We set up a three-firm model of spatial competition to analyse how a merger affects the incentives for relocation, and conversely, how the possibility of relocation affects the profitability of the merger, particularly for the non-participating firm. We also consider the cases of partial collusion in either prices or locations. Under the assumption of mill pricing, we find that a merger will generally induce the merger participants to relocate, but the direction of relocation is ambiguous, and dependent on the degree of convexity in the consumers’ transportation cost function. Furthermore, we identify a set of parameter values for which the free-rider effect of a merger vanishes, implying that the possibility of relocation could solve the “merger paradox”.

Keywords: spatial competition, merger, relocation, partial collusion.


1 Introduction

In imperfectly competitive markets, an important part of the strategic interaction among firms occurs along a spatial dimension. More specifically, the profitability of a given firm is in many cases highly dependent on the firm’s location, relative to its competitors. Thus, to the extent that a firm is able to influence its own location, this is one of the most important decisions to be made. Location can be interpreted in a geographical space, where the locational decision involves the physical location of production plants or outlets, or in the product space. With the latter interpretation, the strategic decision involves the types and ranges of product varieties offered by the firm.
The purpose of this paper is to analyze the strategic importance of spatial competition for firms’ incentives to merge or collude. More specifically, we want to examine how a merger, or partial collusion along one or more dimensions, affects firms’ incentives to relocate from an initial position. The possibility of relocation will, in turn, affect the incentives for merger or collusion.

The importance of relocation in merger analysis is motivated by the casual observation that corporate mergers are often accompanied by some structural changes in the spatial dimension. For instance, we often observe that a merged firm spends considerable resources on rebranding – in order to create a new image in the eyes of consumers – and product repositioning.1,2 Another, more specific, example of spatial location is departure “slots” at airports. Airlines do not only decide the prices, but also the time scheduling of their different flights. Since the profitability of different departure times (i.e., locations) is influenced by the flight schedules of competing airlines, any changes in market concentration, e.g., through mergers, are expected to affect the optimal choices of “slots”.3

In some markets it is also reasonable to expect that mergers affect locations in the geographical space. Strategic relocation of the kind we

1 The possibility of product repositioning is also acknowledged in the European Commission’s recent draft notice on the appraisal of horizontal mergers:

“In some markets it may be relatively easy and not too costly for the active firms to reposition their products... The Commission will examine whether the possibility of repositioning or product line extension by the merging parties or competitors may influence the incentive of the merged entity to raise prices” (European Commission, 2002, paragraph 37).

RBB Economics (2003) provides examples of such post-merger repositioning in the cruise industry.

2 In a related, but quite different paper, Lommerud and Sørgard (1997) analyse the possibilities of introducing a new product, or withdrawing an existing brand, in a context of horizontal merger. In another study, Berry and Waldfogel (2001) analyse empirical evidence of the effect of mergers on variety and product repositioning in US local radio broadcasting markets.

3 Indeed, in an empirical analysis of flight departures in the Norwegian airline market, Salvanes et al. (2004) find that changes in the number of competing firms lead to systematic changes in the location of departures.
are considering in this paper is probably most relevant in retail markets, where consumers’ transportation costs play an important role.\textsuperscript{4} By introducing the possibility of (costly) relocation in a simple model of spatial competition, we show in the present paper that a merger will generally trigger incentives for relocation.

In the literature on purely anti-competitive horizontal mergers, a merger is normally assumed strategically to affect only the firms’ pricing or output decisions. The seminal contributions are Salant et al. (1983) for the case of Cournot competition, and Deneckere and Davidson (1985) for the case of Bertrand competition. A striking feature of these models is the so-called “merger paradox”: a merger between two or more firms is always more beneficial for the firms not participating in the merger. However, these studies do not allow for the possibility of relocations in spatial dimensions. The present paper contributes to the literature on horizontal mergers by showing that relocations of this kind affect the profitability of a merger, also for non-participating firms in the industry. Under some given circumstances, we show that the possibility of relocation could solve the merger paradox.

We set up a model where firms can undertake a costly investment in order to relocate from an initial position. This assumption should fit a broad interpretation of location. If we interpret location in the product space, it is perhaps most natural to think of the relocation cost as investment in product R&D. With this interpretation, our paper is also related to Lin and Saggi (2002), who analyze firms’ incentives to invest in product R&D as a way of increasing the degree of product differentiation in a symmetrically differentiated industry. By assuming a symmetric Chamberlinian demand system, product R&D has two different effects in their model. In addition to the differentiation effect, product R&D by one firm also increases the demand for all products in the industry by an equally large amount, which is a somewhat extreme assumption. In the present paper, we choose a model set-up which focuses exclusively on the differentiation effect.

\textsuperscript{4} Anecdotal examples include the banking and pharmacy sectors in Norway, where recent merger waves have been accompanied by a locational restructuring of branches and outlets. In a related empirical study, Götz and Gugler (2003) analyse how mergers affect product variety – measured as the number of stations per sqkm – in the Austrian retail gasoline market.
With a few exceptions, the effect of mergers on relocation, and vice versa, has received relatively little attention in the literature. Rothschild (2000) and Rothschild et al. (2000) analyze the case where three firms are initially located on a Hotelling line and can relocate in the anticipation of a merger between two of the firms. A problem with this set-up is that the structure of the industry is ex ante asymmetric, so that the choice of merger candidates is somewhat arbitrary. Norman and Pepall (2000a; 2000b) solve this problem by assuming that all firms are initially located at the market centre, which is a Nash equilibrium in the no-merger game. The main result in these studies is that the “merger paradox” could be solved by allowing for the possibility of relocation. However, in addition to the assumption of Cournot competition, all these studies share the common feature that firms are able to price discriminate between consumers at different locations.\textsuperscript{5} Reitzes and Levy (1995) obtain a similar result for the case of price discriminating firms that engage in Bertrand competition, although this is not due to the possibility of relocation. They show that a merger between two neighboring firms is always profitable for the merger participants, while outside firms are unaffected by the merger. Moreover, the assumption of price discriminating firms implies that there are no incentives for relocation.

The present paper adds to this literature by departing from the assumption of price discriminating firms and analyzing the interaction of merger and relocation incentives for the case of mill pricing, which is perhaps a more suitable assumption for spatial competition in product space. We consider a two-firm merger in a model where three price-setting firms are initially equidistantly located on a circle. This set-up resembles the analysis of Levy and Reitzes (1992), who show that a side-by-side merger is always profitable in a model of this kind. However, they do not consider the possibility of relocation, which is the main objective of our paper.

We find that a merger generally gives the merger participants incentives to relocate, but the direction of relocation is crucially dependent on the characteristics of consumers’ transportation costs. Adopting a disutility function with both a linear and a quadratic component, we find that the merger participants will relocate towards the outside firm if the weight attached to the linear part is sufficiently high. In this case, we also identify

\textsuperscript{5} This means that the firms compete in a continuum of segmented markets. Similar assumptions are also used by Matsushima (2001) in a Salop model.
the existence of a set of parameter values for which a merger will be more profitable for an insider than for the non-participant. Thus, we show that the possibility of relocation could possibly solve the “merger paradox” even in the absence of price discrimination. This is the main result of the paper. Regarding welfare considerations, we show that relocation could in some cases improve locational efficiency, thus reducing the negative impact of the merger.

Finally, we also extend the model to consider partial collusion in either location or price setting. In this case we find that partial collusion of either kind will always provide incentives for relocation, and the direction of relocation depends on whether the firms collude in prices or locations.

2 The Model

Consider a population of consumers uniformly distributed, with a constant density of 1, on a circle with circumference 1. Three single-product firms are located on the circle, with the location of firm \( i \) given by \( x_i \). Assuming unit demand, the utility of a consumer located at \( z \in [0, 1] \), and buying from firm \( i \), is given by

\[
U(z, v, x_i, p_i) = v - p_i - t(\psi_i),
\]

where

\[
\psi_i = \min\{|z - x_i|, 1 - |z - x_i|\},
\]

\( v \) is the reservation utility, assumed to be equal for all consumers, \( p_i \) is the price charged by firm \( i \) and \( t(\cdot) \) is a transportation cost function. We also assume that \( v \) is sufficiently high for the market always to be covered, i.e., all consumers are active.

Regarding transportation costs, the standard approach is to assume these costs to be either linear or quadratic in distance. We will adopt a functional form that encompasses both the linear and the quadratic variant as special cases. The costs of travelling a distance \( \Delta \) is given by\(^6\)

\[
t(\Delta) = a\Delta + b\Delta^2, \quad a, b \geq 0.
\]

\(^6\) A similar cost function is used by Lambertini (2001).
We introduce the notation \( \hat{z}_i \) for the location of the consumer who is indifferent between buying the good from the two neighboring firms \( i \) and \( i + 1 \). The location of this consumer is implicitly given by

\[
U(\hat{z}_i, v, x_i, p_i) = U(\hat{z}_i, v, x_{i+1}, p_{i+1}).
\]

Given the locations of the indifferent consumers, the market share of firm \( i \) is given by

\[
M_i = \hat{z}_i - \hat{z}_{i-1}.
\] (4)

We also assume that the firms can undertake an investment in order to change their location. We assume that relocation costs are convex in distance. The cost for firm \( i \) of relocating a distance \( d_i \) is given by \( kd_i^2 \), where \( k \) is a positive constant.

The marginal cost of production is assumed to be constant and equal for all firms and, without loss of generality, equal to zero. Firm \( i \)'s profits are then given by

\[
\pi_i = p_i M_i - kd_i^2.
\] (5)

The game is played in two stages:

Stage 1: The firms simultaneously choose the level of investment, \( d_i \).
Stage 2: The firms simultaneously set prices, \( p_i \).

2.1 Merger

As a benchmark for comparison, we will first consider the case in which all firms make independent decisions about prices and investments. In this case the model is completely symmetric. It is easily shown that each firm, operating independently, would prefer to be located as far away from its competitors as possible. Thus, given initial equidistant locations, the firms have no incentives to invest in relocation.

---

7 Because of the geometry of the model, any firm referred to as \( j \pm 3n \) is the same as firm \( j \), for every \( n \in N \).
Solving for the Nash equilibrium, with $d_i = 0$, yields the following solution for prices and profits:

$$p_i = \frac{3a + b}{9},$$

(6)

$$\pi_i = \frac{3a + b}{27}.$$  

(7)

The main focus of the analysis in this subsection is to investigate how a merger may influence the incentives for relocation. It is well known from the literature (see, e.g., de Frutos et al., 1999) that, with mill pricing, a transportation cost function of the type (3) does not provide pure strategy equilibrium existence in the price subgame for all possible locations. More precisely, a pure strategy price equilibrium fails to exist if the firms are located too closely, since each firm then has an incentive to engage in price-undercutting in order to capture the whole market. Thus, in order to obtain a perfect pure strategy equilibrium of the (re)location-price game, we follow the approach taken in related location models and restrict the strategy space of the relocation game to the set of locations for which a pure strategy equilibrium of the price game exists. Let this set be denoted by $Q$. Following Economides (1986), we define the direction in which $\frac{\partial \pi_i}{\partial d_i}$ is positive as the "relocation tendency" of firm $i$. An equilibrium of the location game must then be at the zero relocation locus, $\frac{\partial \pi_i}{\partial d_i} = 0$, and a perfect equilibrium of the location-price game is defined as the intersection between the zero relocation locus and the existence set $Q$. Formally, a location equilibrium, given by the relocation vector $d^*$, exists if

$$\frac{\partial \pi_i(d^*)}{\partial d_i} = 0; \quad \frac{\partial^2 \pi_i(d^*)}{\partial d_i^2} < 0; \quad x_i + d_i^* \in Q; \quad i = 1, 2, 3.$$  

This condition for equilibrium existence is met if the cost of relocation is sufficiently high. Below we will provide an exact restriction on the parameter $k$ which guarantees equilibrium existence.

---

Given equidistant initial locations, we can assume – without loss of generality – that the merger participants (firms 1 and 2) are located at 0 and $\frac{1}{3}$, with the outsider (firm 3) located at $\frac{2}{3}$. Obviously, any relocation for the merging firms must be symmetric across both outlets (products), thus $d_1 = -d_2$. We will henceforth focus on the relocational incentives for the outlet/product located at 0. To simplify notation, we let $d := d_1 (= -d_2)$ denote the distance of relocation for the firm located at 0, measured in the clockwise direction. Hence, $d < 0$ implies that the merger participants relocate in the direction of the outside firm. Obviously, the outsider has no incentives to relocate.

Since the merger participants coordinate their price setting, the symmetric feature of the model enables us to solve for the equilibrium by identifying the location of one indifferent consumer only. Consider the consumer who is indifferent between buying from firm 1 and firm 3. Her location, $\hat{z}_3$, is found by solving

$$p_1 + t(1 + d - \hat{z}_3) = p_3 + t\left(\hat{z}_3 - \frac{2}{3}\right).$$

Using (3), this yields

$$\hat{z}_3 = \frac{1}{6}(5 + 3d) + \frac{3}{2}\left(\frac{p_1 - p_3}{3a + b + 3bd}\right).$$

Due to symmetry and coordinated price setting, the consumer who is indifferent between buying from either of the merger participants is located at $\hat{z}_1 = \frac{1}{6}$. Furthermore, symmetry also ensures that the market shares of the merged firm and the outsider, respectively, are

$$M_1 + M_2 = 2\left(1 - \hat{z}_3 + \frac{1}{6}\right)$$

and

$$M_3 = 2\left(\hat{z}_3 - \frac{2}{3}\right).$$

9 This assumption of symmetry regarding the relocation distances is made to facilitate the analysis and is not imposed as an exogenous condition. The symmetric outcome can be obtained by explicitly solving the game for $d_i$, $i = 1, 2, 3$.

10 Again, besides being an argument derived from the symmetry of the model, this result can also be obtained as an equilibrium outcome of the relocation game.
Equilibrium prices, as functions of the optimal degree of relocation, is found by inserting (8)–(10) into the profit functions, (5), and maximizing with respect to prices. This yields

\[ p_1 = p_2 = \frac{1}{27} (5 - 3d)(3a + b + 3bd), \]  

(11)

\[ p_3 = \frac{1}{27} (4 + 3d)(3a + b + 3bd), \]  

(12)

with corresponding profits given by

\[ \pi_1 = \pi_2 = \frac{1}{486} (5 - 3d)^2 (3a + b + 3bd) - kd^2, \]  

(13)

\[ \pi_3 = \frac{1}{243} (4 + 3d)^2 (3a + b + 3bd). \]  

(14)

Let us first consider the effects of a merger between two firms, without relocation. With \( d = 0 \) the following result can be stated:\footnote{It can easily be shown that, with \( d = 0 \), a price equilibrium exists for all \( a > 0, b > 0 \).}

**Proposition 1:** With three firms initially located equidistantly from each other, then (i) a merger between two firms is always jointly profitable, (ii) profits are higher for the non-participating firm.

**Proof:** (i) Comparing (13) and (7) we find that

\[ \pi_1|_{d=0} - \pi_i = \frac{7}{486} (3a + b) > 0. \]

(ii) A comparison of (13) and (14) reveals that

\[ \pi_1|_{d=0} - \pi_3|_{d=0} = -\frac{7}{486} (3a + b) < 0. \]

This is a restatement of Levy and Reitzes (1992), and corresponds to the well known results in Deneckere and Davidson (1985). The gain from price
setting coordination, resulting in higher prices, more than outweighs, in terms of profits, the loss of market shares for the merger participants. However, the outside firm enjoys both higher prices and a higher market share, implying that free-rider incentives are present: rather than participating in a merger, each firm would prefer that the other firms merge.

Let us now see how a merger between two firms affects the incentives to relocate. Since the merging firms would only spend resources to relocate their outlets/products if it increases profits, relocation obviously increases the profitability of a merger. The question is, however, whether the merging firms would relocate away from, or in the direction of, the outside firm. The optimal distance of relocation is given by

\[ d^* = \arg \max \{ \pi_1 + \pi_2 \}. \]

Using (13), we find the explicit value of the interior solution to be

\[ d^* = \frac{18b + 108k - 6a - 6\sqrt{4b(a + b + 27k) + (a - 18k)^2}}{18b}. \]

(15)

In order to secure an interior solution\(^{12}\) we make the assumption that relocation is sufficiently costly. More specifically, we impose the assumption

\[ k \geq \bar{k} = \max \left\{ \left( -\frac{1}{2}a + \frac{1}{8}b \right), \left( \frac{3}{4}a - \frac{21}{40}b \right) \right\}. \]

It can also be shown that \( k \geq \bar{k} \) is sufficient to ensure pure strategy equilibrium existence in the location-price game.\(^{13}\)

**Proposition 2:** The merger participants will relocate towards (away from) the outside firm if \( a > (\leq) \frac{1}{2}b \).

\(^{12}\) That means, to ensure that 
\[ |d^*| \leq \min \left\{ \frac{1}{6}, 1 - z_3 \right\}. \]

\(^{13}\) With two firms only, de Frutos et al. (1999) show that a pure strategy equilibrium in the price game is always satisfied if the firms are located at least a distance of 1/4 from each other. This is always the case if \( k \geq \bar{k} \). Moreover, with more than two firms equilibrium prices are lower, due to fiercer competition, so the incentives for price-undercutting is even lower.
Proof: Follows from (15).

The first observation to be made is that \( \frac{d}{C_3} \) is generally nonzero: a merger between two firms creates incentives for relocation. Furthermore, the direction of relocation is generally ambiguous, and depends on the specifics of the transportation cost function. It is easy to verify, though, that \( \frac{\partial d^*}{\partial a} < 0, \frac{\partial d^*}{\partial b} > 0 \) and \( \frac{\partial |d^*|}{\partial k} < 0 \).

The merged firm faces a trade-off in deciding on the direction of relocation: by moving away from the outside firm price competition is reduced, at the expense of a lower market share. Alternatively, the merged firm can gain a larger share of the market by relocating towards its competitor. The nature of this trade-off is determined by the characteristics of the transportation cost function. If there is a relatively high degree of convexity in transportation costs, the degree of price competition is highly dependent on the distance between the firms. The further apart the firms are located, the more costly it is to "steal" market shares from the competitors, implying that the degree of competition is relatively lower. Consequently, relocating further away from their competitor is an effective way for the merger participants to reduce the degree of price competition.

On the other hand, if there is a relatively low degree of convexity in transportation costs, the degree of price competition is not sufficiently reduced to compensate for the reduction of market share by moving further away from the competing firm. In this case, the market share effect dominates the competition effect, and the merged firm can increase profits by moving closer to the outside firm, thereby controlling a larger share of the total market.

2.1.1 A Special Case: Linear Transportation Costs

The transport cost function specified in (3) encompasses the two most commonly used specifications in the literature on spatial competition: linear \((b = 0)\) and quadratic \((a = 0)\) transportation costs. In our model, an interesting result appears for the special case of linear transportation costs.\(^{14}\) From Proposition 2, it follows that linear transportation costs implies relocation towards the outside firm. Comparing the cases with and without relocation, we find that relocation always leads to higher prices for the

\(^{14}\) The relevant equilibrium expressions for this case is easily found by inserting \( \lim_{b \to 0} \frac{d}{C_3} \) into (8)–(14). Note also that linear transportation costs implies \( k = \frac{3}{4} a \).
merged firm and lower prices for the outsider. From the viewpoints of the merging firms, the cost of charging higher prices is a loss of market share to the outsider. However, the merger participants can partly compensate for this effect by moving closer to the outside firm, which enables the merged firms to charge even higher prices. The non-participant, on the other hand, now faces a higher degree of competition, and is forced to reduce its price in order to soften the loss in market share. Thus, the possibility of relocation for the merged firm implies a reduction of both price and market share for the outsider, and this could potentially cause the well-known free-rider effect to vanish.

**Proposition 3:** When transportation costs are linear in distance, a merger participant earns higher profits than a non-participant if the cost of relocation is sufficiently small.

**Proof:** Inserting \( \lim_{b \to 0} d^o \) from (15) into (13)–(14), we find that

\[
\pi_3 - \pi_1 < 0
\]

if

\[
\frac{(119 - 5\sqrt{385})a}{252} < k < \frac{(119 - 5\sqrt{385})a}{252}. 
\]

Imposing the restriction \( k \geq \bar{k} \), we have that

\[
\pi_3 - \pi_1 < 0
\]

if

\[
\frac{3}{4} a \leq k < \frac{(119 - 5\sqrt{385})a}{252} (\approx 0.86a). 
\]

The Proposition identifies a (small) range of \( k \) for which each firm would like to participate in the merger, rather than waiting for the other firms to merge.\(^{15} \)

\(^{15}\) It is not possible to give an exact condition for when the free-rider effect vanishes in the general case. However, if we consider the special case of costless relocation \( (k = 0) \), we find that a merging firm obtains higher profit than the outsider if \( b < 1.56a \). Applying the condition \( k \geq k \), which in this particular case requires \( b \geq 1.43a \), the relevant range is given by \( b \in [1.43a, 1.56a] \).
### 2.1.2 Numerical Examples

A better understanding of the workings of the model can be achieved by studying Table 1, where we present the equilibrium outcomes, in terms of relocation and profits, for specific numerical examples. For a given value of \( a \) we show how the equilibrium outcome varies with the parameter \( b \), which illustrates the effect of increased convexity in transportation costs. The importance of relocation costs is captured by performing this numerical simulation for two different levels of \( k \) – “low” and “high”.

Several general patterns emerge from Table 1. Higher relocation costs obviously reduce the equilibrium distance of relocation, and profits for the merged firm are also reduced. We also see that higher relocation costs lead to increased profits for the outside firm if the merged firm relocate towards the outsider. Otherwise, outsider profits are lower. This is also very intuitive, since the outside firms earn higher profits in equilibrium the further away its competitors are located. A smaller distance of relocation will thus only be an advantage for the outside firm if \( d^* < 0 \).

For a given level of \( a \), a higher value of \( b \) has two different implications: it increases the total costs for consumers of travelling a certain distance, and it also increases the convexity of the transportation cost function. The former implication is reflected in the fact that equilibrium profits – for all firms – increase in \( b \). Higher transportation costs reduce the degree of competition in the market and allows all firms to charge higher prices. More interestingly, though, we clearly see that the degree of convexity in the transportation cost function plays a crucial role in

<table>
<thead>
<tr>
<th>( k = 3/4 )</th>
<th>( k = 2 )</th>
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<tbody>
<tr>
<td>( b )</td>
<td>( d^* )</td>
</tr>
<tr>
<td>0</td>
<td>-0.1333</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.1235</td>
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<tr>
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<tr>
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<td>-0.0535</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.0244</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
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<td>0.0391</td>
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</table>
determining the direction of relocation and, correspondingly, the free-rider effect of a merger. An outsider earns lower profits than a merger participant if the degree of convexity in the transportation cost function and the cost of relocation are both sufficiently low. In the numerical examples of Table 1, we see that the “merger paradox” can only be resolved for the case of “low” relocation costs. If \( k = 3/4 \) and \( a = 1 \), a merger participant earns higher profits than the outside firm if \( b < 0.265 \).

2.2 Welfare

We apply the standard definition of social welfare, \( W \), as the sum of consumers’ and producers’ surplus, which in our case reduces to:

\[
W = v - \sum_{i=1}^{3} \int_{z_{i-1}}^{z_{i}} \frac{t(\psi_i)}{C_0} dz - 2kd^2.
\]

With the assumptions of unit demand and a non-binding reservation price for consumers, social welfare does not depend on prices directly, but is given by the sum of consumers’ gross valuation, \( v \), net of total transportation and relocation costs. Thus, a welfare analysis in this kind of model is basically an analysis along one dimension only, namely locational efficiency.

Using the symmetry properties of the model, the expressions for social welfare in the merger (\( W_m \)) and no-merger (\( W_{nm} \)) cases, respectively, are found to be

\[
W_m = v - \Gamma - 2kd^2,
\]

where

\[
\Gamma = \frac{1674ad^2 - 270bd^3 - 72ad + 486bd^2 + 87a - 18bd + 11b}{972},
\]

and

\[
W_{nm} = v - \frac{9a + b}{108}.
\]
Assume first that relocation is not possible. Comparing (16) and (17), we find that

\[ W_{m|d=0} - W_{nm} = -\frac{3a + b}{486} < 0. \]

Thus, a merger is socially harmful even if it does not lead to any relocation. Post-merger there is a price difference between the merger participants and the non-participant which implies that a larger share of consumers is buying from the outside firm. This causes an increase in the total outlay on transportation costs.

A closer inspection of (16) and (17) also reveals that \( W_{m}/C_0 W_{nm} < 0 \) for the equilibrium value of \( d \), implying that a merger is always socially harmful. However, once two firms have merged welfare is not maximized at \( d = 0 \). Thus, from society’s point of view there are incentives for relocation, as long as this is in the right direction. The possibility of relocation means that the negative impact of a merger, in terms of social welfare, could be reduced if the merger participants relocates away from the outsider. The exact condition is given by the following proposition.

Proposition 4: Given that a merger has taken place, relocation leads to a welfare improvement if \( d^* \in (0, \overline{d}) \).

Proof: From (16) we find that

\[ W_m(d) - W_m|d=0 = \frac{d}{54} (4a + b - 93da + 15d^2b - 27db - 108dk). \]

It follows that

\[ W_m(d) - W_m|d=0 > 0 \text{ iff } 0 < d < \overline{d}, \]

where

\[ \overline{d} = \frac{108k + 27b + 93a - \sqrt{(27b + 108k + 93a)^2 - 60b(4a + b)}}{30b}. \]

This result, which is not immediately obvious, can be explained as follows: consider the location of the consumer who is indifferent between buying from firm 1 and 3, given by \( \hat{z}_3 \). For any set of prices, the optimal location of this indifferent consumer – in terms of locational efficiency –
is midway between firms 1 and 3. With a merger, but without relocation,  \( \hat{z}_3 \) gets too close to firm 1, because of the merger-induced price increase. If firm 1 relocates (marginally) away from firm 3, then  \( \hat{z}_3 \) moves in the same direction, but by a smaller distance than firm 1. This implies that  \( \hat{z}_3 \) gets relatively closer to firm 3, and thus closer to the “new” midpoint, which is a welfare improvement.

Combining Propositions 2 and 4, it is apparent that  \( a < \frac{1}{2} b \) is a necessary condition for welfare improving relocations. It is difficult, though, to provide a further general characterization of the condition given in Proposition 2, in terms of the parameters of the model. However, we can use the expression for  \( \tilde{d} \) to analyze three special cases. If relocation is costless (\( k = 0 \)), we find that  \( d^* \in (0, \tilde{d}) \) if  \( a \in (0.44b, 0.50b) \). If transport costs are linear in distance (\( b = 0 \)), relocation is always welfare detrimental as the condition  \( a < \frac{1}{2} b \) cannot be satisfied. For quadratic transportation costs (\( a = 0 \)), we know that the firms relocate in the “right” direction. However, it turns out that the distance of relocation is always excessive, i.e.,  \( d^* > \tilde{d} \), and thus socially undesirable, for every value of  \( b \) and  \( k \) within the valid ranges.

### 3 Partial Collusion

So far we have assumed that the merger participants coordinate both the price setting and the relocation decisions. These are obvious assumptions if we regard the merged firm as a new fully integrated entity. However, the analysis of mergers when the different outlets/products are maintained post-merger is similar to an analysis of collusion, as long as other effects, like, e.g., cost synergies or defection, are not considered. Thus, the model presented in the previous section might also be interpreted as a cartel where the participants coordinate their decisions with respect to both strategic variables. Therefore, it is also interesting to ask the question of how the analysis would change if firms were able to coordinate decisions with respect to only one of the variables. There are several reasons why partial collusion might be relevant. For example, antitrust legislation may make price coordination infeasible, or at least difficult. It is reasonable to assume, though, that coordination of relocation decisions is much less likely to be prohibited by antitrust authorities. Other examples where partial collusion might be relevant include franchises, or regulation, in which the franchiser/regulator decides locations (prices) of the firms, but
lets these compete in prices (locations). As another example of partial collusion in prices, we can think of a situation in which the firms independently make relocation investments, anticipating that two of the firms might merge or collude in the future.\(^{16}\)

### 3.1 Collusion in Prices but Not Location

To carry out this analysis we should firstly notice that we cannot a priori apply an argument of symmetry for the relocation distances of the colluding firms, since they must be treated as independent variables. Thus, let \(d_i\) denote the distance of relocation, measured in the clockwise direction, with respect to its original position for firm \(i\). Consequently, the location of the indifferent consumers between firm \(i\) and firm \(i + 1\), \(\hat{z}_i\), is found by solving

\[
p_i + t\left(\hat{z}_i - \left(\frac{i - 1}{3}\right) + d_i\right) = p_{i+1} + t\left(\frac{i}{3} + d_{i+1} - \hat{z}_i\right),
\]

while the profits are given by

\[
\pi_i = p_i M_i - kd_i^2 = p_i(\hat{z}_i - \hat{z}_{i-1}) - kd_i^2.
\]

At stage two of the game, firms 1 and 2 are assumed to coordinate their price setting. Profit maximization leads to a system of equilibrium prices \(p_i(d_1, d_2, d_3), i = 1, 2, 3\). By substituting \(p_i(d_1, d_2, d_3)\) back into (18), we can express profits as functions of the relocation distances only.

In the first stage of the game the colluding firms act independently, so that each firm maximizes individual profits by choosing \(d_i\). Using the fact that, by symmetry (a posteriori), \(d_2 = -d_1\) and \(d_3 = 0\), profit maximization yields the following solution for the optimal relocation of firm 1:

\[
d_p := d_1 = \frac{1944ak + 171ab + 648bk + 87b^2 - 9\sqrt{A}}{18b(9a + 5b)},
\]

where \(A > 0\) is a function of the parameters of the model.\(^{17}\)

\(\text{16} \) In a somewhat different setting, the case of partial collusion in prices is also considered in Friedman and Thisse (1993), who analyze a location-then-price game when the firms anticipate collusion in prices.

\(\text{17} \) \(A = 46656a^2k^2 + 8208a^2bk + 31104abk^2 + 121a^2b^2 + 6912ab^2k + 5184b^2k^2 + 154ab^3 + 1392b^3k + 49b^4.\)
Equilibrium prices and profits are found by substituting \( dp \) for \( d \) in (11)–(14). It is straightforward to show that \( dp \) is always nonnegative, \(^\text{18}\) which establishes the following proposition:

**Proposition 5:** Under partial collusion in prices, the colluding firms will relocate, if at all, away from the outside firm.

The intuition is found by comparing with the case of full collusion, or merger. Consider the decision of firm 1 to possibly relocate as a response to price collusion with firm 2. When the firms do not coordinate their location decisions, there is an extra cost associated with moving away from this firm (i.e., moving towards firm 3). The gain in market share vis-à-vis firm 3 is accompanied by a loss of market share to firm 2. Consequently, the competition effect always dominates, and the firms engaged in price collusion will move closer together.

It is worth noting that the special case of linear transportation costs \((b = 0)\) implies no relocation. From (19) we find that

\[
\lim_{b \to 0} dp = 0.
\]

The intuition is relatively straightforward. In this case price competition is not reduced by moving further away from firm 3, and there is no net gain of market share by moving in either direction.

### 3.2 Collusion in Location but Not Prices

When the firms coordinate their location decisions but compete in prices, the analysis is similar. The two main differences are that at the second stage firms maximize individual profits, whereas at the first stage the colluding firms maximize joint profits with respect to the relocation decisions. Following the same procedure as in the previous section, and again applying arguments of symmetry, it is directly shown that prices are given by

\(^\text{18}\) It can be shown that

\[
k > \frac{(5a + b)b}{16(3a + b)}
\]

must be satisfied to ensure an interior solution and equilibrium existence.
\[ p_1 = p_2 = \frac{(5 - 3d_l)(3a + 3bd_l + b)(3a - 6bd_l + b)}{9(15a - 12bd_l + 5b)}, \quad (20) \]

\[ p_3 = \frac{(3a + 3bd_l + b)(15a - 15bd_l + 5b + 18ad_l - 9bd_l^2)}{9(15a - 12bd_l + 5b)}, \quad (21) \]

with corresponding profits

\[ \pi_1 = \pi_2 = \frac{(5-3d_l)^2(6a-3bd_l+2b)(3a-6bd_l+b)(3a+3bd_l+b)}{54(15a - 12bd_l + 5b)^2} - kd_l^2, \quad (22) \]

\[ \pi_3 = \frac{(3a+3bd_l+b)(15a - 15bd_l + 5b + 18ad_l - 9bd_l^2)^2}{27(15a - 12bd_l + 5b)^2}, \quad (23) \]

where \( d_l := d_1(= -d_2) \) is the interior solution of the fifth-degree polynomial defined by \( \partial(\pi_1 + \pi_2)/\partial d_1 = 0 \).

Unfortunately, and due to the fifth-degree nature of the problem, it is impossible to find an explicit expression for the interior solution. It can be shown, though, that \( d_l < 0 \) for every permissible value of the parameters. Again, the intuition is clearly tractable. If the firms do not coordinate their location and price decisions at all, we know that neither firm has any incentive to relocate, since the increased competition with the closer neighboring firm more than offsets, in terms of profits, the decrease in competition with the other neighbour. However, if two of the firms are able to coordinate their location decisions, they can make sure, by both moving in the direction of the third firm, that the decrease in the degree of competition between them is sufficiently reduced to more than compensate for the increase in the degree of price competition with the third firm.\(^{19}\) Moreover, as there is not any agreement between the colluding firms to increase their price, the outsider faces stronger competition and a lower market share, which eliminates any free-riding effect and even lowers its profits compared to the situation with no collusion. The next proposition summarises these results:

\(^{19}\) The unique case which permits tractable analysis is the one with linear transportation costs \((b = 0)\), in which \( d_l = -\frac{5a}{3(25a - 9)}, \) where \( k > \frac{9}{25}a \) ensures an interior solution. The quadratic case \((a = 0)\) with no relocation costs \((k = 0)\) implies \( d_l = -0.027 \).
Proposition 6: With partial collusion in location,

(i) the colluding firms relocate towards the outsider and make higher profits than this firm,
(ii) the outsider makes less profits, compared with the case without collusion.

3.3 Welfare and Profit Comparisons

For the case of partial collusion in locations, it is easily shown that social welfare is maximized at $d = 0$. This is an obvious result. Since prices are set non-collusively, total transportation costs are always minimized with symmetric locations. Furthermore, it is also possible to show that partial collusion in locations is always preferred, from a welfare point of view, to full collusion (or merger). For the special cases of linear and quadratic transportation costs, it is also possible to show that partial collusion in prices is preferred to total collusion. Again, this is not too surprising.

Comparing welfare for the two different kinds of partial collusion, it can also be shown, for the case of linear transportation costs, that partial collusion in locations is socially preferred to partial collusion in prices if the cost of relocation is sufficiently large.\(^{20}\)

The private incentives for the different kind of collusion do not necessarily correspond with the social incentives. For the colluding firms, full collusion is preferred to price collusion, which is preferred to collusion in location. For the outsider, full collusion and price collusion are both preferred to collusion in location. However, collusion in prices might be preferred to full collusion, at least for linear transportation costs.

4 Concluding Remarks

The purpose of this paper has been to analyse how horizontal mergers might create incentives for relocation within a framework of spatial competition, and conversely, how the possibility of relocation might affect both the profitability of a merger and profits of non-participating

\(^{20}\) For the case of $b = 0$ we find that social welfare is higher with partial collusion in locations, compared with partial collusion in prices, if and only if $k > \frac{1}{25}(19 + 6\sqrt{31})a$. 
firms, as well as locational efficiency (social welfare). In order to facilitate analytical tractability, we have used a rather simple set-up, where we consider a two-firm merger in an industry with three price-setting firms initially located in symmetric fashion on a circle. Under the assumption that firms are not able to price discriminate between consumers at different locations, we have considered both a full merger and partial collusion in either price setting or relocation decisions.

We have found that a merger will generally lead the merger participants to relocate, but the direction of relocation is ambiguous, and depends on the characteristics of the transportation cost (disutility) function. Regarding the effects of a merger on the profits of the non-participating firm, the possibility of relocation implies that the well-known free-rider effect could be either mitigated or reinforced, depending on the direction of relocation. If a merger leads to a relocation in the direction of the outside firm, we have shown the existence of a set of parameter values – characterised by low relocation costs and a low degree of convexity in the transportation cost function – for which the free-rider effect vanishes. This paves the way for the main message of the paper, namely that the possibility of relocation could solve the “merger paradox” even in the absence of price discrimination.

Regarding social welfare, the possibility of relocation could dampen the negative effect of a merger, although a merger in this setting (without any cost synergies) is always welfare detrimental. Finally, if firms engage in partial collusion, we have found that (except for the special case of linear transportation costs) incentives for relocation are always present, but the direction of relocation depends on whether the firms collude in prices or location decisions. In the former case, the colluding firms will relocate away from the non-colluding firm.

Due to the complexities involved in performing a joint analysis of the questions of merger and location choices in a spatial framework, we have been forced to consider a fairly particular set of assumptions. Important questions that are not touched upon in our analysis include the possibility of entry to the industry. We have also made the analysis tractable by setting up a three-firm analysis. In this setting, the non-participating firm does not have an incentive to relocate as a response to a merger. Generally, though, with more than three firms, we would expect the relocation incentives of non-participating firms also to be affected by a merger. In particular, we would expect non-participating firms to have incentives to
relocate in a direction that increases the total distance to the neighboring firms. However, a more general analysis would be analytically intractable within the present context. Thus, the present analysis should perhaps be seen as a first stepping stone towards a more comprehensive understanding of the effects of merger and collusion in a spatial framework.

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References


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