Improving durability through probabilistic design

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ABSTRACT: In order to obtain a more controlled durability and long-term performance of concrete structures in chloride containing environment special care is needed in the design phase of reinforced concrete structures. The recent development of probability-based procedures has proven to give a more realistic basis for both durability design and condition assessment of reinforced concrete structures. Although there is still a lack of relevant data, this approach has been successfully applied to several new concrete structures, where requirements to a more controlled durability and service life have been specified.

Since parameters both for concrete durability and environmental exposure typically show a high scatter, a probability-based approach has shown to give a very powerful basis for durability analysis. This approach is primarily being applied for obtaining a more controlled durability and long-term performance of new concrete structures, but it also provides a very valuable basis for condition assessment of existing concrete structures in chloride containing environment.

In the present paper, the probability-based durability analysis is briefly described and used in order to demonstrate the importance and sensitivity of the various parameters affecting and controlling the durability of concrete structures in chloride containing environment. The results show that this procedure provides valuable information concerning the design options, making the decision process more reliable.

1 INTRODUCTION

The recent development of probability-based procedures has proven to give a more realistic basis for both durability design and condition assessment of reinforced concrete structures. Although there is still a lack of relevant data, this approach has been successfully applied to several new concrete structures, where requirements to a more controlled durability and service life have been specified (Gehlen et al 1999, Edvardsen et al 2005).

Since parameters both for concrete durability and environmental exposure typically show a high scatter, a probability-based approach provides a very powerful basis for durability analysis (DuraCrete 2000, McGee 1999). This approach is primarily applied in order to obtain more controlled durability and long-term performance of new concrete structures, but it also provides a very valuable basis for condition assessment of existing concrete structures in chloride containing environments.

The degradation models for corrosion initiation include carbonation and chloride penetration as a function of time and variables related to, among other parameters, concrete composition and the influence of the environment.

In the following article, a generic model is described and applied in order to demonstrate the importance and sensitivity of the various durability parameters affecting and controlling the durability of concrete structures, namely in a chloride containing environment.

A computer software with the model applied using a Monte Carlo simulation is used to perform the durability analysis (Duracon 2004, Ferreira 2004a).
A serviceability limit state of depassivation and onset of steel corrosion is used. The software can express the probability of failure or the reliability index for the serviceability limit state to be reached after a certain period of time. For new concrete structures, this provides an appropriate basis for establishing overall durability criteria for the structure in question (Gjørv 2004). For existing concrete structures, where the chloride front has still not reached the embedded steel, this procedure can be used for estimating the probability of corrosion initiation after a certain period of time (Ferreira et al 2004b).

2 MODEL DESCRIPTION

When corrosion is caused by chloride ingress, the service life is usually assumed to be equal to the initiation time. The period of propagation, which may be of short duration, is traditionally not taken into account because of the uncertainly with regard to the consequences of localized corrosion (Bertolini et al 2004).

The modelling of chloride penetration and time to depassivation is commonly based on Fick’s Second Law of Diffusion. However, as this law does not correctly model the diffusion of ion through concrete, modifications can be made to take into account several phenomena, such as the variation in time of the diffusion coefficient (due to cement hydration and influence of temperature) and the surface concentration of chlorides. The stochastic nature of the individual durability parameters (reflected in mean, standard deviation and type of distribution of relevant variables) require that the model be applied probabilistically.

The initiation time may be calculated as a function of the chloride transport properties of concrete (usually the apparent diffusion coefficient which may either be obtained from accelerated laboratory testing or curve fitting of chloride profiles from existing concrete structures); the time dependence of the diffusion coefficient; the surface chloride concentration influenced by the environment (obtainable from either measurements or previous experience); the thickness of the concrete cover and the critical chloride content for depassivation of embedded steel (both of which may be obtained from existing literature or other experience for the given type of cement and concrete).

The model used in this case only contemplates the dependency of the diffusion coefficient with time and with temperature.

2.1 Rate of chloride penetration

The rate of chloride penetrating into concrete as a function of depth from the surface is given by Fick’s Second Law of Diffusion:

\[
\frac{dC(x,t)}{dt} = D_c \cdot \frac{d^2 C(x,t)}{dx^2}
\]

where \(C(x,t)\) is the chloride ion concentration at a distance \(x\) from the concrete surface after being exposed for a period of time \(t\), and \(D_c\) is the chloride diffusion coefficient. By solving this differential equation for predefined boundary conditions, the following equation is obtained:

\[
C(x,t) = C_s + (C_0 - C_s) \cdot \left[1 - \text{erf}\left(\frac{x}{2\sqrt{D_t}}\right)\right]
\]

where \(C_s\) is the chloride ion concentration on the concrete surface, \(C_0\) is the initial chloride concentration of the concrete mix, and \(\text{erf}\) is the error function.

The time dependency of the diffusion coefficient is well known, ever since Takewaka presented an equation to model this behaviour (Takewaka et al 1988). Much research has been done on this topic (Maage et al 1994, Mangat et al 1994, Duracrete 1998, Bamforth 1999, Gjørv 2002a, Gjørv et al 2002b), and, it has shown that the diffusion coefficient variation with time is dependent on various factors, the most important being the w/c ratio of the mix and the cement type and content, and exposure conditions. The time dependent variation of the diffusion coefficient is introduced by the following equation (Takewaka et al 1988):
\[ D(t) = D_0 \left( \frac{t}{t_0} \right)^\alpha \]  

(3)

where \( D_0 \) is the diffusion coefficient at a given time \( t_0 \), and the exponent \( \alpha \) represents the time dependence of the diffusion coefficient or the increased ability of the concrete to resist chloride penetration over time.

For temperature levels above freezing, the temperature is a decisive factor regarding the rate of certain phenomenon. This factor alone makes hot and tropical environments considerably more aggressive than temperate climates.

The temperature dependence of the viscosity of water is mainly important for other effects such as the concrete’s resistivity and diffusion processes. Diffusion processes are strongly dependent on temperature. In the case of chloride diffusion the situation is somewhat more complicated, because in the diffusion process the chemical and physical interaction of chloride with the cement paste must also be taken into consideration.

Based on the Nernst-Einstein relation, values for the diffusion coefficient at ambient temperature (21°C) are correlated to values at standard temperature according to (Pruckner 2001):

\[ D(T) = D_{294K} \cdot \frac{T}{294} \cdot \exp \left[ -\frac{E_A}{R} \left( \frac{1}{T} - \frac{1}{294} \right) \right] \]  

(5)

where \( T \) is the temperature in Kelvin, \( E_A \) the activation energy, \( R \) the gas constant, and \( D_{294K} \) the diffusion coefficient at the reference temperature (21°C).

By substituting Eq. 3 and Eq. 4 into Eq. 2, the model equation for the prediction of chloride penetration based on the time and temperature dependent diffusion coefficient is obtained, given by:

\[ C(x,t) = C_0 + (C_S - C_0) \cdot \left[ 1 - \text{erf} \left( \frac{x / 2}{\sqrt{D_{0,294K} \cdot \frac{T}{294} \cdot \exp \left[ -\frac{E_A}{R} \left( \frac{1}{T} - \frac{1}{294} \right) \left( \frac{t}{t_0} \right)^\alpha \right]}} \right) \right] \]  

(4)

It should be noted that the rate of chloride penetration may also be controlled by other mechanisms such as capillary suction or crack penetration. However, based on current knowledge, it is assumed that diffusion is a dominating transport process for chloride penetration into concrete structures in marine environments. However, there is still a lack of relevant data and information on the various input parameters. Therefore, a critical interpretation of obtained results and sound engineering judgement are important for experience with the design procedure.

### 2.2 Probabilistic approach

The probabilistic approach is based on the Monte Carlo Method, which can be briefly described as a statistical simulation method, where sequences of random numbers are applied to perform the simulation. In the present application of the simulation, the physical process is simulated directly by use of the modified Fick’s Second Law of Diffusion for describing the transport process. The only requirement is that all the input parameters to the equation be described by a probability density function. Once the probability density functions of the various durability parameters of the system are known, the probability of failure is based on the evaluation of the limit state function for a large number of trials.

When a simulation method is used for calculating the probability of failure, the failure function is calculated for each outcome. If the outcome is in the failure region, then the contribution to the probability of failure is obtained. The probability of failure is estimated by the following expression:

\[ p_f = \frac{1}{N} \cdot \sum_{j=1}^{N} I[g(r_j, s_j)] \]  

(4)

where \( N \) is the number of simulations, \( I[g(r_j, s_j)] \) is the indicator function and \( g(r_j, s_j) \) is the
limit state equation, where $s$ represents the environmental load and $r$ is the resistance of the concrete against chloride penetration.

The standard error of the probability of failure is estimated by (Enevoldsen et al 2000):

$$ s = \sqrt{\frac{p_f(1-p_f)}{N}} $$

(4)

Since the accuracy of the Monte Carlo Method depends mainly on the number of trials undertaken (Di Sciuva et al 2003) and the method is easy to implement, a simulation based on this method appears to be both simple and intuitive.

2.3 Limit state definition

Most service-life approaches calculate the mean service life, which is the service life to be achieved with 50 % probability. From an economic and safety point of view, this is not acceptable. Limit states can be, for example, the need for structural repair because concrete parts are falling off due to corrosion, or the structure collapses. The need for repair is termed a serviceability limit state (SLS). Collapse is termed an ultimate limit state (ULS). Failure probabilities like this are defined by EN 1990, Annex B and C (2002), where the requirements for durability design of structures, for a serviceability limit state should not exceed 7 % for a service-life of 50 years. As the initiation of corrosion does not immediately have extreme consequences, a probability of failure has been proposed for this event of 1:10 (Fluge 2001).

In its simplest form the limit state equation is given by

$$ g = R(t) - S(t) > 0 $$

(5)

where $R(t)$ is the resistance and $S(t)$ is the load, both taken as time dependent. A limit state equation is positive only if the structure considered is fully capable to show the required performance. For chloride induced corrosion, the resistance is defined as the concrete's capacity to resist chloride penetration where as the load is the environment's influence, presented in the form of surface chloride concentration. Equation (5) can also be presented as

$$ g = C_{CR} - C(x,t) > 0 $$

(6)

3 DURABILITY DESIGN OPTIONS

3.1 General

In order to demonstrate the use of this model for durability design (Ferreira 2004a, Ferreira 2005, Duracon 2004) and how it can be used to select appropriate concrete mixtures or concrete cover so as to obtain a more controlled durability, the effects of three different parameters on the durability performance of concrete are simulated below. In the first example, the cement type used in the concrete mixture was investigated. In the second example, the effect of varying temperature was studied, while in the third example, the concrete cover was varied.

3.2 Input durability parameters

As input to the durability analysis, the following durability parameters were used:

- $t$ - exposure period to the chloride containing environment. In all cases, an exposure period of 50 years was selected;
- $C_S$ - surface chloride concentration. In all cases, a normal distribution of surface chloride concentration with an average of 5.4 % by weight of cement and a coefficient of variation (CoV) of 10% were adopted;
- $x_C$ - concrete cover of reinforcement. In the first two cases, a normal distribution of concrete cover with an average of 50 mm and a 5 mm standard deviation were adopted.
- $D_0$ - diffusion coefficient at time $t_0$. For each concrete mixture, this coefficient was determined based on laboratory testing (Årskog et al 2004, Ferreira et al 2002), assuming a normal distribution.
- $t_0$ - the age at which the diffusion coefficient $D_0$ was determined. For all types of concrete, this age was 28 days.

- $C_{CR}$ - critical chloride ion concentration. Based on existing experience, a normally distributed value averaging 0.48 and standard deviation of 0.15 were assumed (HETEK Report n°53 1996);

- $\alpha$ - exponent for time dependence of diffusion coefficient. Based on existing experience for each type of cement, appropriate values for both the exponent and CoV were assumed.

- $T$ – average temperature of concrete environment, in Kelvin. Considered to be the reference temperature for the first and third case.

### 3.3 Effect of cement type

In order to study the effect of cement type, four concrete mixtures with four different types of cement were produced. Based on four types of cement some concrete test mixtures were produced in order to test the chloride diffusivity (NT Build 491 1999). Three types of cement included a high-performance portland cement (CEM I 52.5), a blended fly-ash cement (CEM II/A-V 42.5) and a blast-furnace slag cement with approximately 70% slag (CEM III/B 42.5). The test mixtures had a cement content of 420 kg/m$^3$ and a w/c ratio of 0.45. The forth cement was a high-performance portland cement (CEM I 52.5) mixed with 10% CSF. The test mixture had a cement content and silica fume content of 390 kg/m$^3$ and 39 kg/m$^3$, respectively.

The effect of cement type on the observed diffusion coefficient (m$^2$/s) is shown in Table 1, where the adopted values for $\alpha$ are also included.

<table>
<thead>
<tr>
<th>Cement type</th>
<th>$D_0$ ($10^{-12}$ m$^2$/s)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEM I</td>
<td>*N(10.5; 0.66)</td>
<td>N(0.37;0.07)</td>
</tr>
<tr>
<td>CEM III/B</td>
<td>N(5.3; 0.59)</td>
<td>N(0.60;0.15)</td>
</tr>
<tr>
<td>CEM II/A-V</td>
<td>N(10.1; 0.81)</td>
<td>N(0.51;0.07)</td>
</tr>
<tr>
<td>CEM I+CSF</td>
<td>N(4.7; 0.51)</td>
<td>N(0.39;0.07)</td>
</tr>
</tbody>
</table>

* N- normal distribution (average, standard deviation)

![Figure 1](image1.png)  

Figure 1. Effect of cement type on the probability of corrosion initiation.

As can be seen from Figure 1, the type of cement had a significant effect on the probability of corrosion initiation. Figure 1 clearly demonstrates the big difference in resistance against chloride penetration between the blast furnace slag cement and the pure portland cement, which is in accordance with previous experience (Ferreira 2004c). For the pure portland cement, the probability level of 10% for corrosion would be exceeded within a period of approximately 3 years,
while for the blast furnace slag cement, this level of risk of corrosion initiation would only be exceeded within a period of 32 years. For the fly ash cement and the combination of the portland cement with silica fume, the corresponding risk for corrosion initiation would be exceeded within a period of approximately 9 and 11 years, respectively.

3.4 Effect of temperature

Based on the data previously shown for the Portland cement concrete mix and the Portland cement with 10% silica fume concrete mix, (see example in Table 1), a new durability analysis was carried out in order to find out the effect of temperature on the design procedure (Figure 3). Three average temperatures were used: 21 ºC, 15 ºC and 10 ºC.

As can be seen from Figure 3, the temperature is also of importance for the probability of corrosion initiation. While for the CEM I concrete mix the difference in probability of corrosion initiation is not significant (3 years for 21ºC to 6 years for 10ºC), for the CEM I + CSF concrete mix the difference in probability of corrosion initiation is significant, more than double (12 years for 21ºC to 26 years for 10ºC).

The temperature influences greatly and can optimise the design procedure. For cooler climates, designing for the reference temperature can be on the safe side, however, design optimisation is not achieved. On the other hand, designing for hot climates with the reference temperature can underestimate the diffusion process and seriously undermine the service life intended. A more realistic approach is undertaken by correctly adjusting the temperature during the design procedure, therefore resulting in an optimised design.

3.5 Effect of concrete cover

Based on the data previously shown for the portland cement with 10% silica fume concrete mix, (see example in Table 1), a new durability analysis was carried out in order to find out the effect of increased concrete cover above the minimum requirement of 50 mm (Figure 3). The concrete depths of 50, 60, 70 and 90 mm were used.

As can be seen from Figure 2, the concrete cover is also of great importance for the probability of corrosion initiation. While a nominal concrete cover of 50 mm only would give a service period of approximately 10 years, an effective, nominal concrete cover of 90 mm would give a service period of more than 50 years.
4 CONCLUSIONS

The importance of critical information concerning the durability design of concrete structures prior to the decision making process is invaluable. This paper demonstrates how this information can easily be generated. Several sophisticated numerical methods exist which may be applied for probabilistic approach to durability design, however, a simple software with the model using a Monte Carlo simulation is applied, the results of which are demonstrated in the present paper. The influence of several parameters can be rapidly evaluated and design decisions can be made that improve and optimize durability performance.

As part of the durability design procedure, several parameters where analysed individually so as to understand their influence on concrete performance. As a consequence the following conclusions can be drawn:

- the type of cement influences significantly the probability of chloride-induced corrosion. Thus, for the pure portland cement, a probability level of 10% for corrosion would be exceeded within a period of approximately 4 years, while for the blast furnace slag cement, such a risk of corrosion initiation would only be exceeded within a period of 32 years. For the fly ash cement and the combination of the portland cement with silica fume, the corresponding risk of corrosion initiation would be exceeded within a period of approximately 9 and 11 years, respectively.

- it was further shown that a reduction of the chloride diffusivity of the concrete from 10.5x10^{-12} m^2/s to 5.3x10^{-12} m^2/s would reduce the probability of corrosion initiation by more than 85% over a service period of 50 years. Hence, the chloride diffusivity of a concrete is a very sensitive durability parameter for concrete structures in chloride containing environments.

- the analysis performed at three separate temperatures (10 °C, 15 °C and 21 °C) shows that the choice of temperature is crucial for the durability design as it affects the parameters measured. If the real average temperature is ignored, and the reference temperature used, for cold climates this results in a performance safety margin which can be expensive from an optimization point of view. On the other hand, for hot climates, this results in an under estimation of performance which compromises the service life of the structure.

- the concrete cover also has a large influence on the probability of chloride-induced corrosion. An increase in concrete cover from 50 mm to 70 mm almost triples the durability performance of the concrete (from 12 years to 35 years, respectively).

A more realistic design procedure would find the optimal design parameters by combining the chosen cement type with the appropriate concrete cover for the average environmental temperature. For new concrete structures, this provides an appropriate basis for establishing overall durability criteria for the structures. For existing concrete structures, where the chloride front has still not reached the embedded steel, this procedure can be used for estimating the probability of corrosion initiation after a certain period of time.
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