Flexural and shear response predictions of statically determinate and indeterminate RC structures strengthened with fiber reinforced polymer

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Fiber-reinforced-polymer (FRP) composite materials have the required stiffness and strength properties for the flexural and shear strengthening of reinforced-concrete (RC) structures. However, evaluation of the total deflections of RC structures strengthened with FRP composite materials up to their collapse is relatively difficult due to the complexity in shear mechanism and responses of these strengthened structures. These complexities are even more pronounced when the considered structures are statically indeterminate, since in these cases, the external and internal forces cannot be determined from direct application of the equilibrium equations. Hence, the current study aims to develop a comprehensive simplified analytical model based on the flexibility (force) method to predict the total deflections of statically determinate and indeterminate FRP strengthened structures, composed of the flexural and shear deformations. Moreover, after concrete cracking initiation stage, the proposed analytical model offers the consideration of degrading effects of flexural cracks on the shear stiffness in addition of the flexural stiffness, which is a challenge in this aspect. Furthermore, in this model the shear deformation, which has different level of contribution for the prediction of total deflection, can be separately predicted. The predictive performance of the proposed analytical model for these types of structures is verified through the comparison with the relevant experimental results.

INTRODUCTION

For providing an appropriate performance level for reinforced concrete (RC) structures strengthened with FRP composite materials in serviceability limit state (SLS) conditions, the total deflection composed of flexural and shear deformations should be limited during the design and strengthening process to fulfil the requirements of SLS. Generally, after the initiation of flexural cracks in an RC structure, the element's flexural and shear stiffness decrease [1,2,]. Extensive research was performed to analytically estimate the deflection of RC structures failing in bending [3,4], However, according to the knowledge of the authors, developing a simplified model for designers and engineers to analytically predict the contribution of shear deformation for total deflection of cracked RC structures was not yet comprehensively addressed [5,6]. On the other hand, developing the analytical methodologies, capable of predicting the deflections of statically indeterminate structures, is still a challenging task due to the relevant complexities it involves [7].
In this regard, this study aims to develop a novel simplified analytical model using the flexibility method to simultaneously or separately predict the flexural and shear deformations of statically determinate and indeterminate RC structures strengthened in bending and/or in shear by fiber reinforced polymer (FRP) composite materials up to the failure. In this proposed model, the flexural deflections of a RC structure are determined by using the tangential flexural stiffness of the cross section acquired from the corresponding moment-curvature relationship of the section. For estimating the shear deflections of a RC structure, the tangential shear stiffness of the cross section during the loading process is obtained by assuming that the shear behavior of a FRP strengthened RC structure can be simulated by a three stage diagram representing the pre-cracking, post-cracking and post-diagonal cracking stages, delimited by the following points: concrete crack initiation; diagonal crack initiation; and ultimate shear capacity. The good predictive performance of the proposed model is assessed by determining the force-deflection response registered in the experimental programs composed of determinate and indeterminate RC beams and slabs.

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ANALYTICAL MODEL DESCRIPTION

By using the static equilibrium equations, the external and internal forces can be entirely determined for the statically determinate structures, while in the case of statically indeterminate structures the number of redundant supports is more than the number of static equilibrium equations. In order to derive a system of equations capable of determining the unknowns, displacement compatibility equations should be established. Two methods are available for the analysis of statically indeterminate structures, namely flexibility method (also known as force method) and stiffness matrix method (known as displacement method) [9]. In this study, an analytical model based on the flexibility method is proposed

Flexibility Method

For assisting in the description of proposed analytical model, a two-span element is considered. It is assumed that in each small load increment (ΔF), the principle of superposition can be applied, even in the nonlinear phase response of the structure. Based on this assumption, the structure is decomposed into a number of equilibrium configurations. In the present case, three displacement compatibility equations corresponding to the loaded sections and intermediate support are established. For each equilibrium configuration, the incremental forces (ΔF_1 and ΔF_2) corresponding to the imposed incremental displacements (Δu_1 and Δu_2) and the relevant reaction ΔR are determined (Fig. 1) [7,8].

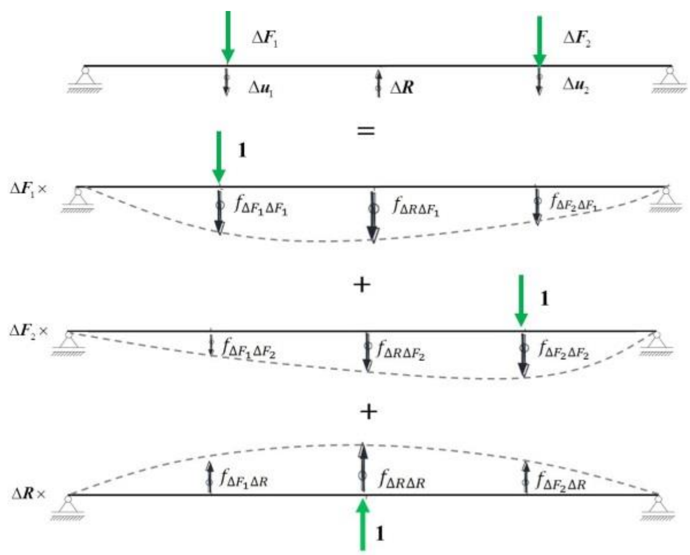


Figure 1. Physical meaning of the terms of the flexibility matrix, based on the displacements for each equilibrium configuration

Regarding the determination of these forces using the flexibility method, the terms of the flexibility matrix with a generic representation of f_{ij} should be calculated by applying the principal of virtual work [7,8]. For example, for the present version of the proposed model, dealing with 2D bar type elements (Fig. 1), each term of flexibility matrix is as follows:

$$f_{ij} = \int_0^L \frac{M_i M_j}{EI} dl + \int_0^L \frac{V_i V_j}{GA^*} dl \tag{1}$$

where f_{ij} is the displacement at coordinate i (in the direction of F_i) due to the application of a real unit load at coordinate j ($F_j = 1$) on the released structure (see Fig. 1). By applying a unit virtual load $F_i = 1$ at coordinate i on the released structure, bending moment an shear force M_i, V_i are introduced at any section of the ith configuration [7]. In Eq. (1) EI and GA^* are the flexural and shear stiffness, respectively. According to the principle of superposition effects, as represented in Fig. 1, the following three equations of displacements compatibility can be established:

$$\begin{Bmatrix} \Delta u_1 = f_{\Delta F_1 \Delta F_1} \times \Delta F_1 + f_{\Delta F_1 \Delta F_2} \times \Delta F_2 + f_{\Delta F_1 \Delta R} \times \Delta R \\ \Delta u_2 = f_{\Delta F_2 \Delta F_1} \times \Delta F_1 + f_{\Delta F_2 \Delta F_2} \times \Delta F_2 + f_{\Delta F_2 \Delta R} \times \Delta R \\ 0 = f_{\Delta R \Delta F_1} \times \Delta F_1 + f_{\Delta R \Delta F_2} \times \Delta F_2 + f_{\Delta R \Delta R} \times \Delta R \end{Bmatrix} \rightarrow \Delta \underline{F} = \underline{f}^{-1} \Delta \underline{u} \tag{2}$$

By solving Eq. (2), the vector of the unknown incremental forces, ΔF is obtained. In this equation, $\underline{f}$ is the flexibility matrix; $\Delta \underline{F}$ is the vector of unknown applied forces (ΔF_1 and ΔF_2) and reaction support (ΔR); and $\Delta \underline{u}$ is the vector of the imposed incremental displacements in the directions of ΔF_1, ΔF_2 and ΔR (the displacement corresponding to ΔR is null).

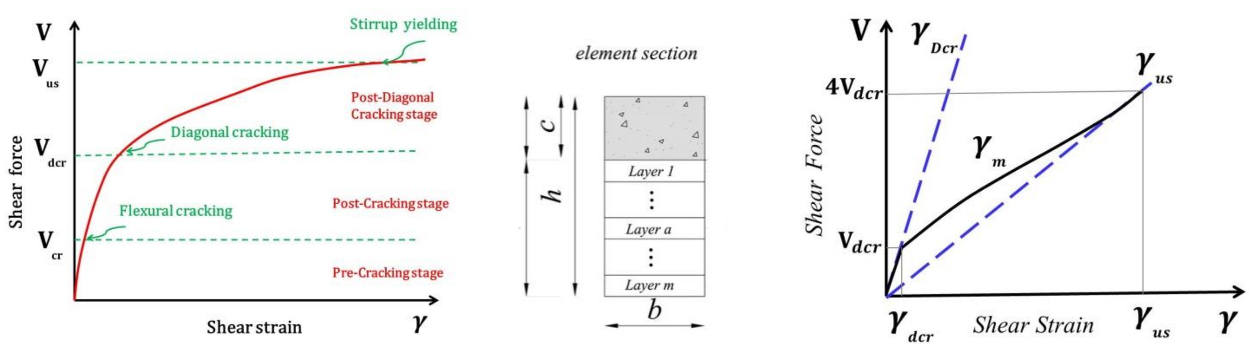
Flexural and shear Part of the Flexibility Matrix

Each term of the flexibility matrix according to the internal work due to bending and shear is obtained by:

$$f_{ij} = \sum_{k=1}^{nel} \left[\frac{M_{i,k} M_{j,k}}{(EI)_k} dl_k \right] + \sum_{k=1}^{nel} \left[\frac{V_{i,k} V_{j,k}}{\left(GA^*\right)_k} dl_k \right] \tag{3}$$

Due to the possible variation of the structure in terms of geometry, bending moments or flexural stiffness and the characteristics that influence the shear part of flexibility matrix, the structure is decomposed in a set of elements (nel) of length dl_k, where in Eq. (3) M_i and M_j are the bending moments, while V_i and V_j are the shear forces in the released structure corresponding to the equilibrium configurations C_i ($F_i = 1$) and C_j ($F_j = 1$), respectively. The bending moments ($M_{i,k}, M_{j,k}$), shear forces ($V_{i,k}$, $V_{j,k}$), the tangential flexural $(EI)_k$ and shear stiffness $\left(GA^*\right)$ of an element with a length dl is calculated in its center [7]. The $M_{i,k}$, $M_{j,k}$, $V_{i,k}$ and $V_{j,k}$ for each *k* element of the structure are updated in each increment of the displacement.

Tangential flexural stiffness $(EI)_k$ is the tangent to the moment-curvature relationship of the cross section. For taking into account the effect of flexural and shear cracks on the shear stiffness degradation during the loading process, the tangential shear stiffness (GA^*) of the cross section of each element is obtained assuming the corresponding shear force versus shear deformation ($V - \gamma$) approach of the cross section schematically represented in Fig. 2a. According to this approach, the $V - \gamma$ response can be regarded as formed by the pre-cracking, post-cracking, and post-diagonal cracking stages, defined by the following points (Fig. 2a): concrete crack initiation (point “cr“); diagonal crack initiation (point “dcr“); and ultimate shear capacity (point “us“).



(a) (b) (c)

Figure2. a) Three stages of shear deformational behavior of RC element, b) tensile zone of the cross section decomposed in layers, c) Shear strain as a function of applied shear force

The pre-cracking stage corresponds to the linear elastic behavior of the section, being the shear stiffness obtained from:

$$G_e A^* = \frac{E_c}{2(1+\upsilon)} \frac{b.d}{f_s} \tag{4}$$

where $G_e = E_c / [2(1+\upsilon)]$ and $A^* = (b.d)/f_s$, with E_c and υ being the the Young modulus and Poisson coefficient of concrete, respectively, while *b* and *d* are the width and height of cross section, and f_s is the shear correction factor according to the Timoshenko theory.

After concrete crack initiation, the cracking process is progressing up to the shear diagonal crack initiation that is assumed limited by the corresponding shear force (V_{dcr}), obtained according to the recommendation of ACI-318 [1]

$$V_{dcr} = V_c = 0.17\sqrt{f_c}\, b\, d \tag{5}$$

where f_c is the concrete compressive strength ; b and d are the web thickness of cross section and internal arm of longitudinal tensile steel bars, respectively. During post-cracking stage, the extension of the flexural cracks reduces not only the flexural stiffness but also the capability of transferring shear forces [5]. For considering the influence of flexural cracking during this stage on the shear stiffness degradation, the contribution of concrete in the cracked tension zone for determining the shear stiffness was taken into account using a shear retention factor (β) that reduces the elastic shear modulus (βG_e) [9], which is function of the applied axial concrete tensile strain (ε_{ct}) and the ultimate concrete tensile strain (ε_{ctu}), as follows:

$$\beta = \left(1 - \frac{\varepsilon_{ct}}{\varepsilon_{ctu}}\right)^p \tag{6}$$

where P is a parameter that determines the shape of reduction of concrete shear modulus with the increase of ε_{ct} [9]. To estimate β with a higher accuracy, the cross section in tension zone is divided in layers of relatively small thickness. Hence, assuming a linear proportionality of strain distribution along the depth of the cross section with regard to the neutral axis level, the mean strain in each layer is taken as a representative concrete tensile strain to calculate the corresponding value of the shear retention factor (β_a) of the layer. Accordingly, during the post-cracking stage the sectional shear stiffness (GA^*) is obtained using Eq. (7) considering the corresponding shear stiffness of compression zone (GA^*_{cc}) and tension zone (GA^*_{ct}) of the cross section (Fig. 2b):

$$GA^* = GA^*_{cc} + GA^*_{ct} = \frac{G_e.b.c}{f_s} + \frac{G_e.b}{f_s}\sum_{a=1}^{m} \beta_a.h_a \tag{7}$$

where *c* is the neutral axis depth of the element cross section, h_a is the thickness of each layer determined by dividing the tension zone depth in a selected number of layers (m) in concrete tension zone (Fig. 2b). The behavior of concrete in the compression zone was assumed to be linear, consequently, the corresponding shear stiffness (GA^*_{cc}) is obtained considering the equation represented for the pre-cracking stage (Eq. (4)).

When the internal shear force of cross section exceeds the corresponding diagonal shear strength, the section is considered in the post-diagonal cracking stage (Fig. 2a). To simulate the shear stiffness degradation of cross section in this stage, two boundary states corresponding to the diagonal shear crack initiation and fully developed diagonal shear crack are considered (Fig. 2c). The proposed mean value of shear strain (γ_m) (Eq. (8)) between these two boundaries is defined according to the shear strains corresponding to the initiation of diagonal shear cracking (γ_{dcr}) and shear strain of fully diagonal shear cracked ($\gamma_{us,eq}$) based on recommendation of CEB Manual by Eqs. (9a,b) [11]:

$$\gamma_m = (1-\zeta)\gamma_{dcr} + \zeta\,\gamma_{us} \Leftrightarrow G_{eff} = \frac{V}{\gamma_m A^*} \tag{8}$$

$$\gamma_{dcr}=\frac{V_{dcr}}{\left(GA^{*}\right)_{dcr}}\ ;\ \gamma_{us,eq}=\frac{V}{0.9\,d\,b}\left(\frac{1}{\rho_{w,eq}\,E_{s,eq}}+\frac{4}{E_c}\right) \tag{9a,b}$$

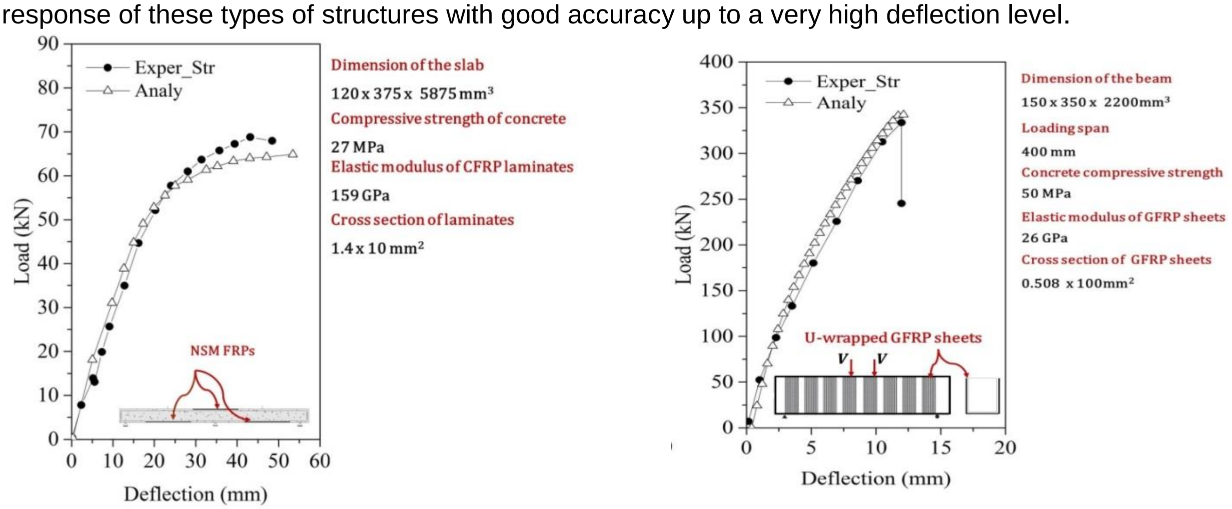
$$\rho_{w,eq}=\rho_s+\frac{E_f}{E_s}\rho_f\ ;\ E_{s,eq}=\frac{A_s.E_s+A_f.\frac{E_f}{E_s}.E_f}{A_s+A_f.\frac{E_f}{E_s}} \tag{9c,d}$$

where $\rho_{w,eq}$ is equivalent shear reinforcement ratio and $E_{s,eq}$ is equivalent elasticity modulus of shear reinforcement. These two parameters ($\rho_{w,eq}$ and $E_{s,eq}$) are adopted to consider the contribution of FRP composite materials applied for the strengthening purpose, and can also be obtained by Eqs.(9 c,d), in which ρ_s and ρ_f are the shear reinforcement ratio with regard to steel and FRP reinforcements, E_s and E_f are the elasticity modulus of steel and FRPs, and A_s and A_f are the cross sectional area of shear steel and FRP reinforcements, respectively [12].

ASSESSMENT of PREDICTIVE PERFORMANCE of ANALYTICAL APPROACH

Prediction of total deflections of strengthened continuous slab

Regarding the assessment of the performance of the model for predicting the response of indeterminate structures, the flexural terms of the model were applied on the prediction of the flexural behavior of the statically indeterminate strengthened two-span RC slab. This slab with one degree of indeterminacy was strengthened by using CFRP strips according to the near surface mounted (NSM) technique applied in both sagging and hogging regions (more details were presented in [8]) (see Fig. 3a). In this case the shear term of the analytical model was neglected, since the contribution of the shear for the response of these types of structures is marginal. In Fig.3a the force-deflection responses obtained analytically using the flexural term of the described model and registered experimentally are compared for the strengthened indeterminate RC slab. This figure evidences that the developed model is capable of predicting the response of these types of structures with good accuracy up to a very high deflection level.



a b

Figure 3: Analytical prediction of load-deflection relationships of: a) strengthened continuous slab, b) RC beam strengthened using U-wrapped GFRP sheets

Assessment of total deflection of strengthened RC beam by using U-wrapped GFRP sheets

To assess the predictive performance of the proposed model, one simply supported RC beam strengthened using Sikawrap Hex-430G GFRP sheets U-wrapped (100 mm wide at a spacing of 200 mm o/c), tested by Baggio et al. [40], was simulated. The characteristics of this beam in terms of the geometry and reinforcement details are available in [13]. This simply supported beam was monotonically tested under four-point loading configuration. Fig. 3b demonstrates that the proposed analytical model is capable of predicting with high accuracy the deflection behavior of this type of structures strengthened in shear using FRP composite materials

CONCLUSIONS

In this study, an analytical model based on the flexibility method is proposed for the prediction of the material nonlinear behavior of determinate and indeterminate RC structures strengthened with FRP composite materials up to their collapse, considering the relevant mechanisms of flexural and shear stiffness degradation due to cracking formation and propagation.

In this proposed model, the flexural deflections of a structure are determined using the tangential flexural stiffness of the representative cross sections obtained from the corresponding moment-curvature relationship. For determining the shear deflections of a structure, the tangential shear stiffness of the representative cross sections was obtained by assuming the shear stiffness evolution can have a pre-cracking, post-cracking, and post-diagonal cracking stages delimited by the concrete crack initiation; diagonal crack initiation; and ultimate shear capacity, respectively.

The results of experimental programs, composed of strengthened RC beam in shear using GFRP composite materials and statically indeterminate flexurally strengthened RC slab using CFRP strips in terms of load versus total deflections, were compared with the ones obtained by the proposed analytical model, and a good predictive performance was evidenced.

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