Application of Fragility Analysis to Timber-Framed Structures for Seismic and Robustness Assessments

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Abstract In the past few years, the construction of multi-storey timber buildings has increased significantly in locations where high-intensity ground motions are likely to occur. On the other hand, the fast development of wood engineered products, as glued-laminated timber (GLT) and cross-laminated timber (CLT), has been challenging researchers to provide adequate guidelines for the design and assessment of structures built in seismic regions. Some guidelines and analysis methods considered in seismic design can improve robustness, commonly described as the ability of structures to sustain limited damage without disproportionate effects. This paper proposes a probabilistic methodology for seismic and robustness assessment of timber-framed structures. The seismic performance and the progressive collapse potential of a three-storey building are here exemplified through the proposed methodology, which accounts for uncertainties in mechanical properties of members and connections, as well as for external loads. The Latin Hypercube Sampling (LHS) was used in each assessment to generate a set of 1000 structural models. Each structural model corresponds to a realization of the random variables used to define the structural model. Incremental dynamic analyses were performed to develop seismic fragility curves for different damage levels. The fragility functions for robustness assessment were developed for distinct damage scenarios, exploiting the results of an alternate load path analysis (ALPA) that involved the performance of nonlinear static analyses (pushdown analyses). The methodology presented is suitable for risk-based assessments that consider the occurrence of different exposures, such as earthquakes, impacts, and explosions, while considering the direct and indirect consequences of failures. However, the methodology involves time-consuming analyses with distinct load scenarios, which can constitute a burdening task within a typical building design phase.

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1 Introduction

The performance of timber structures under intense earthquake ground shaking depends strongly on the type of failure modes, and in particular, their ductility. The failure of timber elements is usually brittle, whereas the failure of connections between timber elements can be ductile. Being constructed in seismic areas, the requirements promoted by the design codes are objectively stricter regarding connections and the sizes of timber members. Seismic design recommendations for timber structures focus on the formation of inelastic deformations of joints by increasing fastener slenderness, guaranteeing that failure occurs after yielding of fasteners. This measure aims to enhance the capacity of joints to withstand large inelastic deformations without rupture. Moreover, seismic resistant multi-storey buildings must have adequate bi-directional resistance and stiffness. Diaphragms are another key component since these allow the inertial forces to be transferred to the different vertical structural elements [1].

Robustness is another essential concept when studying multi-storey timber buildings, described as the ability of structures to avoid progressive collapse due to local damage resulting from an unpredictable event. Some of the aspects sought in seismic resistant systems in design codes include recommendations against progressive collapse due to unforeseen events such as explosions and impacts [2]. For multi-storey timber buildings, robustness is strongly dependent on the structural system and its ability to redistribute loads to undamaged parts, and thus, on its redundancy and ductility. After the loss of a load-bearing element, alternate load paths can be triggered through the capacity of connections to deform along with the development of tension stresses in beams and floors, the so-called catenary action. Moreover, in structures where long continuous beams are used, the catenary action can be substituted by an increment of bending forces, as the connections between these elements and the floor components withstand large deformations without experiencing brittle failure modes such as splitting, due to tension stresses perpendicular to the grain, and block shear. Another mechanism likely to occur is the membrane action of CLT floors, which is dependent on the inter-panel connections [3].

The main objective of this paper is to propose a probabilistic methodology for seismic and robustness assessment of timber structures leading to the development of fragility functions. A three-storey building is evaluated as an application example of the methodology presented.

2 Fragility Functions for Timber Structures

A fragility function F_R is defined as the conditional probability of the structural demand parameter (D) exceeds the structural capacity (C) for a given level of intensity measure (IM) of the hazard:

$$F_R = P[D > C | IM] \quad (1)$$

In the scope of the present work, the fragility curves addressed are obtained through analytical methods that include numerical analysis performance. Timber, as a natural material, presents a high variability in its mechanical properties, which contributes directly to uncertainties associated with the expected strength and deformation capacities of structures. In addition, it is also crucial to account for uncertainties related to external actions [3]. In terms of seismic assessment, the uncertainties in the demands are modelled by including ground motion record-to-record variability. In contrast, robustness assessments are associated with an initial damage imposed to the structure, which are assumed random by considering distinct scenarios. Additionally, in both types of analyses, the applied gravity loads are assumed as random variables. A considerable number of numerical simulations are typically needed to achieve robust fragility function results, requiring high computational costs. Such drawbacks can be partially overcome by using efficient sampling methods, such as Latin Hypercube Sampling, Importance Sampling, or others [4], and the use of high-throughput or high-performance computing [5].

2.1 Fragility Functions for Seismic Assessment

The methodology presented herein starts with the definition of the mechanical properties considered as random variables. Köhler et al. [6] propose probabilistic distributions and their statistical parameters for reference mechanical properties of timber (e.g., bending moment capacity, bending modulus of elasticity, and density). Additionally, it is crucial to consider the uncertainties related to connection response, where the performance of experimental tests can play a central role in the calibration of suitable numerical models [7]. A second phase involves the ground motion selection, which must consider the respective response spectra of the site, which can be retrieved from the standards and local documents. The different time-history records are usually scaled to elastic acceleration response spectra, whereas other sources of uncertainty can be considered, such as phasing and event duration [8]. The third phase includes the numerical modelling of the structural set, using simulation techniques, in a suitable nonlinear finite element analysis software. In the fourth step, the structural models are subjected to distinct accelerograms of increasing intensities (i.e., multi-record IDA). The fifth and final step corresponds to the definition of the

seismic fragility functions, which is accomplished by fitting a lognormal distribution to the values of intensity measure (e.g., spectral acceleration) that caused the exceedance of a predefined demand threshold (e.g., drift, damage, or collapse).

2.2 *Fragility Functions for Robustness Assessments*

It is possible to understand that the concept of robustness is intrinsically connected with the terms “progressive collapse” and “disproportionate collapse.” One considers that a progressive collapse refers to a series of components’ failures that resulted from an unrelated initial component failure. It means that an initial failure triggered a chain reaction of failures throughout the structure. On the other hand, a disproportionate collapse occurs when the subsequent structural collapse is exaggerated (“disproportionate”) compared to the initial failure. Therefore, a progressive collapse can be disproportionate, but the reverse is not necessarily true. To avoid ambiguities, Starossek and Haberland [9] proposed a definition of robustness using the probability of disproportionate collapse, which is given by:

$$P(DC) = P(DC|D) \cdot P(D|EX) \cdot P(EX) \quad (2)$$

where $P(EX)$ is the probability of occurrence of a particular exposure EX , $P(D|EX)$ is the conditional probability of initial damage D given the exposure EX , and $P(DC|D)$ is the conditional probability of disproportionate collapse DC given the initial damage D . In the scope of this work, the fragility functions focus exclusively on robustness analysis, which corresponds to the quantification of the conditional probability $P(DC|D)$. Failure and damage are terms used interchangeably herein when applied to structural components, and both mean that the performance criteria established were not fulfilled; e.g., a connection is damaged when it fails due to excessive deformation, or a column is damaged when the loading exceeds an ultimate shear capacity. When addressing the whole structure, failure shares the definition established for structural components, while the term damage refers to a deviation from the designed configuration that adversely affects functionality due to failure of a structural element.

As per seismic fragilities, the methodology starts with the determination of the mechanical properties considered as random variables. A second phase consists of the numerical modelling of a set of structures, using an adequate simulation technique, in a nonlinear finite element analysis software. The third phase includes the definition of distinct initial damage scenarios that can occur due to the exposures EX considered. The fourth step comprises an alternative load path analysis (ALPA), which aims to evaluate load redistribution through a pushdown analysis that starts with the notional removal of one or more elements to simulate the initial damage D [10]. The numerical analyses can be load controlled or displacement controlled and finish when the damage reaches an extent that is considered disproportionate to the original cause. The fifth and final step corresponds to the definition of the progressive collapse fragility function, which is accomplished by fitting a lognormal

distribution to intensity measure values (e.g., dynamic load factor or vertical load applied) associated with disproportionate collapse.

3 Case Study: A Three-Storey Heavy-Timber Frame with Ring-Doweled Moment-Resisting Beam-Column Connections

The structure under analysis is a three-storey building constructed using two different lateral systems. The structure consists of three moment-resisting timber frames that ensure lateral resistance and stiffness on the x-z plane, as shown in Fig. 1a. Two braced timber frames provide lateral resistance and stiffness in the perpendicular direction, as presented in Fig. 1b. The structural system was designed in Callegari et al. [11], aiming at a ductility level compatible with High Ductility Class (DCH), as defined in EC8 [5]. For this study, the structure was re-evaluated using EC5 [12] and EC8 [5] for a site in Lisbon, Portugal.

3.1 Lateral Resisting Systems

The moment-resisting frames have continuous 13.2m long GL24h beams, as per EN14080 [11], with a 160 by 600 mm cross-section. The columns consist of two posts, each with a rectangular cross-section of 160 by 600 mm. The connections between columns and beams are executed with ring-dowelled joints with two layers of connectors. All dowels are from strength class 4.6 with a diameter of 12 mm. The connection used at the column bases consists of a hinged connection with a low rotational stiffness in the x-z plane. The lateral resisting system uses GL24h members interconnected through concealed steel plates and bolts in the perpendicular direction, with additional details provided in [14]. As presented in Fig. 1b, in the y-z plane, continuous beams with a 200 by 360 mm cross-section span over two 4m long bays. The columns have the same cross-sections as the beams, and their base shoes are built with slotted steel plates that are welded to a pinned support. The diagonals have a cross-section of 140 by 240 mm. The connection between the two different lateral systems is guaranteed through concealed steel plates and M12 bolts. Further information can be obtained in [14].

3.2 CLT Diaphragm

The CLT floor solution presented in Fig. 2a consists of 5-ply panels (nominally 140.0 mm thick) with the major strength along the building's Y-direction. The panels

are manufactured with lumber from strength class C24. The CLT to beam connection is executed with ASSY plus VG 8×260 mm, as presented in Fig. 2b. The fasteners were designed to transfer inertial loads from the diaphragm to the seismic lateral resisting systems. The shear transfer between adjacent CLT panels is guaranteed through half-lap joints with 100 mm wide laps connected with ASSY 3.0 ECOFAST 8×120 mm fastened 100 mm on centre. The diaphragm moments are resisted by chord splices that consist of S235 structural steel plates ($6.35 \text{ mm} \times 50.8 \text{ mm} \times 1500 \text{ mm}$) that meet the requirements of standard EN 10025:2004 [15]. These plates are fastened to the CLT panels with Simpson Strong-Tie 6.4×90 HEAVY-DUTY connectors spaced 60 mm on centre.

Fig. 1 Lateral resisting systems: **a** Moment-resisting frame. **b** Braced frame

Fig. 2 CLT diaphragm: **a** Plan view. **b** CLT-to-beam connection at diaphragm edges. **c** Connection of an interrupted CLT panel to a central beam. **d** Connection of a continuous CLT panel to a central beam

3.3 *Uncertainties in Material Properties*

The Latin Hypercube Sampling (LHS) was used to generate 1000 structural models for each type of analysis performed in this work. Each model corresponds to a realization of the random variables assumed. The inherent uncertainties of timber, as a material, are considered by assigning different mechanical properties to each element. As presented in Table 1, seven random variables are evaluated for each timber element. The distribution parameters of the reference properties (bending strength, bending modulus of elasticity, and density) are computed based on characteristic values, defined in EN14080 [16] for the homogeneous GL24h strength class, and on the coefficients of variation and probabilistic distributions proposed in Köhler [6]. The expected values and the coefficients of variation of the remaining properties are computed according to the expressions proposed by [9] for Nordic softwood. Table 1 also presents the intra-element correlation coefficients considered, which were also taken from the work developed by Köhler [6]. Moreover, variability is also assumed for steel-to-timber connections with multiple bolt joints, overlap connections between CLT and glulam beams. The yielding force and the initial stiffness of each connection are modelled with lognormal distributions. Given the lack of experimental tests, a coefficient of variation equal to 15% is assumed for both properties. However, the uncertainties related to the mechanical models used shall be confirmed through testing in future studies. The remaining values used to establish the backbone curves are calculated by assuming ratios between each key point and the yielding point in terms of displacements and forces. These ratios were obtained through a fitting procedure based on experimental tests performed on similar connections. For the sake of brevity, further information on the calibration of Pinching4 can be obtained from [14]. The CLT floor members are modelled using an orthotropic material with deterministic mechanical properties [17].

4 Seismic Assessment

A set of 24 ground motion records was extracted from the PEER database (PEER 2012) and scaled to the spectra defined in EC8 [1]. Two types of seismic response spectra were considered when designing for a Lisbon site: a large magnitude and far-field earthquake (Type I) and a lower magnitude and near-field earthquake (Type II). The 5% linearly damped response spectra of the scaled ground motions are shown in Fig. 3. Additionally, the range of periods of interest (i.e., those within 0.3 and 3.0 times the median fundamental period of the structure) are also indicated in Fig. 3.

An incremental dynamic analysis (IDA) was performed on a set of 2D finite element models of the X-direction central moment-resisting frame (see Fig. 1a) in Opensees [18]. The beams and columns were modelled using linear elastic frame elements. The ring-dowelled connections were represented using zero-length elements and assigned Pinching4 force-deformation springs, including pinching,

Table 1 Random variables for timber material properties

Distribution parameters				Correlation matrix							
<i>X</i>	Dist	<i>E</i> [<i>X</i>]	<i>CoV</i> [<i>X</i>]		<i>R_m</i>	<i>E_m</i>	<i>ρ_{den}</i>	<i>R_{t,0}</i>	<i>R_{c,0}</i>	<i>G_v</i>	<i>R_v</i>
<i>R_m</i>	LN	31	0.15		<i>R_m</i>	0.8	0.6	0.8	0.8	0.4	0.4
<i>E_m</i>	LN	11,500	0.13		<i>E_m</i>	1.0	0.6	0.6	0.6	0.6	0.4
<i>ρ_{den}</i>	N	420	0.10		<i>ρ_{den}</i>		1.0	0.4	0.8	0.6	0.6
<i>R_{t,0}</i>	LN	18.6	0.18		<i>R_{t,0}</i>			1.0	0.5	0.4	0.6
<i>R_{c,0}</i>	LN	23.4	0.12		<i>R_{c,0}</i>				1.0	0.4	0.4
<i>G_v</i>	LN	718.8	0.13		<i>G_v</i>					1.0	0.6
<i>R_v</i>	LN	3.12	0.15		<i>R_v</i>						1.0

Description: *R_m*—bending strength // to the grain (N/mm²); *E_m*—bending modulus of elasticity (N/mm²); *ρ_{den}*—density (kg/m³); *R_{t,0}*—tension strength // to the grain (N/mm²); *R_{c,0}*—compression strength // to the grain (N/mm²); *G_v*—shear modulus (N/mm²); *R_v*—shear strength (N/mm²)

Fig. 3 Response spectra used to perform multi-record IDA: **a** Type I. **b** Type II

stiffness degradation, and strength degradation phenomena observed in the experimental results [19]. The Pinching4 model is defined by a response envelope, unloading and reloading rules, and three damage rules that control the evolution of deterioration mechanisms based on strength, stiffness, and energy dissipation. A total of 1000 samples were generated based on the assumed random variables and their distributions. Thus, a total of 24,000 curves were developed for the entire structure set. For simplicity, only the results obtained for a median structure are presented in this document. The IDA results obtained for the median structure (i.e., structure considering all random variables set equal to their median value) are shown in Fig. 4a. The fragility curves presented in Fig. 4b were obtained by fitting a lognormal distribution to the spectral accelerations that led to the exceedance of the Life Safety (LS) limit state threshold [13]. From the LS fragility functions, graphically represented in Fig. 4b, it can be observed that the limit state inter-storey drift ratios influence both the expected value and the coefficient of variation, indicating that their uncertainty should also be explicitly considered in future studies.

Fig. 4 Seismic assessment: **a** IDA curves obtained for the analysis of the median structure using the 24 ground motions selected. **b** Life safety fragility curves for distinct inter-storey drift levels θ_{max} .

5 Robustness Assessment

A 3D finite element model was used for the progressive collapse assessment. Linear elastic frame elements represent beams and columns, while elastic orthotropic ShellMITC4 four-node elements modelled the CLT panels. Geometric nonlinearities were incorporated in the form of P- Δ effects for columns and corotational transformation for beams. The connections were represented by zero-length elements that account for their nonlinear behaviour, incorporated through the Pinching4 material available in Opensees. The total number of pushdown analyses is given by the product of the number of models ($N_{sim} = 1000$) and the number of element removal scenarios considered ($N_{ele} = 4$); the four column removal scenarios considered are labelled scenarios D_1 to D_4 in Fig. 5. Thus, the progressive collapse assessment included a total of 4000 nonlinear static pushdown analyses. After each displacement increment step, a global failure assessment. It is worth noting that disproportionate collapse (DC) is assumed to occur when main beams, columns and connections, located at adjacent bays, reach their capacity. Although the structure was designed according to the Eurocodes, for the application of the ALPA method from the U.S. standard, US UFC 4-023-03 [20] was used. This choice was based on the fact that the latter code is more comprehensive and detailed than Eurocode 1-7 [21]. Hence, the load combination assumed for the ALPA method is given in the format:

$$G = \Omega[1.2DL + 0.5LL] \quad (3)$$

where DL is the dead load, LL is the live load, and Ω is a dynamic amplification factor. The dead loads are modelled with a lognormal probability distribution to avoid negative values; their mean value is assumed to be equal to the nominal value. The live load (LL) is modelled by a lognormal distribution with a mean value of 2.0 kN/m^2 and CoV equal to 0.30. From the performance of each pushdown analysis, an overload factor is calculated, which is given by the ratio between the load that a damaged structure is able to sustain (G_R) and its nominal gravity loads G_n ($\Omega = 1.0$).

The majority of the collapse cases were reached after the failure of one ring-dowelled connection. Several analyses returned the failure of one main beam due to excessive bending stresses. From the numerical results obtained, it is possible to state that the initial damage scenarios D_2 and D_3 led to collapse cases that did not exploit the ability of joints to deform. In these cases, the beams located just above the damaged elements sustained the loads and failed due to excessive bending stresses. The primary mechanism used to support the loads due to local failures was the beam action in the main GLT beams. Fragility functions, presented in Fig. 6, were defined for different damage state levels by fitting a lognormal distribution to the overload factor values that caused the exceeding of predefined demand threshold values.

The damage state levels were based on the rotational capacity of floor-to-beam connections and the extent of the damaged area. The first failure state (FF) is reached when any floor-to-beam connection exceeds its capacity. In turn, the collapse prevention (CP) limit state was exceeded when the damage was extended to an adjacent

Fig. 5 Progressive collapse assessment: **a** pushdown procedure; **b** scenario D_1 ; **c** scenario D_2 ; **d** scenario D_3 ; **e** scenario D_4

Fig. 6 Progressive collapse assessment: **a** probability density functions for maximum vertical displacements of the model structure for distinct elements loss scenarios; **b** pushdown curves for distinct scenarios

bay, which was verified when moment-resisting connections and glulam members reached their load bearing capacity. The vertical loads used to perform an ALPA method were multiplied by a dynamic amplification factor [22] equal to 2.0 for timber structures. To assess if the seismic design provisions used to design the structural system also guarantee structural robustness against progressive collapse, the probability of failure associated with an overload factor of 2.0 is evaluated for each damage scenario in Table 2.

From the results presented, it is possible to state that the application of seismic design provisions does not dismiss a progressive collapse assessment during. The main failure modes observed were related to bending stresses of GLT beams, followed

Table 2 Probability of failure (p_f) and respective reliability indexes (β) for an overload factor of 2.0

Damage scenario	Failure state	p_f	β
D_1	$P(CP D_1)$	0.135	1.103
D_2	$P(CP D_2)$	0.002	2.821
D_3	$P(CP D_3)$	0.980	−2.052
D_4	$P(CP D_4)$	0.439	0.153

by the loss of capacity of moment-resisting joints to transfer loads from GLT beams to columns.

6 Conclusions

The paper presented the implementation of a methodology suitable to perform structural assessments for seismic and robustness analyses. A prototype structure designed according to EC5 and EC8 for a site in Lisbon, Portugal, was presented as a case study. The methodology presented is suitable for risk-based assessments that consider different exposures, such as earthquakes, impacts, and explosions, while considering the direct and indirect consequences of failures. However, the methodology involves time-consuming analyses with distinct load scenarios, which can constitute a burdening task within a typical design phase of a building. Thus, the subsequent efforts can focus on developing software packages to alleviate the modelling and analysis time while still allowing for uncertainty propagation throughout the methodology.

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