Review

Seismic Analysis of Slender Monumental Structures: Current Strategies and Challenges

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Abstract: The preservation and seismic risk mitigation of built cultural heritage is considered today as a major priority in the international political agenda. Among the great variety of heritage structures spread worldwide, masonry towers belong to one of the most vulnerable categories against earthquake actions due to their morphological and material singularity. The proper understanding of the structural behavior of these artefacts at the micro, meso and macro scales, combined with a thorough knowledge of the best analysis practices deriving from the shared experience of the scientific community working in this field, is a fundamental prerequisite to appropriately address their seismic assessment. In this context, the present work offers an extensive discussion on the major challenges that slender monumental towers pose in terms of characterization of their actual behavior under seismic actions. A critical appraisal of the principal analysis methods applicable to the study of these structures is also presented along with a brief review of the existing modelling strategies for their numerical representation. Relevant examples are discussed in support of each argument. In spite of being a relatively young discipline, earthquake engineering has made remarkable progress in the last years and appropriate modi operandi have been consolidating to tackle the seismic assessment of unconventional systems, such as slender heritage structures. The work is conceived in a format of interest for both practitioners and researchers approaching the seismic assessment of this type of structures, and for those in need of an overall practical review of the topic.

Keywords: historic towers; dynamics of slender structures; masonry failure mechanisms; masonry modelling; structural analysis methods; seismic assessment

1. Introduction

Earthquakes do not kill people, buildings do. Since 1960, 40% of natural disaster deaths occurred due to earthquake events and 60% of these deaths were caused by collapse of masonry buildings [1]. As is widely known, an earthquake is the result of a sudden release of energy in the Earth's crust that creates seismic waves. These waves result in shaking or rapid movements of the ground, often leading to loss of life and destruction of property. Earthquakes also have the potential to generate a tsunami when the epicenter is located offshore and the seabed abruptly moves, causing displacements of a large amount of water. Past seismic events have clearly shown that structures constructed with strict adherence to seismic safety standards are less likely to be damaged during an earthquake. The Kobe earthquake of 1995, which is counted among the strongest, deadliest, and costliest earthquakes to ever strike Japan and for which the ground shaking in some regions was significantly larger than that considered in the seismic design code of the time, represents an example in this regard. In fact, buildings constructed after the 1981 revision of Japan's building codes were far less likely to collapse than older buildings during this event; whereas all the reinforced concrete buildings which saw a story collapse were built prior to a 1971 code change regarding beam and column ductility [2]. Although some types of ancient

buildings have proved remarkably resistant to earthquake forces for centuries, it must be stressed that their resistance has been achieved only by good conceptual design, with no seismic analysis [3,4]. Besides being conceived in the absence of modern building codes and regulations, more than half of existing heritage buildings are made of unreinforced masonry, thereby featuring a very high seismic vulnerability because of the fragility and inherent complexity of the constituent materials, the frequent use of round unshaped stones, the lack of adequate connections between structural elements or the poor adhesion between units and mortar, to mention a few.

Among the great variety of heritage constructions spread worldwide, slender monumental structures represent a peculiar category still embellishing the skylines and landscapes of many cities. Their main distinguishing feature is the height, which served to guarantee the visibility from the surroundings as a sign of power and to enable the various functions these structures were conceived for, whether as bell or clock towers, watchtowers, chimneys, or minarets. Unlike ordinary masonry buildings, such structures present unique morphological and typological characteristics, which may adversely affect their capability to withstand dynamic actions (e.g., complex geometries, wall thickness, significant dead loads, presence of large openings in elevation, irregularities or contiguity with adjacent buildings) [4–15]. Different geometrical and mechanical properties, as well as different boundary conditions, can result in distinct damage patterns under earthquake forces. As far as ancient masonry towers are concerned, the possible collapse mechanisms that can activate with higher probability—based on past earthquake experiences—concern the following [7–9,15–20]: (1) global overturning of the tower due to the formation of a flexural hinge at the base; (2) vertical splitting of the walls (very frequent in slender towers); (3) diagonal cracking of masonry and overturning of the upper part around the base (Heyman's rocking mechanisms); (4) combination of diagonal overturing and vertical splitting; (5) failure of the belfry; (6) sliding of the structure along a horizonal crack surface located near the base. When a tower is not extremely slender and depending on the frequency content of the input forces, earthquake-induced flexural damages are commonly associated with significant shear cracks; conversely, in the case of isolated slender towers, a cantilever behavior with flexural failure is mainly expected. The adoption of fairly simple and moderate-cost strengthening measures (e.g., improving the wall integrity and the connection between orthogonal walls) can drastically reduce the high seismic vulnerability exhibited by this type of structures, turning unacceptable failures into acceptable damage scenarios in case of earthquakes.

Nowadays, the protection of historic monumental structures is of strategic importance in many countries, yet numerous uncertainties still arise about their behavior against seismic actions, making their structural evaluation a critical issue. This work reviews and examines the main challenges underlying the seismic assessment of slender monumental constructions, beginning with the difficulties associated with the description of the internal structure of the masonry material and with the influence of higher modes in the global dynamic response of these systems, and proceeding with an extensive discussion about the selection of the most adequate structural analysis procedures and modelling strategies. Finally, considerations on the methods and tools that engineering offers to scientifically tackle problems related to the seismic analysis of slender monumental structures are provided.

2. Masonry as a Complex Material with Internal Structure

Masonry is a non-homogeneous material formed by units and joints, with or without mortar, and different bond arrangements [21]. Unlike modern masonry, which mainly consists of regularly arranged units, with or without steel reinforcement, ancient masonry is formed by rather complex three-dimensional arrangements of stone or brick units (Figure 1), usually unreinforced. Such a visible internal structure, combined with the variability of materials, unit shape and surface treatments, makes the behavior of historical masonry structures highly indeterminate and difficult to predict and assess with accuracy [22–26].

Figure 1. Examples of ancient masonry section with complex arrangements of units.

Due to its composite character, masonry mechanics are strongly influenced by the individual properties of its constituents as well as by the bond between them, where the latter is often the weakest link in masonry assemblages. Five basic failure mechanisms can generally occur in masonry [27]: (a) tensile cracking of the joints, (b) shear sliding along the joints, (c) direct tensile cracking of the units, (d) diagonal tensile cracking of the units and (e) compressive crushing (Figure 2). The first two modes are essentially joint mechanisms, where failure occurs at low values of normal stress due to the poor bond strength between joints and units, which is typical for strong unit-weak mortar joint combinations, widely present in ancient stone masonry. The third failure mode is a unit mechanism, which is common in masonry with low-strength units but high-strength mortar, usually with greater tensile bond strength. Instead, the last two failure modes are combined mechanisms involving both units and mortar joints. The preponderance of one failure mode over another or the combination of various failure modes is essentially related to the internal structure of masonry, including the orientation of bed joints with respect to the principal stresses.
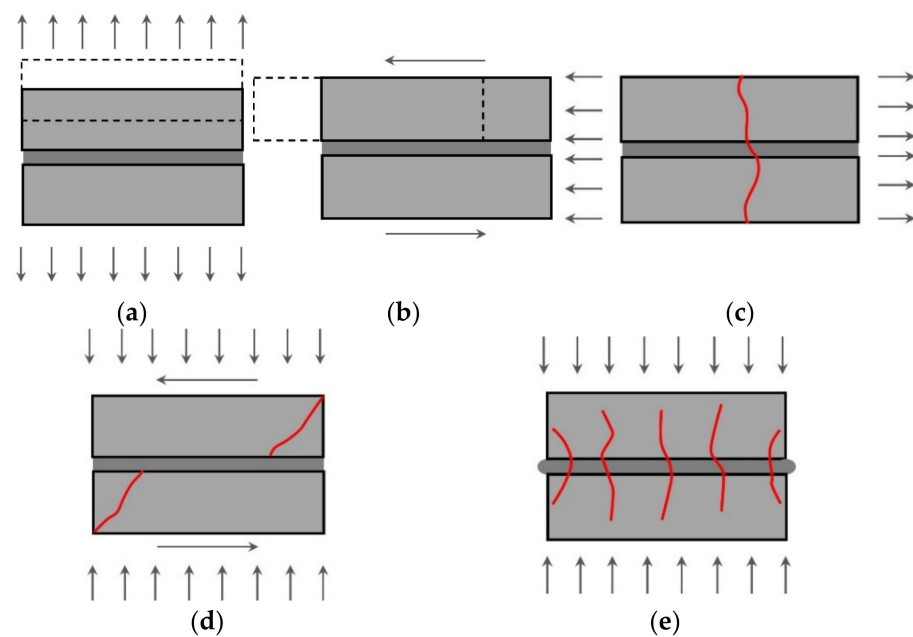

Figure 2. Masonry failure mechanisms (adapted from [27]): (**a**) joint tensile cracking; (**b**) joint slipping; (**c**) unit direct tensile cracking; (**d**) unit diagonal tensile cracking; (**e**) masonry crushing.

Figure 3 shows the results obtained from masonry walls with the same geometry but different units shape and arrangement, under combined vertical and cyclic horizontal in-

plane loading [21]. Three types of walls are considered, namely dry stone (without mortar), coursed masonry and rubble stone masonry (the last two with the same low-strength mortar). While the in-plane behavior of the dry masonry wall with regular large units is characterized by a rocking response with no strength degradation (no damage in the stones) and remarkable lateral displacement capacity, the response of the coursed masonry wall is predominantly flexural, with a slight strength degradation associated with the progressive spreading of flexural cracks through the joints and a considerable lateral displacement capacity, with rocking mechanisms playing a minor role. By contrast, the response of the rubble masonry wall is essentially controlled by shear, showing a significant strength degradation and a low lateral deformation capability, with consequent brittle failure after the formation of diagonal smeared shear cracks. Despite the geometry and material be the same, distinct failure modes, ultimate lateral strengths and hysteretic behaviors are found for the three walls due to aspects such as roughness of the joints, unit dimensions and interlocking. Randomly assembled masonry typically features very low friction as compared to regularly arranged units with sawn or rough surfaces [28]. The connection between leaves, in case of multi-leaf masonry, also influences the masonry behavior. Past seismic events have shown that the systematic presence of headers between leaves plays a key role in the out-of-plane response of masonry walls; analogously, the presence of proper connection between perpendicular walls or between walls and floors is determinant to ensure a monolithic behavior and prevent out-of-plane failures.
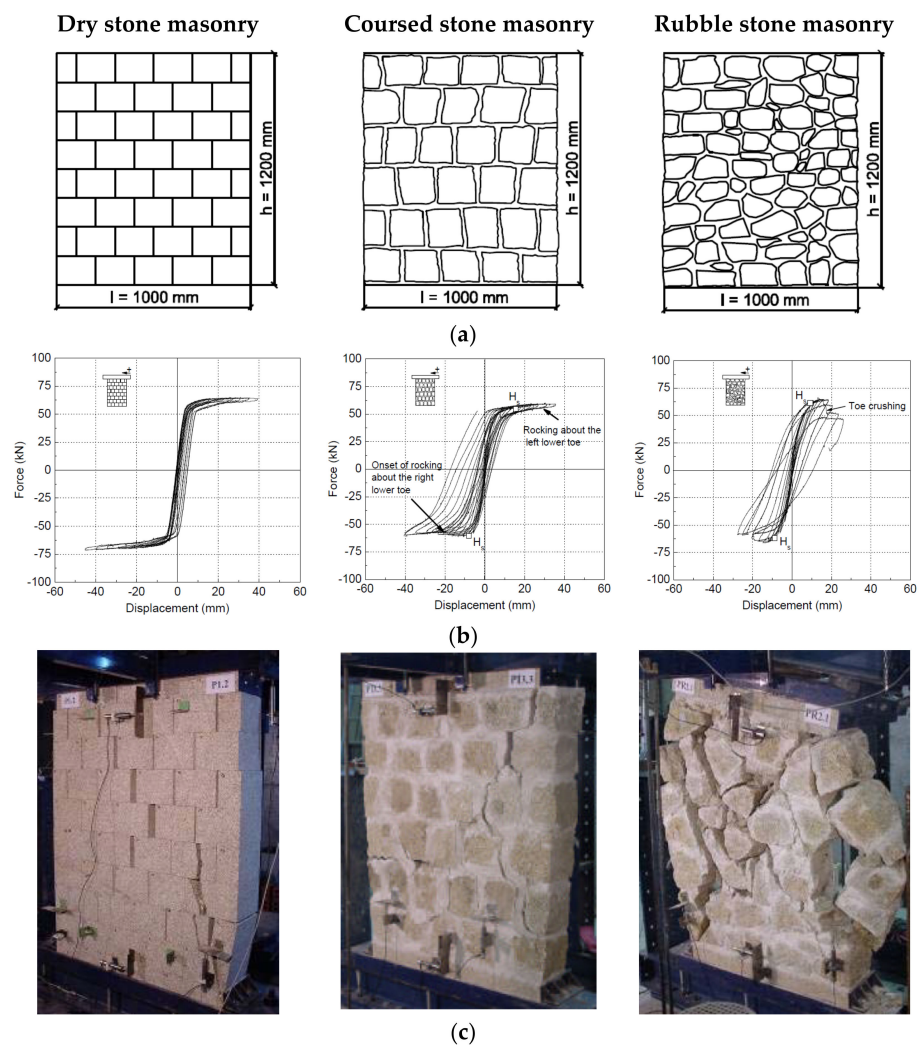

Figure 3. Experimental results on stone masonry shear walls [21]: (**a**) specimen geometries; (**b**) force−displacement diagrams; (**c**) failure mechanisms.

Despite the existence of common features for masonry, i.e., relatively high resistance in compression, very low tensile strength and anisotropy for different loading directions, the complex mechanical behavior of unreinforced masonry structures cannot be univocally stereotyped since there can be as many different responses as the number of masonry types and specific combinations of geometries and materials.

3. Dynamic Amplification and Higher Modes

In the of case of slender monumental historical structures, mostly built using masonry, the problems associated with the accurate modelling of the material internal structure add up to the uncertainties related to their dynamic behavior during earthquakes. Although apparently fixed to the naked eye, each structure does experience small oscillations that are dependent on its physical and mechanical properties, like mass, stiffness and energy dissipation, as well as on its boundary conditions. These small movements allow one to obtain vibration modes, i.e., different patterns in which the system tries to oscillate naturally, described by specific dynamic features, i.e., frequencies, mode shapes and damping ratios. Vibration modes can be tracked over time in a continuous or intermittent way for manifold objectives, including the calibration of realistic numerical models, the validation of strengthening interventions or the assessment of the structural conditions in pre- and post-seismic scenarios [29–37]. Characterizing ancient masonry constructions from a dynamic point of view allows a better interpretation of their structural behavior, which is complex and rather diverse, especially during earthquakes [19,38]. The most common procedure to identify the dynamic properties of historical structures is through operational modal analysis [30,39–42], a dynamic identification technique for estimation that requires only records of the structural response to freely available ambient excitations (like wind, traffic, microtremors and human walking), thereby allowing one to take into account the true operational and boundary conditions of the system. With the input load being unknown, modal parameters are identified from output-only data by applying suitable stochastic modal identification techniques, either in the frequency domain and/or in the time domain [30].

Among the main crucial aspects to consider in the seismic assessment of slender monumental structures is the considerable dynamic amplification that can occur if relevant frequencies of the structure are in the range of the predominant frequencies of the ground motion [43–45]. Indeed, the application of a dynamic force to a vibrating system causes oscillations that will tend to increase in amplitude whenever the frequency of the applied force coincides or approaches one of the system's natural frequencies, leading to the so-called resonance phenomenon. As an example, Figure 4 and Table 1 show the response measured by the strong motion recorders installed in the Church of the Jerónimos Monastery in Lisbon, Portugal (one located at the base of the structure near the chancel, and the other one installed on the nave extrados) during the 6-magnitude earthquake occurred on 12 February 2007. A dynamic amplification occurred in both transversal and vertical directions; indeed, the peaks of the frequency content of the measured response spectra fell exactly within the range of natural frequencies estimated for the church. This phenomenon affected the vibration response of the structure, but the estimated frequency downshifts did not exceed the confidence limits established around each predicted value, meaning that only minor damage took place during the earthquake, see [46,47] for details. In the worst-case scenario, dynamic amplification can originate from an extreme structural response, with large vibrations triggering important nonlinear phenomena and even partial or global collapses (Figure 5).
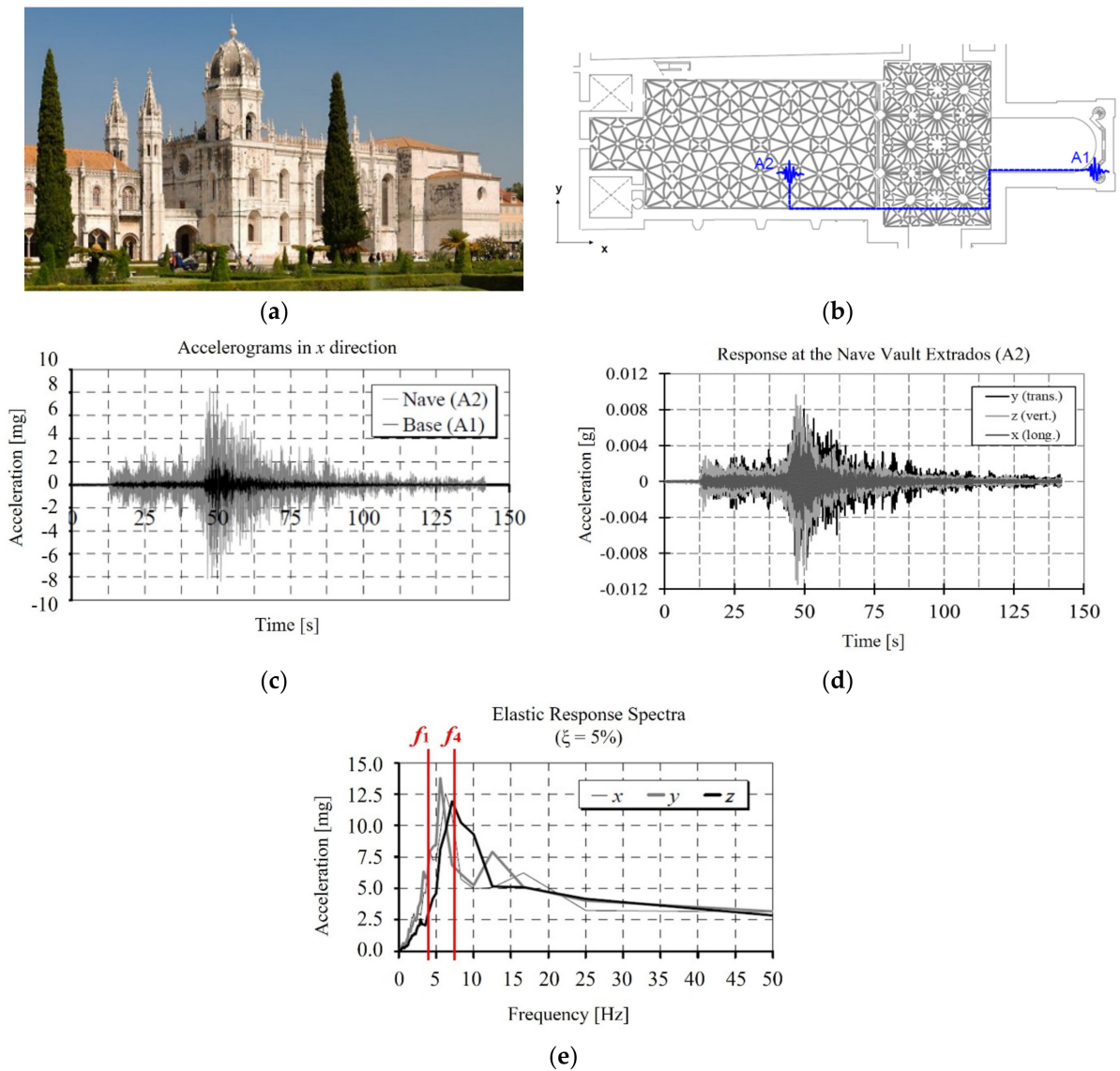
(**a**)

(**b**)

(**c**)

(**d**)

(**e**)

Figure 4. Seismic response of the Church of Jerónimos Monastery, Portugal (adapted from [46,47]): (**a**) exterior view; (**b**) location of strong motion recorders (A1: chancel base; A2: nave vault extrados); (**c**,**d**) acceleration response at the base and at the top of the nave during the earthquake; (**e**) measured response spectra.

Table 1. Dynamic amplification recorded in of the Church of Jerónimos Monastery, Portugal, during the earthquake of 12 February 2007 (maximum results of acceleration in mg).

Event	Sensor A1 (Base)			Sensor A2 (Nave)			A2/A1		
	x	*y*	*z*	*x*	*y*	*z*	*x*	*y*	*z*
Event 1	0.24	0.29	0.26	2.84	8.47	6.75	11.83	29.21	25.96
Event 2	0.12	0.14	0.15	2.67	5.49	6.06	22.25	39.21	40.4

As slender monumental towers do not belong to the category of single-mode dominated structures, the amplification of the dynamic response can interest multiple modes, especially high-frequency modes, whose effects on the seismic behavior of slender systems cannot be neglected. In fact, structures are more prone to experience local damage in case of out-of-phase vibrations due to the unsynchronized movements of the different structural nodes, which is typical for higher modes [48], and slender structures do feature a significantly higher mode response. Figure 6 displays the experimental modes identified

before and after the rehabilitation works of the Mogadouro clock tower, a historic masonry structure located in northeast Portugal. The tower exhibited severe structural damage and a consolidation intervention was carried out to restore its sound state (Figure 7) [46,49].

Figure 5. Amplification effects during the 2016 Bagan earthquake in Myanmar.
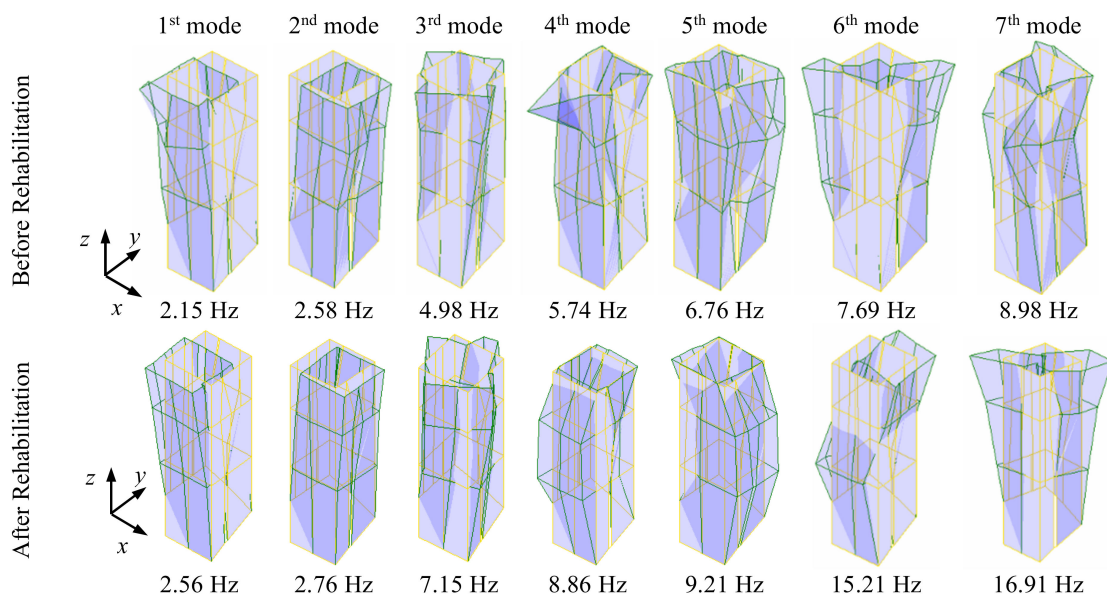

Figure 6. Experimental mode shapes of the Mogadouro clock tower—undeformed structure in yellow wireframe (reprinted with permission from Ref. [49]. Copyright 2018, Elsevier).

The dynamic properties of the damaged tower reflected its structural condition, namely a lower-stiffness system with ongoing non-linear phenomena effects that turned into a higher-stiffness system with reduced non-linear phenomena, as demonstrated by the increase of frequency values along with a decrease of damping ratios after rehabilitation (Table 2). The close inspection of the mode shapes (Figure 6) revealed how the presence of local damage mechanisms was causing large modal displacements in the upper part of the tower, where higher mode effects are particularly important in the case of high-frequency content ground motions. It follows that, as far as slender monumental structures are concerned, depending on the intensity of the seismic action, amplifications may occur in elevation which may be even larger if the structure behaves inelastically. The influence of higher mode effects will depend to a considerable extent on the frequency content of the earthquake and on the magnitude of the plastic deformations of the structure, leading to changes in the dynamic properties that might strongly affect the overall dynamic response of the system to seismic forces.

Figure 7. Overview of the Mogadouro tower before, during and after the rehabilitation works.

Table 2. Dynamic response of Mogadouro Tower before and after rehabilitation.

Mode	Before		After		Δ_f (%)	Before		After		Δ_ξ (%)
	f (Hz)	CV_f (%)	f (Hz)	CV_f (%)		ξ (%)	CV_ξ (%)	ξ (%)	CV_ξ (%)	
1st	2.15	1.85	2.56	0.21	+19.3	2.68	219.5	1.25	0.13	−53.3
2nd	2.58	1.05	2.76	0.30	+6.7	1.71	94.0	1.35	0.17	−21.0
3rd	4.98	0.69	7.15	0.27	+43.7	2.05	65.3	1.20	0.14	−41.3
4th	5.74	1.56	8.86	0.47	+54.4	2.40	24.3	1.31	0.13	−45.7
5th	6.76	1.13	9.21	0.21	+36.1	2.14	31.7	1.16	0.12	−45.6
6th	7.69	2.94	15.21	2.24	+97.9	2.33	56.0	2.54	0.24	+9.1
7th	8.98	1.21	16.91	1.40	+88.3	2.30	46.4	1.49	0.23	−35.1
Avg	−	1.49	−	0.73	+49.5	2.23	76.8	1.47	0.17	−40.3 *

* Average value of damping calculated only for negative differences.

4. Structural Analysis Methods

In the light of the above considerations, it is clear that the inherent characteristics of slender monumental structures considerably affect their behavior under horizontal loads. Most of these structures were built in ancient times, in the absence of specific seismic codes, thus they were essentially conceived to withstand static vertical loads. In recent times, national and international standards [50–52] have imposed the evaluation of their seismic performance, encouraging the use of sophisticated nonlinear analysis methods [6]. However, in spite of the latest computational developments, structural analysis in earthquake engineering remains a complex task because the activated structural behavior is typically nonlinear and uncertain, and the input data concerning the ground motions are random and unpredictable [3,53]. The accuracy of the analysis method is of crucial importance, since a conservative approach may lead to unnecessary expensive interventions [54] (likely with a loss of cultural value), while a non-conservative approach may leave buildings exposed to excessive risk.

In the framework of the seismic analysis of slender structures belonging to the cultural heritage, linear and non-linear methods, static or dynamic, can be used for their global assessment [7,55–57], while limit analysis methods can be adopted to evaluate predefined mechanisms [51,58,59]. The main differences among the various global analysis procedures concern the assumptions about the behavior of the structure, the modelling of the seismic action and the computation of the structural response to that action.

Linear analysis methods assume the structural behavior is linear elastic and the seismic demand is computed through the q-factor approach (defined as seismic behavior factor, in Europe, or seismic response modification factor, in the USA), namely by introducing an elastic force reduction factor which accounts for the typology, energy dissipation capacity and regularity of the structure, where the latter aspect is rather arguable for ancient masonry constructions. In the case of linear static analysis or "lateral force method", the seismic

action may be modelled through equivalent lateral forces proportional to the inertial masses and distributed according to the main vibration mode of the structure in the examined direction, whereas, in the case of linear dynamic analysis or "response spectrum analysis", the effects induced by the seismic action, represented by the elastic response spectrum, are computed—and then combined—for all vibration modes with effective modal mass larger or equal to 5% and activating at least 85% of the overall mass. However, time-dependent response and loading history are not available with this type of analyses.

Among the main issues concerning the seismic behavior of slender monumental structures is the influence of the vertical stresses induced by gravity loads, whose values are often of the same order of magnitude as the ultimate compressive strength of the material [7]. This aspect, combined with the very low tensile strength of masonry, means that the non-linear behavior of these structures can be triggered even by a moderate increase in the stress level in the case of earthquakes, thus the use of inelastic procedures such as non-linear static or dynamic analyses seems far more adequate for their assessment.

In what concerns non-linear static analysis, viz. the classical "pushover analysis", the seismic force is applied to the structure as an incremental static lateral loading, usually assuming a first-modal or mass-proportional force distribution, and results are plotted in the form of a capacity curve (base shear vs. roof top horizontal displacement). Pushover analysis represents one of the most rational analysis methods for practical applications, but it was originally devised for single-mode dominated structures. To account for multiple modes contribution, several proposals have emerged in the literature in the last two decades and the recourse to such methods is explicitly allowed by the EC8 Part 3 [60]. One of these methods is the "modal pushover analysis" (MPA) introduced by [61] that combines classical pushover and response spectrum approaches. The effects induced by the seismic action are computed by combining the results of N pushover analyses, where N is the number of modes chosen based on the amount of activated mass, carried out assuming an invariant inertia force distribution for each mode. Despite the conceptual simplicity and computational attractiveness, MPA relies on the same approximations of all multi-mode methods, such as the superposition of effects and the modal combination rules, which are valid for elastically responding structures but rather questionable in the inelastic range. It follows that the non-linear dynamic analysis or "time history analysis" seems to be a more suitable option for the seismic assessment of masonry structures. Here, the seismic input is represented by spectrum-compatible accelerograms, whose appropriate selection is important to account for the variability associated with different input characteristics in terms of amplitude, energy and frequency content. According to the Italian [50,51] and European [52] codes, at least seven time-history analyses need to be carried out to embrace a wide range of responses, and mean output values are to be considered [62]. Incremental dynamic analysis methods employing multiple non-linear dynamic analyses under scaled ground motion records have also been developed in order to force the structure all the way from elasticity to final global dynamic instability and to have a comprehensive evaluation of the seismic performance of structures [63]. Yet, their application to special constructions, such as ancient monumental structures, is still in its infancy.

Considering the peculiar behavior of slender masonry structures, not all analysis methods result adequate. In this regard, Figure 8 displays the results obtained from different types of analysis carried out for the seismic assessment of the Qutb Minar tower [64], the highest monumental structure in India and one of the tallest masonry towers in the world (Figure 9). It is interesting to note that the nonlinear static and dynamic analyses provide quite a different response of the structure to earthquakes. The non-linear static analysis shows a diffuse cracking in the lowest part of the tower and a base overturning mechanism, but the maximum load factor results are very dependent on the adopted force distribution (Figure 8a): in fact, the load factors proportional to the first mode and to the linear distribution equal, respectively, 35% and 53% of the load factor proportional to the mass. The collapse section varies as well: in the case of forces proportional to the mass, the analysis shows a base collapse mechanism, whereas for the other analyses the collapses

occur at the level of the first balcony. By contrast, the non-linear dynamic analysis proves that the most vulnerable parts of the minaret are the top two stories (Figure 8b), which is in agreement with the damage observed during past earthquakes. Such a difference in the results is due to the influence that higher modes have on the seismic behavior of the tower, modes whose participation is not considered in the standard pushover. To account for this aspect, the seismic assessment was repeated using a modal pushover analysis and considering seven modes for each direction. While displacements and drifts for the first three levels (Figure 8c) are similar to those estimated with the non-linear dynamic analyses, MPA is not able to reproduce satisfactory the failure mechanism of the last two levels. This inconsistency is explained by the variation of the tower's dynamic properties during the damage process, variation that cannot be caught with a modal pushover because the influence of each mode is considered constant throughout the analysis. An alternative is to adopt adaptive pushover analysis [65,66], in which the load distribution depends on the current main mode of vibration, but poor results have been obtained for these complex and relatively brittle structures.
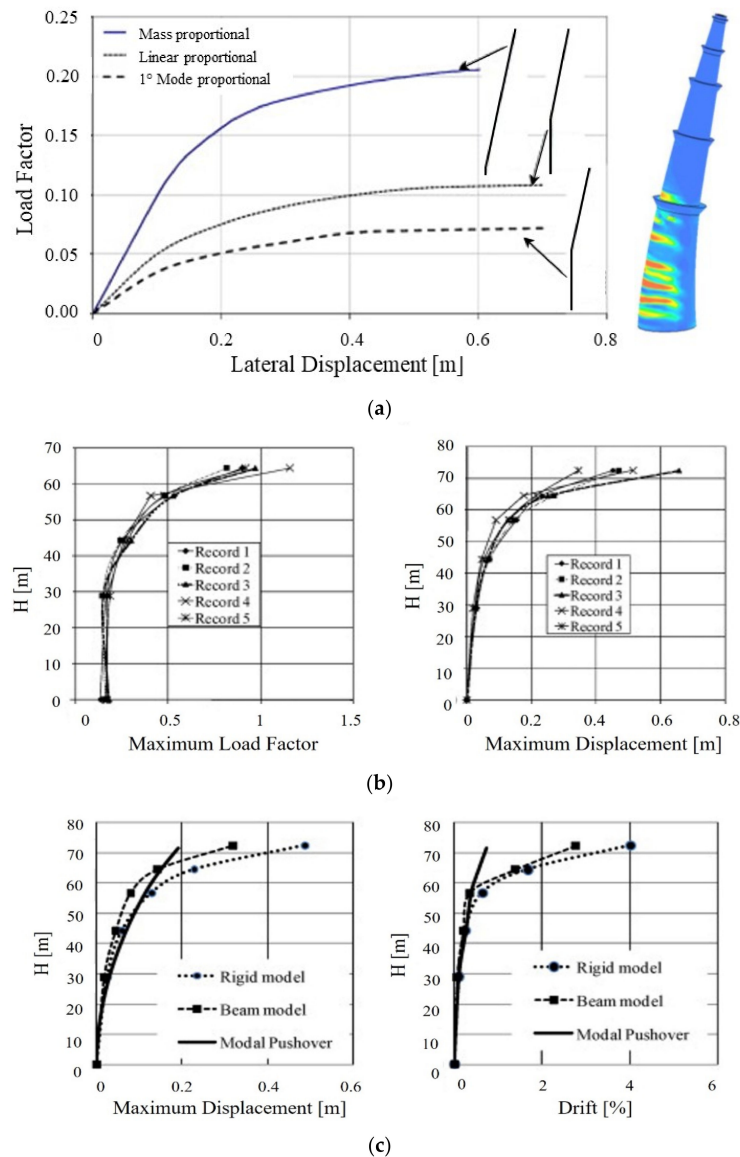

(a)

(b)

(c)

Figure 8. Seismic assessment of the Qutb Minar tower. Comparison of the results from different analysis methods [64]: (**a**) capacity curves for pushover analyses, (**b**) maximum load factor and displacements along the minaret height for dynamic analyses, (**c**) displacements and drifts along the minaret height for modal pushover and non-linear dynamic analyses.
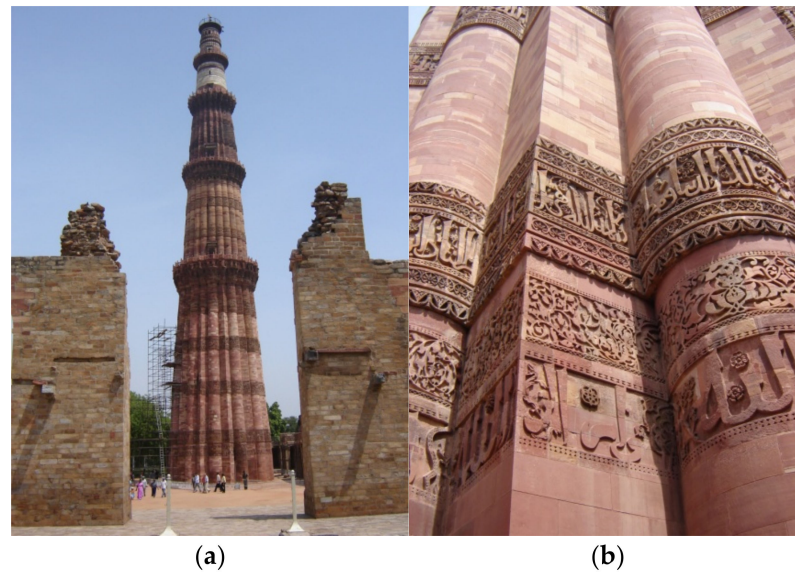
| (a) | (b) |

Figure 9. Qutb Minar tower: (**a**) exterior view and (**b**) detail of the fluted shaft.

Given the time-dependency of the problem, the non-linearity of the material and the influence of higher modes on the seismic behavior of slender monumental structures, non-linear dynamic analyses appear to be the most appropriate numerical procedures to evaluate the performance of these special constructions against earthquakes [67,68]. It is noted that the definition of collapse and other performance levels is quite difficult, and the pros and cons of adopting such a complex tool must be carefully considered. Depending on the prominence of the monument and on the objectives of the analysis, a balance between accuracy and complexity must be achieved.

5. Modelling Strategies

One final aspect deserving attention is the numerical modelling of slender masonry structures, which introduces different challenges due to the inherent difficulties associated with the description of the complex behavior of the masonry material. According to the level of accuracy desired, three main modelling strategies can be adopted, see Figure 10: (1) detailed micro-modelling, in which units and mortar are represented by continuum elements whereas unit-mortar interfaces are represented by discontinuous elements accounting for potential crack and slip planes; (2) simplified micro-modelling, in which expanded units are represented by continuum elements while the behavior of mortar joints and unit-mortar interfaces is lumped into "average" discontinuous elements; (3) macro-modelling, where no distinction among units, mortar and unit-mortar interfaces exists but masonry is treated as a homogeneous anisotropic continuum [69]. Several efforts have also been made in the last decade to improve the so-called homogenization techniques, trying to link micro- and macro-modelling approaches by deriving the constitutive relations of masonry from the individual characteristics of the components of a selected elementary cell [22,28,70–75].

Regardless of the modelling strategy, a thorough experimental description of the material is always required. As for micro-modelling, this must be obtained from laboratory tests in the masonry constituents and small masonry samples, whereas, for macro-modelling, tests must be performed in masonry specimens of sufficient size under homogeneous states of stress and strain [27]. In either case, the task is not trivial considering that the mechanical behavior of masonry is influenced by a large number of factors (see also Section 2), such as the material properties of units and mortar, the shape and dimensions of the units, the bond arrangement, the inner core, the joint thickness, the quality of workmanship, the degree of curing, environment and age.
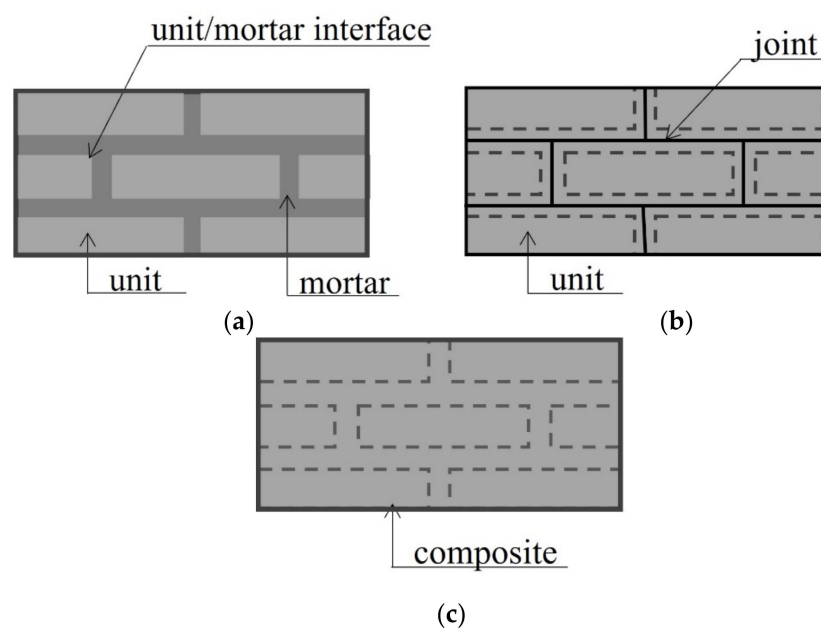
Figure 10. Modelling strategies for masonry structures (adapted from [27]): (**a**) detailed micro-modelling, (**b**) simplified micro-modelling, (**c**) macro-modelling.

In addition to the different modelling approaches in terms of material, three main methods can be adopted for the numerical simulation of heterogeneous masonry constructions, including slender monumental structures: (1) the macro element approach; (2) the discrete element method; and (3) the finite element method. The difference among the various strategies is mainly related to the assumptions about the material and structural behavior, input parameters, modelling effort and computational time required.

The macro element approach relies on the assumption of a no-tension masonry material, usually with infinite compressive strength and no sliding between units. It is a very simple tool that can be applied to evaluate the failure mechanisms of historical masonry structures (kinematic limit analysis), allowing a good estimation of the collapse load factor and displacement capacity of monolithic macro-blocks in which the structure can be ideally subdivided according to the most probable collapse mechanisms. The analysis is not computationally demanding, but it requires an a priori knowledge of the possible disconnections between the rigid blocks composing the kinematic chain, which is not straightforward when a large variety of mechanisms are possible in the structure. Usually, these are selected on the basis of post-earthquake surveyed crack patterns or based on the analyst experience [4,76,77]; yet, an incorrect evaluation of the collapse mechanisms can lead to neglect the mechanism with the lowest load factor and, consequently, to overestimate the real maximum capacity of the structure [70]. Conversely, an incorrect evaluation of the collapse mechanisms may also lead to incorrect overly conservative interventions, if the mechanism with the lowest safety factor assumed by the analyst cannot occur. Moreover, loading history and crack evolution can be hardly included. Remarkable advances have been made towards the automatic assessment of partial failure mechanisms in masonry structures [78–80], but further efforts are needed to fully convert kinematic limit analysis into an automatized process free of arbitrary decisions, which replicates the real conditions of the building.

The distinct element method (DEM) idealizes masonry as a discontinuum material consisting of an assemblage of distinct rigid or deformable bodies whose joints are modelled as contact surfaces with suitable interface laws. Originated from the study of rock mechanics in the 1980s [81], this approach enables to simulate various sources of nonlinearities in masonry structures, including large displacements caused by sliding at the joints with consequent contact update, or the complete separation between blocks. The natural field

of application of DEM is composed of structures formed by regularly shaped masonry or stone blocks, such as stone bridges, columns and pillars, arches, and vaults. DEM is also appropriate for modelling the out-of-plane behavior of multi-leaf masonry walls taking into account the real units' arrangement. Some applications to masonry structures can be found in [82–87]. The application to complex structures of relevant size remains controversial, mainly because of the high number of block elements needed to obtain a realistic discretization of the construction and of the large number of contact points required for the accurate representation of interface stresses, which make the analysis computationally non-viable, especially for 3D problems [23,28].

The finite element method (FEM) remains the most used tool for the advanced analysis of the seismic behavior of masonry constructions, allowing to use different element types and material modelling strategies, including discrete micro-models or continuous macro-models. As mentioned previously, micro-modelling considers the behavior of units, mortar and unit-mortar interfaces separately, thus leading to more accurate results, but mesh preparation can be unwieldy. Owing to the high level of refinement required, its application is mostly limited to the analysis of small structural portions, the computational effort being extremely demanding at large scale [74,88–90]. By contrast, macro-modelling considers masonry as a homogeneous continuum and requires a reduced computational effort, thereby being preferred when dealing with large structures and a compromise between accuracy and efficiency is sought [91]. However, due to the impossibility to execute ad-hoc experimental tests for the characterization of the various parameters necessary to describe the anisotropic behavior of the material in the inelastic range, isotropic macro-models can be adopted for the non-linear analysis of masonry structures, which is an acceptable simplification at the structural level [92–96]. Some recent applications of FEM-based modelling approaches to slender masonry structures can be found in [33,38].

Figures 11 and 12 report the results of the blind predictions provided by several experts on the out-of-plane behavior of two U-shaped masonry structures tested on the shaking table of the LNEC (National Laboratory of Civil Engineering), Lisbon, Portugal, the first specimen being built with granite stones and the second specimen with fired clay bricks, together with lime-based mortar [97]. Several types of analyses were adopted to evaluate the seismic capacity of the structures in terms of failure mechanisms and PGA at collapse, using the three afore-mentioned modelling approaches (Figure 11). The same input parameters, i.e., geometry, material properties, normalized acceleration response spectrum and accelerogram envelope, were provided to the experts for both specimens, see [98]. In the blind predictions of the stone house, 13 idealized collapse mechanisms were proposed, recording an error of about 28% between the average PGA of good predictions and the experimental PGA. Within the good predictions, 80% of the estimated PGAs were lower than or equal to the test PGA. As for the brick house, eight different collapse mechanisms were proposed, and all predictions presented a PGA lower than the experimental one. The high dispersion of the results demonstrates that the peculiar and case-specific character of masonry structures continues to pose significant challenges to the study of these constructions, despite the fact that predictions were generally on the safe side. The most advanced and sophisticated models and analysis methods do not necessarily lead to exact results. Predictions of the structural response greatly depend on the analyst's experience and modelling skills.
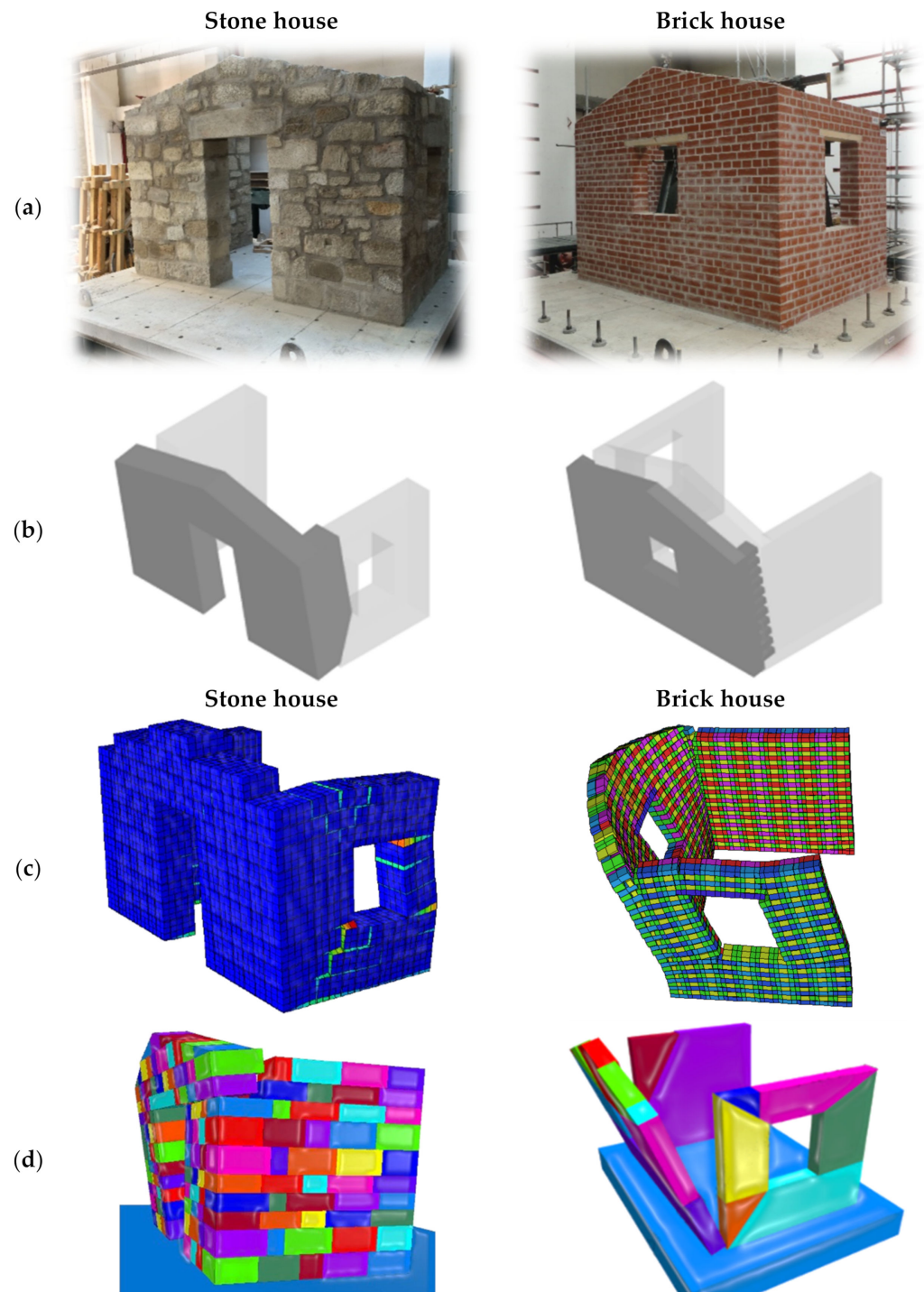
Stone house   **Brick house**

(**a**)

(**b**)

Stone house   **Brick house**

(**c**)

(**d**)

Figure 11. Examples of models adopted for the blind predictions of the (**a**) tested masonry structures: (**b**) models with rigid macro-blocks; (**c**) FEM models and (**d**) DEM models.
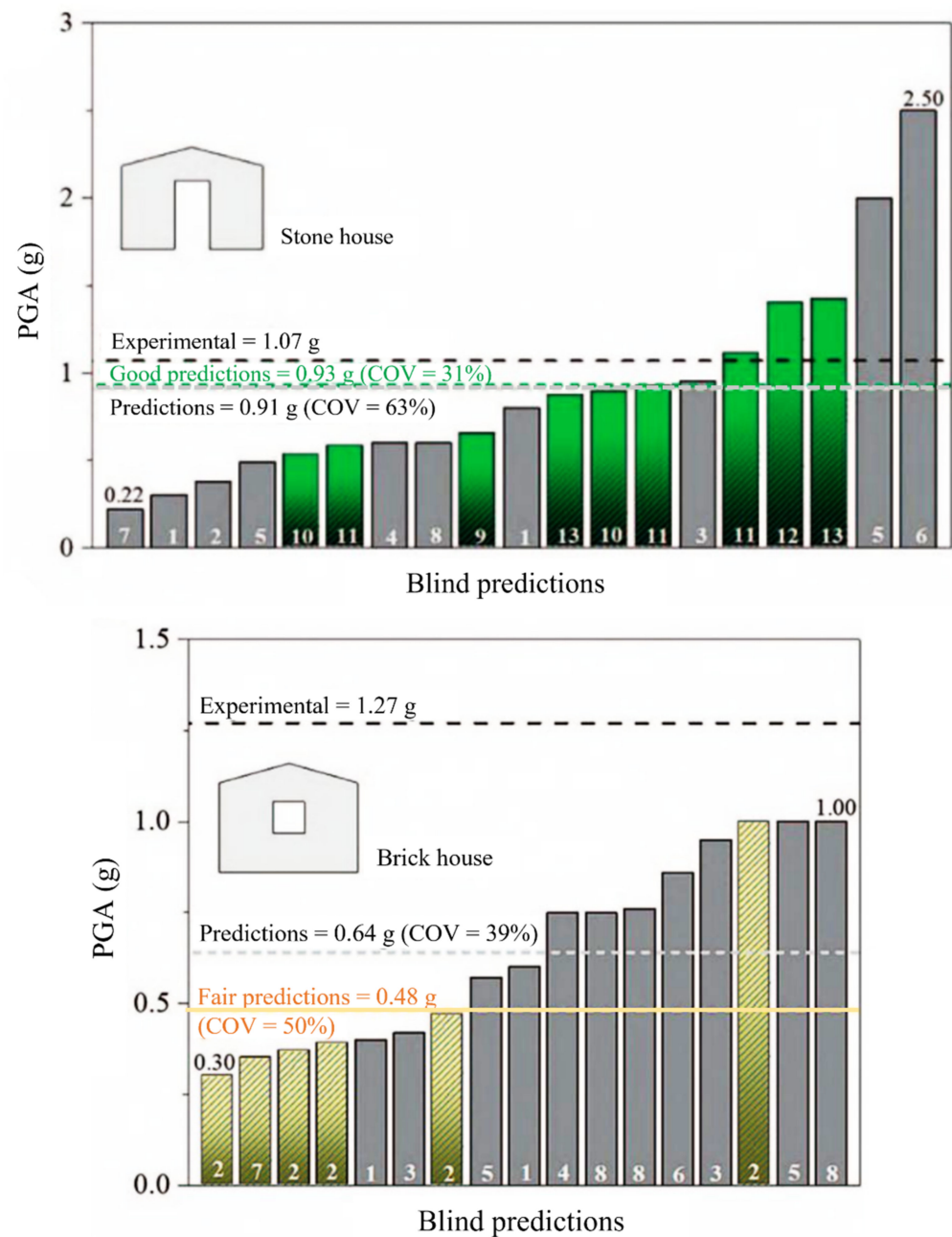
Figure 12. Seismic acceleration capacity (PGA) obtained from the blind predictions [98] (green and yellow bars correspond to good and fair mechanism predictions).

6. Discussion and Conclusions

Historical masonry towers and slender structures may experience many problems during their service life, including soil settlements, compression damage, deterioration processes, lightning strikes, or excessive bell vibrations, but earthquakes remain one of the most challenging problems to face. This is related to the fact that the majority of these peculiar structures were conceived exclusively to withstand gravity loads, thus featuring quite a poor behavior under strong horizontal actions. In many cases, slender structures are simplistically assimilated to cantilever beams made of no-tension material with failure mechanisms occurring due to the formation of a flexural hinge at the base. Yet, past earthquakes have demonstrated that the behavior of these monumental constructions can be much more complex than expected and as mentioned in the Introduction, other mechanisms can be activated with higher probability [8]. Therefore, the accurate assessment of historical

towers and slender monumental structures is of paramount importance to predict their performance during seismic events and to evaluate the real need for strengthening measures aiming at reducing their vulnerability.

To achieve a thorough understanding of cultural heritage structures, a scientific approach combining different diagnosis and analysis tools is required. It is an iterative multi-disciplinary process of knowledge accumulation demanding a variety of complementary activities and integrated analysis methods to correctly interpret the empirical evidence and relate the observed phenomena with the initial hypotheses made on the structure, till reliable and consistent conclusions about its behavior can be drawn (Figure 13). To this end, engineers need to act according to a step-by-step methodology organized in stages similar to those used in medicine [47,99]: condition survey (anamnesis), identification of the causes triggering damage and decay (diagnosis), choice of the remedial measures (therapy) and control of the efficiency of the interventions (control). Anamnesis must involve both a thorough visual inspection and a deep historical survey in order to shed light on the past life of the structure, including its conception, building techniques and alterations, thus allowing a better interpretation of the actual condition. Diagnosis should be based on both qualitative and quantitative investigation methods, where the former implies the direct observation of the structural damage and material decay (whose origins may find support in the historical findings), while the latter requires material experimental tests, structural monitoring and numerical analysis. The afore-mentioned tasks are indispensable to understand the root causes of the observed "symptoms" and eventually drive the selection of the best "therapy", trying to keep interventions to the minimum necessary to guarantee safety and durability without compromising the authenticity of the construction. Appropriate control tools can be finally used to assess the effectiveness of the adopted remedial measures [100].
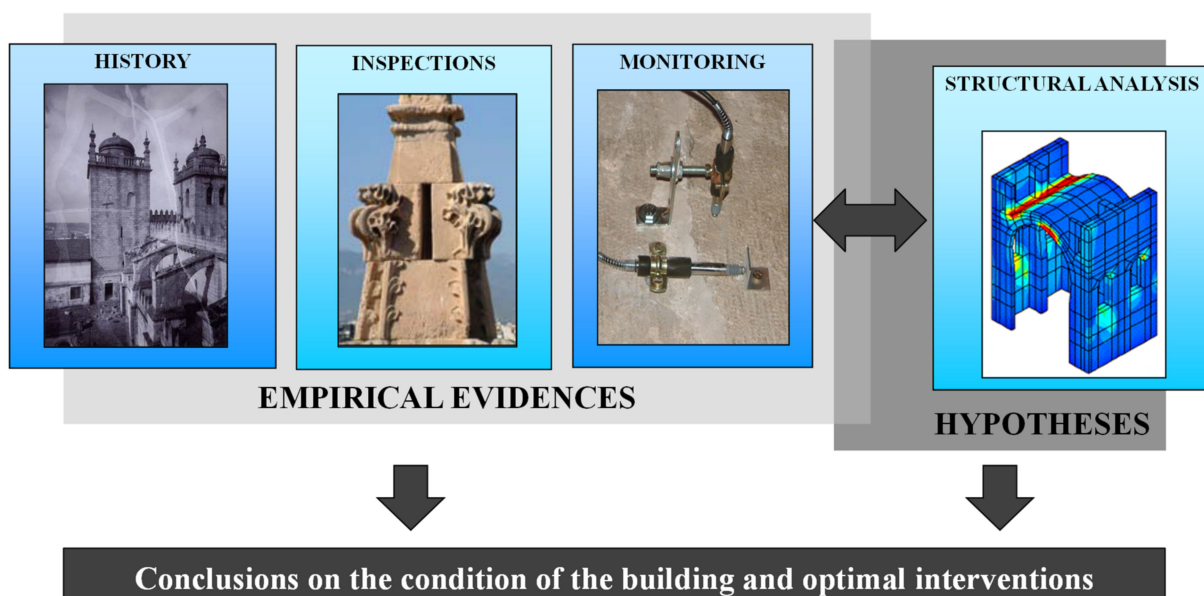

Figure 13. Framework of the scientific approach to the analysis of architectural heritage.

Nevertheless, engineers must be aware that restrictions in the inspection and removal of specimens in structures of historical value, as well as the high costs involved in the execution of experimental characterization tests, often result in limited information about the internal morphology of structural members or the properties of existing materials [99]. These difficulties can be successfully overcome by combining non-destructive diagnostic investigations (e.g., thermography, ground penetrating radar, sonic and ultrasonic tests, or dynamic testing procedures) and monitoring, so that valuable input data at local and global levels can be obtained for the characterization of the analyzed structures as well as for the calibration of realistic numerical models able to predict the structural response. It is

important to bear in mind that, given the remarkable singularity of slender monumental constructions, the most sophisticated methods and tools developed to carry out advanced structural analyses may not always provide sufficiently adequate results. A careful and time-consuming stepped approach is needed.

This manuscript reviewed and discussed the main issues associated with the seismic analysis of slender monumental structures, pointing out—in a format of interest to both practitioners and researchers—relevant aspects that must be considered in the analysis process. The singularity of tall masonry structures, the appropriate reproduction of the constitutive aspects of the material in the dynamic field, and the high computational cost required to perform accurate analyses pose many challenges to the assessment of their actual behavior during earthquakes. Notwithstanding, the significant progresses that occurred in the last decade in the earthquake engineering field with respect to the possibilities of advanced numerical and experimental techniques for cultural heritage structures have allowed us to develop an adequate methodological framework to address the problem.

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