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Behavior-Based Price Discrimination with a General Demand *

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Abstract

This paper offers a complete picture of the impact of behavior-based price discrimination on profits, consumer surplus, and welfare in markets with a general demand function, where consumers and firms can discount the future at different discount factors. Regardless of the demand function considered, in comparison to uniform pricing, BBPD reduces firms' second-period prices and profits. In contrast, we show that new results arise regarding the impact of BBPD on first-period prices. Under perfectly inelastic and CES demand, the firm-side effect is null and the consumer-side effect fully explains the increase in first-period prices. This is no longer the case when the price elasticity of demand varies with price level. Specifically, we show that the firm-side effect can lead firms to raise first-period prices, even when consumers are myopic. We also show that, depending on the demand function considered, the consumer side effect can act to reduce or increase first-period prices. The overall impact of BBPD on first-period prices depends on the interplay between these two effects. Our analysis reveals that the output effect and consumer switching plays an important role in explaining the impact of BBPD on welfare. When discount factors are equal, BBPD may have a positive or negative impact on consumer surplus and social welfare, which contrasts with the result that BBPD is beneficial for consumers under a unit and CES demand. For a linear demand function, we identify the regions for firms and consumers discount factors where BBPD can simultaneously enhance or reduce total discounted profits, consumer surplus, and social welfare.

1 Introduction

In the era of the Internet of Things, Algorithms, Artificial Intelligence, and limitless cloud storage, data collection for price discrimination is virtually infinite. Reports from the OECD and the European Commission on price discrimination in the digital economy (OECD, 2016, 2018a,b and EC, 2018a,b, 2022) identify the growing availability and accessibility of Big Data and Big Analytics as the main factors underlying the increasing use of new forms of price discrimination, such as personalized pricing and behavior-based price discrimination (henceforth, BBPD). This latter form of price discrimination was introduced in the economics literature by Fudenberg and Tirole (2000). For these authors, "behavior" is simply related to a consumer's purchase history,

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therefore, when companies have information on consumers' purchasing behavior, they can use that information to charge different prices to customers with different histories (e.g., old customers versus new customers).

The New York Times ran a widely-read article entitled "How Companies Learn Your Secrets," which delved into Target's attempt to construct a predictive model for identifying pregnant shoppers and the reasoning behind this pursuit.[1] Target uses a "unique code" to keep track of every shopper's purchases, as well as their reactions to prices and location data, which includes the distance between the consumer and Target's store or their competitors' stores, and even the aisle they're in (Ezrachi and Stucke, 2016).

In short, the formula behind firms' price discrimination strategies can be explained by a multitude of variables, including but not limited to consumer demand, shopping habits, product and store preferences, electronic device usage, and even emotions. Additionally, there may be many other variables that we have not yet considered.

As BBPD is expected to become increasingly prevalent (OECD, 2018), it is crucial to have a comprehensive understanding of the markets in which it is being implemented. It is widely recognized that in many markets, consumer demand fluctuates with changes in prices, and consumers' decisions involve not only which firm to buy from but also the quantity of goods to purchase. In most cases, when prices rise, consumers tend to buy less of the product or service, and when prices fall, they tend to buy more. If we assume that consumer demand is invariant to price changes, we are essentially assuming that consumers will purchase the same amount regardless of the price level. This assumption is unlikely to hold in most markets and can lead to inaccurate predictions of market outcomes.

Despite this fact, the literature on BBPD has predominantly relied on the assumption of a unit (perfectly inelastic) demand (e.g. Fudenberg and Tirole, 2000, Esteves, 2010; Choe, et al. 2018, to name a few). Because the aggregate output remains unaffected by price discrimination, the negative effect of BBPD on welfare is simply attributed to inefficient shopping practices, such as incurring "excessive transport costs." In line with this, Armstrong (2006) argues "[...] it is important to extend the analysis beyond the models presented, which involved a relentless use of Hotelling demand specifications with unit demands and uniform distributions. For instance, when consumers have inelastic demand there is no welfare benefit when price discrimination causes prices to fall, and such a benefit would be present in a richer model." Therefore, it is advisable to exercise caution when interpreting the welfare and policy implications of BBPD in unit demand models.

As far as we know, Esteves and Reggiani (2014) are the first to study BBPD relaxing the perfectly inelastic demand assumption. By assuming a CES (constant elasticity of substitution) demand function, they show that if the elasticity of demand is sufficiently high, BBPD boosts overall welfare compared to uniform pricing. Regardless of the elasticity of demand, they show that BBPD intensifies competition, reducing profits at the benefit of consumer welfare. Although the CES formulation has the advantage of being mathematically tractable and yielding a closed-form solution for the two-period BBPD model, it is not exempt from limitations. One important limitation is that regardless of whether there is BBPD or not, consumer surplus is unaltered by the elasticity of demand. Put differently, consumer surplus is the same under a unit and a CES demand function. The reason is that under CES demand preferences the price and demand expansion effects cancel each other out. This property is also important to clarify why, in Esteves and Reggiani (2014), the *firm-side effect of BBP* is null, similar to that of the unit demand model. Hence, in markets with unit and CES demand, behavior based discrimination only affects first-period prices by lowering consumers' price sensitivity. Even though firms are forward looking, the first-order effect of shifting the indifferent consumer equals zero around

[1] Details are available on https://www.nytimes.com/2012/02/19/magazine/shopping-habits.html.

the market center—a firm's marginal gains in profit over one segment are exactly canceled out by losses over the other in period 2. This is an intriguing result because it implies that forward-looking firms' incentive to avoid the unprofitable use of purchase history information has no impact on the first-period market equilibrium.

This paper complements the literature on BBPD by assuming a general downward slope demand function, where the elasticity of demand $\varepsilon(p)$ is an increasing function of prices. In this setting, we demonstrate that the previous conventional result no longer holds true, as the firm side effect can now provide firms with a clear incentive to increase their prices in the initial period. Moreover, depending on the demand function considered, consumers may become either less or more responsive to price changes. Our framework encompasses the perfectly inelastic demand proposed by Fudenberg and Tirole (2000) when $\varepsilon(p) = 0$ and the CES demand proposed by Esteves and Reggiani (2014) when $\varepsilon(p) = \varepsilon$. By doing so, the paper assesses the profit effects of BBPD in homogeneous repeated product markets where consumers are heterogeneous in terms of preferences for firms (stores) and demand $q(p)$ units of the good from one of the firms in the market (one stop shopping).

This work aligns closely to the theoretical literature on behavior-based pricing. Fudenberg and Tirole (2000) and subsequent studies, show that generally BBPD lowers (potentially benefits) firm profitability (consumer welfare) by intensifying competition, unless there are sufficient asymmetries at the firm-or-consumer-level (Chen 1997; Villas-Boas 1999; Fudenberg and Tirole 2000; Pazgal and Soberman 2008; Esteves 2010; Garela, et al, 2021, Laussel and Resende, 2022).[2]

By introducing various asymmetries other studies show how BBPD can improve profitability and harm consumers. Such asymmetries include enhanced services that firms can offer only to loyal customers (Acquisti and Varian 2005; Pazgal and Soberman 2008), quality differences between firms (Jing, 2017), asymmetry in consumer preferences (Chen and Zhang, 2009; Esteves, 2009; Shin and Sudhir 2010; Colombo, 2018), or consumers' fairness concerns (Li and Jain, 2016).

Chen and Pearcy (2010) and Esteves, et al (2022) show that consumer preferences play an important role in determining the profit and welfare effects of BBPD. Chen and Pearcy allow consumer preferences between the two periods to be imperfectly correlated, they show that BBPD raises firms' discounted overall profits when the level of correlation is small. Esteves, et al (2022) show that BBPD can increase industry profits and decrease consumer surplus, even in a symmetric market, if consumers have triangular preferences. They also highlight that the assumption of non-uniform preferences gives rise to a firm side effect that can be positive (triangular preferences) or negative (inverse triangular preferences). This new perspective challenges previous assumptions and shows how market dynamics can affect both industries and consumers.

Finally, our paper also relates to the literature embedding elastic demand into spatial competition models (Nero, 1999) and (Rath and Zhao, 2001) seems to be the first to introduce the linear demand function into Hotelling model,[3] whereas Anderson (2000) seems the first to introduce the CES demand into a spatial competition framework. When consumer demand is perfectly inelastic and the market is fully covered, the total demand is fixed. Price discrimination usually hurts welfare since it lures consumers to buy from the distant firms, which increases the total transport costs. When elastic demand is considered, lower prices caused by price discrimination induce consumers to buy more and thus increase the welfare. When this enlarged demand outweighs the deadweight loss caused by switching, price discrimination can improve welfare under BBPD. The CES demand function is used by Esteves and Reggiani (2014) in a BBPD model, and by Esteves and

[2] See Fudenberg and Villas-Boas (2006, 2012) for more comprehensive reviews.

[3] The difference between Nero (1999) and Rath and Zhao (2001) is that quantity demanded depends on the travel cost in the first paper but not in the second one. Colombo (2011) considers a linear demand model with delivered price, thus consumers do not pay transport cost.

Shuai, (2022) in a personalized pricing model similar to Thisse and Vives (1988).[4] Elastic demand has also been used to exploit other aspects of spatial models, including entry problem (Gu, 2009, 2011), endogenizing price discrimination decision (Zhang, 2019), etc.

By introducing a demand function in which the elasticity of demand is no longer constant, the paper shows that for equal discount factors for consumers and firms, in contrast to Esteves and Reggiani (2014), BBPD increases consumer surplus at the expense of industry profits. However, the analysis highlights that the output expansion effect might be insufficient to raise overall welfare.

Additionally, extending the analysis of Fudenberg and Tirole to different discount factors for firms (δ_f) and consumers (δ_c) reveals that new results can be obtained, even with the assumption of unit demand. Specifically, when demand is perfectly inelastic, it is proven that, for any combination (δ_c, δ_f), price discrimination based on purchase history always has a positive effect on consumer surplus. However, the assumption of different levels of patience for firms and consumers reveals that the impact of BBPD on profits and social welfare can be positive or negative, contrary to the negative effect on profits/social welfare in the case of equal discount factors. If the demand function is linear, the examples presented show that there is a region for (δ_c, δ_f), (low δ_c and high δ_f) where the practice of BBPD results in losses, not only in terms of industry profits but also in terms of consumer surplus and social welfare. As δ_f increases, the firm side effect becomes more important, resulting in an increase in the equilibrium price in the first period. When δ_c is low the negative impact of price increases (reduced consumption) in the first period is more important than the gains from price discrimination in the second period. Therefore, consumers are worse off with price discrimination. Regarding profits, when δ_f is high, the reduction in profits in the second period has a substantial impact on total discounted profits. Consequently, the practice of BBPD negatively affects industry profits. The analysis also shows that if consumers are patient (high δ_c) but firms are impatient (low δ_f), the practice of BBPD can have a positive impact on industry profits, consumer welfare, and social welfare. The economic intuition presented earlier acts in the opposite direction.

The rest of the paper is organized as follows. Section 2 presents the model. The benchmark case of uniform pricing is discussed in Section 3. Section 4 presents the equilibrium analysis for a general demand function. Section 5 discusses the price, quantity, profit and welfare effects of BBPD. Section 6 assesses the results of the model for specific linear demand functions. Final remarks appear in Section 7. All the proofs are relegated to the Appendix.

2 The model

Our basic model is based on Fudenberg and Tirole's (2000) framework. Two firms, A and B, sell a homogeneous good to a unit mass of consumers. There are two periods, 1 and 2, and firms cannot commit to future prices. The marginal production cost is assumed equal to zero. Considering the Hotelling specification, firms are located at the extremes of the interval $[0, 1]$, Firm A is located at point 0 and firm B is located at point 1. Consumers are uniformly distributed on this interval. A consumer of type $x \in [0, 1]$ is at 'distance' $d_A = x$ from firm A and $d_B = 1 - x$ from firm B. Note that x captures firm differentiation, which not only aligns with the geographical interpretation, but also encompasses various other factors. As previously discussed in the introduction, firms can differentiate themselves based on various factors such as their location, layout,

[4] Esteves (2022) e Lu and Matsushima (2022) look at the profits and welfare effects of personalized pricing (PP) in markets where consumers can buy multiple units, showing that in contrast to unit demand models, PP can act to raise profits.

payment methods, design, atmosphere, website design, recommendations, delivery options, availability of chat, and more. It is further assumed that x remains fixed across periods.

A key departure of our paper from the literature is that we allow consumer demand to be a decreasing function of the selling price, and the elasticity of demand to change with price. Specifically, given that a consumer has decided to buy the good at a price p, she buys a quantity $q(p)$ of that product, with $p \in [0, \widehat{p}]$. We also assume that $q(p)$ is continuous and differentiable on $[0, \widehat{p}]$ with $q'(p) < 0$. Basically, $q(p) \geq 0$ if $p \leq \widehat{p}$, while $q(p) = 0$ if $p > \widehat{p}$. Hence, $\widehat{p}$ plays the role of the reservation price of the consumers. In addition, $q(p)$ is identical across firms and consumers. (Although we solve the model for a general demand function, we shall later rely on specific demand functions.)

Let the price elasticity of demand be

$$\varepsilon(p) := -\frac{pq'(p)}{q}. \tag{1}$$

Differentiability of $q(p)$ implies $\varepsilon(p)$ is continuous, and $\varepsilon(0) = 0$. We further assume that $\varepsilon'(p) > 0$ for $[0, \widehat{p}]$.

Let $R(p) := pq(p)$ be the profit per consumer associated with $q(p)$. We can then establish the following lemma.

Lemma 1 *There exists a unique price $\overline{p} \in (0, \widehat{p})$ that maximizes $R(p)$ on $[0, \widehat{p}]$. Furthermore, $R(p)$ is strictly increasing in $[0, \overline{p})$ and $\varepsilon(\overline{p}) = 1$.*

Proof. See the Appendix. ■

Following Anderson and de Palma (2000), Rath and Zhao (2001), Gu and Wenzel (2009) and Esteves and Reggiani (2014) the (indirect) utility for a consumer located at x conditional on buying from firm $i = A, B$ at period $t \in \{1, 2\}$, at price $p_i < \widehat{p}$ is

$$V_i = Y + V + v(p_i) - d_i.$$

Y is the consumer income for the two periods of consumption,[5] $v(p) = \int_p^{\widehat{p}} q(s)ds$ is the utility component that depends on the quantity consumed, or the consumer surplus function associated with $q(p)$, so that $-v'(p_i) = q(p)$. (Recall that preferences are assumed to be quasi-linear.) V is the gross utility from consuming the good, we assume that V is large enough so that the market is covered.[6] Finally, consumers have to incur "transport costs" if the retailer's attributes do not match consumers' preferences. Follow the aforementioned literature, we assume that transport costs do not depend on the quantity consumed. Furthermore, we assume that transport costs are linear in distance.[7]

Hence, a consumer located at x conditional on buying from firm A at period $t \in \{1, 2\}$, at price $p_A < \widehat{p}$ gets a surplus

$$Y + V + \int_{p_A}^{\widehat{p}} q(s)ds - x,$$

while if purchasing from B at price $p_B < \widehat{p}$ she obtains a surplus equal to

$$Y + V + \int_{p_B}^{\widehat{p}} q(s)ds - (1 - x).$$

[5] We shall assume throughout that Y is high enough such that income is never a binding constraint.

[6] This helps us to avoid situations in which a firm could be a local monopoly.

[7] Using a quadratic transport will not affect the determination of the marginal consumer, and thus is equivalent to linear transport cost.

The indifferent consumer between buying from the two firms is located at $\widehat{x}$, such that

$$Y + V + \int_{p_A}^{\widehat{p}} q(s)ds - \widehat{x} = Y + V + \int_{p_B}^{\widehat{p}} q(s)ds - (1 - \widehat{x}).$$

This yields

$$\widehat{x} = \frac{1}{2} + \frac{1}{2}\int_{p_A}^{p_B} q(s)ds. \tag{2}$$

Given the uniform distribution of consumer preferences, each consumer in the market segment $[0, \widehat{x}]$ buys $q(p_A)$ units from firm A and each consumer in the market segment $[\widehat{x}, 1]$ buys $q(p_B)$ units from firm B. The demand of a consumer in either segment does not depend on own location.

We consider the general case when consumers and firms may use different discount factors. The firms' common discount factor is denoted by $\delta_f \in [0, 1]$, while the consumers' discount factor is denoted by $\delta_c \in [0, 1)$.[8] As δ_c and δ_f increase, consumers and firms become more forward looking. Denoting firm i's profit with $i = A, B$ in period $t = 1, 2$ by π_i^t , firm i's problem in the beginning of the game is to maximize its total discounted profit given by $\pi_i^1 + \delta_f \pi_i^2$. Likewise, if consumer x's surplus in period t is u_x^t, then his/her optimal decision in $t = 1$ is to maximize $u_x^1 + \delta_c u_x^2$.

In each period, firms choose their prices simultaneously. A strategy for firm i, with $i = \{A, B\}$, specifies p_i^1 in $t = 1$ and prices (p_i^o, p_i^n) in $t = 2$ based on consumers' previous purchases, where p_i^o and p_i^n are firm i's prices for old and new consumers, respectively.

3 No discrimination benchmark

Before proceeding to the analysis of BBPD, consider first that somehow public policies prohibit any form of price discrimination or that firms cannot track consumers' purchase decisions in period 1. In any case, firms charge a uniform price in both periods. The two-period model reduces to two replications of the static equilibrium. To solve for this equilibrium, consider the one period model, and let p_A and p_B denote the prices set by firms A and B, respectively. Considering for instance the case of firm A, it maximizes its profit π_A^{nd} with respect to its price, where nd stands for no-discrimination:

$$\pi_A^{nd}(p_A, p_B) = p_A q(p_A)\left[\frac{1}{2} + \frac{1}{2}\int_{p_A}^{p_B} q(s)ds\right].$$

Proposition 1 *If price discrimination is not permitted, in the symmetric SPNE, in each period each firm charges $p^{nd} \in (0, \overline{p})$ implicitly defined by*

$$R(p^{nd}) = 1 - \varepsilon(p^{nd}) \tag{3}$$

with $\varepsilon(p^{nd}) \in (0, 1)$, and overall equilibrium profits are equal to

$$\pi^{nd} = \frac{(1 + \delta_f)\left[1 - \varepsilon(p^{nd})\right]}{2}. \tag{4}$$

Proof. See the Appendix. ■

[8] The condition $\delta_c < 1$ is necessary to guarantee that first-period market share is well behaved.

Before proceeding we compute welfare under uniform pricing. In contrast to models with completely inelastic demand, where welfare only depends on average transport costs, say Ω, we now have to consider the prices because they have an impact on the quantity purchased and hence on welfare. Let Ω be the equilibrium average transport costs, then overall consumer surplus in each period is

$$CS^{nd} = V + \int_{p^{nd}}^{\widehat{p}} q(s)ds - \Omega^{nd} \text{ with } \Omega^{nd} = 2\int_0^{\frac{1}{2}} xdx.$$

Social welfare (W) is defined by the sum of consumer surplus (CS) and industry profits. This yields

$$W^{nd} = CS^{nd}(1+\delta_c) + 2(1+\delta_f)\pi^{nd}.$$

4 Behavior-based price discrimination

As usual we solve the game by backward induction. In equilibrium, the firms' strategies must induce a Nash equilibrium at any second-period subgame as well as a Nash equilibrium in period 1. Because firms are ex-ante symmetric, we only look for symmetric (pure strategy) equilibria.

As a preliminary step, a standard revealed-preference argument implies that at any pair of first-period prices such that all consumers purchase and both firms have positive sales, there will be a first-period cutoff x_1 such that all consumers with $x < x_1$ buy from firm A in the first period, and all consumers with $x > x_1$ buy from firm B.

4.1 Second period

Given the existence of a first-period cutoff x_1, consumers at the left of x_1 lie in firm A's 'turf' and those to the right lie in firm B's. Provided x_1 is not too large, the second-period equilibrium has this form: Both firms poach some of their rival's first-period customers, so that some consumers do switch providers. These switchers are in the middle of the preference line, types between x_A and x_1 switch from A to B, and types between x_1 and x_B switch from B to A.

Look first at firm A's turf on $[0, x_1]$. These are firm A's old customers, and thus will observe the following second period prices: p_A^o and p_B^n. Given these prices, some consumers will buy again from A while others will be willing to switch to firm B. Let x_A denote the marginal consumer who is indifferent between staying with A paying p_A^o and consuming $q(p_A^o)$ units, and switching to B, paying p_B^n and consuming $q(p_B^n)$. Then

$$Y + V + \int_{p_A^o}^{\widehat{p}} q(s)ds - x_A = Y + V + \int_{p_B^n}^{\widehat{p}} q(s)ds - (1 - x_A)\,.$$

This yields:

$$x_A = \frac{1}{2} + \frac{1}{2}\int_{p_A^o}^{p_B^n} q(s)ds. \tag{5}$$

Therefore in the turf of firm A, firm A's market share from old customers is x_A while firm B's share from poached customers is $(x_1 - x_A)$, with $0 \leq x_A < x_1 \leq 1$. Total demand from these segments for firm A and B, respectively given by D_A^o and D_B^n, depends on the market share and on the quantity per consumer. Therefore:

$$D_A^o = q(p_A^o)x_A,$$

while

$$D_B^n = q(p_B^n)\,(x_1 - x_A)\,.$$

Look next at firm B's turf on $[x_1, 1]$. The group of firm B's old consumers will be offered second period prices p_A^n and p_B^o. Given these two prices, some consumers will buy again from B while others will be willing to switch. As we have done for consumers in A's turf, the indifferent consumer between staying with B and switching to A is located at x_B such that:

$$V + \int_{p_A^n}^{\widehat{p}} q(s)ds - x_B = V + \int_{p_B^o}^{\widehat{p}} q(s)ds - (1 - x_B)\,.$$

This yields

$$x_B = \frac{1}{2} + \frac{1}{2}\int_{p_A^n}^{p_B^o} q(s)ds. \tag{6}$$

Doing the same in the turf of firm B, firm B's market share from old customers is $(1 - x_B)$, while firm A's market share from poached customers is $(x_B - x_1)$, as long as $0 \leq x_1 < x_B \leq 1$. Firm A's total demand from new customers is denoted by D_A^n and firm B's total demand from retained old consumers is denoted by D_B^o. It follows that

$$\begin{aligned} D_A^n &= q(p_A^n)\,(x_B - x_1) \\ D_B^o &= q(p_B^o)\,(1 - x_B) \end{aligned}$$

Therefore, both firms' profits from old and new customers are:

$$\pi_A^o(p_A^o, p_B^n) = p_A^o D_A^o = p_A^o q(p_A^o)\left[\frac{1}{2} + \frac{1}{2}\int_{p_A^o}^{p_B^n} q(s)ds\right] \tag{7}$$

$$\pi_A^n(p_A^n, p_B^o) = p_A^n D_A^n = p_A^n q(p_A^n)\left(\frac{1}{2} + \frac{1}{2}\int_{p_A^n}^{p_B^o} q(s)ds - x_1\right) \tag{8}$$

$$\pi_B^n(p_A^o, p_B^n) = p_B^n D_B^n = p_B^n q(p_B^n)\left[x_1 - \left(\frac{1}{2} + \frac{1}{2}\int_{p_A^o}^{p_B^n} q(s)ds\right)\right] \tag{9}$$

$$\pi_B^o(p_A^n, p_B^o) = p_B^o D_B^o = p_B^o q(p_B^o)\left(\frac{1}{2} - \frac{1}{2}\int_{p_A^n}^{p_B^o} q(s)ds\right) \tag{10}$$

In period 2, firm i chooses (p_i^o, p_i^n) in order to maximize $\pi_i^o(p_i^o, p_j^n)$ and $\pi_i^n(p_i^n, p_j^o)$ with respect to p_i^o and p_i^n, respectively. We may establish the next proposition.

Proposition 2 *When firms can employ BBPD, at an interior solution* $(0 < x_A < x_1 < x_B < 1)$*, the second-period equilibrium prices* $p_i^{o*} \in (0, \overline{p})$ *and* $p_i^{n*} \in (0, \overline{p})$ *with* $\varepsilon(p_i^{o*}) \in (0,1)$ *and* $\varepsilon(p_i^{n*}) \in (0,1)$ *are the solution to the following system of implicit equations:*

$$R_A^o(p_A^{o*}) = 2x_A(p_A^{o*}, p_B^{n*})\,[1 - \varepsilon(p_A^{o*})] \tag{11}$$

$$R_B^n(p_B^{n*}) = 2\,[x_1 - x_A(p_A^{o*}, p_B^{n*})]\,[1 - \varepsilon(p_B^{n*})] \tag{12}$$

$$R_B^o(p_B^{o*}) = 2\,[1 - x_B(p_A^{n*}, p_B^{o*})]\,[1 - \varepsilon(p_B^{o*})] \tag{13}$$

$$R_A^n(p_A^{n*}) = 2\,[x_B(p_A^{n*}, p_B^{o*}) - x_1]\,[1 - \varepsilon(p_A^{n*})] \tag{14}$$

Proof. See the Appendix. ■

From this proposition we can establish the following well-know result in the literature.

Corollary 1. *Regardless of the demand function considered, in a symmetric pure strategy equilibrium, BBPD leads to an all-out competition in the second period (i.e.,* $p_A^{o*} < p^{nd}$ *and* $p_B^{n*} < p^{nd}$*).*

Proof. See the Appendix. ■

The equilibrium second-period profits can be written as:

$$\pi_A^o(p_A^{o*}, p_B^{n*}) = 2\left[x_A(p_A^{o*}, p_B^{n*})\right]^2 \left[1 - \varepsilon(p_A^{o*})\right], \tag{15}$$

$$\pi_B^n(p_A^{o*}, p_B^{n*}, x_1) = 2\left[x_1 - x_A(p_A^{o*}, p_B^{n*})\right]^2 \left[1 - \varepsilon(p_B^{n*})\right], \tag{16}$$

$$\pi_B^o(p_A^{n*}, p_B^{o*}) = 2\left[1 - x_B(p_A^{n*}, p_B^{o*})\right]^2 \left[1 - \varepsilon(p_B^{o*})\right], \tag{17}$$

$$\pi_A^n(p_A^{n*}, p_B^{o*}, x_1) = 2\left[x_B(p_A^{n*}, p_B^{o*}) - x_1\right]^2 \left[1 - \varepsilon(p_A^{n*})\right]. \tag{18}$$

Second-period overall profits are:

$$\pi_A^2 = 2\left[x_A(p_A^{o*}, p_B^{n*})\right]^2 \left[1 - \varepsilon(p_A^{o*})\right] + 2\left[x_B(p_A^{n*}, p_B^{o*}) - x_1\right]^2 \left[1 - \varepsilon(p_A^{n*})\right], \tag{19}$$

$$\pi_B^2 = 2\left[1 - x_B(p_A^{n*}, p_B^{o*})\right]^2 \left[1 - \varepsilon(p_B^{o*})\right] + 2\left[x_1 - x_A(p_A^{o*}, p_B^{n*})\right]^2 \left[1 - \varepsilon(p_B^{n*})\right]. \tag{20}$$

4.2 First-period

Consider now the equilibrium first-period pricing and consumption decisions. Due to the absence of commitment power, firms's market shares in the first period will affect their second period pricing and profits. Consequently, forward looking firms take this interdependence into account when setting their first period prices. As consumers are non myopic they also anticipate the firms' second period pricing behavior. Suppose first-period prices lead to a cut-off x_1 that is in the interior of the interval $[0, 1]$. Then the marginal consumer must be indifferent between buying from A in the first period at price p_A^1 and consuming $q(p_A^1)$ units, and buying from B in the next period at the poaching price p_B^n and consuming $q(p_B^n)$ units; or buying from B in the first period at price p_B^1 and consuming $q(p_B^1)$ units, and switching to firm A in the second period at the poaching price p_A^n and consuming $q(p_A^n)$ units. Hence, at an interior solution:

$$V + \int_{p_A^1}^{\widehat{p}} q(s)ds - x_1 + \delta_c \left(V + \int_{p_B^n}^{\widehat{p}} q(s)ds - (1 - x_1)\right) = V + \int_{p_B^1}^{\widehat{p}} q(s)ds - (1 - x_1) + \delta_c \left(V + \int_{p_A^n}^{\widehat{p}} q(s)ds - x_1\right),$$

this simplifies to

$$x_1 = \frac{1}{2} + \frac{1}{2(1 - \delta_c)} \int_{p_A^1}^{p_B^1} q(s)ds + \frac{\delta_c}{2(1 - \delta_c)} \int_{p_B^n}^{p_A^n} q(s)ds \tag{21}$$

with $\delta_c < 1$. Firm A's first-period overall demand given the price p_A^1 is $D_A^1 = x_1 q(p_A^1)$. Similarly, firm's B overall demand in period 1 is $D_B^1 = (1 - x_1)q(p_B^1)$. Firm A and B's overall profits are respectively:

$$\Pi_A = p_A^1 q(p_A^1) x_1 + \delta_f \left(\pi_A^o + \pi_A^n\right) \tag{22}$$

and

$$\Pi_B = p_B^1 q(p_B^1)(1 - x_1) + \delta_f \left(\pi_B^o + \pi_B^n\right). \tag{23}$$

Note that second period prices depend on x_1, which in turn depends on first period prices. Then firms' overall profits are defined as a function of first period prices as well. Consequently, each firm maximizes its overall profit with respect to its first period price. Consider, for example, firm A's decision. Because firm A's second period prices are chosen to maximize its second period profit, we can use the Envelope theorem to simplify the first-order condition, $\frac{\partial \Pi_A}{\partial p_A^1} = 0$ where Π_A is defined in equation (22) and x_1 is defined in equation (21). This yields:

$$x_1 q(p_A^1) + p_A^1 q'(p_A^1) x_1 + p_A^1 q(p_A^1) \frac{\partial x_1}{\partial p_A^1} + \delta_f \left[\frac{\pi_A^o}{\partial p_B^n} \frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o} \frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1} \right] \frac{\partial x_1}{\partial p_A^1} = 0$$

Let $\Lambda = \frac{\partial \pi_A^2}{\partial x_1} = \left[\frac{\pi_A^o}{\partial p_B^n} \frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o} \frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1} \right]$. Then

$$x_1 q(p_A^1) + p_A^1 q'(p_A^1) x_1 + p_A^1 q(p_A^1) \frac{\partial x_1}{\partial p_A^1} + \delta_f \Lambda \frac{\partial x_1}{\partial p_A^1} = 0$$

Dividing by $q(p_A^1)$ yields

$$\frac{x_1 q(p_A^1)}{q(p_A^1)} + \frac{p_A^1 q'(p_A^1) x_1}{q(p_A^1)} + \frac{p_A^1 q(p_A^1)}{q(p_A^1)} \frac{\partial x_1}{\partial p_A^1} + \delta_f \frac{\Lambda}{q(p_A^1)} \frac{\partial x_1}{\partial p_A^1} = 0$$

from which we obtain

$$p_A^1 q(p_A^1) = -\frac{x_1 q(p_A^1)}{\frac{\partial x_1}{\partial p_A^1}} \left[1 - \varepsilon(p_A^1) \right] - \delta_f \Lambda. \tag{24}$$

Lemma 2 *From the expression that defines the indifferent consumer in period 1, given by equation (21), it follows that under symmetry:*

$$\left(\frac{\partial x_1}{\partial p_A^1} \right)^{\text{BBPD}} |sym = \frac{-q(p_A^1)}{2\left[(1-\delta_c) + 2\delta_c R(p_B^n) \rho_{Bx1}^n \right]},$$

where $\rho_{Bx1}^n = \frac{\partial p_B^n}{\partial x_1} \frac{x_1}{p_B^n} > 0$. W*ith uniform pricing or when* $\delta_c = 0$ *it is given by*

$$\left(\frac{\partial x_1}{\partial p_A^1} \right)^{\text{u}} |sym = -\frac{q(p_A^1)}{2}.$$

Proof. See the Appendix. ■

Based on lemma 2, we can establish the following result.

Proposition 3 *In the symmetric subgame perfect nash equilibrium:*

(i) the first-period price is implicitly defined as follows

$$R(p_A^1) = \left[(1-\delta_c) + 2\delta_c R(p_B^n) \rho_{Bx1}^n \right] \left[1 - \varepsilon(p_A^1) \right] + \delta_f (-\Lambda_{sym}). \tag{25}$$

(ii) second-period prices are the solution to the system of implicit equations*:*

$$R(p^o) = \left(1 - \int_{p^n}^{p^o} q(s) ds \right) \left[1 - \varepsilon(p^o) \right] \tag{26}$$

$$R(p^n) = \left(\int_{p^n}^{p^o} q(s) ds \right) \left[1 - \varepsilon(p^n) \right] \tag{27}$$

(iii) first and second-period profits from consumers who belong to the old and new segment are, respectively:

$$\begin{aligned}\pi^1 &= \frac{1}{2}R(p_A^1)\\ \pi^o &= \left(\frac{1}{2}+\frac{1}{2}\int_{p^o}^{p^n} q(s)ds\right)R(p^o)\\ \pi^n &= \left(\frac{1}{2}\int_{p^n}^{p^o} q(s)ds\right)R(p^n)\end{aligned}$$

We can substitute the expression for $\left(\frac{\partial x_1}{\partial p_A^1}\right)$ defined in lemma 2 into equation (24). The resulting equation provides a straightforward proof for part (i) of the proposition.

5 Effects of price discrimination

In what follows let us consider $\Delta\Gamma = \Gamma^D - \Gamma^U$, with $\Gamma = \{p, Q, \pi_{ind}, CS, W, \Omega\}$.

5.1 Prices and quantities

Regarding second-period prices we have seen $p^n < p^o < p^{nd}$, hence $q^n > q^o > q^{nd}$. Hence, the quantity consumed by any switching consumer in period 2 exceeds the quantity consumed by any loyal consumer. Furthermore, second-period prices (quantities) are expected to be lower (higher) in markets where the price elasticity of demand $\varepsilon(p)$ is higher.

Next, we illustrate how BBPD affects first period prices. We should take into account the expression that defines implicitly the equilibrium no discrimination price (cf. Proposition 1) equal to

$$R(p^{nd}) = \left[1-\varepsilon(p^{nd})\right]$$

and the expression that defines implicitly the first-period equilibrium price in the SPNE with BBPD given in part (i) of Proposition 3:

$$R(p_A^1) = \underbrace{\left[(1-\delta_c)+2\delta_c R(p_B^n)\rho_{Bx1}^n\right]}_{\text{Consumer-side effect}}\left[1-\varepsilon(p_A^1)\right] + \underbrace{\delta_f(-\Lambda_{sym})}_{\text{Firm-side effect}}, \tag{28}$$

Let Ψ_{sym} and $-\Lambda_{sym}$ represent the consumer and firm side effects under symmetry.

Lemma 3. *When moving from uniform to behavior-based pricing it follows that:*

$$\begin{aligned}\Psi_{sym} &= (1-\delta_c)+2\delta_c R(p_B^n)\rho_{Bx1}^n\\ \Lambda_{sym} &= R^n\left[R^o\left(\rho_{Bx}^n+\rho_{Bx}^o\right)-1\right]\end{aligned}$$

with $\rho_{Bx}^n > 0$ *and* $\rho_{Bx}^o < 0$.

(i) $\Psi_{sym} > 1$ *(consumer side effect increases first-period prices) as long as* $R(p_B^n)\rho_{Bx1}^n > \frac{1}{2}$. *Otherwise,* $\Psi_{sym} < 1$ *and the reverse happens.*

(ii) $\Lambda_{sym} < 0$ (i.e.,$-\Lambda_{sym} > 0$ and firm side effect increases first-period prices) as long as $(\rho^n_{Bx1} + \rho^o_{Bx1}) < \frac{1}{R^o}$. Otherwise the reverse happens.

Proof. See the Appendix. ■

Part (i) of lemma 3 suggests that when condition $R(p^n_B)\rho^n_{Bx1} > \frac{1}{2}$ holds $p^1 > p^{nd}$. Consider the marginal consumer in period 1 and firm A's price decision. He/she anticipates that a reduction in p^1_A increases x_1. This induces firm B to offer a higher p^n in the second-period. Then the anticipation of a lower poaching price (higher consumption) in period 2 induces consumers to become less price sensitive in the first-period when $R(p^n_B)\rho^n_{Bx1}$ is higher than the transport cost incurred by the consumer located at $\frac{1}{2}$. ρ^n_{Bx1} can be understood as firm B's new customer price elasticity with respect to x_1. When firm B's new customer price is very responsive to a rise in x_1, then it is more likely that the LHS is higher than the RHS, suggesting that the consumer side effect increases firs-period prices.

The intuition for the consumer side effect can be understood as follows. Consider the marginal consumer located at x_1. If she buys from firm A in the first period, she will switch to firm B in the second period. Then $R(p^n_B)$ measures her total payment to firm B, and ρ^n_{Bx1} is firm B's new customer price elasticity with respect to x_1. Particularly, a larger ρ^n_{Bx1} implies a larger rise in p^n_B in response to a rise in x_1. As $p^n_B < \overline{p}$ an increase in p^n_B increases $R(p^n_B)$. The consumer anticipates that an increase in x_1 increases p_n ($\frac{\partial p^n_B}{\partial x_1} > 0$) and so $R(p^n_B)$. The the larger the LHS ($R(p^n_B)\rho^n_{Bx1}$) the more the marginal consumer needs to pay in the second period when it chooses firm A in the first period. When the LHS is greater than $\frac{1}{2}$ (the transport cost supported by the indifferent consumer), the consumer becomes less sensitive to price changes in period 1. The lowered price sensitivity leads to a higher first period price. In contrast, when the LHS is lower than $\frac{1}{2}$, the opposite happens and the consumer becomes more sensitive to price changes. The higher price sensitive leads to lower first-period price.

Consider the two linear demand cases presented in section 6.3, where $q(p) = a - p_i$. In this case the demand elasticity depends only on a (the vertical intercept) and not on the slope. Specifically, $\varepsilon(p) = \frac{p}{a-p}$ with $\frac{\partial \varepsilon(p)}{\partial a} < 0$. The examples considered are $q(p) = 2-p$ and $q(p) = 1-p$. As $R(p^n_B) = \frac{1}{2}(x_1 - x_A)(1-\varepsilon(p^n_B))$, it is straightforward to conclude that with the same price p^n_B, the former results in larger per consumer revenue $R(p^n_B)$ than the latter. In section 6.3 we can see that in equilibrium, $R(p^n_B) = 0.2848$ when $q(p) = 2-p$, and $R(p^n_B) = 0.1398$ when $q(p) = 1-p$. A larger per consumer revenue gives a firm larger incentive to sell to more consumers. Thus compared to the small demand ($q(p) = 1-p$), under a larger demand ($q(p) = 2-p$), when x_1 increases, p^n_B rises slowly, resulting in a lower price sensitivity with response to x_1, i.e., ρ^n_{Bx} is smaller. And here a larger demand (higher a) has two opposing effects on $R(p^n_B)\rho^n_{Bx1}$. On one hand, it raises $R(p^n_B)$, on the other hand it reduces ρ^n_{Bx1}. Specifically, when $a = 1$ we obtain $\rho^n_{Bx} = 3.086$; when $a = 2$ we get $\rho^n_{Bx} = 2.083$. The final result of these two opposing forces depends on which is larger. We will see that when $a = 1$, $R(p^n_B)\rho^n_{Bx1} < \frac{1}{2}$ and consumers become more price sensitive to changes in first-period prices. The CSE gives firms an incentive to reduce the first-period. When $a = 2$, $R(p^n_B)\rho^n_{Bx1} > \frac{1}{2}$. Consumers become less sensitive to price changes in first-period prices. The CSE gives firms an incentive to increase the first-period.

Part (ii) of lemma 3 implies that if $|\rho^o_{Bx1}| > |\rho^n_{Bx}|$, as $\rho^o_{Bx1} < 0$ and $R^o > 0$, it is always the case that $\Lambda_{sym} < 0$. Otherwise, $\Lambda_{sym} < 0$ as long as condition $(\rho^n_{Bx1} + \rho^o_{Bx1}) < \frac{1}{R^o}$ holds.

In general, BBPD generates two effects on first-period prices: a consumer-side effect (CSE) and a firm-side effect (FSE). Under a general demand function, the CSE suggests that forward-looking consumers, who correctly anticipate lower second-period prices, can become less or more price-sensitive in period 1. With a uniform distribution of consumer preferences, Fudenberg and Tirole (2000) and Esteves and Reggiani (2014) have shown that, under symmetry, the CSE always leads consumers to become less price sensitive, inducing firms to raise their first-period prices. Because in their analysis the firm-side effect plays no role (i.e., $\Lambda_{sym} = 0$) the consumer-side effect fully determines the result that BBPD raises first-period prices.

However, our analysis reveals that the firm-side effect can drive up first-period prices, even when consumers are myopic. When firms are forward-looking, they consider that changes in first-period prices impact the first-period marginal consumer, which, in turn, alters the nature of second-period competition.

Consider the FSE and the case of firm A. An increasing in x_1 has two opposing effects on firm A's second period profit. On the one hand, a rising in x_1 will make firm B prices less aggressively to its new customers ($\frac{\partial p_B^n}{\partial x_1} > 0$), this improves firm A's profit from its old customers; on the other hand, a rising in x_1 makes firm B prices more aggressively to its old customers ($\underset{-}{\frac{\partial p_B^o}{\partial x_1}} < 0$), this hurts firm A's profit from its new customers. The overall effect depends on the balance of these two. In particular, on the left hand side, ρ_{Bx1}^n can be understood as firm B's new customer price elasticity with respect to x_1, and ρ_{Bx1}^o is firm B's old customer price elasticity with respect to x_1. When ρ_{Bx1}^o absolute value is large such that a firm B's old customer price is very responsive to an increasing in x_1, and ρ_{Bx1}^n is small such that a firm B's new customer price is not responsive to a rise in x_1, the negative effect dominates the positive effect, and an increasing in x_1 hurts firm A's second period profit. The right hand side is the threshold ($\frac{1}{R^o}$), which is determine by R^o, firm A's old customer per point (consumer) revenue. When R^o is small (so the threshold is large), the benefit of a lower p_B^n is small, an increasing in x_1 is more likely to hurt firm A (the LHS is more likely to be lower than the RHS).

Proposition 4 *In the symmetric SPNE with BBPD, in comparison to no discrimination:*

(i) if $\delta_f = 0$ or $\Lambda_{sym} = 0$, the firm-side effect is null and only the consumer side effect affects first-period prices. If $\delta_c > 0$, then BBPD increases the first-period price as long $2R(p_B^n)\rho_{Bx1}^n > 1$.

(i) if $\delta_c = 0$ and $\delta_f > 0$, the consumer-side is null and only the firm-side effect affects first-period prices. If $\Lambda_{sym} = 0$ then $p^1 = p^{nd}$. Otherwise if $(\rho_{Bx1}^n + \rho_{Bx1}^o) < \frac{1}{R^o}$, then $\Lambda_{sym} < 0$ and $p^1 > p^{nd}$, the reverse happens if $(\rho_{Bx1}^n + \rho_{Bx1}^o) > \frac{1}{R^o}$. The firm side effect is stronger the higher is δ_f.

(iii) if $(\delta_c, \delta_f) \in [0, 1) \times [0, 1]$, depending on the dimension of consumer and firm side effects, BBPD can increase or decrease first-period prices.

Consider first the strategic decisions of firms when their decisions change the information that firms gain in the current period. Note that if no information is gained by the consumers' choices, there are no dynamic effects and the firm's decision is exactly as considered in the uniform price benchmark.

We should take into account that $\Lambda_{sym} = \frac{\partial \pi^2}{\partial x_1}(x_1 = \frac{1}{2})$. Fudenberg and Tirole (2000) find that with inelastic demand and consumer preferences uniformly distributed, if firms employ BBPD, a change in the first period price has no effect on second period profit because with uniform distribution a firm's marginal gain in one market is exactly offset by its loss in the other ($\Lambda_{sym} = \frac{\partial \pi_2}{\partial x_1} = 0$). Thus, in Fudenberg and Tirole (2000), under symmetry ($x_1 = \frac{1}{2}$), the firm-side effect plays no role and the consumer-side effect fully determines the result that BBPD raises first period prices. (The same happens under the CES demand function, i.e., $\Lambda_{sym} = \frac{\partial \pi_2}{\partial x_1} = 0$.)

In contrast, with a linear demand, a change in the first period price does not cancel out in the neighborhood of $x_1 = \frac{1}{2}$. Specifically, we find that $\frac{\partial \pi_2}{\partial x_1} < 0$ suggesting that as $\frac{\partial x_1}{\partial p_A^1} < 0$ then $\frac{\partial \pi_A^2}{\partial p_A^1} = \frac{\partial \pi_2}{\partial x_1}\frac{\partial x_1}{\partial p_A^1} > 0$. Thus, firm A's marginal gain in one second period market is higher than its loss in the other market. In other words, if firm A becomes more aggressive in the first period (increasing x_1) firm B can increase p_B^n in period 2 which has a positive impact on p_A^o and thus on π_A^o ($\frac{\pi_A^o}{\partial p_B^n}\frac{\partial p_B^n}{\partial x_1} > 0$). However, firm A will need to price more aggressively to to poach consumers from the rival and an increase in x_1 reduces the segment of new customers too (i.e., $\frac{\pi_A^n}{\partial p_B^o}\frac{\partial p_B^o}{\partial x_1} < 0$ and $\frac{\pi_A^n}{\partial x_1} < 0$). Thus aggressive pricing in the first period reduces first-period profit as well as second-period. To avoid this, firms want to raise their first period prices above the uniform price. Therefore, even if consumers are naive (i.e., no consumer-side effect), the firm-side effect can still lead to higher first period prices under BBPD. We will see that this is the case for instance for the linear demand function. Specifically we will consider two examples of linear demands, a low and a high demand, respectively: (i) $q(p) = 1 - p$ (consumer-side effect leads to a reduction in first-period price but firm side effect leads to an increase) and (ii) $q(p) = 2 - p$ (both effects increase first-period prices).

We will see that the demand function considered as well as consumers and firms' patience play an important role in the effect of BBPD on first-period prices, and hence on profits.

Prices clearly affect the quantity demanded by each type of consumer and the overall output supplied. In the perfectly inelastic benchmark case ($q(p) = 1$), any given consumer demands one unit of the good with and without price discrimination. Elastic demand, instead, implies an inverse relation between price and demand. The consequence is that switching consumers are demanding a higher quantity $q(p^n) > q(p^{nd})$, both individually and on aggregate. Loyal consumers, despite consuming less than switchers $q(p^o) < q(p^{nd})$, get more of the good than in case discrimination did not take place $q(p^o) > q(p^{nd})$. Let Q_2^{BBPD} and Q_2^{nd} represent the aggregate quantity exchanged on the market under BBPD and no discrimination, in period 2, respectively. So basically

$$Q_2^{BBPD} = 2x_A q(p^o) + 2(\frac{1}{2} - x_A)q(p^n)$$

$$Q_2^{nd} = q(p^{nd})$$

$$\begin{aligned}
\Delta Q_2 &= Q_2^{BBPD} - Q_2^{nd} = 2x_A q(p^o) + 2(\frac{1}{2} - x_A)q(p^n) - q(p^{nd}) \\
&= 2x_A q(p^o) + 2(\frac{1}{2} - x_A)q(p^n) - \left(2x_A + 2(\frac{1}{2} - x_A)\right) q(p^{nd}) \\
&= 2x_A \left[q(p^o) - q(p^{nd})\right] + 2(\frac{1}{2} - x_A)\left[q(p^n) - q(p^{nd})\right] > 0
\end{aligned}$$

If $p^1 < p^{nd}$, $q(p^1) > q(p^{nd})$ suggesting that aggregate quantity exchanged on the market under BBPD is greater than under no discrimination (i.e., $Q_1^{BBPD} > Q_1^{nd}$). The more interesting case occurs when $p^1 > p^{nd}$ and $q(p^1) < q(p^{nd})$. In this situation, (i.e., $Q_1^{BBPD} < Q_1^{nd}$). If the demand increase in period 2 more than compensates the higher price and lower consumption in period 1, overall consumption should increase over the two periods. In other words, if $|\Delta Q_2| > |\Delta Q_1|$, overall demand increases when moving from uniform to BBPD. Otherwise, the reverse happens.

5.2 Profits

We can now attempt to answer the question: can BBPD be a winning strategy for firms?

As $R(p)$ is strictly increasing in $[0, \overline{p})$ and $\varepsilon(\overline{p}) = 1$, in comparison to uniform pricing, profits increase with price discrimination (say D) as long as $\Delta p = p^D - p^U > 0$ and $p^D, p^U < \overline{p}$. Otherwise, if $p^D < p^U$, profits fall. This explains why in the next proposition we state that profits fall in period 2, but can increase or fall in period 1.

In general, we will see that conclusions about the impact of BBPD on aggregate discounted profits depend on two factors: first, the specific demand function $q(p)$ that is being used to model the market, and second, the degree of foresight exhibited by consumers and firms, which can be quantified using their respective discount factors(δ_c, δ_f).

Proposition 5 Comparing profits under BBPD and uniform pricing and let $\Delta\pi = \pi - \pi^{nd}$:
(i) Second-period profits are always below the no discrimination profits: $\Delta\pi^2 < 0$
(ii) First-period profit increases with δ_c *if* $2R(p_B^n)\rho_{Bx1}^n > 1$, *and increases with* δ_f *if* $(\rho_{Bx1}^n + \rho_{Bx1}^o) < \frac{1}{R^o}$.
(iii) The impact of BBPD on overall profits is equal to $\Delta\Pi = \Delta\pi^1 + \delta_f\Delta\pi^2$. *If* $p^1 < p^{nd}$ *then* $\Delta\Pi < 0$, *otherwise if* $p^1 > p^{nd}$ *then* $\Delta\Pi > 0$ *as long as* $\left|\Delta\pi^1(\delta_c, \delta_f)\right| > \left|\delta_f\Delta\pi^2\right|$. *As* $\Delta\pi^2 < 0$ *an increase in* δ_f *has a positive impact on overall profits due to the positive impact on first period profits but a negative impact because BBPD lowers second period profits.*

Proof. See the Appendix. ■

5.3 Consumer surplus and social welfare

We can now examine the welfare effects of BBPD. Aggregate consumer, which we take as the measure of consumer welfare, in period $t = 1, 2$ is CS. Ω_t represents the average transport costs in period t.

$$CS_2 = Y + V + 2x_A \int_{p^o}^{\widehat{p}} q(s)ds + 2\left(\frac{1}{2} - x_A\right)\int_{p^n}^{\widehat{p}} q(s)ds - \Omega_2$$

with

$$\Omega_2 = 2\int_0^{x_A} xdx + 2\int_{x_A}^{\frac{1}{2}} (1-x)dx = \frac{3}{4} - 2x_A(1-x_A).$$

Under no discrimination in each period:

$$CS^{nd} = Y + V + \int_{p^{nd}}^{\widehat{p}} q(s)ds - \Omega^{nd} \text{ with } \Omega^{nd} = 2\int_0^{\frac{1}{2}} xdx = \frac{1}{4}$$

It follows that for $p^{nd} > p^o > p^{nd}$:

$$\Delta CS_2 = \underbrace{2x_A\left[\int_{p^o}^{pnd} q(s)ds\right] + 2\left(\frac{1}{2} - x_A\right)\int_{p^n}^{pnd} q(s)ds}_{(+)} - \Delta\Omega_2$$

In period 1, all consumers buy from the preferred retailer:

$$CS_1 = Y + V + \int_{p^1}^{\widehat{p}} q(s)ds - \Omega^1 \text{ with } \Omega_1 = 2\int_0^{\frac{1}{2}} xdx = \frac{1}{4}$$

$$\Delta CS_1 = \int_{p^1}^{pnd} q(s)ds \text{ with } p^1 \gtrless p^{nd}$$

We may establish the following result.

Corollary 2. *(Consumer surplus effect): In comparison to uniform pricing:*

(i) BBPD increases consumer surplus in period 2;

(ii) BBPD boosts consumer surplus in period 1 as long as $p^1 < p^{nd}$, otherwise the reverse happens.

(iii) If $p^1 < p^{nd}$, BBPD boosts overall consumer surplus, for any $(\delta_c, \delta_f) - values$. If $p^1 > p^{nd}$ then BBPD benefits consumers as long as $|\Delta CS_1| > |\delta_c \Delta CS_2|$; otherwise the reverse happens.

Proof. See the Appendix. ∎

Note that the overall impact of BBPD on consumer surplus is given by $\Delta CS = \Delta CS_1 + \delta_c \Delta CS_2$. Part (i) of Corollary 2 suggests that the reduction in prices and the output expansion effect of BBPD more than compensates the increase in average transport costs in period 2. Look next at period 1. When the consumer and the firm side effects (which depend on (δ_c, δ_f)) are such that $p^1 < p^{nd}$ then first-period consumption increases. In this case, BBPD boosts overall discounted consumer surplus for all $(\delta_c, \delta_f) - values$. Conclusions are less clear cut when $p^1 > p^{nd}$. In this situation, overall first-period consumption falls so $\Delta CS_1 < 0$; in period 2 $\delta_c \Delta CS_2 > 0$. If consumers are myopic or when $\delta_c = 0$ or low, the negative impact of the price increase with associated reduced consumption in the first period ($\Delta CS_1 < 0$) is more significant than the gains resulting from price discrimination in the second period ($\Delta CS_2 > 0$). Overall consumer surplus falls. As δ_c increases the reverse might happen: the output expansion effect in period 2 more than compensates the reduction in first-period consumption (due to higher prices) and the higher average transport costs in period 2. In aggregate consumers are better off with BBPD.

Consider next the impact of BBPD on period $t = 1, 2$ social welfare $W_t = \pi_{tind} + CS_t$. Consider a strictly decreasing demand function $q(p)$ with $\varepsilon(\overline{p}) = 1$. Let p^D be the discrimination price and p^U the uniform pricing, with $p^D, p^U < \overline{p}$, with associated output Q^D and Q^U, respectively. If $\Delta\Omega = 0$ and $p^D > p^U$ it follows that

$$\Delta Q p^U < \Delta W < \Delta Q p^D$$

Hence, $\Delta W > 0$ as long as $\Delta Q > 0$. As in period 1, $\Delta\Omega_1 = 0$, welfare increases with BBPD as long as the consumer and firm side effects reduce the first-period price. Otherwise, when BBPD leads firms to raise the first-period price, the output reduction effect fully explains the social welfare reduction in period 1.

As stated by Stole, 2007, in unit demand models, because price discrimination has no role to increase aggregate output, one must be careful when interpreting the welfare results obtained. Indeed, we can see that under unit demand $\Delta W_t = -\Delta\Omega$. This suggests that social welfare is only affected by inefficient switching caused by price discrimination.

Consider next the second-period prices under BBPD. The average price under price discrimination can be written as $Ap^D = 2x_A p_A^o + 2(\frac{1}{2} - x_A)p_B^n$. As $Ap^D < p^U$ then BBPD boots social welfare in period 2 as long as the output expansion effect more than compensates inefficient switching.

To compute the overall impact of BBPD on aggregate discounted welfare we should take into account the consumers and firm's discount factors equal to:

$$\Delta W = \Delta\pi_{1ind} + \Delta CS_1 + \delta_f \Delta\pi_{2ind} + \delta_c \Delta CS_2$$

We can readily obtain the following result.

Corollary 3 *(Social welfare effects)*: *In comparison to uniform pricing:*

(i) BBPD boosts social welfare in period 1 if total output increases ($p^1 < p^{nd}$), otherwise welfare falls.

(ii) BBPD boosts social welfare in period 2 if the output expansion effect more than compensates the increase in average transport costs due to inefficient switching.

(iii) If the consumer and firm side effects are such that $p^1 < p^{nd}$ (output increases), *BBPD boosts overall discounted welfare for any* $(\delta_c, \delta_f) - values$*. If* $p^1 > p^{nd}$ *then depending on* $(\delta_c, \delta_f) - values$*, BBPD can enhance or reduce overall discounted welfare.*

Next we will see that the assumption of different demand functions and possibly different discount factors for firms and consumers might produce different welfare results.

6 Specific demand functions

This section investigates whether BBPD can benefit consumers, profits, and overall welfare relative to uniform pricing for specific demand functions, namely (i) perfect inelastic unit demand, (ii) CES demand and (iii) linear demand for $(\delta_c, \delta_f) \in [0, 1) \times [0, 1]$.

6.1 Unit demand

As in the base model of Fudenberg and Tirole (2000), each consumer only buys one unit of the good in each period. Hence $q(p) = 1$, $\varepsilon(p) = 0$. Using the results derived in Proposition 1 it is easy to show that with no discrimination $p^{nd} = 1$ and overall profits are $\pi^{nd} = \frac{1}{2}(1 + \delta_f)$; $CS^{nd} = \left(V - \frac{5}{4}\right)(1 + \delta_c)$ and $W^{nd} = \left(V - \frac{5}{4}\right)(1 + \delta_c) + \frac{1}{2}(1 + \delta_f)$. Using equations (11) and (12) defined in Proposition 2, we get $p_A^o = \frac{2}{3}x_1 + \frac{1}{3}$ and $p_B^n = \frac{4}{3}x_1 - \frac{1}{3}$. Doing the same to obtain second-period prices in turf B we get $p_A^n = 1 - \frac{4}{3}x_1$ and $p_B^o = 1 - \frac{2}{3}x_1$. With uniform distribution with unit demand, in the symmetric equilibrium with $x_1 = \frac{1}{2}$ it is straightforward to prove that the firm-side effect plays no role, i.e. $\Lambda_{sym} = 0$. The firm side effect is null. Thus, changes in x_1 (and hence in p_A^1) have no net effect on firm A's second-period profit. Even though firms are forward looking, the first-order effect of shifting the indifferent consumer equals zero around the market center—a firm's marginal gains in profit over one segment are exactly canceled out by losses over the other in period 2. This is an intriguing result because it implies that forward-looking firms' incentive to avoid the unprofitable use of purchase history information has no impact on the first-period market equilibrium. From Proposition 3 it follows that the first-period equilibrium price is obtained from $p^1 = \left(1 - \delta_c + \delta_c \frac{\partial p_B^n}{\partial x_1}\right) - \delta_f \Lambda_{sym}$. Hence only the consumer-side effect affects the first-period price with BBPD. Because $\frac{\partial p_B^n}{\partial x_1} = \frac{4}{3}$ it immediately follows that $p^1 = 1 + \frac{1}{3}\delta_c > p^{nd}$, $p_A^o = \frac{2}{3}$ and $p_B^n = \frac{1}{3}$. Therefore, overall profits with BBPD are $\pi^{BBPD} = \frac{1}{2}\left(1 + \frac{1}{3}\delta_c\right) + \frac{5}{18}\delta_f$.

From our previous computations it is straightforward to show that

$$\Delta CS = (CS_1 - CS_{nd}) + \delta_c (CS_2 - CS_{nd}) = \frac{1}{18}\delta_c$$

Thus $\Delta CS > 0$ for all δ_c and it is higher the greater is δ_c, and

$$\Delta W = \frac{7}{18}\delta_c - \frac{4}{9}\delta_f$$

Therefore following result ensues.

Proposition 6. (*Effects of BBPD with unit demand*)

With perfectly inelastic demand:

(i) When $\delta_f = \delta_c = \delta$*, BBPD boots overall consumer surplus but reduces industry profits and social welfare.*

(ii) BBPD boots consumer surplus for all $(\delta_c, \delta_f) - values$*.*

(iii) Profits are higher with BBPD as long as $\delta_f < \frac{3}{4}\delta_c$, otherwise profits fall with BBPD.

(iv) In the region where $\delta_f < \frac{3}{4}\delta_c$ BBPD boosts profits and welfare. If $\frac{3}{4}\delta_c < \delta_f < \frac{7}{8}\delta_c$ profits fall with BBPD but welfare increases.

The comparison with our results with FT (2000) shows that BBPD benefits consumers for all values (δ_c, δ_f). When firms and consumers discount the future at equal rates $\delta_c = \delta_f = \delta$, BBPD boost consumer surplus but reduces industry profits and social welfare. The same happens in our model if δ_c is low enough compared to δ_f. As δ_c increases, the consumer surplus increases more with BBPD. If $(\delta_c, \delta_f) = (1, 1)$ consumer surplus increase at the expense of profits and welfare. It also shows that at the point $(\delta_c, \delta_f) = (0, 1)$ which may have some empirical appeal, addressing the case of naive customers and forward looking firms, BBPD is bad for profits and welfare. Interestingly, our results highlight that there is a domain of values of (δ_c, δ_f), where BBPD can benefit industry profits, consumer surplus and social welfare. This is the case when δ_c is sufficiently high compared to δ_f (that is, $\delta_f < \frac{3}{4}\delta_c$).

6.2 CES demand

Following Esteves and Reggiani (2014) $q(p) = p^{-\varepsilon}$, $\varepsilon(p) = \varepsilon$, with $\varepsilon \in (0, 1)$. Using the results derived in Proposition 1 it is easy to show that with no discrimination $p^{nd} = (1-\varepsilon)^{\frac{1}{1-\varepsilon}}$ and overall profits are $\pi^{nd} = \frac{(1-\varepsilon)}{2}(1+\delta_f)$; $CS^{nd} = Y + \left(v - \frac{5}{4}t\right)(1+\delta_c)$, *and social welfare equals* $W^{nd} = Y + (1-\varepsilon)(1+\delta_f) + \left(v - \frac{5}{4}\right)(1+\delta_c)$. Using equations (11) and (12) defined in Proposition 2, we get the second-period prices defined in Proposition 2 of Esteves and Reggiani (2014). With uniform distribution with a CES demand, in the symmetric equilibrium with $x_1 = \frac{1}{2}$ it is straightforward to prove that the firm-side effect plays no role, i.e. $\Lambda_{sym} = 0$. The reason is that under CES demand preferences the price and demand expansion effects cancel each other out. The equilibrium first-period price is $p_1 = \left[t(1-\varepsilon)\left(1+\frac{\delta}{3}\right)\right]^{\frac{1}{1-\varepsilon}}$, which is clearly above the no-discrimination counterpart. Therefore, the increase in the first-period price due to BBPD can be attributed solely to the consumer side effect. In this case, industry discounted profits and consumer surplus with BBPD are, respectively $\pi_{ind} = t(1-\varepsilon)\left(1+\frac{\delta_c}{3}\right) + \frac{5}{9}t\delta_f(1-\varepsilon)$; $CS = Y + v(1+\delta_c) - \frac{5}{4}t - \frac{43}{36}t\delta_c$. Hence

From our previous computations it is straightforward to show that $\Delta\pi_{ind} = \frac{1}{9}t(1-\varepsilon)(3\delta_c - 4\delta_f)$; $\Delta CS = \frac{1}{18}\delta_c$ and $\Delta W = \frac{1}{18}\left[\delta_c(7-6\varepsilon) - 8\delta_f(1-\varepsilon)\right]$. Therefore, we can establish the following result.

Proposition 7. (*Effects of BBPD with CES demand*)

With a CES demand:

(i) BBPD boosts consumer surplus for all $(\delta_c, \delta_f) -$ values. Consumer surplus under a CES demand function is equal to its counterpart under a unit demand.

(iii) Profits are higher with BBPD as long as $\delta_f < \frac{3}{4}\delta_c$, otherwise profits fall with BBPD.

(ii) BBPD boosts welfare as long as $\delta_f < \frac{(7-6\varepsilon)}{8(1-\varepsilon)}\delta_c$.

Propositions 6 and 7 show that assuming a CES demand instead of a unit demand only affects the impact of BBPD on welfare. Although the CES formulation offers the advantage of being mathematically tractable and providing a closed-form solution for the two-period BBPD model, it has limitations. One significant limitation is that One important limitation is that regardless of whether there is BBPD or not, consumer surplus is unaltered by the elasticity of demand. Additionally, consumer surplus does not vary when shifting from a unit to a CES demand function. Obviously, when $\varepsilon = 0$, the results align with Proposition 6. The main difference is the impact on discounted social welfare. As the elasticity increases, it becomes more likely that

BBPD will increase welfare. This is also the case when δ_f is sufficiently low compared to δ_c. If $\delta_c = \delta_f = \delta$, BBPD boosts overall welfare as long as $\varepsilon > \frac{1}{2}$.

The limitations of the CES formulation emphasize the need to consider other demand functions to obtain a complete understanding of the impact of BBPD on profits, consumer surplus, and welfare.

6.3 Linear demand

In this section we assume that demand is linear of form $q(p) = a - p$. Under the linear demand function of form $q(p) = a - p$; $\varepsilon(p) = \frac{p}{a-p}$ and $\varepsilon'(p) = \frac{1}{(a-p)^2} > 0$. $\varepsilon(\overline{p}) = 1$ at $\overline{p} = \frac{a}{2}$. To illustrate our main findings, we consider two we consider two examples: (i) $q(p) = 1 - p$ and (ii) $q(p) = 2 - p$. In case (ii), consumers demand more units for the same price. We chose these examples for specific reasons. First, under condition (i) of lemma 3, $q(p) = 1 - p$, our analysis indicates that $R(p_B^n)\rho_{Bx1}^n < \frac{1}{2}$, which suggests that the consumer-side effect makes consumers more sensitive to price changes in the first period. If the firm-side effect is null ($\delta_f = 0$), the consumer-side effect leads to a reduction in first-period prices. On the other hand, in case (ii) where $q(p) = 2 - p$, condition (i) of lemma 3 satisfies $R(p_B^n)\rho_{Bx1}^n > \frac{1}{2}$, indicating that the consumer-side effect raises first-period prices under BBPD.[9]

6.3.1 Example 1 (First-period prices can either decrease or increase with BBPD)

Consider the case where demand is of form $q(p) = 1 - p$. In contrast to the case of Fudenberg and Tirole (2000), where first-period prices always increase with BBPD, this example highlights the importance of considering both the consumer and firm-side effects. Depending on the relative strengths of these factors, first-period prices can either decrease or increase with BBPD. When the firm-side effect is absent, i.e. $\delta_f = 0$, and the consumer-side effect fully explains the impact of BBPD on first period prices, we conclude that BBPD reduces prices across all periods relative to uniform pricing.

From proposition 1 and using the fact that $\varepsilon(p) = \frac{p}{1-p}$ we we can summarize the main findings under uniform pricing in the following proposition.

Proposition 8. *For the linear demand function of form $q(p) = 1 - p$, if firms quote a uniform pricing in both periods, each firm equilibrium price is*

$$p^{nd} = \frac{2}{3} + \frac{-\frac{5}{9}}{\sqrt[3]{-\frac{11}{54} + \sqrt{\frac{23}{108}}}} + \sqrt[3]{-\frac{11}{54} + \sqrt{\frac{23}{108}}} \simeq 0.43$$

and each consumer demands $q(p^{nd}) \simeq 0.57$ units. *Industry profit is* $\Pi^{nd} \simeq 0.2451(1 + \delta_f)$. *Consumer surplus is* $CS^{nd} \simeq (V - 0.0876)(1 + \delta_c)$ *and overall welfare is* $W^{nd} \simeq (V - 0.08756)(1 + \delta_c) + 0.2451(1 + \delta_f)$.

When firms can employ BBPD, at an interior solution, the second-period equilibrium prices $p_i^{o*} \in \left(0, \frac{1}{2}\right)$ and $p_i^{n*} \in \left(0, \frac{1}{2}\right)$ with $\varepsilon(p_i^{o*}) \in (0, 1)$ and $\varepsilon(p_i^{n*}) \in (0, 1)$ are the solution to the system of implicit equations defined in equations (11), (12), (13) and (14). Therefore, we can summarize some important findings in the following Corollary.

[9] The Corollaries and Propositions presented in this section are derived by substituting the specific demand functions into the results obtained for a general demand function. The proofs for these results are available upon request from the authors.

Corollary 4. *When $q(p) = 1 - p$, under symmetry $\left(x_1 = \frac{1}{2}\right)$:*

(i) The price targeted to old and new customers are respectively equal to $p^o = 0.414$ $(q^o = 1.586)$ and $p^n = 0.168$ $(q^n = 1.832)$. Therefore, $R^o = 0.2430$ and $R^n = 0.1398$.

(ii) Consumers with preferences for retailer $x \in [0, 0.4120] \cup [0.588, 1]$ buy from the same firm, while consumers with $x \in [0.4120, 0.588]$ switch retailers from one period to the next.

(iii) $\rho^n_{Bx1} = \underbrace{\frac{\partial p^n_B}{\partial x_1}}_{-} \frac{x_1}{p^n_B} = 3.086$ and $\rho^o_{Bx1} = \underbrace{\frac{\partial p^o_B}{\partial x_1}}_{-} \frac{x_1}{p^o_B} = -0.111$.[10]

(iv) Since $R(p^n_B)\rho^n_{Bx1} = 0.4314 < \frac{1}{2}$, the consumer side effect ($\Psi_{sym} < 1$, c.f. lemma 3) *acts to decrease first-period prices in comparison to uniform pricing.*

(v) Because $\Lambda_{sym} = -0.0388 < 0$, the firm side effect, i.e., $\delta_f\left(-\Lambda_{sym}\right) > 0$, acts to increase first-period price in comparison to uniform pricing.

Proof. See the Appendix. ■

The equilibrium outcome in the first period is determined by the levels of patience exhibited by both firms and consumers. As demonstrated in parts (iv) and (v), there are two opposing forces at play that influence first-period prices under BBPD. When $R(p^n_B)\rho^n_{Bx1} < \frac{1}{2}$ forward-looking consumers respond more to first-period price changes. Consequently, first-period prices fall as consumer patience increases. Conversely, an increase in firms' patience results in higher first-period prices. Therefore, whether first-period prices will be lower or higher than the no discrimination counterpart will depend on which of these effects dominates, which in turn also depend on the (δ_c, δ_f) −values.

It's essential to note that, with linear demand, BBPD has the potential to intensify competition in the first period beyond the static benchmark, a phenomenon that diverges from Fudenberg and Tirole's (2000) findings that first-period prices are higher than the static level.

We will now demonstrate that for high $\delta_c - values$ and low $\delta_f - values$, the consumer side effect is the dominant one. As a result, the first-period price falls in comparison to its non-discrimination counterpart. Moreover, we find that as firms become more patient, first-period prices actually increase. This contradicts the idea that firm patience has no impact on first-period prices under unit demand, and BBPD only affects first-period prices by lowering consumers' price sensitivity.

From proposition 3, the first-period price is implicitly defined as follows

$$R(p^1_A) = \Psi_{sym}\left[1 - \varepsilon(p^1_A)\right] + \delta_f(-\Lambda_{sym}). \tag{29}$$

Using the values for Ψ_{sym} and $-\Lambda_{sym}$ presented above we can obtain the following proposition.

Proposition 9. With $q(p) = 1 - p$, *there is a symmetric equilibrium in which $p^1_A = p^1_B = p^1$, with*

$$p^1 = \frac{2}{3} + \frac{\frac{2277}{25000}\delta_c - \frac{6427}{500\,000}\delta_f - \frac{5}{9}}{\sqrt[3]{\Theta(\delta_c, \delta_f)}} + \sqrt[3]{\Theta(\delta_c, \delta_f)}$$

$\Theta(k, \delta)$ is defined in the Appendix. As expected at $\Theta(0, 0)$, $p^1 = p^{nd}$.

Proof. See the Appendix. ■

The next pictures plot the first-period price and quantity demanded at that price with BBPD (red) and with no discrimination (grey).

[10] Note that $\frac{\partial p^n_B}{\partial x_1} = -\frac{\partial p^n_A}{\partial x_1} = 1.037$ *and* $\frac{\partial p^o_A}{\partial x_1} = -\frac{\partial p^o_B}{\partial x_1} = 0.092$.

Figure 1: First-period price with BBPD and Uniform pricing

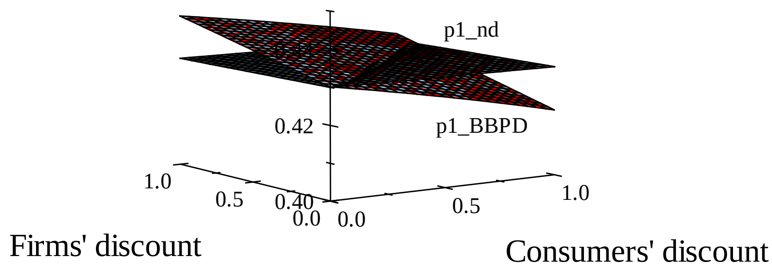
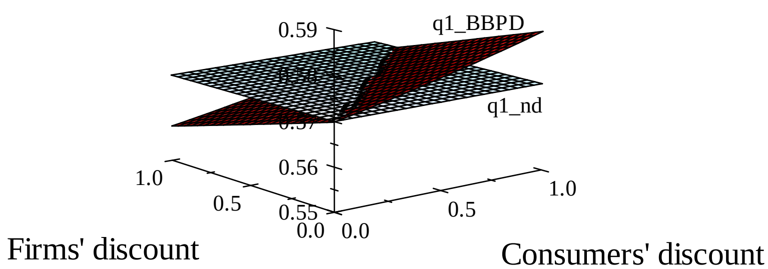


Figure 2: First-period demand with BBPD and Uniform pricing



The pictures demonstrate that the consumer side effect causes a decrease (increase) in first-period price (quantity), whereas the firm side effect results in an increase (decrease) in first-period price (quantity). This occurs when δ_c is high and δ_f is low. Conversely when δ_f is high and δ_c is low, the opposite is true.

When $\delta_c = \delta_f = \delta$, we observe that $p^1 > p^{nd}$, indicating that the firm-side effect dominates the consumer side effect. The same holds true in special cases where consumers are myopic, when $(\delta_c, \delta_f) = (0, \delta_f)$ or when δ_c is low compared to δ_f. Note that, under unit demand, BBPD would have no effect on first-period price when consumers are myopic or when $(\delta_c, \delta_f) = (0, \delta_f)$, as demonstrated by FT, 2000.

If $p^1 > p^{nd}$ due to the values of (δ_c, δ_f), then we conclude that $\Delta\pi_{1ind} > 0$,meaning that the price effect more than compensates for the reduction in output. Since all consumers buy efficiently in period 1, $\Delta\Omega = 0$, the welfare impact of BBPD on CS and W is solely explained by the output reduction effect (which affects profits and consumer welfare). Thus, $\Delta CS_1 < 0$ and $\Delta W_1 < 0$. In contrast, when δ_c is significantly higher than δ_f,then the consumer side effect causes $p^1 < p^{nd}$. Consequently, $\Delta\pi_{1ind} < 0$, indicating that the output expansion effect is not sufficient to offset the price effect. As all consumers buy efficiently in period 1, $\Delta\Omega = 0$. Thus, $\Delta CS_1 > 0$ and $\Delta W_1 > 0$ due to the output expansion effect.

Now let's consider period 2. In contrast to BBPD with unit demand, where roughly one-third of consumers switch from period 1 to period 2, our analysis using the linear demand function $q(p) = 1 - p$, suggests that a lower proportion of consumers switch retailers in equilibrium, around 17.6% of all consumers. This implies that the detrimental impact of BBPD on second-period consumer surplus and welfare, caused by inefficient switching, is expected to be less significant.

Therefore, since BBPD lowers all second-period prices and the average second-period price is below its non-discrimination counterpart,[11] industry profits decrease while consumer surplus and social welfare increase. In the next section, we analyze the overall effect of BBPD on discounted profits, consumer surplus, and welfare, which depend on the patience levels of both consumers and firms.

6.3.2 Example 2 (First-period prices always increase with BBPD)

We now consider $q(p) = 2 - p$, with $p \leq 2$. In this case, consumers purchase more units at a given price p compared to the previous example. Furthermore, the price elasticity of demand is higher. Applying the same approach as before, we can derive the main results under uniform pricing from proposition 1, using the fact that $\varepsilon(p) = \frac{p}{2-p}$. These results are summarized in the following proposition.

Proposition 10. *For the linear demand function of form $q(p) = 2 - p$, if firms quote a uniform pricing in both periods, each firm equilibrium price is*

$$p^{nd} = \frac{4}{3} + \frac{-\frac{2}{9}}{\sqrt[3]{-\frac{17}{27} + \sqrt{\frac{11}{27}}}} + \sqrt[3]{-\frac{17}{27} + \sqrt{\frac{11}{27}}} \simeq 0.4563$$

and each consumer demands $q(p^{nd}) \simeq 1.5437$ units. *Industry profit is* $\Pi^{nd} \simeq 0.7044(1 + \delta_f)$. *Consumer surplus is* $CS^{nd} \simeq (V + 0.9415)(1 + \delta_c)$ *and overall welfare is* $W^{nd} \simeq (V + 0.9415)(1 + \delta_c) + 0.7044(1 + \delta_f)$.

When firms can employ BBPD, at an interior solution, the second-period equilibrium prices $p_i^{o*} \in (0, 1)$ and $p_i^{n*} \in (0, 1)$ with $\varepsilon(p_i^{o*}) \in (0, 1)$ and $\varepsilon(p_i^{n*}) \in (0, 1)$ are the solution to the system of implicit equations defined in equations (11), (12), (13) and (14). Therefore, we can summarize our main findings in the following Corollary.

Corollary 5 *When $q(p) = 2 - p$, under symmetry ($x_1 = \frac{1}{2}$):*

(i) $p^o = 0.3311$ *and* $p^n = 0.1543$. *Thus,* $R^o = 0.5526$ *and* $R^n = 0.2848$.

(ii) Consumers with preferences $x \in [0, 0.3446] \cup [0.6554, 1]$ buy from the same firm, while consumers with $x \in [0.3446, 0.6554]$ switch retailers from period 1 to period 2.

(ii) $\rho^n_{Bx1} = 2.083$ *and* $\rho^o_{Bx1} = -0.452\,88$.[12]

[11] The average second-period price is $Ap^D = 2x_A p^o + 2(\frac{1}{2} - x_A)p^n = 2(0.412)(0.414) + 2(0.5 - 0.412)0.168 = 0.371 < p^{nd}$

[12] The derivatives $\frac{\partial p^n_B}{\partial x_1} = -\frac{\partial p^n_A}{\partial x_1} = 0.6427$ and $\frac{\partial p^o_A}{\partial x_1} = -\frac{\partial p^o_B}{\partial x_1} = 0.2999$

(iii) Since $R(p_B^n)\rho_{Bx1}^n = 0.5932 > \frac{1}{2}$*, the consumer side effect* ($\Psi_{sym} > 1$, c.f. lemma 3) *acts to increase first-period prices in comparison to uniform pricing.*

(iv) Because $\Lambda_{sym} = \frac{\partial \pi_A^2}{\partial x_1} = -0.0286$*, the firm side effect*, i.e. $\delta_f(-\Lambda_{sym}) > 0$*, acts to increase first-period price in comparison to uniform pricing.*

Proof. See the Appendix. ■

Once again, based on parts (iii) and (iv), we can conclude that the overall impact of BBPD on first-period prices hinges on the values of δ_c and δ_f. In the case where both consumers and firms are fully myopic, the equilibrium price would be the same as the non-discrimination price. However, under the demand function $q(p) = 2 - p$, as long as δ_c and δ_f are greater than 0, we can assert that first-period prices will always exceed the static level of p^{nd}. This is due to the combined effect of consumer patience, which reduces price sensitivity and increases first-period prices directly, and firm patience, which also increases prices as discussed earlier.

From equation (29) and using the values for Ψ_{sym} and $-\Lambda_{sym}$ presented in Corollary 5, we can write the following proposition.

Proposition 11. With $q(p) = 2 - p$, *there is a symmetric equilibrium in under which* $p_A^1 = p_B^1 = p^1$, where

$$p^1 = \frac{4}{3} - \frac{\frac{18\,631}{150\,000}k + \frac{28\,261}{3000000}\delta + \frac{2}{9}}{\sqrt[3]{\Phi(\delta_c, \delta_f)}} + \sqrt[3]{\Phi(\delta_c, \delta_f)}$$

where $\Phi(\delta_c, \delta_f)$ *is defined in the Appendix. As expected at* $\Phi(0,0)$, $p^1 = p^{nd}$.

Proof. See the Appendix. ■

Figure 3: First-period price with BBPD and uniform pricing.

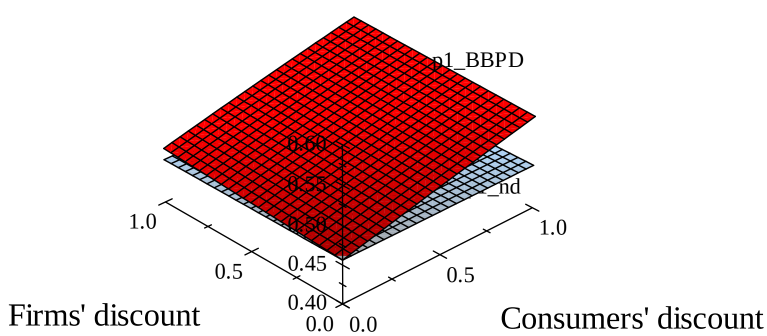
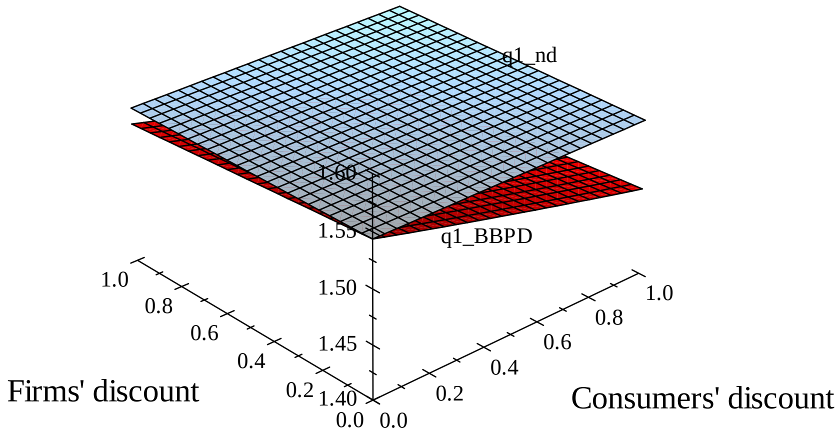


Figure 4: First-period demand with BBPD and uniform pricing.



Figures 3 and 4 illustrate how changes in $(\delta_c, \delta_f)-$values affect the first-period price and quantity. The consumer side effect is stronger when $q(p) = 2-p$; the firm side effect is stronger when $q(p) = 1-p$ (as $|\Lambda_{sym}|$ is higher in this case).

When the demand function takes the form of $q(p) = 2 - p$, BBPD results in around 15.54% of consumers in each turf switching to a new retailer in period 2, which translates to approximately 31% of all consumers switching retailers in period 2. This suggests that there is a greater degree of inefficient switching in example 2 compared to example 1. However, as we have already established, as long as $\frac{1}{4} < x_A < \frac{1}{2}$, (which is the case here), the consumer surplus gains offset the loss associated with "higher transport costs". Therefore, consumer surplus increases in period 2. Additionally, since the average second-period price is lower (or the quantity demanded is higher) under example 2 than under example 1, the increase in second-period CS is expected to be higher when $q(p) = 2 - p$. The same holds true for social welfare. When the output expansion is greater than the increase in "excessive transport costs," the increase in welfare from moving from uniform pricing to BBPD is expected to be higher under example 2 compared to example 1. In period 1, firms benefit from the first-period price increase, but consumers and welfare are worse off. Comparing the two examples, we observe that firms benefit more when $q(p) = 2 - p$, while CS and W are better off $q(p) = 1 - p$.

6.4 Managerial and policy implications

While managers often ask themselves when it would be profitable to employ BBPD, policymakers are interested in knowing when BBPD can enhance consumer surplus and social welfare. In what follows we will show that the answer to these questions depends on industry-specific factors like consumer demand behavior and the foresight of firms and consumers. By taking these factors into consideration, managers (and policymakers alike) can make more informed decisions about when to implement (allow) BBPD in imperfectly competitive markets.

Next we will explore how BBPD affects total discounted profits, consumer surplus, and welfare, taking into consideration the unit and the linear demand functions discussed above and the discount factors space. The following pictures are plotted for $(\delta_c, \delta_f) \in [0, 1]^2$. They summarize our main findings for the demand cases:

$q(p) = 1$; $q(p) = 1 - p$; and $q(p) = 2 - p$.

Figure 5: Total discounted profits, consumer surplus and welfare when $q(p) = 1$

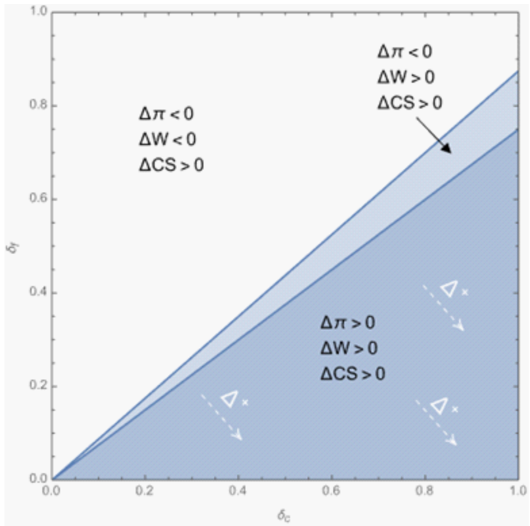


Figure 6: Total discounted profits, consumer surplus and welfare when $q(p) = 1 - p$

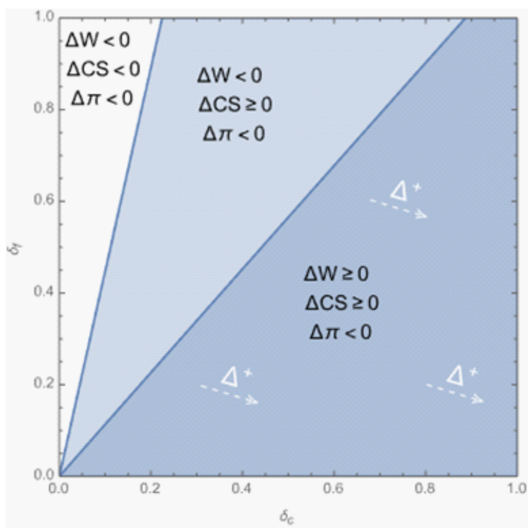
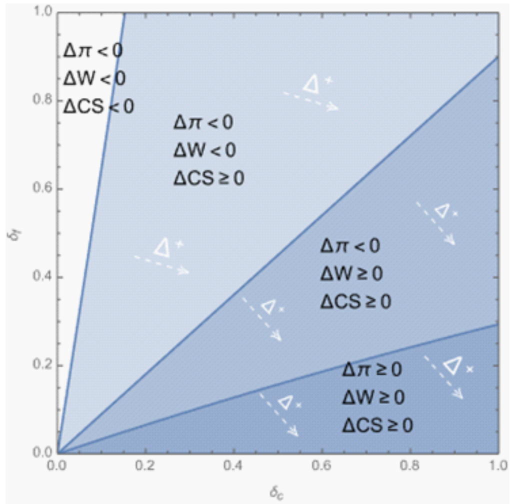


Figure 5: Total discounted profits, consumer surplus and welfare when $q(p) = 2 - p$



Figures 5-7 illustrate when $\delta_c = \delta_f = \delta$ (45-degree line) the practice of BBPD leads to an increase in total discounted consumer surplus at the expense of total discounted industry profits, regardless of the demand function considered. However, the two linear demand examples depicted in the figures 6 and 7 demonstrate that depending on the magnitude of the output effect and the "excessive transport costs" effect caused by inefficient switching, welfare can either increase or decrease. In the low demand case ($q(p) = 1 - p$), approximately 17.6% of consumers switch firms in period 2. When consumers demand more units at the same price level ($q(p) = 2 - p$), this proportion almost doubles, reaching about 33%. As a result, if $\delta_c = \delta_f = \delta$ total welfare increases when $q(p) = 1 - p$, but decreases when $q(p) = 2 - p$.

By extending the Fudenberg and Tirole's analysis to different discount factors for firms and consumers, we discover new insights even when demand is perfectly inelastic. We demonstrate that for any combination of (δ_c, δ_f), price discrimination based on purchase history consistently results in an increase in consumer surplus. However, its impact on industry profits and total social welfare can be positive or negative, depending on the specific combination of discount factors. This is in contrast to the negative effect on profits and welfare that is observed when firms and consumers have equal discount factors.

When the demand function is linear and given by $q(p) = 2 - p$, Figure 7 highlights that if the discount factor for customers is high while the discount factor for firms is low, managers can use BBPD to increase profits. In contrast, if $q(p) = 1 - p$, BBPD will harm profits, for all combinations of discount factors.

In sharp contrast to the perfectly inelastic case, both figures 6 and 7 show that under linear demands there is a small region in the discount factor space− δ_c is sufficiently low and δ_f is sufficiently high− where BBPD is also bad for consumer surplus and welfare. Although this sub-domain is a relatively small area, one should notice that it includes for instance the point $(\delta_c, \delta_f) = (0, 1)$ which may have some empirical appeal, addressing the case of naïve customers and forward-looking firms.

As firms become more forward looking (δ_f increases), the firm side effect becomes more important, resulting in an increase in the equilibrium price in the first period. If δ_c is low, consumers value more the negative impact of a price increase (reduced consumption) in the first period ($\Delta CS^1 < 0$) than the gains from price discrimination in the second period ($\Delta CS^2 > 0$). Thus, total discounted consumer surplus falls with price discrimination. With respect to profits, if δ_f is high, the reduction in profits in the second period has a substantial impact on discounted total profits. Consequently, the practice of BBPD has a negative effect on total discounted profits.

When we allow the intercept of the demand function to increase from $a = 1$ to $a = 2$, consumers demand more units for the same price. Figure 7 shows that there arises a fourth region where profits can be increased by BBPD. This happens when δ_f is sufficiently low compared to δ_c. There are several factors that can contribute to firms being more impatient than consumers. One such factor is the disparity between loan rates and deposit rates which is a reality in many markets. When loan rates are significantly higher than deposit rates, firms may be more inclined to discount the future compared to consumers. Alternatively, if firms are more pessimistic about the future than consumers, they may not even be sure if there will be a second period profit. This is often the case with starting businesses, or when firms have access to more (negative) information about the market than consumers. In such cases, firms may be more inclined to prioritize short-term gains over long-term ones, which can impact their decision-making processes.

It is noteworthy that in this region, BBPD not only increases industry profits, but also enhances consumer surplus and overall welfare. When δ_f is small it indicates that profits with price discrimination are largely discounted (the firm side effect is less important due to the low discount factor). As consumers become more forward-looking (higher δ_c), they tend to place more value on the long-term benefits of price discrimination, making them less price sensitive in the short term (strong consumer side effect). Therefore, having a sufficiently low value of δ_f is necessary to achieve higher profits with BBPD. Importantly, when this happens it's not just the profits that benefit, but consumers and overall welfare also experience positive impacts when this condition is met.

In sum, when evaluating the profit and welfare effects of price discrimination based on purchase history, managers and policymakers alike should consider the consumer demand behavior in specific markets as well as the discount factor space.

7 Final remarks

This paper has tried to enrich our understanding of the profit and welfare effects of behavior-based price discrimination taking into account consumers' demand behavior in different markets. It is crucial to note that the nature of demand is dependent on the specific product or service under scrutiny and the particular market in which it is being sold. In line with previous research we have studied a two-period model of differentiated duopoly where firms compete à la Hotelling in the first period and offer distinct prices to repeat and new customers in the second period. (Consumers are heterogeneous with respect to their preferences for retailers.) However, our paper distinguishes itself from previous studies by incorporating a general downward sloping demand function, where the elasticity of demand is not invariant to price changes. Assuming that consumer demand falls with price, rather than remaining constant, is important because it reflects the reality of consumer behavior in many markets.

Although certain markets may exhibit unit or perfectly inelastic demand characteristics, it is more common for demand to be represented by a downward sloping demand curve. As stated by Armstrong (2006), neglecting the possibility of a downward sloping demand function can result in inaccurate predictions about the overall welfare implications of BBPD. Thus, it is critical to assume a downward sloping demand function when modeling consumer behavior and assessing the impact of behavior-based price discrimination in oligopolistic markets.

Another distinctive feature of this paper, in comparison to existing studies of BBPD, is the assumption that firms and consumers can have different discount factors. It is essential to consider different discount factors for firms and consumers because they have distinct objectives and time horizons when making decisions. By recognizing these differences in discount factors, we can better model the behavior of firms and consumers and accurately predict the different impact of firm versus consumer patience on market outcomes. A common finding in most models of BBPD with a unit or a constant elasticity demand, is that first-period prices increase with consumer patience but are invariant to firm patience (Fudenberg and Tirole, 2000; Esteves and Reggiani, 2014). Our analysis reveals that this classic result no longer holds true. If, for instance, consumers demand is linear, the firm side effect gives firms an incentive to raise first-period prices; and the prospect of BBPD in the second-period can make consumers less or more sensitive to price changes in period 1. Depending on the relative strength of these effects, first-period prices may either increase or decrease compared to uniform pricing.

Our analysis reveals that taking into account different discount factors for firms and consumers leads to new results compared to the assumption of unit demand and equal discount factors. Specifically, in markets relatively well represented by a perfectly inelastic demand, we show that price discrimination consistently results in an increase in consumer surplus for any combination of discount factors. As usual, compared to uniform pricing BBPD only impact second period welfare due to inefficient switching. With unit demands, depending on the specific combination of discount factors, we show that the impact of BBPD on total discounted industry profits and welfare can be positive (high δ_c compared δ_f) or negative (low δ_c compared to δ_f).

In markets where consumer demand behavior is better represented by a linear demand function we show that the output effect plays a crucial role in determining the impact of price discrimination on overall welfare. Depending on the strength of this latter effect (compared to excessive switching), BBPD can either increase or decrease social welfare. In markets with a low demand function (e.g. $q(p) = 1 - p$) three regions are identified depending on the discount factor space (δ_c, δ_f). For all combinations of discount factors (δ_c, δ_f), profits fall with discrimination. In contrast to unit demand models, when δ_c is sufficiently low and δ_f is sufficiently high, BBPD harms consumer surplus and welfare. Although this sub-domain is a relatively small area, one should notice that it includes for instance the point $(\delta_c, \delta_f) = (0, 1)$ which may have some empirical appeal, addressing the case of naïve customers and forward-looking firms. In contrast, when δ_c is sufficiently high compared to δ_f, we find that BBPD boosts consumer surplus and overall welfare.

In markets where consumers demand more units (e.g. $q(p) = 2 - p$) we can observe a fourth region where BBPD can increase total discounted profits. This happens when δ_f is sufficiently low compared to δ_c. It is noteworthy that in this region, BBPD not only increases profits, but also enhances total discounted consumer surplus and overall welfare. When δ_f is small it indicates that profits with price discrimination are largely discounted. As consumers become more forward-looking (higher δ_c), they tend to place more value on the long-term benefits of price discrimination, making them less price sensitive in the short term. Therefore, under this demand function, having a sufficiently low value of δ_f is necessary to achieve higher profits with BBPD. Importantly, it's not just the profits that benefit, but consumers and overall welfare also experience positive impacts when this condition is met.

Notwithstanding the model addressed in this paper is far from covering all complex aspects of real markets, it highlights that managers and policymakers need to be aware of the different outcomes that can arise when consumer data is used for behavior-based pricing. Specifically, it is important to consider specific market features like consumer demand shape and discount factor conditions. Further directions for future research might be to relax the assumption that transport costs do not depend on quantity. By doing so, the model could explore the possibility of two-stop shopping (as explored by Lu and Matsushima, 2022, in a personalized pricing static model where consumers demand is perfectly inelastic but consumers buy multiple units). As these business practices continue to be important in the digital economy, there is plenty of potential for further research in this area. Finally, as the theoretical model also provides testable hypotheses for empirical research, we hope it can be used to further validate and refine the findings.

Appendix

This appendix collects the proofs that were omitted from the text.

Proof of Lemma 1:

From $R'(p) = q(p) + pq'(p) = q(p)\left[\frac{q(p)+pq'(p)}{q(p)}\right] = q(p)\left[1 - \varepsilon(p)\right]$. Since $\varepsilon(p)$ is continuous and $\varepsilon'(p) > 0$ on $[0, \widehat{p}]$, there exists a unique $\overline{p} \in (0, \overline{p})$ such that $\varepsilon(\overline{p}) = 1$ and hence $R'(\overline{p}) = 0$. Additionally, $R(0) = R(\widehat{p}) = 0$ and $R'(p) > 0$ for $p \in (0, \overline{p})$ and $R'(p) < 0$ for $p \in (\overline{p}, \widehat{p})$. Therefore, $R(p)$ is strictly quasi-concave and lemma 1 follows.

Proof of Proposition 1:

Considering with no loss of generality the case of firm A, it maximizes its profit π_A^{nd} with respect to its price, with

$$\pi_A^{nd}(p_A, p_B) = p_A q(p_A) \underbrace{\left[\frac{1}{2} + \frac{1}{2}\int_{p_A}^{p_B} q(s)ds\right]}_{\widehat{x}}$$

From the FOC we get

$$q(p_A)\left[\frac{1}{2} + \frac{1}{2}\int_{p_A}^{p_B} q(s)ds\right] + p_A q'(p_A)\left[\frac{1}{2} + \frac{1}{2}\int_{p_A}^{p_B} q(s)ds\right] - \frac{p_A\left[q(p_A)\right]^2}{2} = 0$$

$$\begin{aligned}
p_A\left[q(p_A)\right] &= 2\left[\frac{1}{2} + \frac{1}{2}\int_{p_A}^{p_B} q(s)ds\right]\left[\frac{q(p_A) + p_A q'(p_A)}{q(p_A)}\right] \\
p_A q(p_A) &= 2\left[\frac{1}{2} + \frac{1}{2}\int_{p_A}^{p_B} q(s)ds\right]\left[(1 - \varepsilon(p_A))\right]
\end{aligned}$$

From $R(p) := pq(p)$ it follows that under symmetry $p_A = p_B = p^{nd}$ satisfies the following condition

$$R(p^{nd}) = 1 - \varepsilon(p^{nd}).$$

Proof of Proposition 2:

Consider the firms' second-period profits,

$$\pi_A^o(p_A^o, p_B^n) = p_A^o D_A^o = p_A^o\left[\frac{1}{2} + \frac{1}{2}\int_{p_A^o}^{p_B^n} q(s)ds\right] q(p_A^o),$$

$$\pi_A^n(p_A^n, p_B^o) = p_A^n D_A^n = p_A^n\left(\frac{1}{2} + \frac{1}{2}\int_{p_A^n}^{p_B^o} q(s)ds - x_1\right) q(p_A^n),$$

$$\pi_B^n(p_A^o, p_B^n) = p_B^n D_B^n = p_B^n\left[x_1 - \left(\frac{1}{2} + \frac{1}{2}\int_{p_A^o}^{p_B^n} q(s)ds\right)\right] q(p_B^n),$$

$$\pi_B^o(p_A^n, p_B^o) = p_B^o D_B^o = p_B^o \left(\frac{1}{2} - \frac{1}{2} \int_{p_A^n}^{p_B^o} q(s)ds \right) q(p_B^o).$$

Look first on prices targeted to consumers in A's turf. The FOC of $\pi_A^o(p_A^o, p_B^n)$ wrt p_A^o is equal to:

$$q(p_A^o) \left[\frac{1}{2} + \frac{1}{2} \int_{p_A^o}^{p_B^n} q(s)ds \right] + p_A^o q'(p_A^o) \left[\frac{1}{2} + \frac{1}{2} \int_{p_A^o}^{p_B^n} q(s)ds \right] - \frac{1}{2} p_A^o \left[q(p_A^o) \right]^2 = 0$$

dividing both terms by $q(p_A^o)$ and using the fact that $\varepsilon(p_A^o) = -\frac{p_A^o q'(p_A^o)}{q}$ it can be shown that:

$$\left(1 + \int_{p_A^o}^{p_B^n} q(s)ds \right) (1 - \varepsilon(p_A^o)) - p_A^o q(p_A^o) = 0$$

using the fact that $R(p_A^o) = p_A^o q(p_A^o)$, the previous condition can be written as

$$\begin{aligned} R(p_A^o) &= \left(1 + \int_{p_A^o}^{p_B^n} q(s)ds \right) (1 - \varepsilon(p_A^o)) \\ R(p_A^o) &= 2x_A(p_A^o, p_B^n)\,(1 - \varepsilon(p_A^o)) \end{aligned}$$

The FOC of $\pi_B^n(p_A^o, p_B^n)$ wrt p_B^n is:

$$q(p_B^n) \left[x_1 - \frac{1}{2} - \frac{1}{2} \int_{p_A^o}^{p_B^n} q(s)ds \right] + p_B^n q'(p_B^n) \left[x_1 - \frac{1}{2} - \frac{1}{2} \int_{p_A^o}^{p_B^n} q(s)ds \right] - \frac{1}{2} p_B^n \left[q(p_B^n) \right]^2 = 0$$

dividing by $q(p_B^n)$ it can be shown that

$$\left[2x_1 - 1 - \int_{p_A^o}^{p_B^n} q(s)ds \right] (1 - \varepsilon(p_B^n)) - p_B^n q(p_B^n) = 0$$

and therefore:

$$R_B^n(p_B^{n*}) = \left(2x_1 - 1 - \int_{p_A^{o*}}^{p_B^{n*}} q(s)ds \right) \left[1 - \varepsilon(p_B^{n*}) \right]$$

or

$$R(p_B^{n*}) = 2\left[x_1 - x_A(p_A^{o*}, p_B^{n*}) \right] (1 - \varepsilon(p_B^{n*}))$$

Look next on firms' prices targeted to consumers in B's turf. The FOC of $\pi_A^n(p_A^n, p_B^o)$ with respect to p_A^n is given

$$q(p_A^n) \left(\frac{1}{2} + \frac{1}{2} \int_{p_A^n}^{p_B^o} q(s)ds - x_1 \right) + p_A^n q'(p_A^n) \left(\frac{1}{2} + \frac{1}{2} \int_{p_A^n}^{p_B^o} q(s)ds - x_1 \right) - \frac{1}{2} p_A^n \left[q(p_A^n) \right]^2 = 0$$

dividing by $q(p_A^n)$ we obtain:

$$\left(1 + \int_{p_A^n}^{p_B^o} q(s)ds - 2x_1 \right) (1 - \varepsilon(p_A^n)) - p_A^n q(p_A^n) = 0$$

which can be written as follows:

$$R(p_A^n) = \left(1 + \int_{p_A^n}^{p_B^o} q(s)ds - 2x_1 \right) (1 - \varepsilon(p_A^n))$$

or

$$R(p_A^n) = 2\left[x_B(p_A^n, p_B^o) - x_1\right](1 - \varepsilon(p_A^n))$$

Finally the FOC of $\pi_B^o(p_A^n, p_B^o)$ with respect to p_B^o is:

$$q(p_B^o)\left(\frac{1}{2} - \frac{1}{2}\int_{p_A^n}^{p_B^o} q(s)ds\right) + p_B^o q'(p_B^o)\left(\frac{1}{2} - \frac{1}{2}\int_{p_A^n}^{p_B^o} q(s)ds\right) - \frac{1}{2}p_B^o\left[q(p_B^o)\right]^2 = 0$$

dividing by $q(p_B^o)$ yields:

$$\left(1 - \int_{p_A^n}^{p_B^o} q(s)ds\right)(1 - \varepsilon(p_B^o)) - p_B^o q(p_B^o) = 0$$

which can be written as

$$R(p_B^o) = 2\left[1 - x_B(p_A^n, p_B^o)\right](1 - \varepsilon(p_B^o)).$$

Proof of Corollary 1:

From Proposition 2 the price targeted to old consumers is independent of x_1. In contrast, p_A^n falls with x_1 and p_B^n increases with x_1. Additionally, as the LHS of equations (11) and (12) is the same and the same happens for the expression $[1 - \varepsilon(p)]$ in the RHS, then $p_A^{o*} > p_B^{n*}$ as long as $x_A(p_A^{o*}, p_B^{n*}) > x_1 - x_A(p_A^{o*}, p_B^{n*})$. At an interior solution at turf A, $x_A < x_1$, hence the previous condition implies that at an interior equilibrium where some consumers switch from A to B in period 2, because $p_A^{o*} > p_B^{n*}$, it must be the case that $\frac{1}{2}x_1 < x_A(p_A^{o*}, p_B^{n*}) < x_1$. If firms share equally the market in period 1, i.e., if $x_1 = \frac{1}{2}$ then in equilibrium $\frac{1}{4} < x_A(p_A^{o*}, p_B^{n*}) < \frac{1}{2}$. Additionally, comparing Proposition 1 and 2, as long as $x_A(p_A^{o*}, p_B^{n*}) \le \frac{1}{2}$ then $x_1 - x_A(p_A^{o*}, p_B^{n*}) \le \frac{1}{2}$. As this is the case at an interior solution in the interval $[0, x_1]$ then, regardless of $q(p)$, second period prices fall in comparison to uniform pricing (i.e., $p_A^{o*} < p^{nd}$ and $p_B^{n*} < p^{nd}$).

Proof of Lemma 2:

Since x_1 is implicitly defined as:

$$x_1 - \frac{1}{2} - \frac{1}{2(1-\delta_c)}\int_{p_A^1}^{p_B^1} q(s)ds - \frac{\delta_c}{2(1-\delta_c)}\int_{p_B^n}^{p_A^n} q(s)ds = 0$$

we can find $\frac{\partial x_1}{\partial p_A^1}$ implicitly, as follows:

$$\frac{\partial x_1}{\partial p_A^1} - \frac{-q(p_A^1)}{2(1-\delta_c)} - \frac{\delta_c}{2(1-\delta_c)}\left\{\frac{\partial\left(\int_{p_B^n}^{p_A^n} q(s)ds\right)}{\partial p_A^n}\frac{\partial p_A^n}{\partial x_1} - \frac{\partial\left(\int_{p_B^n}^{p_A^n} q(s)ds\right)}{\partial p_B^n}\frac{\partial p_B^n}{\partial x_1}\right\}\frac{\partial x_1}{\partial p_A^1} = 0$$

$$\frac{\partial x_1}{\partial p_A^1}\left[1 - \frac{\delta_c}{2(1-\delta_c)}\left(q(p_A^n)\frac{\partial p_A^n}{\partial x_1} - q(p_B^n)\frac{\partial p_B^n}{\partial x_1}\right)\right] = -\frac{q(p_A^1)}{2(1-\delta_c)}. \tag{30}$$

This yields:

$$\frac{\partial x_1}{\partial p_A^1} = \frac{-q(p_A^1)}{2(1-\delta_c) - \delta_c\left[q(p_A^n)\frac{\partial p_A^n}{\partial x_1} - q(p_B^n)\frac{\partial p_B^n}{\partial x_1}\right]}$$

Under symmetry $p_A^1 = p_B^1$ and $x_1 = \frac{1}{2}$. Consequently, $p_A^n = p_B^n$, $q(p_A^n) = q(p_B^n)$ and $-\frac{\partial p_A^n}{\partial x_1} = \frac{\partial p_B^n}{\partial x_1}$, with $\frac{\partial p_B^n}{\partial x_1} > 0$. Thus,

$$\frac{\partial x_1}{\partial p_A^1}|\text{sym} = \frac{-q(p_A^1)}{2\left[(1-\delta_c) + \delta_c q(p_B^n)\frac{\partial p_B^n}{\partial x_1}\right]}.$$

Let $\rho^n_{Bx1} = \frac{\partial p^n_B}{\partial x_1}\frac{x_1}{p^n_B} > 0$ represent the elasticity of firm's B poaching price with respect to x_1. The previous condition can be written as follows:

$$\frac{\partial x_1}{\partial p^1_A}|\text{sym} = \frac{-q(p^1_A)}{2\left[(1-\delta_c) + 2\delta_c R(p^n_B)\rho^n_{Bx1}\right]}.$$

Proof of Lemma 3: Given

$$R(p^n_B) = 2(x_1 - x_A)(1 - \varepsilon(p^n_B)),$$

consider if x_1 rises, and p^n_B may go up or go down. Holding p^o_A fixed, if p^n_B decreases, then x_A decreases, thus $(x_1 - x_A)$ increases. And given $\varepsilon' > 0$, $\varepsilon(p^n_B)$ decreases, and $(1 - \varepsilon(p^n_B))$ increases. Combined, RHS increases. And LHS decreases since $R(p)$ increases in $[0, \bar{p})$. Thus if p^n_B decreases, the equation will not balance. It can only go up. Similar arguments apply to show that $\frac{\partial p^o_A}{\partial x_1} > 0$ if we use the expression that defines implicitly p^o_A. However, due to the complementarity when p^n_B increases due to an increase in x_1 the same might happen to p^o_A. Following similar arguments it is straightforward to prove that $\frac{\partial p^n_A}{\partial x_1} < 0$ and $\frac{\partial p^o_B}{\partial x_1} < 0$. This yields $\rho^o_{Bx1} = \frac{\partial p^o_B}{\partial x_1}\frac{x_1}{p^o_B} < 0$ and $\rho^n_{Bx1} = \frac{\partial p^n_B}{\partial x_1}\frac{x_1}{p^n_B} > 0$. From $\Lambda = \frac{\pi^o_A}{\partial p^n_B}\frac{\partial p^n_B}{\partial x_1} + \frac{\pi^n_A}{\partial p^o_B}\frac{\partial p^o_B}{\partial x_1} + \frac{\pi^n_A}{\partial x_1}$, it follows that

$$\frac{\partial \pi^o_A}{\partial p^n_B} = \frac{p^o_A q(p^o_A) q(p^n_B)}{2} = \frac{1}{2} R^o_A q(p^n_B)$$

$$\frac{\pi^n_A}{\partial p^o_B} = \frac{p^n_A q(p^n_A) q(p^o_B)}{2} = \frac{1}{2} R^n_A q(p^o_B)$$

and

$$\frac{\pi^n_A}{\partial x_1} = -R^n_A.$$

Therefore:

$$\Lambda = \left(\frac{1}{2}R^o_A q(p^n)\right)\left(\frac{\partial p^n_B}{\partial x_1}\right) + \left(\frac{1}{2}R^n_A q(p^o_B)\right)\left(\frac{\partial p^o_B}{\partial x_1}\right) - R^n_A$$

which can be written as

$$\Lambda = \left(\frac{1}{2}R^o_A\right)\left(\frac{R^n_B}{x_1}\rho^n_{Bx1}\right) + \left(\frac{1}{2}R^n_A\right)\left(\frac{R^o_B}{x_1}\rho^o_{Bx1} - 2\right)$$

Under symmetry $R^o_A = R^o_B$ and $R^n_B = R^n_B$, $x_1 = \frac{1}{2}$. This yields:

$$\Lambda_{sym} = R^n \left(R^o \left(\rho^n_{Bx} + \rho^o_{Bx}\right) - 1\right)$$

As $R^n > 0$, $\Lambda_{sym} < 0$ as long as $(\rho^n_{Bx} + \rho^o_{Bx}) - 1 < \frac{1}{R^o}$.

Next we prove how to determine $\frac{\partial p^o_A}{\partial x_1}$, $\frac{\partial p^n_B}{\partial x_1}$, $\frac{\partial p^o_B}{\partial x_1}$ and $\frac{\partial p^n_A}{\partial x_1}$. Consider the system of implicit equations defined by expressions (11), (12), (14) and (13). To simplify make $p^o_A = x, p^n_B = y, p^o_B = z$ e $p^n_A = w$. x_1 is the indifferent consumer in period 1. We get that:

$$F_1 \quad : \quad \left(1 + \int_x^y q(s)ds\right)(1 - \varepsilon(x)) - xq(x) = 0 \tag{31}$$

$$F_2 \quad : \quad \left(2x_1 - 1 - \int_x^y q(s)ds\right)(1 - \varepsilon(y)) - yq(y) = 0 \tag{32}$$

$$F_3 \quad : \quad \left(1 - \int_w^z q(s)ds\right)(1 - \varepsilon(z)) - zq(z) = 0 \tag{33}$$

$$F_4 \quad : \quad \left(1 - 2x_1 + \int_w^z q(s)ds\right)(1 - \varepsilon(w)) - wq(w) = 0 \tag{34}$$

Hence:

$$\begin{bmatrix} \frac{\partial F_1}{\partial p_A^o} & \frac{\partial F_1}{\partial p_B^n} & \frac{\partial F_1}{\partial p_B^o} & \frac{\partial F_1}{\partial p_A^n} \\ \frac{\partial F_2}{\partial p_A^o} & \frac{\partial F_2}{\partial p_B^n} & \frac{\partial F_2}{\partial p_B^o} & \frac{\partial F_2}{\partial p_A^n} \\ \frac{\partial F_3}{\partial p_A^o} & \frac{\partial F_3}{\partial p_B^n} & \frac{\partial F_3}{\partial p_B^o} & \frac{\partial F_3}{\partial p_A^n} \\ \frac{\partial F_4}{\partial p_A^o} & \frac{\partial F_4}{\partial p_B^n} & \frac{\partial F_4}{\partial p_B^o} & \frac{\partial F_4}{\partial p_A^n} \end{bmatrix} \begin{bmatrix} \frac{\partial p_A^o}{\partial x_1} \\ \frac{\partial p_B^n}{\partial x_1} \\ \frac{\partial p_B^o}{\partial x_1} \\ \frac{\partial p_A^n}{\partial x_1} \end{bmatrix} = - \begin{bmatrix} \frac{\partial F_1}{\partial x_1} \\ \frac{\partial F_2}{\partial x_1} \\ \frac{\partial F_3}{\partial x_1} \\ \frac{\partial F_4}{\partial x_1} \end{bmatrix}$$

where

$$\begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} & \frac{\partial F_1}{\partial w} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} & \frac{\partial F_2}{\partial w} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} & \frac{\partial F_3}{\partial z} & \frac{\partial F_3}{\partial w} \\ \frac{\partial F_4}{\partial x} & \frac{\partial F_4}{\partial y} & \frac{\partial F_4}{\partial z} & \frac{\partial F_4}{\partial w} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & 0 & 0 \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial z} & \frac{\partial F_3}{\partial w} \\ 0 & 0 & \frac{\partial F_4}{\partial z} & \frac{\partial F_4}{\partial w} \end{bmatrix}.$$

Thus,

$$\begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & 0 & 0 \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial z} & \frac{\partial F_3}{\partial w} \\ 0 & 0 & \frac{\partial F_4}{\partial z} & \frac{\partial F_4}{\partial w} \end{bmatrix} \begin{bmatrix} \frac{\partial p_A^o}{\partial x_1} \\ \frac{\partial p_B^n}{\partial x_1} \\ \frac{\partial p_B^o}{\partial x_1} \\ \frac{\partial p_A^n}{\partial x_1} \end{bmatrix} = \begin{bmatrix} 0 \\ -2(1-\varepsilon(y)) \\ 0 \\ 2(1-\varepsilon(w)) \end{bmatrix}$$

This yields:

$$\begin{aligned} \frac{\partial F_1}{\partial x}\frac{\partial p_A^o}{\partial x_1} + \frac{\partial F_1}{\partial y}\frac{\partial p_B^n}{\partial x_1} &= 0 \\ \frac{\partial F_2}{\partial x}\frac{\partial p_A^o}{\partial x_1} + \frac{\partial F_2}{\partial y}\frac{\partial p_B^n}{\partial x_1} &= -2(1-\varepsilon(y)) \end{aligned}$$

$$\begin{aligned} \frac{\partial F_3}{\partial z}\frac{\partial p_B^o}{\partial x_1} + \frac{\partial F_3}{\partial w}\frac{\partial p_A^n}{\partial x_1} &= 0 \\ \frac{\partial F_4}{\partial z}\frac{\partial p_B^o}{\partial x_1} + \frac{\partial F_4}{\partial w}\frac{\partial p_A^n}{\partial x_1} &= 2(1-\varepsilon(w)) \end{aligned}$$

It is straightforward to show that:

$$\frac{\partial p_A^o}{\partial x_1} = \frac{2(1-\varepsilon(y))\frac{\partial F_1}{\partial y}}{\frac{\partial F_2}{\partial y}\frac{\partial F_1}{\partial x} - \frac{\partial F_1}{\partial y}\frac{\partial F_2}{\partial x}}$$

$$\frac{\partial p_B^n}{\partial x_1} = \frac{-2(1-\varepsilon(y))\frac{\partial F_1}{\partial x}}{\frac{\partial F_2}{\partial y}\frac{\partial F_1}{\partial x} - \frac{\partial F_1}{\partial y}\frac{\partial F_2}{\partial x}}$$

$$\frac{\partial p_B^o}{\partial x_1} = -\frac{2(1-\varepsilon(w))\frac{\partial F_3}{\partial w}}{\frac{\partial F_3}{\partial z}\frac{\partial F_4}{\partial w} - \frac{\partial F_3}{\partial w}\frac{\partial F_4}{\partial z}}$$

$$\frac{\partial p_A^n}{\partial x_1} = \frac{2(1-\varepsilon(w))\frac{\partial F_3}{\partial z}}{\frac{\partial F_3}{\partial z}\frac{\partial F_4}{\partial w} - \frac{\partial F_3}{\partial w}\frac{\partial F_4}{\partial z}}$$

with:

$$\begin{bmatrix} \frac{\partial F_1}{\partial x} \\ \frac{\partial F_2}{\partial x} \\ \frac{\partial F_3}{\partial x} \\ \frac{\partial F_4}{\partial x} \end{bmatrix} = \begin{bmatrix} -q(x)\left(2-\varepsilon(x)\right) - \varepsilon'(x)\left(1+\int_x^y q(s)ds\right) - xq'(x) \\ q(x)(1-\varepsilon(y)) \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial y} \\ \frac{\partial F_3}{\partial y} \\ \frac{\partial F_4}{\partial y} \end{bmatrix} = \begin{bmatrix} q(y)\left(1-\varepsilon(x)\right) \\ -q(y)(2-\varepsilon(y)) - \varepsilon'(y)\left(2x_1 - 1 - \int_x^y q(s)ds\right) - yq^{'}(y) \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial z} \\ \frac{\partial F_3}{\partial z} \\ \frac{\partial F_4}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -q(z)(2-\varepsilon(z)) - \varepsilon'(z)\left(1 - \int_w^z q(s)ds\right) - zq'(z) \\ q(z)(1-\varepsilon(w)) \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial F_1}{\partial w} \\ \frac{\partial F_2}{\partial w} \\ \frac{\partial F_3}{\partial w} \\ \frac{\partial F_4}{\partial w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ q(w)(1-\varepsilon(z)) \\ -q(w)(2-\varepsilon(w)) - \varepsilon'(w)\left(1 - 2x_1 + \int_w^z q(s)ds\right) - wq'(w) \end{bmatrix}.$$

This allows us to get $\frac{\partial p_A^o}{\partial x_1}$, $\frac{\partial p_B^n}{\partial x_1}$, $\frac{\partial p_B^o}{\partial x_1}$ and $\frac{\partial p_A^n}{\partial x_1}$ for any specific demand function.

Proof of Proposition 5:

To prove part (i) note that under symmetry, $\pi^{nd} = \frac{1}{2}p^{nd}q(p^{nd})$ and $\pi^1 = \frac{1}{2}p^1q(p^1)$. Because $\frac{\partial}{\partial p}\left[pq(p)\right] > 0$ for $p \in (0, \overline{p})$, then $\Delta\pi^1 = \pi^1 - \pi^{nd} > 0$ whenever $p^1 > p^{nd}$, otherwise the reverse happens. As p^1 depends on δ_c, δ_f due to the consumer and firm side effects, $\Delta\pi^1$ is also a function of δ_c, δ_f (i.e., $\Delta\pi^1(\delta_c, \delta_f)$).

To prove part (ii) look next at second-period profits

$$\pi^2 = 2\left[x_A(p_A^{o*}, p_B^{n*})\right]^2\left[1-\varepsilon(p_A^{o*})\right] + 2\left[x_B(p_A^{n*}, p_B^{o*}) - \frac{1}{2}\right]^2\left[1-\varepsilon(p_A^{n*})\right]$$

We have seen before that if firms share equally the market in period 1, i.e., if $x_1 = \frac{1}{2}$ then in equilibrium $\frac{1}{4} < x_A(p_A^{o*}, p_B^{n*}) < \frac{1}{2}$. Additionally, as $p^o > p^n$ and $\varepsilon(p^o) > \varepsilon(p^n)$ we can show that $\frac{\partial\pi^2}{\partial x_A} > 0$. Note that $\frac{\partial\pi^2}{\partial x_A} = 2x_A\left(2-\varepsilon(p^o)-\varepsilon(p^n)\right) - 1 + \varepsilon(p^n)$. $\frac{\partial\pi^2}{\partial x_A} > 0$ implies that $x_A > \frac{1-\varepsilon(p^n)}{2(2-\varepsilon(p^o)-\varepsilon(p^n))}$. Then $\frac{1-\varepsilon(p^n)}{2(2-\varepsilon(p^o)-\varepsilon(p^n))} > \frac{1}{4}$ as long as $2\varepsilon(p^o) > 2\varepsilon(p^n)$, which is always true for $p^o > p^n$. Now we need to check that $\frac{1-\varepsilon(p^n)}{2(2-\varepsilon(p^o)-\varepsilon(p^n))} < \frac{1}{2}$. This yields $1 < 2 - \varepsilon(p^o)$, which is always true in equilibrium. Hence, $\frac{\partial\pi^2}{\partial x_A} > 0$. Using the expression of second-period profits,

$$\pi^2 = 2\left[x_A(p_A^{o*}, p_B^{n*})\right]^2\left[1-\varepsilon(p_A^{o*})\right] + 2\left[x_B(p_A^{n*}, p_B^{o*}) - \frac{1}{2}\right]^2\left[1-\varepsilon(p_A^{n*})\right]$$

taking into account that under symmetry $1 - x_A(p_A^{o*}, p_B^{n*}) = x_B(p_A^{n*}, p_B^{o*})$ and evaluating at the $\frac{1}{4} < x_A < \frac{1}{2}$ yields:

$$\pi^2 = 2\left[x_A(p_A^{o*}, p_B^{n*})\right]^2\left[1-\varepsilon(p_A^{o*})\right] + 2\left[\frac{1}{2} - x_A(p_A^{o*}, p_B^{n*})\right]^2\left[1-\varepsilon(p_A^{n*})\right]$$

$$\frac{2}{16}\left[1-\varepsilon(p^{o*})\right] + \frac{2}{16}\left[1-\varepsilon(p^{n*})\right] < \pi^2 < \frac{2}{4}\left[1-\varepsilon(p^{o*})\right]$$

Additionally, when $x_A = \frac{1}{2}$, from Proposition 1 and 2, it is straightforward to see that $p^o = p^{nd}$ and so $\pi^{nd} = \pi^2$ and

$$\frac{1}{8}\left[1-\varepsilon(p^{o*})\right] + \frac{1}{8}\left[1-\varepsilon(p^{n*})\right] < \pi^2 < \frac{1}{2}\left[1-\varepsilon(p^{o*})\right] = \frac{1}{2}\left[1-\varepsilon(p^{nd})\right]$$

Therefore we only need to prove that

$$\frac{1}{8}\left[1-\varepsilon(p^{o*})\right] + \frac{1}{8}\left[1-\varepsilon(p^{n*})\right] < \pi^2 < \frac{1}{2}\left[1-\varepsilon(p^{o*})\right]$$

$$\begin{aligned}\frac{1}{8}\left[1-\varepsilon(p^{o*})\right]+\frac{1}{8}\left[1-\varepsilon(p^{n*})\right] &< \frac{1}{2}\left[1-\varepsilon(p^{nd})\right]\\ 2-\varepsilon(p^{o*})-\varepsilon(p^{n*}) &< 4\left[1-\varepsilon(p^{nd})\right]\end{aligned}$$

As $\varepsilon(p^j)\in(0,1)$ with $j=o,n,nd$ the previous condition is always true. Therefore, $\Delta\pi_2=\pi^2-\pi^{nd}<0.\blacksquare$

Finally, look at the difference in overall profits with BBPD and no discrimination:

$$\underbrace{\Delta\pi_1(\delta_c,\delta_f)}_{(+)\text{ if } p^1>p^{nd};\ (-)\text{ if } p^1<p^{nd}}+\underbrace{\delta_f\Delta\pi_2}_{(-)}$$

Proof of Corollary 2:

The proof of part (i) is straightforward. To prove part (ii) note that:

$$\begin{aligned}\Delta CS_2 &= V+2x_A\int_{p^o}^{\widehat{p}}q(s)ds+2\left(\frac{1}{2}-x_A\right)\int_{p^n}^{\widehat{p}}q(s)ds-\Omega_2-\left(2x_A+2\left(\frac{1}{2}-x_A\right)\right)\left(V+\int_{p^{nd}}^{\widehat{p}}q(s)ds-\Omega_2^{nd}\right)\\ &= 2x_A\left[\int_{p^o}^{pnd}q(s)ds\right]+2\left(\frac{1}{2}-x_A\right)\int_{p^n}^{pnd}q(s)ds-\Delta\Omega_2\\ &= \int_{p^n}^{p^{nd}}q(s)ds+2x_A\left[\int_{p^o}^{p^{nd}}q(s)ds-\int_{p^n}^{p^{nd}}q(s)ds\right]-\Delta\Omega_2\\ &= \int_{p^n}^{p^{nd}}q(s)ds+2x_A\int_{p^o}^{p^n}q(s)ds-\Delta\Omega_2.\end{aligned}$$

We know that $x_A=\frac{1}{2}-\frac{1}{2}\int_{p^n}^{p^o}q(s)ds$ and $\Delta\Omega_2=\frac{1}{2}-2x_A\left(1-x_A\right)$. To simplify make $\int_{p^n}^{p^o}q(s)ds=\alpha>0$ and $\int_{p^n}^{p^{nd}}q(s)ds=\beta>0$. Then $x_A=\frac{1}{2}-\frac{1}{2}\alpha$. Note that as $\frac{1}{4}<x_A<\frac{1}{2}$ then from $\frac{1}{2}-\frac{1}{2}\alpha>\frac{1}{4}$ we obtain $\alpha<\frac{1}{2}$.

$$\begin{aligned}\Delta CS_2 &= \beta+2\left(\frac{1}{2}-\frac{1}{2}\alpha\right)\alpha-\left(\frac{1}{2}-2\left(\frac{1}{2}-\frac{1}{2}\alpha\right)\left(1-\left(\frac{1}{2}-\frac{1}{2}\alpha\right)\right)\right)\\ &= \beta+\frac{1}{2}\alpha\left(2-3\alpha\right)>0.\blacksquare\end{aligned}$$

Proof of Corollary 4:

To prove part (i), in period 2, the FOC give rise to the following system of implicit equations. To simplify make $p_A^o=x, p_B^n=y, p_B^o=z$ and $p_A^n=w$, where $x_1=\theta_1$ is the indifferent consumer in period 1.

$$F_1 : \left(1+\int_x^y(1-s)ds\right)\left(1-\frac{x}{1-x}\right)-x(1-x)=0 \tag{35}$$

$$F_2 : \left(2\theta_1-1-\int_x^y(1-s)ds\right)(1-\frac{y}{1-y})-y(1-y)=0 \tag{36}$$

$$F_3 : \left(1-\int_w^z(1-s)ds\right)(1-\frac{z}{1-z})-z(1-z)=0 \tag{37}$$

$$F_4 : \left(1+\int_w^z(1-s)ds-2\theta_1\right)(1-\frac{z}{1-z})-w(1-w)=0 \tag{38}$$

Note that under symmetry in period 1 the second-period prices in A's turf are the solution to the following system of implicit equations:

$$\begin{aligned}\left(1+\int_x^y(1-s)ds\right)\left(1-\frac{x}{1-x}\right)-x(1-x) &= 0\\ \left(1-1-\int_x^y(1-s)ds\right)(1-\frac{y}{1-y})-y(1-y) &= 0\end{aligned}$$

From which we get $x = 0.413\,86$ and $y = 0.167\,59$. To simplify: $p^o = 0.414$ and $p^n = 0.167$. From $q(p) = 1 - p$, we easily obtain q^o, q^n, R^o and R^n.

Next we prove part (iii). Given the total discounted profit $\Pi_A = p_A^1 x_1 q(p_A^1) + \delta_f\left(\pi_A^o + \pi_A^n\right)$, the derivative wrt p_A^1 is:

$$x_1 q(p_A^1) + p_A^1 q'(p_A^1) x_1 + p_A^1 q(p_A^1)\frac{\partial x_1}{\partial p_A^1} + \delta_f\left[\frac{\pi_A^o}{\partial p_B^n}\frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o}\frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1}\right]\frac{\partial x_1}{\partial p_A^1} = 0$$

Let $\Lambda = \left[\frac{\pi_A^o}{\partial p_B^n}\frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o}\frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1}\right]$. Then

$$x_1 q(p_A^1) + p_A^1 q'(p_A^1) x_1 + p_A^1 q(p_A^1)\frac{\partial x_1}{\partial p_A^1} + \delta_f \Lambda \frac{\partial x_1}{\partial p_A^1} = 0$$

The indifferent consumer in period 1 is:

$$x_1 = \frac{1}{2} + \frac{1}{2\left(1-\delta_c\right)}\int_{p_A^1}^{p_B^1} q(s)ds + \frac{\delta_c}{2\left(1-\delta_c\right)}\int_{p_B^n}^{p_A^n} q(s)ds$$

With linear demand, making $p_A^1 = a, p_B^1 = b$

$$x_1 = \frac{1}{2} - \frac{(a-b)\left(2-a-b\right)}{4\left(1-\delta_c\right)} + \frac{\delta_c\left(w-y\right)\left(2-w-y\right)}{4\left(1-\delta_c\right)}$$

from which we get

$$\frac{\partial x_1}{\partial a}\left(1 - \frac{1}{2}\frac{\delta_c}{1-\delta_c}\left(1-w\right)\frac{\partial w}{\partial x_1} + \frac{1}{2}\frac{\delta_c}{1-\delta_c}\left(1-y\right)\frac{\partial y}{\partial x_1}\right) = -\frac{(1-a)}{2\left(1-\delta_c\right)}$$

Make $\frac{\partial y}{\partial x_1} = B$ and $\frac{\partial w}{\partial x_1} = D$

$$\frac{\partial x_1}{\partial a} = -\frac{(1-a)}{\left(2\left(1-\delta_c\right) - \delta_c\left(\left(1-w\right)D - \left(1-y\right)B\right)\right)}$$

We need the derivatives of second-period prices with respect to θ_1, namely $\frac{\partial p_A^o}{\partial \theta_1} = \frac{\partial x}{\partial \theta_1} = A$, $\frac{\partial p_B^n}{\partial \theta_1} = \frac{\partial y}{\partial \theta_1} = B$, $\frac{\partial p_B^o}{\partial \theta_1} = \frac{\partial z}{\partial \theta_1} = C$, $\frac{\partial p_A^n}{\partial \theta_1} = \frac{\partial w}{\partial \theta_1} = D$.We can get them as follows:

$$\begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & 0 & 0 \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial z} & \frac{\partial F_3}{\partial w} \\ 0 & 0 & \frac{\partial F_4}{\partial z} & \frac{\partial F_4}{\partial w} \end{bmatrix}\begin{bmatrix} \frac{\partial x}{\partial \theta_1} \\ \frac{\partial y}{\partial \theta_1} \\ \frac{\partial z}{\partial \theta_1} \\ \frac{\partial w}{\partial \theta_1} \end{bmatrix} = -\begin{bmatrix} 0 \\ \frac{\partial F_2}{\partial \theta_1} \\ 0 \\ \frac{\partial F_4}{\partial \theta_1} \end{bmatrix}$$

Hence

$$\begin{aligned} \frac{\partial F_1}{\partial x}\frac{\partial x}{\partial \theta_1} + \frac{\partial F_1}{\partial y}\frac{\partial y}{\partial \theta_1} &= 0 \\ \frac{\partial F_2}{\partial x}\frac{\partial x}{\partial \theta_1} + \frac{\partial F_2}{\partial y}\frac{\partial y}{\partial \theta_1} &= -\frac{\partial F_2}{\partial \theta_1} \end{aligned}$$

$$\begin{aligned} \frac{\partial F_3}{\partial z}\frac{\partial z}{\partial \theta_1} + \frac{\partial F_3}{\partial w}\frac{\partial w}{\partial \theta_1} &= 0 \\ \frac{\partial F_4}{\partial z}\frac{\partial z}{\partial \theta_1} + \frac{\partial F_4}{\partial w}\frac{\partial w}{\partial \theta_1} &= -\frac{\partial F_4}{\partial \theta_1} \end{aligned}$$

$$
\begin{aligned}
\frac{\partial F_1}{\partial x} &= \frac{1}{2\,(x-1)^2}\left(8x^3-21x^2+18x+y^2-2y-6\right)\\
\frac{\partial F_1}{\partial y} &= \frac{1}{x-1}\,(2x+y-2xy-1)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial F_2}{\partial x} &= \frac{1}{y-1}\,(x+2y-2xy-1)\\
\frac{\partial F_2}{\partial y} &= -\frac{1}{2\,(y-1)^2}\left(-x^2+2x-8y^3+21y^2-18y+4\theta_1+2\right)\\
\frac{\partial F_2}{\partial \theta_1} &= \frac{4y-2}{y-1}
\end{aligned}
$$

From

$$
\begin{aligned}
\frac{\partial F_1}{\partial x}\frac{\partial x}{\partial \theta_1}+\frac{\partial F_1}{\partial y}\frac{\partial y}{\partial \theta_1} &= 0\\
\frac{\partial F_2}{\partial x}\frac{\partial x}{\partial \theta_1}+\frac{\partial F_2}{\partial y}\frac{\partial y}{\partial \theta_1} &= -\frac{\partial F_2}{\partial \theta_1}
\end{aligned}
$$

Make $\frac{\partial x}{\partial \theta_1}=A$ and $\frac{\partial y}{\partial \theta_1}=B$

$$
\begin{aligned}
\frac{\partial F_1}{\partial x} &: \frac{d}{dx}\left[\left(1+\int_x^y(1-s)ds\right)\left(1-\frac{x}{1-x}\right)-x(1-x)\right]=\frac{1}{2\,(x-1)^2}\left(8x^3-21x^2+18x+y^2-2y-6\right)\\
\frac{\partial F_1}{\partial y} &: \frac{d}{dy}\left[\left(1+\int_x^y(1-s)ds\right)\left(1-\frac{x}{1-x}\right)-x(1-x)\right]=\frac{1}{x-1}\,(2x+y-2xy-1)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial F_2}{\partial x} &: \frac{d}{dx}\left[\left(2\theta_1-1-\int_x^y(1-s)ds\right)(1-\frac{y}{1-y})-y(1-y)\right]=\frac{1}{y-1}\,(x+2y-2xy-1)\\
\frac{\partial F_2}{\partial y} &: \frac{d}{dy}\left[\left(2\theta_1-1-\int_x^y(1-s)ds\right)(1-\frac{y}{1-y})-y(1-y)\right]=-\frac{1}{2\,(y-1)^2}\left(-x^2+2x-8y^3+21y^2-18y+4\theta_1+2\right)\\
\frac{\partial F_2}{\partial \theta_1} &: \frac{d}{d\theta_1}\left[\left(2\theta_1-1-\int_x^y(1-s)ds\right)(1-\frac{y}{1-y})-y(1-y)\right]=\frac{4y-2}{y-1}
\end{aligned}
$$

From

$$
\begin{aligned}
\frac{1}{2\,(x-1)^2}\left(8x^3-21x^2+18x+y^2-2y-6\right)A+\frac{1}{x-1}\,(2x+y-2xy-1)\,B &= 0\\
\frac{1}{y-1}\,(x+2y-2xy-1)\,A+-\frac{1}{2\,(y-1)^2}\left(-x^2+2x-8y^3+21y^2-18y+4\theta_1+2\right)B &= -\frac{4y-2}{y-1}
\end{aligned}
$$

$$
A = \frac{\left[\begin{array}{c}-24x-32y-120xy^2-64x^2y+48xy^3\\+80x^2y^2-32x^2y^3+96xy+16x^2+40y^2-16y^3+8\end{array}\right]}{\left[\begin{array}{c}-8x-88y+24\theta_1-72x\theta_1+8y\theta_1-300xy^2-300x^2y+112xy^3+112x^3y+\\84x^2\theta_1-32x^3\theta_1-4y^2\theta_1+342x^2y^2-128x^2y^3-128x^3y^2+48x^3y^3\\+264xy-20x^2+52x^3-37x^4+68y^2+8x^5+20y^3-37y^4+8y^5+8\end{array}\right]}
$$

$$
B = -\frac{\left[\begin{array}{c}72x+64y+144xy^2+252x^2y-96x^3y-168x^2y^2+\\64x^3y^2-216xy-84x^2+32x^3-20y^2-28y^3+8y^4-24\end{array}\right]}{\left[\begin{array}{c}-8x-88y+24\theta_1-72x\theta_1+8y\theta_1-300xy^2-300x^2y+112xy^3+\\112x^3y+84x^2\theta_1-32x^3\theta_1-4y^2\theta_1+342x^2y^2-128x^2y^3-128x^3y^2+\\48x^3y^3+264xy-20x^2+52x^3-37x^4+68y^2+8x^5+20y^3-37y^4+8y^5+8\end{array}\right]}
$$

Evaluating now A and B at the 2nd period equilibrium prices under symmetry we get: $A = 9.238\,5 \times 10^{-2}$ and $B = 1.0372$. Therefore, using the fact that $\theta_1 = x_1$, under symmetry the derivatives $\frac{\partial p_B^n}{\partial x_1} = 1.037$ and $\frac{\partial p_A^o}{\partial \theta_1} = 9.238\,5 \times 10^{-2}$. Following the same reasoning we can check that $\frac{\partial p_A^n}{\partial x_1} = -\frac{\partial p_B^n}{\partial x_1}$ and $\frac{\partial p_A^o}{\partial x_1} = -\frac{\partial p_B^o}{\partial x_1}$.

Therefore, under symmetry:

$$\rho_{Bx1}^n = \underset{-}{\frac{\partial p_B^n}{\partial x_1}} \frac{x_1}{p_B^n} = 1.037 \frac{\frac{1}{2}}{0.168} = 3.0863$$

$$\rho_{Bx1}^o = \underset{-}{\frac{\partial p_B^o}{\partial x_1}} \frac{x_1}{p_B^o} = \left(-9.2385 \times 10^{-2}\right) \frac{\frac{1}{2}}{0.414} = -0.111\,58$$

Part (iv) is easily proven. To prove part (v) note that:

$$\Lambda = \frac{\pi_A^o}{\partial p_B^n} \frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o} \frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1}$$

To simplify make $p_A^o = x, p_B^n = y, p_B^o = z$ e $p_A^n = w$, where $x_1 = \theta_1$. Given $q(p) = 1 - p$, it is straightforward to obtain:

$$\frac{\partial \pi_A^o}{\partial p_B^n} = \frac{x\,(a-x)}{2}\,(a-y)$$

$$\frac{\partial \pi_A^o}{\partial p_B^n} = \frac{x\,(1-x)\,(1-y)}{2}$$

$$\frac{\pi_A^n}{\partial x_1} = -p_A^n q(p_A^n)$$

$$\frac{\pi_A^n}{\partial p_B^o} = \frac{w(a-w)(a-z)}{2}$$

$$\frac{\pi_A^n}{\partial x_1} = -w(a-w)$$

$$\frac{\partial \pi_A^o}{\partial p_B^n} = \frac{x\,(1-x)\,(1-y)}{2}$$

$$\frac{\pi_A^n}{\partial p_B^o} = \frac{w(1-w)(1-z)}{2}$$

$$\frac{\pi_A^n}{\partial x_1} = -y(1-y)$$

From $\Lambda = \frac{\pi_A^o}{\partial p_B^n} \frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o} \frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1}$, and using the fact that under symmetry $\frac{\partial p_B^n}{\partial x_1} = 1.0372$ and $\frac{\partial p_B^o}{\partial x_1} = -9.238\,5 \times 10^{-2}$, we get $\Lambda = \left(\frac{x(1-x)(1-y)}{2}\right)(1.0372) + \left(\frac{w(1-w)(1-z)}{2}\right)\left(-9.238 \times 10^{-2}\right) - y(1-y)$. Using the fact that under symmetry $x = z = 0.414$ and $w = y = 0.168$, yields:

$$\Lambda_{Sym} = -3.88 \times 10^{-2}$$

Alternatively, from $\Lambda_{sym} = R^n \left[R^o \left(\rho_{Bx}^n + \rho_{Bx}^o\right) - 1\right]$, we obtain:

$$\Lambda_{Sym} = 0.1398\,(0.243\,(3.086 - 0.111) - 1) = -3.875 \times 10^{-2}.$$

Proof of Corollary 5: The proof is similar to the proof of Corollary 4. We only need to use $q(p) = 2 - p$ rather than $q(p) = 1 - p$. Details are available from the authors upon request.

Proof of Proposition 9:

When $q(p) = 1 - p$, the first-period price is implicitly defined as follows

$$R(p_A^1) = [1 - \delta_c + 2\delta_c R(p_B^n)\rho_{Bx1}^n] \left[1 - \varepsilon(p_A^1)\right] - \delta_f \Lambda_{sym}$$

Make $\delta_f = \delta$ and $\delta_c = k$, we obtain:

$$\left[p\,(1-p) = (1 - k + 0.86338k)\left(1 - \frac{p}{1-p}\right) - \delta\left(-3.856\,2 \times 10^{-2}\right)\right]$$

from which we get:

$$p^1 = \left[\begin{array}{l} \frac{2}{3} - \dfrac{-\frac{2277}{25\,000}k + \frac{6427}{500\,000}\delta + \frac{5}{9}}{\sqrt[3]{\frac{2277}{100\,000}k + \frac{6427}{1000\,000}\delta - \frac{11}{54} + \sqrt{\begin{array}{l} -\frac{9361}{100\,000}k + \frac{83\,551}{9000\,000}\delta - \frac{180\,489\,441}{50000\,000\,000}k\delta \\ + \frac{950\,045\,567}{3000\,000\,000\,000}\delta^2 + \frac{265\,475\,776\,483}{125\,000\,000\,000\,000\,000}\delta^3 \\ - \frac{282\,163\,533\,399}{6250\,000\,000\,000\,000}k\delta^2 + \frac{99\,966\,759\,849}{312\,500\,000\,000\,000}k^2\delta \\ + \frac{143\,444\,169}{10\,000\,000\,000}k^2 - \frac{11\,805\,627\,933}{15\,625\,000\,000\,000}k^3 + \frac{23}{108} \end{array}}}} \\ + \sqrt[3]{\frac{2277}{100\,000}k + \frac{6427}{1000\,000}\delta - \frac{11}{54} + \sqrt{\begin{array}{l} -\frac{9361}{100\,000}k + \frac{83\,551}{9000\,000}\delta - \frac{180\,489\,441}{50\,000\,000\,000}k\delta \\ + \frac{950\,045\,567}{3000\,000\,000\,000}\delta^2 + \frac{265\,475\,776\,483}{125\,000\,000\,000\,000\,000}\delta^3 \\ - \frac{282\,163\,533\,399}{6250\,000\,000\,000\,000}k\delta^2 + \frac{99\,966\,759\,849}{312\,500\,000\,000\,000}k^2\delta \\ + \frac{143\,444\,169}{10\,000\,000\,000}k^2 - \frac{11\,805\,627\,933}{15\,625\,000\,000\,000}k^3 + \frac{23}{108} \end{array}}} \end{array}\right]$$

Let's define $\Theta(k, \delta)$ as follows:

$$\Theta(k,\delta) = \frac{2277}{100\,000}k + \frac{6427}{1000\,000}\delta - \frac{11}{54} + \sqrt{\begin{array}{l} -\frac{9361}{100\,000}k + \frac{83\,551}{9000\,000}\delta - \frac{180\,489\,441}{50\,000\,000\,000}k\delta \\ + \frac{950\,045\,567}{3000\,000\,000\,000}\delta^2 + \frac{265\,475\,776\,483}{125\,000\,000\,000\,000\,000}\delta^3 \\ - \frac{282\,163\,533\,399}{6250\,000\,000\,000\,000}k\delta^2 + \frac{99\,966\,759\,849}{312\,500\,000\,000\,000}k^2\delta \\ + \frac{143\,444\,169}{10\,000\,000\,000}k^2 - \frac{11\,805\,627\,933}{15\,625\,000\,000\,000}k^3 + \frac{23}{108} \end{array}}$$

Using the fact that $\delta_c = k$ and $\delta = \delta_f$:

$$p^1 = \frac{2}{3} - \frac{-\frac{2277}{25\,000}\delta_c + \frac{6427}{500\,000}\delta_f + \frac{5}{9}}{\sqrt[3]{\Theta(\delta_c,\delta_f)}} + \sqrt[3]{\Theta(\delta_c,\delta_f)}.$$

with $\Phi(0,0) = \frac{1}{18}\sqrt{69} - \frac{11}{54}$. Therefore

$$\begin{aligned} p^1(\Phi(0,0)) &= \frac{2}{3} - \frac{\frac{5}{9}}{\sqrt[3]{\frac{1}{18}\sqrt{69} - \frac{11}{54}}} + \sqrt[3]{\frac{1}{18}\sqrt{69} - \frac{11}{54}} \\ &= 0.430\,16 = p^{nd}. \end{aligned}$$

Proof of Proposition 11:

When $q(p) = 2 - p$, the first-period price is implicitly defined as follows

$$R(p_A^1) = [1 - \delta_c + 2\delta_c R(p_B^n)\rho_{Bx1}^n] \left[1 - \varepsilon(p_A^1)\right] - \delta_f \Lambda_{sym}$$

Making $\delta_f = \delta$ and $\delta_c = k$, we obtain:

$$p\,(2-p) = (1 - k + 1.18631k)\left(1 - \frac{p}{2-p}\right) + \delta\left(2.826\,1 \times 10^{-2}\right)$$

which yields:

$$p_1 = \left[\frac{4}{3} - \frac{\frac{18\,631}{150\,000}k + \frac{28\,261}{3000\,000}\delta + \frac{2}{9}}{\sqrt[3]{-\frac{18\,631}{300\,000}k + \frac{28\,261}{3000\,000}\delta - \frac{17}{27} + \sqrt{\begin{array}{c}\frac{130\,417}{1350\,000}k - \frac{28\,261}{2700\,000}\delta + \frac{526\,530\,691}{1350\,000\,000\,000}k\delta \\ + \frac{798\,684\,121}{5400\,000\,000\,000}\delta^2 + \frac{22\,571\,611\,943\,581}{27\,000\,000\,000\,000\,000\,000}\delta^3 \\ + \frac{14\,880\,283\,858\,351}{450\,000\,000\,000\,000\,000}k\delta^2 + \frac{9809\,793\,304\,021}{22\,500\,000\,000\,000\,000}k^2\delta \\ + \frac{3818\,255\,771}{270\,000\,000\,000}k^2 + \frac{6467\,083\,933\,591}{3375\,000\,000\,000\,000}k^3 + \frac{11}{27}\end{array}}}} + \sqrt[3]{-\frac{18\,631}{300\,000}k + \frac{28\,261}{3000\,000}\delta - \frac{17}{27} + \sqrt{\begin{array}{c}\frac{130\,417}{1350\,000}k - \frac{28\,261}{2700\,000}\delta + \frac{526\,530\,691}{1350\,000\,000\,000}k\delta \\ + \frac{798\,684\,121}{5400\,000\,000\,000}\delta^2 + \frac{22\,571\,611\,943\,581}{27\,000\,000\,000\,000\,000\,000}\delta^3 \\ + \frac{14\,880\,283\,858\,351}{450\,000\,000\,000\,000\,000}k\delta^2 + \frac{9809\,793\,304\,021}{22\,500\,000\,000\,000\,000}k^2\delta \\ + \frac{3818\,255\,771}{270\,000\,000\,000}k^2 + \frac{6467\,083\,933\,591}{3375\,000\,000\,000\,000}k^3 + \frac{11}{27}\end{array}}}\right]$$

Let

$$\Phi(\delta_c, \delta_f) = -\frac{18\,631}{300\,000}k + \frac{28\,261}{3000\,000}\delta - \frac{17}{27} + \sqrt{\begin{array}{c}\frac{130\,417}{1350\,000}k - \frac{28\,261}{2700\,000}\delta + \frac{526\,530\,691}{1350\,000\,000\,000}k\delta \\ + \frac{798\,684\,121}{5400\,000\,000\,000}\delta^2 + \frac{22\,571\,611\,943\,581}{27\,000\,000\,000\,000\,000\,000}\delta^3 \\ + \frac{14\,880\,283\,858\,351}{450\,000\,000\,000\,000\,000}k\delta^2 + \frac{9809\,793\,304\,021}{22\,500\,000\,000\,000\,000}k^2\delta \\ + \frac{3818\,255\,771}{270\,000\,000\,000}k^2 + \frac{6467\,083\,933\,591}{3375\,000\,000\,000\,000}k^3 + \frac{11}{27}\end{array}}$$

This yields

$$p^1 = \frac{4}{3} - \frac{\frac{18\,631}{150\,000}k + \frac{28\,261}{3000000}\delta + \frac{2}{9}}{\sqrt[3]{\Phi(\delta_c, \delta_f)}} + \sqrt[3]{\Phi(\delta_c, \delta_f)}$$

with $\Phi(0,0) = \frac{1}{9}\sqrt{33} - \frac{17}{27}$. Therefore

$$\begin{aligned} p^1\left(\Phi(0,0)\right) &= \frac{4}{3} - \frac{\frac{2}{9}}{\sqrt[3]{\frac{1}{9}\sqrt{33} - \frac{17}{27}}} + \sqrt[3]{\frac{1}{9}\sqrt{33} - \frac{17}{27}} \\ &= 0.4563 = p^{nd}. \end{aligned}$$

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