An Evolving Cognitive Approach to Dyscalculia Screening and Therapeutics
Filipa Tinoco Ferraz
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University of Minho
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PhD Thesis

Doctoral Program in Biomedical Engineering

Work carried out under the supervision of

Professor Doutor José Carlos Ferreira Maia Neves

Professor Doutor Victor Manuel Rodrigues Alves

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DECLARATION

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STATEMENT OF INTEGRITY

I hereby declare having conducted this academic work with integrity. I confirm that I have not used plagiarism or any form of undue use of information or falsification of results along the process leading to its elaboration.

I further declare that I have fully acknowledged the Code of Ethical Conduct of the University of Minho.

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RESUMO

Uma Abordagem Cognitiva Evolutiva para o Despiste e Terapêutica da Discalculia

A crescente taxa de insucesso escolar e o aumento da aversão à matemática têm geralmente origem na discalculia. A sua incidência na população é de cerca de 6 a 7%, e o facto de que é praticamente desconhecida faz com que a sua associação à discalculia seja difícil de se realizar, tendo um impacto negativo no quotidiano dos indivíduos. Uma vez que esta desordem existe desde a conceção, é crucial iniciar a terapêutica nas fases iniciais da vida para mitigar a sua severidade.

Atualmente não existe um teste standard para o diagnóstico da discalculia, e a sua origem ainda não está bem definida. Adicionalmente, os sistemas de apoio à aprendizagem para crianças com discalculia apenas existem sob a forma de papel ou software para computador, somente na língua inglesa. Por isso, é uma lacuna a sua não-consciencialização no sistema de educação especial português, assim como a inexistência de ferramentas de apoio direcionadas para este tipo de dificuldade de aprendizagem. É relevante criar um sistema de apoio à aprendizagem português adequado à discalculia, em linha com as novas tendências tecnológicas. A falta de sensibilização para este tipo de desordem, assim como a inexistência de sistemas de apoio resultam nos maus resultados observados nos exames nacionais de matemática.

É importante criar conhecimento nesta área para permitir o desenvolvimento de ferramentas eficientes de apoio compatível com qualquer dispositivo móvel e que visa auxiliar as crianças a evoluir os seus conhecimentos matemáticos, permitindo-lhes ao mesmo tempo divertirem-se enquanto jogam. Doravante, a inclusão de características inteligentes no *software* que ajusta o jogo às dificuldades das crianças, e o estudo da atividade cerebral enquanto usam a aplicação, são dois potenciais ganhos diferenciadores para o estudo e dedução de novas perspetivas sobre a discalculia.

O objetivo principal é desenvolver e extrair conhecimento suficiente de modo a se conseguir uma solução inovadora, atrativa e dinâmica para terapêutica e diagnóstico de discalculia, direcionada para ajudar crianças a melhorar as suas capacidades de ordem matemática enquanto jogam. A inovação surge da inclusão de competências inteligentes e evolutivas na solução de acordo com as dificuldades das crianças e do seu progresso.

Palavras-Chave: Discalculia para Crianças; Despiste de Discalculia; Sistemas Inteligentes; Terapêutica de Discalculia

Abstract

An Evolving Cognitive Approach to Dyscalculia Screening and Therapeutics

The growing scholar failure rate among pupils and the increasing aversion to mathematics generally have their origins in dyscalculia. The incidence is around 6-7% and the fact that it is hardly known makes its association hard to accomplish, having a negative impact on everyday life. As it is a disorder that exists since conception, it is important to start therapy early in life to mitigate its severity.

Currently, there are no standard tests to detect dyscalculia and its origins are not yet well-identified. In addition, learning support systems for children with dyscalculia exist only in English in the form of paper or computer software. Therefore, it is a big gap that there is no awareness of this disorder in Portuguese special education, as well as the non-existence of support tools targeted at this type of learning disability. It is important to create an appropriate Portuguese learning support system for dyscalculia, in line with new technological trends. The non-awareness of this disorder and the non-existence of support systems contribute to the tragic results seen in the national mathematics tests.

It is important to create knowledge in this area to allow the development of efficient support tools that can be adapted to any mobile device and that aims to help children develop their mathematical knowledge, allowing them to have fun while they play it. Henceforth, incorporating intelligent features into the software according to children's difficulties and studying brain activity while playing are two potential gains in studying and deriving new perspectives about dyscalculia.

The main goal is to develop and extract enough knowledge to achieve an innovative, engaging, and dynamic solution for dyscalculia therapeutics and screening, aimed at helping children to improve their mathematical skills while playing. Innovation comes from incorporating intelligence and developing skills into the solution according to children's difficulties and progress.

Keywords: Dyscalculia in Children; Dyscalculia Screening; Dyscalculia Therapeutics; Intelligent Systems

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List of Abbreviations and acronyms

A

AC	Ant Colony
ADHD	Attention-Deficit/Hyperactivity Disorder
AI	Artificial Intelligence
AN	Anergy
ANN	Artificial Neural Networks
API	Application Programming Interface
APK	Android Application Package
App	Application
AWMF	Association of the Scientific Medical Societies In German *"Arbeitsgemeinschaft der Wissenschaftlichen Medizinischen Fachgesellschaften"*

B

BCI	Brain-Computer Interface
BCS	Best-Case Scenario

C

CADIn	Child Development Support Centre In Portuguese *"Centro de Apoio ao Desenvolvimento Infantil"*
CBR	Case-Based Reasoning
cs	Child's Symptoms
CSV	Comma-Separated Values

D

DB	Database
DD	Developmental Dyscalculia
DoC	Degree of Confidence
DSC	Dyscalculia Symptoms in Children
DSM	Diagnostic and Statistical Manual

E

ECM	Eindhoven Classification Method
EEG	Electroencephalogram
ERP	Event-Related Potential
EX	Exergy

F

FK	Foreign Key
fMRI	Functional Magnetic Resonance Imaging

G

GP	Genetic Programming
GUI	Graphical User Interface

H

HTML	Hypertext Markup Language
HTTP	Hypertext Transfer Protocol

I

ICD	International Statistical Classification of Diseases and Related Health Problems
ICVS	Life and Health Sciences Research Institute
	In Portuguese *"Instituto de Investigação em Ciências da Vida e Saúde"*
IIS	Internet Information Services
IPS	Intraparietal Sulcus
IQ	Intelligence Quotient

K

KRR	Knowledge Representation and Reasoning

L

LD	Learning Disability

LI	Learning Impairment
LP	Logic Programming

M

MLD	Mathematical Learning Disability
MRI	Magnetic Resonance Imaging

N

NN	Not Null

O

OS	Operative System

P

PK	Primary Key
PS	Particle Swarm

Q

QoI	Quality of Information

R

RDBMS	Relational Database Management System
REST	Representational State Transfer
RPROP	Resilient Backpropagation

S

SQL	Structured Query Language
SWOT	Strengths, Weaknesses, Opportunities and Threats Analysis

U

UML	Unified Modelling Language

V

VA	Vagueness

W

WCS	Worst-Case Scenario

X

XOR	Exclusive OR

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GLOSSARY

APK – Android Application Package	Package file format used by the Android operating system for distribution and installation of mobile applications and middleware. This file is first compiled by the developer and then all its parts are packaged into a container file. It contains resources, assets and other configuration files that identify the software. Once created, this file can be installed on Android-powered devices and subsequently executed.
API – Application Programming Interface	Set of definitions, communication protocols, and tools for building software. A strong API allows for an easier underlying understanding of the overall program exposing only the objects or actions the computer programmer needs.
AI – Artificial Intelligence	Branch of computer science; a set of algorithms that simulate human intelligence in machines, e.g., learning, problem-solving and decision-making processes.
ANN – Artificial Neural Networks	Model that is inspired by the brain and that performs the mapping from a received input to the desired output using the "neurons" of the network, also referred to as a unit and nodes that are weighted interconnected.
Back-end	Data access layer responsible for all the business logic and data structure. It establishes the connection to the server and programs all interface components.
Cache	Fast access device integrated into a system that acts as an intermediary between a process and the storage device. It functions as a repository for information stored in a compartment of an operating system and avoids accessing the storage device when information is needed, which can take a long time. It stores enough fragments or information to run processes.

Deep Learning

ML technique that uses a biologically inspired neural network architecture, ANN, to process information. This ''learning by immersion and example'' system helps computers to quickly recognise and process images and language. The neural networks contain several hidden layers that can have different functions and process data, make connections, and weigh inputs to get the best results.

Fragile X Syndrome

Genetic condition that causes intellectual disabilities, ranging from learning disabilities to cognitive impairment. It tends to affect more the male sex and is associated with physical features such as a long and narrow face, large ears, and flexible fingers. The developmental problems are usually observed at the age of two, with late development of speech and language.

Front-end

The presentation layer of the application provides a user-friendly interface to the user. On a client-server model, the front-end is the client as it is the component that can be manipulated by the user.

fMRI – Functional Magnetic Resonance Imaging

Variation of MRI, which relies on blood flow in the brain to measure brain activity. Since cerebral blood flow and neural activation are linked, it enables the areas of the brain that are active in certain tasks to be mapped. The use of the Blood-Oxygen-Level Dependent (BOLD) contrast agent helps to identify brain problems, such as the effects of a stroke, and to map the brain to, for example, brain surgeries, epilepsy, or tumours. The resulting brain activation can be displayed graphically by colour-coding the strength of the activation in the entire brain or the examined region.

GUI – Graphical User Interface

User interface that allows for user interaction with the graphical content within.

Likert-type Scales	Psychometric approach to measuring people's points of view. It is a 5- or 7-point ordinal scale that rates the degree of agreement or disagreement with a statement, helping to measure attitudes, opinions, or feelings. It is very widely used in customer satisfaction surveys and marketing research.
ML – Machine Learning	AI subset that uses labelled and unlabelled data, statistical techniques and “trial and error” methods to give the systems the ability to learn and optimize processes without ever having to be coded specifically for the task.
MRI – Magnetic Resonance Imaging	Type of scan that uses strong magnetic fields to align the protons present in the body's water molecules in the same direction. It also sends out short radio waves to try to destabilize the alignment of the protons in the desired areas. When the protons realign themselves, they send radio signals that make it possible to determine the exact position of the protons in the body and the type of tissue in that area, depending on the speed at which the protons realign themselves. The signals from the millions of protons in the body are combined to create a detailed picture of the inside of the body in a non-invasive and painless process. MRI, which may or may not be used on different parts of the body, helps diagnose disease, monitor treatments, and evaluate the effectiveness of therapeutics.
Turner’s Syndrome	Genetic condition that causes health and developmental problems, including short stature, lack of ovarian development and heart defects, and lack of spatial awareness. It only affects females due to one of the X chromosomes being missing or partially missing.
Williams’ Syndrome	Rare genetic disorder resulting in pre- and postnatal growth retardation and manifested by short stature, varying degrees of mental weakness, pronounced facial features that typically increase with age, dental and heart problems, and high blood calcium levels.

1 Introduction

Key points:

- Developmental Dyscalculia is poorly understood, very common and interferes with personal and academic life in relation to mathematics.
- It makes sense to create an innovative framework that screens the student, adapts, and evolves accordingly.
- The study of dyscalculia is multidisciplinary and requires efforts and knowledge from different areas.
- There is lack of research and solutions on this topic.

1.1 CONTEXT AND MOTIVATION

Back to the beginning of times, mathematics was not always considered a beloved or well-accepted science, i.e., it was eternally despised or adored. Only a few found themselves in the middle of these extremes, a result that can be attributed to an individual's personality. Indeed, until Henschen (Henschen, 1926) and Kosc (Kosc, 1974), Mathematical Learning Disabilities (MLD) were unknown, especially Development Dyscalculia (DD).

DD, or simply *"dyscalculia"*, is a disorder that affects the acquisition of concepts related to the numerical magnitude and other related senses. Unfortunately, it cannot be diagnosed according to National Health Service standards, since there are no standard screening tests for Learning Disabilities (LD) specialists, and affects 6 to 7% of the population (Berch & Mazzocco, 2007). Moreover, it is often misdiagnosed (i.e., subject to medical malpractice) or not even considered when someone has difficulty grasping mathematical skills (Brian Butterworth et al., 2011; Mazzocco & Räsänen, 2013). As it is a permanent disorder that is inherent in the individual, it is advisable to make efforts as early as possible to mitigate the severity of the disorder (National Joint Committee on Learning Disabilities, 1991).

Besides, there is also a need to demystify the preconceived notion that people have towards the subject, i.e., mathematics. There is also a need for learning support systems that can help young children develop a proper understanding of mathematical concepts while informing educators about children's progress and their greatest difficulties. With the help of such tools, the teaching system can be reoriented and lead to better prospects in terms of students' performance in mathematics, physics, and chemistry examinations.

1.2 GOALS AND CHALLENGES

Given the problem outlined, the question naturally arises as to whether it is possible to create a cognitive approach to dyscalculia and, with the use of current and new technologies, an evolving approach to support the cognitive approach.

Therefore, this research aims to give a formal knowledge account of dyscalculia in children. It includes the identification of their difficulties and the analysis of the results of appropriate therapies. In order to support and test this thesis, the creation and development of a therapeutic and screening solution of dyscalculia addressed to children were carried out.

The goals and challenges include an intelligent and cognitive proposal of the dyscalculia status in Portuguese children under logical formalisms. It also includes the conception and design of an innovative and dynamic solution for the therapy and diagnosis of dyscalculia (Figure 1.1) in the form of a mobile game.

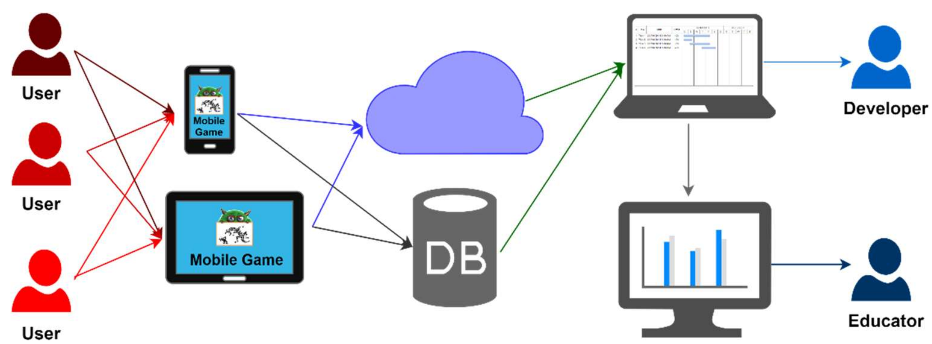

Figure 1.1 The game-based solution architecture *(Image by Author)*

To achieve the intended results and answers, the adopted methodology is the deductive-inductive one (Eisenhardt & Graebner, 2007), where several conceptual theories are analysed, followed by tests and practical solutions, as well as an attempt to reconstruct and add value to the conceptual structure considered at the beginning.

On a larger level, the free availability of the software of the game-based solution is envisaged for children, educators, and the public in general.

1.3 COLLABORATIONS

Efforts were made to establish partnerships and/or knowledge exchange between the Department of Informatics[1] and other departments and institutes of the University of Minho, as dyscalculia is multidisciplinary.

Therefore, the Research Centre on Child Studies[2] was contacted, a professor known for her studies in early literacy, reading comprehension and assessment in young children, and language acquisition. Thus, it was important to consider the assimilation process of learning in young children in order to properly construct and design tasks for children with dyscalculia. After a single appointment, the professor was able to redirect this research into other areas and shed light on the need to explore the construction of numbers in children, the genesis of logical structures and even mental operations.

On referral, a professor from the School of Psychology[3] was contacted as he specialised in the psychology of education. Although he had led several research projects, those related to mathematical learning and self-regulatory learning were more relevant to the situation. Thus, in a single session, the professor contributed by presenting a website created by a research team he led and aimed at children in 9th grade. This website includes an online game to help students develop their mathematical skills.

In addition, a professor with experience in educational technology was contacted from the Research Centre on Education[4] of the University of Minho. He has worked on several projects combining attractive technology and education, mainly for children. Therefore, his knowledge in this field helped in the re-design of the game-based solution proposed in this work. As part of the same research centre, another professor was contacted who has expertise in special education, particularly dyslexia. Her long experience in this field helped to make some contacts who also work with dyscalculia and recommended a Child Development Support Centre in Portugal – CADIn[5]. As part of this centre, she had contacted several technicians who are experts in special education and rehabilitation. She got a different perspective on how to approach and deal with children with dyscalculia and what tools are used in this centre.

Within the University of Minho network, contact was also made with researchers that were investigating the use of Magnetic Resonance Imaging (MRI), functional MRI (fMRI), and Electroencephalogram (EEG) to study the human brain, both its structure and function. A researcher from the Institute for Research in

[1] https://www.di.uminho.pt/eng
[2] http://www.ciec-uminho.org
[3] https://www.psi.uminho.pt/en
[4] https://www.cied.uminho.pt
[5] https://www.cadin.net

Life and Health Sciences[6] (ICVS) explained the recent studies on children's brains, focusing on special needs. He also addressed the use of EEG in minors to study learning disabilities. So far, no research projects have been initiated in Portugal using fMRI, MRI and/or EEG in children with dyscalculia.

On the other hand, the help of medical experts is relevant, at least when it comes to children with learning disabilities. In this sense, a child psychiatrist from the Department of Child and Adolescent Psychiatry and Mental Health[7] of the University Hospital of Porto was contacted and helped to consolidate the constructed state-of-art about dyscalculia in the northern part of Portugal.

Finally, the role of schools in the study of dyscalculia in children is extremely relevant. Therefore, two different primary schools from the north of Portugal were visited and contacted: one from a rural area and the other from an urban area. On the rural side, the school director and the first cycle coordinator not only provided insights into identifying and dealing with children with dyscalculia but also allowed the game-based solution developed in this work to be tested in two classes. On the urban side, the director of the primary school allowed testing the mobile game with children from this area. It helped to understand the educational process targeting children with learning difficulties and the level of the children.

1.4 Overview

This document is divided into six chapters that are interconnected. The first chapter introduces the topic and gives a contextualization. In this chapter, the proposed hypothesis is presented and, to support this outline, some goals and challenges have been drawn to be achieved.

The second chapter presents DD from a clinical point of view, looking at the concept from several scales. The origin, causes, symptoms, diagnosis procedures, and treatments are also analysed from the perspective of several experts on the subject.

In the third chapter, a cognitive outline of DD is proposed. It relies on Knowledge Representation and Reasoning (KRR) as a problem solver and works on energy imbalance. This approach also uses Artificial Neural Networks (ANN) and Case-Based Reasoning (CBR) as decision-making tools to give a formal representation of the disability, support the screening procedures and validate the therapeutic guidance.

The fourth chapter presents an evolving solution to support DD therapeutics and screening that establishes the theoretical foundations described in the previous chapter. This solution has in consideration all potential users who may benefit from it, and its design is described in detail in this chapter.

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6 http://icvs.uminho.pt

7 https://www.chporto.pt/departamento.php?cod=0B0B0P

In the fifth chapter is presented the materialisation of the solution proposed in the previous chapter. It entails the process of the design, implementation, and evaluation of the developed system.
Finally, the sixth chapter consists of the discussion of the developed work, the dissemination of the scientific research, the conclusions, and the unfolded possibilities of discovery.

2 Developmental Dyscalculia

Key points:

- Developmental Dyscalculia is a learning disability that exists on the individual from birth and cannot be treated, only attenuated.
- Its origin is attributed to different areas and affects the subject's cognition in dealing with numbers and quantities, spatial-temporal sense, naming directions, retrieving from memory.
- There is no standard test to detect dyscalculia.
- There are several tools that help to mitigate the degree of dyscalculia, but none is both standard and consensus.

2.1 CONCEPT

Dyscalculia was primarily defined as *"a structural disorder of mathematical abilities"*. Therefore, and through some consequent studies, it can be defined as a mathematical learning disability that affects the ability to perform operations and make the proper use of arithmetic. Frequently described as *"dyslexia or blindness for numbers"*, the dyscalculia is hard to be well-diagnosed, despite the incidence of 6 to 7% in the population. Nevertheless, it is relevant to distinguish the type of dyscalculia under analysis:

- The person in question suffered a trauma, like an injury or a stroke, and developed this difficulty in leading with numbers, then it is designated *"acalculia"*; and
- The disorder exists since birth, with the absence of accidents, then it is designated developmental dyscalculia since it will accompany the individual through ages. This last type is the most common and the one that is referred to as *"dyscalculia"* (Ferraz, Vicente, et al., 2016).

LD interfere in the individual's daily life, as a matter of social interaction, personal confidence, and professional opportunities. Hopefully, nowadays it is possible to remediate some of these LD, or even attenuate their severity, which helps to increase their quality of life. Development dyscalculia, or just dyscalculia, is a specific LD, belonging to the MLD group. The DD's name is because LDs exist from the conception of the individual. On the contrary, acalculia is what is designated acquired dyscalculia due to a brain injury or other accident (Ferraz et al., 2017). To a proper understanding of the concept (dyscalculia), it is wise to highlight its definition, i.e., dyscalculia is an *"inability to perform mathematical operations"*, which can be seen *"as development impairment in (...) mathematics"*. According to the Diagnostic and Statistical Manual-IV (DSM-IV), the screening criteria describes that the mathematical ability in the individuals is *"substantially below that expected, given the person's chronological age,* *measured intelligence, and age-appropriate education"*, which *"significantly interferes with academic achievement or activities of daily living that require mathematical ability"*. As noted in the various official definitions of dyscalculia, it may be subdivided into categories according to the type of affected fields or according to the brain's immaturity, which reflects on the severity of the disorder (Ferraz et al., 2017). Since this specific developmental disorder can be reflected in various areas of mathematic, dyscalculia may be set in six sub-areas taking into account the most affected areas (Ferraz, Neves, et al., 2016), namely:

- Lexical dyscalculia – troubles in reading mathematical symbols;
- Verbal dyscalculia – troubles in naming mathematical quantities, numbers and symbols;
- Graphic dyscalculia – troubles in writing mathematical symbols;
- Operational dyscalculia – troubles in performing mathematical operations and calculus;
- Practognostic dyscalculia – troubles in enumerating, manipulating, and comparing real objects and pictures;
- Ideagnostic dyscalculia – troubles in mental operations and understanding mathematical concepts.

Following Kosc (Kosc, 1974), he distinguishes that one may be faced with six distinct types of dyscalculia that comprehend the lexical one, which concerns troubles reading and understanding mathematical symbols and numbers, as well as mathematical expressions or equations. The children who have lexical dyscalculia can understand spoken views, but have trouble in writing or understanding them, presenting difficulty in reading symbols, such as numerals, and cannot understand them when they occur in number sentences or equations; verbal dyscalculia, in which children have problems in naming and comprehending the mathematical concepts exposed verbally. The children can read or write a number but cannot recognize them when they are revealed verbally – they present some strain when talking about mathematical concepts or relationships; graphical dyscalculia, manifested as no easy task when writing mathematical symbols. The children that have this type of dyscalculia can understand the mathematical concepts but cannot read, write, or use the mathematical symbols – difficulty with writing such icons including but not limited to numbers; operational dyscalculia, which presents itself with a difficulty to complete arithmetical operations or mathematical computations, both written and verbal. Someone with operational dyscalculia will be able to understand the numbers and the relationships between them but will have trouble manipulating numbers and mathematical symbols in the computational process; practognostic dyscalculia, which denotes difficulty in the process of translating their abstract-mathematical concepts to real and ideal aspects of human life. These children can understand mathematical concepts but have trouble in the listing, comparison, or manipulation of mathematical equations, demonstrating difficulty in translating their abstract mathematical knowledge into real-world actions or procedures; and ideagnostic dyscalculia, a snag when carrying out mental operations without using numbers to arrive at a solution or to understand concepts or ideas related to mathematics or arithmetic. These children have a challenging time remembering mathematical concepts once having learned them and difficulties with tasks that require an understanding of mathematical notions and relationships, such as identifying which sequence of numbers is larger or smaller (Ferraz, Neves, et al., 2019).

According to Romagnoli (Romagnoli, 2008), dyscalculia can also be classified according to the state of neurological immaturity, i.e.:

- A former state, related to the individuals that react favourably to therapeutically intervention;
- A second one, associated with the individuals who have other learning disabilities;
- A last one, linked to the individuals that feature an intellectual deficit caused by a neurological injury(ies).

This classification is rarely used (Barkley, 1982; Ferraz, Neves, et al., 2016).

2.2 Origin

Some experts on the human brain state that there is, indeed, a theoretical foundation of numerical cognition and development. Dehaene (Dehaene, 1992) proposed a triple-code model to justify this underpinning, assuming that, mentally, numbers are manipulated in three magnitude codes: Arabic, verbal and analogical, which is selected accordingly to the mental problem. He states that is only available the anagogical magnitude code to animals and preverbal infants.

On the other hand, von Aster and Shalev (Michael G von Aster & Shalev, 2007) propose four-magnitude codes: cardinality, verbal, Arabic and ordinality. The cardinality helps to conceive the meaning of *"number"*, the linguistic magnitude (verbal) is seen as a precondition in which the *"number"* is translated in *"words"*, the Arabic number system provides the symbolism, and the ordinality aids to develop working memory skills, to manipulate and expand mental number line. Further needed interventions can be based on this model, on the therapeutic and educational levels, constituting as a ground code (Figure 2.1).

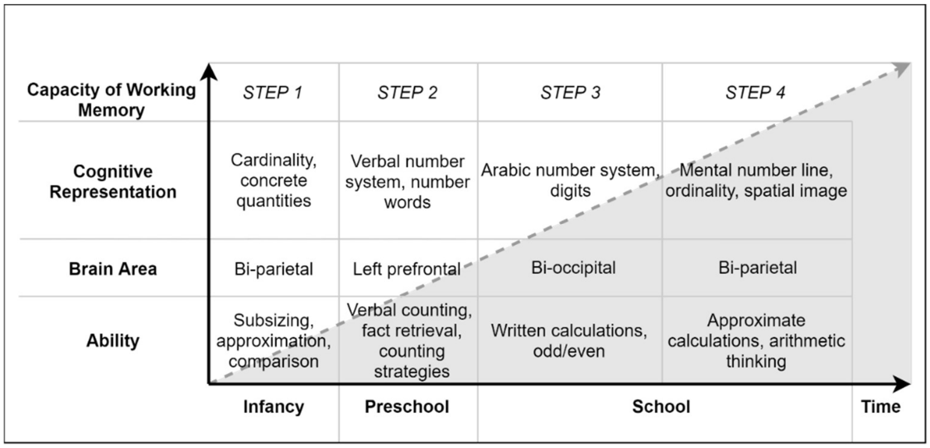

Figure 2.1 Four-step developmental model of numerical cognition, enhancing in grey the increasing working memory *(Image by Author)*

Data processing is very challenging and requires interaction between several brain functions. Children with dyscalculia show decreased activity in brain areas related to the neural network of number and quantity processing during activities of simple arithmetic tasks (Ashkenazi et al., 2012; Kucian et al., 2006). This may be due to a genetic deficiency of congenital core skills, which leads to an underdevelopment of some cognitive functions (Brian Butterworth, 2005; J. Campbell, 2005; Michael von Aster et al., 2007). For example, the association between an Arabic number (e.g., *8*) and the equivalent set is normally activated spontaneously. Experimental studies have shown that this is not the case in people with dyscalculia (Rubinsten & Henik, 2006): for them, numbers seem like *"meaningless words"*.

Other fundamental math skills are compromised when there is a calculation error, like quick estimates of small quantities, quantities and numbers comparison, identifying and reducing numbers, and mental number line development (Gaupp et al., 2004; Karin Landerl et al., 2004; M. von Aster et al., 2005). It was also reported deficits in general cognitive functions, such as memory, processing speed and visual-spatial functions.

In terms of causes for this disorder there exist several approaches, viz (Ferraz et al., 2017).

- Geneticists believe on a heritage base, especially when their parents present the same LD or genetic diseases (e.g., Turner's, Williams', and Fragile X syndromes);
- Linguistics suggest that there is a misapprehension of the language and its formalities, affecting the understanding of mathematics concepts;
- Neurologists think that the responsible areas for the number of senses in the brain have malformations or are not entirely developed;

- Pedagogues interpret that DD is a consequence of an inefficient teaching-learning system;
- Paediatricians consider medical conditions as causes, like poor intrauterine growth or exposition to harmful substances or high levels of lead during the pregnancy;
- Psychologists state that environmental factors can contribute to DD, like poor education or traumatic episodes. Unfortunately, DD is usually associated with other disorders like attention deficit, hyperactivity, and dyslexia, which confuses the diagnosis and therapeutics.

There are studies with family, twins included, that evidence a genetic component of dyscalculia (Alarcón et al., 1997; Alom et al., 2019; Docherty et al., 2010; Petrill et al., 2012; Ruth S Shalev et al., 2001). Studies on neurophysiological correlate using MRI, fMRI and Event-Related Potentials (ERP) have identified regions of the brain that are associated with different computing activities (overview in (Vogel & Ansari, 2012)). For example, it has been observed activity of the Intraparietal Sulcus (IPS) of the two hemispheres on the ability to distinguish numerical voids between numbers. On the other hand, the activation of the left angular gyrus is related to the linguistic handling of numbers in computational processes.

A few studies with children with dyscalculia that reinforced the importance of IPS in comparing and reckoning quantities (Kucian et al., 2006), stated less activation in a numerical estimation task (e.g., *7 + 2 = 3* or *8*), meaning underdevelopment of their number sense (Ferraz, Vicente, et al., 2016). It was found that children with dyscalculia were significantly reduced in a functionally planned number (e.g., *4 + 3 = 9* or *6*). Additional studies confirmed the relation between basic numerical abilities and lower IPS activation (Kaufmann et al., 2009). Regarding the medical field, there have been neurological studies aiming to detect which brain areas are responsible for the numerical sense, using several types of exams, like fMRI and EEG.

Finally, the coexistence with comorbid disorders in most cases must not be disregarded. Attention-Deficit/Hyperactivity Disorder (ADHD) and Dyslexia are the most common, having an incidence of 40% and 25-40%, respectively (Karin Landerl & Moll, 2010).

2.3 SYMPTOMS

Naturally, several previous studies associated language to word problem-solving, but never to calculations (e.g., (Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2008; Fuchs, Geary, Compton, Fuchs, Hamlett, & Bryant, 2010; Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, et al., 2010; Fuchs et al., 2013)). However, language has a more significant role than forecasted, since it is the main method to give instructions to children in early elementary grades, such as mastering the number-word sequence and learning to count, and to articulate processes, such as verbalizing an answer to a calculation problem (e.g., *1 + 1 = ?*). Moreover, the level of a child's vocabulary is directed related to basic calculation competence (Durand et al., 2005; Geary, 2010; Hecht et al., 2001), as language aptitude affecting more basic calculation competence than cognitive skills like nonverbal reasoning, quantitative skills and working memory (Cowan et al., 2005; LeFevre et al., 2010). Hence, this suggests that children with strong language capability may improve deeper comprehension of calculation concepts when compared with those with poor language ability. As theoretical and practical understandings influence each other, better understanding of calculation concepts may easing the procedural calculations (Rittle-Johnson et al., 2001; Rittle-Johnson & Siegler, 1998).

Children with dyscalculia present difficulties in comparisons of sets (more/less) and number (larger/smaller), identifying and writing numbers, and counting skills (Gaupp et al., 2004; Karin Landerl et al., 2004). Regarding this last one, a few researchers stated that these individuals cannot count in steps larger than one or backwards, and can only achieve that task if the object is placed in a certain order (Gaupp et al., 2004; Moser Opitz et al., 2004).

Another warning sign is the difficulty in understanding the decimal system, leading to number turners (e.g., *"fifty-eight"* is written *85*), position errors (e.g., *"one hundred two"* is written *1002*) and bundling errors (e.g., *"How many bundles of ten can be made with 56 tiles?"*) (K Landerl & Kaufmann, 2008; Moser Opitz et al., 2004). Even though children between the ages of 8-9 years old tend to mistake less, it is observed increased response time in reading and writing Arabic numerals (e.g., *38* is written *"thirty-eight"*, and vice-versa) (Karin Landerl et al., 2004).

Additionally, children with dyscalculia present a substantial impairment in learning and retrieving arithmetic facts (e.g., *3 + 3 = 9*) (Gaupp et al., 2004; Geary et al., 2012; Geary & Hoard, 2001; Gersten et al., 2005). In other words, they fail to memorize and later retrieve the results of basic computational tasks. Consequently, these children tend to use counting strategies, often using their fingers, to solve simple tasks longer than expected, which affects them negatively, as it requires a lot of time and it is error-prone (Gonzalez & Espinel, 1999; Moser Opitz et al., 2004; A. K. Patwary et al., 2019; Pixner & Kaufmann, 2008).

The mathematical procedure - missing or erroneous thoughts on the required computational steps to perform specific tasks it is also other symptoms presented by children with dyscalculia (Geary & Hoard, 2001). The procedure for multi-digit subtraction tasks, e.g., is, among others, a mathematical procedure that children learn intuitively but do not perceive due to their deficits in recognizing quantities and numbers (decimal system included). Therefore, although the need to adapt the procedures for a new task, they perform the same computational steps and in the same order (Moser Opitz et al., 2004).

Beyond their educational development, children with dyscalculia can also present psychological abnormalities (Auerbach et al., 2008; Prior et al., 1999; M. G. Von Aster, 1996), like attention deficits. Some abnormalities can happen independently of this disorder, others may be a response to school problems and failures, as depressive traits, aggressive conduct or anxiety (Auerbach et al., 2008; Huntington & Bender, 1993). Commonly, these children develop specific anxieties towards mathematics and assessments, in general, that can last for years and affect other subjects and/or even the acceptance of the problem (Krinzinger & Kaufmann, 2006; Rubinsten & Tannock, 2010).

The stability of symptoms presented by children with dyscalculia is relatively low in the first two school years, improving towards the end of the fourth year (Ruth S Shalev et al., 2005). Around a third of children who present poor computational skills in the first grade, achieve the expected performance a year later (Geary & Hoard, 2001). An Israeli longitudinal study analysed the computational ability of children who were diagnosed with dyscalculia in the fifth grade, and later when they were in the eleventh grade. Six years later, most of the group presented difficulties in numeracy – 95% were on the weakest quarter of their age group, but only 40% still met the criteria for having a computational disorder. It reinforces that deficits in arithmetic fact knowledge are a stable symptom of dyscalculia (Gaupp et al., 2004; Geary et al., 2012). Regarding the first school years, an American study observed primary school children between the middle of their second grade until the end of their third grade. As a result, it was stated that children with low numeracy skills presented little progress in retrieving arithmetical facts, while the other children have their knowledge of arithmetic facts increased (Jordan et al., 2003).

2.4 Consequences

Dyscalculia is irreversible, i.e., the disorder cannot be treated, but it can be worked out in order to decrease its degree of severity (Ferraz, Neves, et al., 2016).

Therefore, it is primary that a diagnosis must be obtained as soon as possible, in order to start an early therapy rather than later therapy. It is already known that children begin acquiring knowledge from a very tender age. So, as soon as they start their first school years, it is the best time to proceed to a screening test and, with a confirmed diagnosis of dyscalculia, invest in a directed therapy. This will prevent the children from being delayed on school performance and follow their classmates.

Basic mathematical skills must be mastered so children can acquire the basic arithmetic methods, as well as mastering the counting skills, i.e., know the numbers, the ability to assign numbers to correspondent objects, and associating mentioned numbers to counted quantities (Krajewski, 2008). Pre-school skills like counting and number knowledge (e.g., *"Which number is greater? 5 or 4?"*), according to longitudinal studies, can forecast the later computational power (Aunola et al., 2004; Locuniak & Jordan, 2008), and, therefore, weaker mathematical basic skills are often observed in children with lower computational reasoning in kindergarten (Gersten et al., 2005; Krajewski & Schneider, 2009; A. Patwary et al., 2020; Stock et al., 2010; Weißhaupt et al., 2004).

A later intervention will certainly affect social development as much as the cognitive one. When growing up, children start to notice each other and, naturally, compare. When abnormalities are presented, comparisons tend to negatively affect the acceptance of each other, leading to depressive symptoms, in a worst-case scenario. On the other hand, bullying can be a daily basis, affecting how children define their social interactions. Reaching forward on the timeline will delay their education level, providing difficulties in integrating into the society and working environment.

Social Entropy may be understood as the force that not only strives to dissolve the higher or medium institutions of any society, namely by reducing the individual learning impairments to its component forms. Being left unchecked, it may act as a destructive force that may cause lawlessness, riots, unpredictability, and social concern. Our societies are dependent on what people are learning and following, either looking at the norms in terms of getting up, getting to work, getting to learn, or not stealing or vandalizing property or hurting others. Indeed, while the underlying cause of Learning Impairments (LI) remains a mystery, experts agree that it is neurologically based, meaning that it results from differences in the way that the brain is wired and processes information. Although LI cannot be outgrown, it can be compensated and remediated. The better one understands the nature of individuals' learning difficulties, the greater the chances of helping them achieve scientific and social success (José Neves, Ferraz, et al., 2019).

One's approach focuses on the examination of sensory data from individuals diagnosed with specific LI, whereby the perception of visual stimuli is of the utmost importance, i.e., special attention should not only be devoted to the perception of the brain's physical characteristics but also their thoughts and emotions. Those are set in terms of an environment that brings together individuals and technology, interacting and producing deeds and information that would not be possible to extract without interaction among all. Indeed, in terms of issues that may affect LI, the focus should be on economic and social policies, i.e., attending to the conditions under which an individual lives and learns. This integrated perspective entails a flexible approach to emotional behaviour, i.e., the expressive comportment, the action tendencies and the cultural influence and feelings, all of which have their place in understanding the individuals' sensitive changes. LI may involve difficulties with mathematical skills (e.g., addition, subtraction, multiplication) and perceptions (e.g., sequencing of numbers). Remembrance of mathematical facts, perceptions of time, money and musical may be also induced. However, language and other skills may be improved, such as severe positioning capabilities, problems with reading cards, punctuality, grappling with mechanical processes, dealing with abstract concepts like time and direction, schedules, the course of time or the sequence of past and future events (Ferraz, Neves, et al., 2019).

2.5 Screening

As mentioned in the previous section, early intervention is an asset to children with dyscalculia.

Regarding the identification of the risk of having dyscalculia, or another learning impairment, there are several test methods. Targeting preschool-age children is highlighted ZAREKI-K (M von Aster et al., 2009) and TEDI-MATH (Mann et al., 2013). On an indirect way to evaluate this risk, it can be used playful exercises, e.g., dice or board games, as it helps to perceive the numerical knowledge and efficient counting strategies of the children (Griffin & Case, 1997; Siegler & Ramani, 2008; Wilson et al., 2009).

Indubitable, the implementation of a standardized computing test is the core of diagnostics. A divergence of the computing power, when compared with the standard for that children's age, indicates the need for the diagnosis of a computational disorder, as recommended by ICD-10 (Dilling et al., 2011). In parallel, when it is observed an age discrepancy, it should be performed an intelligence test to verify the Intelligence Quotient (IQ) discrepancy, as well.

Several standardized test methods differ in terms of contents, execution (solo or group test, test's duration) and standardization's quality, and can examine computing power. They can be primarily based on the mathematics curricula (e.g., DEMAT series) or focusing on the known impaired competencies of the computational disorder – criterion-oriented test procedures (e.g., ZAREKI-R (Moser Opitz & Ramseier, 2012; Michael von Aster et al., 2007)). Ideally, it must be chosen a test method that not only compiles the teaching materials, but also assembles the main symptoms of the computational disorder, i.e., a test method that covers both fields, like Heidelberg Computer Test (HRT 1-4) (Haffner et al., 2005) for primary school children, and BASISMATH 4-8 for the consequence school grades (Moser Opitz & Ramseier, 2012). The Dyscalculia Screener, the most common commercial screener used in the United Kingdom, has been a key screener for children with dyscalculia and wide-used, by its focus and its applicability in the school environment (Brian Butterworth, 2005).

It should be noted that not all difficulties in mathematics and dealing with numbers may bridge into a computational disorder. Children whose impairments are a consequence of environmental factors (inadequate instruction, support's absence, etc.) do not meet the criteria for a computational disorder. Likewise, deficits in hearing or seeing, neurological, mental, or psychiatric disorders may cause difficulties in computing but are conditioned to discard the diagnosis.

Such as other disorders and accordingly to ICD-10, the basic criterion for diagnosing dyscalculia is the child's performance when it differs from the general intelligence, age and grade level attended. A profiled test method is the more accurate one, since the different types of tasks are judiciously selected, and the performance is suitable to the individuals. WISC-IV is one of these kind of test batteries (Franz Petermann & Petermann, 2011).

Although being a measure for some, the IQ-discrepancy criterion has generated a lot of controversies, not being recommended by several scientific literature authors (Ehlert et al., 2012; Moser Opitz & Ramseier, 2012; Michael von Aster et al., 2007). The methodological problems are one of the reasons since many tests include subtests that record the computational power, putting children with low computational power at a disadvantage. On the other hand, the most common tests approach mathematical competencies differently, creating different results for the same individual (Ehlert et al., 2012). Therefore, the IQ-discrepancy criterion is also dependent on the computing test used, subjecting it to a bias. Additionally, numerically weak children who meet the IQ-discrepancy criterion have the same difficulties in arithmetic as numerically weak children who do not meet the criterion. Both groups of children apply similar problem-solving strategies and provide comparable services, making the IQ-discrepancy criterion a not so good approach (Akter et al., 2020; Ehlert et al., 2012; Gonzalez & Espinel, 1999; Moser Opitz et al., 2004; A. Patwary & Omar, 2016). Also, there is evidence stated in an American longitudinal study that a diagnosis of dyscalculia is more stable over time when based on the age discrepancy criterion instead of the IQ-discrepancy criterion (Mazzocco & Myers, 2003).

Children from second and third grades onwards with affected skills must be, usually, presented in clinical practices (Claus Jacobs et al., 2009), because they can hardly recompense their arithmetic deficits with growing school requirements by other neuropsychological competencies (Franz Petermann & Toussaint, 2009; Tischler et al., 2010). According to AWMF (Association of the Scientific Medical Societies in Germany, *"Arbeitsgemeinschaft der Wissenschaftlichen Medizinischen Fachgesellschaften"* in German) (Becker et al., 2011), the diagnosis of an arithmetic disorder can be built based on a dynamic procedure of four levels.

On the first level, it is analysed the extension of deficits in language, motor skills and visual information processing with an anamnestic initial interview with the primary caregivers of the child. It is intended to sketch a profile, giving attention to the child's milestones (or lack of them) from early childhood development and family environment (social, financial, and emotional conditions).

On the second level, it is requested to primary caregivers and educators to complete problems checklists, providing information about the child's socio-emotional status. It is implemented a complex intelligence test that allows to characterize the cognitive performance level of the child and trace the diagnosis of potential partial performance disorders (such as attention deficit disorder, working memory disorders and the ability to learn and remember) (Franz Petermann & Petermann, 2011). A commonly used test is the mosaic test, which targets disorders of visual information processing (i.e., visual-cognitive, spatial-perceptive, spatial-cognitive and spatial-constructive disorder) (F Petermann, Knievel, & Tischler, 2010; Becker et al., 2011). It is also recorded the behaviour and motivation status of the child along with the examination procedures. The implemented method can use a single and individual test (M von Aster et al., 2009; C. Jacobs & Petermann, 2005) or group test; either choice must have in account the purpose: a quick diagnosis (C. Jacobs & Petermann, 2005) or a slower diagnosis but with underlying causes known and/or useful information to plan the therapy (Claus Jacobs & Petermann, 2012). The most suitable is the class-level standardized individual test procedures that approach arithmetic and basic mathematical skills, followed by a reading and spelling performance assessment, for a more differentiated procedure and complete diagnostic clarification. The third level is reached if the suspicion continues after the second level.

On the third level, given the test results and collected information, it is outlined the potential disorder, whether it is neuropsychological, emotional, or behavioural. It is recommended the implementation of disorder-specific questionnaires and neuropsychological diagnostics (Becker et al., 2011).

On the fourth level, it is discussed the diagnosis with the primary caregivers, along with an interpretation of the test results and possibilities of therapy (costs, requirements, methods, etc.).

The use of EEG and/or fMRI to screen a child with dyscalculia can be a possibility in the future, but not now. Currently, there are not enough studies that identify a specific trace of dyscalculia on the brain, like detecting a mass to identify a tumour. Likewise, the costs, bureaucracy and ethical issues associated to perform a neurological exam in a child are an obstacle to taking this as a support screening tool.

There are other screening tests for dyscalculia that are non-official, subjective, paid and online. The current method used by psychologists and psychiatrists implies the use of a screening standard test for dyslexia and, with the results, infer the existence of dyscalculia in the individual. Although being a non-standard method to screen dyscalculia by psychologists, it is understood that it must be done and recommended by experts, especially when the child presents weak responses to mathematical exercises. Since there are many classification processes, requiring multiple tests and exams, as well as experts from different fields (e.g., psychology, paediatric, education) to perform the assessment, there is no standard test to screen this disorder (Ferraz et al., 2017).

2.6 THERAPEUTICS

When a child is diagnosed with dyscalculia, it is outlined a therapeutic plan. However, not all the children that have dyscalculia get to be diagnosed – most of the cases are dealt with as if they have low IQ.

The majority of the therapeutic plans involve the methodical habit to solve exercises regarding dyscalculia's issues, like exercising memory, counting amounts, and so on, leading the subject to evolve the child weaknesses (Ferraz, Vicente, et al., 2016). Most of the therapeutics use neuroplasticity to re-educate the individual's cognitive flaws with brain training, which can be applied in didactic support tools, either manual or digital. Alternative therapies consist of adapting the teaching system *"Special Needs in Education"* (Ferraz et al., 2017). Whether a therapeutic plan is defined, all options have in account the mitigation of dyscalculia's effects on the daily basis of the child.

Neuroplasticity is the ability of the nervous system to reshape and readapt when subjected to new experiences, on the structural and functional levels of neural development. Therefore, when individual works and tries to learn the same concept over and over, even using different strategies, he/she may succeed thanks to this neuronal process. This method explains what happens to children with dyscalculia when they try to acquire the same concept repeatedly.

So, regarding the support tools, it can be used books to guide and work the child's neuroplasticity, like Dyscalculia Toolkit (Causier, 2008), Dyscalculia Guidance (Bryan Butterworth & Yeo, 2004), or

Dyscalculia Solution (Emerson & Babtie, 2014); board games, like Cuisenaire rods, dice, domino, lotus or cards; worksheets and handwork kits; software, or the combination of several.

There has been an evolution towards technology, nevertheless, there are no records of free mobile applications directed to dyscalculia and MLD. Some of the most completed support systems are Dyscalculia Screener (Brian Butterworth, 2003), DynamoMaths (Esmail, 2010), LearningRX (Carpenter et al., 2016), Calcularis (Rauscher et al., 2016), Cognifit (Minchew, 2016), Number Sense (Education.com, 2016), and Number Race (Unit, 2004). These are technologies directed for enhancing numerical cognition for children with dyscalculia, and most of them have a set of training and test instruments, giving a detailed analysis of the child's performance. Except for the last two mentioned, these tools are online paid software to help children with dyscalculia, being difficult for many people.

3 A Cognitive Approach to Dyscalculia

Key points:

- Artificial Intelligence has contributed to decision-making solutions in several fields, improving workflow and helping to achieve the desired outcomes.
- A formal representation of problems for the application of reasoning techniques has a positive influence on clinical decisions.
- A cognitive approach to dyscalculia screening and therapeutics represented under formalisms led to the application of reasoning algorithms with high accuracy levels, such as KRR and CBR.
- This approach helped create decision-making models that aid in the screening and therapeutics of dyscalculia.

3.1 KNOWLEDGE REPRESENTATION AND REASONING

Artificial intelligence (AI) is widespread nowadays, leading to a more computerised and digital world. This leads to a more computerised and digital world. As a result, decision-making solutions are more technological and rely on the help of a machine to speed up and simplify the process. In the medical field, medical professionals rely on machines to observe deeper and speed up a diagnosis (Saltzman, 2015). AI targets problem solvers that mimic the highest level of human expertise on a given task - expert systems such as medical diagnostics.

KRR is the branch of AI that represents information about the world in a way that computer systems can understand and use to perform complex tasks. It incorporates results from psychology about human behaviour in problem-solving strategies and knowledge representation, and from logic about automating reasoning so that it can design formalisms that make it easier to design complex systems and apply rules and relationships of sets and subsets. Applications of KRR include the representation of semantic networks, system architectures, frames, rules and ontologies, machine inference, theorem provers and classifiers, natural language dialogue construction, medical condition diagnosis, etc. (Lakemeyer & Nebel, 1994).

It is undeniable that AI is present in many fields. KRR is one of these areas, a fundamental one that is essential to design, develop and/or build an expert system to simulate human performance.

3.2 COGNITIVE REASONING

3.2.1 ARTIFICIAL NEURAL NETWORKS

ANNs are computing techniques inspired by studies of the human brain and nervous system. They are mathematical models that simulate such systems as are understood today. ANNs were first introduced in 1943 (McCulloch & Pitts, 1943), which it was presented a simplified model of the neuron (artificial neuron or node) – the basic unit of an ANN –, that has two states: active or inactive, corresponding to the true or false in logic clauses, as well as zero or one in Boolean algebra.

Significant developments occurred until 1969, with the emergence of single-layer perceptron (Rosenblatt, 1958) originating the feed-forward neural network. However, some limitations (e.g., the fact that a single-layer perceptron cannot solve the XOR problem) led to a decline in interest in ANNs between 1969 and 1986. The above drawbacks were addressed in the 1980s and research on ANNs resumed with the advent of the back-propagation algorithm in 1986 (Rumelhart et al., 1986) and subsequent stimulation through the development of numerous fast gradient-based variants (e.g., RPROP) (Riedmiller, 1994). Despite the evolution of concepts, one of the main characteristics of ANNs is their ability to learn. It is important to note that ANNs are not traditional computer programs. One learns from examples through a process designated training, in which ANNs self-organise to adjust an internal set of parameters (i.e., synapses weights) that are used to collect the information contained in the data.

Compared to traditional methods, ANNs handle imprecise and/or incomplete data, provide approximate results and are less prone to outliers. Moreover, it is not necessary to accept constraints or know the relationships between variables a priori. Multi-Layer Perceptron is one of the most widely used ANN architectures, where neurons are layered and only forward connections exist (Haykin, 2009). Multi-Layer Perceptron design is typically done by trial and error with an ascending approach, starting with an initial architecture that is adjusted to minimise internal error (e.g., Mean Square Error) (Cortez et al., 2004; Haykin, 2009).

3.2.2 Case-Based Reasoning

CBR method of problem-solving represents an act of finding and reasoning about a solution to a given problem based on consideration of similar past problems by reprocessing and/or adapting their data or knowledge (Aamodt & Plaza, 1994; Richter & Weber, 2013). In CBR, the cases are stored in a Case-Based, and those cases that are similar (or close) to a new problem are used in the problem-solving process.

There are several examples in the literature of the use of CBR as a problem-solving method for problems in Medicine. Different researchers have reviewed more than thirty CBR systems/projects (Begum et al., 2011; Blanco et al., 2013) and found that CBR is widely used in the medical field, including disease diagnosis, classification, treatment and management. Indeed, the CBR approach used in mental health problems to predict the effect of treatments of patients with anxiety disorders showed 65% correct predictions in the absence of similarity constraints, while for the scenario with similarity constraints (i.e. under the condition that prediction was based only on cases with a similarity of at least 0.62), accuracy increased to 80% (Janssen et al., 2015).

Another study presents a fuzzy ontology based semantic CBR system for a decision support system to answer complex medical queries related to a semantic understanding of medical concepts and dealing with vague terms in diabetes diagnosis. The proposed system has an overall accuracy of 97.7%, which is higher than the accuracies obtained with other artificial intelligence-based tools such as a k-nearest neighbour, with k = 3 (68.3%), decision trees (90.0%), support vector machines (76.7%), Bayesian classifiers (76.7%) and ANN (71.7%) (El-Sappagh et al., 2015).

A study by Shen (Shen et al., 2015) combines CBR and multi-agent systems. The multi-agent architecture aims to address the full cycle of clinical decision-making, i.e., adaptable to different medical aspects such as diagnosis, prognosis, treatment, and therapeutic monitoring of gastric cancer. In the multi-agent architecture, the ontological agent type uses the knowledge domain to ensure the extraction of similar clinical cases and provide treatment suggestions to patients and physicians. CBR, in turn, is used to store and retrieve experience data to solve similar problems (Shen et al., 2015).

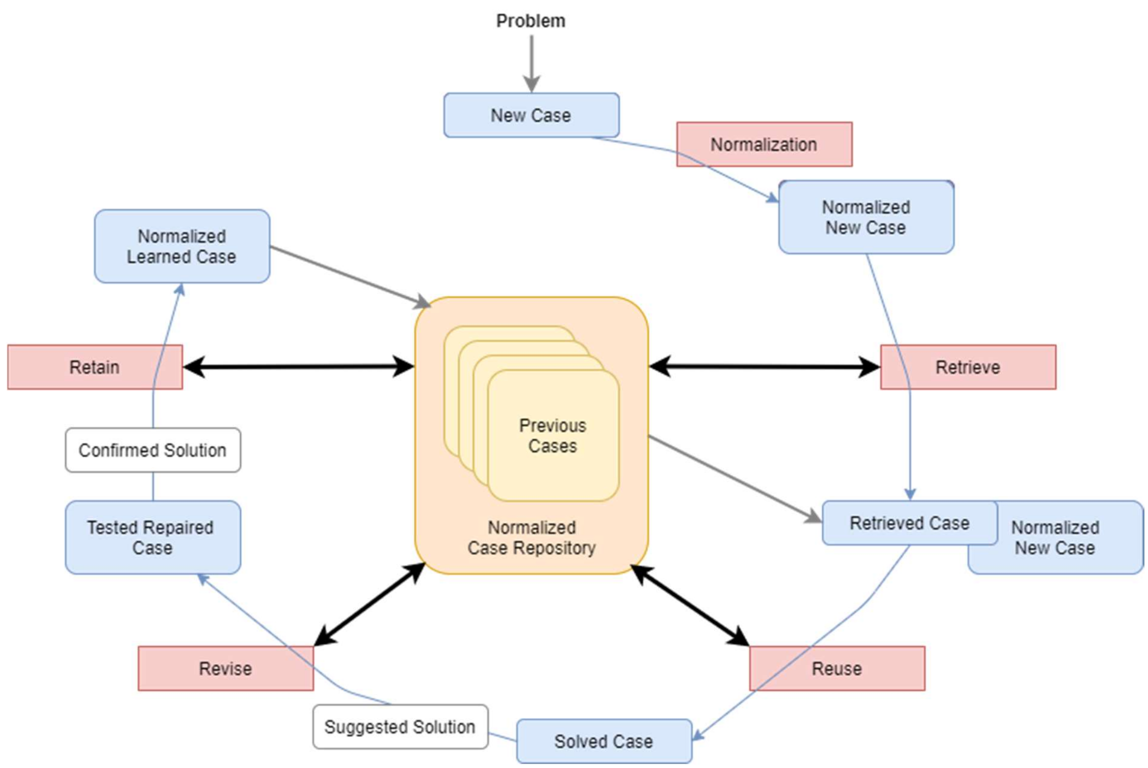

Figure 3.1 Typical CBR cycle *(Adapted from* (Ferraz, Vicente, et al., 2016)*)*

The typical CBR cycle (Figure 3.1) represents an initial phase in which the problem is described and characterized. The new case is defined and, after a *Normalization* phase, used to retrieve one or more cases from the repository. In the *Retrieve* phase, it must be considered that the cases with a higher degree of similarity are retrieved. Then, in the *Reuse* phase, the case is solved, and a suggested solution is obtained by merging the new case with the retrieved ones. The suggested solution is adapted (reused) to the new case, but it is important to get feedback from the user to validate the automatic adaptation. Therefore, in the *Revise* phase, the suggested solution is tested and, if necessary, modified, resulting in a tested repaired case. This is an iterative process as the solution needs to be tested and adjusted while it is inconclusive. In the *Retain* (or learning) phase, the case is learned and the repository is updated with the normalized learned case (Aamodt & Plaza, 1994; Richter & Weber, 2013).

Despite the promising results, the typical CBR system is neither complete nor adaptable enough to be used in all domains. For example, the user cannot always choose the similarity methods to use, and the ones used by the system are not always the more appropriate. Furthermore, in real-world problems, not all the necessary information is available and the CBR system cannot deal with unknown, incomplete, or self-contradictory information. To fulfil these gaps, an extended version of the CBR cycle was induced (Figure 3.2).

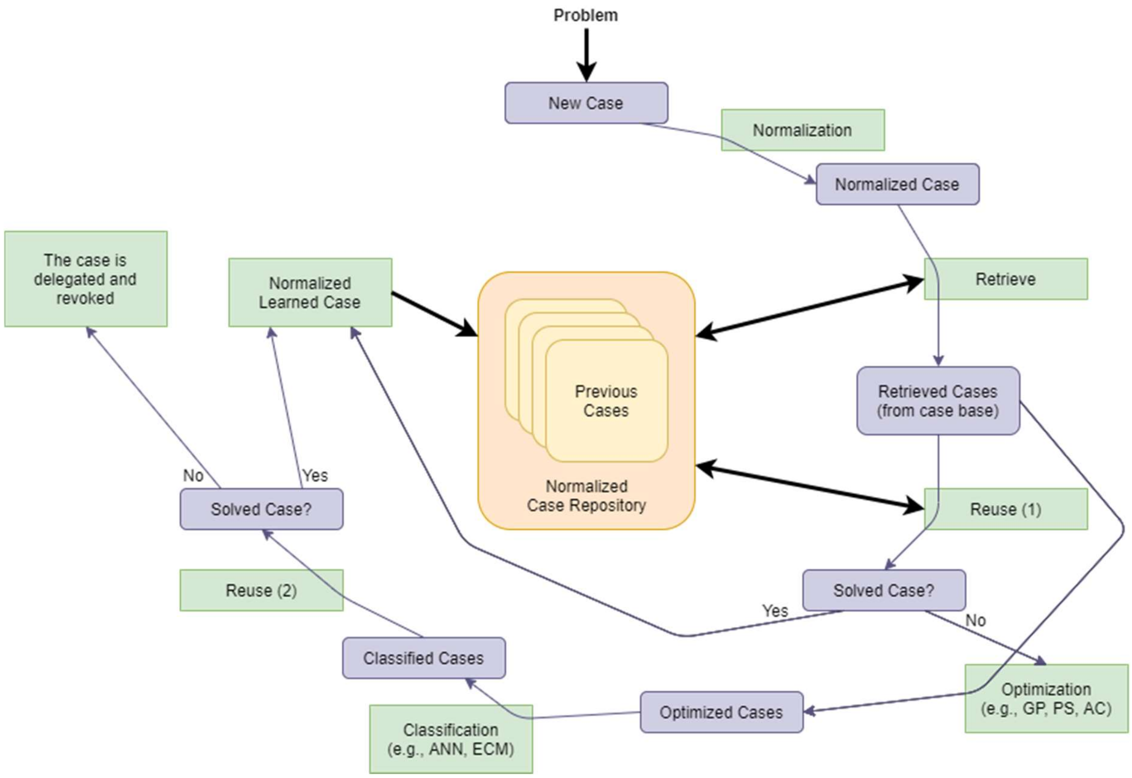

Figure 3.2 Extended CBR cycle *(Adapted from* (Ferraz, Vicente, et al., 2016)*)*

The extended view of the CBR cycle is equal to the typical cycle in the first three phases. With the retrieved cases from the repository, a suggested solution is given in a *Reuse (1)* phase. The metrics Quality-of-Information (QoI) and Degree of Confidence (DoC) of the case are considered. If that suggested solution solves the case, the case is learned, and the repository is updated. Otherwise, the suggested solution enters in an *Optimization* phase, where some algorithms such as Genetic Programming (GP) (José Neves et al., 2007), Particle Swarm (PS) (Mendes et al., 2003) or Ant Colony (AC) (Mendes et al., 2004), can be applied to optimize and generate a set of cases. In the Optimization phase, the generated set of new cases is used in the diagnostic that match the invariant:

$$\bigcap_{i=1}^{n} (A_i D_i) \neq \emptyset$$

which means that the intersection of the values ranges of the attributes in the cases that form the Case-Based repository, and that was selected as a first approximation to solve the problem (A_i), and the cases to be generated (D_i) must not be empty. n stands for the cases that were selected from the Case-Based repository. Then, in the *Classification* phase, the cases are classified using algorithms such as ANN (Haykin, 2009; Vicente et al., 2012) and the Eindhoven Classification Method (ECM) (Pereira et al., 2019). In the *Reuse (2)* phase, the classified cases are evaluated and, if the new suggest solution solves the case, then this case is learned and the repository updated; if the new solution does not solve the case, the case is delegated and revoked.

3.3 Energy Degradation

The problem-solving methodology presented is based on thermodynamics and aims to describe KRR practices as a process of energy degradation (Fernandes et al., 2019; José Neves, Maia, et al., 2019; Wenterodt & Herwig, 2014). To explain the basic rules of the proposed approach, the First and Second Laws of Thermodynamics are considered, which state that a system moves from one state to another over time. The first, also known as the law of conservation of energy, states that the total energy of an isolated system is constant, i.e., cannot change over time. This means that energy can be converted but cannot be generated or destroyed. The latter deals with entropy, a property that quantifies the ordered state of a system and its evolution. These properties fit the proposed vision of KRR practices, as they must be understood as a process of energy degradation. Indeed, it is assumed that a data item is in an entropic state whose energy can be decomposed and used in the sense of degradation, but never in the sense of destruction, viz.

- *exergy*, sometimes designated *"available energy"* or more precisely *"available work"*, is that part of the energy that can be arbitrarily used by a system after a transfer operation, or in other words, giving a measure of its entropy. In Figure 3.3 it is given by the grey-coloured areas;
- *vagueness*, i.e., the corresponding energy values that may or may not have been consumed. In Figure 3.3 are given by the grey-coloured areas with spheres;
- *anergy*, which stands for an energetic potential that was not yet consumed, being therefore available, i.e., all the energy that is not exergy. In Figure 3.3 it is given by given by the grey-coloured areas with asterisks.

On the other hand, there are many approaches to KRR using the epitome of Logic Programming (LP), namely in the areas of Model Theory and Proof Theory. In this approach, the Proof Theoretical methodology for problem-solving has been adopted and expressed as an extension of the LP language (Ferreira Maia Neves & Neves, 1984) to set in an LP approach to the assessment of Dyscalculia Symptoms in Children (DSC). Under this setting, a DSC program is based on a finite set of clauses in the form presented in Program 3.1.

$\{$

$\neg p \leftarrow not\ p, not\ exception_p$

$p \leftarrow p_1, \cdots, p_n, not\ q_1, \cdots, not\ q_m$

$?\,(p_1, \cdots, p_n, not\ q_1, \cdots, not\ q_m)\ \ (n, m \geq 0)$

$exception_{p_1}, \ \cdots \ , exception_{p_j}\ \ (0 \leq j \leq k), \ \ being\ j\ and\ k\ integer\ numbers$

$\}$

Program 3.1 The archetype of a logic DSC program

The first sentence denotes the closure of the predicate, *“,”* denotes “*logical and*”, while *“?”* is a domain atom denoting *“falsity”*, the p_i, q_j, and p are classical ground literals, i.e., either positive atoms or atoms preceded by the classical negation sign ¬ (Re:Work, 2016). Indeed, ¬ stands for a strong statement that speaks for itself, while not denotes negation-by-failure, i.e., a failure to prove a particular statement because it has not been explicitly declared. According to this line of thinking, a set of abducibles is present in every program (Kakas et al., 1998). These are given in the form of exceptions to the extensions of the predicates that make the program, i.e., clauses of the form, viz.

$$exception_{p_1}, \ \cdots \ , exception_{p_j}\ \ (0 \leq j \leq k), \ \ being\ j\ and\ k\ integer\ numbers$$

that denote data, information or knowledge that cannot be excluded. On the other hand, clauses of the type, viz.

$$?\,(p_1, \cdots, p_n, not\ q_1, \cdots, not\ q_m)\ \ (n, m \geq 0)$$

are invariants that allow specifying the context under which the universe of discourse is to be understood (Fernandes et al., 2019; José Neves, Maia, et al., 2019).

3.4 DYSCALCULIA'S DATA ACQUISITION AND PROCESSING

In order to collect information on DSC, a questionnaire of sixteen questions was considered, from which seven (7) items were taken as examples, viz.

- *Q1 – Does the child find it difficult to learn to count?*
- *Q2 – Does the child find it difficult to understand money, and has difficulty changing money or sticking to a budget?*
- *Q3 – Does the child have difficulty telling time on an analogue clock?*
- *Q4 – Does the child find it difficult to relate the concept of numbers to real objects? For example, when asked how many cookies are left, does he/she seem confused by the question or answer incorrectly?*
- *Q5 – Does the child find it difficult to sort objects by shape, colour, or size?*
- *Q6 – Does the child seem uninterested in keeping score or playing any game that involves math, no matter how indirectly?*
- *Q7 – Does the child seem not to understand the difference between adding and subtracting? Does he/she confuse the symbols + and – when solving math problems?*

They denote all possible occurrences in the universe of discourse, whose purpose is to assess dyscalculia symptoms in children in schools. In order to make the process understandable, it is presented graphically. The scale used is based on an extension of Likert-type scales to include the concept of entropy, viz.

always agree (4), sometimes agree (3), rarely agree (2), never agree (1), rarely agree (2), sometimes agree (3), always agree (4)

Moreover, it is included a neutral term, neither agree nor disagree, which stands for uncertain or vague. The reason for the individual's answers is concerning the query, viz.

As a teacher, how much would you agree with each of the above seven questions?

The responses given by a randomly selected individual to the questionnaire about a single child are presented in Table 3.1, which shows that the individual gave a *vague* answer to Q2, Q4 and Q6. Concerning Q1, the individual answered that he/she *always agree*s *(4)* and *sometimes agree*s *(3)*; regarding Q3, the individual answered that he/she *sometimes agree*s *(3)*; concerning Q5, the individual answered that he/she *sometimes agree*s *(3)*; and regarding Q7, the individual answered that he/she *never agrees (1)* and *rarely agrees (2).*

Table 3.1 Answers to seven questions of the DSC questionnaire about a single child

Questions	Scale							
	4	3	2	1	2	3	4	*vagueness*
Q1	×	×						
Q2								×
Q3		×						
Q4								×
Q5						×		
Q6								×
Q7				×	×			

Applying these answers to the adopted problem-solving methodology, the DSC entropic states are calculated for the Best-Case Scenarios (BCS) and Worst-Case Scenarios (WCS), as shown in Table 3.2. The entropic states obtained for each question are shown graphically in Figure 3.3 and Figure 3.4, for BCS and WCS, respectively.

Table 3.2 Evaluation of the DSC entropic states for the best-case and worst-case scenarios

	BCS	WCS
Q1	$exergy_{Q1} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{1}{4}\sqrt{\frac{1}{\pi}}} = \frac{1}{7}\pi\left(\frac{1}{4}\sqrt{\frac{1}{\pi}} - \frac{0}{4}\sqrt{\frac{1}{\pi}}\right)^2 = 0.01$	$exergy_{Q1} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0.04$
	$vagueness_{Q1} = \frac{1}{7}\pi r^2\Big]_{\frac{1}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.08$	$vagueness_{Q1} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.04$
	$anergy_{Q1} = \frac{1}{7}\pi r^2\Big]_{\frac{1}{4}\sqrt{\frac{1}{\pi}}}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0.01$	$anergy_{Q1} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0$
Q2	$exergy_{Q2} = \frac{1}{7}\pi r^2\Big]_{0}^{0} = 0$	$exergy_{Q2} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.25$
	$vagueness_{Q2} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$	$vagueness_{Q2} = \frac{1}{7}\pi r^2\Big]_{\frac{4}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$anergy_{Q2} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$	$anergy_{Q2} = \frac{1}{7}\pi r^2\Big]_{\frac{4}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0$

	$exergy_{Q3} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0.04$	$exergy_{Q3} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0.04$
Q3	$vagueness_{Q3} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0$	$vagueness_{Q3} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$anergy_{Q3} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.04$	$anergy_{Q3} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.04$
	$exergy_{Q4} = \frac{1}{7}\pi r^2\Big]_{0}^{0} = 0$	$exergy_{Q4} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$
Q4	$vagueness_{Q4} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$	$vagueness_{Q4} = \frac{1}{7}\pi r^2\Big]_{\frac{4}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$anergy_{Q4} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$	$anergy_{Q4} = \frac{1}{7}\pi r^2\Big]_{\frac{4}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$exergy_{Q5} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0.04$	$exergy_{Q5} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0.04$
Q5	$vagueness_{Q5} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0$	$vagueness_{Q5} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{2}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$anergy_{Q5} = \frac{1}{7}\pi\frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.04$	$anergy_{Q5} = \frac{1}{7}\pi r^2\Big]_{\frac{2}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.04$
	$exergy_{Q6} = \frac{1}{7}\pi r^2\Big]_{0}^{0} = 0$	$exergy_{Q6} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$
Q6	$vagueness_{Q6} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} =0.14$	$vagueness_{Q6} = \frac{1}{7}\pi r^2\Big]_{\frac{4}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$anergy_{Q6} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$	$anergy_{Q6} = \frac{1}{7}\pi r^2\Big]_{\frac{4}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$exergy_{Q7} = \frac{1}{7}\pi r^2\Big]_{\frac{3}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.01$	$exergy_{Q7} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0.14$
Q7	$vagueness_{Q7} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{3}{4}\sqrt{\frac{1}{\pi}}} = 0.08$	$vagueness_{Q7} = \frac{1}{7}\pi r^2\Big]_{\frac{3}{4}\sqrt{\frac{1}{\pi}}}^{\frac{3}{4}\sqrt{\frac{1}{\pi}}} = 0$
	$anergy_{Q7} = \frac{1}{7}\pi r^2\Big]_{0}^{\frac{3}{4}\sqrt{\frac{1}{\pi}}} = 0.08$	$anergy_{Q7} = \frac{1}{7}\pi r^2\Big]_{\frac{4}{4}\sqrt{\frac{1}{\pi}}}^{\frac{4}{4}\sqrt{\frac{1}{\pi}}} = 0$

Leading to → Fig. 3.3 and Fig3.4 ← Leading to

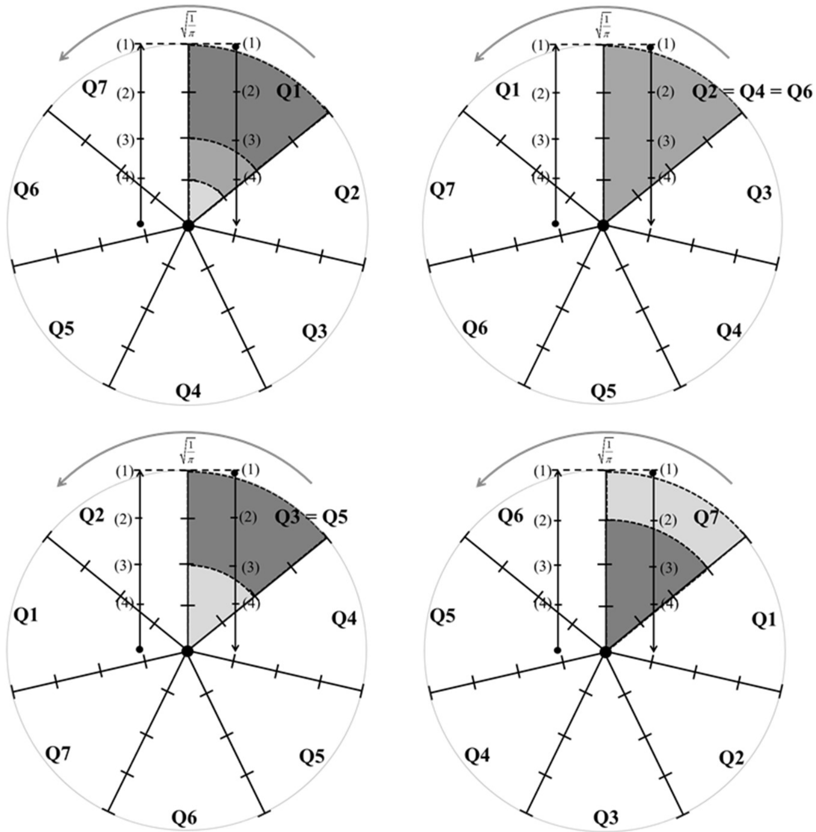

Figure 3.3 Graphical representation of a child's symptoms assessment in the best-case scenario *(Image by Author)*

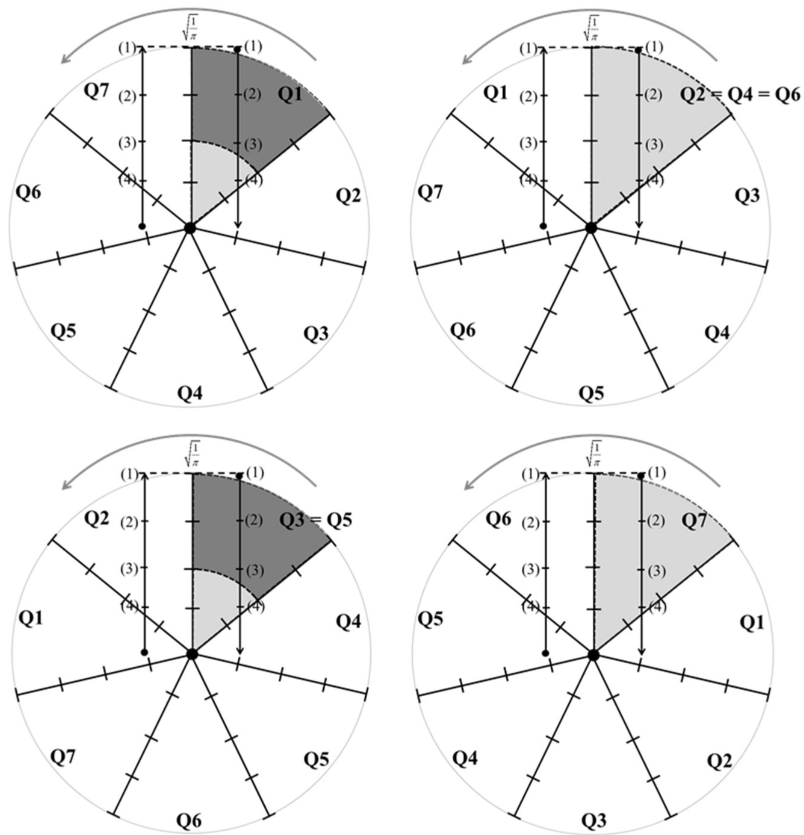

Figure 3.4 Graphical representation of a child's symptoms assessment in the worst-case scenario *(Image by Author)*

The data collected above is now structured in terms of the extent of predicate *child's symptoms* (cs), which is given in the form of the following clause and whose abstract view may take the form, viz.

$$cs\colon \boldsymbol{EX}ergy, \boldsymbol{VA}gueness, \boldsymbol{AN}ergy, \boldsymbol{D}yscalculia\ \boldsymbol{S}ymptoms\ in\ Children, \boldsymbol{QoI} \rightarrow \{True, False\}$$

where the DoC and QoI for the various logical terms or clauses forming the DSC are given in the form, viz.

Once the input for Q1 exactly matches (4) → (3), the system tends to deteriorate, i.e., the input for Q1 (4) → (3) states that there is a tendency for the system to deteriorate. The inputs are read from left to right (e.g., from (4) → (1) with increasing entropy, or (1) → (4) with decreasing entropy). The markers on the axis correspond to any of the possible scale options, which may be used from the bottom → top (i.e., from (4) → (1)), indicating that the performance of the system decreases with increasing entropy, or used from the top → bottom (i.e., from (1) → (4)), indicating that the performance of the system increases with decreasing entropy.

The DSC is generally expressed as $\sqrt{1-(EX+VA)^2}$; it represents the sum of social, emotional and cognitive skills that enable the child to cope with mathematical skills, i.e., a DSC truth value that lies in the interval of values (IL) between 0 and 1, viz (Figure 3.5).

$$DSC_{BCS} = \sqrt{1-EX^2} = \sqrt{1-0.01^2} = 1$$
$$DSC_{WCS} = \sqrt{1-(EX+VA)^2} = \sqrt{1-(0.11+0.01)^2} = 0.99$$

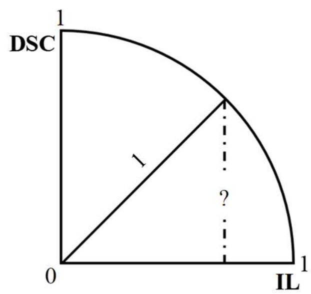

Figure 3.5 DSC evaluation *(Image by Author)*

QoI is generally evaluated in the form, viz.

$$QoI = 1 - \frac{EX+VA}{Interval\ Lenght\ (IL)}$$

which stands for the sustainability degrees of DSC, i.e., it shows how the child has dealt with the mathematical problem, i.e., a DSC truth value that lies in the interval of values between 0 and 1, viz.

$$QoI_{BCS} = 1 - \frac{EX+VA}{1} = 1-(0.01+0.08) = 0.91$$
$$QoI_{WCS} = 1 - \frac{EX+VA}{1} = 1-(0.11+0.01) = 0.88$$

Table 3.3 provides an assessment of the energy consumption per person with the answers to the above questionnaire for the *best-case* and *worst-case* scenarios, where EX refers to *"exergy"*, VA to *"vagueness"* and AN to *"anergy"*.

Table 3.3 BCS and WCS of the answers to seven questions of the DSC questionnaire about a single child

Questions	**BCS**					**WCS**				
	EX	*VA*	*AN*	*DSC*	*QoL*	*EX*	*VA*	*AN*	*DSC*	*QoL*
Q1	0.01	0.08	0.01	1	0.91	0.04	0.04	-	1	0.92
Q2	-	0.14	0.14	1	0.86	0.25	-	-	0.97	0.75
Q3	0.04	-	0.04	1	0.96	0.04	-	0.04	1	0.96
Q4	-	0.14	0.14	1	0.86	0.14	-	-	0.99	0.86
Q5	0.04	-	0.04	1	0.96	0.04	-	0.04	1	0.96
Q6	-	0.14	0.14	1	0.86	0.14	-	-	0.99	0.86
Q7	0.01	0.08	0.08	1	0.91	0.14	-	-	0.99	0.86
Overall	*0.01*	*0.08*	*0.08*	*1*	*0.91*	*0.11*	*0.01*	*0.01*	*0.99*	*0.88*

On the other hand, looking at Table 3.3, the tendency for system deterioration or system improvement can not only be set but also quantified. Indeed, the partition starting from (4) → (1) tends to set the tendency to system deterioration, while the partition starting from (1) → (4) determines the system improvement. Now looking at Table 3.3, one can measure the entropy associated with each partition. For the best-case scenario, one may have, viz.

$$entropy_{(4)\rightarrow(1)} = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + E_7 = 0.01 + 0 + 0.04 + 0 + 0 + 0 + 0 = 0.05$$

$$entropy_{(1)\rightarrow(4)} = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + E_7 = 0 + 0 + 0 + 0 + 0.04 + 0 + 0.01 = 0.05$$

Therefore, and once 0.05 = 0.05, the system tends to develop evenly. To evaluate the worst-case scenario, the procedures to follow are similar.

$$entropy_{(4)\rightarrow(1)} = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + E_7 = 0.04 + 0 + 0.04 + 0 + 0 + 0 + 0 = 0.08$$

$$entropy_{(1)\rightarrow(4)} = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + E_7 = 0 + 0 + 0 + 0 + 0.04 + 0 + 0.14 = 0.18$$

In this scenario, the system tends to develop negatively (0.08 < 0.18).

The extension of the predicate DSC represented by the child's symptoms (cs) for the best-case and worst-case scenarios is presented in Program 3.2.

$\{$

$$\neg cs(EX, VA, AN, DSC, QoI) \leftarrow not\ cs(EX, VA, AN, DSC, QoI), not\ exception_{cs}(EX, VA, AN, DSC, QoI)$$

$cs(0.01, 0.08, 0.08, 1, 0.91)_{BCS}$.

$cs(0.11, 0.01, 0.01, 0.99, 0.88)_{WCS}$.

$\}$

Program 3.2 The extension of the predicate cs for the best-case and worst-case scenario

If all the possible scenarios for the given answers to seven questions of the DSC questionnaire are considered, there can be two best-case scenarios and two worst-case scenarios for Q1, two best-case scenarios and one worst-case scenario for Q3, one best-case scenario for Q5, two best-case scenarios and two worst-case scenarios for Q7. In total, twelve more scenarios can be analysed on the given answers, as shown in Table 3.4, to infer the child's symptoms.

Table 3.4 Answers to seven questions of the DSC questionnaire about a single child, in all possible scenarios

Questions	Scale							
	4	3	2	1	2	3	4	*vagueness*
$Q1$	×	×						
$Q1_{BCS_1}$	×		×					
$Q1_{BCS_2}$	×			×				
$Q1_{WCS_1}$		×	×					
$Q1_{WCS_2}$		×		×				
$Q2$								×
$Q3$		×						
$Q3_{BCS_1}$		×	×					
$Q3_{BCS_2}$		×		×				
$Q3_{wcs}$			×	×				
$Q4$								×
$Q5$						×		
$Q5_{BCS}$						×	×	
$Q6$								×
$Q7$				×	×			
$Q7_{BCS_1}$				×		×		
$Q7_{BCS_2}$				×			×	
$Q7_{WCS_1}$					×	×		
$Q7_{WCS_2}$					×		×	

When evaluating the DSC entropic states for all possible scenarios (Table 3.5), the overall QoI in BCS is higher than in WCS (0.98 > 0.89), as well as the overall vagueness (0.07 > 0) and the overall anergy (0.07 < 0.01). The overall exergy is higher in the WCS than in BCS (0.10 > 0.01). The tendency of the overall values obtained in these twelve scenarios is the same as the tendency of the overall values in the seven initial scenarios (Table 3.3).

Table 3.5 BCS and WCS of the answers to seven questions of the DSC questionnaire about a single child, in all possible scenarios

Questions	BCS					WCS				
	EX	*VA*	*AN*	*DSC*	*QoI*	*EX*	*VA*	*AN*	*DSC*	*QoI*
$Q1$	0.01	0.08	0.01	1	0.91	0.04	0.04	-	1	0.92
$Q1_{BCS_1}$	-	0.02	0.01	1	0.98	0.02	-	-	1	0.98
$Q1_{BCS_2}$	-	0.02	0.02	1	0.98	0.04	-	-	1	0.96
$Q1_{WCS_1}$	0.01	0.01	-	1	0.98	0.02	-	-	1	0.98
$Q1_{WCS_2}$	0.01	0.01	0.01	1	0.98	0.04	-	-	1	0.96
$Q2$	-	0.14	0.14	1	0.86	0.25	-	-	1	0.75
$Q3$	0.04	-	0.04	1	0.96	0.04	-	0.04	1	0.96
$Q3_{BCS_1}$	0.01	0.01	-	1	0.98	0.03	-	-	1	0.97
$Q3_{BCS_2}$	0.01	0.01	0.01	1	0.98	0.05	-	-	1	0.95
$Q3_{wcs}$	0.03	-	-	1	0.97	0.05	-	-	1	0.95
$Q4$	-	0.14	0.14	1	0.86	0.14	-	-	0.99	0.86
$Q5$	0.04	-	0.04	1	0.96	0.04	-	0.04	1	0.96
$Q5_{BCS}$	0.01	0.01	0.04	1	0.98	0.04	-	-	1	0.96
$Q6$	-	0.14	0.14	1	0.86	0.14	-	-	0.99	0.86
$Q7$	0.01	0.08	0.08	1	0.91	0.14	-	-	0.99	0.86
$Q7_{BCS_1}$	0.01	0.01	0.04	1	0.98	0.04	-	-	1	0.96
$Q7_{BCS_2}$	0.02	-	0.04	1	0.98	0.04	-	-	1	0.96
$Q7_{WCS_1}$	-	0.01	0.02	1	0.99	0.02	-	-	1	0.98
$Q7_{WCS_2}$	0.01	-	0.02	1	0.99	0.02	-	-	1	0.98
Overall	*0.01*	*0.07*	*0.07*	*1*	*0.93*	*0.10*	*0*	*0.01*	*1*	*0.89*

Measuring the entropy associated with each partition, it is obtained for the best-case scenario:

$$entropy_{(4)\rightarrow(1)} = 0.07$$
$$entropy_{(1)\rightarrow(4)} = 0.09$$

Once 0.07 < 0.09, the system tends to develop negatively. To evaluate the worst-case scenario, the procedures are similar:

$$entropy_{(4)\rightarrow(1)} = 0.19$$
$$entropy_{(1)\rightarrow(4)} = 0.22$$

In this scenario, the system also tends to evolve negatively (0.19 < 0.22). Since the system tends towards increasing entropy, both in the best-case and worst-case scenarios, it is possible to infer the presence of symptoms of dyscalculia in this child. In order to describe this particular system by a formalism, the extension of predicate cs is given in Program 3.3 and Program 3.4 for the best-case and worst-case scenarios, respectively.

$\{$

$\neg cs(EX, VA, AN, DSC, QoI)$

$\leftarrow not\ cs(EX, VA, AN, DSC, QoI), not\ exception_{cs}(EX, VA, AN, DSC, QoI)$

$cs(0.01, 0.08, 0.01, 1, 0.91).$

$exception_{CS}(0, 0.02, 0.01, 1, 0.98).$

$exception_{CS}(0, 0.02, 0.02, 1, 0.98).$

$exception_{CS}(0.01, 0.01, 0, 1, 0.98).$

$exception_{CS}(0.01, 0.01, 0.01, 1, 0.98).$

$cs(0, 0.14, 0.14, 1, 0.86).$

$cs(0.04, 0, 0.04, 1, 0.96,).$

$exception_{CS}(0.01, 0.01, 0, 1, 0.98).$

$exception_{CS}(0.01, 0.01, 0.01, 1, 0.98).$

$exception_{CS}(0.03, 0, 0, 1, 0.97).$

$cs(0, 0.14, 0.14, 1, 0.86).$

$cs(0.04, 0, 0.04, 1, 0.96).$

$exception_{CS}(0.01, 0.01, 0.04, 1, 0.98).$

$cs(0, 0.14, 0.14, 1, 0.86).$

$cs(0.01, 0.08, 0.08, 1, 0.91).$

$exception_{CS}(0.01, 0.01, 0.04, 1, 0.98).$

$exception_{CS}(0.02, 0, 0.04, 1, 0.98).$

$exception_{CS}(0, 0.01, 0.02, 1, 0.99).$

$exception_{CS}(0.01, 0, 0.02, 1, 0.99).$

$\}$

Program 3.3 The extension of the predicate cs for the best-case scenario

$\{$

$\neg cs(EX, VA, AN, DSC, QoI)$

$$\leftarrow not\ cs(EX, VA, AN, DSC, QoI), not\ exception_{cs}(EX, VA, AN, DSC, QoI)$$

$cs(0.04, 0.04, 0, 1, 0.92).$

$exception_{CS}(0.02, 0, 0, 1, 0.98).$

$exception_{CS}(0.04, 0, 0, 1, 0.96).$

$exception_{CS}(0.02, 0, 0, 1, 0.98).$

$exception_{CS}(0.04, 0, 0, 1, 0.96).$

$cs(0.25, 0, 0, 1, 0.75).$

$cs(0.04, 0, 0.04, 1, 0.96).$

$exception_{CS}(0.03, 0, 0, 1, 0.97).$

$exception_{CS}(0.05, 0, 0, 1, 0.95).$

$exception_{CS}(0.05, 0, 0, 1, 0.95).$

$cs(0.14, 0, 0, 0.99, 0.86).$

$cs(0.04, 0, 0.04, 1, 0.96).$

$exception_{CS}(0.04, 0, 0, 1, 0.96).$

$cs(0.14, 0, 0, 0.99, 0.86).$

$cs(0.14, 0, 0, 0.99, 0.86).$

$exception_{CS}(0.04, 0, 0, 1, 0.96).$

$exception_{CS}(0.04, 0, 0, 1, 0.96).$

$exception_{CS}(0.02, 0, 0, 1, 0.98).$

$exception_{CS}(0.02, 0, 0, 1, 0.98).$

$\}$

Program 3.4 The extension of the predicate cs for the worst-case scenario

4 An Evolving Approach to Dyscalculia

Key points:

- Early therapeutics can improve the quality of life for individuals with dyscalculia.
- Learning through games is very effective in children.
- The evolving solution for dyscalculia includes a mobile game with exercises that adapt to the needs of the child and a web platform with analysis of the child's performance and therapeutic suggestions for those responsible for the child.
- The basic metrics of the mobile game to be assessed relate to the child's academic development.

4.1 CONTEXT

It is recommended to start therapy for children with dyscalculia as early as possible, namely in primary and middle school. At these stages, the children are between 5 and 10 years old, and it is at these ages that they develop the concept of number, i.e., differentiate between symbolic and non-symbolic representations of quantities. Therefore, the maturity of conceptualization achieved during these years will influence their adult life. In addition, different types of exercises, either mental, written, or oral, need to be designed and endorsed towards this purpose, i.e., to help the child develop and improve his/her numeric sense, as stated in section 2.6.

Nevertheless, these drills must be appealing to attract the child and well-planned to work on the significant areas of the brain. Hence, and considering the technological era we are going through, where children are always playing with smartphones and tablets everywhere, the app creation came up as a means of enforcing therapeutics for dyscalculia and other mathematical learning disabilities. Although the use of games to get children to learn different mathematical concepts is not a novelty, the difference between now and the past is evident in the type of games they prefer. In past, children assembled puzzles, played board games and even domino; now they prefer it in a digital appearance, as well as games of increasing difficulty where they must go through different levels that are not easy to finish.

As outlined in (Ferraz et al., 2017), the opportunity arose to develop a game that helps children with dyscalculia to improve mathematical outcomes in the educational system. Therefore, an evolving approach to dyscalculia may include a web application that allows to manage, analyse, and visualize data collected from children playing the mentioned game, towards dyscalculia's screening and abilities' evolution tracking. The whole conceptualization aids immediate users, teachers, parents, educators, and experts to profile and assess the development of longitudinal use of the mobile game.

4.2 Implementation Process

The structure of this platform is divided into six main components: the mobile application, which contains all the interactive tasks; the application programming interface, which connects the other components; the web server; the web application itself; the cognitive engine, which applies reasoning algorithms; and the database, which stores all the collected data (Figure 4.1).

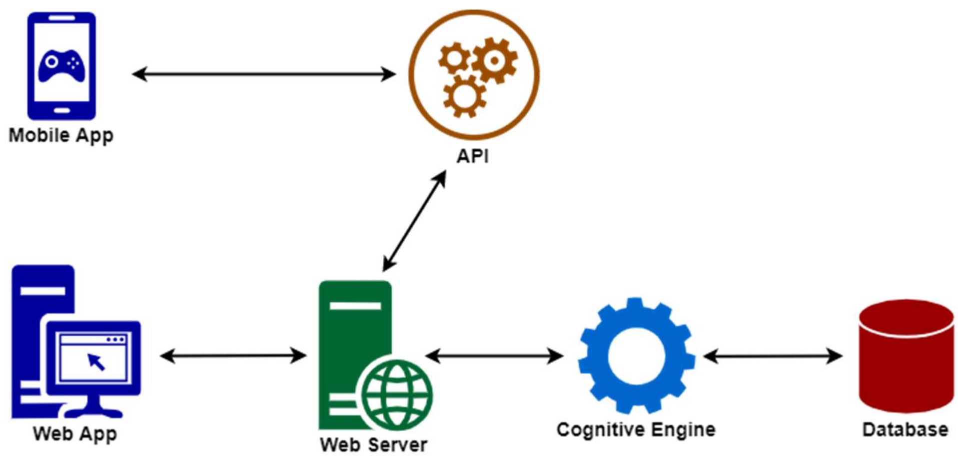

Figure 4.1 Game-based solution's components *(Image by Author)*

Therefore, the mobile application is an application that consists of a game for children with tasks that test their mathematical abilities.

The application programming interface is the component of the platform that connects the mobile application to the web server. Whenever the mobile application needs to perform an operation that requires access to one of the other components, be it data access, addition, removal, or modification, it connects to the application programming interface, which requests said operation from the target component and processes its response, if necessary.

The web server is responsible for delivering the web pages to the users of the platform, replying to the request of the web browser application or the Application Programming Interface (API). This allows the information to be online.

On the other hand, the web application displays the data existing in the database and/or the inference results from the cognitive engine.

As for the cognitive engine, it uses the collected data and subjects it to the reasoning processes mentioned in chapter 3, i.e., it helps to detect dyscalculia in students and identify the areas of mathematics where it is more prevalent.

the main objective of the database is to store all the data that the applications use. this data consists of information about students, teachers, experts, mobile games registered in the application, scores and times achieved by the student in each task. Currently, only one mobile game is registered for this platform.

4.3 Mobile Game Design and Task Design Process

In terms of content, the mobile application cycles through ten tasks approaching laterality, direction, size, memory, measures, time, orientation, and quantities, and is organised by level of difficulty. This organisation is illustrated in Figure 4.2 in activities, where the game's flow is represented, i.e., following an entry page, a menu is presented with the options *"About"*, *"Help"* and *"Levels"*. In the *"About"* option, the player learns about the purpose of the application and who developed it; in the *"Help"* option, the user learns how to play the game; and in the *"Levels"* option, the user switches to another view where he/she can see the score achieved in each level and the level in which he/she can continue the game. Each level represents one task per view. After the last task, the user returns to the *"Levels"* view.

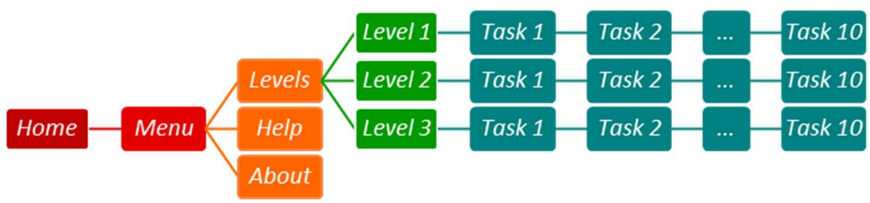

Figure 4.2 Game-based solution's sketch activities scheme *(Image by Author)*

To properly understand the usability of the application, the diagram of the architecture of the game-based system is shown in Figure 4.3. As can be seen, multiple users play the application on either smartphones or tablets. Once a user completes a level, there is an option to send the displayed behaviour to a cloud (if there is an internet connection), where inference algorithms are applied to the collected data and the results are stored in the database. Whenever there is an internet connection, the developer can access and process the results of the session, i.e., understand both the reasons for frequent misbehaviour and the actions to be taken. After this pre-processing and analysis, the information is available to educators, i.e., it is an easy-to-use reference tool that helps them define and correct student behaviour.

Presented in a clear form, one can say:

- Identifies the root cause of each student behaviour;
- Explains how each behaviour affects teachers, other students, and the learning environment;

- Suggests methods for dealing with each student behaviour;
- Identifies common mistakes teachers often make when trying to correct the behaviour;
- Cross-references other related behaviours (Ferraz et al., 2017).

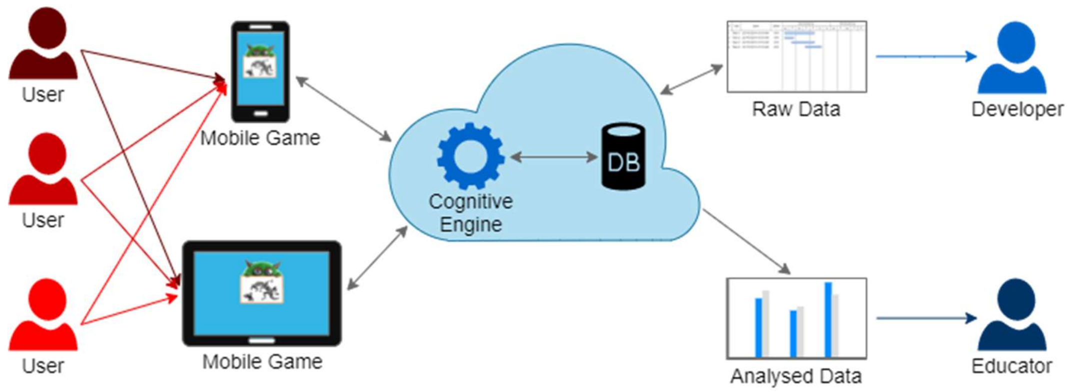

Figure 4.3 Game-based solution's architecture *(Image by Author)*

The development of a mobile game for the children to perform mathematical tasks to improve and consolidate their knowledge involves the creation of a mobile application. The mobile application thus provides mathematical tasks, whose interaction results in the registration of times, scores, and actions/choices of the users. Therefore, its purpose includes:

- Displaying mathematical exercises in the form of game tasks;
- Enabling interaction between the user and the application;
- Enabling automatic registration of various metrics.

The child is assessed based on the score per task, the response time in each task, the total task time, the score per level and the total level time. An overall analysis gives us the child's weaknesses and strengths, especially when the results can be compared with classmates. In addition, children answer a form where they identify their main obstacles, give their opinion about the application and suggest changes (Ferraz et al., 2017).

In addition, this system was designed with English as the main language in the back-end and Portuguese as the main language for each user interface. This decision concerns the main objective and the parts involved - Portuguese children and their educational system.

Regarding the game design, it is inspired by several children's games and other animated multimedia, most notably Cognifit (Tests et al., 2017), a series of cognitive stimulation programs available online, including online games to assess and train specific dysfunctions. The evolving game consists of a variety of mathematical game exercises, the purpose of which is to address children to creatively train their mathematical skills.

Regarding the exercises, or tasks, the following requirements must be met:

- Be appropriate to the level of difficulty of children aged 5-9 years;
- Provide an enjoyable experience with fun and engaging exercises.

In order to extract appropriate metrics for assessment from these tasks, some considerations are made:

- Objective answers with the mechanism for wrong or correct answers and subsequent actions;
- Timing of each answer, as well as storage of the scores, obtained;
- Sound implementation to stimulate the experience.

4.4 Application Programming Interface Design

The API allows a user to perform operations on a database's content. So, its purpose entails:

- Allowing a user, with permissions, to visualize requested data existing on the database;
- Allowing a user, with permissions, to edit requested data existing on the database.

This component is responsible for requests between the web application and the database, in both directions. These requests regard the consult of information. Therefore, if every condition is met, the API must let a certain user visualize the requested information. Not every user must have the same permissions and objectives, hence the conditions should differ among users:

- <u>Student</u> – this is the target user of the web application. So, when a student accesses the platform, he/she must have access to information about him/her, his/her teacher, his/her expert, his/her class, all apps, tasks, and levels engaged, as well as his/her and his/her class' score records. This user must have also the ability to add score records to the database via a registered application and to change his/her password.
- <u>Teacher</u> – this user is directly linked to the student, being responsible for monitoring and enriching his/her learning process. Consequently, the teacher must have access to information about himself/herself, his/her students, his/her classes, the experts, and administrators linked to his/her students, all apps, tasks, and levels engaged by his/her students, as well as his/her

students' score records. This user must not have the ability to add nor remove information from the database, since it is not needed. He/she must be able to change his/her password if wanted.

- Expert – this user is responsible for supervising and analysing the student's performance. As so, the expert must have access to information about him/her, his/her students and their teachers, the administrators linked to his/her students, all apps, tasks, and levels engaged by his/her students, as well as his/her students' score records. Like the teacher, the expert must not have the ability to add nor remove information from the database but have permission to change his/her password.
- Administrator – this user manages the other users and comprehends the developers of every component of this platform. Hence, this user must be focused on assuring every aspect of the platform is working as expected. The administrator must be accessible to all information existing in the database, including users, group of users, apps, levels, tasks, and scores. This user must be able to add other administrators to the database, but only the database administrator should be permissions to add students, classes, teachers, and experts. Unlike the other users, the administrator should be the only user with clearance to remove information from the database. He/she must have the ability to alter any detail of every user, including the passwords, except for the identification codes.

4.5 COGNITIVE ENGINE DESIGN AND CONTENT

The data produced in the mobile application must be sent to a data cloud where it is processed and stored. The Cognitive Engine is one of the components that should be hosted in the cloud, receives the data (obtained scores, response times, etc.), and, with the decision-making models mentioned in chapter 3, completes the results' analysis with therapeutic guidance and screening outcome. The Cognitive Engine must be a self-learning system that improves its outcomes as the amount of data received increases. This component will constitute an automatic tool for the experts who will later analyse and validate the produced reasoning.

The other component of the cloud is the database. The registration of the child's performance must be stored so that can be evaluated by experts, and consulted by teachers, educators, and the student himself/herself. Hence, the database is used to store and register all data exchanged between the web application and the mobile application, and within components. Therefore, its purpose includes:

- Storing information concerning the web application;
- Storing information concerning the mobile application;
- Enabling the management (addition, deletion, edition) of said information.

Since the database must be able to store both personal information and function-related information, such as identification codes and classes, on a mesa-level it has tables:

- to store information regarding students, teachers, experts, and administrators, but also regarding classes of students that a teacher has, and a group of students that the expert is responsible for;
- to store information related to the mobile application, such as its name, identification code, game levels, game tasks and the field of dyscalculia concerning them;
- to store information regarding the score and time achieved by a student for the level and for the task of the mobile application, as well as the date and time of completion.

The modelling of the database is described in Appendix A1.1.

4.6 Web Components Design

4.6.1 Web Server

The analysis of the child's performance must be available regardless of the location. The web server regards for storing, processing, and delivering web pages to several clients, namely students, teachers, educators, experts, and administrators. The web browser requests a specific resource using the Hypertext Transfer Protocol (HTTP) protocol and the web server responds with the requested content or a message indicating the error. In addition, the web server can accept and store files, if it is also configured to do so. Hence, this data flow is illustrated in Figure 4.4, and its purpose involves:

- Storing the information that the user agent requests;
- Processing requests from the web application;
- Delivery resources requested by the web application;
- Hosting the web application to make it available to anyone over the internet.

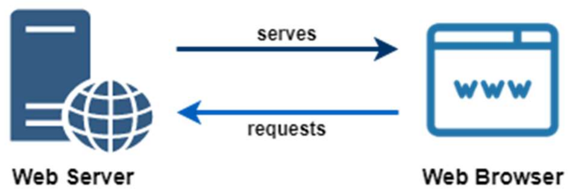

Figure 4.4 Web server and web server interaction *(Image by Author)*

4.6.2 Web Application

The web application is used to visualize and manipulate the data collected in the mobile application while the children are playing the game. Hence, its purpose implies:

- Allowing all users to view the information;
- Allowing users with permissions to edit privileged information and other users;
- Presenting easily understandable analysis to all users.

The web application allows not only the visualization of the data gathered by the mobile application but also the structuring of this data, the work with it and the presentation of a more proper analysis to the user.

The students who use the mobile application, the teacher who is responsible for a certain group of students (classes), the expert who analyses the performance of a group of students, and the administrator who manages them all, are the main targets of the web application to be developed.

Regarding the student, a history of the score obtained per task and per level must be visualized, as well as the time is taken to complete the tasks. This user must have permission to see an indiscrete ranking list in which his/her performance is inserted and seen on a global view.

The teacher must have permission to see the performance of his/her group of students. This user must have a graphical comparison of his/her students to assess their development and target their attention. Additionally, with the students' permission, the teacher should visualize an analysis of the class's results.

Regarding the expert, this user must be able to see all students and all classes to which he/she has been given access. The expert should be able to access the student's performance as an individual but also as part of the class. He/she must have permission to edit the automatic analysis.

The administrator must have access to all users and analyses.

The modelling of the web application is described in Appendix A1.2.

4.7 ASSESSMENT METRICS

To evaluate the students' performance, it is important to establish the metrics that can provide an overall analysis of these children's struggles, weaknesses and strengths when dealing with mathematical concepts. These metrics should be easily collected, clear to interpret, transversal for all students, and representative of this disorder.

4.7.1 AGE

This metric helps to characterize the user in terms of affective, physical, and cognitive development. Placing the student on a developmental scale helps to classify his/her difficulties (R S Shalev et al., 2000). For instance, children with dyscalculia of preschool age have difficulties with basic counting and number understanding, while children with dyscalculia in middle school age are more likely to have difficulties with measurement and mathematical structures (Ruth S Shalev et al., 2005).). Age is not only helpful in assessing the development of the disorder in a student, but also in providing a comparative analysis of students under the same conditions.

4.7.2 TASK AND LEVEL OVERALL SCORE

Since the mobile application is based on the concept of a game with different levels of difficulty, the score achieved by the user is an important metric. Each level must have different tasks, each corresponding to a vulnerable mathematical area of dyscalculia. Therefore, if a certain task is not completed, it can be assumed that there are difficulties in understanding and/or answering that task due to its subject[8]. The score achieved per task helps to associate the area of difficulty and analyze whether it is related to dyscalculia. Additionally, it helps to trace the progress of the student along with the level and field of struggle, when combined with his/her age and duration of the performance.

[8] When performing the tasks per level, it is assumed that the user has followed the instructions and the game flow. The exceptions to these rules consider anomalies such as skipping the task by mistake.

4.7.3 Task and Level Overall Time Completion

Another metric to consider is the time taken to complete a task and a level. The moment the user starts a level, the timer should count the seconds the user takes to answer the task presented, also recording the time of the start and end of each task. This metric helps to understand whether a task is skipped, meaning that the user does not want to take time to complete it; if the user spends an unusually long time completing a particular task because he/she is having difficulties doing it; if the user completes a specific task quickly, the user is proficient in that field.

An overall comparison of the level's completion time with other users of the same age can help figure out the evolution of a certain user and infer his/her main difficulties.

5 disMAT – A Gaming Therapeutic Tool

Key points:

- Technologies are attractive to both adults and children.
- Working with the child under the gamification concept can improve the child's development if the game has educational purposes.
- The developed gaming therapeutic tool consists of a mobile game, disMAT, with evolving exercises whose records are linked to a web application, Discalculia Web, where the person in charge can view the child's performance, evolution, screening, and guidance.
- The solution developed has demonstrated to be useful and appropriate for the group of study.

5.1 Gamification

In today`s technological era, where gadgets are the trend, an electronic and mobile application is an attraction to the kids. Indeed, this application has the purpose to assist children from the age range 5-10 with mathematical learning disabilities, and dyscalculia, in an attractive and fun way. The developed application was designated *"disMAT"*, a combination of the words *"dyscalculia"* (in Portuguese, *"discalculia"*) and *"mathematics"*. It is an application whose purpose entails to appeal children to train their mathematical abilities in a three levels game, where the difficulty is distributed across a set of tasks (being used ten at each level) that intend to stimulate the brain`s areas affected by dyscalculia (Ferraz, Vicente, et al., 2016).

Additionally, it is available in two languages - English and Portuguese. Each screen view with written words has an associated button that reads all the content of that view, turning this application valuable for children who can read, who cannot read and who have difficulties in reading but want to test their knowledge in mathematic. Furthermore, it is important to refer that this application runs under the Android Operative System (OS) since it is the OS most common in the target population (Ferraz et al., 2017).

Although DD has an incidence rate of 6 to 7%, its awareness is an object of concern. Additionally, the lack of innovative learning support systems to compete with the existent ones – computer software and outdated paper guides – constitutes the required motivation to develop a specific and didactic mobile application for individuals with dyscalculia. The next section is about this gaming therapeutics tool, regarding disMAT's architecture, implementation, and assessment procedures (Ferraz et al., 2017).

5.1.1 Task Development Process

The process to work on brain plasticity has to be planed, calculated, and assessed in order to be effective. As so, the need to construct assignments that work directly and indirectly in fragile areas of Dyscalculia was a lagging process with extensive research.

To maintain the essence of a game, the tasks had to be creative and appealing, so the children would have fun and enrol freely and willing. Translating mathematical problems into simple and direct tasks was completed with colourful environments, sympathetic cartoons and encouraging sounds. The choice of colours, graphics, sounds, and images was based on existing therapeutic tools of several areas, mainly the ones mentioned in chapter 2.

Some of the struggling fields that dyscalculia affects considered to be approached in these tasks:

- Memory: this skill manifests by the poorly capacity to remember information regardless of the amount of time passed by. On the mathematical field, it is translated in presenting difficulties in calculating and performing problems, as remembering quantities, spoken and written instructions, multiplication tables, etc.;
- Shapes and sizes: comparing and identifying different forms, shapes, sizes, and quantities is another issue common in people with Dyscalculia. Therefore, classifying algorisms and objects in *"bigger"*, *"smaller"*, and *"equal"* when compared to others can be challenging, as well as distinguish algorisms that look similarly (e.g., number *"3"* with the number *"8"*);
- Spatial orientation: this notion directly affects directions and positions. It is decoded in troubles placing the cardinal points, distinguish left side from the right one, confusing sequences of directions and easiness to get lost;
- Measures: the assigning of a quantity measure can be tough and messy. The need to memorize scales and then allocate properly makes the comparison between measures very challenging, even more, when varies from greatness. For instance, it is extremely difficult for people with Dyscalculia to compare greatness as weight and length, and performing conversions;
- Mathematical expressions and equations: the symbols that materialize these operations are frequently mistaken, by their graphical similarity (*"+"* with *"x"*), by their meaning. This represents issues on calculating and understanding basic mathematical structures, sometimes aggravated when there are difficulties in comprehending numbers in the expressions, as well.

After identifying the main areas to work on, it was concretized in several tasks per level, having a total of three levels of different difficulty. Initiating in a basic level, the tasks are presented in a not specific order, each one approaching one of the fragile areas referred to before and selected according to the child's difficulty. A successful answer is accompanied by a cheerful sound and a wrong answer by a stimulating sound, as well as a correspondent message. For each task and level, it is made a local registry of the time took to complete it, and the scores achieved. Later, these metrics are uploaded to a cloud database and can be analysed in a subsequent phase when the responsible, for this evaluation, access the web application. Following are presented some examples of tasks presented to the user of the mobile application (Figure 5.1, Figure 5.2 and Figure 5.3).

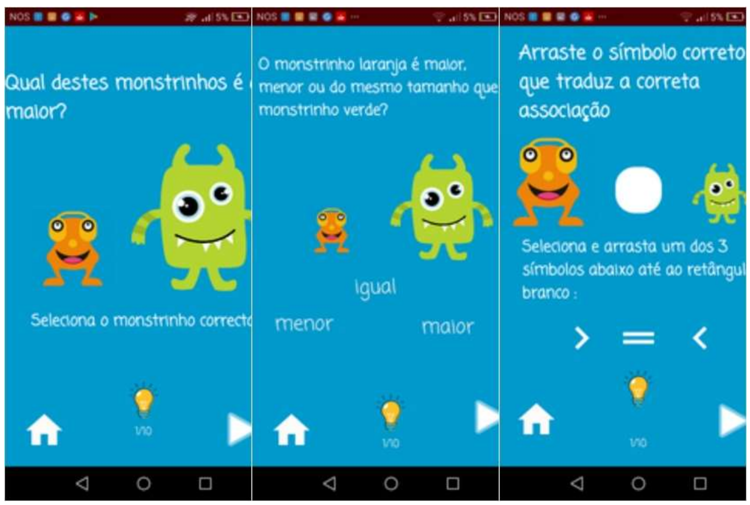

Figure 5.1 Tasks approaching shapes and sizes in different levels of difficulty *(Image by Author)*

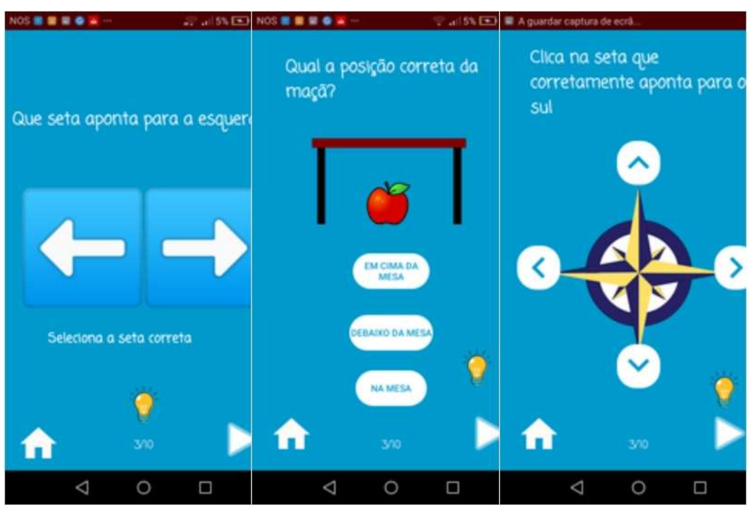

Figure 5.2 Tasks approaching spatial orientation in different levels of difficulty *(Image by Author)*

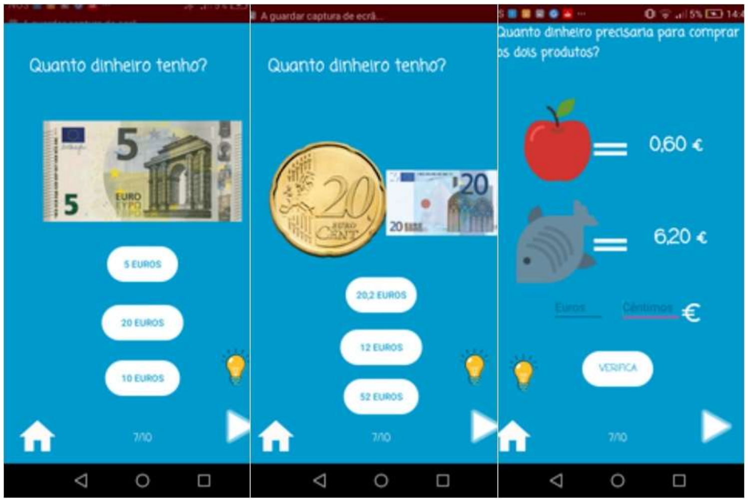

Figure 5.3 Tasks approaching measures and mathematical equations in different levels of difficulty *(Image by Author)*

5.1.2 Components

In order to develop a mobile game, it is necessary to work on several components that interact and deliver the desired outcome (Figure 5.4).

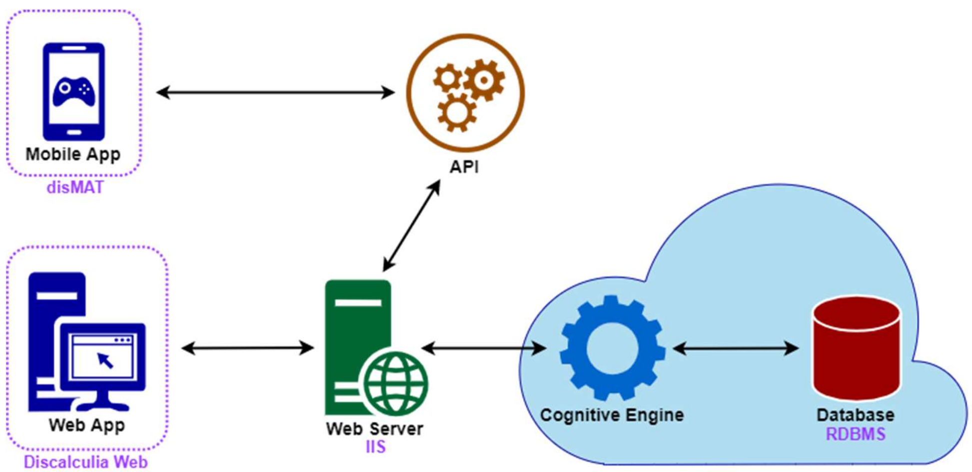

Figure 5.4 Interaction of the disMAT's components *(Image by Author)*

The main component of a mobile game is the mobile application, shown in the top left side of Figure 5.4 in indigo blue, where the whole concept of the game is presented. Even though it can be the only component if its purpose does not require external resources, it is usually accompanied by several other components that interact with each other. In this system, the mobile application was designated *"disMAT"*, as explained in the previous section

To connect the mobile application to other components like the database, an API is needed, represented in dark orange in the top centre of Figure 5.4. This component is an interface that requests and provide access to the information. The API also has a set of settings and protocols that allow services to be requested and communicate between components without the need to know how the other software has been implemented.

A web server is a component that receives requests from the API, or another application, and contacts the database to order them, and is shown in green in the lower part of Figure 5.4. The web server functions through internet protocols, overcoming the obstacles of having a physical server, in logistic terms, and providing services as the information needed from the database. In this system, the used web services are the Internet Information Services (IIS), which hosts the database and the web application.

The web application, represented in indigo blue on the lower left side of Figure 5.4, is like a web browser. It is a component that allows users to visualize the information gathered per student in several forms and under different analyses appropriate to the situations. It allows selected users to visualize make comparisons of information not only about one but about several students. This component retrieves information from the database, via the web server. In this system, this component has been assigned as *"Discalculia Web"*, where *"discalculia"* is the Portuguese translation of *"dyscalculia".*

Usually, the web server contacts the database directly, but, in this system, this connection goes through another component, the Cognitive Engine, which is shown in turquoise blue in the bottom centre of Figure 5.4. The Cognitive Engine consists of a set of reasoning algorithms, mentioned in chapter 3, that are applied to the data received. Subsequently, this component provides a complete analysis regarding the data given, where it is given screening observation and suggested therapeutic exercises, which are later validated by the expert.

If there is a need to store data from other components, which is the case, a database is required. The register of users, the scores obtained during the game and the timestamps between game phases are some items that require a platform that holds information, whether online or offline. It also stores the processed data sent by the Cognitive Engine. The used system is a Relational Database Management System (RDBMS), in which the data is stored in tables.

Organising the system and the components that come together, we have the mobile application - disMAT, which interacts directly with the API, which interacts with the web server. The web server interacts directly with the web application - Discalculia Web, the API, and the Cognitive Engine. The latter interacts directly with the web server and the database. Information flows in both directions between the components, characterizing request/answer connections (Figure 5.4).

5.2 MOBILE APPLICATION DISMAT

The conceptualization of the game and its development were built in a framework chosen by the mobile application's OS in mind. Some frameworks generate applications that are suitable for multi-operating systems, including a wider range of target users, but, on the other hand, do not provide a wider range of functionalities and personalization tools that can be implemented in the application.

The development of the mobile application was made towards Android devices since Android is the preferred operating system in Portuguese primary schools. Therefore, the chosen framework only generates applications for this operating system.

The construction of the tasks was described in a previous section and the design of the mobile game considers bright and vivid colours, cheerful icons and encouraging sounds. It was designed to be simple but containing the main matter. This serves to keep the children focuses on the exercises.

In the first contact with the mobile application, the user registers the basic details: user's name, age, and class that belongs. If it is not the first interaction, the user can choose his/her name from the registry of users that played the game on that device (Figure 5.5 (a)). This primary information is stored locally and, when the internet connection is available, is sent to the database.

Figure 5.5 First disMAT's preview screens: (a) registry menu, (b) levels' menu, (c) first level entry screen and (d) random first task on level 1 *(Image by Author)*

After the registry or logging in, the user is presented with a levels' menu where he/she can choose which level of difficulty prefers to play. The option to consult the scores' history is presented in this menu, as well, which will be empty if it is the user's first interaction (Figure 5.5 (b)). With the level chosen, it is presented the level's identification screen, followed by tasks in random order (Figure 5.5 (c) and (d)), and his/her performance is tracked on every task in terms of correct answers (score) and completion time. This last metric is calculated in seconds from the start of the level to its end and from the start of each task until an answer is given.

Once reached the end level, the results achieved are calculated and stored. When the user plays the game one or more times without an internet connection (offline; the green circuit in Figure 5.6), the registry of scores and times is saved locally (in cache) until further internet connection and in a Comma-Separated Values (CSV) file in the device (internal storage in a dedicated folder to the user) in case of cache is cleared. Upon application deletion, this file is kept intact and later useful to update the database content. As soon as the user connects to the internet, every registry is updated to the database (online; the blue circuit in Figure 5.6) and the user has the option to send his/her scores and times history stored in the CSV file by e-mail to the assigned teacher (the purple circuit in Figure 5.6). A specific email was created to receive the results and it will appear on the screen with the information auto completed.

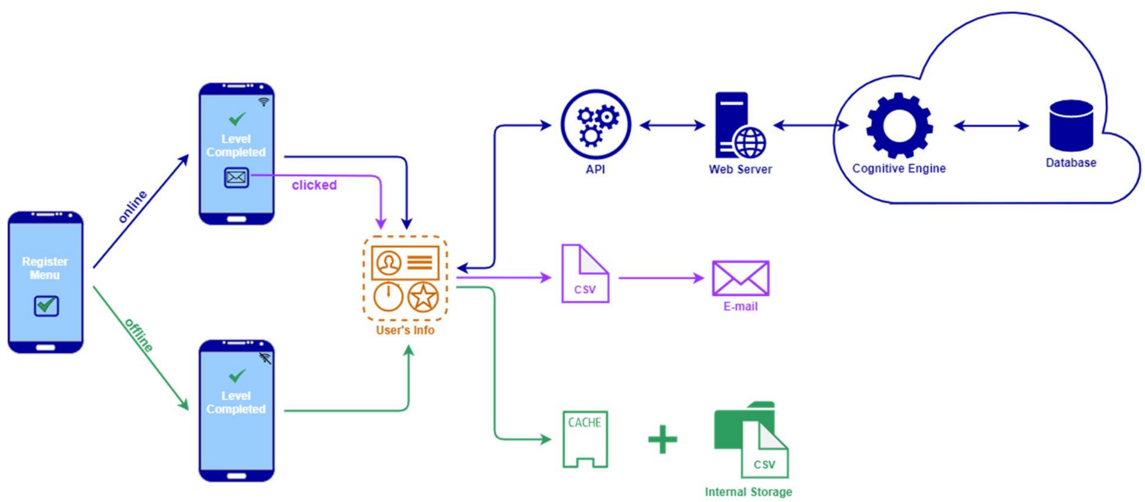

Figure 5.6 Data flow using mobile application disMAT *(Image by Author)*

The information flow regarding the mobile application was decided in order not to create constraints to the use of the game by the students. Internet access is not always available since it is not secure for children. Therefore, at the end of every level and the levels' menu, the achieved scores and times taken can be consulted by the user.

5.3 APPLICATION PROGRAMMING INTERFACE FOR DISMAT

The connection of the mobile application and the other components are supported by the API, an interface that provides a communication channel between the components. The API has a set of settings and protocols for building and integrating application software, regardless of how it was created and where it is hosted. In other words, it facilitates the integration of data into the application and delivers and requests data to the other components, namely to the web server. It operates by responding to HTTP requests under HTTP protocols, which are responsible for structuring requests and replies between the two entities. In this system, a Representational State Transfer (REST) API has been implemented, i.e., an API that corresponds to the constraints of the REST architectural style and allows interaction with RESTful web services.

Figure 5.7 shows the data flow to and from the API, represented in dark orange, with the interrupted arrow representing an indirect connection. Additional information about the API is described in Appendix A2.1.

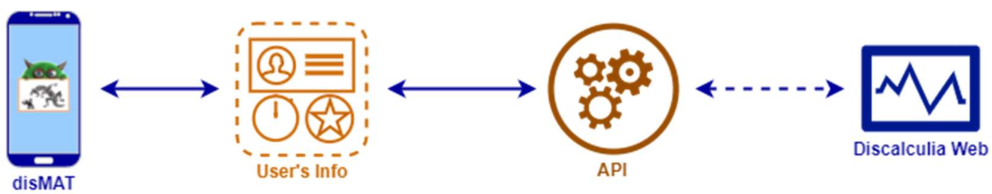

Figure 5.7 Data flow to and from the API *(Image by Author)*

5.4 COGNITIVE ENGINE AND DATABASE FOR DISMAT AND DISCALCULIA WEB

As referred before, the children's performance is measure in correct answers – the claimed *"score"*, - and in how fast the tasks and levels are completed – the *"time".* Every child must register with name, age, class (grade and letter (number) and gender (this last one can be optional). Additionally, a child is associated with a school, a teacher, and an expert. All this information is analysed by the Cognitive Engine and represented under KRR formalisms.

The Cognitive Engine, represented in bright blue in Figure 5.8, is hosted in a cloud computing platform and comprises reasoning algorithms such as ANN and CBR. These decision-making algorithms analyse all the information delivered by the mobile application and the web application and, based on the knowledge previously introduced, provide inferences about the children, such as the possibility of having or not having dyscalculia, qualifying this possibility in a scale of certainty, and, if necessary, guidance for therapeutics. The algorithms used to analyse the data are selected based on the accuracy of each model to that data; the model with the best accuracy is selected, applied, and registered. Even though the processes regarding the Cognitive Engine should be automatic, they were manually implemented in this solution due to external constraints.

The information resulting from the Cognitive Engine needs to be stored so that it can be added, accessed, edited and/or deleted. Since it can be retrieved from the mobile application, the web application or the Cognitive Engine, and its size is cumulative, the database where it can be stored must be accessible from anywhere; a physical database is very limiting. Therefore, it was implemented a database, represented in dark red in Figure 5.8, that is hosted in the same cloud computing platform as the Cognitive Engine. The system to manage and store the information is an RDBMS, in which the implemented database is a relational database, and the information is store in form of tables.

Figure 5.8 shows the data flow to and from the Cognitive Engine and database, with the interrupted arrow representing an indirect connection.

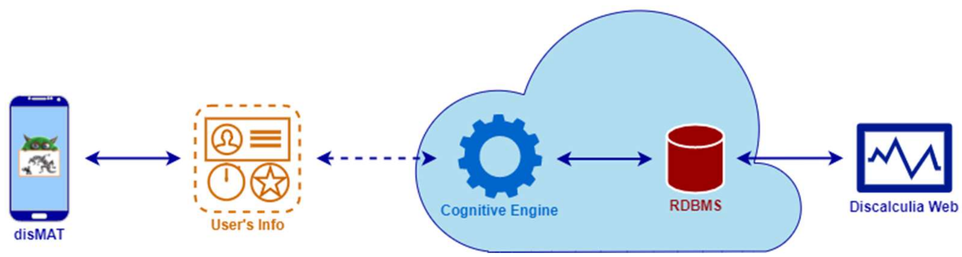

Figure 5.8 Data flow to and from the Cognitive Engine and database *(Image by Author)*

The testing of the database is described in Appendix A2.2.

5.5 Web Server to Support disMAT and Discalculia Web

The flow of information from the mobile application goes through the API. This last one makes requests to retrieve and deliver information. These requests are designated *"services"* and, since the process is online, those are *"web services".* The web server is the component responsible for handling these requests, whether they come from the API, the web application, or the Cognitive Engine (in this system). The web server acts as an intermediary between disMAT, the database and Discalculia Web. If the web server cannot deliver the requested data, it can store it until the connection is re-established.

The IIS is the web server on this system and handles Hypertext Markup Language (HTML) pages and files and hosts other components such as the API and the web application.

Figure 5.9 shows the data flow to and from the web server, represented in dark green, with the interrupted arrow representing an indirect connection.

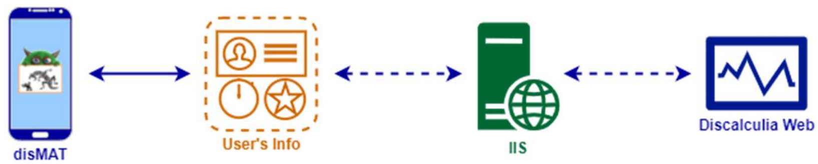

Figure 5.9 Data flow to and from the web server *(Image by Author)*

5.6 Web Application Discalculia Web

The data resulting from the use of the mobile application, whether it is raw data or processed data, is displayed in the web application, designated *"Discalculia Web"*. This web application allows the user to visualize the data but also to add, delete or edit information. Of course, the user must have the appropriate permissions to perform an action.

If the data to be consulted is the data produced in the mobile application, without any changes, it is designated *"raw data".* This information is only useful for developers and the Cognitive Engine. On the other hand, if the data has some alterations or is presented in a different form, it is designated *"analysed data".* This type of data is useful for the student, as well as for the teacher for the expert since it is presented in a "friendlier" and more targeted way. Both types of data can be stored in the database. Figure 5.10 shows the data flow to and from Discalculia Web, represented on the right side of the image, with the interrupted arrow representing an indirect connection.

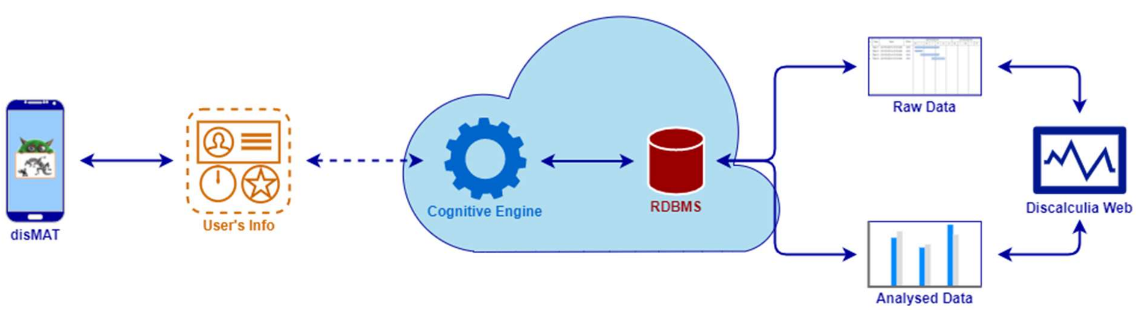

Figure 5.10 Data flow to and from the Discalculia Web *(Image by Author)*

The web application is the framework in which users can consult their personal details, such as name, age, class, etc., and information related to the students' performance, such as completion times and scores per task and per level. Therefore, this information is presented in an appealing and easy-to-understand interface, with straightforward vocabulary. The analysis of the student's results is completed with simple graphs that allow teachers, educators, psychologists, and parents to easily compare a student's evolution with other students in the same age percentile, and with his/her performance over a timeline. They can also access more detailed information, such as the score achieved in a specific task, the total time spent during a level, or the areas of mathematics in which the student has the greatest difficulty.

To access Discalculia Web, an IIS hosting program has been implemented to make it available online. The address is public, but as it is not easy to acquire a personalized public domain, the address of Discalculia Web is not so *"user-friendly".*

Each assigned user, - student, teacher, expert, or developer, - has defined permissions and different functionalities available, with a login mode requiring a password. Therefore,

- The student can view his/her performance measured in scores obtained for each task and level completed, as well as the time is taken to complete that assignment. This user can also see an evaluation of his/her evolution over time and a comparative analysis of the performance of his/her classmates. Finally, he/she can also consult his/her performance across the several domains of mathematics. A screening result is also available, along with advice on therapeutic interventions, if needed;
- The teacher can consult all the information on the performance of a student or a class if he/she is responsible for that student or class. This user can also see a comparative analysis of the performance between classes. If one of his/her students presents difficulties, a screening result with therapeutic guidance is provided;
- The expert can visualize any type of information on a student if the student has been assigned to him/her for supervision. The related information is available to this user under a comparative analysis. This user can consult all the screening analyses provided and edit them if there is the need to change;
- The developer can access any type of information concerning any user.

Every user can view information about the project and instructions on how to use the platform. There is also the contact of the developer and the researcher available for each user.

Lastly, further details and preview images of the web application are described in Appendix A2.3.

5.7 EVALUATIONS

To evaluate if the developed solutions are appropriate, functional, and effective, it was needed to perform tests and assessments. Therefore, it was necessary to approach the target users – children in early school years, – and allow them to interact with disMAT in real-time under the designed conditions.

Firstly, it was contacted several basic schools within a radius of 15 km. From the schools that had conditions to accept the test, it was selected the ones that had resources and availability to perform these tests after classes-hours. The resources constituted a few android devices (at least two) where the mobile application could be installed and an internet connection. Regarding the availability, it was needed a school's teacher or responsible to be schedule-free on the arranged times, as well as children, attending one of the four first school years, available and with parental consent[9] to attend the tests after classes. Given those conditions, few basic schools attended the requirements and were allowed to conduct the study with students aged 6 to 11 years old.

Initially, there was a session with the teachers responsible for the students who would take part in the study, where was explained the project and how the tests would be performed. After, the target users had an accompanied session when playing the game for the first time in which every functionality was explained. Most of the time, the students discarded this help since consider the mobile application very intuitive.

The subsequent session regarded the first tests, which were performed in a total of 53 students, of which 23 were female students. The targeted users constituted a sample of children aged 7 to 11 years old, with an average age of 9, participating in classes from second to fourth grades.

In the referred session, the students were asked their first name (because of the data protection law), their age, the school year that was enrolled. This information was inserted into the application menu selected for that purpose before starting the game. Next, it was given the students the chance to play the game, with a supervisor (a teacher or the researcher) in the same room so they could have the help to understand the tasks if needed through the game. At the end of each level, the students were asked about the main difficulties felt during that level, the suggestions that they would change in the game, and the overall performance was registered. At the end of their interaction of the game, their results – the scores and times completion, - were recorded and sent by email to the school's electronic address. These results were later analysed and displayed on the web application.

[9] It was given a consent form to the parents/educators where the project and the details of the tests were explained.

The teachers and educators took part in a session where the web application was explained: the ways to access it, their logins were made available, and the displayed analysis was described.
Lastly, it was made the third session with the students six months after the second session, so their development could be traced.

5.8 Results And Discussion

As mentioned earlier, the tests were administered in a few schools and analyses were drawn from the samples collected over some time.
In (Ferraz, Neves, et al., 2016), it is presented a study with 148 children from a primary school, aged between 5 to 10 years old, that tested disMAT. With an average age of 8,6 years and gender distribution of 59,2% female students and 40,8% male students, for each participant, it was registered the age, the number of levels completed, the completion time for each level, and the classification of the difficulties they have through the game.
Case-Based Reasoning techniques, also used in the Cognitive Engine, were applied to this sample to identify students with difficulties. The model created had an overall accuracy of 86.4%, indicating that the model is well suited to estimate children that present evidence of dyscalculia, based on disMAT.
In (Ferraz, Vicente, et al., 2016), a primary evaluation of disMAT is presented in which the sample had only one contact with the game. In this study, the sample consists of 203 children from two primary schools, aged between 5 and 10 years, with an average age of 8,2 years. About 57,6% were female students and 42.4% were male. Before this test, 48 students, i.e., 23,6% of the cohort, were signalled as having difficulties with numbers and mathematical concepts. The children participated in the study voluntarily and without pressure or coercion and were informed that their grades would not be affected. Each of the participants gave informed consent to participate in the study. The study was conducted following institutional guidelines. For each participant was recorded the age, the number of levels completed, the minimum score obtained, as well as the maximum score, the completion time in each of the three levels and the classification of difficulty in understanding and acting through the game.

Table 5.1 The coincidence matrix for the proposed model (Ferraz, Vicente, et al., 2016)

Target	**Predictive**	
	True (1)	False (0)
True (1)	*43*	5
False (0)	16	*139*

As this was a large sample, Case-Based Reasoning techniques, the same techniques used in the Cognitive Engine, were used to identify students with difficulties and create a model. The model showed a performance accuracy of 89,7%, as seen in Table 5.1), and an area under the ROC curve near to 0,9, as seen in Figure 5.11, which means that the model performs well in signalize evidence of dyscalculia.

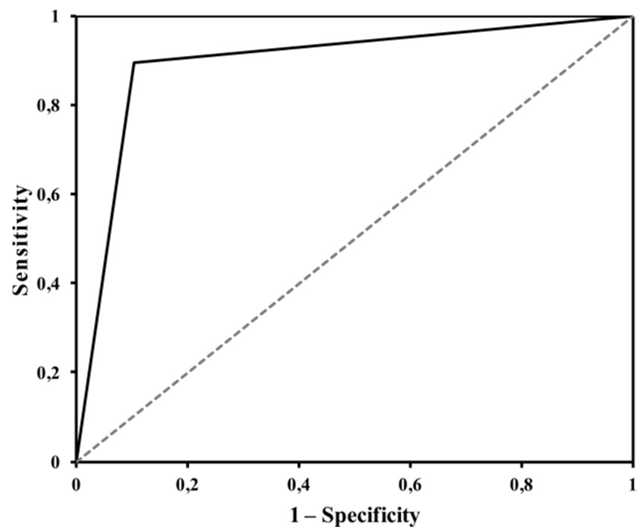

Figure 5.11 The ROC curve for the proposed model (Ferraz, Vicente, et al., 2016)

In (Ferraz et al., 2017), an analysis was conducted on a sample of children from two classes of 3rd and 4th grades, with 19 and 26 students, respectively, with 47,83% of the population being female and 52,17% male, with an age average of 9,18 years old. Table 5.2 shows the gender relation and the average of metrics such as age, achieved score per level, and completion time per level for 3rd grade class, 4th grade class and both classes combined.

Table 5.2 Comparison of relevant metrics of 3rd and 4th grades classes using disMAT

			Level 1		Level 2		Level 3	
	Avg. Age (years)	**Gender Relation**	**Avg. Score (%)**	**Avg. Completion Time (min)**	**Avg. Score (%)**	**Avg. Completion Time (min)**	**Avg. Score (%)**	**Avg. Completion Time (min)**
3rd Grade	8,68	7F/12M	83,68	01:42	63,68	02:09	70,00	04:29
4th Grade	9,54	15F/11M	93,85	01:31	65,38	02:19	74,23	04:06
Both Classes	9,18	22F/23M	89,56	01:36	64,67	02:14	72,44	04:16

From the main statistics on the two classes, 3rd, and 4th grades, it is highlighted:

- For Level 1, an average score of 89,56% and an average response time of 01:36 minutes;
- For Level 2, an average score of 64,67% and an average response time of 02:14 minutes;
- For Level 3, an average score of 72,44% and an average response time of 04:16 minutes.

This group of students had a lower performance in Level 2 in terms of the score achieved, and interpretation difficulties in Level 3, which resulted in a higher completion time. Table 5.3 shows the difficulties encountered, making a comparison between the two classes in terms of the number of students and the respective percentage, by field of difficulty. It should be noted that on average there are 18 students with difficulties, with the most affected fields being those dealing with measures related to size (63,04%), telling the hours (82,61%), measures related to weight (60,87%) and positioning/orientation (58,70%).

Table 5.3 Comparison of fields of difficulties of 3rd and 4th grades classes using disMAT

Field of Difficulties	No. Students			% Students		
	3rd Grade	**4th Grade**	**Both Classes**	**3rd Grade**	**4th Grade**	**Both Classes**
Size	5	3	8	26,32	11,54	17,39
Laterality	11	3	14	57,89	11,54	30,43
Memory (Pairs)	4	3	7	21,05	11,54	15,22
Measures (Height)	15	14	29	78,95	53,85	63,04
Time	18	20	38	94,74	76,92	82,61
Memory (Logo)	5	2	7	26,32	7,69	15,22
Measures (Weight)	17	11	28	89,47	42,31	60,87
Positioning/Orientation	12	15	27	63,16	57,69	58,7
Money	14	8	22	73,68	30,77	47,83
Memory (Puzzle)	0	1	1	0	3,85	2,17
Average	*10*	*8*	*18*	*53,16*	*30,77*	*39,35*

According to these results, it is found that dealing with measures, hours and orientation are the weak points of these students, perhaps because the teaching-learning system is not adequate or efficient, or because the tasks presented in the mobile game were not clear enough for them. Otherwise, there is a need to conduct more tests and different types of assessments in both classes to identify the failure of the system in these children. To conclude, each student in the 45-member group filled out a form, with 10 students answered that they had difficulties, 15 answered that they did not know if they had difficulties and 20 answered that they had no difficulties at all. Of the 25 who had difficulties or were in doubt about it, 14 answered that they had difficulties understanding what was being asked and 11 had difficulties understanding the game.

In (Ferraz, Neves, et al., 2019), the study was conducted with 1st, 2nd and 3rd grades teachers regarding the difficulties dealing with mathematical problems of their 5 to 8 years-old students. Teachers were asked to rate children's difficulties using a list of criteria aimed at both diagnosed and undiagnosed students. The assessment of the children's difficulties was made based on prior knowledge and daily interaction. The criteria list contained mixed tasks on the six types of dyscalculia and provided a qualitative overview. The Case-Based techniques used in the Cognitive Engine were applied to this sample and a measure of entropy was calculated, indicating a direct association between higher values of entropy to students with dyscalculia and lower values of entropy to students with no or few difficulties.

In (José Neves, Ferraz, et al., 2019), it is presented a study of learning impairments in students under an entropic perspective. The sample regards 100 students, and it assesses the mathematical skills and perceptions of these students. Although it is not directly related to dyscalculia, this study may be considered as a support to understand how difficulties in learning affect specific learning disabilities. With the application of Artificial Neural Networks to this approach, it is obtained an accuracy of 0,88 to 0,94, as seen in the coincidence matrix in Table 5.4, meaning that this a good model that assesses the influence of learning impairments under a social entropic perspective.

Table 5.4 The coincidence matrix for the proposed model (José Neves, Ferraz, et al., 2019)

Target	**Predictive**	
	True (1)	False (0)
True (1)	*[70, 72]*	[4, 7]
False (0)	[2, 5]	*[18,22]*

The sample of 53 students who participated in the three sessions described in the previous section, of whom 43,40% were female children and 56,6% were male, showed a positive progression between sessions. Students, aged 7 to 11 years-old, revealed an increase in average score achieved in Level 1 of about 7% and a decrease in average completion time in this level of about 33%. In Level 2, the average score achieved increased by 16% and the average completion time decreased by 14%. Lastly, in Level 4, the average score achieved increased by 15% and the average completion time decreased by 10%, indicating that the six-month practice helped to develop their knowledge and learning skills. The overall results are shown in Table 5.5.

Table 5.5 Comparison of relevant metrics of the 53 students in both sessions using disMAT

			Level 1		Level 2		Level 3	
	Avg. Age (years)	**Gender Relation**	**Avg. Score (%)**	**Avg. Completion Time (min)**	**Avg. Score (%)**	**Avg. Completion Time (min)**	**Avg. Score (%)**	**Avg. Completion Time (min)**
Session 2	9,00	23F/30M	89,80	01:33	68,50	02:12	71,90	04:12
Session 3			96,40	01:02	81,30	01:53	84,60	03:46

Regarding disMAT, the final version of the designed solution was classified as a Minimum Viable Product (Nguyen-Duc et al., 2019), a lean concept that classifies it as the version that allows collecting the maximum amount of validated knowledge about target users with the least amount of effort. In other words, this version provides a balance of the product's value to the end-users, the benefits, and the useful features.

Therefore, a Strengths, Weaknesses, Opportunities, and Threats (SWOT) (Milgram, 2009) analysis was conducted to assess this version, which helps to focus on strengths, minimize weaknesses, avoid threats, and explore opportunities (Table 5.6).

Table 5.6 SWOT analysis for disMAT

	Strengths	**Weaknesses**
Internal Origin	• Portuguese and English mobile game towards dyscalculia • Trendy support tool for children • Tasks with evolving difficulty • Intelligent task selection • Possibility to continue to play anywhere and at any time • Schools and/or educators have access to children's achievements • No competitor on the Portuguese market	• Android-oriented application • Implemented for Portuguese schools only • Online connection required to access students' performance • Modest design • Only ten tasks and three levels
	Opportunities	**Threats**
External Origin	• Lack of new technologies to implement in the education field • Lack of learning support systems to screen and accompany the children learning difficulties	• Use in the classroom can be a problem if the educators do not accept it • Arising of similar support tools as there are more companies investing in the mobile applications field

Alpha-beta testing (Mohd & Shahbodin, 2015) was also conducted: a few small groups of students tested and evaluated the solution. As their evaluation was positive, the solution went into a beta test and was tested by a larger group of students in the desired context over some time. Based on their analysis, some improvements were made to the solution (e.g., changes in the design, increasing/decreasing the difficulty of the tasks, etc.).

6 Conclusions

6.1 Discussion

As Rubinsten and Henik (Rubinsten & Henik, 2006) have shown in experimental studies, it was found in this study that children who were classified as having dyscalculia did indeed have difficulty associating Arabic numerals with the corresponding set. It was also observed that they took an unusually long time to identify the algorism corresponding to the number of objects in small groups, a difficulty also mentioned by Gaupp et al., Landerl et al., Opitz et al., and von Aster et al. (Gaupp et al., 2004; Karin Landerl et al., 2004; Moser Opitz et al., 2004; M. von Aster et al., 2005). To bridge this gap, they usually use their fingers and hands to count and make sure they are not mistaken, a strategy that is usually error-prone as Gonzalez et al., and Pixner et al. (Gonzalez & Espinel, 1999; Pixner & Kaufmann, 2008) explain. Another difficulty observed was the ability to remember objects and numbers observed seconds before. If they were then presented in a different order, this would be a task doomed to failure in most cases. Recalling arithmetic facts is often associated with children with dyscalculia, as is using the same arithmetic steps and in the same order, as studied by Geary et al., and Gersten et al. (Geary et al., 2012; Gersten et al., 2005).

The children who could express themselves well, both orally and in writing, had fewer or no difficulties during the sessions. The children who could express themselves well, both orally and in writing, had fewer or no difficulties during the sessions. In contrast, the children who had difficulty expressing themselves or who had problems interpreting did not perform well in the game tasks. As Cowan et al., and LeFevre et al. (Cowan et al., 2005; LeFevre et al., 2010) noted, this is a direct correlation between the children's language skills and their basic numeracy skills.

It was also observed that some children suspected of having dyscalculia, has a diagnosis of comorbid disorders such as dyslexia an ADHD, or mental health problems, an association noted by Auerbach et al., Landerl et al., Prior et al., and von Aster (Auerbach et al., 2008; Karin Landerl & Moll, 2010; Prior et al., 1999; M. G. Von Aster, 1996).

Similar to what Geary et al., and Jordan et al. (Geary & Hoard, 2001; Jordan et al., 2003) observed in their studies, in the six months between the last two sessions, the students improved their scores, both in terms of reaction time and the number of correct answers. Although the children with suspected dyscalculia made little progress in recalling arithmetic facts, their performance showed that, as mentioned by Ferraz et al. (Ferraz, Vicente, et al., 2016), working on their neuroplasticity has a positive impact on their learning difficulties.

On the other hand, children with learning disabilities did, indeed, have difficulties in social integration and failed in their academic performance. The social entropy resulting from their learning disabilities directly affected their daily interactions and social acceptance, and indirectly their growth, an association fact confirmed by Neves et al. (José Neves, Ferraz, et al., 2019).

Finally, the integration of AI can be helpful in different domains, but in education, it is undoubtedly interesting and compensatory, as Saltzman (Saltzman, 2015) mentions, especially when KRR, ANN and CBR are used to build decision models that are accurate and efficient, like those used in the Cognitive Engine. The studies of Begum et al., Blanco et al., Janssen et al., and Lakemeyer et al. (Begum et al., 2011; Blanco et al., 2013; Janssen et al., 2015; Lakemeyer & Nebel, 1994) helped to achieve the desired level of quality of the decision techniques used.

6.2 CONTRIBUTIONS

Throughout this thesis, several contributions were made, being possible to group them by lines of conducted research.

On a cognitive approach to Dyscalculia

The absence of awareness and information about Dyscalculia led to an attempt to conceptualize the issue, moreover, represent it on a knowledge representation and reasoning level. This gap was fulfilled with a cognitive approach based on intelligent decision support systems like ANNs and CBR, which aimed to estimate the evidence of dyscalculia (and other mathematical learning disabilities) considering the child's symptoms, performance or behaviour. Not only was given a formal representation of the matter but also was demonstrated the applicability of artificial intelligence on supporting dyscalculia's screening (Ferraz, Dias, et al., 2019; Ferraz, Neves, et al., 2016, 2019; Ferraz, Vicente, et al., 2016).

On Dyscalculia and Social Impairment

The social interactions with a child are truly relevant to his/her development, intellectual and behavioural. Analysing children's impairments from a social entropic perspective aided to cement the previous statement (José Neves, Ferraz, et al., 2019).

On using gaming in Dyscalculia

Creating appealing and efficient therapies for young ache children is a challenge. The developed game, disMAT, presents itself as an attractive therapy for children with or suspects of having dyscalculia since it aids to assess the children's reasoning, detect traces of dyscalculia, and attenuate the symptoms. The positive evaluation of this learning support tool was supported with intelligent decision support systems (Ferraz et al., 2017; Ferraz, Neves, et al., 2016; Ferraz, Vicente, et al., 2016).

On comorbidities and Dyscalculia

Dyscalculia is commonly associated with other comorbidities and learning impairments. The study of these connections can speed up screening and therapeutics (Ramalhosa et al., 2018).

On Healthcare and Artificial Intelligence

Artificial Intelligence's resources can be widely applied to improve processes and workflows in healthcare, as to produce a quicker diagnosis and adequate therapeutic plans. Even though it is necessary to approval from the technician or medical practitioner, the use of Deep Learning in this field can certainly enhance the quality of given healthcare (José Neves, Vicente, Esteves, Ferraz, Abelha, Machado, et al., 2018, 2018).

KRR, with processes as CBR and architectures as ANN, can be applied to detect tumours, to identify anomalies and other relevant features that help the medical people speed the healing process of the patient. The applicability of this AI's field to different areas of expertise proves the viability of this representation system (Dias et al., 2019; Faria et al., 2017; Mendonça et al., 2018; Neto et al., 2017; J. Neves, Alves, et al., 2019; J. Neves, Dias, et al., 2019; Joao Neves et al., 2017; João Neves, Faria, Alves, Ferraz, Vicente, & Neves, 2018; José Neves, Dias, Morais, Fonseca, Loreto, Alves, et al., 2018; José Neves, Vicente, Esteves, Ferraz, Abelha, Machado, et al., 2018; José Neves, Vicente, Ferraz, Leite, Rodrigues, Cruz, et al., 2018).

All the results and achievements of the research were disseminated by participating in conferences and publishing in journals of the area. Additionally, it was taken participation in projects, events, courses, and other academic functions.

Lastly, it is disclosed in two GitHub repositories (Discalculia Web[10] and disMAT[11]) the source code of the web application and the mobile application, respectively, for public use. Both repositories have numerous views and several downloads.

[10] https://github.com/disMAT/DiscaluliaWeb
[11] https://github.com/disMAT/disMAT_app

6.3 CONCLUSIONS

This research demonstrated that it is possible to create a cognitive approach to dyscalculia and support it with an evolving approach based on emerging technologies.

A formal knowledge representation of dyscalculia in children allows the identification of children's difficulties and an analysis of the outcomes of appropriate therapeutics. It also led to an intelligent and cognitive proposal of the dyscalculia status in Portuguese children under logical formalisms. Then, this representation allows the application of decision-making algorithms and techniques, such as KRR, ANN and CBR, to provide inferences from the analysis of the results. This reasoning system was designated the Cognitive Engine.

To support this representation, an evolving solution was designed, developed, tested, and built: disMAT. This solution represents a therapeutic tool in the form of a mobile game that has a dynamic and intelligent decision-making system responsible for selecting the tasks suitable for the child's needs using the Cognitive Engine. disMAT also provides a non-medical diagnosis of dyscalculia by analysing the child's performance and presenting it within the Discalculia Web, accessible to educators, teachers and experts associated with the child.

The developed solution, mobile and web, has been disclosed on an open-source framework (Discalculia Web's repository[12] and disMAT's repository[13]). This software of game-based solution is freely available for children, educators, teachers, and the public in general.

In addition to the system developed, a parallel study of the children's brains while engaging with this solution would be interesting and promising. Mapping the structure and functions of the brain in children with dyscalculia using non-invasive and painless exams, such as EEG, fMRI, and MRI, would provide more accurate information about the origin, causes and consequences of dyscalculia. However, the moral and ethical implications have been a major barrier for other researchers to conduct this type of clinical research, as has the cost of the exams and the lack of awareness of dyscalculia in most countries.

Another relevant approach in this area would be the integration of a Brain-Computer Interface (BCI) system[14] in children with dyscalculia, along with the use of the developed solution or not. After a confirmed diagnosis of dyscalculia and a description of the therapeutics, the child, using BCI, would have the opportunity to visibly train the neuroplasticity of the brain, making this system an encouraging tool to

[12] https://github.com/disMAT/DiscalculiaWeb
[13] https://github.com/disMAT/disMAT_app
[14] BCI system acquires brain signals, analyses them, and translates into commands transmitted to an output device.

complement the therapeutics. On the other hand, the experts in the field would have a precise overview of the severity of the dyscalculia and its weakening over time.

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APPENDIXES

A1 Evolving Solution

A1.1 Database Modelling

To understand how the system is structured, it is relevant to present a conceptual model. It represents the entities and the relationships between them.

The entities defined for the database component of this platform are as follows:

- Student – the entity responsible for storing data related to the students associated with the platform;
- Teacher – the entity responsible for storing data related to the teachers associated with the platform;
- Expert – the entity responsible for storing data related to the experts associated with the platform;
- Developer – the entity responsible for storing data related to all users associated with the platform;
- Class – the entity responsible for storing data related to the existing classes;
- Application – the entity responsible for storing data related to applications and mobile games associated with the platform;
- Application-level – the entity responsible for storing data related to application levels;
- Application-task – the entity responsible for storing data related to application tasks;
- Student score on application-level – the entity responsible for storing data related to the scores and times achieved by students at a level of an application;
- Student score on application task – the entity responsible for storing data related to the scores and times achieved by students on a task of an application.

The relationships defined for this step of the database creation process are:

- Class and student – a relationship aimed at understanding which students belong to which class;
- Class and teacher – a relationship aimed at representing the teacher that is responsible for a class;
- Class and expert – a relationship aimed at representing the experts who are responsible for a class;
- Application and application-level – a relationship aimed at representing the levels present in an application;

- <u>Application-level</u> and <u>application-task</u> – a relationship aimed at representing the tasks that exist within a level of an application;
- <u>Student</u> and <u>student score on application-level</u> – a relationship used to understand which student has achieved a score at a level of an application;
- <u>Student score on application-level</u> and <u>application-level</u> – a relationship that aims to represent the level of an application to which a score on a level relates;
- <u>Student score on an application-task</u> and <u>student score on application-level</u> – a relationship whose main objective is to make understandable to which level the score of an application task refers;
- <u>Student score on application-task</u> and <u>application-task</u> – a relationship whose main aim is to show to which task of an application-level a task score refers.

To outline the conceptual model, it was considered useful to draw a UML class diagram representing the database and its defined entities and relationships between them (Figure A.1).

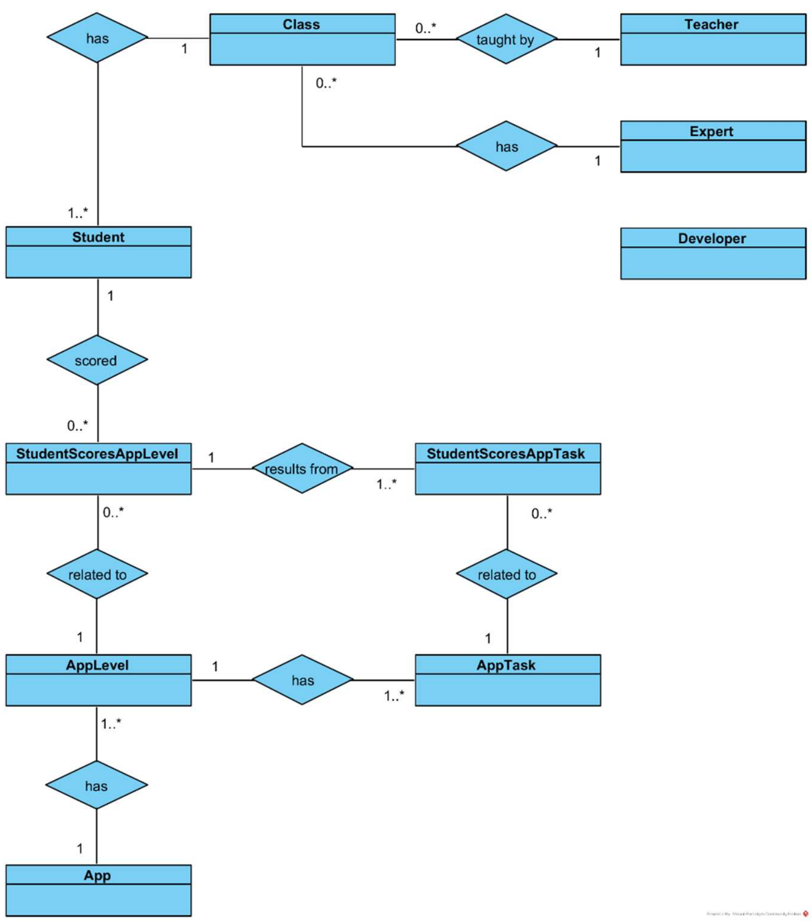

Figure A.1 Conceptual model of the database represented in a class diagram *(Image by Author)*

After creating the conceptual model, it is relevant to create a logical model, which is a more detailed description of the data flow in the system. In addition to the conceptual model, the attributes and definition keys of the entities and their relationships are represented in the logical model.

The referred attributes of a class or entity are features that give more detailed information about it, such as *name*, *age,* and *gender* if the entity is a *person*. On the other hand, the keys can be primary if they define the entity, or foreign if they define the relationships between entities. If we take a *person* as an entity as an example, its primary key can be the *identification number* because it is unique. If the other entity is an *object* and the entity *person* owns it, the attribute *owner* in the *object* is a foreign key that describes the relationship between the *object* and the *person*.

The analysis of all entities can be defined by the following primary keys:

- Student – an identification code: *studentID*;
- Teacher – an identification code: *studentID*;
- Expert – an identification code: *expertID*;
- Developer – an identification code: *developverID*;
- Class – an identification code: *classID*;
- Application – an identification code: *applicationID*;
- Application-level – a key composed of multiple attributes: the identification code of the application and the level number;
- Application task – a key composed of multiple attributes: the identification code of the application, the level and the task numbers;
- Student score on the application level – a key composed of multiple attributes: the identification codes of the student and the application, as well as the number of the level to which it refers, and the date (including time) when the score and time were reached;
- Student score on the application task – a key composed of multiple attributes: the identification codes of the student and the application, as well as the numbers of the level and the task to which it refers, and the date (including time) when the score and the time of the level were registered.

Regarding the relationships between entities, the following foreign keys can be defined:

- Class and student – the identification code of the class to which the student belongs: *Class_classID;*

- Class and teacher – the identification code of the teacher responsible for the class: *Teacher_teacherID;*
- Class and expert – the identification code of the expert responsible for the class: *Expert_expertID;*
- Application and application-level – the identification code of the application: *App_appID;*
- Application-level and application task – the identification code of the application and the level: *App_appID* and *AppLevel_levelNumber;*
- Student and student score on the application level – the student's identification code: *Student_studentID;*
- Student score on the application level and application level – the application's identification code and the number of the level to which the score refers: *App_appID* and *AppLevel_levelNumber;*
- Student score on the application task and student score on the application level – the identification code of the student and the application, as well as the level's number and the date when the scores were obtained: *Student_studentID, App_appID, AppLevel_levelNumber* and *StudentScoresAppLevel_scoreDate*;
- Student score on the application task and application task – the application's identification code and the level and task numbers: *App_appID, AppLevel_levelNumber* and *AppTask_taskNumber.*

All the primary and foreign keys and attributes of the defined entities and their relationships are shown in Table A.1.

Table A.1 Primary and foreign keys, and attributes of defined entities and their relationships

Entity	**Primary Key (PK)**	**Attributes**	**Foreign Key (FK)**
Student	*studentID*	*studentPassword* *studentName* *studentNumber* *studentAge* *studentGender*	*Class_classID*
Teacher	*teacherID*	*teacherPassword* *teacherName*	
Expert	*expertID*	*expertPassword* *expertName*	
Developer	*develoeprID*	*developerPassword* *developerName*	
Class	*classID*	*scholarshipLevel* *schoolName*	*Teacher_teacherID* *Expert_expertID*

Application	*appID*	*appName*	
Application-level	*App_appID* *levelNumber*		*App_appID*
Application task	*App_appID* *AppLevel_levelNumber* *taskNumber*		*App_appID* *AppLevel_levelNumber*
Student score on the application level	*Student_studentID* *App_appID* *AppLevel_levelNumber* *scoreDate*	*levelScore* *levelTime*	*Student_studentID* *App_appID* *AppLevel_levelNumber*
Student score on the application task	*Student_studentID* *App_appID* *AppLevel_levelNumber* *AppTask_taskNumber* *StudentScoresAppLevel_scoreDate*	*taskScore* *taskTime*	*Student_studentID* *App_appID* *AppLevel_levelNumber* *AppTask_taskNumber* *StudentScoresAppLevel_scoreDate*

The logical model can be translated into a UML class diagram as shown in Figure A.2.

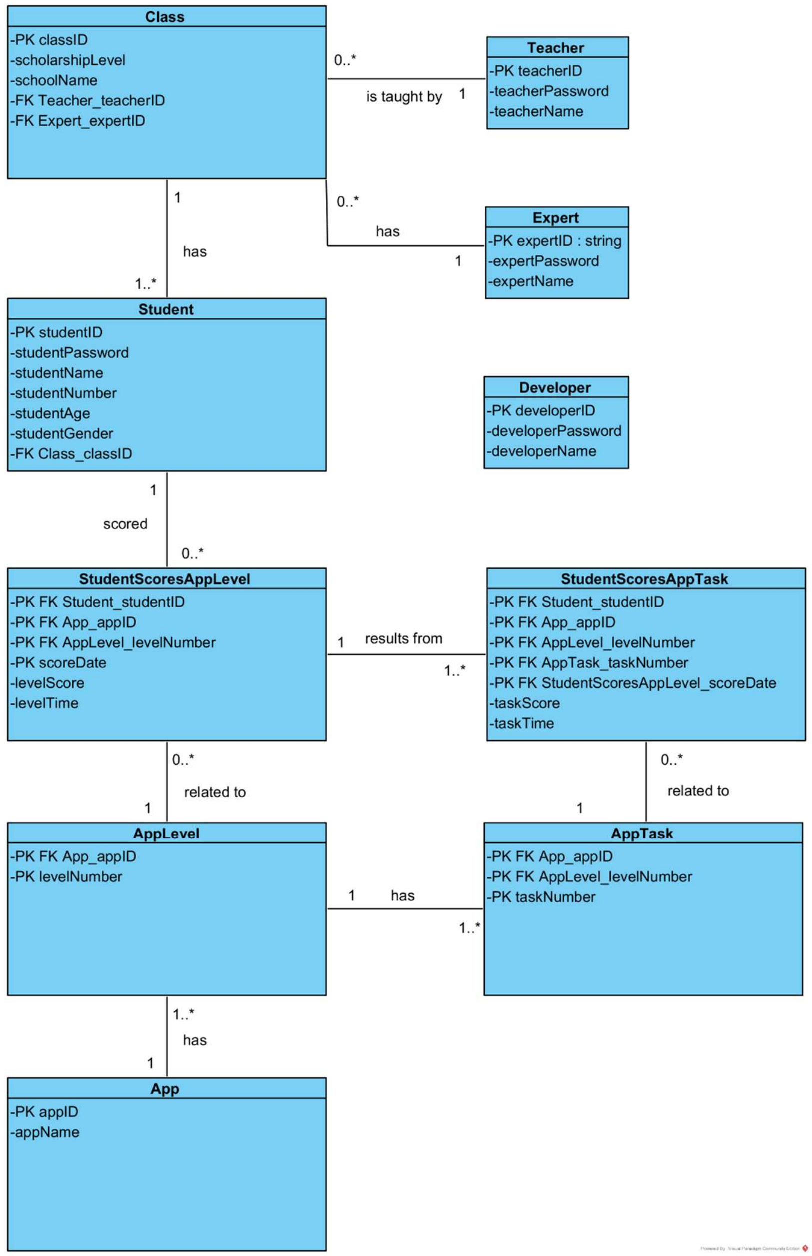

Figure A.2 Logical model of database represented in a class diagram *(Image by Author)*

The last modelling phase involves the construction of the physical model. This is a more detailed model than the logical model, converting entities and relationships, keys, and attributes into filled tables. So, with this nomenclature, tables represent entities and relationships, column names represent entities' attributes, column constraints detail the relationships between entities. It is also added the type of data of each attribute. As result, the physical layout is shown in Table A.2:

Table A.2 Physical layout of database

	Column name	Data type	Constraints
student	*studentID*	nvarchar(20)	PK, NN
	studentPassword	nvarchar(256)	NN
	studentName	nvarchar(200)	NN
	studentNumber	nvarchar(20)	NN
	studentAge	int	NN
	studentGender	nvarchar(1)	NN
	class_classID	nvarchar(20)	FK, NN
teacher	*teacherID*	nvarchar(20)	PK, NN
	teacherPassword	nvarchar(256)	NN
	teacherName	nvarchar(200)	NN
expert	*expertID*	nvarchar(20)	PK, NN
	expertPassword	nvarchar(256)	NN
	expertName	nvarchar(200)	NN
developer	*developerID*	nvarchar(20)	PK, NN
	developerPassword	nvarchar(256)	NN
	developerName	nvarchar(200)	NN
class	*classID*	nvarchar(20)	PK, NN
	scholarshipLevel	int	NN
	schoolName	nvarchar(200)	NN
	teacher_teacherID	nvarchar(20)	FK, NN
	expert_expertID	nvarchar(20)	FK, NN
applicat	*appID*	nvarchar(20)	PK, NN
	appName	nvarchar(200)	NN
applicat	*app_appID*	nvarchar(20)	PK, FK, NN
	levelNumber	int	PK, NN
application task	*app_appID*	nvarchar(20)	PK, FK, NN
	appLevel_levelNumber	int	PK, FK, NN
	taskNumber	int	PK, NN
	dyscalculiaArea	nvarchar(20)	NN
student score	*student_studentID*	nvarchar(20)	PK, FK, NN
	app_appID	nvarchar(20)	PK, FK, NN
	appLevel_levelnumber	int	PK, FK, NN

	scoreDate	datetime	PK, NN
	levelScore	int	NN
	levelTime	int	NN
student score on application	*student_studentID*	nvarchar(20)	PK, FK, NN
	app_appID	nvarchar(20)	PK, FK, NN
	appLevel_levelnumber	int	PK, FK, NN
	scoreDate	datetime	PK, NN
	levelScore	int	NN
	levelTime	int	NN

The data type of all names, identification codes and passwords are *nvarchar* characters with 200, 20 and 256 characters, respectively. The password must be encrypted to be stored in the database. Therefore, its length depends on the encryption algorithm chosen.

The student's number can contain letters and numbers, so it is also *nvarchar* data, as well as the student's gender, the school's name, and the area of dyscalculia.

The scholarship level represents the class's grade, which is numeric data, as it is the student's age, the level's number, the task's number, the level's score, and the level's time.

Lastly, the data of the register of the score is *datetime* data because it is important to record the data format with hours, minutes, and seconds.

The physical model is shown in Figure A.3 in the form of a UML class diagram.

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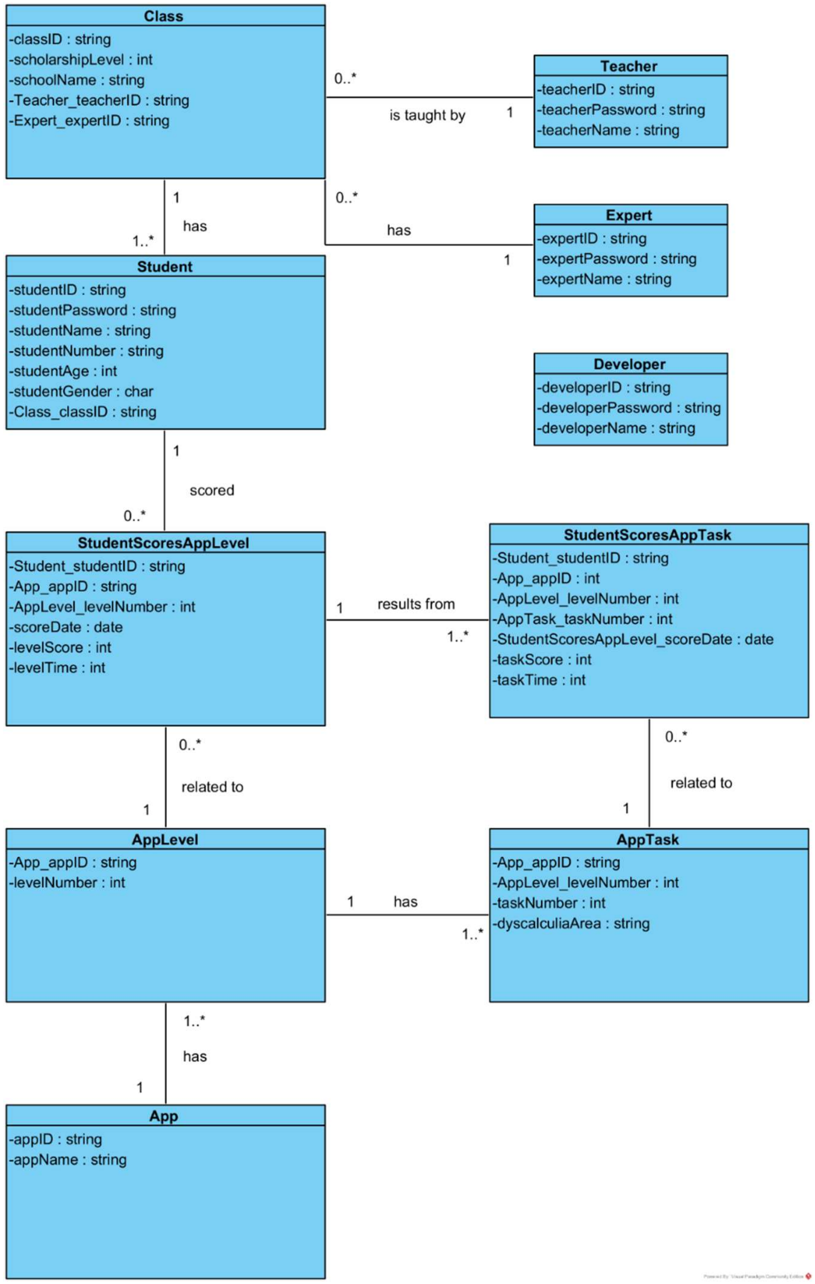

Figure A.3 Physical model of database represented in a class diagram *(Image by Author)*

A1.2 Web Application Modelling

In order to proper behaviour, the web application must have its accesses correctly assigned, as well as its functions. So, to analyse and grant the appropriate permissions and access to certain functionalities, it is useful to draw Use Case diagrams for each user.

For the student, the user with fewer permissions should be granted access to a general analysis of his/her performance, with an option to visualize more details on it. This user should also have access to all the scores achieved and registered times with a correspondent evaluation of these metrics (Figure A.4).

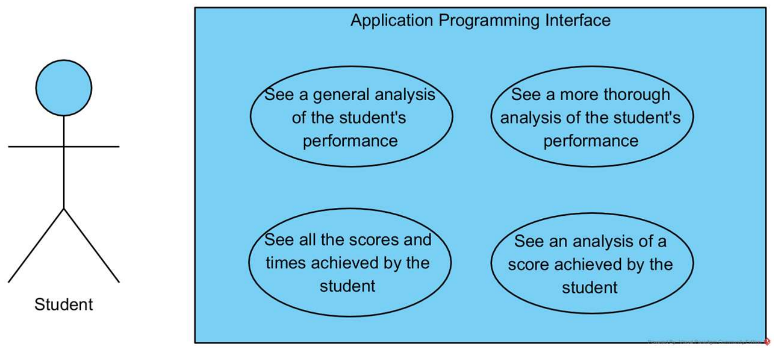

Figure A.4 Student's use case diagram for the web application *(Image by Author)*

Regarding the teacher, he/she must have access to three levels of information: the classes that the user is responsible for, each class itself, and the students enrolled in each class. Therefore, this user must be able to see the overall performance of every class that teaches, as well as the comparison of every student within each class. Lastly, this user must be able to see every student's performance on an individual level, and on the class level (Figure A.5).

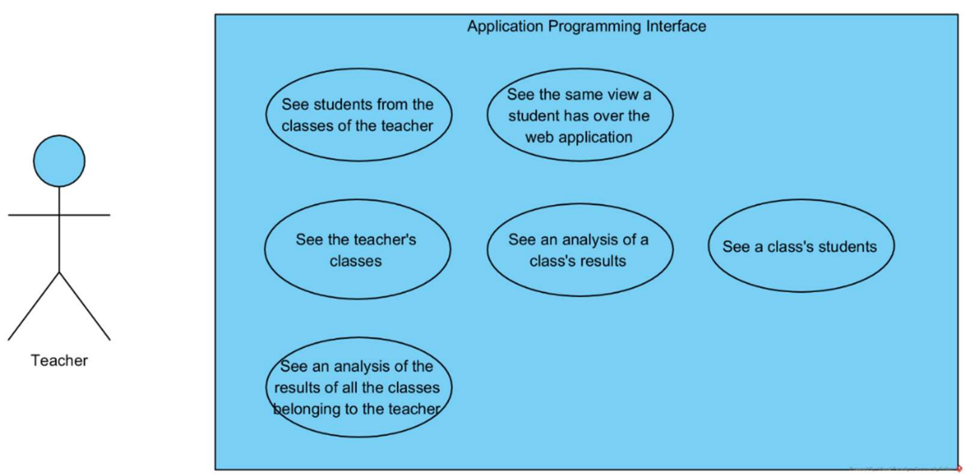

Figure A.5 Teacher's use case diagram for the web application *(Image by Author)*

With almost the same permissions as the teacher, the expert should be able to visualize the metrics on the student's level and a general level regarding a group of students. This user, similarly, to the teacher, should be able to access all the classes assigned to him/her, and, within each class, to each student enrolled in. The expert must be able to see the analysis of the results compared between classes, within classes and between students. Finally, this user must have access to every student's performance and alter the automatic analysis resultant from the gathering process (Figure A.6).

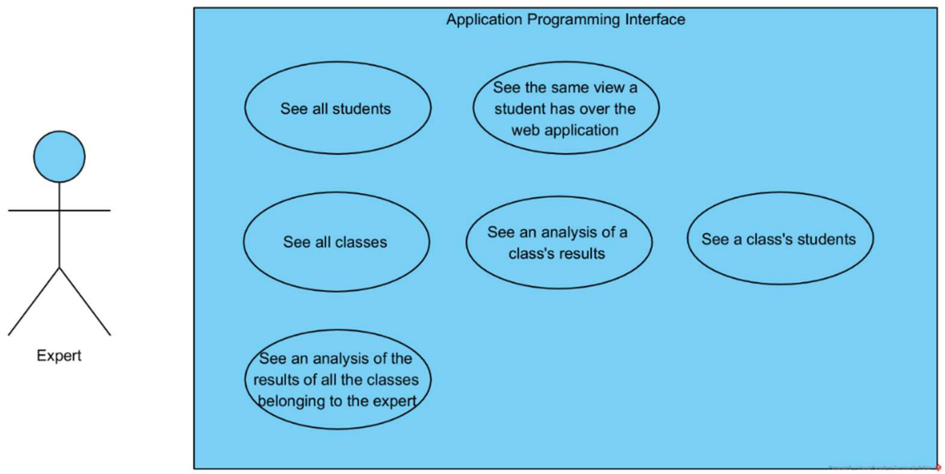

Figure A.6 Expert's use case diagram for the web application *(Image by Author)*

Regarding the administrator, this is a super user since he/she must have access to all functionalities and information. As so, the administrator must be able to add, delete and edit users (students, teachers, experts, and administrators), as well as the information concerning each one. Nonetheless, the administrator is a case-sensitive user, whereby this access should be carefully assigned (Figure A.7).

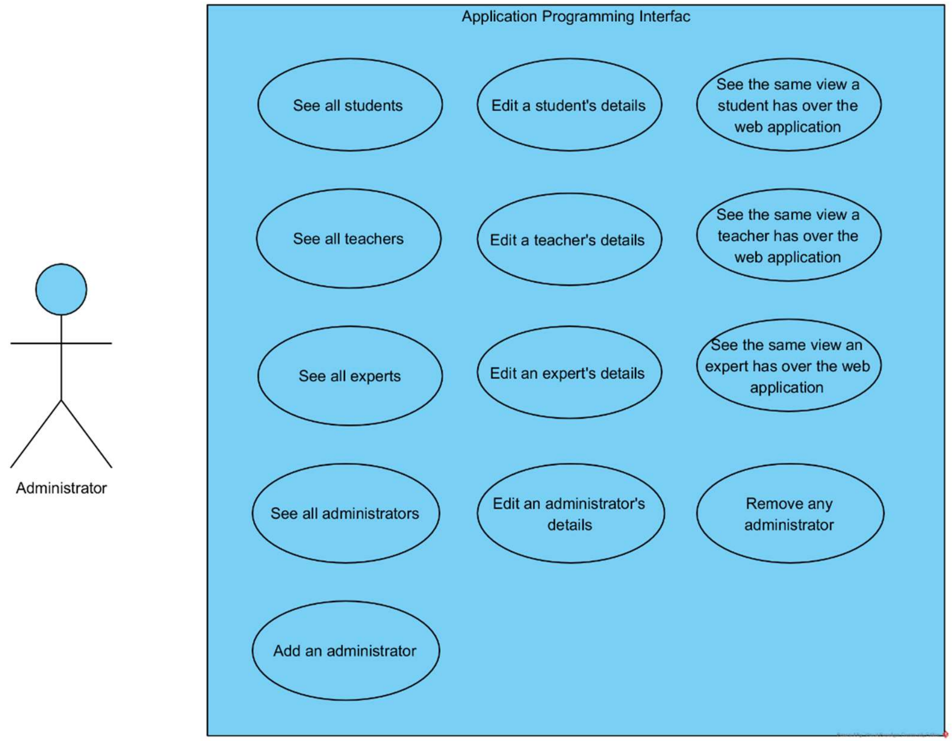

Figure A.7 Administrator's use case diagram for the web application *(Image by Author)*

After designated the functionalities and web application accesses, comes the modelling phase of designing the Graphical User Interface (GUI). In addition to the consult users' details and results' analysis, all the users must interact with a login interface, a personal information lookup and an "About" division (*"Acerca"* in Portuguese) where the users have access to the context of the web page and contact information for support.

Starting with the initial screen (Figure A.8) the user is presented when accessing the web application, which allows the user to log in with the credentials previously provided. If the user has forgotten the password, must have an option to remind to contact an administrator (information presented in speech balloon).

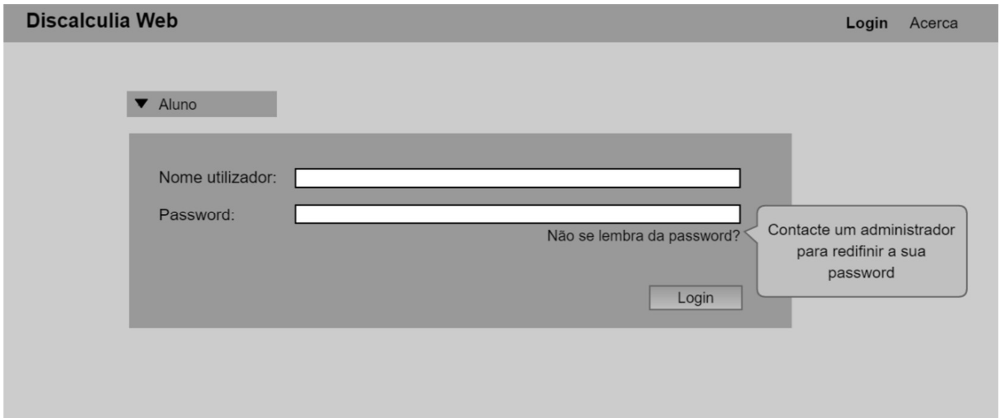

Figure A.8 Initial web page presented on the web application *(Image by Author)*

As referred before, there must have an *"About"* option, introducing the user to the context of this web application, and briefly informing what dyscalculia is (Figure A.9). This text must be presented in Portuguese.

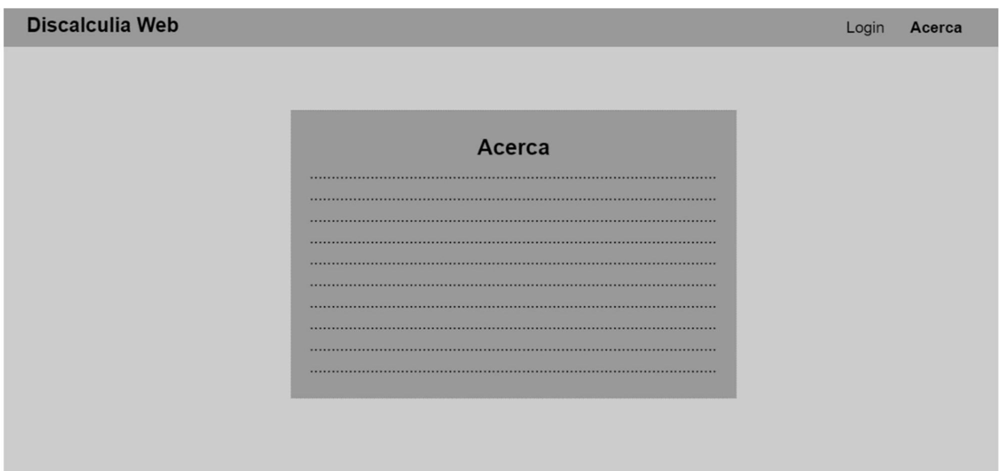

Figure A.9 *"About"* web page presented on the web application *(Image by Author)*

After login, the student must be presented with all options: consult a general analysis of his/her results, consult the historic of scores, consult his/her profile, change the password, or log out (Figure A.10).

Figure A.10 Student's opening web page presented on the web application *(Image by Author)*

When choosing the option to consult a general analysis of his/her results, it must be presented a graphical assessment completed with written recommendations, suggestions and/or conclusions (Figure A.11). The graphical analysis must contemplate the student's scores and times, as well the student's comparison with the class's overall.

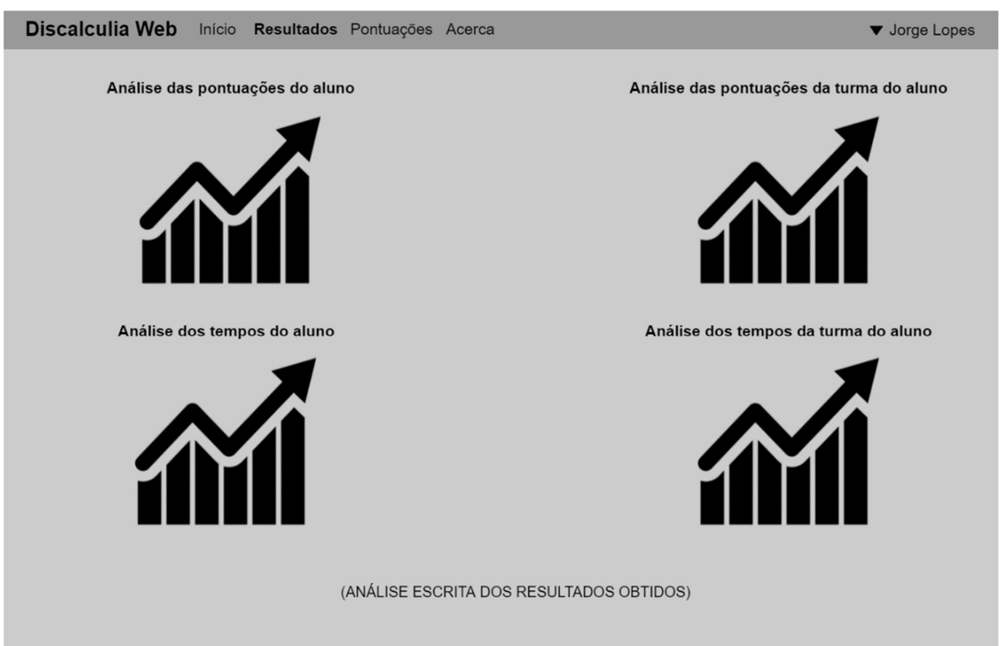

Figure A.11 Student's performance general analysis web page presented on the web application *(Image by Author)*

From the student's opening web page, the option to consult the historic of scores should lead to a web page with his/her scores obtained by level of difficulty, with the registry date displayed (Figure A.12):

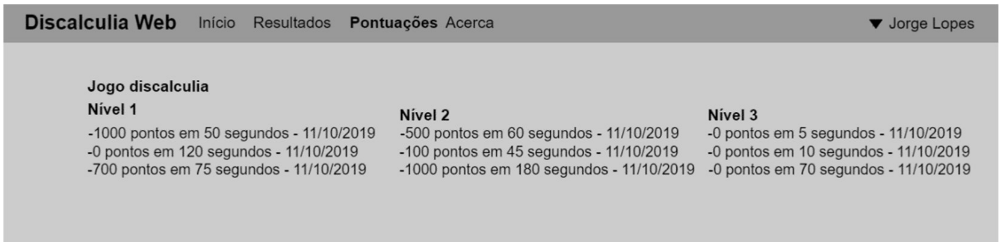

Figure A.12 Student's history of the score's web page presented on the web application *(Image by Author)*

If the student selects a score, must be presented a detailed registry of his/her scores per task and respective obtained time, as well the comparison with the class's overall for that tasks and level (Figure A.13). The level and date of the registry must be referred to, as well as a graphical view of the scores.

Figure A.13 Student's analysis of scores of a selected registry web page presented on the web application *(Image by Author)*

Regarding the teacher's view of the web application, after logging in, it must be presented options like consult his/her students, his/her classes, the results' analysis, along, his/her profile, change the password or log out (Figure A.14).

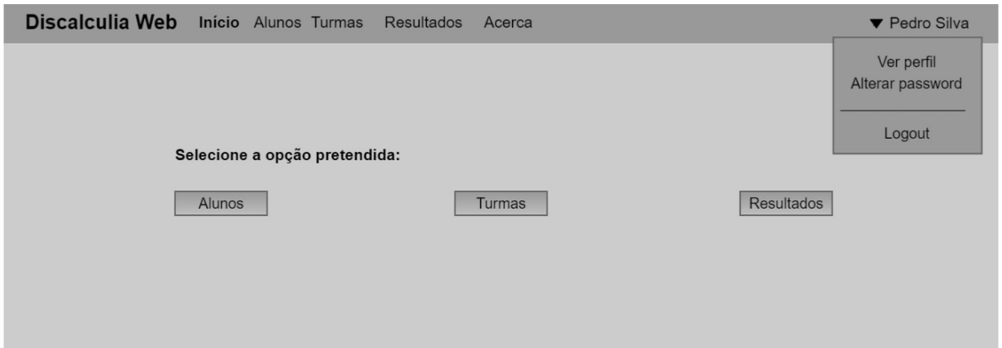

Figure A.14 Teacher's opening web page presented on the web application *(Image by Author)*

Selecting the option to consult the students, must be presented to the students by class, allowing each student's name to redirect to the child's results and performance (Figure A.15).

Figure A.15 Teacher's group of students per class web page presented on the web application *(Image by Author)*

If the teacher had chosen the option to see his/her classes, it must be presented a list of those classes, being each one linked to that class's student. If the teacher chooses to select that class's name again, it must be presented the class's results and overall performance (Figure A.16, Figure A.17 and Figure A.18).

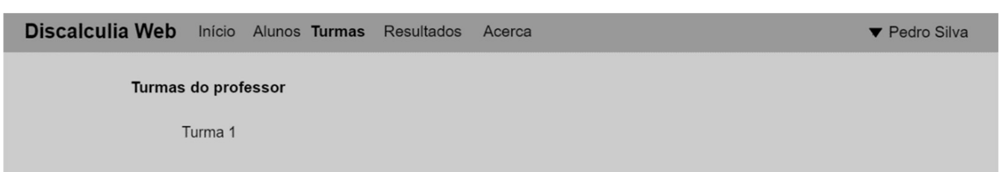

Figure A.16 Teacher's group of the classes' web page presented on the web application *(Image by Author)*

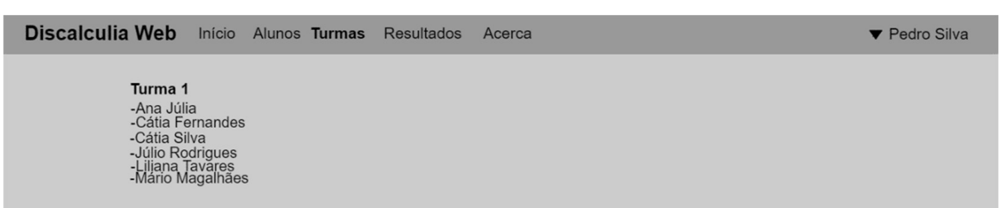

Figure A.17 Teacher's group of students of a specific class web page presented on the web application *(Image by Author)*

Figure A.18 Teacher's general performance per class and between classes web page presented on the web application

(Image by Author)

Lastly, the teacher may be able to consult the position of his/her classes compared to all classes on the system. Therefore, should be presented an overall graphic illustration representative of his/her classes' results in score and time, and a graphic representation of all the classes' performance, so it can be made a comparison (Figure A.19).

Figure A.19 Teacher's general performance of his/her classes compared to all classes' performance web page presented on the web application *(Image by Author)*

The expert must have access to similar web pages to the teacher does. Hence, the options to consult students, classes, results, personal profile or logging out must be presented after the expert logs in (Figure A.20).

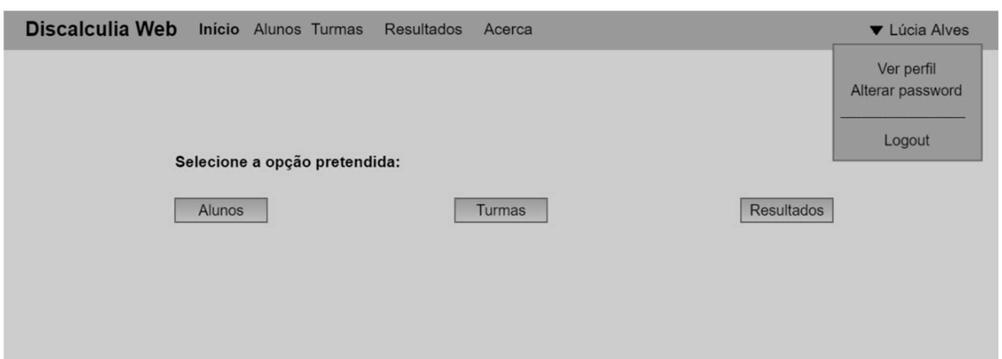

Figure A.20 Expert's opening web page presented on the web application *(Image by Author)*

Selecting the option "students", the expert must have access to the view where all the students in the system are grouped by class (Figure A.21).

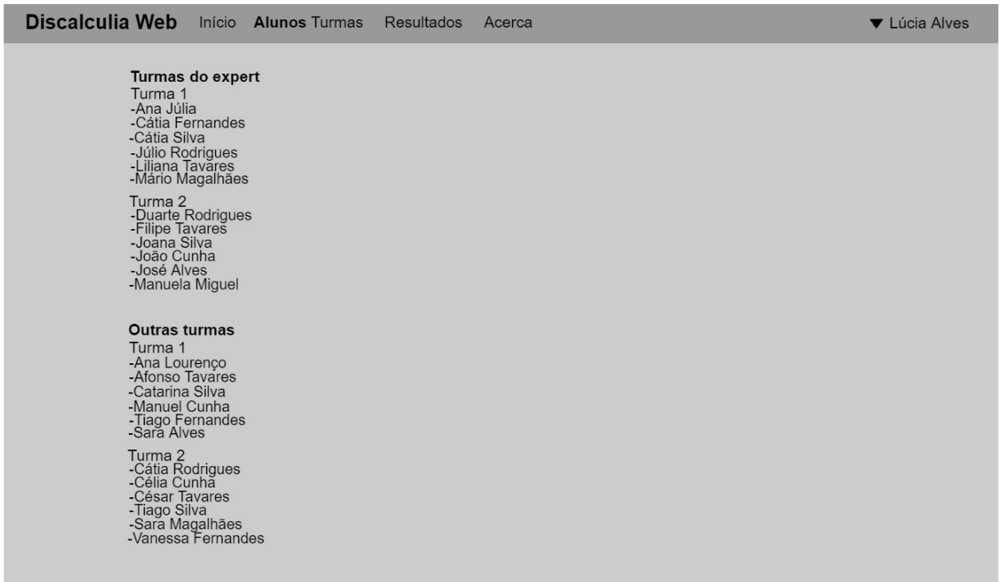

Figure A.21 Expert's group of students per class web page presented on the web application *(Image by Author)*

Each students' name must be redirected, when selected, to this student's analysis of results. Contrary to the teacher, the expert must be able to visualize the performance of any student.

To visualize the analysis of the results per class, the expert must select that option and be presented with the list of classes he/she is responsible for, and the rest of the classes registered on the database (Figure A.22). Each class must be linked to a web page with the overall performance of that class (Figure A.23).

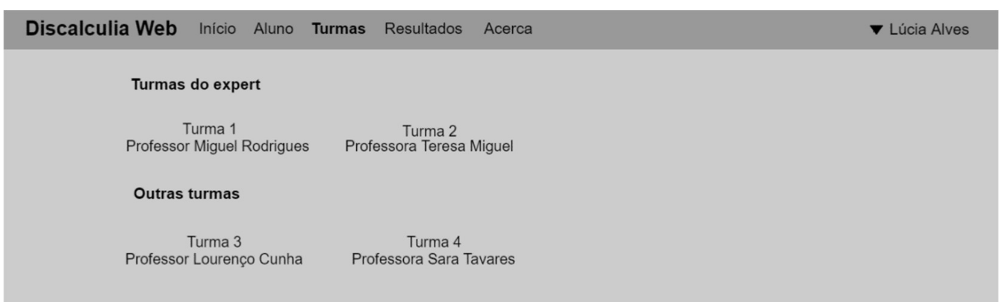

Figure A.22 Expert's group of classes' web page presented on the web application *(Image by Author)*

Figure A.23 Expert's general performance per class and between classes web page presented on the web application *(Image by Author)*

The expert may also be able to visualize a graphical illustration comparing his/her classes to all classes in the database. This comparison must be supported by a written analysis (Figure A.24).

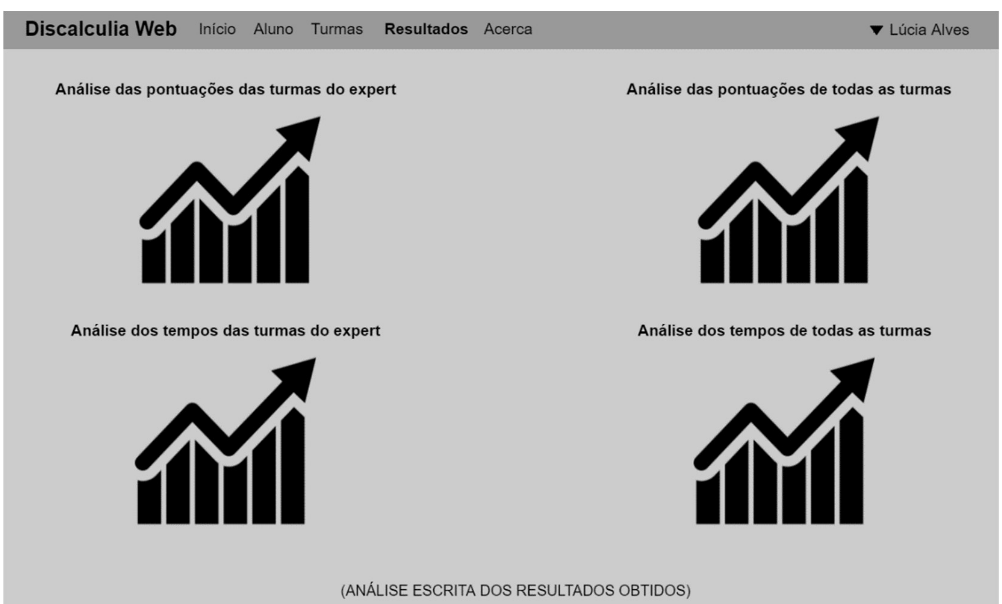

Figure A.24 Expert's general performance of his/her classes compared to all classes' performance web page presented on the web application *(Image by Author)*

The last user, the administrator, must have access to is the one presented in Figure A.25, where every type of user is presented and can be selected to access its information, whether to consult, edit, add, or delete. This screen must emerge after an administrator logs in.

Figure A.25 Administrator's opening web page presented on the web application *(Image by Author)*

After selecting one type of user, the administrator must have access to the registry of:

- All students if the user selected is "students" (Figure A.26);
- All the teachers if the user selected is "teachers" (Figure A.27);
- All experts if the user selected is "experts";
- All administrators if the user selected is "administrators".

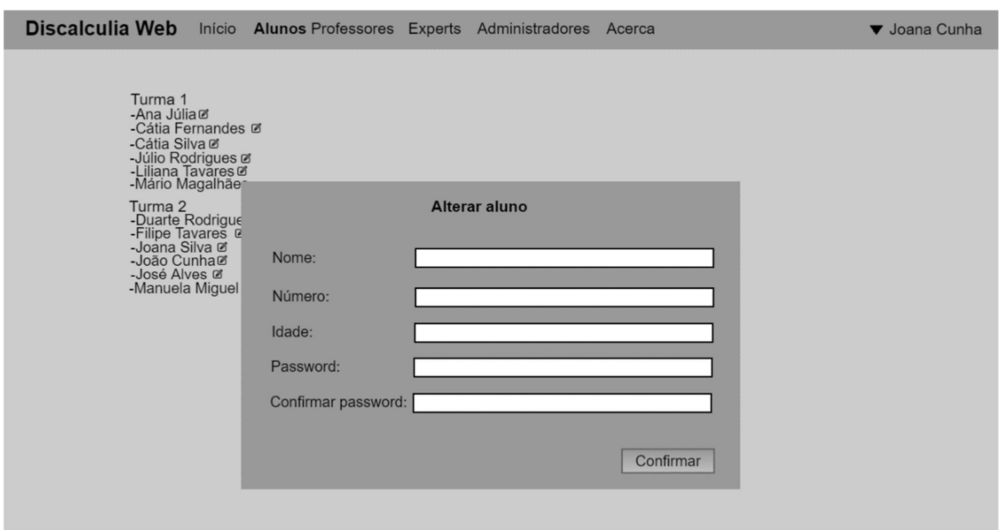

Figure A.26 Administrator's group of students' web page presented on the web application *(Image by Author)*

If a user's name is selected, the administrator must have permission to edit that user's personal information, such as the name and password. If the user selected is a student, the administrator must be able to edit the student's number and age.

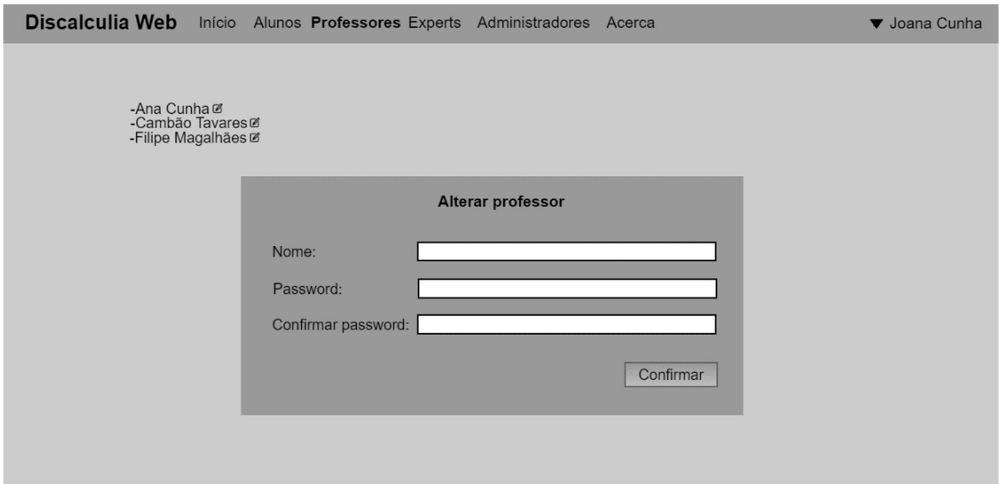

Figure A.27 Administrator's group of teachers' web page presented on the web application *(Image by Author)*

The administrator must have access to all the web pages the other users have access to, presented the same way.

A2 DISMAT

A2.1 API USE CASE DIAGRAMS

In order to implement the API and to confirm the requirements for each user, it was made simplified use case diagrams. Regarding the student, the information is presented in Figure A.28; for the teacher, the requirements are presented in Figure A.29; for the expert, it is represented in Figure A.30; and for the administrator or developer, it is shown in Figure A.31.

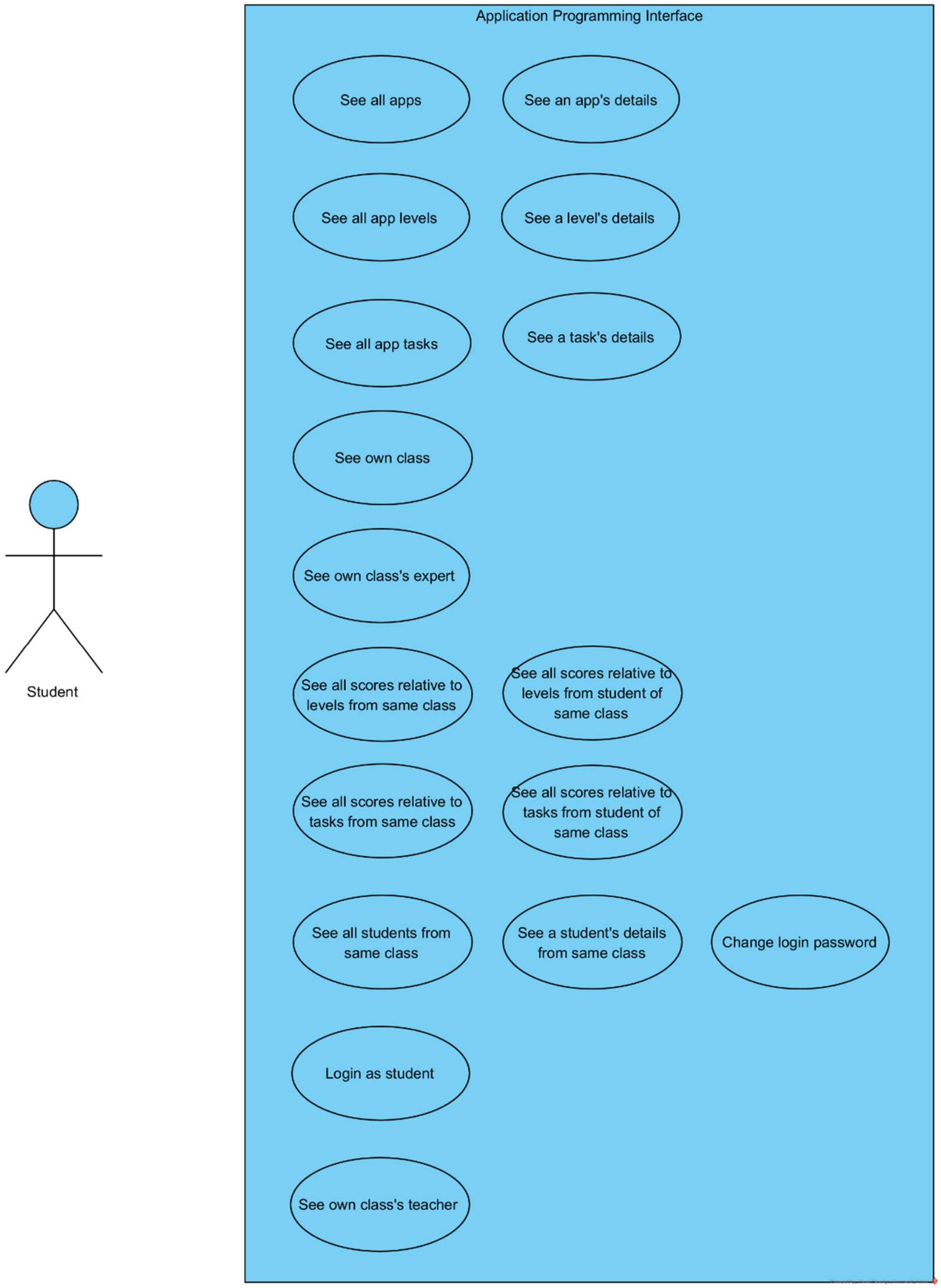

Figure A.28 Student's use case diagram for the API *(Image by Author)*

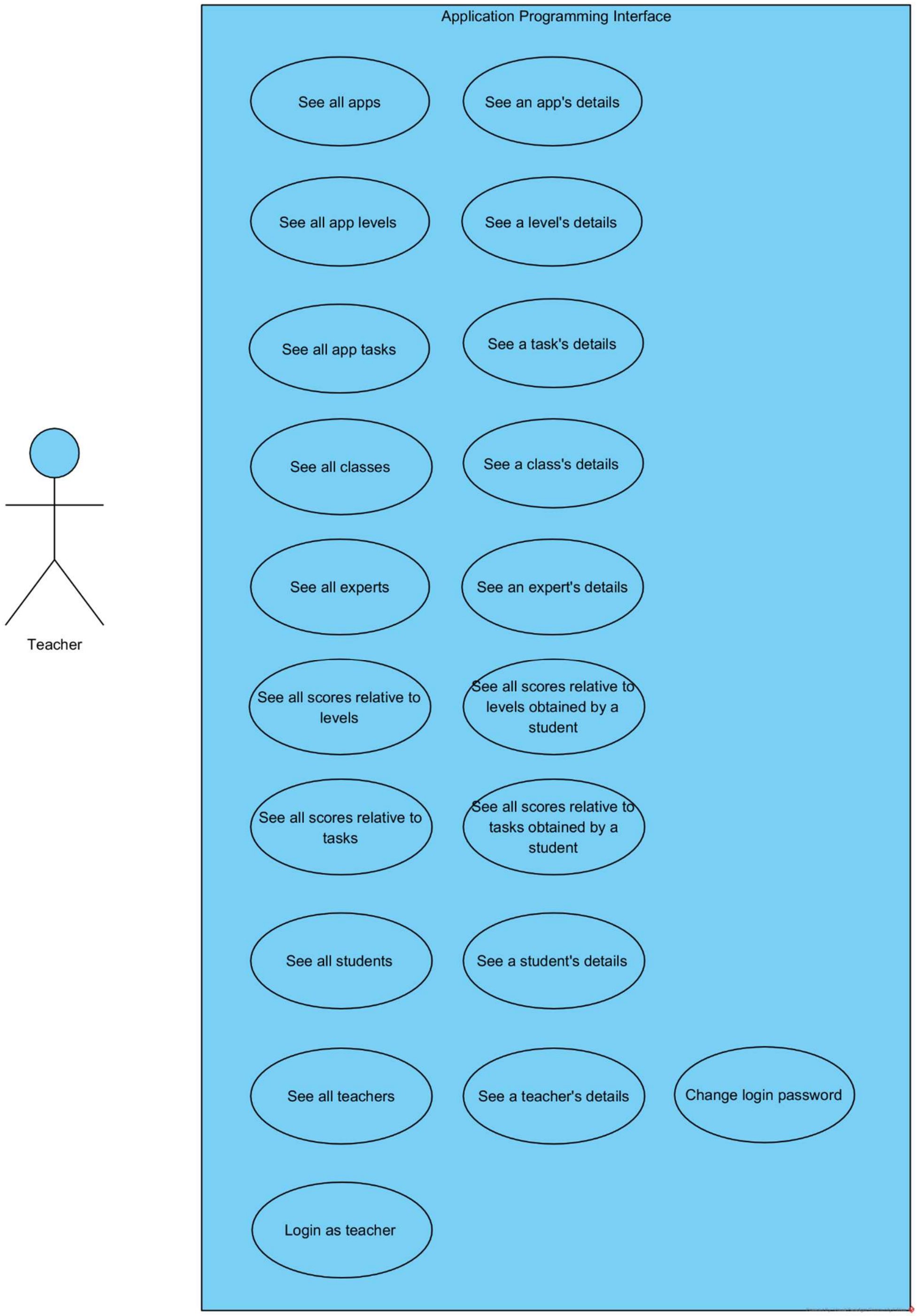

Figure A.29 Teacher’s use case diagram for the API *(Image by Author)*

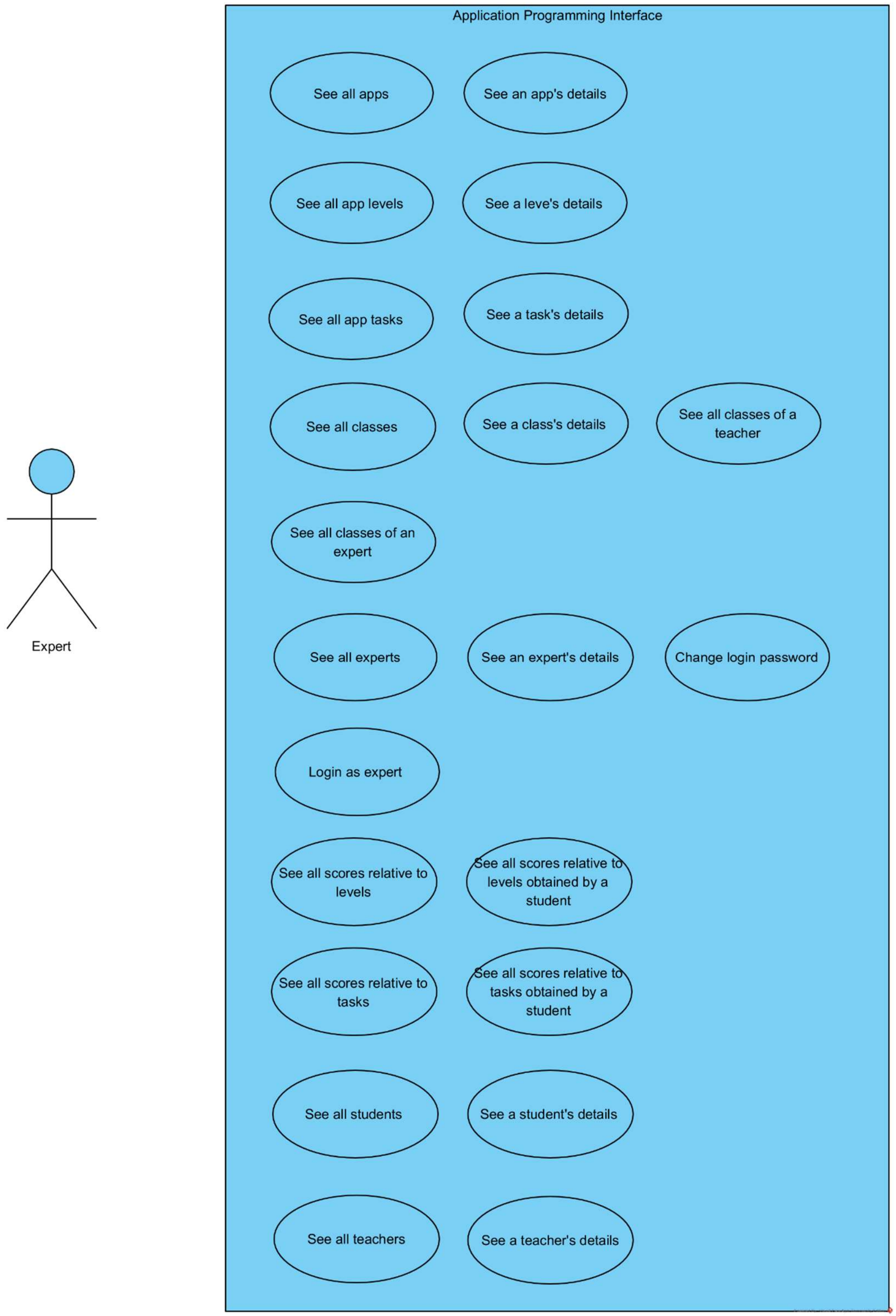

Figure A.30 Expert's use case diagram for the API *(Image by Author)*

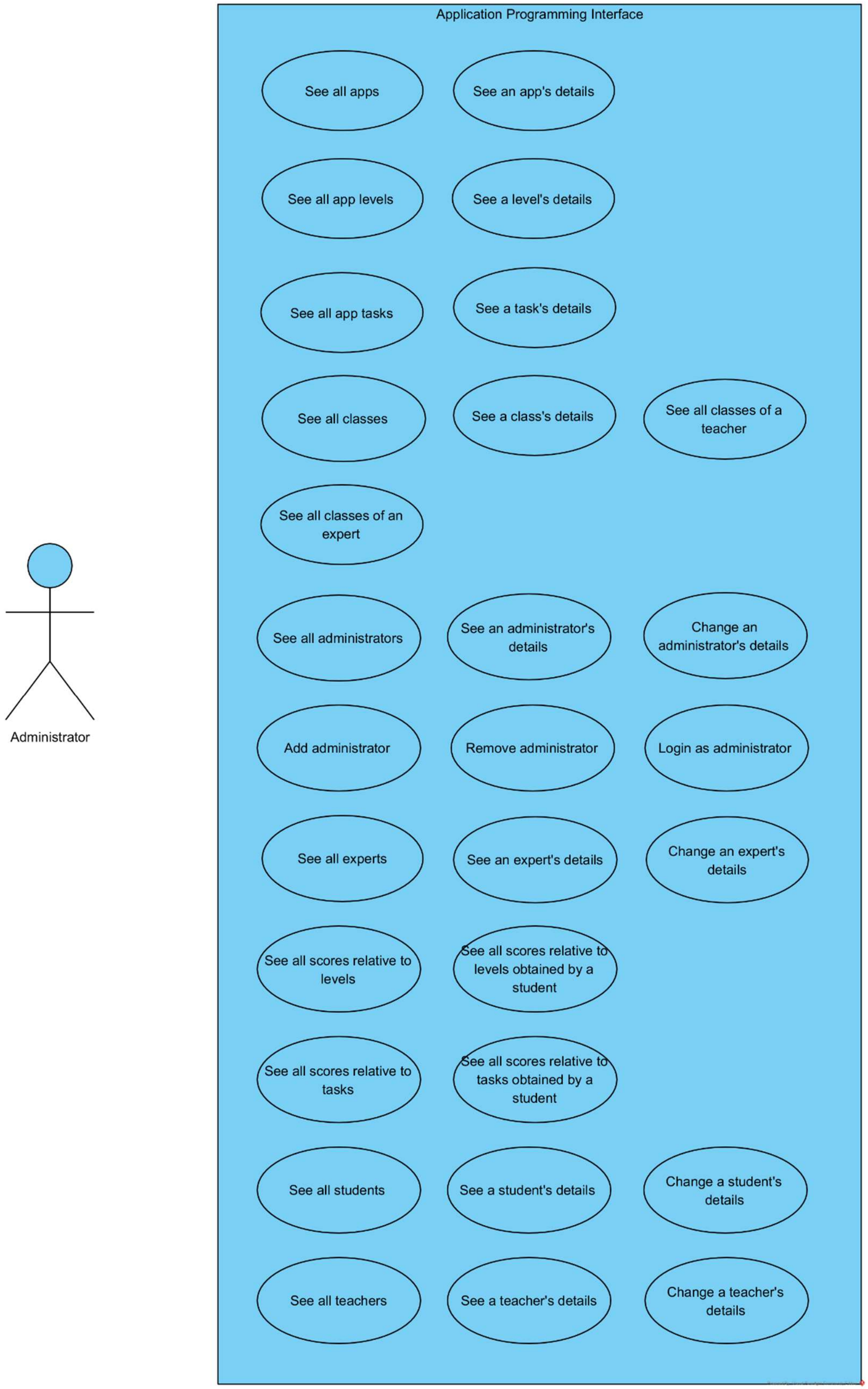

Figure A.31 Administrator's use case diagram for the API *(Image by Author)*

A2.2 Database Physical Model

The construction of the database resulted in the following diagram generated by this Microsoft SQL Server, shown in Figure A.32.

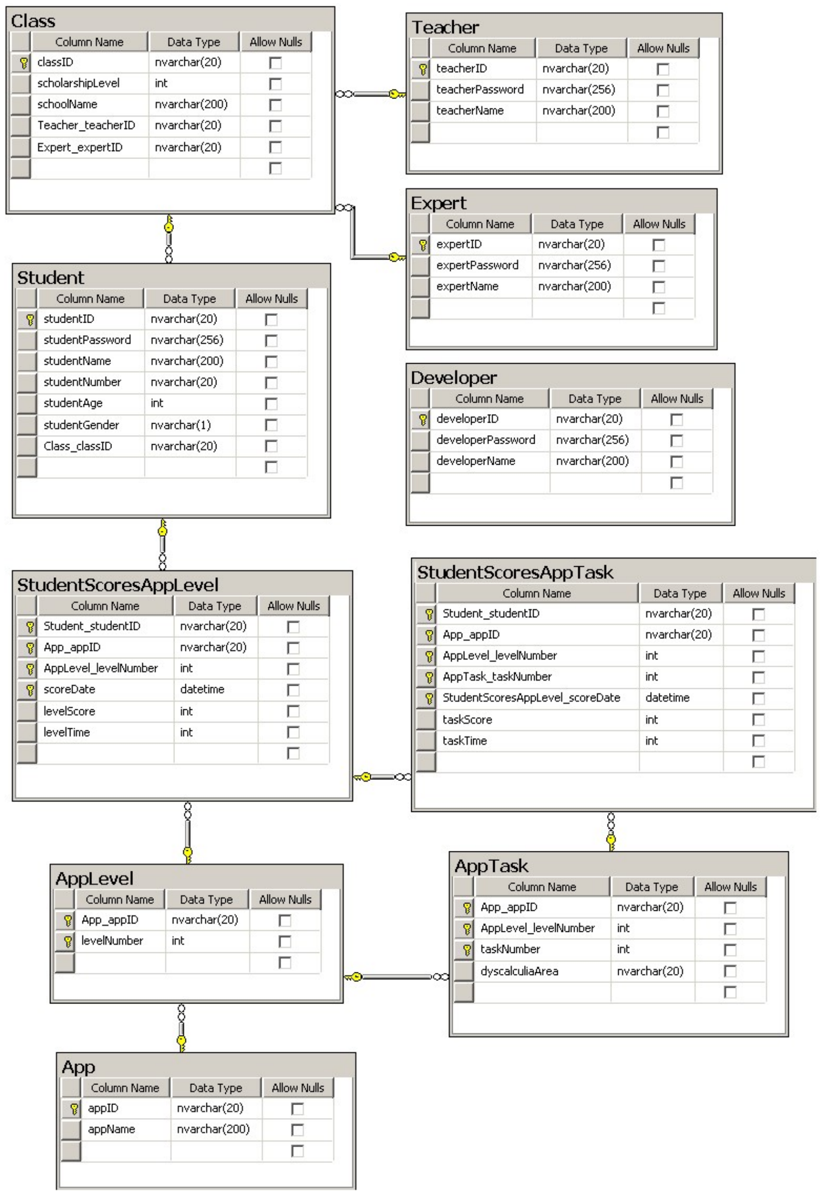

Figure A.32 Physical model of database generated by Microsoft SQL Server *(Image by Author)*

A2.3 Discalculia Web Previews

Discalculia Web has different web pages that are displayed when the user selects a certain option and if the user has the permissions to visualize it.

After connecting to the web application via its public internet address, the initial web page, shown in Figure A.33, is displayed with the platform's logo in the upper left corner.

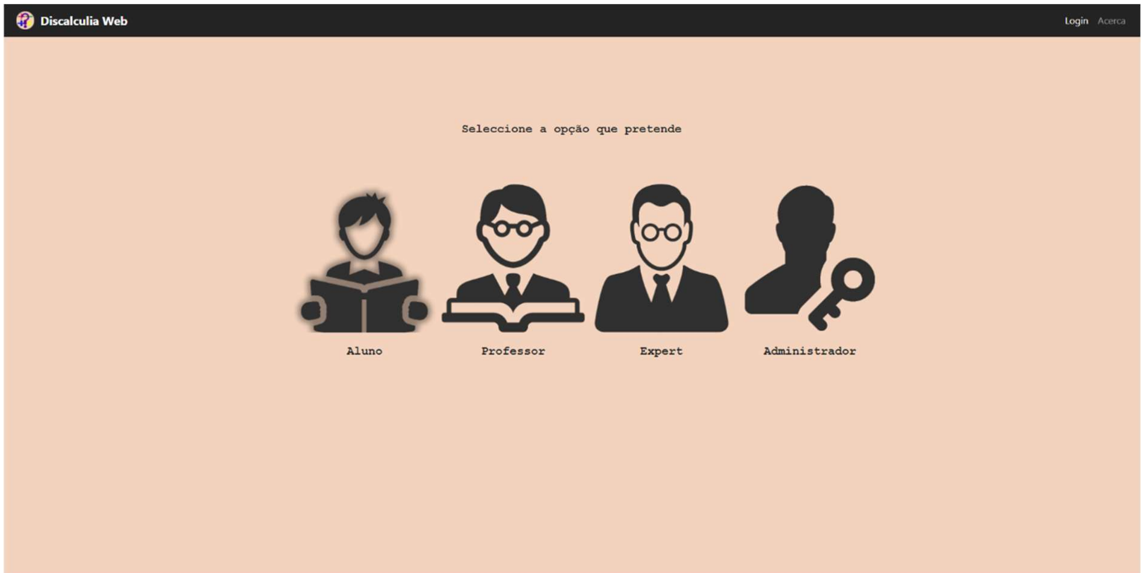

Figure A.33 Preview of the initial web page on Discalculia Web *(Image by Author)*

On this first page, the user is offered four options – the four types of users who can log in: student, teacher, expert, and administrator. Each image is clickable and leads to the login page for that user. At the top right of the page, the user has the options *"Login"* and *"Acerca"* (*"About"* in English). The last one presents information about the project, the web application, and instructions on how to use the platform, as shown in Figure A.34, and is accessible to all users.

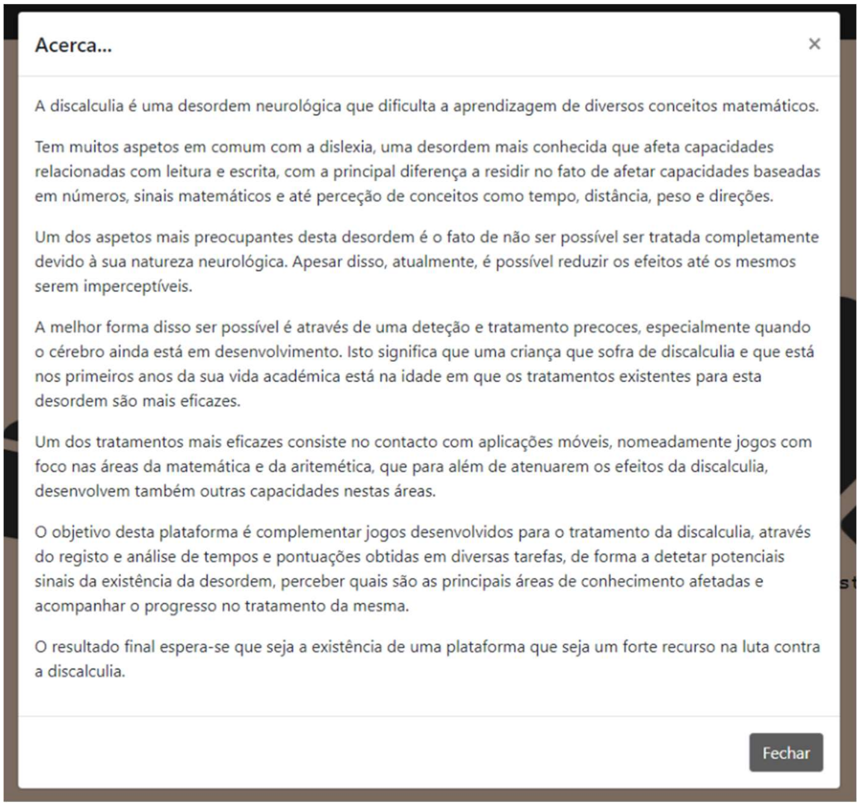

Figure A.34 Preview of the *"Acerca"* pop-up on Discalculia Web *(Image by Author)*

Selecting one of the options presented on the initial page, the login page is presented to the user, as shown in Figure A.35.

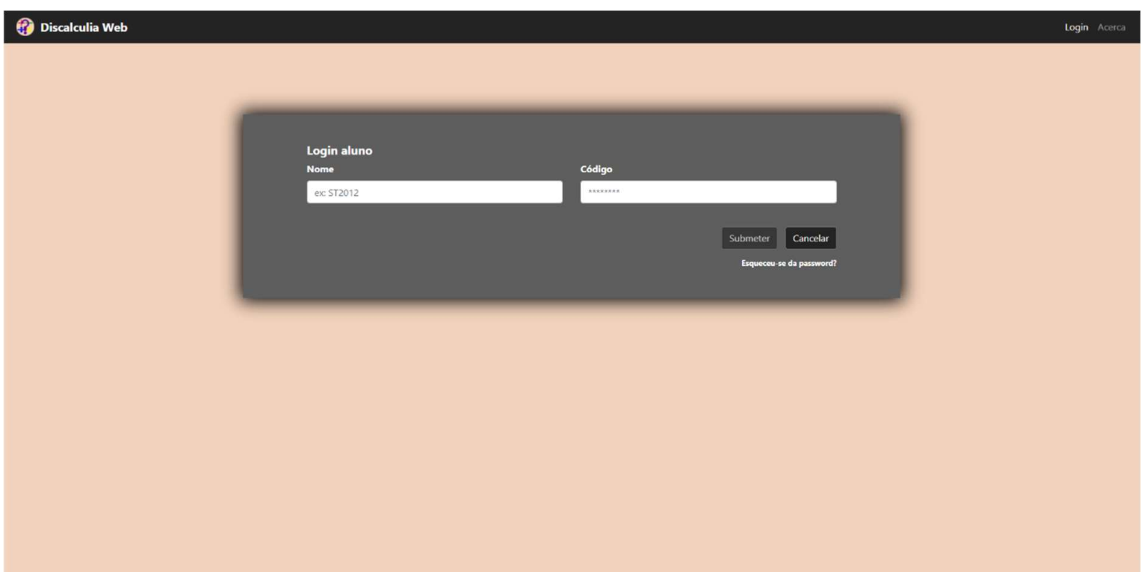

Figure A.35 Preview of the login page on Discalculia Web *(Image by Author)*

The previous figure shows the student’s login page, but it is identical for each user. The left field is for entering the name and the right field is for the corresponding password. The user had two options: *“Submeter”* (*“Submit”* in English) and *“Cancelar”* (*“Cancel”* in English). Under these options, the user can recover the lost password or contact the developer via the email given there.

After logging in as a student, the student’s results page is displayed, as shown in Figure A.36.

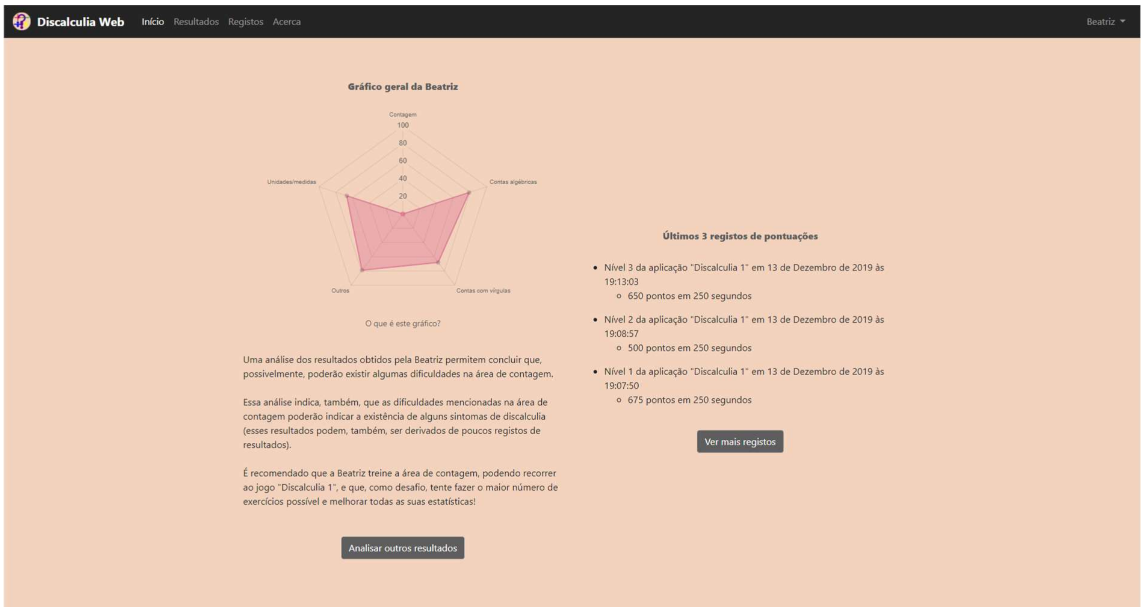

Figure A.36 Preview of the student’s results page on Discalculia Web *Image by Author)*

On the left-hand side of the page, as shown in Figure A.37, is a radar chart showing the percentage score concerning the tasks of the different areas of dyscalculia on which the student was tested. There is also a written analysis of these results discussing possible areas of knowledge where there may be dyscalculia symptoms, areas where the student has difficulty, and therapeutic recommendations based on the student's difficulties, as well as an *"Analisar outros resultados"* (*"Analyse other results"* in English) button that takes the user to a page with a more detailed analysis of the student's performance.

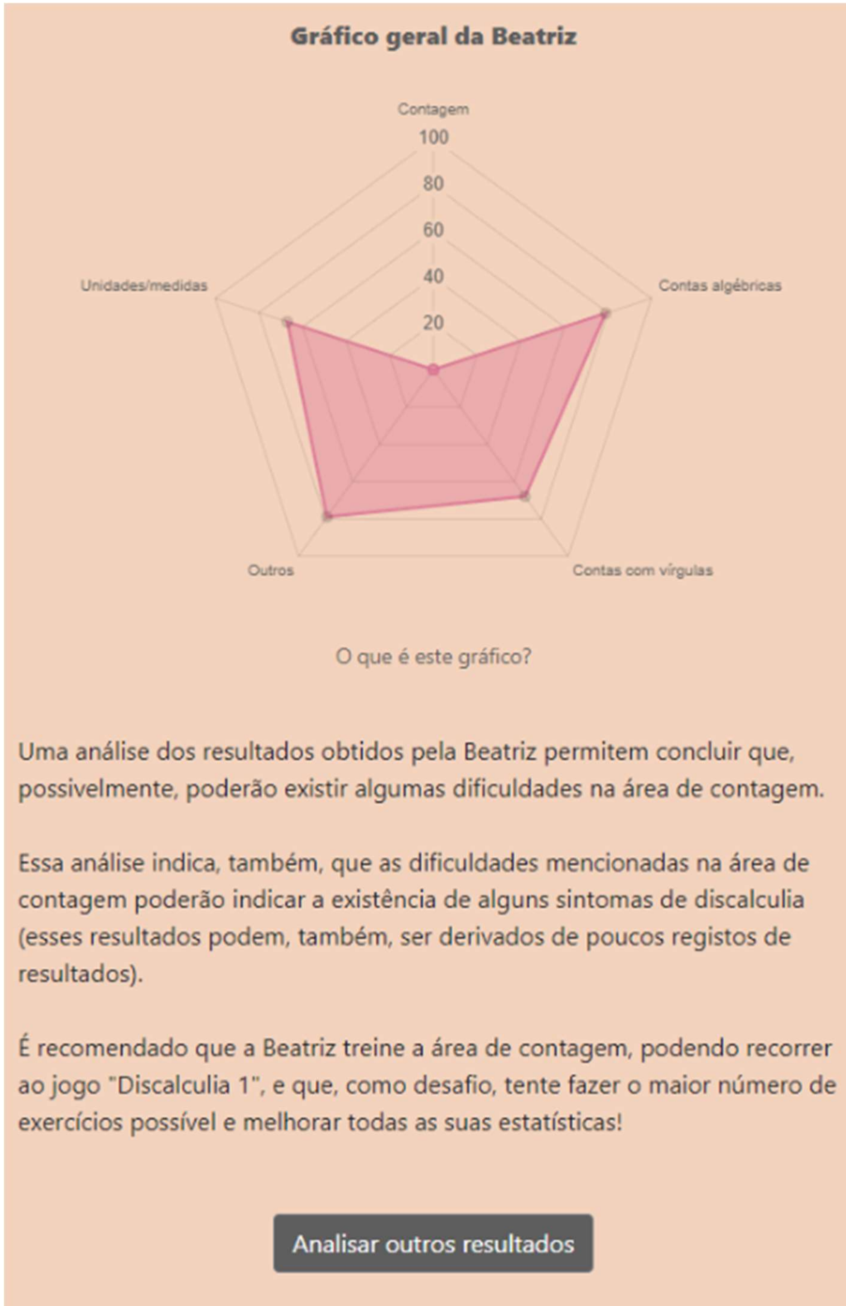

Figure A.37 Left side close-up of the student's results page on Discalculia Web *(Image by Author)*

On the right-hand side of the page, as shown in Figure A.38, the last three results achieved by the student are displayed. Each of these results shows the task and the level at which the score was achieved, the date of registration, the score, and the completion time. Clicking on any result takes the user to a page with a detailed analysis of that result. There is also a button *"Ver mais registos"* (*"See more results"* in English), which shows the user a page with all the results achieved by the student.

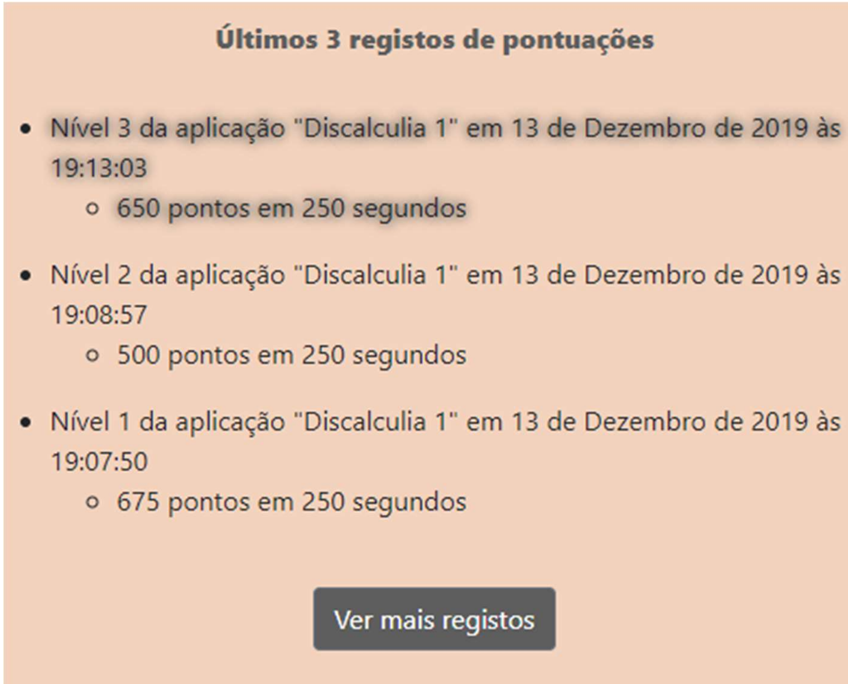

Figure A.38 Right side close-up of the student's results page on Discalculia Web *(Image by Author)*

At the top left of the page, as shown in Figure A.39, the user is presented with the logo of the web application and several options: *"Início"* (*"Start"* in English), *"Resultados"* (*"Results"* in English), *"Registos"* (*"Records"* in English) and *"Acerca".* The first option leads to the student's initial page, the second option leads to the student's results page, the third option leads to the page of the student's history of scores, and the fourth option leads to the pop-up with the information regarding the web application, as mentioned before.

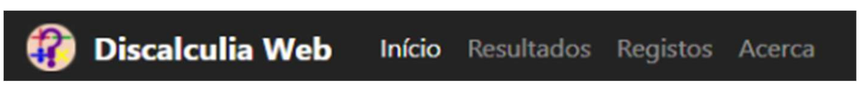

Figure A.39 Top left close-up of the student's page on Discalculia Web *(Image by Author)*

When the student clicks on his/her name at the top right side of the page, the options to consult his/her profile or to log out are displayed. When the user selects the option to view his/her profile, a pop-up is displayed, as shown in Figure A.40, where the student's ID, name, class, number, age, and gender can be consulted.

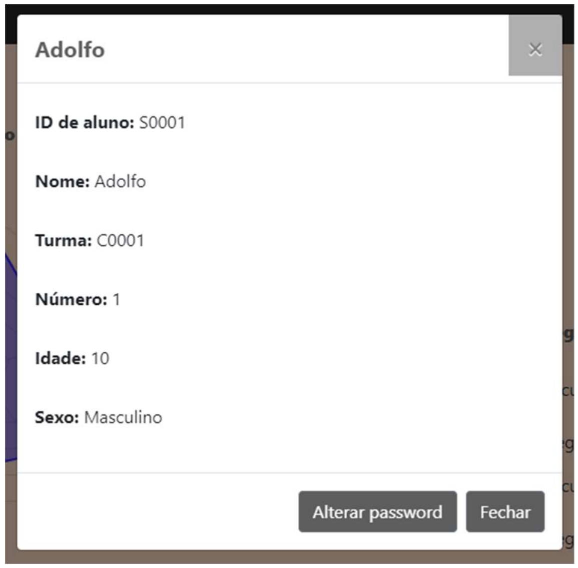

Figure A.40 Preview of the student's profile pop-up on Discalculia Web *(Image by Author)*

In this pop-up, the user has the option to change the password or to close the window. Clicking on the option to change the password (*"Alterar password"* in Portuguese), is displayed a pop-up as shown in Figure A.41, which asks for the current password and the new password.

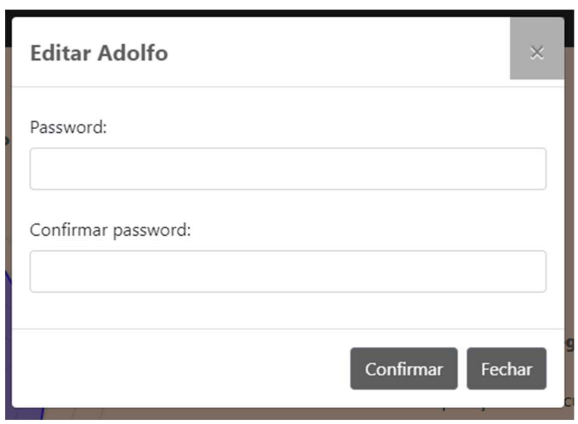

Figure A.41 Preview of the student's password edition pop-up on Discalculia Web *(Image by Author)*

When the user clicks on *"Resultados"* at the top of the page, it is presented a page with a comparative analysis of the student's difficulties with those of their classmates, as shown in Figure A.42. This is the same page that is displayed if the student selects the option *"Analisar outros resultados"* on the initial page.

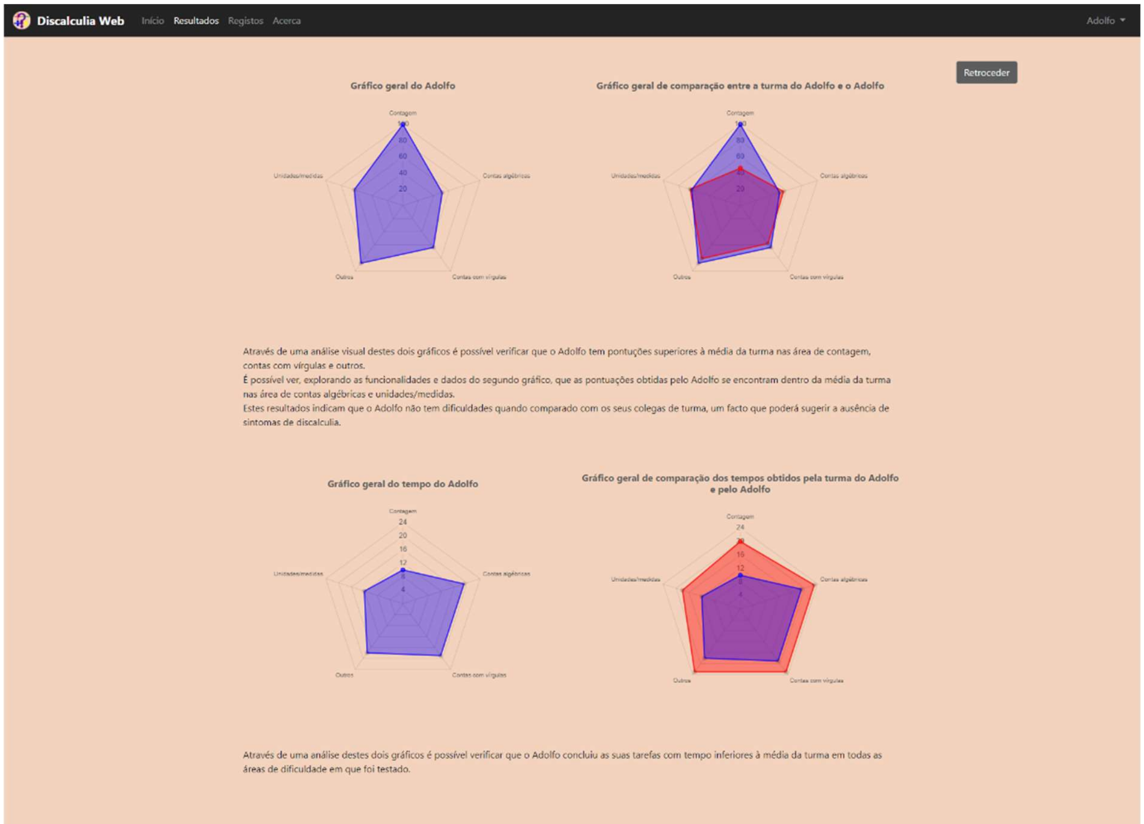

Figure A.42 Preview of the student’s comparative analysis of difficulties with his/her class on Discalculia Web *(Image by Author)*

This page presents a comparative analysis of the student’s difficulties with the overall performance of his/her class in the same areas. Screening analysis of the student and therapeutic instructions, if needed, are also presented. This analysis is divided into two sections: one comparing scores and the other comparing completion times, as shown in Figure A.43 and Figure A.44, respectively.

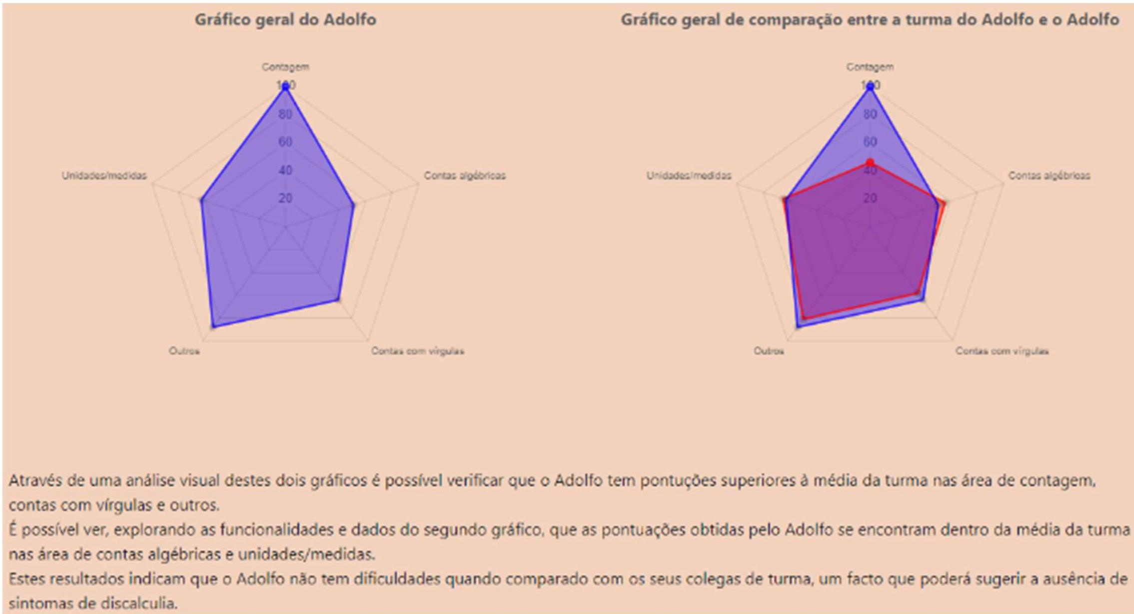

Figure A.43 Close-up 1 of the student's comparative analysis of difficulties with his/her class page on Discalculia Web

(Image by Author)

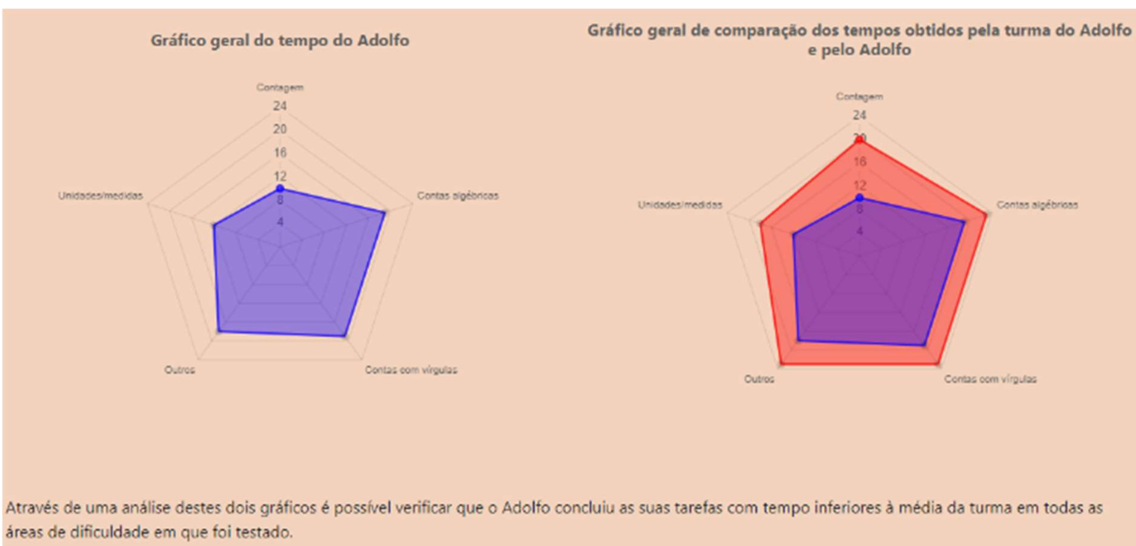

Figure A.44 Close-up 2 of the student's comparative analysis of difficulties with his/her class page on Discalculia Web

(Image by Author)

By clicking on the option *"Ver mais registos"* on the initial page or *"Registos"* at the top of the page, the student can visualize a page with a history of achieved scores per level, per task and in chronological order, as shown in Figure A.45.

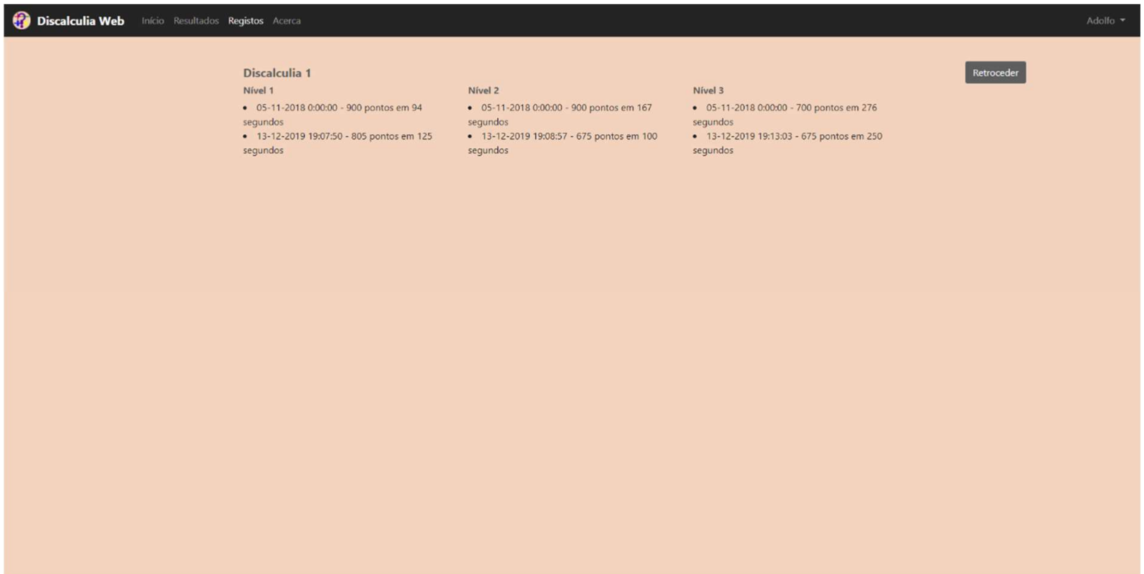

Figure A.45 Preview of the student's history of scores page on Discalculia Web *(Image by Author)*

Every record is clickable and leads to a page with a comparative analysis of the student's scores and completion times over time, as shown in Figure A.46. Figure A.47 and Figure A.48 show the recordings of the selected day and the comparison of these results with those of his/her class, as well as a similar analysis with the number of correct answers.

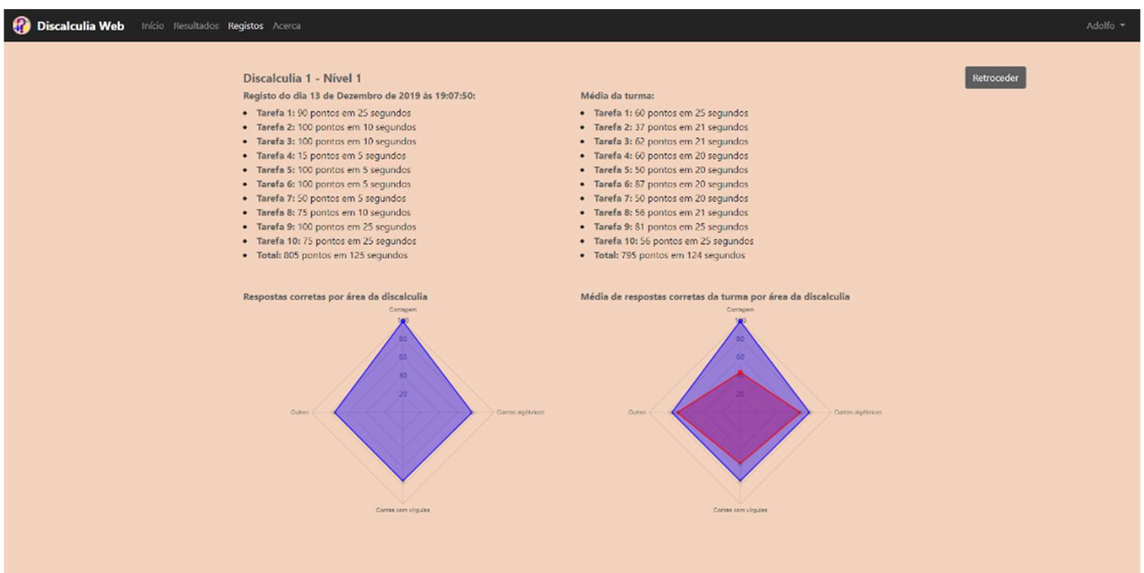

Figure A.46 Preview of the student's comparative analysis of scores and completion times with the class page on Discalculia Web *(Image by Author)*

Discalculia 1 - Nível 1

Registo do dia 13 de Dezembro de 2019 às 19:07:50:

- **Tarefa 1:** 90 pontos em 25 segundos
- **Tarefa 2:** 100 pontos em 10 segundos
- **Tarefa 3:** 100 pontos em 10 segundos
- **Tarefa 4:** 15 pontos em 5 segundos
- **Tarefa 5:** 100 pontos em 5 segundos
- **Tarefa 6:** 100 pontos em 5 segundos
- **Tarefa 7:** 50 pontos em 5 segundos
- **Tarefa 8:** 75 pontos em 10 segundos
- **Tarefa 9:** 100 pontos em 25 segundos
- **Tarefa 10:** 75 pontos em 25 segundos
- **Total:** 805 pontos em 125 segundos

Média da turma:

- **Tarefa 1:** 60 pontos em 25 segundos
- **Tarefa 2:** 37 pontos em 21 segundos
- **Tarefa 3:** 62 pontos em 21 segundos
- **Tarefa 4:** 60 pontos em 20 segundos
- **Tarefa 5:** 50 pontos em 20 segundos
- **Tarefa 6:** 87 pontos em 20 segundos
- **Tarefa 7:** 50 pontos em 20 segundos
- **Tarefa 8:** 56 pontos em 21 segundos
- **Tarefa 9:** 81 pontos em 25 segundos
- **Tarefa 10:** 56 pontos em 25 segundos
- **Total:** 795 pontos em 124 segundos

Figure A.47 Close-up 1 of the student's comparative analysis of scores and completion times with the class page on Discalculia Web *(Image by Author)*

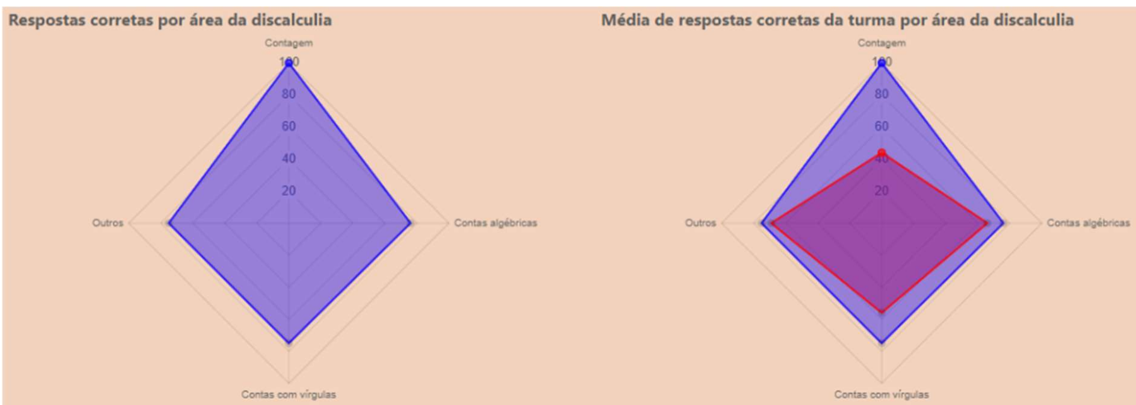

Figure A.48 Close-up 2 of the student's comparative analysis of scores and completion times with the class page on Discalculia Web *(Image by Author)*

The teacher has access to similar pages as the student. After logging in, an initial page is presented with three options: *"Alunos"* (*"Students"* in English), *"Turmas"* (*"Classes"* in English) and *"Resultados gerais"* (*"Overall results"* in English), as shown in Figure A.49.

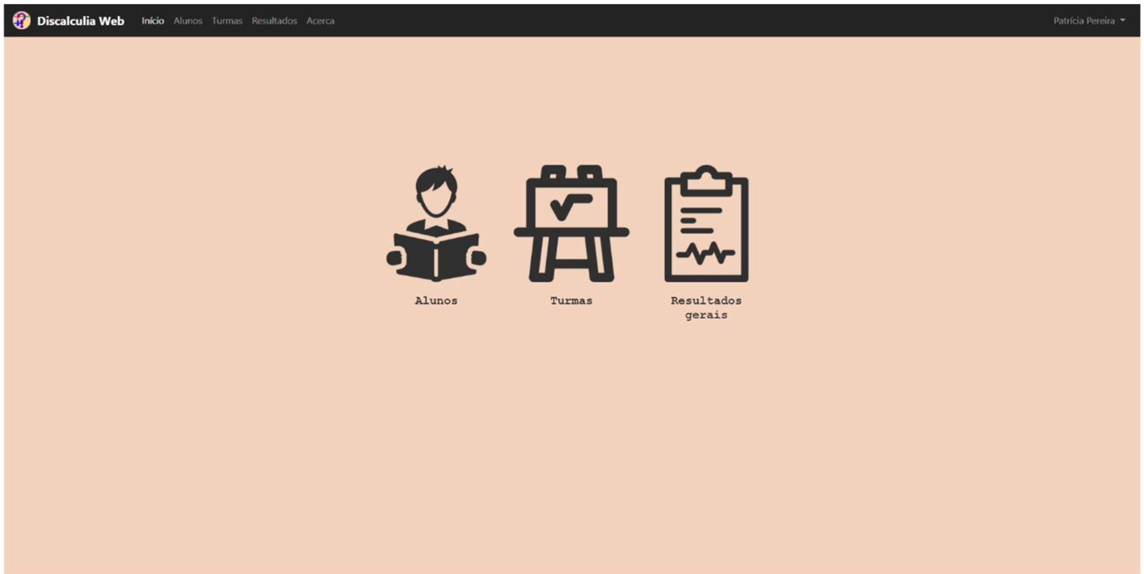

Figure A.49 Preview of the teacher's initial page on Discalculia Web *(Image by Author)*

At the top left of the page, after the logo, the user has five options to click on at any time: *"Início"* (*"Start"* in English), *"Alunos"*, *"Turmas"*, *"Resultados"* (*"Results"* in English) and *"Acerca"*, as shown in Figure A.50.

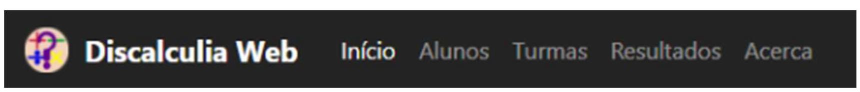

Figure A.50 Top left close-up of the teacher's page on Discalculia Web *(Image by Author)*

The option *"Início"* leads to the teacher's initial page, the option *"Alunos"* leads to the page where the teacher can visualize his/her students, the option *"Turmas"* leads to the page where the teacher can visualize his/her classes, the option *"Resultados"* leads to the page where the teacher can see an analysis of the results of his/her classes, and *"Acerca"* leads to the pop-up that explains the web application.

As the student, the teacher's name is displayed at the top right of the page and when it is clicked on, the options to view the profile or to log out appear. Selecting the option to view the profile, a pop-up appears with the teacher's name, his/her ID, and the option to change the password, as the student.

Clicking on the option *"Alunos"* on the initial page or at the top of the page, it takes the user to the page where he/she can view his/her students per class, as shown in Figure A.51.

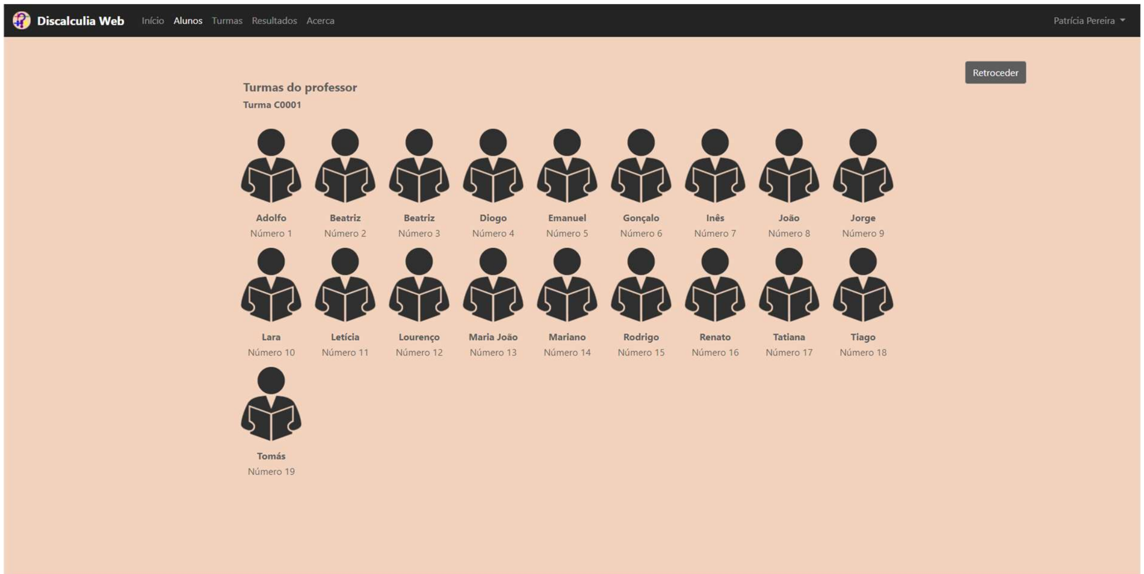

Figure A.51 Preview of the teacher's students page on Discalculia Web *(Image by Author)*

The teacher can see each student from his/her classes, identified by the name and class number. After clicking on a student, the teacher can access all the student's pages, starting with the student's initial page, as shown in Figure A.52. This page displays the same results' analysis page that is displayed to the student.

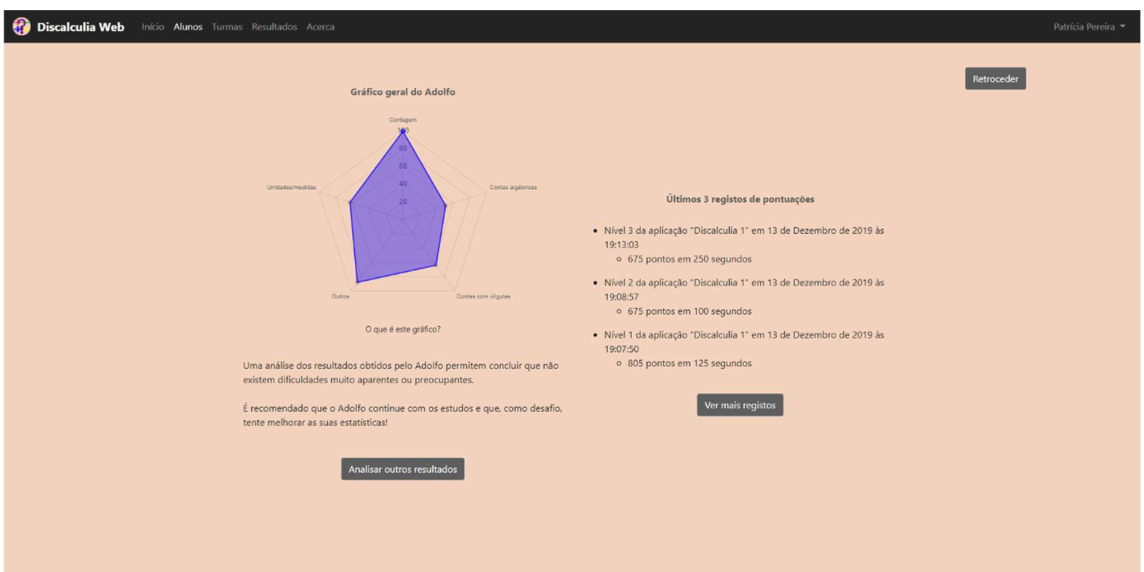

Figure A.52 Preview of the student's results on the teacher's view on Discalculia Web *(Image by Author)*

If the teacher clicks on the option *"Turmas"* in the top bar of the page or on the initial page, he/she will be taken to the page where he/she can see all his/her classes with the class number, the name of the teacher in charge and the scholarship level, as shown in Figure A.53.

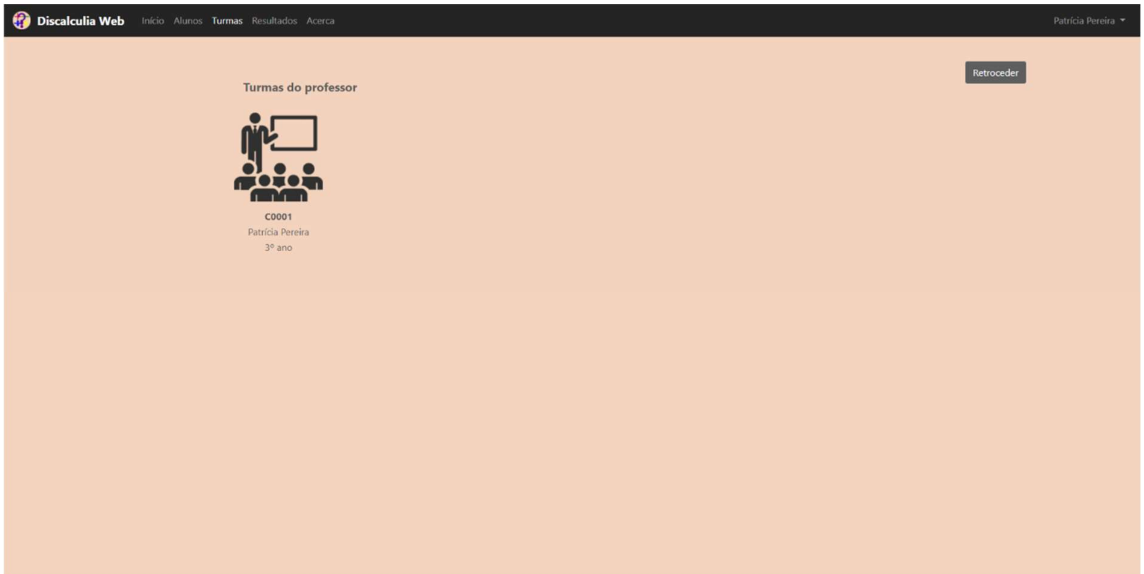

Figure A.53 Preview of the teacher's classes page on Discalculia Web *(Image by Author)*

When the teacher clicks on a class, he/she is redirected to the page where a comparative analysis of the performance of that class with other classes is presented, as shown in Figure A.54, Figure A.55 and Figure A.56.

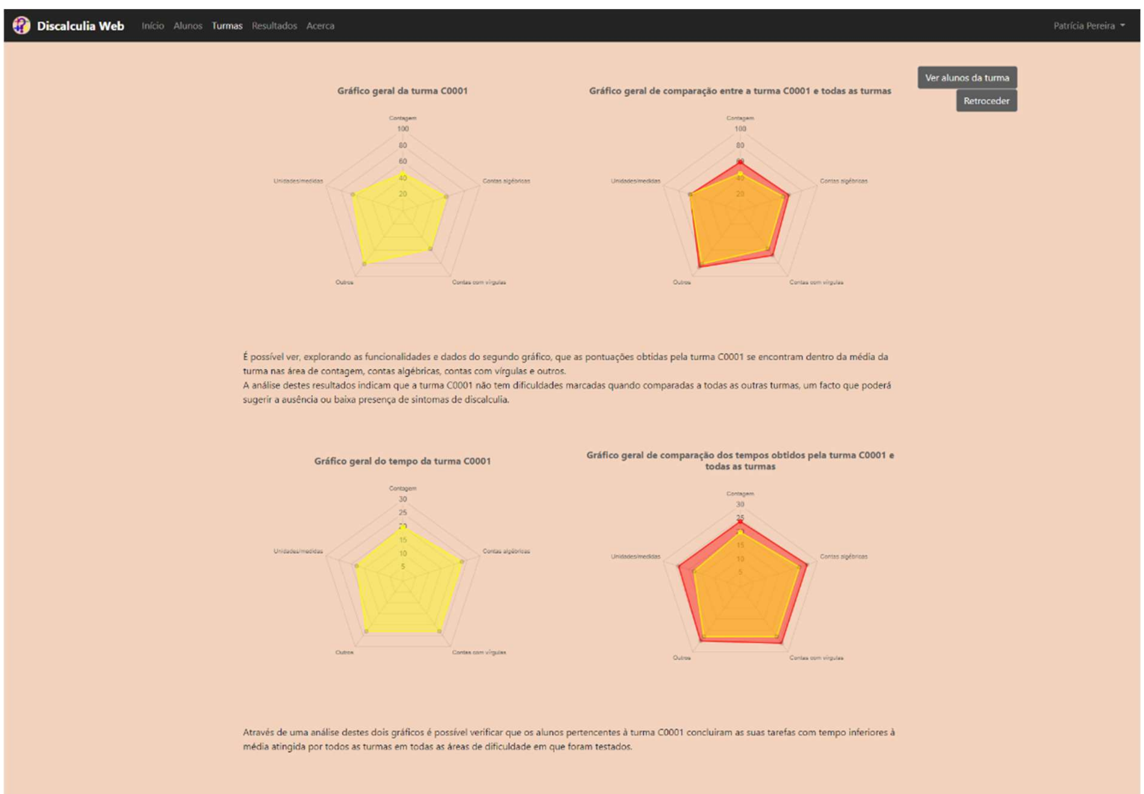

Figure A.54 Preview of the comparative analysis of one of the teacher's classes with other classes page on Discalculia Web *(Image by Author)*

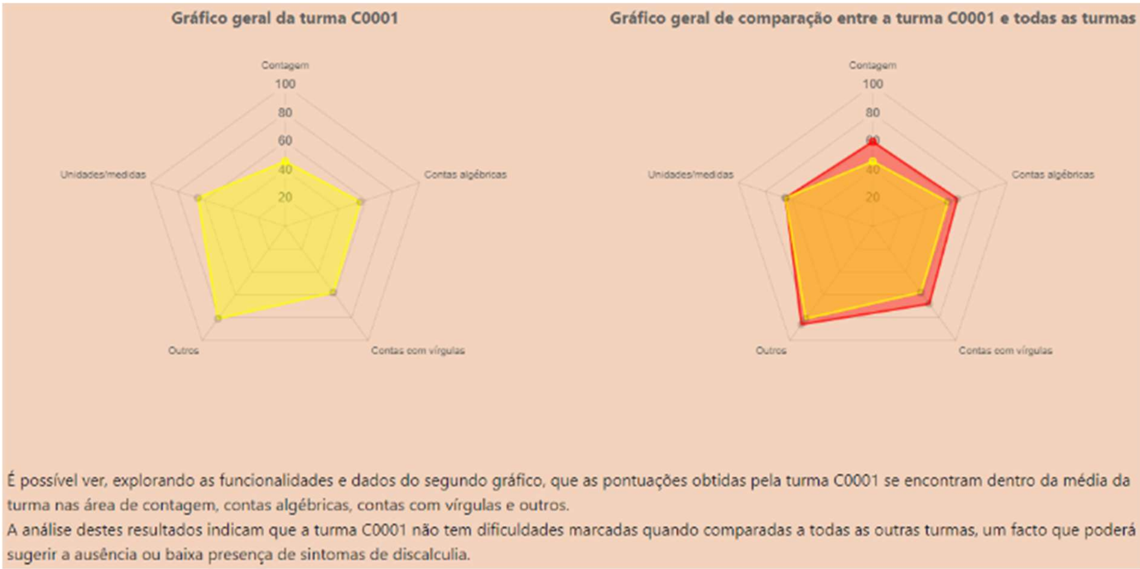

Figure A.55 Close-up 1 of the comparative analysis of one of the teacher's classes with other classes page on Discalculia Web *(Image by Author)*

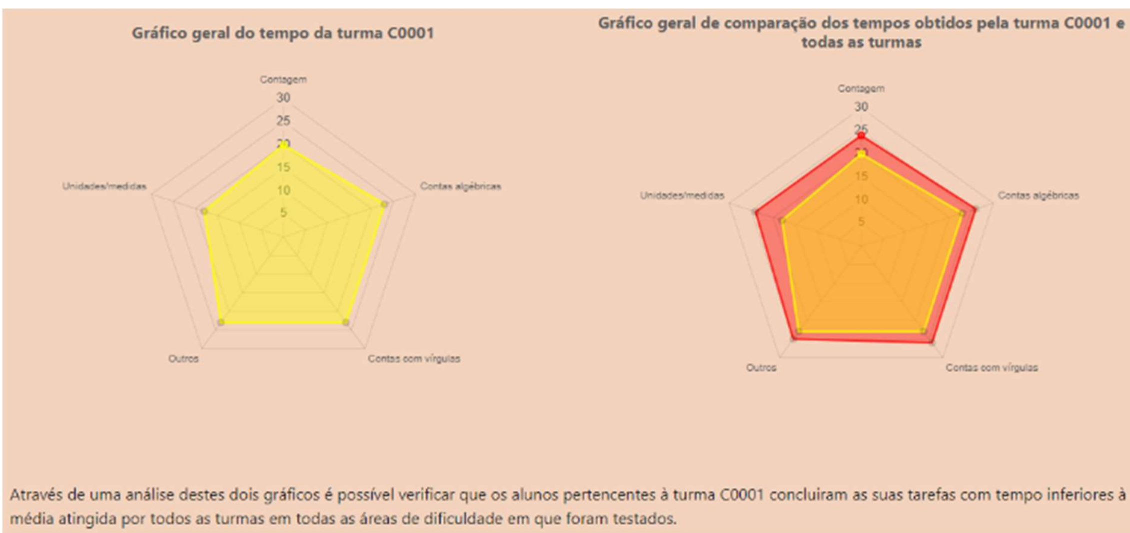

Figure A.56 Close-up 2 of the comparative analysis of one of the teacher's classes with other classes page on Discalculia Web *(Image by Author)*

By clicking on the option *"Ver alunos da turma",* which is displayed at the top right of the page of the selected class, the teacher is taken to a page where he/she can see all the students belonging to that class, as shown in Figure A.57. The difference between this page and the page shown in Figure A.51 is that this page only presents the students regarding that class, while the other page presents all the teacher's students.

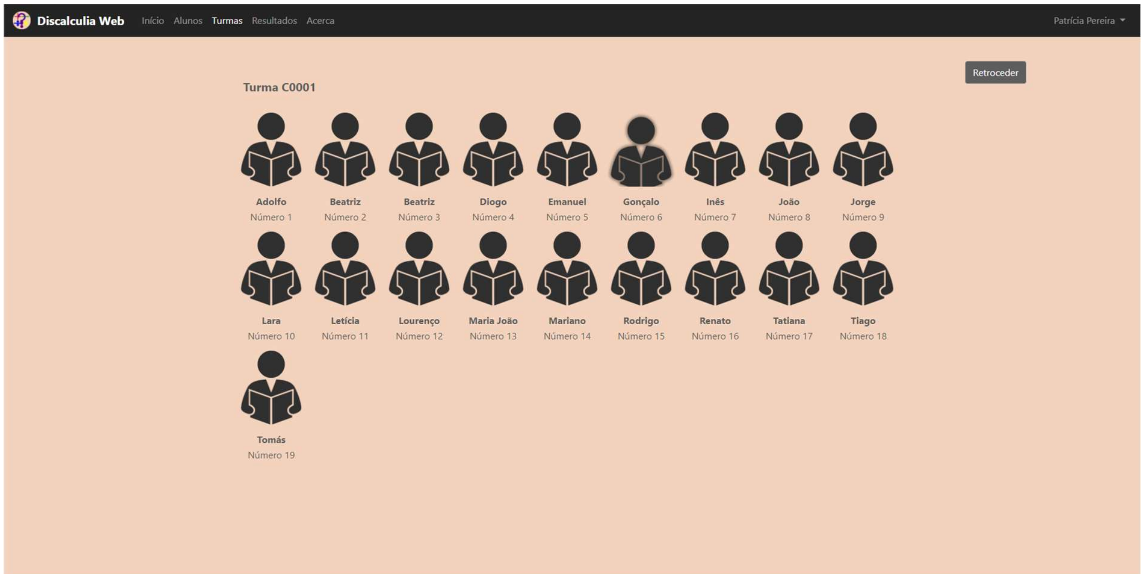

Figure A.57 Preview of the teacher's students per class page on Discalculia Web *(Image by Author)*

By clicking on the option *"Resultados"* in the top bar of the page or on the option *"Resultados gerais"* on the initial page, the teacher is redirected to the page where he/she is presented with the comparative analysis of his/her classes with all other classes registered in the web application, as shown in Figure A.58, Figure A.59 and Figure A.60.

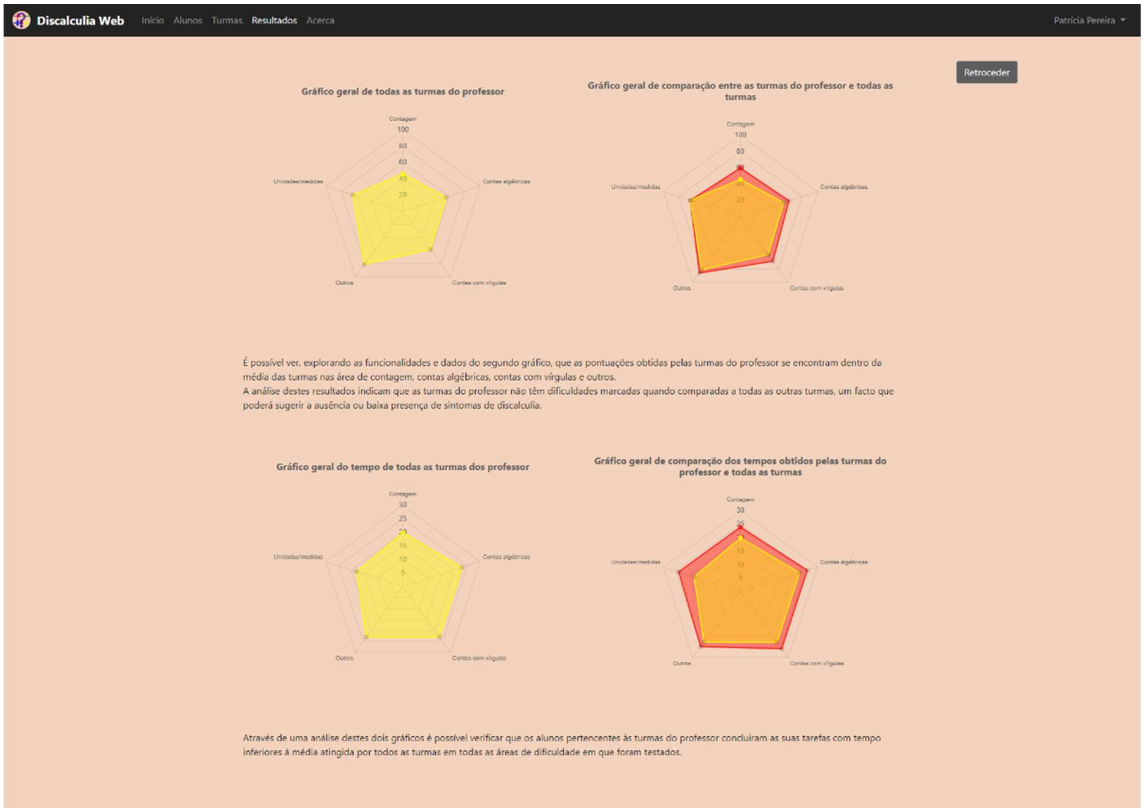

Figure A.58 Preview of the comparative analysis of the teacher's classes with all other classes page on Discalculia Web

(Image by Author)

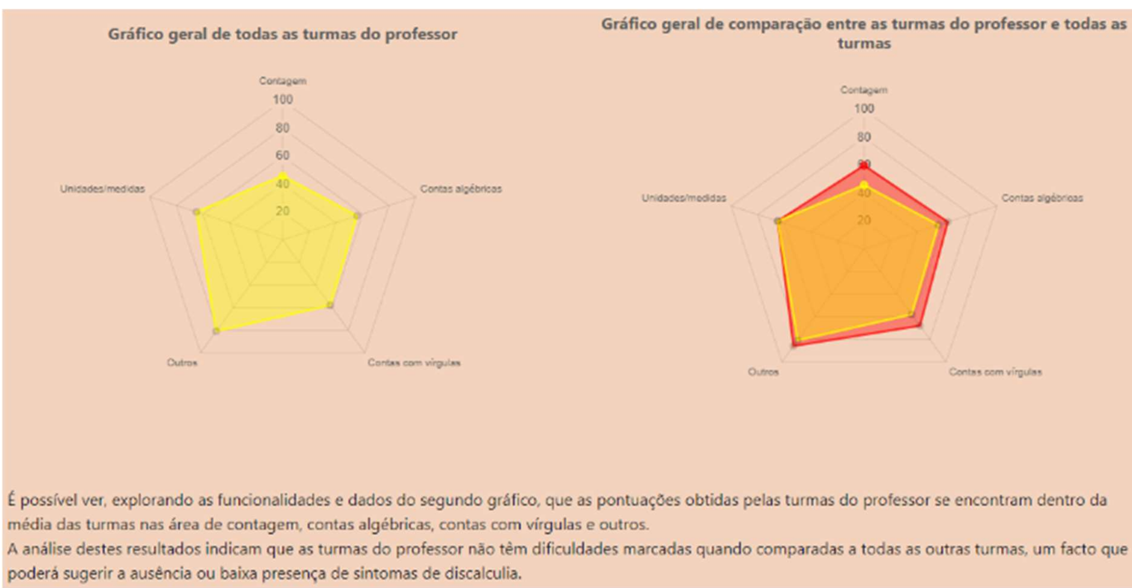

Figure A.59 Close-up 1 of the comparative analysis of the teacher's classes with all other classes page on Discalculia Web

(Image by Author)

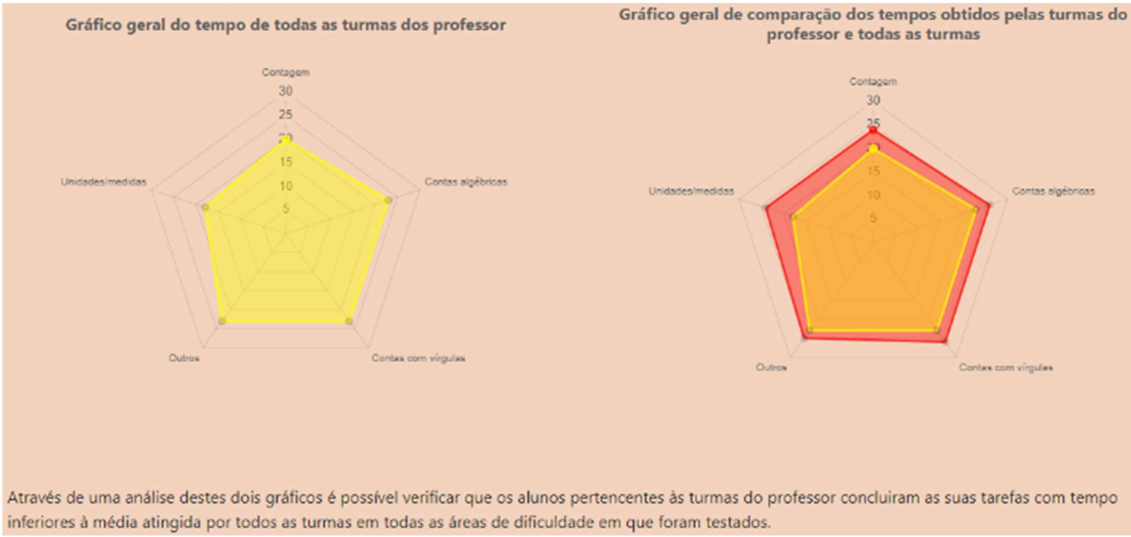

Figure A.60 Close-up 2 of the comparative analysis of the teacher's classes with all other classes page on Discalculia Web *(Image by Author)*

The expert has access to similar pages as the teacher, with permission to change the analysis if necessary. After logging in, the expert is presented with the three options: *"Alunos"*, *"Turmas"* and *"Resultados gerais"*, the same options presented to the teacher, as shown in Figure A.61. Like the teacher, the expert has five options at the top left of the page: *"Início"*, *"Alunos"*, *"Turmas"*, *"Resultados"*, and *"Acerca"*. These options lead to the initial page, the page where the expert can see his/her students, the page where the expert can see his/her classes, the page where the expert can see an analysis of the results of his/her classes, and the pop-up with information about the web application, respectively.

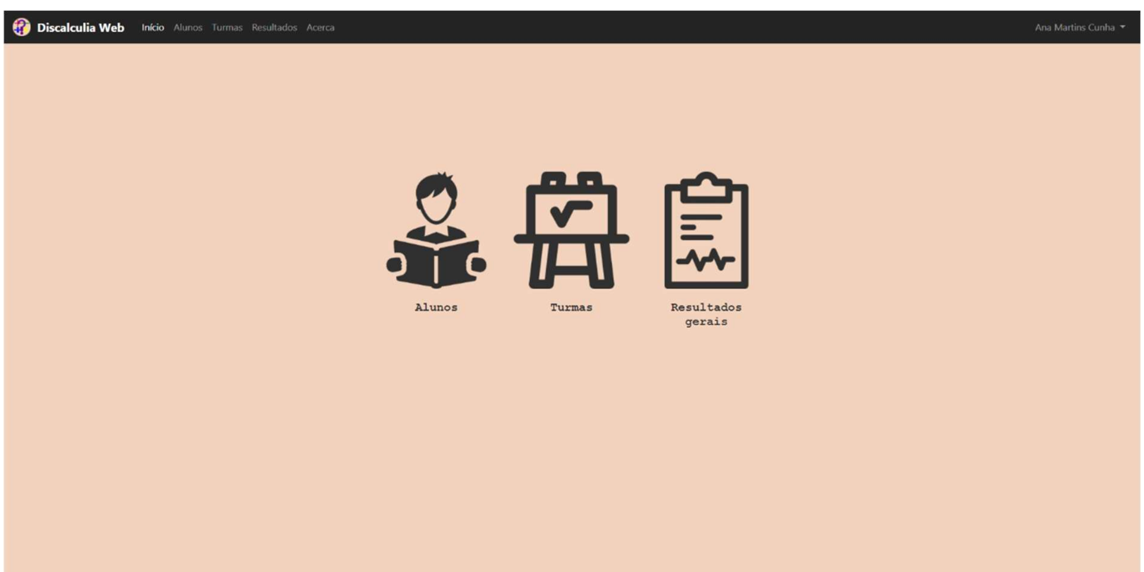

Figure A.61 Preview of the expert's initial page on Discalculia Web *(Image by Author)*

Just like the student and the teacher, the expert can see his/her name at the top right of the page, which is clickable and leads to the option to see his/her profile. If the expert selects this option, he/she can see the profile where the name, ID code and the option to change the password are displayed.

By clicking on the option *"Alunos"*, which is on the initial page or at the top bar of the page, the expert is shown all his students, divided by class, as shown in Figure A.62.

Figure A.62 Preview of the expert's students page on Discalculia Web *(Image by Author)*

When the expert clicks on a student, he/she is redirected to the page with the overall analysis of that student's results, as shown in Figure A.63.

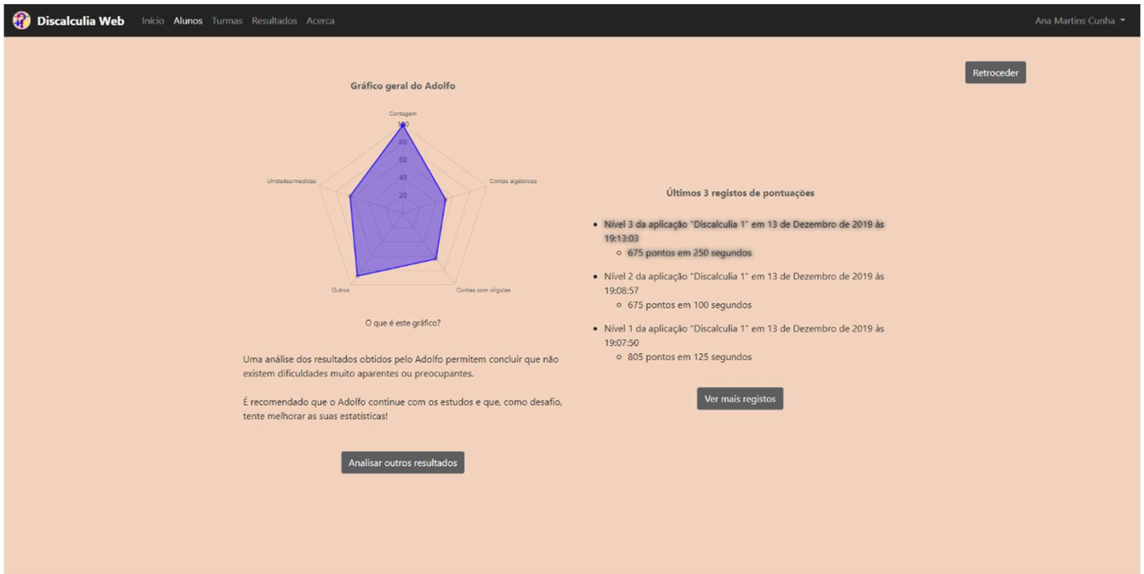

Figure A.63 Preview of the student's results on the expert's view on Discalculia Web *(Image by Author)*

After clicking on the option *"Turmas"* on the initial page or at the top bar of the page, the expert is redirected to the page shown in Figure A.64, where he/she can see all his/her classes and also the other classes registered in the web application.

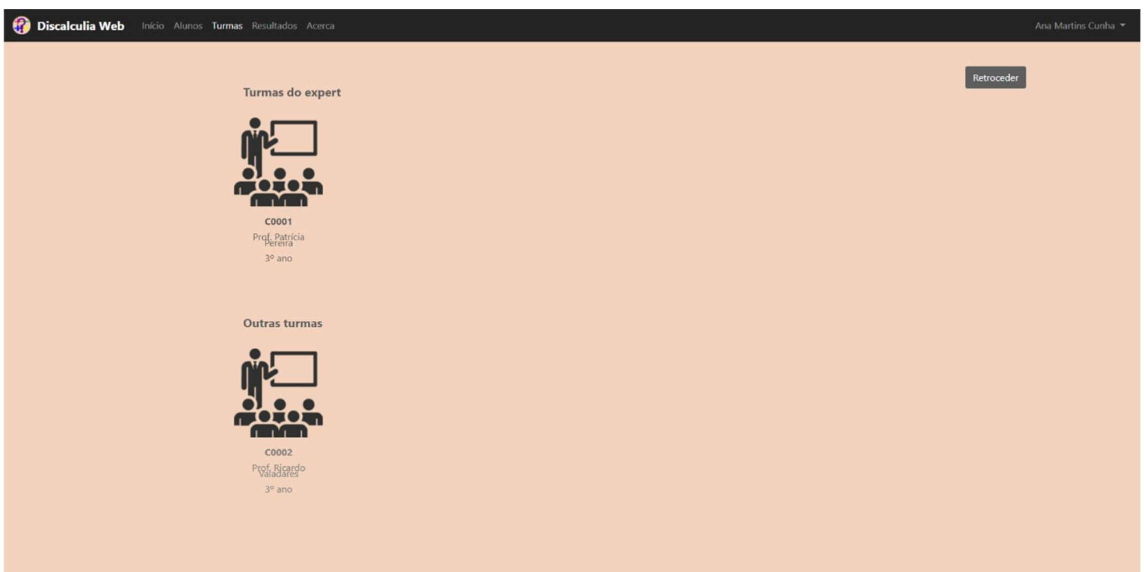

Figure A.64 Preview of the expert's classes page on Discalculia Web *(Image by Author)*

Each class, when clicked, leads to a page where it is presented a comparative analysis of the performance of that class with all other classes, as shown in Figure A.65, Figure A.66 and Figure A.67.

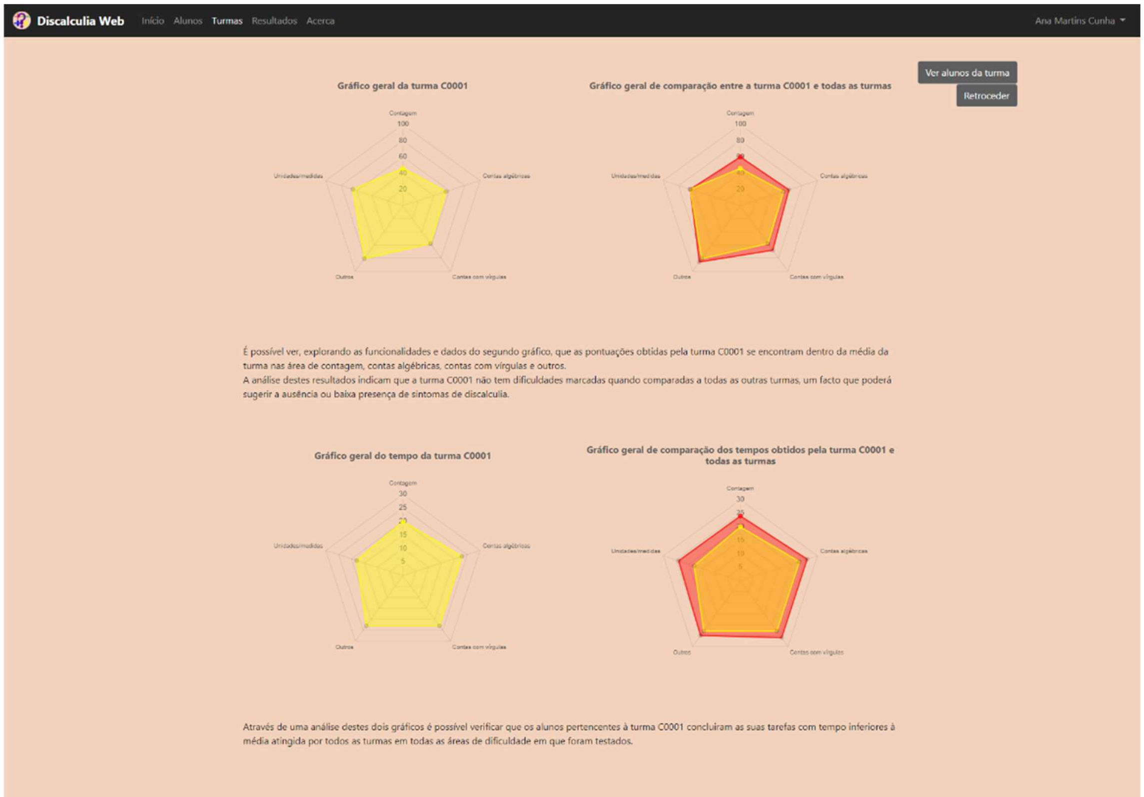

Figure A.65 Preview of the comparative analysis of one of the expert's classes with other classes page on Discalculia Web

(Image by Author)

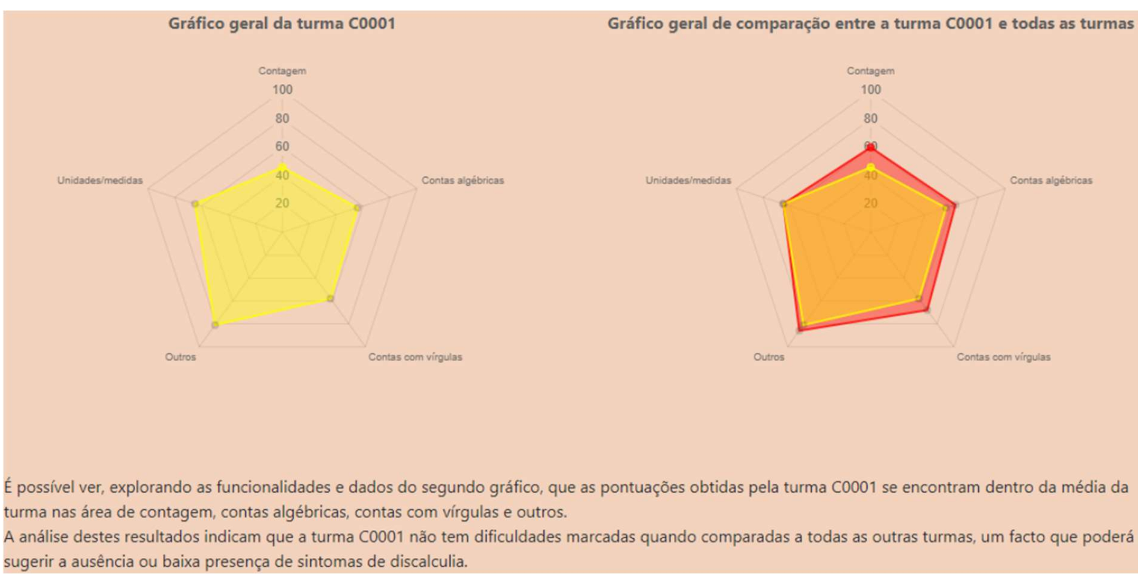

Figure A.66 Close-up 1 of the comparative analysis of one of the expert's classes with other classes page on Discalculia Web

(Image by Author)

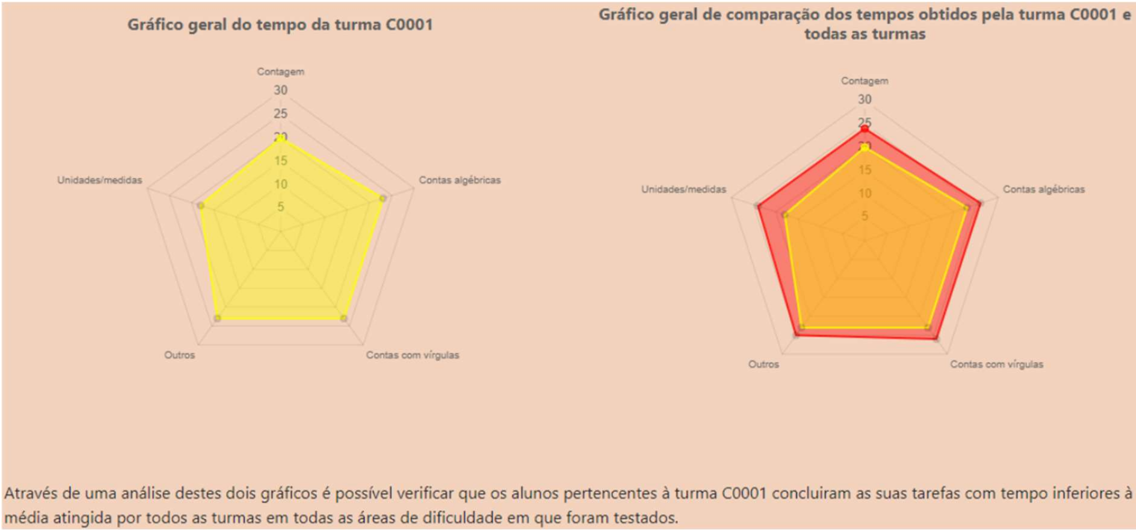

Figure A.67 Close-up 2 of the comparative analysis of one of the expert's classes with other classes page on Discalculia Web *(Image by Author)*

On the other hand, by clicking on the option *"Resultados"* at the top of the page or the option *"Resultados gerais"* on the home page, the expert is redirected to a page where he/she is presented with a comparative analysis of the expert's classes with all the other classes, as shown in Figure A.68, Figure A.69 and Figure A.70.

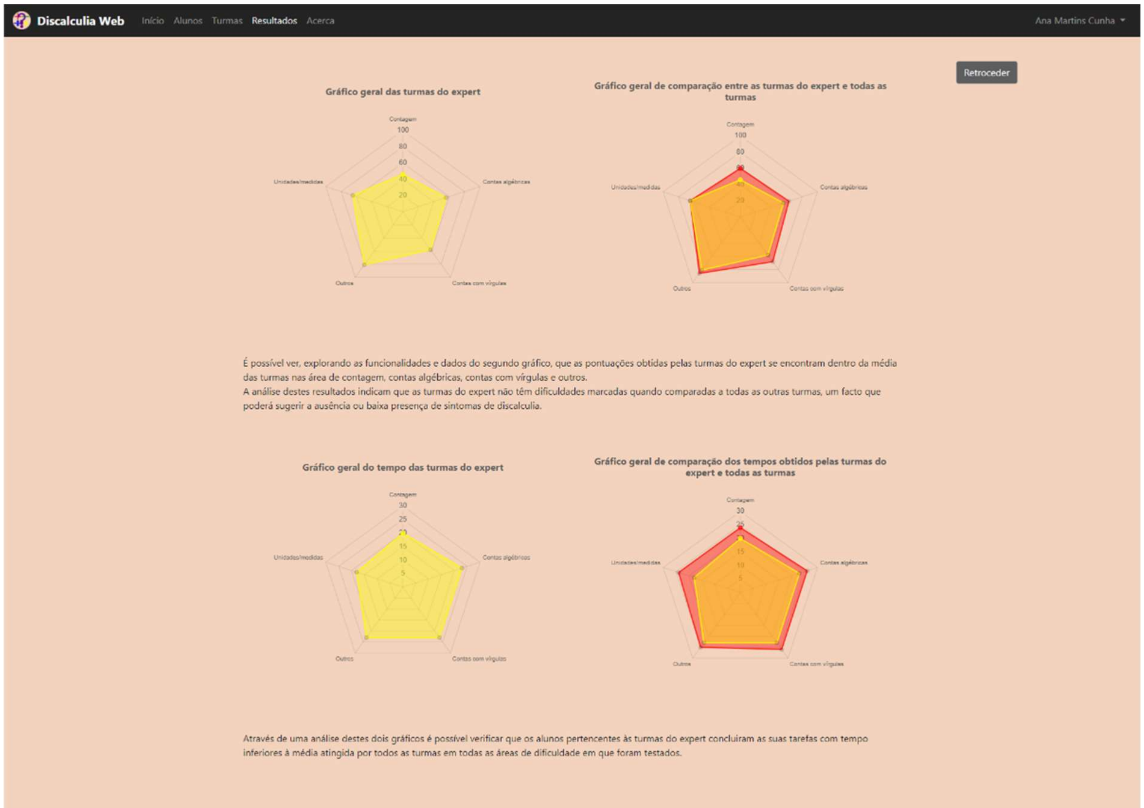

Figure A.68 Preview of the comparative analysis of the expert's classes with all other classes page on Discalculia Web

(Image by Author)

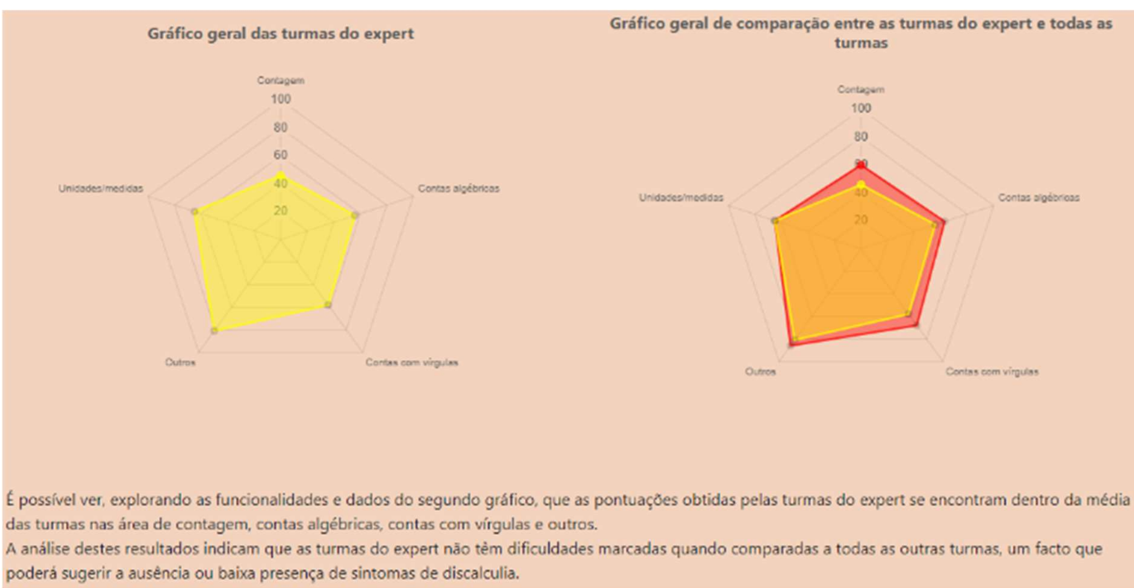

Figure A.69 Close-up 1 of the comparative analysis of the expert's classes with all other classes page on Discalculia Web

(Image by Author)

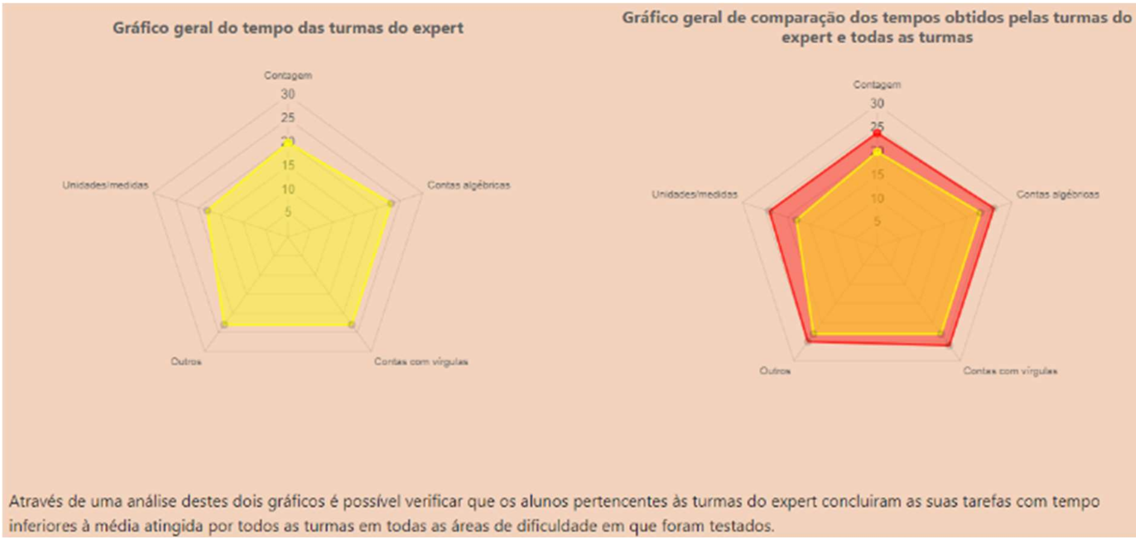

Figure A.70 Close-up 2 of the comparative analysis of the expert's classes with all other classes page on Discalculia Web *(Image by Author)*

Like all previously mentioned comparative analysis, an assessment of the student or the class in question is presented with screening results and therapeutic instructions that can be edited if the expert does not agree with the observation made.

The administrator, or developer, is the user with the most permissions and has the ability to change the details of all other users. After logging in, the administrator is offered four options: *"Alunos"*, *"Professores"* (*"Teachers"* in English), *"Experts"* and *"Administradores"* (*"Administrators"* in English), as shown in Figure A.71. The top bar of the page displays the options: *"Início"*, *"Alunos"*, *"Professores"*, *"Experts"*, *"Administradores"*, and *"Acerca"*.

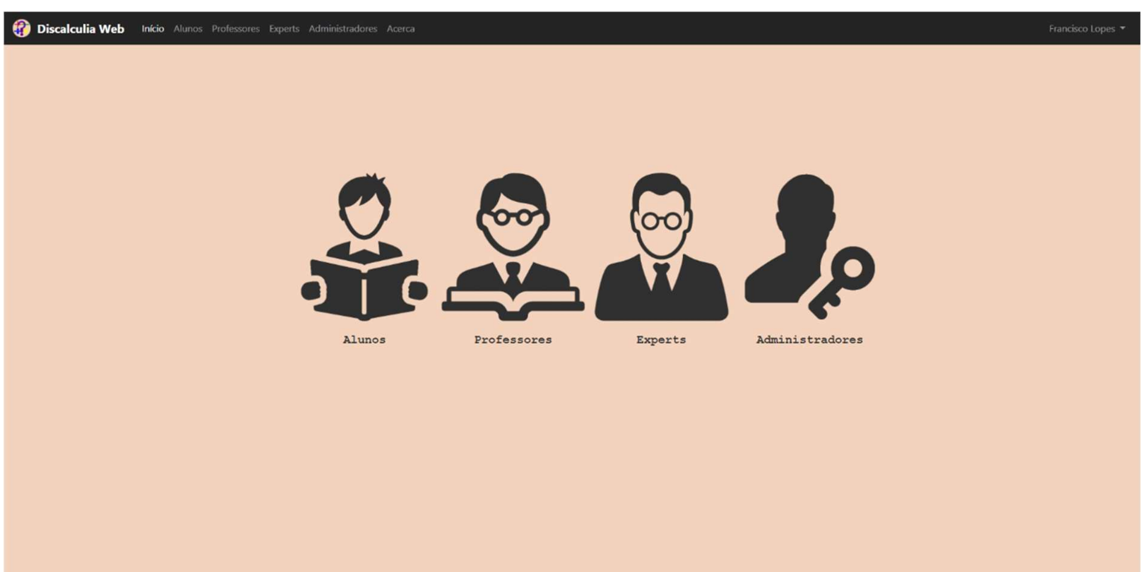

Figure A.71 Preview of the administrator's initial page on Discalculia Web *(Image by Author)*

The option *"Início"* leads the administrator to the initial window, the option *"Alunos"* leads to the page with all students, the option *"Professores"* leads to the page with all teachers, the "Experts" option leads the page with all experts, the option *"Administradores"* leads to the page with all administrators, and the option *"Acerca"* makes the pop-up explaining the platform appear in the window.

At the top right side of the page, the name of the administrator is displayed, which, when clicked, leads to the options to view the profile or to log out. If the option to view the profile is selected, the administrator will be shown his/her name, the ID code, and the option to change the password. The process to change the password is similar to that for the other users.

If the administrator selects the option *"Alunos"* at the top bar of the page or on the initial page, he/she can see a list of students grouped by class, as shown in Figure A.72.

Figure A.72 Preview of the administrator's students page on Discalculia Web *(Image by Author)*

In addition, to see the page with the analysis of results of a student, the administrator can also edit the student's details. By clicking on the student, a pop-up appears where he/she can edit the student's name, class number, age, and password, as shown in Figure A.73.

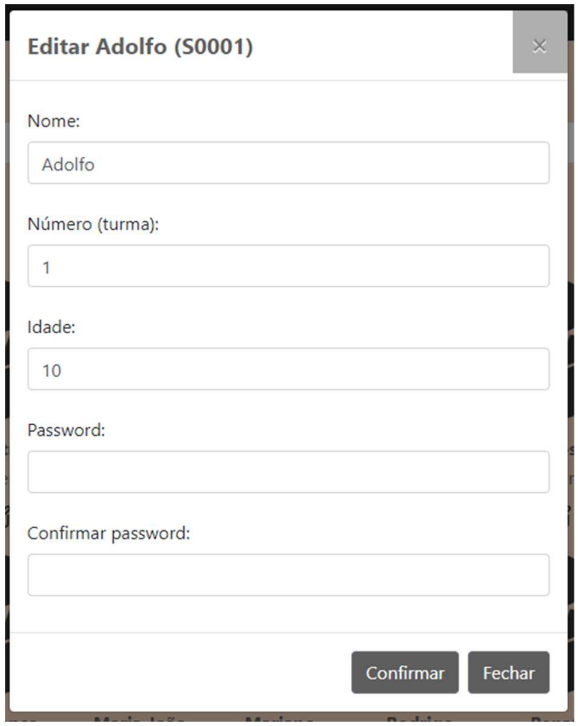

Figure A.73 Preview of the administrator's view on student edition pop-up on Discalculia Web *(Image by Author)*

Choosing the option *"Professores"* at the top bar of the page or on the initial page, the administrator sees all teachers, as shown in Figure A.74.

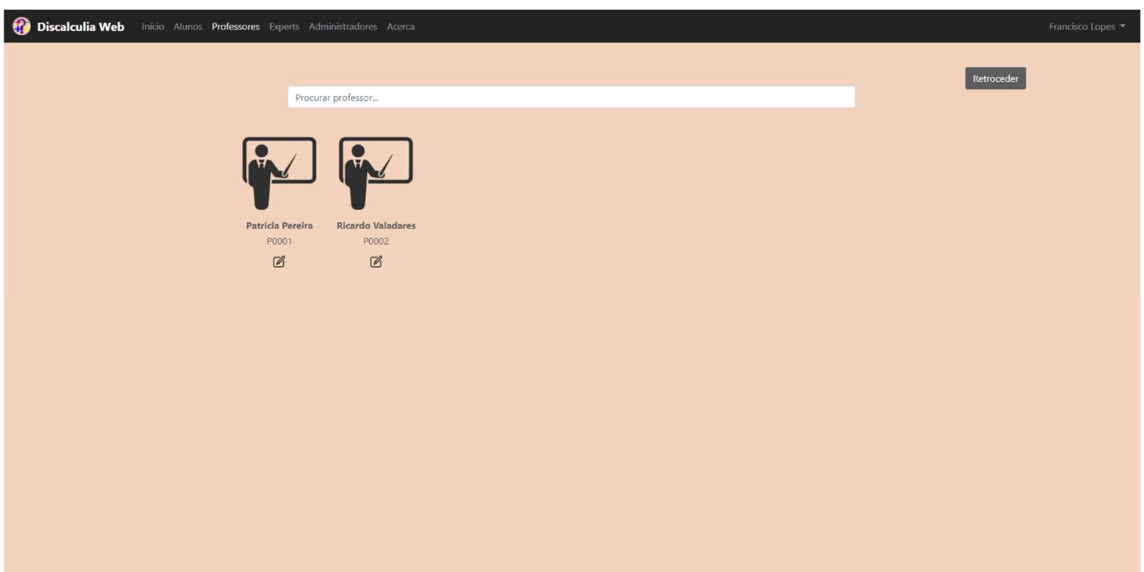

Figure A.74 Preview of the administrator's teachers page on Discalculia Web *(Image by Author)*

Selecting one of the teachers allows the administrator to navigate through the web application from a teacher's perspective and edit details such as the teacher's name and password, as shown in Figure A.75.

Figure A.75 Preview of the administrator's view on teacher edition pop-up on Discalculia Web *(Image by Author)*

Clicking on the option *"Experts"* at the top bar of the page or on the initial page, the administrator is presented with the page that lists all experts, as shown in Figure A.76.

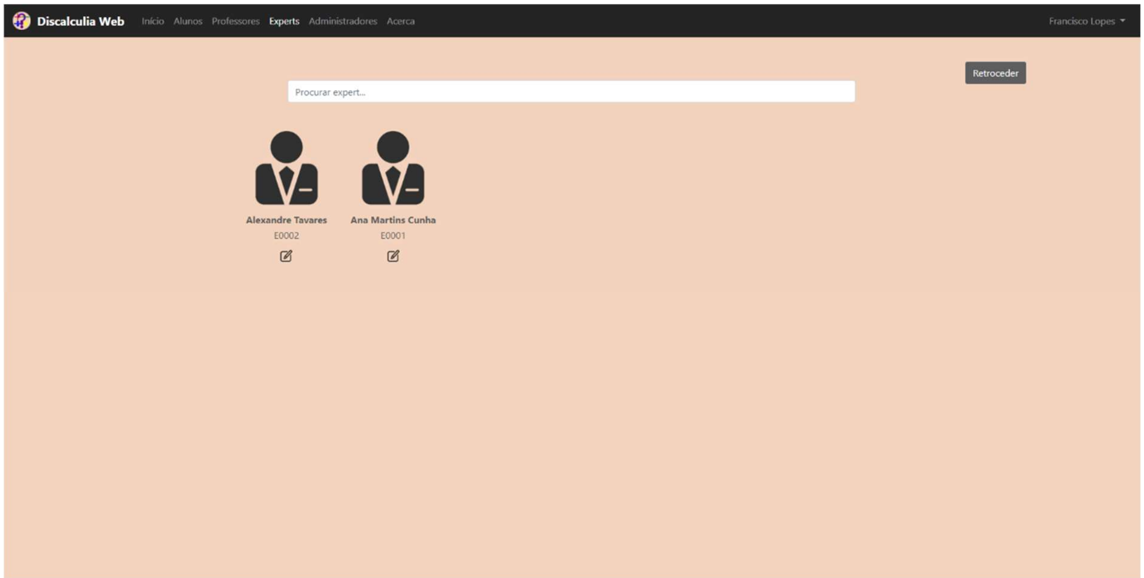

Figure A.76 Preview of the administrator's experts page on Discalculia Web *(Image by Author)*

Like to the other users, the administrator can navigate through the web application under the expert's perspective and can edit the expert's details by clicking on the chosen expert. The pop-up, as shown in Figure A.77, appears with the option to edit the expert's name and password.

Figure A.77 Preview of the administrator's view on expert edition pop-up on Discalculia Web *(Image by Author)*

By selecting the option *"Administradores"* at the top bar or on the initial page, the administrator is lead to the page with all administrators, as shown in Figure A.78.

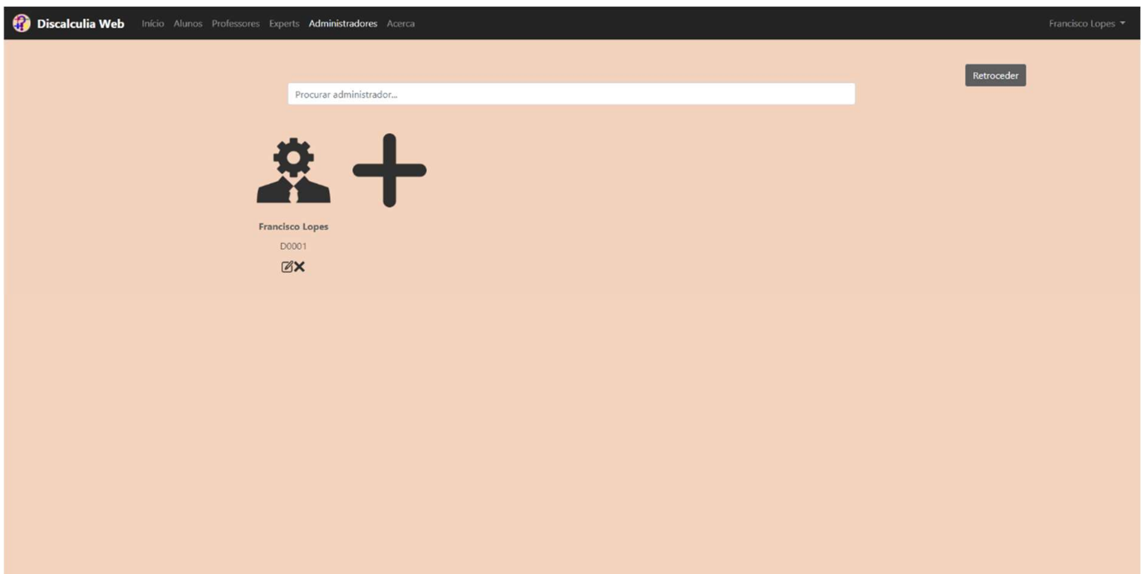

Figure A.78 Preview of the administrator's administrators page on Discalculia Web *(Image by Author)*

Contrary to the other users, the administrator cannot navigate through the web application from another administrator's perspective. However, the administrator can edit the details of other administrators, remove them or add others, as shown in Figure A.79, Figure A.80, and Figure A.81, respectively.

Figure A.79 Preview of the administrator's edition pop-up on Discalculia Web *(Image by Author)*

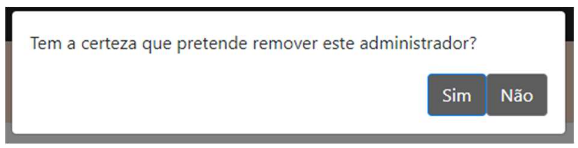

Figure A.80 Preview of the administrator's removal pop-up on Discalculia Web *(Image by Author)*

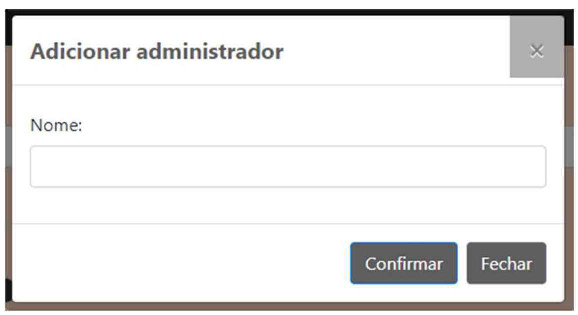

Figure A.81 Preview of the administrator's addition pop-up on Discalculia Web *(Image by Author)*

Most of the pages of the web application require a logged-in and validated user. Therefore, these pages cannot be accessed just by inserting the correct address and route.