MATHEMATICAL MODELLING AND SIMULATION OF THE PHYSICAL SYSTEM OF PULLEYS IN MATHEMATICAL LEARNING

José Magalhães[1], Floriano Viseu[2] and Paula Mendes Martins[3]

[1]DMA- School of Engineering, Polytechnic of Porto, Porto, Portugal, jdm@isep.ipp.pt
[2]CIEd-University of Minho, Braga, Portugal, fviseu@ie.uminho.pt
[3]DMAT – University of Minho, Braga, Portugal, pmendes@math.uminho.pt

Abstract. *The standards of the CDIO model (Conceiving, Designing, Implementing, and Operating) include the existence of project development experiences that involve different curricular courses and that provide team activities from the 1st year of engineering degrees. In this sense, it was implemented a teaching experience in the engineering degree in Electrical Power Systems at School of Engineering of the Porto Polytechnic (ISEP) based on a multidisciplinary project that involved modulation, simulation and construction of a prototype based on the concept of pulleys. Here we present the phases of the task development, the objectives to be achieved with its realisation, the results obtained by the students and their perceptions about the realization of the Project-based learning (PBL). Here we present the phases of the task development, the objectives to be achieved with its realisation, the results obtained by the students and their perceptions about the realization of the Project-based learning (PBL) challenge.*

Key words: Simulation; Modelling; PBL; CDIO; EUR-ACE; Learning; Mathematics.

INTRODUCTION

From the 1st year of engineering degrees, the existence of project activity is advocated in the CDIO (Concept-Design-Implement-Operate) model (Crawley et al., 2007) and in the EUR-ACE degree recognition system (Malmqvist, 2009). This type of activity allows students to develop skills in technical problem solving, project realisation, research activity development, communication, leadership, and ethical principles.

THEORETICAL FRAMEWORK

The CDIO educational framework is implemented in various schools worldwide (Figure 1) and approved 12 standards to describe CDIO programmes in 2004 (Malmqvist, 2009).

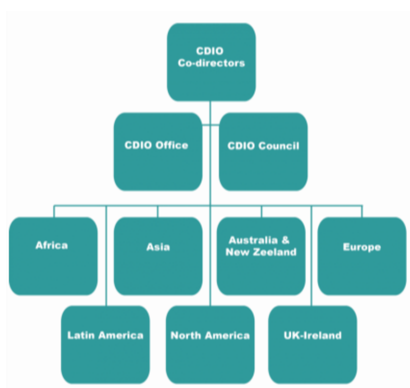

Figure 1: CDIO Organisation Diagram (www.cdio.org).

These norms include "Design-Implement Experiences": it demands the realization of complex projects in teams and in an environment that simulates professional activity; there should be activities of this type already in the 1st year of the course; the projects have to be "fitted" into Curricular Units (CUs), usually integrative CUs; It is intended an organized curriculum based on courses that support each other; based on design-build projects developed by students; active and experimental learning.

Project-based learning (PBL) (Masson et al., 2012)**:** application (growing) in Engineering degrees; it involves problem or research situations based on real problems; it allows students to acquire scientific and technological knowledge, including environmental and social issues (Figure 2).

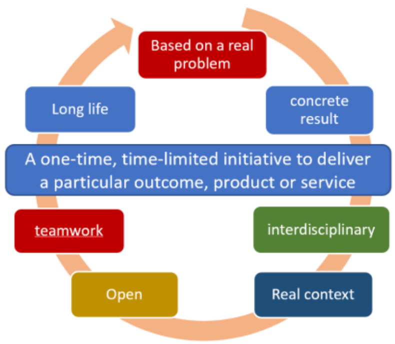

Figure 2: Features of the PBL activity.

PBL develops students' skills in: carrying out projects; problem solving; communication; teamwork; self and peer assessment; values attitudes of ethics, responsibility towards colleagues and society.

Mathematical modelling is a model-building process that transforms a real situation into a mathematical situation (Biembengut & Hein, 2000; Blum, 2002), according to the cycle represented in Figure 3.

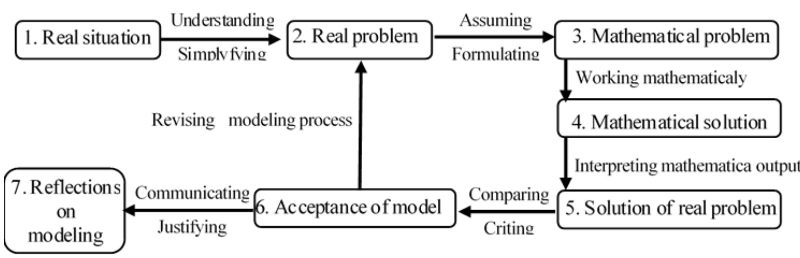

Figure 3: Mathematical modelling cycle (Takaci & Budinski, 2013, adapted)

The modelling method is based on problematization, observation of phenomena, management of doubts about them and the construction of a mathematical model that expresses the relationships between the quantities observed.

Simulators can take many forms, be presented in different fields of knowledge, and can have the following characteristics: they are exploratory learning environments; they are discovery spaces where students can manipulate, explore and experiment, developing their skills and taste for discovering the objects of the simulated world; they allow limited simulations of real-world phenomena. In this domain, **GeoGebra** is a freely distributed algebraic and dynamic geometry computer software, with good characteristics to be used in a classroom environment; it allows constructions to be performed dynamically.

METHODOLOGY AND GOALS

In the school year 2014/15, the unit Mathematics Laboratories I (LMAT1) of the first year of Electrical Power Systems degree from ISEP, followed the CDIO model along with the CUs of Mathematics I (MATE1) and Working Methods in Engineering (MTENG). A Project was developed involving mathematical modelling and simulation of a system based on the physics of the "Pulleys" and the creation of an experimental physical prototype. The project was composed of four tasks with the objectives of this activity were: to develop mathematical calculation skills; to stimulate problem modelling and simulation; to build reports and communicate results in public; project management and student integration in work teams.

Students delivered, in two stages, preliminary written answers to parts of the project and then a final report. These written documents were analysed at each of these delivery phases and the students were informed of their deficiencies in order to correct them. Afterwards, the students made a public presentation of the project results and showed the physical prototypes they made. The information presented in this paper is the result of data collected from the students' reports and public oral presentations.

RESULTS

This section describes the main tasks proposed to students and exemplifies solutions created by the professor and students in response to them. In **Task 1** it was asked to build an applet, in GeoGebra, to simulate a model with one, two and three pulleys, where the user could vary the length of the wire L, radius, positions and masses of the pulleys, the values of the mass M or the force F1 (Figure 4).

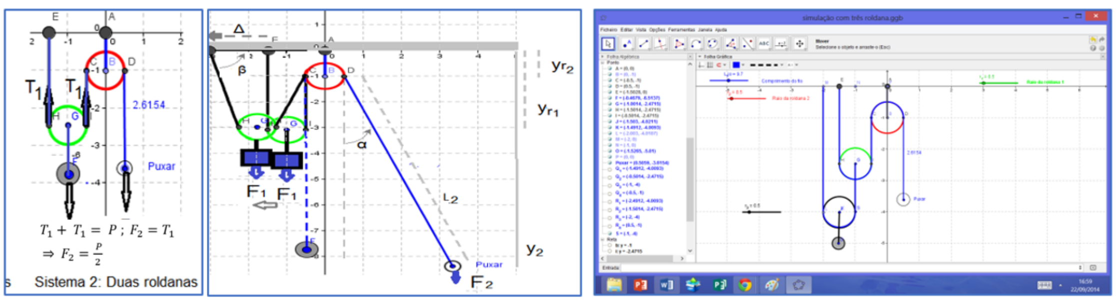

Figure 4: Modelling and simulation of the physical model of the pulleys

The simulator allowed students to answer a set of questions for different conditions in the physical context of pulleys, such as: visualisation of the motion of the pulleys; determination of forces to maintain the equilibrium of the system; determination of velocities of the pulleys motion and angles relative to these conditions.

The goal of **Task 2** was the elaboration of the Taylor series simulator (Figure 5a) that allowed to support the students: in the understanding of the conception of function series; to help them in the answers to the questions in the mathematical and physical context of the pulleys. **Task 3** aimed at the physical construction of a system including the pulley(s) to analyse its functioning and **Task 4** was the public presentation of results as shown in the pictures in Figure 5b) with final prototypes by some groups of students. In the comments of the student groups, it was perceptible that the accomplishment of the tasks of this PBL type provided them with a good motivation for learning.

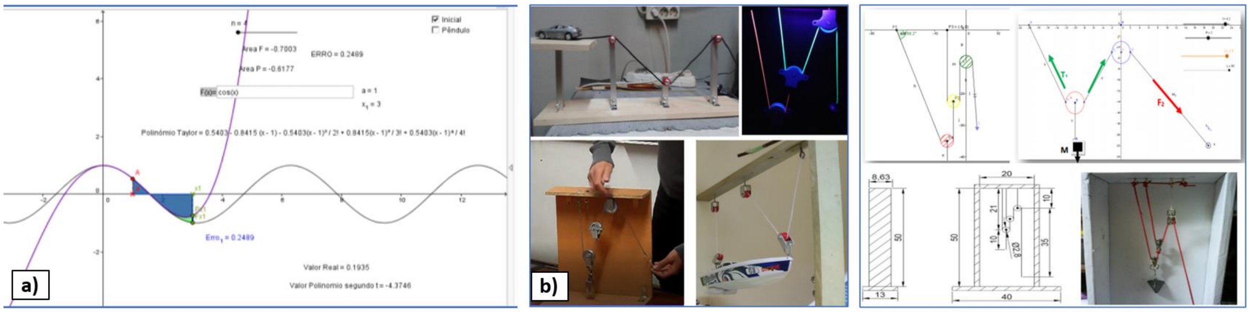

Figure 5: Public presentations of student groups' results in the pulleys project task.

CONCLUSIONS

The project was, par excellence, a task that allows engineering students integrating multidisciplinary transversal skills - Mathematics, Physics and Engineering - of working in a social context (team) and integrating new students (Magalhães et al, 2018). Modelling and simulation of physical processes are tasks that contribute to enhance more meaningful engineering learning, as advocated by Malmqvist, Edström et al. (2020). As advocated in the CDIO standards (Crawley et al., 2007; Malmqvist, 2009), in the project task most transversal skills were recognised by the students, such as project management, report writing and oral communication of results. Regarding GeoGebra software, it facilitates the rapid development of simulations and the construction of dynamic applets, develops intuition and visualisation skills.

References

Biembengut, M. & Hein, N. (2000). *Modelagem Matemática no Ensino*. São Paulo: Editora Contexto.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education - Discussion document. *Educational Studies in Mathematics*, 51(1–2), 149–171.

Crawley, E. & al. (2007). *Rethinking engineering education the CDIO approach*. Springer Science+Business Media, LLC. New York, USA. ISBN 978-0-387-38287-6.

Magalhães, J., Pinto, A., Costa, M. & Sá, C. (2018). Implementation of a PBL/CDIO methodology at ISEP-P. PORTO Systems Engineering Course. *3rd International Conference of the Portuguese Society for Engineering Education* (CISPEE 2018), 27-29 June, Aveiro, Portugal.

Malmqvist, J. (2009). A comparison of the CDIO ANDEUR-ACE quality assurance systems. In Proceedings of the *5th International CDIO Conference*, Singapore Polytechnic, June 7 – 10.

Malmqvist, J., Edström, k., & al. (2020). Optional cdio standards: sustainable development, simulation-based mathematics, engineering entrepreneurship, internationalisation & mobility. Proceedings of the *16th International CDIO Conference*, hosted on-line by Chalmers University of Technology, Gothenburg, Sweden, 8-10 June 2020.

Masson, T. & al. (2012). Metodologia de ensino: Aprendizagem baseada em projetos (PBL). In *XL Congresso Brasileiro de Educação em Engenharia*. 3 a 6 de Setembro de 2012. Belém PA.

Takaci, Dj., & Budinski, N. (2013). Using Computers and Context in the Modeling-Based Teaching of Logarithms. *Computers in the Schools*, 30:1-2, 30-47, DOI:10.1080/07380569.2013.764275.