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Does it pay off?”**

Rosa Branca Esteves

NIPE WP 12 / 2008

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS
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Price Discrimination with Partial Information: Does it pay off?*

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Abstract

This paper investigates the profit effects of price discrimination when firms have partial information about consumer preferences. Using a two-dimensional model of product differentiation it shows that price discrimination can boost industry profit if firms have access to the right kind of information about consumer preferences while remaining ignorant of other relevant information.

1 Introduction

While advances in information technology and the increasing use of the Internet mean that sellers nowadays have access to more information about individual customer preferences, thereby increasing their ability to price discriminate, it is reasonable to admit that sellers may remain ignorant of other relevant information about their customers when setting their prices.

This paper investigates the profit effects of competitive price discrimination in markets where firms have access to partial information about consumer preferences.¹ The paper addresses a two-dimension Hotelling model² where each firm offers a single product. The location of a consumer on the unit square represents his two-dimensional preference for a brand name/product. The model assumes that consumers are heterogeneous because they have different tastes for the different brand names and products. Based on past behaviour, consumers are more or less loyal to one firm (or brand name) than to the other. However, when consumers are offered two differentiated products, their product preferences do not always coincide with their brand name preferences.³

Clearly, in this model, where we have a firm offering a unique product, the distinction between “brand” and “product” preferences may seem somewhat redundant. There are, nonetheless, two reasons why the analysis is worth pursuing.

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¹For a complete survey of price discrimination in competitive environments see Stole (2007).

²A two-dimension Hotelling model is also presented by Matutes and Regibeau (1988).

³It is important to stress that the model could equally embrace other types of two-dimension heterogeneities (e.g. switching costs/product preferences; geographical location of firms/product preferences).

First, this method of analysis allows us to investigate the profitability of price discrimination when firms have the ability to partially observe consumer preferences. Armstrong (2006) argues that the crucial feature for discrimination to intensify competition and to depress industry profit is best response asymmetry.⁴ In a standard Hotelling fashion model, Thisse and Vives (1988) suppose that firms can observe the location of individual consumers on the line and quote personalised prices. In this setting, each firm’s strong market is the rival’s weak market, so they show that firms are worse off if they can observe all the information of consumers and set prices accordingly.⁵ This paper extends the Thisse and Vives model and shows that price discrimination may boost profits when firms have partial information about consumer preferences.

Second, the analysis is a useful step towards a fuller treatment of the “brand name vs. product” dichotomy. Consider the following example. While some consumers have a preference for the *Coca-Cola* brand over the *Pepsi*, when they consider buying a diet drink, some loyal *Coca-Cola* consumers may prefer *Diet Pepsi Max* rather than *Diet Coke Plus* because *Diet Pepsi Max* contains, for instance, more caffeine.

The remainder of the paper is organised as follows. Section 2 lays out the formal model. Section 3 presents the benchmark case where price discrimination cannot occur either because firms have no information about consumer preferences or because it is illegal. Section 4 analyses price discrimination with partial information and concluding remarks are presented in Section 5.

2 The Model

Suppose two firms, A and B, sell competing brands of a differentiated good produced at constant marginal cost c . The total number of consumers in the market is normalised to one. A consumer wishes to buy a single unit either from firm A or B. The consumer’s valuation for the product, v , is sufficiently high so that nobody stays out of the market. Consumers are uniformly distributed on the unit square, and the location of a consumer is denoted $(\theta, l) \in [0, 1]^2$. Firm A and product A are located at the point of coordinates $(0, 0)$, while firm B and product B are located at the point of coordinates $(1, 1)$. It is assumed that product A is exclusively offered by firm A and product B is exclusively offered by firm B.

Each consumer is located at a point (θ, l) . It is assumed that the consumers’ relative preference for products (or product tastes) A and B is represented by a parameter θ uniformly distributed in the horizontal line segment $[0, 1]$. Assuming that firm A’s product is located at zero and firm B’s product is located at one, $t\theta$ is the “transport cost” of choosing product A and $t(1-\theta)$ is the “transport cost” of choosing product B. Additionally, the consumers’ relative loyalty for brand names (firms) A and B is represented by a

⁴According to Corts (1998), the market exhibits best response asymmetry when one firm’s “strong” market is the other’s “weak” market. In the literature on price discrimination, a market is designated as “strong” if in contrast to uniform pricing a firm wishes to increase its price there. The market is said to be “weak” if the reverse happens.

⁵Fudenberg and Tirole (2000) investigate the profit effects of price discrimination in a market where a firm can observe whether a consumer belongs to its turf or rather to the rival’s one and set prices accordingly. A similar analysis is carried out by Chen (1997) in a market where firms offer a homogenous product and consumers have switching costs to switch to a different supplier. In both papers the market exhibits best-response asymmetry and price discrimination is bad for industry profit.

parameter l uniformly distributed in the vertical line segment $[0, 1]$. Placing firm A at zero and firm B at one, λl is the “transport cost” of buying from firm A and $\lambda(1 - l)$ is the “transport cost” of buying from firm B.

When firms can observe the location of a consumer in one of the two dimensions (thereby, the term of partial information) they simultaneously and non-cooperatively choose a personalised price to that consumer. When firms have no information they simultaneously set a uniform price to all consumers.

3 No Discrimination Case

Suppose firms are not allowed to price discriminate, either because price discrimination is illegal or because firms cannot observe the location of consumers. Both firms charge a uniform price p_i where $i = A, B$. The consumer with preferences (θ, l) is indifferent between buying product A or product B if

$$v - p_A - t\theta - \lambda l = v - p_B - t(1 - \theta) - \lambda(1 - l).$$

Hence, a consumer located at (θ, l) is indifferent between buying product A or product B if

$$\theta = \frac{1}{2} + \frac{p_B - p_A + \lambda(1 - 2l)}{2t} \quad (1)$$

or,

$$l = \frac{1}{2} + \frac{p_B - p_A + t(1 - 2\theta)}{2\lambda} \quad (2)$$

Case I: $t > \lambda$

When $t > \lambda$, using (1) it is clearly observed that the most loyal consumer to firm A, located at $l = 0$, buys product A whenever $\theta < \bar{\theta} = \frac{1}{2} + \frac{p_B - p_A + \lambda}{2t}$. In contrast, the least loyal consumer to firm A, with $l = 1$, buys product A whenever $\theta < \underline{\theta} = \frac{1}{2} + \frac{p_B - p_A - \lambda}{2t}$.

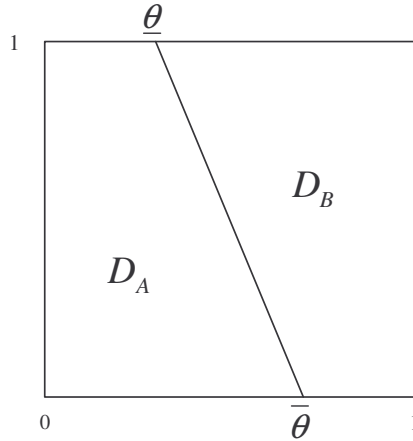


Figure 1: Possible Market Areas when $t > \lambda$

Hence, let $D_i^I(p_i, p_j)$ represent firm i 's demand when $t > \lambda$, it follows that:

$$D_A^I(p_A, p_B) = \underline{\theta} + \frac{1}{2} (\bar{\theta} - \underline{\theta}) = \frac{1}{2} + \frac{p_B - p_A}{2t}, \quad (3)$$

and

$$D_B^I(p_B, p_A) = 1 - D_A^I(p_A, p_B) = \frac{1}{2} + \frac{p_A - p_B}{2t}. \quad (4)$$

Case II: $t < \lambda$

Similarly, when $t < \lambda$, it follows that

$$D_A^{II}(p_A, p_B) = \underline{l} + \frac{1}{2}(\bar{l} - \underline{l}) = \frac{1}{2}(\bar{l} + \underline{l}) = \frac{1}{2} + \frac{p_B - p_A}{2\lambda}, \quad (5)$$

and $D_B^{II}(p_B, p_A) = 1 - D_A^{II}(p_A, p_B)$.

Thus, firm i 's profit is equal to:

$$\pi_i^I = (p_i - c) \left(\frac{1}{2} + \frac{p_j - p_i}{2t} \right), \text{ if } t \geq \lambda \quad (6)$$

or

$$\pi_i^{II} = (p_i - c) \left(\frac{1}{2} + \frac{p_j - p_i}{2\lambda} \right), \text{ if } t \leq \lambda \quad (7)$$

Proposition 1. *When price discrimination cannot occur the Nash equilibrium (NE) in prices is given by*

$$p_i^* = \begin{cases} c + t & \text{if } t \geq \lambda \\ c + \lambda & \text{if } t \leq \lambda \end{cases} \quad (8)$$

with equilibrium profit equal to

$$\pi_i^* = \begin{cases} \frac{t}{2} & \text{if } t \geq \lambda \\ \frac{\lambda}{2} & \text{if } t \leq \lambda \end{cases}. \quad (9)$$

Note that in both cases the resulting NE coincides with that obtained when consumers are heterogeneous only on the basis of one dimension. In a Hotelling standard fashion model, the resulting NE in prices is given by the marginal production cost plus the transport cost. Since here the corresponding transport cost is t in the product differentiation dimension and λ in the loyalty dimension, the NE only depends on the relation between t and λ . When t is greater than λ , it can happen that the most loyal consumer to brand B may decide to buy product A if he has a taste parameter that satisfies $\theta < \underline{\theta}$. Thus, all consumers with $\theta < \underline{\theta}$ buy from firm A even if they are strongly loyal to brand B (i.e. with $l = 1$). In this case the cost of a consumer not buying the product that is closer to his tastes is greater than the cost of not buying from his preferred firm.⁶ Hence, when $t > \lambda$, firms set their equilibrium prices according to the transport cost associated with the product tastes dimension. Conversely, when $t < \lambda$, it is more expensive for consumers to not buy from their preferred firm than not buy the product that is closer to their tastes. In this second case, equilibrium prices are set according to the transport cost λ . This suggests that when firms cannot price discriminate they will set prices according to the highest transport cost.⁷

⁶It is straightforward to confirm that in equilibrium a consumer with $\theta < \underline{\theta}$ and $l = 1$, always buys from firm A because: $t\theta + \lambda < t(1 - \theta) \Rightarrow 2t\theta < t - \lambda = 2t\underline{\theta}$.

⁷It is important to note that this happens because firms are located symmetrically in each dimension. If, for instance, firm A had a greater fraction of loyal consumers in the market than firm B, the price would include this asymmetry.

4 Price Discrimination with partial information

Consider now the case where firms can observe the location of consumers in one of the two dimensions. Suppose, for instance, that based on their data on individuals' past purchasing behaviour firms can observe a consumer brand loyalty degree l but not a consumer product preference θ and then set their prices accordingly. Thus, firms have partial information about consumer preferences. If firms know a consumer's loyalty parameter, they can target a personalised price to that consumer, taking into account, however, that the consumer will be closer or more distant from the firm's product in the tastes line.

Let $p_i(l)$ denote firm i 's price for a consumer with brand loyalty parameter l . Consider, for instance, the perspective of firm A. The consumer with preferences (θ, l) buys product A rather than product B when

$$p_A(l) + \lambda l + t\theta < p_B(l) + \lambda(1-l) + t(1-\theta)$$

Firm A's demand is given by

$$D_A(p_A, p_B, l) = \frac{1}{2} + \frac{p_B - p_A + \lambda(1-2l)}{2t}, \quad (10)$$

Firm B's demand is $D_B(p_A, p_B, l) = 1 - D_A(p_A, p_B, l)$. Firm i 's profit, $i = A, B$, is $\pi_i(p_i, p_j, l) = (p_i - c) D_i(p_i, p_j, l)$.

Proposition 2. *When firms can observe the brand loyalty parameter of each individual consumer and price discriminate, the pure strategy NE in prices $(p_A^d(l), p_B^d(l))$ is defined as:*

$$p_A^d(l) = \left\{ \begin{array}{ll} c + t + \frac{\lambda}{3}(1-2l) & \text{if } l < \frac{1}{2} + \frac{3t}{2\lambda} \\ c & \text{if } l \geq \frac{1}{2} + \frac{3t}{2\lambda} \end{array} \right\} \quad (11)$$

$$p_B^d(l) = \left\{ \begin{array}{ll} c & \text{for } l \leq \frac{1}{2} - \frac{3t}{2\lambda} \\ c + t + \frac{\lambda}{3}(2l-1) & \text{for } l > \frac{1}{2} - \frac{3t}{2\lambda} \end{array} \right\} \quad (12)$$

and each firm equilibrium profit equals:

$$\pi_i^d = \left\{ \begin{array}{ll} \frac{t}{2} + \frac{\lambda^2}{54t} & \text{if } t \geq \frac{\lambda}{3} \\ \frac{(3t+\lambda)^3}{108\lambda t} & \text{if } t \leq \frac{\lambda}{3} \end{array} \right\}. \quad (13)$$

Proof. See the Appendix.

When firms can observe a consumer's brand loyalty degree they price high to those consumers who prefer their brand name but price low to consumers who prefer the rival's brand. As expected, when consumers have different brand preferences but see the firms' products as perfect substitutes, i.e. $t = 0$, as in Thisse and Vives (1988) firm A's price is equal to c to all consumers with $l > \frac{1}{2}$. Here, in contrast, for other values of t , firm A's price to a strong loyal consumer to firm B (with $l = 1$) is $c + t - \frac{\lambda}{3}$, greater than c whenever $t > \frac{\lambda}{3}$.

Price Discrimination Based on Product Preferences Consider now the case where firms observe a consumer product preference θ , but not the parameter l and set their prices accordingly. Again firms have partial information. Given the similarity between this case and the previous one, it is straightforward to deduce that the equilibrium results inherent to this case are simply obtained by interchanging t and λ .

5 Concluding Remarks

Table 1 shows a firm's equilibrium profit with uniform pricing and with price discrimination based on partial information. In order to facilitate the analysis, equilibrium profits are measured on the basis of λ .

Table 1: Equilibrium Profits with and without price discrimination

	No Disc.	Disc. according to l	Disc. according to θ
$t < \frac{\lambda}{3}$	$\pi_i^* = 0.5\lambda$	$\pi_i^d < 0.22(2)\lambda$	$\pi_i^d > 0.502\lambda$
$\frac{\lambda}{3} \leq t < \lambda$	$\pi_i^* = 0.5\lambda$	$0.22(2)\lambda \leq \pi_i^d < 0.518\lambda$	$0.502\lambda \leq \pi_i^d < 0.518\lambda$
$\lambda \leq t < 3\lambda$	$0.5\lambda < \pi_i^* < 1.5\lambda$	$0.518\lambda \leq \pi_i^d < 1.506\lambda$	$0.518\lambda \leq \pi_i^d < 0.66(6)\lambda$
$t \geq 3\lambda$	$\pi_i^* \geq 1.5\lambda$	$\pi_i^d \geq 1.506\lambda$	$\pi_i^d \leq 0.66(6)\lambda$

Table 1 shows that under certain conditions competitive price discrimination with partial information can boost firm and industry profit. However, the profitability of price discrimination is strongly dependent on the kind of information available to firms. When t is sufficiently low in comparison to λ , firms are better off when they have the required information to price discriminate on the basis of a consumer's product taste preference. Conversely, when t is higher than λ , discrimination can raise profits if firms have access to information about a consumer's brand loyalty. In contrast, if firms can observe the brand loyalty degree of individual consumers but $\lambda > t$, they are better off under the uniform pricing policy. (The same happens when firms observe a consumer taste θ but $t > \lambda$.)

One important implication of the simple model addressed in this paper is that price discrimination may not necessarily lead to the prisoners' dilemma result that generally follows in markets that exhibit best-response asymmetry. The model predicts that price discrimination can boost industry profit (i) if firms have access to the right kind of information about consumer preferences and (ii) if firms have partial information, i.e. if they don't know everything about consumers. In other words, price discrimination can increase industry profit if firms have information about the location of consumers in the less important differentiation dimension (i.e. with lower transport cost) while they remain ignorant of their preferences in the other dimension (i.e. with high transport costs). While some information may benefit firms, ignorance acts to help competing firms to use price discrimination in a profitable way.⁸

⁸Esteves (2008) shows that when firms price discriminate on the basis of their private and imperfect information about consumer preferences, they are better off when information is more imprecise. Further, profits are higher under private information than public information.

Appendix

Proof of Proposition 1: In equilibrium firm i solves the following problem:

$$\underset{p_i \geq 0}{Max} (p_i - c) D_i(p_i, p_j^*).$$

When $t \geq \lambda$, using (6) the first order condition⁹ is

$$\frac{\partial \pi_i}{\partial p_i} = \left(\frac{1}{2} + \frac{p_j^* - p_i}{2t} \right) - \frac{1}{2t} (p_i - c) = 0,$$

Therefore, the symmetric pure strategy Nash Equilibrium is given by $p_i^{I*} = c + t$, with equilibrium profit equal to $\pi_i^{I*} = \frac{t}{2}$. It is straightforward to verify that when $t \leq \lambda$, equilibrium price is $p_i^{II*} = c + \lambda$, and each firm equilibrium profit equals $\pi_i^{II*} = \frac{\lambda}{2}$. *Q.E.D.*

Proof of Proposition 2: Given the observed value of l , it follows that each firm's demand from its p -group is respectively,

$$D_A(p_A, p_B, l) = \frac{1}{2} + \frac{p_B - p_A + \lambda(1 - 2l)}{2t},$$

$$D_B(p_B, p_A, l) = \frac{1}{2} + \frac{p_A - p_B - \lambda(1 - 2l)}{2t}.$$

With correspondent profit

$$\pi_A(p_A, p_B, l) = (p_A - c) \left(\frac{1}{2} + \frac{p_B - p_A + \lambda(1 - 2l)}{2t} \right),$$

and

$$\pi_B(p_B, p_A, l) = (p_B - c) \left(\frac{1}{2} + \frac{p_A - p_B - \lambda(1 - 2l)}{2t} \right).$$

From the first and second order conditions for a maximum it follows that firm A and B's reaction functions are

$$p_A(p_B^*) = \frac{p_B^* + c + t + \lambda(1 - 2l)}{2}$$

and

$$p_B(p_A^*) = \frac{p_A^* + c + t - \lambda(1 - 2l)}{2}.$$

It is straightforward to verify that the pure strategy NE in prices is $(p_A^d(l), p_B^d(l))$ defined as

$$p_A^d(l) = c + t + \frac{\lambda}{3}(1 - 2l) \tag{14}$$

and

$$p_B^d(l) = c + t + \frac{\lambda}{3}(2l - 1). \tag{15}$$

⁹Note that $\frac{\partial^2 \pi_i}{\partial p_i^2} = -\frac{1}{t} < 0$.

However, since neither firm sets in equilibrium a price below marginal cost, it follows that each firm's equilibrium price is respectively

$$p_A^d(l) = \left\{ \begin{array}{ll} c + t + \frac{\lambda}{3}(1 - 2l) & \text{for } l \leq \frac{1}{2} + \frac{3t}{2\lambda} \\ c & \text{for } l \geq \frac{1}{2} + \frac{3t}{2\lambda} \end{array} \right\}, \quad (16)$$

$$p_B^d(l) = \left\{ \begin{array}{ll} c & \text{for } l \leq \frac{1}{2} - \frac{3t}{2\lambda} \\ c + t + \frac{\lambda}{3}(2l - 1) & \text{for } l \geq \frac{1}{2} - \frac{3t}{2\lambda} \end{array} \right\}. \quad (17)$$

For high enough values of t (i.e. $t \geq \frac{\lambda}{3}$) a firm's price is given by (14) and (15). Therefore, total equilibrium profit for firm A is:

$$\pi_A^d = \int_0^1 \left(t + \frac{\lambda}{3}(1 - 2l) \right) \left(\frac{3t + \lambda(1 - 2l)}{6t} \right) dl = \frac{1}{2}t + \frac{\lambda^2}{54t}.$$

In contrast, when $t \leq \frac{\lambda}{3}$, firm's A profit equals:

$$\begin{aligned} \pi_A^d &= \int_0^{\frac{1}{2} + \frac{3t}{2\lambda}} \left(t + \frac{\lambda}{3}(1 - 2l) \right) \left(\frac{3t + \lambda(1 - 2l)}{6t} \right) dl \\ &= \frac{(3t + \lambda)^3}{108\lambda t} \leq \frac{2\lambda}{9}. \quad Q.E.D. \end{aligned}$$

References

- ARMSTRONG, M. (2006), "Recent Developments in the Economics of Price Discrimination," in *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society*, ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.
- CHEN, Y. (1997), "Paying Customers to Switch", *Journal of Economics and Management Strategy*, 6, 877-897.
- CORTS, K. (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment", *Rand Journal of Economics*, 29, 306-323.
- ESTEVEZ, R.B. (2008), "Price Discrimination with Private and Imperfect Information." NIPE working paper 13/2008.
- FUNDENBERG, D. AND TIROLE, J. (2000), "Customer Poaching and Brand Switching", *Rand Journal of Economics*, 31(4), 634-657.
- MATUTES, C. AND REGIBEAU, P. (1988), "Mix and Match: Product Compatibility without Network Externalities", *Rand Journal of Economics*, 19(2), 221-234.
- STOLE, L. (2007), "Price Discrimination and Competition". In Armstrong, M. and R. Porter, eds., *The Handbook of Industrial Organization*, Vol.3, Amsterdam: North-Holland.
- THISSE, J. AND VIVES, X. (1988), "On the Strategic Choice of Spatial Price Policy", *American Economic Review*, 78, 122-137.

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