Universidade do Minho
Escola de Economia e Gestão

The performance of mean-variance optimal portfolios' extended methods with restrictions – evidence from Germany.

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Master Dissertation
Master in Finance

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Professor Ph.D. Nelson Manuel Pinho Brandão Costa Areal

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Without you, nothing would be possible.

Statement of Integrity

I hereby declare having conducted this academic work with integrity. I confirm that I have not used plagiarism or any form of undue use of information or falsification of results along the process leading to its elaboration.

I further declare that I have fully acknowledged the Code of Ethical Conduct of the University of Minho.

The performance of mean-variance optimal portfolios' extended methods with restrictions – evidence from Germany.

Abstract

This study aims to compare different strategies for creating diversified portfolios by implementing various methods and the associated variants. A comparison of portfolio building strategies such as the minimum variance and the tangency portfolio, the equal- and value-weighted portfolios will be studied using different estimation time windows of 30, 60 and 120 months. Additionally short-selling restrictions as well as restrictions for constraining the portfolio weights will be considered. The different portfolio building strategies were applied to stocks data from the German market over the period from the 1st of January 2000 to the 30th of September 2021. Ex-post performance was measured using the Sharpe ratio, the Certainty-Equivalent return ratio, and the 5-factor Fama and French model. The main conclusion to be drawn from this study, and according to DeMiguel et al. (2009), is that the performance of the constructed portfolios can hardly outperform the equal-weighted portfolios simultaneously in terms of Sharpe ratio and Certainty Equivalent return ratios. Minimum variance portfolios have a similar performance. The same happens with value-weighted portfolios and when analysing the Sharpe ratio, sometimes superior. Also, the portfolios that generally have the worst performance are the tangency portfolios.

Keywords: Portfolio Optimization, Covariance Matrix, Expected Returns, Asset Allocation, Portfolio Performance.

O desempenho de métodos de carteiras de variância-média ótima com restrições – evidência da Alemanha.

Resumo

Este estudo visa comparar diferentes estratégias para a criação de carteiras diversificadas, implementando vários métodos e as variantes associadas. Uma comparação de estratégias de criação de carteiras, tais como a variância mínima e a carteira tangencial, as carteiras ponderadas por igual e valor serão estudadas utilizando diferentes janelas temporais de 30, 60 e 120 meses. Além disso, serão consideradas restrições de venda a descoberto, bem como restrições para limitar os pesos das carteiras. As diferentes estratégias de construção de carteiras foram aplicadas aos dados de stocks do mercado alemão durante o período de 1 de janeiro de 2000 a 30 de setembro de 2021. O desempenho ex-post foi medido utilizando o rácio de Sharpe, o rácio de rendimento Certainty-Equivalent, e o modelo Fama and French de 5 factores. A principal conclusão a tirar deste estudo, em conformidade com os resultados obtidos por DeMiguel et al. (2009), é que o desempenho das carteiras construídas dificilmente pode superar as carteiras ponderadas por igual simultaneamente em termos de rácio de Sharpe e rácio de rendimento Certainty-Equivalent. As carteiras de variância mínima têm um desempenho semelhante. O mesmo acontece com as carteiras ponderadas pelo valor e, quando analisado o rácio de Sharpe, por vezes superior. Além disso, as carteiras que geralmente têm o pior desempenho são as carteiras de tangência.

Palavras-chave: Optimização de Carteira, Matriz de Covariância, Retornos Esperados, Alocação de Activos, Desempenho da Carteira.

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Chapter 1 – Introduction

The current research was conducted as part of the Master's in Finance program at the School of Economics and Management at the University of Minho. It aims to implement several portfolio construction methods and consider some variants to compare them and see which performs best with data from the German market.

Despite the vast amount of research on the predictability of returns, this study was conducted with the intention to make an addition to the currently existing literature and studies and specifically to the work of DeMiguel et al. (2009). The authors analysed the sample-based mean-variance model's out-of-sample performance and any improvements that minimise estimation error. Also, they compared several portfolio construction methodologies.

The construction of optimal portfolios has attracted the interest of academics since Markowitz (1952) proposed the portfolio theory that allows to identify efficient portfolios. It relies on the covariance matrix and the expected security returns estimations. The covariance matrix plays a determinant role in the construction of efficient portfolios. The real covariance matrix is frequently unknown in practical applications, such as Markowitz portfolio selection, and must be inferred from data instead.

Investors have always tried to answer the question of how to best invest their wealth. The answer can be found in portfolio theory when trying to minimise the risk and maximise the expected return. Investors seek a reward for taking on risk. Therefore, riskier assets have a greater expected return. The risk premium is the difference between a risky asset's expected return and the risk-free rate of return. Fewer investors would have possible invested in riskier assets if risk premiums were not available.

1.1. Relevance of the Study and Academic and Practical Contributions

DeMiguel et al. (2009) examined the out-of-sample performance of the sample-based mean-variance model and any adjustments that reduce estimate error compared to a benchmark, the naive 1/N portfolio. Understanding the circumstances in which mean-variance optimum portfolio models may be expected to perform favourably even in the presence of estimation risk is the main goal of their work. They concluded that none of the portfolios tested out-preforms their benchmark portfolio in terms of Sharpe and Certainty-Equivalent ratios.

Thus, the primary purpose of this research is to understand, when using the methodology adapted by DeMiguel et al. (2009), which portfolios perform best by analysing them in three different time windows and with some restrictions imposed. The portfolios examined will be the minimum variance and the tangency portfolios. Also, the equal and value-weighted portfolios. These are strategies that, despite being simple, end up having a satisfactory ex-post results compared to others. Finally, the Sharpe ratio, Certainty-Equivalent ratio and Fama-French five-factor portfolio were used to compare the performance of all these strategies. Some results are expected to be in accordance with the existent literature on portfolio performance, such as, the good performance of the naive equal-weighted portfolios.

A non-linear shrinkage estimator to assess the covariance matrix of individual stock returns was developed by Ledoit and Wolf (2017) to solve problems that occur when estimating large-dimension covariance matrices. This will be the method used in this study for estimating the covariance matrix.

This study uses data from the German market because its economy is the largest in the European Union and the world's fourth largest, having small and medium-sized companies as its engine and displaying large economic centres. As a result, it plays a significant role in the global financial picture.

1.2 Research Framework

Regarding structure, this dissertation is organised into the following six chapters. The present introduction constitutes Chapter 1.

Chapter 2 presents the literature review. Chapter 3 describes the methodology used in this research, consisting of the adaption of the one used by DeMiguel et al.'s (2009) to the German market. Then, the construction of the alternative portfolios referred previously. The computation of the portfolio's weights and returns is also presented. Finally, the methodology for ex-post performance is reviewed. Chapter 4 illustrates the sample construction methods and data cleaning process. In Chapter 5, the results are presented and discussed in the context of the empirical evidence from the studies. As a conclusion for this work, Chapter 6 emphasises the dissertation's principal findings. The study's theoretical and practical implications will be addressed, along with its limitations and suggestions for further studies.

Chapter 2 - Literature Review

Sharpe (1992) demonstrated the importance of the asset allocation decision, established through studies revealing that investment decision dominates the return of a portfolio. This asset allocation is based on designating an investor's portfolio across several major asset classes.

Maximizing expected return and minimizing risk, determined by the standard deviation of expected return, are the two core principles of portfolio theory, which offers a solution to the question of how individuals should invest in their wealth (Ruppert & Matteson, 2011).

For many years, the conventional mean-variance optimization (Markowitz, 1952) has been a central concept in contemporary finance theory. If future asset returns are known and portfolio risk and return are the only pertinent factors, it gives the investor the ideal asset allocation (Bessler et al., 2017).

2.1 Markowitz's (1952) Portfolio Theory

The pioneering work of Markowitz (1952) proposes a model for constructing efficient portfolios that depends on the estimation of the security returns and covariance matrix. In fact, several methods in financial economics are based on Markowitz's (1952) work, and it is still one of the most extensively utilized quantitative techniques for portfolio construction.

This method combines a vector of expected returns estimates and an estimated covariance matrix of returns. The investor's decision challenge in the mean-variance framework is to select a vector of asset weights that minimizes the portfolio's variance given a target rate of return on a portfolio of n assets.

Estimates of each stock's expected returns as well as a covariance matrix are required to construct Markowitz (1952) mean-variance optimum portfolios. Portfolio weights are determined based on such estimates to optimize the predicted portfolio return while adhering to a risk restriction. The investor should select the portfolio with the lowest risk for a certain target return, or, conversely, the portfolio with the highest expected return for a given level of risk.

The implementation of a mean-variance optimization is challenging due to unreliable expected return estimations and covariance matrix estimation, as mentioned by Jorion's (1985) and Menchero's (2019) research. Due to estimating error, mean-variance portfolios built using the sample mean and covariance matrix of asset returns behave poorly out-of-sample (DeMiguel et al., 2009).

Because practitioners mistrust portfolios that are not naively diversified, mean-variance approaches are often developed with substantial sets of restrictions that guarantee diversification. Even though actual data frequently contradicts this, in the lack of estimation errors, Green and Hollifield (1992) suggest that the existence of an equity return single factor would lead to significant negative weights in mean-variance efficient portfolios. The authors use a factor structure for the covariance matrix estimator, reducing the number of parameters that would be estimated. Also, Green and Hollifield (1992) demonstrate that a robust factor structure is more relevant then diversity to completely remove residual risk.

2.1.1 Minimum variance portfolios

It is extremely difficult to estimate expected returns using time series data. For this reason, several recent studies advise using the global minimal variance portfolio (Kempf & Memmel, 2006). The weights in this portfolio are independent of expected returns and solely rely on return variances and covariances. The real return covariance matrix is often replaced by its time series estimator to determine the weights of the global lowest variance portfolio.

Several recent articles (see, for example, Ledoit and Wolf (2003) and Jagannathan and Ma (2003)) advise against estimating expected returns. They consider that all stocks have equivalent predicted returns. According to this supposition, the only way stock portfolios vary from one another is in terms of risk. As a result, the global minimal variance portfolio, which has the lowest risk, is the only effective stock portfolio. This portfolio and the risk-free asset should therefore be combined by all investors whose portfolios are designed to maximize the trade-off between expected return and risk (Kempf & Memmel, 2006).

The stock return covariance matrix is the single factor that influences the global minimum variance portfolio's construction. By focusing on this portfolio, the estimation risk of the investor is predicted to be lowered since the covariance matrix can be calculated with a lot more accuracy than the expected returns.

Historical correlations and stock-return volatility can be utilized as a starting point to reach reasonable estimations of the covariance matrix, even when historical returns do not offer relevant predictions for future returns. Because it is the only effective portfolio that can be created using only the covariance matrix estimator and does not require estimates of expected returns, the ex-ante global minimum-variance portfolio is the base of several portfolio managers (Mostowfi & Stier, 2013).

DeMiguel et al. (2009) proposed a broad approach, when estimation error is present, for identifying portfolios that behave efficiently out-of-sample. Because of what was said before about the previous work of Markowitz (1952), and the existence of the estimation error, the researchers try to solve the traditional minimum-variance problem.

Numerous solutions to the issue of estimating the covariance matrix's have been put forth in the literature. One strategy is to switch from monthly returns to higher frequency data, such as daily returns (see Jagannathan and Ma, 2003). Another strategy, by Ledoit and Wolf (2003, 2004), is using the weighted average of the sample covariance matrix and another estimator, such as the identity matrix or the 1-factor covariance matrix. Finally, the last one, minimise the number of parameters that need to be estimated by applying some factor structure to the estimator of the covariance matrix (Chan et al. 1999; Green and Hollifield 1992).

Lyle and Yohn (2021) based their study on creating fully optimised fundamental portfolios using mean-variance portfolio optimisation that incorporates fundamental analysis information. In this study, to develop mean-variance optimal portfolios following Markowitz (1952), the authors based the estimation of the covariance matrix of individual stock returns in the non-linear shrinkage estimator of Ledoit and Wolf (2017). This decision comes from the fact that it is simpler to use and has also been found to produce portfolios with lower variance than those created using a factor-based covariance matrix.

Ledoit and Wolf (2003, 2004) introduced a method to obtain an estimate of the sample covariance matrix. The authors proposed a versatile approach for attributing structure to calculating the covariance matrix of a large number of stock returns, by using a shrinkage method, that suggests using an optimally weighted average of two known estimators – one is a weighted average of the sample covariance matrix and the other could be the identity matrix.

The authors, years later, created a non-linear shrinkage estimator with the acceptable number of free parameters, making it more versatile than earlier linear shrinkage estimators (Ledoit and Wolf, 2017). The number of free parameters to estimate is the fundamental distinction between this idea and the prior technique for calculating the covariance matrix. Once the number of assets equals the sample size, this method is asymptotically ideal for portfolio selection.

Furthermore, linear shrinkage is easier to calculate and apply. Non-linear shrinkage, on the other hand, can increase performance even more when combined with stylized facts like time-varying co-volatility or factor models.

Chan et al. (1999) performed a study where the predicting capability of several covariance models is assessed. Forecasts of future covariances and the out-of-sample volatility of each model's optimized portfolios are used to contrast information. The basic covariance structure is captured by a small number of components but adding additional factors does not increase forecasting accuracy. Also, substantial differences are shown when the tracking error volatility criteria is used.

According to Green and Hollifield (1992), even with the lack of estimate errors, the presence of a dominating factor would cause extremely negative weights in mean-variance efficient portfolios. In such scenario, enforcing no-short-sale restrictions should be disadvantageous, yet actual data frequently shows the opposite. Jagannathan and Ma (2003) provide an explanation of how requiring portfolio weights to be nonnegative can lower the risk in computed optimal portfolios. The sample covariance matrix performs as well as covariance matrix estimations based on factor models, shrinkage estimators, and daily data when no-short-sale restrictions are in place.

When portfolio weights are restricted to be nonnegative, factor models and shrinkage estimators perform substantially worse (Jagannathan & Ma, 2003). The performance of minimum variance and minimum tracking error portfolios created using the sample covariance matrix, however, is nearly equal to that of portfolios generated using factor models, shrinkage estimators, or daily returns when the nonnegativity constraints on portfolio weights are used.

Mostowfi and Stier (2013) used daily return data from the German stock market to examine the out-of-sample performance of ex-ante minimum variance portfolios based on four alternative estimators for the covariance matrix. According to their empirical investigation, all four covariance estimators provide minimum-variance portfolios that beat the benchmark DAX index, the representative index of the German market.

In the presence of estimating error, DeMiguel et al., (2009) offers a generic methodology for identifying portfolios that outperform their study sample. The foundation of this framework is the standard minimum-variance problem, but with the extra restriction that the portfolio-weight vector's norm must be below a certain limit. Also, in this study, the authors present a new class of portfolios that are more stable than typical minimal variance portfolios. These portfolios are built using specific robust estimators and could be generated by executing a unique non-linear program that includes both robust estimation and portfolio optimization.

In fact, a variety of empirical findings demonstrates that the minimum-variance portfolio consistently outperforms all mean-variance portfolios, even when employing a performance metric that takes into account both the portfolio mean and variance.

2.1.2 Equal and Value-Weighted Portfolios

DeMiguel et al. (2009) compared the sample-based mean-variance model and its extensions aiming to decrease estimate error with the naive 1/N portfolio in terms of out-of-sample performance. The researchers tested many models and found that none outperform the 1/N rule in terms of Sharpe ratio, certainty-equivalent return, or turnover, indicating that the benefit of optimal diversification is very much compensated by estimation error out of sample. Their findings are consistent with the notion that, to use the mean-variance model, estimations of the variance-covariance matrix of returns as well as the vector of predicted excess returns over the risk-free rate are required.

The authors describe the naive rule as one in which each of the N assets that are available for investment at each rebalancing date receive a percentage of wealth equal to 1/N of wealth. This rule is used as a standard for two reasons: firstly, it doesn't rely on either estimation of the moments of asset returns or optimization and second, it is simple to implement. Moreover, investors tend to divide their money across various assets using straightforward allocation methods. The 1/N rule has the added benefit of being simple to apply to a large number of assets, as opposed to optimization models, which often require more parameters as the number of assets rises. So, the 1/N is used as a benchmark for asset allocation strategies.

DeMiguel et al. (2009) concluded that between the models that were tested (Sharpe ratio, certainty-equivalent return, and turnover), the 1/N rule outperformed the other alternatives, proving that the advantagegiven by optimum diversification is offset by estimation error out-of-sample. Additionally, the unconstrained policies that attempt to take estimating error into account perform far worse than any of the short sales constraint techniques, as well as significantly worse than the 1/N approach.

An equal-weighted investment strategy seems to be better than mean-variance optimization (Allen et al., 2019). The finding appears to be based on two significant hypotheses –

the investor is incapable of predicting and the estimated error in the covariance matrix is immutable.

In a value-weighted portfolio, the percentage of each stock is equal to the market capitalization of the stock, which is determined by multiplying the price per share by the number of outstanding shares and then, dividing by the sum of the market capitalization of all stocks (Bodie et al., 2014).

Mostowfi and Stier (2013) show that this market-weighted portfolio was outperformed by the minimum variance portfolio in relation to return and volatility and this might mean that in long-term, low-variance portfolio strategies make sense for all investors, not just those who are extremely risk-averse. Furthermore, Mostowfi and Stier (2013) demonstrate that owning a typical market-weighted portfolio, in a figure of a representative index of a market, like DAX, would be considerably worse than adopting a minimum-variance approach.

According to Plyakha et al. (2021), equal-weighted portfolios beat value-weighted portfolios in terms of total mean return, four-factor alpha, and Sharpe ratio with monthly rebalancing. The study goes on to state that the equal-weighted portfolio's larger exposure to systematic risk variables is a contributing reason for this outperformance. The value-weighted market portfolio has performed a significant role in asset pricing, such as in Sharpe's Capital Asset Pricing Model (1964). But on the other hand, equal-weighted mean returns are widely used in quantitative finance. A value-weighted portfolio is the CAPM market portfolio, where the market value of each security is divided by the total market value of all securities to determine how much of each is held (Boodie et al., 2014).

The fact that market-value-weighted and price-weighted index returns resemble those of simple portfolio strategies is a great advantage. The value-weighted index would accurately reflect capital gains on the underlying portfolio in the case that shares in each company were purchased in proportion to its current market value. As such, a price-weighted index monitors the performance of a portfolio composed of an equal number of shares from each company (Bodie et al., 2014).

Using these two portfolios as benchmarks — the equal-weighted (EW) portfolio and the value-weighted (VW) portfolio — Zanello (2021) examines the performance of the optimized portfolio created by parametric portfolio rules. To minimize the variations in performance to only the asset weighting strategy, these two are built from the identical pool of stocks in the optimized portfolio. While it seems obvious that the market portfolio should be included since it mirrors how national indexes are created, the literature supports the inclusion of the naive portfolio.

2.1.3 Other portfolio construction strategies

In respect to the out-of-sample Sharpe ratio, tangency portfolios, whether limited or not, do not outperform the global minimum variance portfolios (Jagannathan & Ma, 2003).

For the weights of the sample tangency portfolio's mean and variance, Jobson and Korkie (1980) propose approximation formulas. Also, Jobson and Korkie (1981) come to the conclusion that the sample mean and variance of the sample tangency portfolio is an extremely poor estimator of its out-of-sample performance for estimation windows with typical length. They conclude that the sample tangency portfolio can be significantly outperformed even over a naive evenly weighted portfolio.

The gains of optimization are outweighed by an estimate error, and the sample tangency portfolio doesn't perform significantly better than the sample global minimum-variance portfolio. Parametric portfolio policies is an approach to obtain portfolio weights from stock attribute, with the the advantage of not requiring the estimation of the covariance matrix and expected returns immediately. This approach is provided by Brandt et al. (2009) and Hand and Green (2011). Parametric portfolios are created by combining the long-short portfolios related to each of the firm-specific attributes studied with a benchmark portfolio (DeMiguel et al., 2009). In this method, the expected portfolio weight function is parameterized applying a non-linear estimation and a power utility function. The weights in the portfolio are expected to represent a linear parametric merge of stock properties. This function's coefficients are determined by maximizing the investor's average utility of the portfolio's return across the sample period.

2.2 Fama and French 5-factor model (2015)

The connection between risk and return has long been a source of debate and investigation. Researchers have concentrated on the cross-section of individual stock returns as opposed to anticipating predicted stock returns in the time-series. In the past, it was believed that the market beta, or the slope of the regression of a security's return on the market, was suitable for describing predicted stock returns in the cross-section. In their research, Fama and French (1992) demonstrated that two readily measurable variables, size (price per share multiplied by the number of common shares outstanding) and book-to-market ratio (book value per share divided by price per share) capture the cross-sectional fluctuation in average stock returns better than market beta.

Expected stock returns and systemic market risk are positively and linearly correlated. However, Artmann et al. (2012) note that the CAPM has suffered over the years because empirical data reveals that betas do not sufficiently account for cross-sectional variations in average returns. Several multifactor models have been proposed to capture these return patterns, with Fama and French's (2015) model being the most well-known. The model is applicable to tasks like assessing portfolio performance or calculating the cost of capital that call for estimations of expected returns.

The Fama and French model, however, performs poorly in describing the cross-section of average stock returns in Germany, claim Artmann et al. (2012). The majority of prior research on the German stock market has focused on firm characteristics as predictors of expected returns, which is related to the study's major conclusion that book-to-market equity, earnings-to-price, market leverage, return on assets, and momentum all raise average stock returns. Additionally, Artmann et al. (2012) demonstrate in multivariate Fama and MacBeth (1973) regressions that only momentum and the two value characteristics - book-to-market equity and earnings-to-price - have predictive power for the cross-section of stock returns.

Chapter 3 - Methodology

As previously mentioned, this research focus on comparing portfolio creation using various strategies and using different time windows to estimate the model parameters. It will follow DeMiguel et al. (2009) and explore these different portfolio creation methods in the German stock market. The authors use, as a benchmark to assess the performance of the various portfolio rules proposed in the literature, the naive 1/N portfolio, that is, the equal-weighted portfolio.

Both long-short and long-only portfolios will be created since taking short positions is not always possible, and even when it is, the execution costs might be significant, as found by Beneish et al. (2015).

The present study will also consider unconstrained and constrained portfolio weights. That is, limiting the weights for long-short portfolios to between -2.5 percent and 2.5 percent, and long-only portfolios to be within 0 percent and 2.5 percent to see how sensitive the outcomes are to these allocation limits.

Also, different time windows will be used as a term of comparison. I will estimate time windows of 30, 60 and 120 months.

In addition to the equal-weighted portfolios, that is, the benchmark used in the article by DeMiguel et al. (2009), value-weighted portfolios will be performed. Another type of portfolios, the minimum variance portfolios and the tangency portfolio - when the Sharpe ratio is maximized - will also be created.

Finally, the Sharpe ratio and the certainty-equivalent return ratio will be used as performance analysis. Also, I will use the 5-factor models of Fama and French (2015).

3.1. Efficient Portfolios.

This study starts by defining the notation and describe the cross-section of stock returns in a matrix form, r:

$$(1)\ r = \mu + \sqrt{\Sigma \in}\ ;$$

$$(2)\ \mu = (\mu_{1,t}, \mu_{2,t}, \dots \mu_{N,t})^T\ ;$$

$$(3)\ \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_{NN} \end{bmatrix};$$

$$(4)\ \in = (\in_{1,t+1}, \in_{2,t+1}, \dots, \in_{N,t+1})^{T}.$$

Where μ – Nx1 vector of expected returns; $\sum$ - NxN covariance matrix; $\in$ - Nx1 vector of independent mean zero, unit variance noise terms.

$$(5)\ r_{p,t+1} = w^{T} r;$$

$$(6)\ w = (w_1, w_2, \dots, w_N)^{T};$$

$$(7)\ E_t\left[r_{p,t+1}^{2}\right] = w^{T}\mu;$$

$$(8)\ E_t\left[r_{p,t+1}^{2}\right] - E_t\left[r_{p,t+1}\right]^{2} = w^{T} \textstyle\sum w.$$

Where equation (5) is the return of the portfolio and (6) a vector of portfolio weights. Then, equation (7) and (8) are, respectively, the expected return of the portfolio and the portfolio variance.

3.1.1. Markowitz (1952) mean-variance optimal stock portfolio.

Estimates of each stock's predicted returns, as well as individual stock return variances and covariances, are required to build Markowitz (1952) mean-variance optimum portfolios.

Here we have the approach of Markowitz (1952) to create mean-variance optimal stock portfolio, that is, an efficient portfolio construction model, which involves distributing wealth, w, among stocks to optimize the portfolio's anticipated return under a certain risk, the portfolio variance, Σp.

$$(9)\ \max_{w} w^{T}\ \mu;$$

$$(10)\ s.t.\, w^{T}\Sigma w = \Sigma p;$$

$$(11)\ w^{T}e = 1;$$

$$(12)\ w = \frac{\Sigma^{-1}\mu}{e^{T}\,\Sigma^{-1}\mu}.$$

Where e being a Nx1 vector of ones. The equation number (12) gives an optimal portfolio policy that is obtained by a maximum Sharpe ratio. This optimization problem can be solved numerically in R.

3.1.2. Minimum-variance portfolios - MVP

Beginning with the most straightforward and, when limited, effective application of mean-variance models: minimizing the variance of a portfolio. Where if investor believes that predicted returns are cross-sectionally consistent, meaning that $\mu = e * c$ (where c is a constant), therefore cross-sectional variations in covariances will be the only source of variation between stocks. In this minimum variance portfolios, we do not take into account expected returns.

$$(13)\ w_{MVP} = \frac{\Sigma^{-1} e}{e^T \Sigma^{-1} e}.$$

The portfolio that has the lowest variance is said to be the minimum-variance portfolio. A quadratic objective function with linear constraints is minimized using quadratic programming. The variance of the portfolio returns serves as the goal objective function.

As it was stated before, the covariance-only portfolios are the minimum variance portfolios researched in the finance literature (Jagannathan and Ma, 2003), in which projected returns are omitted and portfolio risk is decreased. The package *nlshrink* is used to implement the non-linear shrinkage method of Ledoit and Wolf (2017). Also, I used the package *quadprog* to determine the minimum-variance portfolio's weights.

The efficient portfolio will be the one with least variance so finding the efficient portfolio is the purpose. Based on Ruppert and Matteson (2011), to create efficient portfolios with unlimited number of assets, they employ quadratic programming. The goal is to minimize the function that determines portfolio variance with weights, w:

$$(14)\ w^T \sum w\ ;$$

The asset weights in the portfolio are represented by w (Nx1 vector), while the asset returns are represented by $\sum$ the covariance matrix (NxN).

The sum of weights is equal to 1 so:

$$(15)\ w^T 1 = 1$$

A quadratic objective function with linear constraints is minimized using quadratic programming. The variance of the portfolio return is the target function in portfolio optimization methods. The weights of N assets, which are expressed by a Nx1 vector x, are the example of the N variables that make up the objective function used by the authors.

Using the function solve.Qp in the quadprog package in R, Ruppert and Matteson (2011) minimized the following quadratic objective function:

$$(16)\ \frac{1}{2} x^T D x - d^T x;$$

$$(17)\ A^T x \geq b_{neq}.$$

Where D is a NxN matrix, d is a Nx1 vector and $\frac{1}{2}$ is to maintain the consistence with R. Moreover, inequality and equality constraints are two different forms of linear constraints on x. Thus, where m is the number of inequality constraints, A^T is a m x N matrix and b_{neq} is a m x 1 vector.

Given that $x = w$, d is a Nx1 vector of zeros, and $D = 2\sum$:

$$(19)\ w^T \textstyle\sum w \text{ with } A^T x \geq b_{neq}.$$

Then, using the function *nlshrink_cov* in the program I estimated the population eigenvalues. The weight is set to reduce quadratic loss. A one-factor model provides the prior and the cross-sectional average of all random variables is used to calculate the factor. So, if the number of variables exceeds the number of observations, this estimator is assured to be invertible and well-conditioned.

3.1.3 Tangency portfolios - TANG

The portfolio that optimizes the ratio of excess return to portfolio volatility is known as a tangency portfolio (Tobin, 1958). The portfolio can be estimated by using the following objective function:

$$(20)\ \max_{m} \frac{m'w - rf}{\sqrt{w' \sum w}},$$

$$\text{With } w^T = 1$$

The risk-free rate, or rf, is taken to be zero (see Feldman & Reisman, 2003 and Eun & Resnick, 1988).

The same procedure as the one explained before for minimum variance portfolios will be used for obtaining the tangency portfolios. In this case, and since they are based both on the covariance matrix and in the expected returns, they will be formed maximizing the Sharpe ratio.

The portfolio that optimizes the Sharpe ratio is on the mean–variance efficiency frontier, according to Market Portfolio Theory. This portfolio is known as the tangency portfolio because it coincides with the point where the Capital Market Line is tangent to the frontier (Kourtis, 2016).

Because it is impossible to have expected returns larger than the expected return of the company with the greatest expected return without short sales, the efficient frontier will differ for portfolios with and without short sales (Ruppert and Matteson, 2011). When short sales are permitted, however, there is no maximum limit on the predicted return (or on the risk).

3.1.4 Value-weighted market index portfolios – VAL

The value-weighted market portfolio is the most advantageous method in the CAPM framework (DeMiguel et al., 2009). The fraction of each stock in this portfolio is calculated by dividing its market value (price per share multiplied by the number of shares outstanding) by the total market value of all the stocks (Bodie et al., 2014).

The value-weighted portfolios have different weights that are calculated according to the market capitalization of the shares. In this case, the weight is determined by the market capitalization of the stock. It is a weighted average in which the weighting factors correspond to the proportion of the market capitalization of that asset in the set of all assets in that portfolio. All this, taking into account that the market capitalization at the moment to which it refers, and it varies over time and therefore must take into account the information available at the time the portfolio is being built. Market capitalization effectively quantifies a company's size.

The model assumes that market prices accurately reflect all available information and represent a security's "fair" worth. This makes it clear that the market portfolio is a portfolio that is weighted according to market capitalization. The market capitalization of a corporation is divided by the aggregate market capitalization of all securities to determine its weight.

3.1.5 Naive Equal-Weighted Portfolios - EW

Finally, we have the case in which the investor assumes that expected returns and covariances are both cross-sectionally constant. So, we have $\mu = e * c$ and $\sum = C * I$.

$$(21)\ w_{EW} = \frac{1}{N} e.$$

Equal-weighted portfolios invest evenly in each accessible stock without concern for projected returns or covariances. DeMiguel et al. (2009) conclude that outperforming EW portfolios is extremely difficult. According to the authors, the naive rule is allocating 1/N of wealth to each of the N assets available for investment at each rebalancing date. No limits were imposed on the weights assigned to each stock in the portfolios evaluated in this section, save that the weights must amount to one.

To obtain these portfolios, without regard to expected returns or covariances, it is required to invest equally in each available stock. In the studies of DeMiguel et al. (2009), as it was mentioned before, the authors employ EW portfolios as a naive benchmark to measure out-of-sample performance increases of optimized portfolios and demonstrate that outperforming EW portfolios is extremely difficult. The equal-weighted portfolios have the same weight, so it is an arithmetic average, that is, it is giving equal weight to each of the returns.

3.2 Large Dimension Covariance Estimation – Ledoit and Wolf (2017)

In this section will be presented the method for estimating the covariance matrix, as it was used by Lyle and Yohn (2021). This study explores a well-conditioned and asymptotically more accurate estimator than the sample covariance matrix.

The sample covariance matrix is the only estimator that is both well-conditioned and accurate. The linear shrinkage method of Ledoit and Wolf (2003, 2004) discovers a covariance matrix estimator that is optimal in the one-dimensional space of convex linear combinations of the sample covariance matrix with the correctly scaled identity matrix. The estimator presented in the research's study is well-conditioned and asymptotically more reliable than the sample covariance matrix. It is the asymptotically best convex linear combination of the identity matrix and the sample covariance matrix. This because as its number of observations and the number of variables approach infinity simultaneously and it is defined with regard to a quadratic loss function.

Later, the researchers created the non-linear shrinking method (Ledoit & Wolf, 2017). This implies that the sample covariance matrix's smaller eigenvalues are pushed upward, and the bigger ones are dragged downward by an amount that is distinct for each eigenvalue. This provides the necessary N degrees of freedom because there are N eigenvalues. The more difficult aspect is determining the ideal shrinking intensity for each eigenvalue. The authors suggest employing a loss function that represents an investor's or researcher's goal when utilizing portfolio selection. Kan and Smith (2008) have already taken this into consideration.

The algorithm used to implement the non-linear shrinkage method was provided by the package *nlshrink*. This package is based on Ledoit and Wolf (2004, 2015, 2016) work, non-linear shrinkage estimates of population eigenvalues and covariance matrices.

3.3 Performance analysis

The present study will use the Sharpe ratio (SR), the Certainty-Equivalent (CE) and the 5-factor model of Fama and French (2015) as distinct performance metrics:

3.3.1 Sharpe ratio - SR

The Sharpe Ratio (SR) is the primary performance metric and is calculated by dividing the average portfolio excess return during a sample period by the standard deviation of those returns. It calculates the reward-to-volatility-total trade-off (Sharpe, 1998).

$$(22)\ SR = \frac{\overline{r_{p,t+1}} - \overline{r_f}}{\sqrt{\sigma^2_{p,t+1}}}$$

Where r_p is the average return of portfolio p, r_f is the average risk-free rate and σ_p the standard deviation of portfolio return.

3.3.2 Certainty-Equivalent ratio - CE

The Certainty-Equivalent ratio provides the real return that investors would be ready to take to create a risk-free investment, instead of getting a greater return from a riskier portfolio. The rate that a risk-free investment would have to have the same utility value as the risky portfolio is known as the Certainty-Equivalent rate (Bodie et al., 2014). The utility values of rival portfolios may be naturally compared using the confidence equivalent rate of return. It is a widely used evaluation metric in this context:

$$(23)\ CE = \hat{\mu}_p - \frac{\gamma}{2}\hat{\sigma}^2$$

Where:

- $\frac{\gamma}{2}$ is the coefficient of risk aversion;
- $\hat{\mu}_p$ and $\hat{\sigma}^2$ are the mean and variance of the portfolio,
- The value of γ will be 1, as is defined by the DeMiguel et al. (2009).

3.3.3 Five factor model (Fama & French, 2015)

Following Fama and French (2015) I will use the five-factor model, to compare the performance of these portfolios using:

$$(24)\ r_{p,t} - r_{f,t} = \alpha_p + \beta_{p1}\left(r_{m,t} - r_{f,t}\right) + \beta_{p2}\ SMB_t + \beta_{p3}\ HML_{\ t} + \beta_{p4}\ RMW_t + \beta_{p5}\ CMA_t + \varepsilon_p.$$

Where:

$r_{p,t}$ - Return of portfolio p on period t;

$r_{f,t}$ - Risk-free rate on period t;

$r_{m,t}$ - Return of the market portfolio on period t;

SMB_t - difference in returns between a portfolio of small stocks and a portfolio of large stocks;

$HML_{\ t}$ - difference in returns between a portfolio of high book-to-market stocks and a portfolio of low book-to-market stocks;

RMW_t - Profitability factor - the difference between the returns on diversified portfolios of stocks with robust and weak profitability;

CMA_t - Investment factor - difference between the returns on diversified portfolios of the stocks of low and high investment firms;

$\beta_{p1}, \beta_{p2}, \beta_{p3}, \beta_{p4}, \beta_{p5}$ - factor coefficients;

ε_p – zero-mean residual;

Treating the parameters in (24) as true values instead of estimates, if the factor coefficients take all variation in expected returns, the intercept α_p is zero for all securities and portfolios. In terms of performance evaluation, alpha refers to the excess return, as a result of market, size, value, profitability, and investment risk characteristics, generated over the expected return (Sarwar et al., 2018).

Chapter 4 - Data

4.1 Data Extraction

Landis and Skouras (2021) propose a way of extracting data from the Refinitiv Eikon to prevent survivorship bias and reduce problems with data accuracy. The authors concentrate their efforts on Refinitiv Eikon since it is the most extensively used international data source amongst academics. Beginning in the late 1990s, the XETRA platform/exchange took over the trading of stocks having initially listed on the Frankfurt Stock Exchange.

Because Refinitiv Eikon only covers the most recent stock exchange classifications, eliminating outdated exchanges would result in the removal of stocks which are in those exchanges since they were dismissed from a major exchange due to a bad performance. When assessing the viability of Refinitiv Eikon to be used in research that included large numbers of individual stocks in markets outside the United States, Ince and Porter (2006) outlined numerous large firms for which the platform preserves data, but the data is not included on the relevant constituent list and therefore will not be retrieved by the researcher.

4.2 Read static data and select instruments

Using Refinitiv Eikon and following Landis and Skouras (2021), the first step when gathering raw data is to collect securities from the German market, searching in the constituent lists. The most used lists, that are the ones that include both active and dead stocks, and the Worldscope lists (supplied by RE). Taking this into account, I have selected the lists FDEALL1-8, DEADDE1-9 and WSSECBD, WSBD1-2. For each one of these lists, I have downloaded the instruments with the variables: "Type", "DSCD", "BDATE", "ENAME", "EXMNEM", "GEOGN", "ISIN", "ISINID", "LOC", "PCUR", "TRAC" and "TYPE" and started with 56936 observations.

Constituent lists incorporate instruments that many researchers do not want to use in their studies and generally seek to exclude, for example, instruments that are not equities and not contained in the country of interest but are traded there. So, I used the filters prepared by Landis and Skouras (2021) to identify common stock securities in the German market.

The first step is to apply stock filters based on static information and, for that, select between securities the ones that are equities, that is, select securities with type “EQ” and there were 53986 observations, excluding 2950. The following step is to select instruments from the German stock exchange by filtering observation where the variable "EXNEM" is one of "BER", "FRA", "STU", "HAM", "DUS", "MUN", "XET". This excluded 47 observations.

After that, it is necessary to exclude non-common stocks. A benefit of employing this filter is that it cannot induce survivorship bias because stock classification as common does not shift over time. In this way and following the author's work, I have filtered all stocks with security type code - datatype "TRAC" – for not having "ORD", "ORDSUBR", "FULLPAID", "UKNOWN", "UNKNOW", "PRF", "PART" and "KNOW", and resulting in 52766 observations.

I have utilized stocks' extended names and filtered out any stocks with country-specific identifiers in their "ENAME", such as "BONUS RIGHT", "NIL PAID", "REIT", "GENUSSSCHEINE, PREFERENCE", "DEPOSITORY RECEIPTS", "CHESS DEPOSITORY INTEREST", "DEFERRED", "PARTICIPATE CERTIFICATE", "LIMITED PARTENERSHIP", "SWAP", "NPV", "GS", "PF", "SUB RIGHTS", "CDI", "RSP", "UNIT", "GDRS", "TRUST", "REIT" and "REFINERY" ending with 48967 observations. This filter was applied to all stocks except the ones in which the "EXMNEM" was equal to "XET", because the authors mentioned that they do not apply it on stocks traded on the Xetra exchange. The reason is that the mentioned authors aim to maintain data from stocks that were cross-listed on Xetra and Frankfurt so that their time series can be combined. According to their findings, Germany is the only foreign market where choosing an exchange to analyse stock data primarily based on the stock's leading listing exchange is inaccurate.

Also, it is important to exclude securities for which the Refinitiv Eikon geographical categorisation - datatype "GEOGN" - differs from the country's identity and all securities traded in a currency except for the country's local currency – "E" and "DM". These two steps left 6272 observations. The final step is to test if there are duplicated local codes, and after removing them, this study was left with 4601 observations because 1671 were duplicated.

4.3 Clean the return index and prices series and compute monthly returns

The following step is to, extract from RefinitivEikon, Return Index and Prices time series data. I have obtained time series data for the variables RI, RI#S and RI#T.

The second two variables are used to solve padding problems. In qualifier #S, instead of padding with the most recent value, NA is always displayed. In the #T qualifier, once the series has stopped trading, it displays NA instead of padding, although it still pads for non-trading days.

As a result, instead of padding the last considerable value, N/A is shown. The series will remain padded when the exchange does not report a stock's closing price.

To deal with the Frankfurt/Xetra stock exchange problem, Landis and Skouras (2021) generated a single return index by merging cross-listed stocks. This was caused due to the fact that, in the late 1990s, the XETRA platform took over the trading of companies having primary listings on the Frankfurt Stock Exchange. By cross-listed stocks, it is meant stocks with datatype "EXMNEM" equal to "FRA" and "XET" that had equal "LOC" or "ISIN" datatypes. So, I started my data in 2000 to avoid this problem and filtered the variable "ds_code" for not having duplicated "ISIN" and "LOC" when they were both from Xetra and Frankfurt stock exchange, ending up with 3398 stocks.

I have extracted the data from 01-01-2000 to 30-09-2021. Since Refinitiv Eikon adjusts the data to the second decimal, to prevent this, I obtained data with 6 decimal points.

The first thing to do is to identify the last trading dates and, for that, to compute the continuous returns. To do this, it is necessary to have the return index and the return index #T, both with their return the prior period and the respective logarithm. Also it is necessary to have the return index #S.

$$(25)\ return = \frac{ri - ri_{t-1}}{ri_{t-1}};$$

$$(26)\ continuous\ return = \log{(ri - ri_{t-1})}$$

When looking if there are observations from the RI#T series with a last non-zero return date more significant than the RI data set the result was zero observations. Considering observations from the RI series with a final non-zero return date bigger than the RI#T data collection, the result was 305 observations. Then I used the most recent non-zero return as the latest trading day.

Following Landis and Skouras (2021), I excluded stocks for which the return estimated from the return index is zero in at least 95% of the sample, which is 2511 observations. Then, I imposed an implausibility filter for eliminating stocks with non-zero daily returns that are either non-negative or non-positive for more than 98% of the time. There were 74 observations to exclude.

Then, applying the filter for outlier errors that accounts for excessive returns. If the return on date t is more than 100% (or less than -50%) and the return on day t+1 is less than -50% (or more than 100%), both days are removed. There were 2199 observations of this type.

To verify if the RI#S series has non-trading days, I have determined which days have a small proportion of non-zero returns which could indicate a holiday. The authors removed days for which non-missing or non-zero returns account for less than 0.5 percent of the total of stocks listed for that country over all days, excluding expected holidays or days on which markets were typically closed on a country-by-country basis. I used a less strict with 20% and eliminated 153 observations. Also, I have limited my focus to stocks with more than a minimum number of observations to prevent stocks with a short record, which might be due to Refinitiv Eikon limits.

Since all datatypes are calculated using the price at the latest trading date in Refinitiv Eikon, missing values are presented - it has padded prices. For this reason, it is necessary to avoid staleness and apply a filter for eliminating all observations after 30 consecutive identical prices until the following change. For that, I downloaded prices in the same way as the return indexes collection, that is, collecting prices and prices with the qualifiers #S and #T.

Applying this staleness filter to prices and then filtering the return indexes based on the dates and "ds_codes" of these obtained prices, results in 4408931 stocks. Then, as recommended by the authors, they removed penny stocks, which are stocks with share prices below one dollar. This filter resulted in sample of 3791449 observations.

Finally, it would be possible to look for adjustment inconsistencies and sort out situations where the adjusted prices are offered. The prices inferred from unadjusted prices and adjustment factors differ significantly (by more than 5%). If the adjusted price differs from the unadjusted price multiplied by the adjusted factor, it was almost likely a mistake, thus eliminating it. According to the authors, this standard filter has a small impact, eliminating only 0.4 percent of global stock days. Therefore, I have not applied it.

The final step is to create monthly return series and, after that, calculate monthly returns as a sum of the continuous daily returns, then convert them to a discrete rate of return. Monthly returns are calculated from ongoing daily returns, converted to a discrete compounded rate at that rate. After that, check for outliers in the monthly series, removed 127 observations. With a final sample of 3791449 daily observations and 185023 monthly observations.

To measure performance using the FF multifactor model, stock returns should be calculated in dollars since the factors were also calculated in dollars. Thus, it was necessary to get the last exchange rate per month, compute the exchange rate monthly returns and convert monthly returns in euros into USD.

4.4 Rolling window estimation

To obtain the portfolio weights I started by identifying the first and last monthly observations, the first being February 2000 and the latest in September 2021. I have defined the last trading date as August 2021 because there is only complete information until there.

Then, I created a month-by-month data set containing the start and end dates of the estimation window. That is, the portfolios where reviewed monthly with estimation windows of 30, 60 and 120 months.

Additionally, it is important to note that all data regarding the 5-factor model of Fama and French (2015), was taken from the website Professor Kenneth R. French Data Library.

Chapter 5 – Results

5.1 Portfolio Performance

The portfolio weights are computed with and without short sales using estimation windows of 5-year (60 months) information starting in February 2000 and ending in August 2021, which results in 200 periods. There are also periods of 30 and 120 months which gives 230 and 140, respectively.

After that, I calculated the ex-post portfolio returns. The same will be performed for constrained portfolios, that is, constrain the weights for long-short portfolios to between -2.5% and 2.5% and long-only portfolios to between 0% and 2.5%. This can only be applied in the minimum variance portfolios and the tangency portfolios with the long-short restriction. In the long-only portfolios, this cannot be solved using quadratic programming. Also, it is not possible to apply restrictions on weights on equal and value-weighted portfolios.

The performance of the EW, VAL, TANG, and MVP portfolios is shown in Table 1, panels A and B. Both panels' columns (1-6) give the outcomes for the naive EW and the VAL portfolios. Results for the MVP and TANG portfolios, both long-short and long-only, are shown in columns (7) through (18). Unlike long-only portfolios, long-short portfolios permit short selling. The only restriction Panel A places on the weights assigned to each stock is that they must add up to one. The weights for long-short portfolios are restricted in Panel B. Within each portfolio, the mean, volatility (Std.), Sharpe ratio (SR), and Certainty-Equivalent (CE) ratio are presented. All portfolios, in both tables, have their results presented for the three-time windows, that is, 30, 60 and 120-months periods.

Table 1 shows that the value weighted portfolios have a significant positive performance. They present the higher Sharpe ratio except for the 30-months time window. The results show not to be in accordance with what is mentioned in the literature as Mostowfi and Stier (2013) defend that this value-weighted portfolio was outperformed by the minimum variance portfolio. Plyakha et al. (2021), also report that the equal-weighted portfolios beat the value-weighted portfolios in terms of average total return, four factor alpha, and monthly rebalancing Sharpe ratio.

Analysing the Sharpe ratio values of the tangency portfolios, we can observe that these are the ones that present the worst results, especially when they are long-short portfolios, as they have a very poor performance. This conclusion is in line with Jagannathan and Ma (2003) who conclude that tangency portfolios do not outperform the overall minimum variance portfolios and that they can be significantly outperformed by the other strategies.

Regarding the Certainty-Equivalent ratio, the values are quite identical in both value and equal-weighted portfolios. The second ones have a slightly better performance but both are very positive.

Overall, the portfolios that show greater stability with changing time windows are the equal-weighted portfolios. We can also observe that, except for these portfolios and the value-weighted where there is no such distinction, the difference in results between constrained and unconstrained portfolios is relatively similar.

Finally, and now looking at the distinction between long-short and long-only portfolios, we can directly compare minimum variance portfolios with tangency portfolios. It can be seen that overall, especially for the second ones, the values are much better when it comes to long-only portfolios.

Concerning the observed results, no model consistently outperforms the equal-weighted portfolios from a global results prespective, as demonstrated by DeMiguel et al. (2009). This shows that the advantage from optimum diversification is significantly compensated by estimating error out of the sample. Both tangency and minimum variance portfolios perform very similarly to EWs with respect to these ratios.

Tangency portfolios perform much better in terms of Sharpe ratio when a no short sales restriction is imposed on them. As studied by Jagannathan and Ma (2003), minimum variance portfolios perform quite well in terms of Sharpe ratio. These are not superior values, as are the Certainty-Equivalent ratio values. Overall, looking at the time windows, the portfolios that perform least well are those that are estimated with 30-month windows.

Table 1 - Panel A: Portfolio Summary Statistics – UNCONSTRAINED

							Long-short						Long-Only					
Portfolio:	EW			VAL			MVP			TANG			MVP			TANG		
	30	60	120	30	60	120	30	60	120	30	60	120	30	60	120	30	60	120
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Mean	0.021	0.016	0.019	0.023	0.019	0.017	0.013	0.012	0.017	0.028	-0.034	0.015	0.015	0.011	0.017	0.020	0.017	0.017
Std.	0.038	0.035	0.033	0.049	0.028	0.019	0.030	0.030	0.035	0.087	0.474	0.147	0.030	0.029	0.038	0.043	0.037	0.032
SR	0.547	0.462	0.588	0.473	0.679	0.868	0.427	0.402	0.492	0.329	-0.071	0.105	0.497	0.371	0.459	0.458	0.468	0.533
CER	0.019	0.015	0.019	0.022	0.019	0.017	0.012	0.012	0.017	0.025	-0.146	0.005	0.015	0.010	0.017	0.019	0.016	0.017

Table 1 - Panel B: Portfolio Summary Statistics –CONSTRAINED

	Long-short					
Portfolio:	MVP			TANG		
	30	60	120	30	60	120
	(7)	(8)	(9)	(10)	(11)	(12)
Mean	0.012	0.012	0.017	0.028	-0.033	0.015
Std.	0.030	0.030	0.034	0.088	0.474	0.146
SR	0.427	0.401	0.491	0.329	-0.071	0.105
CER	0.012	0.012	0.017	0.025	-0.146	0.005

Table 1: The EW, VAL, MVP, and TANG portfolios' monthly performance measures are shown in the table. It provides information on important portfolio performance measures and weighted attributes. The out-of-sample monthly mean, standard deviation, Sharpe Ratio and Certainty-Equivalent ratios are represented by Mean, Std., SR and CER, respectively.

5.2 Ex-post portfolio performance

5.2.1 Differences between Sharpe ratios

Table 2 shows tests for the differences in Sharpe ratios, and the significance levels obtained using a two-tailed Wald test.

The differences between the value weighted portfolios and the minimum variance portfolios for the 60- and 120-month windows are positive and significant. The same is true for the differences between the Sharpe ratios of value weighted portfolios and equal weighted portfolios for the 120-month window.

The differences between tangency portfolios and the other three portfolios are negative and significant when it comes to tangency portfolios with short-sales allowed and in the 60- and 120-month windows.

The difference between minimum variance portfolios and equal weighted portfolios is always negative. Again, for all portfolios, the differences between constrained and unconstrained results are not significant.

Finally, and showing that tangency portfolios are again the worst performers, the difference between these and value weighted portfolios is always negative and the difference between these and equal-weighted portfolios is mostly negative as well.

DeMiguel et al., (2009) demonstrate that no other portfolio construction outperforms the 1/N rule in terms of Sharpe ratio, certainty-equivalent return, or turnover, indicating that the benefit of optimal diversification is very much compensated by estimation error out of sample. In this study, and relatively to the Sharpe ratio results, we cannot say that this is totally true because of the results related to value-weighted portfolios. These portfolios present results that, although very similar, turn out to be higher except for the 30-month windows.

5.2.2 Fama and French 5-factor models

The portfolio performance estimated using the 5-factor Fama and French model is shown in Table 6. Panel A, B and C correspond to the different time windows, 60, 30 and 120, respectively.

The alpha is very significant in the 120-month window for almost all portfolios. The values for the SMB variable are always negative for all portfolios and in all time windows. The same happens for the HML variable, with the exception of the 60-month window. The difference in overall values between time windows is very notorious.

The results related to the sensitivity to market are not very common except for the tangency long-short portfolio for the 120-months time window. However, no conclusions were reached as to what caused this, in order to possibly justify such results.

Finally, the minimum variance portfolios show very significant values in the 120-month window, both in long-only and long-short portfolios.

Lyle and Yohn (2021) demonstrate that the EW portfolios tend to have higher sensitivity to the market portfolio. Looking at the portfolio beta (sensitivity to the market portfolio, β), the values vary a lot considering the time window and the restrictions on short sales. It is not possible to identify a pattern. The highest value corresponds to the tangency portfolio, with short-sales and relative to the 120-month window.

Table 2 - Panel A: Differences between ratios – UNCONSTRAINED

Portfolios:			MVP - EW	VAL- EW	VAL - MVP	TANG - MVP	TANG – VAL	TANG - EW
Time window			(1)	(2)	(3)	(4)	(5)	(6)
30	SR	Long-short	-0.1191	-0.0734	0.0458	-0.0977	-0.1435	-0.2169
		Long-only	-0.049	-0.0734	0.1811	-0.0392	-0.0149	-0.0882
60	SR	Long-short	-0.0606	0.2163	0.2769*	-0.4726**	-0.7495***	-0.5332***
		Long-only	-0.0915	0.2163	0.3078**	0.0972	-0.2106	0.0057
120	SR	Long-short	-0.0960	0.2802*	0.3763**	-0.3868**	-0.7631***	-0.4829***
		Long-only	-0.1284	0.2802*	0.4086***	0.0733	-0.3353**	-0.0551

Table 2 - Panel B: Differences between Sharpe ratios –CONSTRAINED

Portfolios:			MVP - EW	VAL- EW	VAL - MVP	TANG - MVP	TANG – VAL	TANG - EW
Time window			(1)	(2)	(3)	(4)	(5)	(6)
30	SR	Long-short	-0.1191	-0.0734	0.0458	-0.0977	-0.1435	-0.2169
		Long-only	-0.0647	-0.0734	-0.0086	-0.0647	-0.0561	-0.1295
60	SR	Long-short	-0.0606	0.2163	0.2769*	-0.4726**	-0.7496***	-0.5332***
		Long-only	-0.0953	0.2163	0.3117**	0.1011	-0.2106	0.0057
120	SR	Long-short	-0.0960	0.2802*	0.3763**	-0.3868**	-0.7631***	-0.4829***
		Long-only	-0.1652	0.2802*	0.4454***	0.1152	-0.3301**	-0.0499

Table 2: The table represents testing the difference between Sharpe ratios. Significance levels are * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$ and are based on a two-tailed Wald Test.

Table 3 – Panel A: Factor sensitivities (60-month estimation window)

Portfolio:	EW	VAL	MVP(LS)	TANG(LS)	MVP(LO)	TANG(LO)
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept (α)	0.009+	0.019***	0.010*	-0.011	0.009*	0.012*
Sensitivity to Market	0.255+	0.001	0.040	-1.736	0.010	0.148
Sensitivity to SMB	0.125	-0.069	-0.034	-2.191	-0.075	0.140
Sensitivity to HML	-0.227	-0.173	-0.318	-3.752	-0.422	-0.492
Sensitivity to RMW	0.379	-0.325	0.289	-6.765	0.219	0.294
Sensitivity to CMA	-0.136	-0.485*	0.131	-7.448	0.112	-0.184

Table 3 – Panel B: Factor sensitivities (30-month estimation window)

Portfolio:	EW	VAL	MVP(LS)	TANG(LS)	MVP(LO)	TANG(LO)
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept (α)	0.016*	0.024***	0.011*	0.021	0.014*	0.012
Sensitivity to Market	0.052	-0.033	0.098	0.070	-0.027	0.094
Sensitivity to SMB	0.377	-0.046	-0.467	-0.445	-0.068	0.495
Sensitivity to HML	0.227	-0.266	0.055	1.436	0.068	0.413
Sensitivity to RMW	0.801	-0.489+	0.375	2.221	0.422	1.276
Sensitivity to CMA	0.702	-0.778**	-0.299	-2.988+	0.061	-0.244

Table 3 – Panel C: Factor sensitivities (120-month estimation window)

Portfolio:	EW	VAL	MVP(LS)	TANG(LS)	MVP(LO)	TANG(LO)
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept (α)	0.017***	0.016***	0.018***	0.006	0.017***	0.014***
Sensitivity to Market	0.052	-0.008	-0.192*	0.646	-0.126	0.064
Sensitivity to SMB	-0.117	-0.056	-0.521*	-0.042	-0.478+	-0.029
Sensitivity to HML	0.103	0.025	-0.102	-3.202*	-0.006	-0.354
Sensitivity to RMW	0.605	-0.195	0.597	0.385	0.844+	0.435
Sensitivity to CMA	0.275	-0.556**	0.337	2.467	0.377	0.347

Table 3: Sensitivities to each of the five factors identified by Fama and French (2015) are represented by the terms Sensitivity to Market, HML, SMB, RMW, and CMA.

Significance levels are + $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. LS and LO correspond to long-short and long-only portfolios, respectively.

Chapter 6 - Conclusions

This dissertation evaluates the performance of portfolios constructed under different methodologies and restrictions using data from the German market. Portfolios are constructed with restrictions on the existence of short sales, in the weights and for different estimation windows. The Sharpe ratio, Certainty-Equivalent ratio and the Fama and French 5-factor model are used as performance measures. Data was retrieved from January 2000 to September 2021 for stocks from the German market.

One of the main conclusions drawn was that there is no portfolio construction strategy that consistently outperforms the equal-weighted strategy, as studied by DeMiguel et al. (2009). Sharpe ratios for the equal-weighted approach are often higher (or statistically similar) then those of the limited approaches. Nevertheless, both tangency and minimum variance portfolios perform very similarly to EWs with respect to these ratios, except for the tangency portfolio when short sales are allowed.

The sample-based mean-variance strategy's Sharpe ratio is significantly worse than that of the 1/N approach. This demonstrates that the inaccuracies in estimating means and covariances remove all profits from the optimal, as opposed to naive, diversification. Additionally, it can be observed that most sample-based mean-variance model modifications that have been suggested in the literature to address the issue of estimate error fail comparing to the equal weighted portfolio strategy.

In summary, there is no single model in the literature that constantly produces a Sharpe ratio and a Certainty-Equivalent return simultaneously larger than that of the EW portfolio. However, it should be noted in this study that although the equal weighted portfolios are the most stable in all the restrictions imposed, the value weighted portfolios show very positive results especially when analysing the Sharpe ratio results for the 60- and 120-month windows.

The construction of efficient portfolios has advanced significantly, as demonstrated by DeMiguel et al. (2009), but more work must be done to enhance the estimation of the moments, particularly expected returns.

For future research it would be interesting to study this topic by developing some of the portfolio constructions suggested by DeMiguel et al., (2009), such as optimal combinations of portfolios. A portfolio constructed with a mixture of equally weighted and minimum-variance portfolios or applying a Bayesian approach to estimation error could be an interesting topic. It would also be remarkable to study the ex-post performance taking into account a turnover ratio.

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