0164 & 0.0359 & 0.0351 & 0.0000 & 0.0034 \\ 0.0337 & 0.0352 & 0.0109 & 0.0237 & 0.0034 \end{bmatrix}.$$

- [Step 8] Evaluate the values of ρ_{ij} from the matrix $\rho_{ij} \rho_{ij} = G_{p_{ij}} - G_{r_{ij}}$ given by:

$$\begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} \begin{bmatrix} 0.0337 & 0.0359 & 0.0281 & 0.0124 & 0.0000 \\ 0.0153 & 0.0139 & 0.0140 & 0.0320 & 0.0536 \\ 0.0336 & 0.0286 & 0.0351 & 0.0000 & 0.0108 \\ 0.0172 & 0.0000 & 0.0000 & 0.0418 & 0.0502 \\ 0.0000 & 0.0007 & 0.0242 & 0.0180 & 0.0502 \end{bmatrix}.$$

- [Step 9] Compute $\theta_i = \sum_{i=1}^{m} \rho_{ij}$ and allocate the rank. The rank of the SMPs can be seen in Table 6.

The results derived from the application of the three distinct methods are compiled in Table 7. As illustrated in this table and Figure 3, e_5 (representing Facebook) consistently stands out as the premier choice across all methodologies, highlighting Facebook as the dominant SMP for devising digital marketing strategies.

Table 6
Values of θ_i and ranks obtained with the BPF CRITIC MAIRCA method for the indicated SMP.

SMP	θ_i	Rank
e_1	0.1100	4
e_2	0.1288	5
e_3	0.1082	2
e_4	0.1092	3
e_5	0.0931	1

Table 7
Comparative analysis of the methods.

Method	Rank	Best SMP
BPF CRITIC COPRA	$e_5 > e_4 > e_2 > e_3 > e_1$	e_5
BPF CRITIC MARCOS	$e_5 > e_4 > e_3 > e_1 > e_2$	e_5
BPF CRITIC MAIRCA	$e_5 > e_3 > e_4 > e_1 > e_2$	e_5

To further elucidate the significance of our proposed method, consider a global digital marketing agency navigating various international markets, employing multiple SMPs to cater to diverse audience segments. The selection criteria would encompass audience demographics, user behavior, ad expenses, engagement rates, and ROI. Navigating through the intricacies and cultural variances across regions, data becomes fraught with uncertainties. Existing methodologies often falter amidst such uncertainties, demanding precise data, which may be elusive or costly, especially on a global scale.

Our innovative approach excels by adeptly managing ambiguous or incomplete data. It operates within a spectrum of possibilities, proving invaluable in complex scenarios characterized by intricate data. By harnessing matrix operations, it evaluates multiple criteria simultaneously, offering a more nuanced decision-making framework compared to traditional methods. This sophistication leads to informed decisions, contributing to successful marketing campaigns and enhanced ROI.

In essence, our method is strategically tailored for the complexities of global SMP selection, overcoming the shortcomings of traditional methodologies.

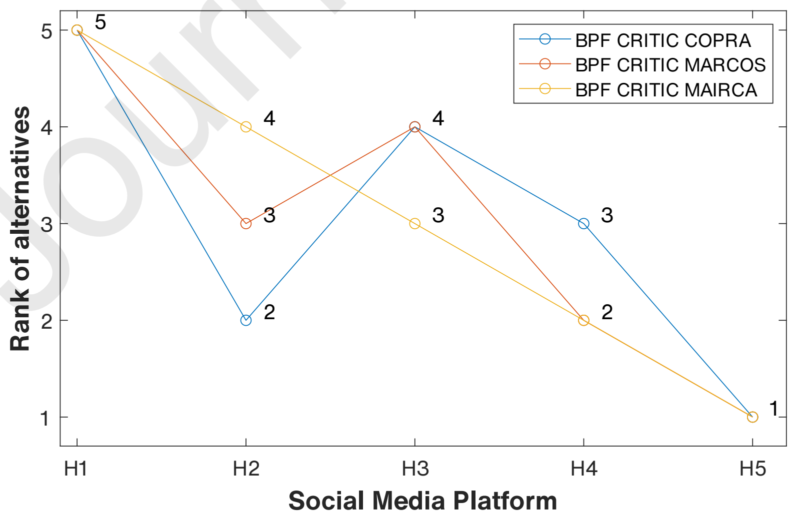

Figure 3: Behavior of five SMP with the indicated method.

5. Conclusions

In this study, we explored the potential of bipolarity in Pythagorean fuzzy soft regular generalized closed matrices within the framework of their corresponding topological spaces. This innovative exploration presented a unique approach to problem-solving in the field of digital marketing.

To select optimal social media strategies, we leveraged three decision-making methods: bipolar Pythagorean fuzzy CRITIC COPRA, BPF CRITIC MAIRCA, and BPF CRITIC MARCOS namely. Through our analysis, Facebook consistently emerged as the most suitable platform for digital marketing strategies, thereby underscoring the applicability of our approach. The contributions of this research are twofold. Firstly, our work provides a comprehensive understanding and application of bipolar Pythagorean fuzzy soft matrix topological spaces and the BPFSRGOM method, illuminating their potential in intricate decision-making scenarios. Secondly, we introduced groundbreaking algorithms tailored for the nuances of digital marketing: bipolar Pythagorean fuzzy CRITIC, CRITIC COPRA, CRITIC MARCOS, and CRITIC MAIRCA. Together, these methods show the innovative essence of our study in reshaping digital marketing strategies. However, we must recognize certain limitations. The deployment of our approach requires an in-depth grasp of bipolar Pythagorean fuzzy sets and soft matrix topological spaces, which might pose challenges for those unfamiliar with these concepts. Furthermore, while our methodology is tailored for digital marketing, its application to other domains might necessitate modifications. The efficacy of our methods is intrinsically tied to the quality of input data, emphasizing the significance of meticulous data collection and analysis.

Looking ahead, we see potential in integrating fuzzy set theory with recent advancements in the field. Models such as (2,1)-Fuzzy sets, SR-fuzzy sets, (3,2)-Fuzzy sets, and new generalizations of fuzzy soft sets could significantly enhance the applicability and utility of decision-making methodologies across various fields, including image processing, pattern recognition, and artificial intelligence [34, 35]. Also, the use of other statistical distributions can be considered instead of triangular distributions when applying fuzzy theory [36]. Additionally, we plan to develop an `R` package or a `Python` library to encapsulate the proposed algorithms, offering a practical resource for researchers and practitioners. In conclusion, this work successfully blends theoretical advancements with practical utility, providing novel tools for making optimal decisions in intricate and uncertain environments. Despite its limitations, this study shiows the path for further research and applications in fuzzy set theory and digital marketing.

CRediT authorship contribution statement

Vishalakshi Kuppusamy: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Writing – original draft. Maragathavalli Shanmugasundaram: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Writing – original draft. Prasantha Bharathi Dhandapani: Conceptualization, Formal analysis, Methodology, Writing – original draft. Carlos Martin-Barreiro: Conceptualization, Formal analysis, Methodology, Writing – original draft. Xavier Cabezas: Conceptualization, Formal analysis, Methodology, Writing – original draft. Víctor Leiva: Conceptualization, Formal analysis, Investigation, Methodology, Supervision, Writing – review and editing. Cecilia Castro: Conceptualization, Data curation, Formal analysis, Methodology, Writing – review and editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

Data availability

Data will be made available on request from the authors.

Acknowledgments

The authors would like to thank the editors and reviewers for their constructive comments, which improved the presentation of this article. Our research is partially funded by FONDECYT, grant number 1200525 (V. Leiva) from the National Agency for Research and Development (ANID) of the Chilean government under the Ministry of Science, Technology, Knowledge, and Innovation; as well as by Portuguese funds through the CMAT—Research Centre of Mathematics of the University of Minho, within projects UIDB/00013/2020 and UIDP/00013/2020 (C. Castro).

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Declaration of Interest Statement and Author Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

All persons who meet authorship criteria are listed as authors, and all authors certify that they have participated sufficiently in the work to take public responsibility for the content, including participation in the concept, design, analysis, writing, or revision of the manuscript:

On a decision problem using a bipolar Pythagorean fuzzy approach: New methodology with application in digital marketing

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