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Quality competition in mixed oligopoly*

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Abstract

We study quality competition in a mixed oligopoly (with applications to health care and education) where a welfare-maximising public provider competes with two profit-maximising private providers that differ with respect to the regulatory regime they face, with only one of the private providers being included in the public funding scheme. We find that changes in the funding scheme or in the degree of competition have differential effects on quality provision across the different types of providers and thus generally ambiguous effects on average quality provision. In terms of social welfare, we find that the two policy instruments in the funding scheme – price and copayment – are policy complements (substitutes) for sufficiently low (high) levels of the copayment rate. We also identify a welfare trade-off between the public funding scheme's generosity (price level) and the extent (number of private providers included).

Keywords: Quality; Competition; Mixed oligopoly.

JEL Classification: H44; I11; L13; L33

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1 Introduction

There are many services, among them health and education, which are provided by a mix of public and private providers, but where the relative share of these types of providers varies considerably across different countries. In such mixed markets, where public and private providers coexist, competition typically takes place among providers with different objectives and which are subject to different regulatory schemes. This raises several policy issues. For example, should private providers be included in public funding schemes? And if so, should such providers be allowed to distribute profits? In education markets, for example, many countries do not give public funding to for-profit private schools, while several others, the US included, permit publicly funded charter schools to be operated by for-profit providers (Boeskens, 2016). Furthermore, in health and education markets quality is a key concern, and designing policies to ensure a satisfactory provision of quality requires an understanding of how public and private providers strategically interact, and how they respond to different funding schemes.

In this paper we analyse the effects of mixed oligopolistic competition on quality provision in regulated markets where three different types of providers interact: (1) public providers, (2) publicly funded private providers, and (3) private providers without public funding. Providers of type 1 and 2 both face a regulated price (paid by the public funder) and a copayment rate (paid by the providers' consumers), but are assumed to differ in their objectives, with private providers being more profit-oriented than their public counterparts. On the other hand, providers of type 2 and 3 are similar in terms of objectives, but differ in terms of the regulatory environment in which they operate. Whereas publicly funded providers receive (part of) their revenues from the public funder, private providers without public funding must raise all their revenues from the market by charging a price for their services. Thus, while providers of type 1 and 2 only choose the quality of the service they provide, type 3 providers choose both quality and price.

Within this framework, we study three different (but related) set of issues. First, we study the nature of strategic interaction among these three different types of providers and how their quality provision depends on the characteristics of the funding scheme, which in turn determines the ranking of equilibrium quality provision across the three types of providers. Second, we analyse the effect of (intensified) competition on the quality provision of each type of provider and on the average quality provision in the market. Finally, we include a welfare analysis where we characterise the normative relationship between the regulated price and the copayment rate as policy instruments, and where we also study the optimal degree of public funding coverage in the market.

Although our model is not tailor-made to fit one particular industry, our analysis applies in particular to regulated markets such as health care and education. In the health care markets of many European countries, patients can choose between public and private providers within the national health system, where prices and copayments are regulated, or alternatively choose a private provider outside the national health system and pay the expenses either out-of-pocket or via private health insurance.[1] A similar mix of provider options is present in education markets, where tuition fees in publicly funded schools tend to be either absent or regulated, while independent private schools rely on the fees charged to their students. In such markets, publicly funded private schools have become a prominent feature across OECD countries (Boeskens, 2016). Average OECD figures for 2012 show that 14.2% of 15-year-old students attended government-dependent private schools, 81.7% attended public schools, while 4.1% attended independent private schools (OECD, 2013).

Both in health care and education markets, the extent of public funding coverage for private providers is a contentious issue in many countries. In education markets, for example, proponents of extending funding to private providers argue that this stimulates inter-school competition and offers incentives for innovation and quality improvements. On the contrary, opponents argue that funding private education might lead to public sector resource depletion and ultimately result in a reduction in educational quality (Boeskens, 2016).

In order to analyse competition among three different types of providers, as explained above, we use a spatial competition framework with three providers – one of each type – equidistantly located on a Salop circle. We consider a two-stage game where all three providers choose quality in the first stage, followed by the price choice of the unregulated private provider in the second stage. Within this game-theoretic framework we derive three sets of results: two sets of positive results and one set of normative results. First, regarding the relationship between the characteristics of the funding scheme and the equilibrium quality provision in the market, we find that a higher regulated price or a higher copayment rate will reduce the quality provision of the public provider while increasing the quality provision of at least one of the private providers. The resulting effect on average quality is generally ambiguous. Furthermore, the highest quality in the market is provided by one of the publicly funded providers, unless the copayment rate is very high. Second, regarding the effect of competition on quality provision, we find that stronger competition stimulates the quality provision

[1] See for example Siciliani et al. (2017) for an overview of the scope for competition between health care providers in five different European countries

of the publicly funded private provider but has a generally ambiguous effect on the quality provision of the other two providers. However, numerical simulations suggest that the relationship between competition intensity and average quality provision is positive. Finally, regarding the welfare effects of different funding policies, we find that the regulated price and the copayment rate are policy complements (substitutes) for sufficiently low (high) levels of the copayment rate. Furthermore, when extending the analysis to consider the optimal degree of public funding coverage, we find that this depends on the level of the regulated price, where welfare is maximised when both, one and no private providers are funded for low, intermediate and high values, respectively, of the regulated price. Thus, there exists a welfare trade-off between funding generosity and funding coverage.

The rest of the paper is organised as follows. In the next section we present a relatively brief summary and discussion of related literature, before presenting the model in detail in Section 3. The main analysis, both positive and normative, is conducted in Section 4 for a given market structure in terms of public funding. In Section 5 we extend the analysis to consider the welfare effects of either removing public funding from the private provider or extending public funding to both private providers. Some concluding remarks are offered in Section 6.

2 Related literature

Our paper is related to the literature on mixed oligopoly in general and on quality competition between public and private providers in health care and education markets in particular. In the theory of mixed oligopolies, a sizeable literature has grown out of the seminal contributions by De Fraja and Delbono (1989) and Cremer et al. (1989). Later contributions include Cremer et al. (1991), Matsumura (1998), Bennett and La Manna (2012) and Haraguchi and Matsumura (2016). A main message from this literature is that the presence of public firms might yield welfare improving effects in oligopolistic industries, and a key issue has been to determine the optimal degree of public ownership (e.g., Matsumura, 1998). A common assumption in this literature is that firms compete either in prices or quantities, and quality is generally not an issue.

There is however a smaller and more specialised literature dealing with quality competition in mixed oligopolies. Grilo (1994) produced what is probably the earliest contribution in this literature, studying quality and price competition in a vertically differentiated mixed duopoly. A later contribution building on this work is Lutz and Pezzino (2014), who find that a mixed duopoly is generally welfare superior to a private duopoly. Laine and Ma (2017) also study quality and price competition in a vertically differentiation framework and show the existence of multiple equilibria that differ with respect to the identity of the high-quality firm (public or private). The latter result has some parallels to the present paper, where we show that the public provider may or may not produce the highest quality in the market, depending on the details of the funding scheme. However, one of several important differences between our paper and all of the above mentioned papers on quality competition in mixed oligopolies is that the latter papers apply a vertical differentiation framework, whereas our study is conducted in a setting of horizontal differentiation. Our paper is therefore more closely related to the type of analysis conducted by Ishibashi and Kaneko (2008), who study quality and price competition between a welfare-maximising state-owned firm and a profit-maximising private firm in a Hotelling model. They find that, absent any cost efficiency differences, the public firm chooses a lower quality than the private firm in equilibrium, which is similar to the quality ranking result in our paper for a sufficiently high regulated price. Furthermore, they show that social welfare is maximised if the public firm's objective is a weighted average of welfare and profits, thus indicating that partial privatisation of the state-owned firm would be welfare improving.

Common for all the above mentioned papers is that competition takes place in an unregulated setting, which is another key difference from the present paper, in which two of the three competing providers face regulated prices. In this respect, our paper is more closely related to papers that study quality competition in *regulated* mixed oligopolies, often applied to health care markets. An early study is Barros and Martinez-Giralt (2002) who analyse quality and price competition between a public and a private health care provider under different reimbursement rules. Sanjo (2009) and Herr (2011) also study quality competition between a public and a private health care provider, but under the assumption that prices for both providers are regulated. These studies are all conducted within a horizontal differentiation (Hotelling) framework.[2] More recent studies of mixed duopoly quality competition with fixed prices have addressed issues such as soft budgets (Levaggi and Montefiori, 2013), partial privatization policies (Chang et al., 2018) and location choices (Hehenkamp and Kaarbøe, 2020). A broader review of the merits of mixed markets in health care, presented in a unified framework, is given by Levaggi and Levaggi (2020).

A similar type of study, using a Hotelling-type framework, but applied to the education sector, is Brunello and Rocco (2008), who analyse a mixed duopoly game between a public school choosing

[2] A similar study using instead a vertical differentiation framework is Stenbacka and Tombak (2018).

quality ('educational standard') and a private school choosing quality and price (tuition fee). As in the present paper, they find that the public agent can provide either the highest or the lowest quality in equilibrium. Overall, our paper can be seen as an extension of the above described literature on quality competition in regulated mixed oligopolies, where we include a richer set of provider types that differ not only in their objectives but also in terms of regulatory constraints.[3]

Finally, our paper is related to the literature on the relationship between competition and quality provision, which has become an increasingly prominent strand of the health economics literature in particular. The empirical evidence of this relationship in hospital markets with regulated prices is somewhat mixed, with both positive (e.g., Cooper et al., 2011, Gaynor et al., 2013) and negative (e.g., Skellern, 2017, Moscelli, et al., forthcoming) effects being reported. This should probably not come as a surprise, though, given the ambiguous nature of the theoretical predictions (Brekke et al., 2011).

3 The model

Consider a market for a good (e.g., health care or education) that is supplied by three different providers that are equidistantly located on a circle with circumference equal to 1. Each of the three providers is of a different kind. Provider 1 is publicly owned, Provider 2 is a publicly funded private provider, whereas Provider 3 is a private provider without public funding. The two providers that are either publicly owned or publicly funded receive a fixed price $p_1 = p_2 = \overline{p}$ per unit of the good supplied. A fraction s of this price is paid by the consumers as copayment, whereas the remaining share is paid by a public funder. However, these two providers are assumed to differ with respect to their objectives. We follow the standard assumption in the mixed oligopoly literature that the public provider maximises social welfare while the private provider is a profit maximiser. The third provider also maximises profits, but has to raise revenues in the market by charging a price p_3 per unit of the good supplied.

Consumers are uniformly distributed on the same circle. Each consumer demands one unit of the good from the most preferred provider and the total mass of consumers is normalised to 1. The utility of a consumer located at x who buys the good from Provider i, located at z_i, is given by

[3] Our paper is more directly an extension of Ghandour (2019) who studies quality competition in a mixed duopoly where the public provider is subject to price regulation while the private provider is not.

$$u(x, z_i) = v + \beta q_i - r_i - t\left|x - z_i\right|; \; i = 1, 2, 3, \tag{1}$$

where q_i is the quality offered by Provider i and r_i is the price paid by Provider i's consumers. In line with our previously stated assumptions, $r_1 = r_2 = s\overline{p}$ and $r_3 = p_3$. The parameters $\beta > 0$ and $t > 0$ measure, respectively, the marginal willingness to pay for quality and the marginal transportation cost. The latter can be interpreted either as the marginal cost of travelling in geographical space or the marginal mismatch cost in product space. We also assume that the utility parameter $v > 0$ is sufficiently large to ensure full market coverage for all quality and price configurations.

Suppose that every consumer in the market makes a utility-maximising choice of provider. Let $\widehat{x}_i^{i+1}$ denote the distance between the location of Provider i and the location of the consumer who is indifferent between Provider i and the neighbouring Provider $i+1$. When each consumer maximises utility, this distance is given by

$$\widehat{x}_i^{i+1}\left(q_i, q_{i+1}; r_i, r_{i+1}\right) = \frac{1}{6} + \frac{\beta\left(q_i - q_{i+1}\right) - \left(r_i - r_{i+1}\right)}{2t}. \tag{2}$$

Since each provider has two neighbours, the demand for Provider i is given by

$$D_i\left(q_i, q_{i-1}, q_{i+1}; r_i, r_{i-1}, r_{i+1}\right) = \widehat{x}_i^{i+1}\left(q_i, q_{i+1}; r_i, r_{i+1}\right) + \widehat{x}_i^{i-1}\left(q_i, q_{i-1}; r_i, r_{i-1}\right). \tag{3}$$

Substituting from (2), this yields

$$D_i\left(q_i, q_{i-1}, q_{i+1}; r_i, r_{i-1}, r_{i+1}\right) = \frac{1}{3} + \frac{\beta\left(2q_i - q_{i-1} - q_{i+1}\right) - \left(2r_i - r_{i-1} - r_{i+1}\right)}{2t} \tag{4}$$

The Salop model is generally characterised by localised competition, implying that the demand of each provider only depends on the prices and qualities of that provider and its two neighbours. However, with only three providers, each provider has all the remaining providers in the market as neighbours. Thus, all providers compete directly with each other.

We assume that the cost of provision is separable in quantity and quality, with the cost function of Provider i given by

$$C\left(D_i, q_i\right) = cD_i + \frac{k}{2}q_i^2. \tag{5}$$

The profits of Provider i are thus given by

$$\pi_i = (p_i - c)\, D_i - \frac{k}{2} q_i^2. \tag{6}$$

Whereas the private providers (2 and 3) are assumed to maximise profits, the publicly owned provider is assumed to maximise social welfare, denoted W, which is given by aggregate consumer utility, denoted U, plus total profits, net of public funding:

$$W = U + \sum_{i=1}^{3} \pi_i - (1-s)\, \overline{p} \sum_{i=1}^{2} D_i. \tag{7}$$

With a slight abuse of notation, aggregate consumer utility is given by[4]

$$U = \sum_{i=1}^{3} \left(\int_0^{\widehat{x}_i^{i+1}} (v + \beta q_i - r_i - tx)\, dx + \int_0^{\widehat{x}_i^{i-1}} (v + \beta q_i - r_i - tx)\, dx \right). \tag{8}$$

Since total demand is fixed, which implies that social welfare does not depend directly on prices and other monetary transfers, we can more conveniently reformulate the welfare expression as

$$W = v + \beta \overline{q} - T - c - \frac{k}{2} \sum_{i=1}^{3} q_i^2, \tag{9}$$

where

$$\overline{q} := \sum_{i=1}^{3} D_i q_i \tag{10}$$

is average quality and

$$T := \frac{t}{12} + \frac{\sum_{i=1}^{3} r_i \left(r_i - r_{i+1}\right) + \beta \left(\beta \sum_{i=1}^{3} q_i \left(q_i - q_{i+1}\right) + \sum_{i=1}^{3} q_i \left(r_{i-1} + r_{i+1}\right) - 2 \sum_{i=1}^{3} q_i r_i \right)}{2t} \tag{11}$$

is aggregate transportation costs.[5] The last two terms in (9) represent the total cost of provision in the market. It is immediately obvious from (11) that aggregate transportation costs are minimised (at $T = t/12$) for a symmetric outcome, where $r_i = r_j$ and $q_i = q_j$, for all i and j, $i \neq j$.

Our subsequent analysis is based on different versions (or subgames) of the following three-stage game:

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[4] Notice that, if $i = 1$, then $i - 1 = 3$, and if $i = 3$, then $i + 1 = 1$.

[5] Notice that subscripts $i + 1$ and $i - 1$ refer to the two neighours of Provider i located in the clockwise and anticlockwise direction, respectively. Keep also in mind that $r_1 = r_2 = s\overline{p}$ and $r_3 = p_3$.

Stage 1 A welfare-maximising regulator chooses its policy parameter(s), either $\overline{p}$ (s) or both $\overline{p}$ and s.

Stage 2 Each of the three providers chooses its level of quality provision, q_i.

Stage 3 The private Provider 3 chooses its price, p_3.

The separation of Stage 2 from Stage 3 is motivated by the implicit assumption that the level of quality provision is more of a long-term decision than the price choice. Furthermore, in versions of the game where we include Stage 1, we implicitly assume that the regulator is able to precommit to a particular regulatory policy as a long-term decision. Finally, in order to ensure equilibrium existence in all versions of the game considered, we assume that the quality cost parameter k is bounded from below:[6]

$$k \geq \underline{k} := \frac{3\beta^2}{2t}. \tag{12}$$

In order to rule out a negative price-cost margin for the publicly funded private provider, we also assume that $\overline{p} \geq c$.

4 Analysis

In this section we derive and characterise the subgame-perfect Nash equilibrium. In particular, we are interested in comparing the equilibrium quality provision across the three different providers, and how this quality provision depends on the design of the funding scheme and on the degree of competition in the market. We start out by considering the subgame that starts at Stage 2 of the above described game, which allows us to analyse optimal provider behaviour under an exogenously given regulatory regime. This is arguably the most realistic scenario, given that prices and copayment rates might be based on considerations that lie outside the scope of the present model. However, we will subsequently introduce Stage 1 to the game and analyse the optimal choice of regulated price for a given copayment rate, before endogenising both policy variables ($\overline{p}$ and s) and derive the subgame-perfect Nash equilibrium of the full game.

[6] See Appendix A for a derivation of the lower bound $\underline{k}$.

4.1 Fixed price and copayment

Suppose that the publicly funded providers face an exogenous price, $\overline{p}$, and an exogenously given copayment rate, s. The game is solved by backwards induction, so we start out by considering the optimal price chosen by Provider 3.

4.1.1 Optimal private price

At the third stage, the private provider without public funding chooses a price that maximises the provider's profits. By maximising π_3, as given by (6), with respect to p_3, we find that the profit-maximising price is given by[7]

$$p_3\left(q_1, q_2, q_3; s, \overline{p}\right) = \frac{t}{6} + \frac{c + s\overline{p}}{2} - \beta\left(\frac{q_1 + q_2}{4}\right) + \frac{\beta q_3}{2}. \tag{13}$$

We see that the optimal price of the private provider is decreasing in the quality levels of each of the two rival providers (q_1 and q_2). A higher quality by a rival provider leads to a drop in demand, which makes demand more price elastic, all else equal. This reduces in turn the profit-maximising price. Thus, the price of the private provider is a *strategic substitute* to the quality of a rival provider.

On the other hand, the optimal price of the private provider is increasing in the provider's own quality (q_3). All else equal, a higher quality provision leads to higher demand, which makes demand less price elastic. Consequently, the profit-maximising price increases. In other words, price and quality are *complementary strategies* for the private provider.

Finally, notice that Provider 3's optimal price is increasing in both the regulated price ($\overline{p}$) and the copayment rate (s). This is due to prices being *strategic complements* for given quality levels. A higher $\overline{p}$ or a higher s implies, all else equal, that the good supplied by either of the publicly funded providers becomes more expensive for consumers. This leads to higher, and thus less price-elastic, demand for Provider 3, who optimally responds by increasing the price.

4.1.2 Quality competition

Anticipating the price choice of Provider 3, all providers simultaneously and independently choose qualities in order to maximise their objective functions. It is instructive to carefully study the

[7] The second-order condition is trivially satisfied, since $\partial^2\pi_3/\partial p_3^2 = -2/t < 0$.

nature of the strategic interaction between the different providers. Maximising (6)-(7) with respect to q_i, the best response functions are given by[8]

$$q_1(q_2, q_3) = \beta \frac{2(c - s\overline{p}) + 6t - 3\beta(3q_2 + 2q_3)}{16kt - 15\beta^2}, \tag{14}$$

$$q_2 = \frac{7\beta(\overline{p} - c)}{8kt}, \tag{15}$$

$$q_3(q_1, q_2) = \beta \frac{2t + 6(s\overline{p} - c) - 3\beta(q_1 + q_2)}{6(2kt - \beta^2)}. \tag{16}$$

For each provider, the optimal quality level balances marginal benefits against marginal costs. Whereas the marginal cost of quality provision is by assumption equal for all providers, and given by kq_i, the marginal benefits are not.

Consider first the two profit-maximising providers. The marginal revenue of quality provision for the publicly funded provider (Provider 2) is given by

$$(\overline{p} - c)\left(\frac{\partial D_2}{\partial q_2} + \frac{\partial D_2}{\partial p_3}\frac{\partial p_3}{\partial q_2}\right) = (\overline{p} - c)\left(\frac{\beta}{t} - \frac{\beta}{8t}\right). \tag{17}$$

The profitability of quality provision depends on the size of the price-cost margin $(\overline{p} - c)$ and on the quality responsiveness of demand. All else equal, a higher price-cost margin and/or a more quality responsive demand will increase the incentives for quality provision. However, notice that a quality increase by Provider 2 has a direct and an indirect effect on the provider's demand. The positive direct effect is counteracted by the fact that a quality increase triggers a price reduction by the competing private provider (Provider 3) in the subsequent stage. This indirect effect dampens the incentives for quality provision by Provider 2, all else equal. However, because of the linearity of the demand function, neither the direct nor the indirect effect of quality on demand depends on the quality levels chosen by the competing providers. Thus, q_2 is strategically independent of the rivals' qualities.

Consider next the private Provider 3. The marginal revenue of quality for this provider is given by

[8] For second-order and stabilitiy conditions, see Appendix A.

$$(p_3 - c)\left(\frac{\partial D_3}{\partial q_3} + \frac{\partial D_3}{\partial p_3}\frac{\partial p_3}{\partial q_3}\right) + \frac{\partial p_3}{\partial q_3}D_3 = (p_3 - c)\left(\frac{\beta}{t} - \frac{\beta}{2t}\right) + \frac{\beta}{2}D_3. \tag{18}$$

The difference between the two private providers is that Provider 3 chooses its price, p_3. The resulting effect on the incentives for quality provision is captured by the last term in (18). Since price and quality are complementary strategies for the provider, a higher quality level will have an additional positive effect on revenues through a higher price. Notice, however, that the magnitude of this effect depends on Provider 3's demand (D_3), which is decreasing in the quality levels of the provider's rivals (q_1 and q_3). All else equal, a higher quality level by Provider 1 or Provider 2 will reduce the demand of Provider 3, which in turn reduces the latter provider's revenue gain of a higher price, with a corresponding reduction in the provider's incentives for quality provision. Thus, the quality decision of Provider 3 is a *strategic substitute* to the qualities chosen by the provider's rivals. Notice also that the optimal quality level chosen by Provider 3 is increasing in the regulated price $\overline{p}$. The reason is that a higher regulated price increases the optimal price chosen by Provider 3 in the last stage of the game, all else equal, which in turn increases the profitability of quality provision for this provider at the previous stage.

Finally, consider the public provider, which by assumption maximises social welfare. Using (9), the marginal benefit of quality for the public provider is given by

$$\beta\left(\frac{\partial \overline{q}}{\partial q_1} + \frac{\partial \overline{q}}{\partial p_3}\frac{\partial p_3}{\partial q_1}\right) - \left(\frac{\partial T}{\partial q_1} + \frac{\partial T}{\partial p_3}\frac{\partial p_3}{\partial q_1}\right). \tag{19}$$

Once more, the marginal benefit is a sum of direct and indirect effects. Consider first the *direct* effect of higher quality provision by the public provider, which is given by

$$\beta\frac{\partial \overline{q}}{\partial q_1} - \frac{\partial T}{\partial q_1}. \tag{20}$$

The first term is unambiguously positive, since a unilateral increase in the quality provision of the public provider increases average quality in the market. However, the sign of the second term is *a priori* indeterminate and depends on relative market shares, which in turn depend on the distribution of qualities and consumer prices across the three providers. Generally, a higher quality provision by the public provider increases (reduces) aggregate transportation costs if it leads to a more (less) asymmetric distribution of market shares.

Using the definitions of $\overline{q}$ and T, we derive

$$\frac{\partial \overline{q}}{\partial q_1} = D_1 + \frac{\beta}{2t}(2q_1 - q_2 - q_3) > 0 \tag{21}$$

and

$$\frac{\partial T}{\partial q_1} = \beta \frac{p_3 - s\overline{p} + \beta(2q_1 - q_2 - q_3)}{2t} \gtrless 0, \tag{22}$$

where p_3 is given by (13). A higher quality by rival providers (i.e, an increase in q_2 or q_3) implies that the public provider has a lower market share, which in turn reduces the effect of q_1 on average quality. On the other hand, a lower market share for the public provider increases the scope for a negative sign of $\partial T/\partial q_1$, which implies that aggregate transportation costs can be reduced by an increase in q_1. A similar ambiguity applies to the regulated price, $\overline{p}$. As long as $s > 0$, a lower price $\overline{p}$ increases the market share of the public provider, thus making q_1 a more effective instrument to increase average quality provision. On the other hand, the scope for a detrimental effect of a quality increase on aggregate transportation costs also increases. Summing these two potentially counteracting effects, we obtain

$$\beta \frac{\partial \overline{q}}{\partial q_1} - \frac{\partial T}{\partial q_1} = \beta \left(\frac{1}{3} + \frac{\beta(2q_1 - q_2 - q_3)}{2t} \right). \tag{23}$$

We see that the sum of the two effects does not depend on the regulated price, which implies that the two counteracting effects of a price change exactly cancel each other. On the other hand, the direct marginal benefit of quality depends negatively on rivals' qualities, implying that the above described effect related to average quality dominates.

In order to explain how the public provider's quality provision depends on the regulated price $\overline{p}$, we need to turn to the effects that work through subsequent changes in the private price p_3. The effects of p_3 on average quality and aggregate transportation costs are given by, respectively,

$$\frac{\partial \overline{q}}{\partial p_3} = \frac{q_1 + q_2 - 2q_3}{2t} \tag{24}$$

and

$$\frac{\partial T}{\partial p_3} = \frac{2(p_3 - s\overline{p}) + \beta(q_1 + q_2 - 2q_3)}{2t}. \tag{25}$$

The *indirect* marginal benefit of quality provision by the public provider is thus given by

$$\left(\beta\frac{\partial\overline{q}}{\partial p_3}-\frac{\partial T}{\partial p_3}\right)\frac{\partial p_3}{\partial q_1}=\left(\frac{s\overline{p}-p_3}{t}\right)\frac{\partial p_3}{\partial q_1}, \tag{26}$$

where p_3 is given by (13), and where $\partial p_3/\partial q_1 = -\beta/4$. Thus, the public provider's incentive for quality provision in order to induce a desired change in p_3 depends *negatively* on $s\overline{p}$, and the intuition for this follows directly from (25).[9] A lower value of $s\overline{p}$ reduces the market share of Provider 3, which turn increases the scope for a reduction in aggregate transportation costs as a result of a decrease in p_3. And a reduction in p_3 can be induced by higher public quality provision.

The above decomposition of direct and indirect effects explains why the public provider's optimal choice of quality depends negatively on q_2, q_3 and $s\overline{p}$. A higher quality by any of the rival providers leads to a reduction in the market share of the public provider, which implies that q_1 becomes a less effective instrument to increase average quality. Consequently, the optimal quality level of the public provider goes down. A quality reduction by the public provider also results from an increase in $s\overline{p}$, but for a different reason, which is related to the objective of reducing aggregate transportation costs by inducing a change in the price set by the private Provider 3, as explained above.

If the subgame perfect Nash Equilibrium is an interior solution, the equilibrium outcome is given by

$$q_1^*=\beta\frac{\beta^2\left(42\beta^2\left(\overline{p}-c\right)+kt\left(79c-63\overline{p}\right)\right)+16kt\left(kt\left(c+3t-s\overline{p}\right)-\beta^2\left(2t+s\overline{p}\right)\right)}{8kt\left(kt\left(16kt-23\beta^2\right)+6\beta^4\right)}, \tag{27}$$

$$q_2^*=\frac{7\beta\left(\overline{p}-c\right)}{8kt}, \tag{28}$$

$$q_3^*=\beta\frac{21\beta^2\left(3\beta^2\left(\overline{p}-c\right)+2kt\left(3c-\overline{p}\right)\right)+4kt\left(8kt\left(t-3c+3s\overline{p}\right)-3\beta^2\left(4t+7s\overline{p}\right)\right)}{12kt\left(kt\left(16kt-23\beta^2\right)+6\beta^4\right)}, \tag{29}$$

$$p_3^*=\frac{\left(2kt-3\beta^2\right)\left(16kt^2+3c\left(16kt-\beta^2\right)-21\beta^2\overline{p}\right)+12ks\overline{p}t\left(8kt-7\beta^2\right)}{12\left(kt\left(16kt-23\beta^2\right)+6\beta^4\right)}. \tag{30}$$

In the following, we perform a thorough characterisation of the equilibrium and show how the equilibrium quality provision depends on the characteristics of the funding scheme and on the

[9] Although p_3 depends positively on $s\overline{p}$, it is straightforward to verify, by using (13), that $s\overline{p}-p_3$ is monotonically increasing in $s\overline{p}$.

intensity of competition, as inversely measured by the parameter t.

4.1.3 The relationship between the funding scheme and equilibrium quality provision

In our model, the funding scheme consists of two elements: the regulated price ($\overline{p}$) and the copayment rate (s). In the following, we analyse the effects of a change in each of these instruments on the equilibrium quality provision.

The effects of a change in the regulated price $\overline{p}$ are given as follows:[10]

Proposition 1 *A higher regulated price $\overline{p}$ leads to*

(i) lower quality for the public provider,

(ii) higher quality for the publicly funded private provider,

(iii) lower (higher) quality for the private provider without public funding if the copayment rate s is sufficiently low (high).

The intuition for these results is directly linked to the nature of the strategic interaction in the quality game. Notice that the two publicly funded providers respond to changes in the regulated price in a completely opposite fashion, which is caused by the assumed differences in the objective functions. The profit-maximising provider (Provider 2) responds to a higher price by increasing quality, because a higher price-cost margin makes it more profitable to attract demand by providing a higher quality level. For the publicly owned provider, on the other hand, such a concern is irrelevant because of the assumption that the provider is a welfare maximiser. On the contrary, a higher regulated price gives this provider an incentive to *reduce* its quality in order to induce a price increase by the private Provider 3, with the objective of reducing aggregate transportation costs through a more equal distribution of market shares.[11]

Finally, for the private provider without public funding, the effect of a higher regulated price on quality provision depends crucially on the magnitude of the copayment rate that applies to the publicly funded providers. If the copayment rate is sufficiently low (high), the provider will respond to a higher regulated price by reducing (increasing) quality provision. In order to understand this result, notice that the mechanisms through which a change in the regulated price affects quality

[10] The proof of this an all subsequent propositions (apart from those that are trivially proved) are given in Appendix B.

[11] More precisely, a higher regulated price will either *dampen* the public provider's incentive to reduce aggregate transportation costs by offering *higher* quality, or it will *reinforce* the provider's incentive to reduce aggregate transportation costs by *lowering* its quality provision. In either case, a higher regulated price results in lower public quality provision, all else equal.

provision are very different for the two private providers. Whereas the regulated price directly determines the price-cost margin of Provider 2, the effect on Provider 3 goes through demand. If the copayment rate is sufficiently low, a higher regulated price will shift demand from Provider 3 to Provider 2 because of the increase in quality offered by the latter provider.[12] This makes Provider 3's demand more price elastic and the provider will therefore respond by reducing both price and quality. However, a higher copayment rate will dampen (and might ultimately reverse) the demand shift from Provider 3 to Provider 2 due to a higher regulated price, because of a larger increase in the consumer copayment. Thus, if the copayment rate is sufficiently high, a higher regulated price will *increase* the demand of Provider 3 and therefore lead to a higher price and quality offered by this provider.

In sum, a higher regulated price has a strongly heterogeneous effect on quality provision across the different providers, with a negative effect for the publicly owned provider, a positive effect for the publicly funded private provider, and an *a priori* ambiguous effect for the private provider without public funding. It might therefore be useful to consider the effect on *average* quality, $\overline{q}$, as defined by (10). On general form, the effect of a marginal increase in the regulated price on average quality is given by

$$\frac{\partial \overline{q}}{\partial \overline{p}} = \sum_{i=1}^{3} \left(\frac{\partial q_i^*}{\partial \overline{p}} D_i + q_i^* \frac{\partial D_i}{\partial \overline{p}} \right). \tag{31}$$

Proposition 2 *(i) Suppose that s is sufficiently close to either zero or one. In this case, $\partial\overline{q}/\partial\overline{p} > 0$ for all $k > \underline{k}$ if $(\overline{p} - c)$ is sufficiently high relative to t. (ii) Suppose that k is sufficiently close to $\underline{k}$. In this case, $\partial\overline{q}/\partial\overline{p} < 0$ for all $s \in [0, 1]$ if $(\overline{p} - c)$ is sufficiently small or if t is sufficiently high.*

Unsurprisingly, given the heterogeneous results presented in Proposition 1, the relationship between the size of the regulated price and average quality provision is *a priori* ambiguous. In Proposition 2 we have identified different parameter sets for which this relationship is either positive or negative. The characteristics of these parameter sets suggest that the scope for a *positive* effect of a price increase on average quality provision is larger if the regulated price is relatively high to begin with, and if the intensity of competition is also relatively high (i.e, if t is relatively low), which magnifies the demand responses to changes in prices and qualities. In such a scenario, if the copayment rate is sufficiently small, a higher regulated price leads to a higher average quality because of the quality increase by Provider 2, whereas, if the copayment rate is sufficiently large,

[12] A higher price will also reduce the quality provision of Provider 1, but this effect is not large enough to prevent a demand loss for Provider 3.

a similar effect is enabled by the quality increase by Provider 3.

The effects of a change in the copayment rate s are summarised below:

Proposition 3 *A higher copayment rate s leads to*

(i) lower quality for the public provider,

(ii) no change in the quality of the publicly funded private provider,

(iii) higher quality for the private provider without public funding.

A higher copayment rate implies that the good supplied by either of the publicly funded providers become more expensive for consumers. But this has no effect on the quality offered by the publicly funded private provider. Notice that Provider 2 maximises profits and a higher copayment rate does not influence the profit margin, nor does it influence the demand responsiveness to quality. In other words, both the marginal benefit and the marginal cost of quality provision for Provider 2 are unaffected by the copayment rate.

The incentives are different for the welfare-maximising public provider. Since a higher copayment rate reduces the market share of the public provider, this reduces the effect of the public provider's quality on average quality (cf. (21)), which all else equal gives Provider 1 an incentive to reduce its quality provision.

Since a higher s leads to lower quality of the public provider, the private provider without public funding experiences higher, and thus less price-elastic, demand. This, in turn, gives the private Provider 3 an incentive to increase the price and therefore also leads to higher quality (because price and quality are complementary strategies).

Therefore, in our model, each provider responds differently to a higher copayment rate, with a negative effect for the publicly owned provider, a positive effect for the private provider without public funding, and no effect for the publicly funded private provider. Thus, it is important to assess the effect of the copayment rate s on *average* quality $\overline{q}$, which is generally given by

$$\frac{\partial \overline{q}}{\partial s} = \sum_{i=1}^{3} \left(\frac{\partial q_i^*}{\partial s} D_i + q_i^* \frac{\partial D_i}{\partial s} \right). \tag{32}$$

Proposition 4 *Suppose that the regulated price is not very high nor very low. In this case, there exists a threshold value $\widehat{s} \in (0,1)$ such that $\partial \overline{q}/\partial s < (>) 0$ if $s < (>) \widehat{s}$.*

Not surprisingly, given the results in Proposition 3, the relationship between the copayment rate and average quality provision is *a priori* ambiguous. However, for a large set of parameter values, we are able to establish a convex relationship between the copayment rate and average quality, where a marginal increase in the copayment rate leads to a reduction (increase) in average quality if the initial level of the copayment rate is sufficiently low (high). In other words, average quality is minimised for an intermediate degree of consumer copayment.

This convex relationship has a relatively intuitive explanation. Notice first that a change in the copayment rate affects average quality directly through an increase (decrease) in the quality of Provider 1 (Provider 3). In addition, there is an indirect effect through demand reallocation from Provider 1 to Provider 3. Since a higher copayment rate leads to lower (higher) quality for Provider 1 (Provider 3), this demand reallocation is more likely to contribute to higher average quality the higher the copayment rate is to begin with.

4.1.4 Equilibrium quality ranking

We proceed to identify the characteristics of the market that can explain the distribution of the quality provision across the different providers. For this purpose, it is convenient to define three different threshold values of the regulated price:

$$p^* := \frac{32t\left(2kt-3\beta^2\right)+3c\left(48kt-77\beta^2\right)}{21\left(16kt-19\beta^2\right)+24s\left(7\beta^2-8kt\right)}, \tag{33}$$

$$p^{**} := \frac{8t\left(3kt-2\beta^2\right)+c\left(64kt-41\beta^2\right)}{7\left(8kt-7\beta^2\right)+8s\left(\beta^2+kt\right)}, \tag{34}$$

$$p^{***} := \frac{16kt^2+3c\left(16kt-\beta^2\right)}{3\left(7\beta^2+8s\left(2kt-\beta^2\right)\right)}. \tag{35}$$

Using these definitions, we are able to state the following:[13]

Proposition 5 *(i) Suppose that* $s < \frac{21c+7t}{21c+8t}$*, which implies* $p^* < p^{**} < p^{***}$*. The equilibrium quality ranking is then given by*

$$\begin{aligned} q_1^* &> q_3^* > q_2^* \quad if \quad \overline{p} < p^*, \\ q_1^* &> q_2^* > q_3^* \quad if \quad p^* < \overline{p} < p^{**}, \\ q_2^* &> q_1^* > q_3^* \quad if \quad p^{**} < \overline{p} < p^{***}, \\ q_2^* &> q_3^* > q_1^* \quad if \quad \overline{p} > p^{***}. \end{aligned}$$

[13] The proof of this proposition relies on a straightforward comparison of equilibrium expressions and is therefore omitted.

(ii) Suppose that $s > \frac{21c+7t}{21c+8t}$*, which implies* $p^{***} < p^{**} < p^{*}$*. The equilibrium quality ranking is then given by*

$$\begin{aligned} q_1^* &> q_3^* > q_2^* \quad if \quad \overline{p} < p^{***}, \\ q_3^* &> q_1^* > q_2^* \quad if \quad p^{***} < \overline{p} < p^{**}, \\ q_3^* &> q_2^* > q_1^* \quad if \quad p^{**} < \overline{p} < p^{*}, \\ q_2^* &> q_3^* > q_1^* \quad if \quad \overline{p} > p^{*}. \end{aligned}$$

(iii) If $s = \frac{21c+7t}{21c+8t}$*, which implies* $p^{***} = p^{**} = p^{*}$*, then* $q_1^* = q_2^* = q_3^*$*.*

Although any possible ranking of quality levels across the three providers can arise in equilibrium, the above proposition nevertheless reveals some clear patterns, which can be described as follows:

Corollary 1 *(i) If the regulated price* $\overline{p}$ *is sufficiently low (high), the publicly funded private provider offers the lowest (highest) quality and the publicly owned provider offers the highest (lowest) quality. (ii) The private provider without public funding offers the highest quality in the market only if the copayment rate (s) is sufficiently close to 1.*

These patterns are explained by looking at the results derived in Propositions 1 and 3. The quality of the public provider is decreasing in the regulated price and copayment rate while the quality of the publicly funded private provider is only increasing in $\overline{p}$. This explains why Provider 1 offers higher quality than Provider 2 if the regulated price is sufficiently low, and *vice versa* if the regulated price is sufficiently high.

For the private provider without public funding, we have seen from Proposition 1 that the relationship between the regulated price and equilibrium quality provision for this provider depends crucially on the size of the copayment rate s. The quality provision of Provider 3 is increasing in $\overline{p}$ only if s is sufficiently high, which explains why the private provider without public funding might offer the highest quality in the market only if both $\overline{p}$ and s are sufficiently high.

4.1.5 Competition intensity and quality provision

In spatial competition models, a standard competition measure is the (inverse of) transportation costs. Lower transportation costs increase the degree of substitutability between the goods offered by different providers, which intensifies competition. In our model, the publicly funded providers only choose their qualities for a given regulated price. On the contrary, the private provider without public funding chooses both quality and price. Hence, more competition makes demand more responsive to changes in qualities and prices.

Generally, more competition has two countering effects on quality. The direct effect is that increased competition makes demand more responsive to a marginal increase in quality for given prices. However, if prices are endogenous there is also an indirect effect due to the fact that increased competition makes consumers more responsive to price changes, which all else equal leads to lower prices and thus reduces providers' marginal return to quality investments. This indirect effect counteracts the aforementioned direct effect and makes the relationship between competition and quality provision *a priori ambiguous* for the private provider without public funding.

Proposition 6 *More competition (lower t) has the following effects on the quality provision of each provider:*

(i) The public provider increases (decreases) quality if $\overline{p}$ is sufficiently low (high).

(ii) The publicly funded private provider increases quality.

(iii) If the regulated price is not very high nor very low, there exists a threshold value $\widetilde{s} \in (0,1)$ such that the private provider without public funding reduces (increases) quality if $s < (>)\, \widetilde{s}$.

For the private provider with public funding, the effect of more competition is unambiguous and standard. Lower transportation costs make the provider's demand more responsive to quality changes and, given a positive price-cost margin, the provider increases its quality provision in order to attract more demand.

For the two other providers, though, increased competition has an ambiguous effect on the incentives for quality provision. We find that the public provider has an incentive to increase (decrease) its quality provision in response to more competition if the regulated price is sufficiently low (high). In order to explain the intuition behind this result, we focus on the public provider's incentive to use its quality provision as an instrument to increase average quality in the market. The effectiveness of this instrument depends on relative market shares. More specifically, the larger the market share of the public provider, the larger is the effect of an increase in the provider's quality on average quality in the market. More competition (lower t) makes demand more quality and price elastic. In an asymmetric equilibrium (with quality and price differences), more competition therefore leads to a reallocation of demand towards providers with higher quality and/or lower price. If $\overline{p}$ is sufficiently low (high), the public provider is the high (low) quality provider in the market (cf. Proposition 5). Therefore, for a sufficiently low $\overline{p}$, increased competition leads to an inflow of consumers towards the public provider, which, in turn, expands its market share (higher D_1). This makes q_1 a more effective instrument to increase average quality, resulting in stronger incentives for quality provision by the public provider. The reverse result (i.e., $\partial q_1/\partial t > 0$) requires that $\overline{p}$ is sufficiently high.

For the private provider without public funding, there are two main channels through which more competition affects the provider's incentives for quality investments. The first channel is a strategic response to the other private provider. A reduction in t triggers a quality increase by Provider 2, which in turn leads to lower, and thus more price-elastic, demand for Provider 3, who optimally responds by decreasing the price. This reduces the profitability of quality provision for Provider 3 and leads to lower quality (because p_3 and q_3 are complementary strategies). On the other hand, competition leads to a demand reallocation, which depends on relative quality levels, as previously explained. The higher s is, the higher is the equilibrium quality provision of Provider 3 relative to the other providers (cf. Proposition 3). Thus, for a sufficiently high s, increased competition leads to a demand reallocation towards Provider 3, who experiences higher, and thus less price-elastic, demand. This gives Provider 3 an incentive to increase the price and in turn quality, thus counteracting the effect of the aforementioned strategic response to the other private provider. If s is sufficiently high, the effect working through demand reallocation is the dominating effect, leading to an overall increase in quality provision by Provider 3. On the other hand, if s is sufficiently low, the effect of demand reallocation reinforces the strategic response effect, leading to a reduction in q_3.

In sum, the relationship between competition and quality provision for Provider 2 is positive, while it has an indeterminate sign for the two other providers. Therefore, it is *a priori* not clear whether the effect of more competition on *average* quality, $\overline{q}$, is positive or negative. While it is not possible to determine the sign of this effect analytically, numerical simulations suggest that the effect of more competition (a reduction in t) on average quality provision is unambiguously positive, implying that the increase in quality provision by the publicly funded private provider is always sufficient to outweigh any quality reduction by the other providers.

4.2 Optimal price for a given copayment rate

We now turn to the normative part of our analysis. Suppose that the copayment rate is exogenously given, but that the public payer, at an initial stage of the game, chooses a welfare-maximising price for the two publicly funded providers.[14] Given the equilibrium outcomes in (27)-(30), we maximise the welfare function in (7) with respect to $\overline{p}$ for a given copayment rate to find the optimal price level, given by[15]

$$\overline{p}(s) = \frac{8kst\left(\beta^2+16kt\right)\left(kt-\beta^2\right)\left(16kt^2+3c\left(16kt-\beta^2\right)\right)+\Lambda}{3\Delta}, \tag{36}$$

where

$$\begin{aligned}\Lambda \;:\; &= 56t\beta^2\left(12\beta^4\left(17kt-3\beta^2\right)+k^2t^2\left(144kt-311\beta^2\right)\right)\\ &+21c\beta^2\left(\beta^4\left(1427kt-252\beta^2\right)+16k^2t^2\left(64kt-137\beta^2\right)\right)\end{aligned} \tag{37}$$

and

$$\begin{aligned}\Delta \;:\; &= 16kst\left(kt-\beta^2\right)\left(\beta^2+16kt\right)\left(7\beta^2+4s\left(2kt-\beta^2\right)\right)\\ &+49\beta^2\left(8k^2t^2\left(16kt-37\beta^2\right)+\beta^4\left(205kt-36\beta^2\right)\right).\end{aligned} \tag{38}$$

The relationship between the copayment rate and the welfare-maximising price can be described as follows:

Proposition 7 *A marginal increase in the copayment rate, s, leads to an increase (decrease) in the optimal price, $\overline{p}(s)$, if the copayment rate is initially sufficiently low (high).*

In other words, there is a positive relationship between the price and the copayment rate if the copayment rate is sufficiently low, while this relationship is negative for sufficiently high values of the copayment rate. In order to trace the intuition behind this result, notice that social welfare is maximised at a price which balances marginal social (net) benefit of improved quality against marginal costs, which implies that quality can be either underprovided or overprovided from a social

[14] We can think of this scenario as the level of the copayment rate being set to satisfy considerations that are not explicitly modelled in our framework. For example, the copayment rate might be set at a relatively low level to ensure broad access to the good offered by the two publicly funded providers.

[15] The assumption in (12) ensures that the second-order condition of the welfare-maximising problem is satisfied (see Appendix A for details).

welfare perspective. Whereas the marginal cost of quality is by assumption equal for all providers, the marginal benefits are not. On the one hand, if s is relatively low to begin with, we know that average quality decreases in response to a higher copayment rate due to the convex relationship established by Proposition 4. Thus, if s increases from a sufficiently low initial value, the regulator would like to *stimulate* quality provision, and this can be done by increasing the price, as indicated by the first part of Proposition 2. On the other hand, for a sufficiently high initial value of s, the effect of a further increase in the copayment rate on average quality is positive (cf. Proposition 4). In this case, the regulator would like to *dampen* incentives for quality provision, which can be achieved by lowering the price (once more, given the result in the first part of Proposition 2).

As in the previous section, we proceed by ranking the equilibrium quality levels across the three providers, but now setting the regulated price at the welfare-maximising level. In other words, we compare the equilibrium quality across providers given the welfare-maximising price level, $\overline{p}(s)$, which produces the following ranking:[16]

Proposition 8 *Suppose that the regulated price is set at the welfare-maximising level, given by (36). In this case,*

(i) if $s < \frac{21c+7t}{21c+8t}$*, the equilibrium quality ranking is given by*

$$q_2^*(s) > q_1^*(s) > q_3^*(s);$$

(ii) if $s > \frac{21c+7t}{21c+8t}$*, the equilibrium quality ranking is given by*

$$q_3^*(s) > q_1^*(s) > q_2^*(s).$$

For any given copayment rate, the quality offered by the public provider always lies between the qualities offered by the high-quality and low-quality providers, respectively. The highest and lowest quality in the market is always offered by a private provider. Unless the copayment rate is very close to one, the publicly funded private provider has the highest quality, whereas the private provider without public funding has the lowest quality in the market, but these roles are reversed if the copayment rate is sufficiently close to one. Notice that the two regimes detailed in Proposition 8 correspond to two of the several regimes detailed in Proposition 5 and the intuition behind this

[16] The proof of this proposition relies on a straightforward comparison of equilibrium expressions and is therefore omitted.

quality ranking mirrors the one discussed in relation to Proposition 5.

4.3 Optimal price and copayment rate

Finally, suppose that, at an initial stage of the game, the public payer chooses both the copayment rate and the price (applying to the publicly funded providers) in order to maximise social welfare. We start out by deriving the first-best solution and subsequently show how this solution can be implemented by optimal choices of the price and the copayment rate.

4.3.1 The first-best solution

Suppose that the regulator is able to control quality and demand directly. Given the symmetry of the model, transportation costs are clearly minimised if each consumer attends the nearest provider, implying equal market shares for all providers. Maximising (9) with respect to the quality of each provider under this symmetry assumption, the first-best quality level – equal for each provider – is found to be given by

$$q_i^{FB} = \frac{\beta}{3k}. \tag{39}$$

Intuitively, the first-best quality level is increasing in the consumers' marginal willingness to pay for quality (β) and decreasing in the marginal cost of quality provision (captured by k).

4.3.2 Implementation of the first-best solution

Suppose that the regulator cannot set quality directly, but is able to commit to a particular funding scheme as a long term decision. In other words, we let the regulator set both the price and the copayment rate at the first stage of the game. Formally, the regulator maximises (9) with respect to $\overline{p}$ and s. The unique solution to this problem is stated in the next proposition:

Proposition 9 *If the regulator can commit to a funding scheme before the providers make their decisions, the first-best solution is implemented by setting the price*

$$\overline{p}^* = c + \frac{8}{21}t \tag{40}$$

and the copayment rate

$$s^* = \frac{21c + 7t}{21c + 8t}. \tag{41}$$

The proof of this proposition is left to the interested reader, who can easily verify that the first-best solution is implemented by plugging (40)-(41) into (27)-(30).

Social welfare is maximised by considering two different dimensions: minimising total transportation costs and ensuring quality provision at a level where the marginal benefit is equal to the marginal cost. Because of the two-dimensionality of the problem, two different instruments are needed to implement the first-best solution, and this implies some degree of cost-sharing between consumers and the public funder.

5 Extension: Public funding coverage

In this section we extend our analysis by introducing another policy variable, namely the degree of public funding coverage among the providers in the market. Taking the above analysis as a benchmark, we consider the effects (on quality provision and welfare) of (i) extending public funding to include the third (private) provider, or (ii) restricting public funding only to the publicly owned provider.

5.1 Public funding of all private providers

Suppose that all providers in the market, whether they are public or private, are subject to the same funding scheme. This amounts to setting $p_3 = \overline{p}$ and $r_3 = s\overline{p}$, which implies complete symmetry between the two private providers (2 and 3). It also implies that all providers now only compete along the quality dimension. The Nash equilibrium at the second stage of the game (described in Section 2) is in this case given by

$$q_1^{PF} = \frac{\beta\left(kt^2 - 3\beta^2\left(\overline{p} - c\right)\right)}{3kt\left(kt - \beta^2\right)}, \tag{42}$$

$$q_2^{PF} = q_3^{PF} = \frac{\beta\left(\overline{p} - c\right)}{kt}. \tag{43}$$

The quality of the public provider is decreasing in the regulated price $\overline{p}$ whereas the quality of publicly funded private providers is increasing in $\overline{p}$. In addition, it follows immediately that more competition (lower t) leads unambiguously to higher quality for the private providers, whereas the relationship between the degree of competition and quality of the public provider is *a priori* indeterminate and depends on the size of the regulated price. In particular, if $\overline{p} - c$ is sufficiently high (low) relative to t, more competition leads to lower (higher) quality. All these results mirror the previously derived results for Provider 1 and Provider 2 in the benchmark model.

What are the effects on equilibrium quality provision of extending the public funding coverage? If copayment rates are relatively low, which is arguably the most relevant case, we are able to state the following results:

Proposition 10 *Suppose that s is sufficiently low and that $\overline{p} - c$ is sufficiently high relative to t. In this case, an extension of public funding to all private providers leads to lower quality provision by the public provider ($q_1^{PF} < q_1^*$) and higher quality provision by both private providers ($q_2^{PF} > q_2^*$ and $q_3^{PF} > q_3^*$).*

Thus, within the range of parameters considered, a public funding extension tends to stimulate quality provision for both private providers while lowering quality provision for the public provider. As long as the incentives for quality provision among the private providers are sufficiently strong (i.e., as long as $\overline{p} - c$ is sufficiently large relative to t), a public funding extension induces higher quality for these providers. However, since the quality choice of the public provider is a strategic substitute to the quality choices made by the private providers (see analysis and discussion in Section 3.1.3), the former provider will respond by reducing its quality provision.

Since public and private providers respond differently to a public funding extension, the implication for average quality provision is *a priori* indeterminate. Numerical simulations suggest that average quality tends to increase if the regulated price is sufficiently high and decrease otherwise. This is quite intuitive, since the level of the regulated price determines the market shares of the providers. If $\overline{p}$ is relatively high, the market share of the public provider is relatively low (cf. Proposition 1), implying that the effect on average quality is dominated by the quality increase of the private providers. The opposite logic applies if $\overline{p}$ is relatively low.

5.2 No public funding of private providers

An alternative policy option is to abstain from funding private providers. Suppose instead that only the public provider faces a regulated price and copayment rate, whereas each of the two private providers must raise funds in the market by charging a price for the good provided. Thus, we assume a two-stage game similar to the one considered in the main analysis, but where the two private providers simultaneously choose price at the second stage of the game. As before, we solve the game by backwards induction.

If the subgame perfect Nash Equilibrium is an interior solution, the equilibrium outcome is given by[17]

$$q_1^{NPF} = \beta \frac{t\left(55kt - 42\beta^2\right) + 2\left(14\beta^2 + 15kt\right)(c - s\overline{p})}{27kt\left(5kt - 6\beta^2\right)}. \quad (44)$$

$$q_2^{NPF} = q_3^{NPF} = 14\beta \frac{t\left(2kt - 3\beta^2\right) + \left(3kt - 2\beta^2\right)(s\overline{p} - c)}{27kt\left(5kt - 6\beta^2\right)}. \quad (45)$$

$$p_2^{NPF} = p_3^{NPF} = \frac{5t\left(2kt - 3\beta^2\right) + 2c\left(15kt - 22\beta^2\right) + 5s\overline{p}\left(3kt - 2\beta^2\right)}{9\left(5kt - 6\beta^2\right)}. \quad (46)$$

Both private providers offer the same quality and price in equilibrium, both of which react positively to an increase in the regulated price $\overline{p}$ or in the copayment rate s. We proceed to compare the quality levels in (44)-(45) with our benchmark in (27)-(29). Once more we restrict attention to the case of a relatively low copayment rate.

Proposition 11 *Suppose that s is sufficiently low and that $\overline{p} - c$ is sufficiently high relative to t. In this case, a removal of public funding for private providers leads to higher quality provision for the public provider ($q_1^{NPF} > q_1^*$) and the private provider without previous public funding ($q_3^{NPF} > q_3^*$), and lower quality provision by the private provider with previous public funding ($q_2^{NPF} < q_2^*$).*

The effects on quality provision of a funding removal are to a large extent the opposite of the effects of a funding extension. Given that the private providers have sufficiently strong incentives to compete for demand (i.e., given that $\overline{p} - c$ is sufficiently high relative to t), a removal of funding reduces the incentives for quality provision for the private provider that loses its public funding. However, because of strategic substitutability, the two other providers respond by increasing their quality provision. Once more, the effect on average quality provision is *a priori* ambiguous, but numerical simulations suggest that average quality will increase if the regulated price is sufficiently low and decrease otherwise. In qualitative terms, this is the opposite of the effect of a funding

[17] The second-order conditions are reported in Appendix A.

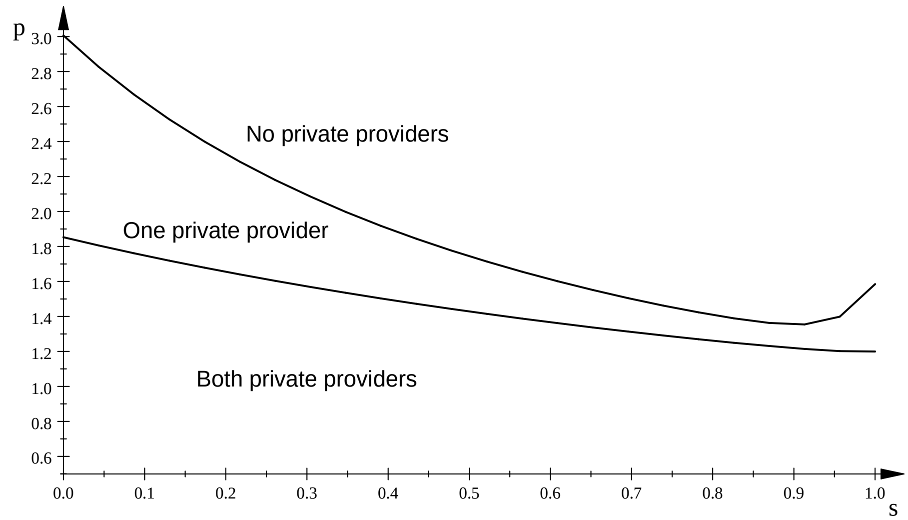

Figure 1: Optimal funding coverage

extension. Intuitively, this is once more related to the relationship between the regulated price and the market shares of the three providers. If the regulated price is low, the public provider has a high market share and the average quality effect is driven by the quality increase of the public provider. On the other hand, if the regulated price is sufficiently high, the average quality effect is driven by the quality reduction of the previously funded private provider, which has a high market share in the pre-reform equilibrium.

5.3 Optimal degree of funding coverage

A natural extension of the above analysis is to consider the optimal degree of funding coverage. For a given price and copayment rate, is welfare maximised by funding one or both private providers, or by funding none of them? Of course, as shown in the previous section, the first-best outcome can always be implemented by an appropriate choice of $\overline{p}$ and s, regardless of funding coverage. Thus, an underlying assumption of the analysis in this section is that the price and the copayment rate are exogenously determined by out-of-the-model considerations and do not coincide with the first-best levels.

For analytical tractability reasons, our analysis is performed numerically. In Figure 1 we indicate the optimal degree of funding coverage in $(s, \overline{p})$-space when the other parameters are given by $\beta = 3$, $k = 10$, $t = 2$, $c = 0.5$ and $v = 1$. Although the figure is drawn for a particular set of parameters, a similar picture emerges for alternative parameter configurations. The different regimes depicted in Figure 1 reveal that there exists a trade-off between the *generosity* of the funding (the size of $\overline{p}$)

and the *extension* of public funding (how many private providers that are included in the public funding scheme). If the regulated price is relatively low, welfare is maximised by extending funding to both private providers. On the other hand, for a sufficiently high price, it is optimal not to fund any private provider. However, for intermediate ranges of $\overline{p}$, the welfare optimal funding extension is given by our benchmark case, where only one of the private providers is included in the public funding scheme. This conclusion holds for all values of s, although the benchmark case is optimal for a larger range of parameters if the copayment rate is relatively low.

6 Concluding remarks

In this paper we have analysed quality competition among a welfare-maximising public provider and two profit-maximising private providers, where the public and one of the private providers face regulated prices and copayment rates, while the second private provider is free to set the price of its good. This is a market structure that applies to health care and education markets in many countries.

A common pattern among our findings is a differential response (in terms of quality provision) across providers to changes in the parameters of the funding scheme or in the intensity of competition. The details of these results are described elsewhere. In this final section of the paper we would like to briefly highlight some of the potential policy implications of our analysis. First, if we take the presence of publicly funded provision with (relatively low) copayment rates as given, we find that the welfare-maxmising price (given to the publicly funded providers) is increasing in the copayment rate, as long as the copayment rate is at a sufficiently low level. This indicates that these two funding instruments are *policy complements*. In other words, if policy makers wish to increase the copayment rate (from a sufficiently low level), such a policy change should optimally be accompanied by a corresponding increase in the regulated price, and *vice versa*. Second, we find that the welfare effects of either extending public funding to more private providers, or removing funding from currently funded providers, depend on the level of the regulated price. More precisely, we find that more (fewer) providers should be publicly funded if the regulated price is sufficiently low (high). This suggests that the extent of the funding coverage (i.e., how many private providers to include in the public funding scheme) and the generosity of the funding (i.e., the regulated price level) are *policy substitutes*.

Our analysis is obviously not without limitations, and we would here like to mention two of them. Importantly, we have conducted the model in a framework where consumer preferences are heterogeneous only along a horizontal dimension. This means that we are not able to capture effects that might result from vertical preference differentiation, where some consumers have higher willingness to pay for quality than others, for example. However, our model already includes asymmetries along two different dimensions (provider objectives and public funding coverage), and adding asymmetry along a third dimension would simply render the model intractable. Another limitation is that we do not allow for any (exogenous or endogenous) differences in cost efficiency across public and private providers. There are several reasons why public versus private ownership might lead to different incentives for cost-efficient provision, for example the presence of soft budgets associated with public ownership. Potential explorations along these lines are left for further research.

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Appendix

A. Equilibrium existence

Benchmark model

In the quality subgame, there are two conditions that do not trivially hold. First, the problem of the welfare-maxmising public provider is well-behaved if

$$\frac{\partial^2 W}{\partial q_1^2} = -\frac{\left(16kt - 15\beta^2\right)}{16t} < 0, \tag{A1}$$

which requires $k > 15\beta^2/16t$. Second, the Nash equilibrium is locally stable if the Jacobian of the system of first-order conditions is negative definite, which requires

$$\frac{\partial^2 W}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 W}{\partial q_1 \partial q_2} = \frac{k}{16t}\left(16kt - 15\beta^2\right) > 0 \tag{A2}$$

and

$$\begin{vmatrix} \frac{\partial^2 W}{\partial q_1^2} & \frac{\partial^2 W}{\partial q_1 \partial q_2} & \frac{\partial^2 W}{\partial q_1 \partial q_3} \\ \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_2}{\partial q_2^2} & \frac{\partial^2 \pi_2}{\partial q_2 \partial q_3} \\ \frac{\partial^2 \pi_3}{\partial q_3 \partial q_1} & \frac{\partial^2 \pi_3}{\partial q_3 \partial q_2} & \frac{\partial^2 \pi_3}{\partial q_3^2} \end{vmatrix} = -\frac{k\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)}{16t^2} < 0. \tag{A3}$$

(A2) holds if (A1) holds, while (A3) holds if $kt\left(16kt - 23\beta^2\right) + 6\beta^4 > 0$. Notice that

$$kt\left(16kt - 23\beta^2\right) + 6\beta^4\Big|_{k=\frac{15\beta^2}{16t}} = -\frac{3}{2}\beta^4 < 0 \tag{A4}$$

and

$$\frac{\partial\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)}{\partial k} = t\left(32kt - 23\beta^2\right) > 0 \text{ for } k > \frac{15\beta^2}{16t}, \tag{A5}$$

which implies that the condition in (A3) holds if k is above some threshold value higher than $15\beta^2/16t$, which in turn implies that (A1) and (A2) always hold if (A3) holds.

Furthermore, the regulator's optimal pricing problem (for a given copayment rate) is well-behaved if

$$\frac{\partial^2 W}{\partial \overline{p}^2} = -\frac{\left(2kt - 3\beta^2\right)\Theta}{64kt^2\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)^2} < 0, \tag{A6}$$

where

$$\begin{aligned} \Theta \ :\ &= 16kst\left(\beta^2 + 16kt\right)\left(kt - \beta^2\right)\left(7\beta^2 + 4s\left(2kt - \beta^2\right)\right) \\ &+49\beta^2\left(8k^2t^2\left(16kt - 37\beta^2\right) + \beta^4\left(205kt - 36\beta^2\right)\right). \end{aligned} \tag{A7}$$

Assuming that $\Theta > 0$, the condition in (A6) holds if $k > 3\beta^2/2t$. Evaluating the numerator in (A3) at $k = 3\beta^2/2t$ yields

$$kt\left(16kt - 23\beta^2\right) + 6\beta^4\Big|_{k=\frac{3\beta^2}{2t}} = \frac{15}{2}\beta^4 > 0. \tag{A8}$$

Thus, the condition in (A3) always holds if (A6) holds. It remains to show that $\Theta > 0$. To do so, we derive

$$\frac{\partial^3 \Theta}{\partial k^3} = 768t^3 \left(7\beta^2 \left(2s+7\right) + s^2 \left(64kt - 23\beta^2\right)\right). \tag{A9}$$

Notice that $\partial^3\Theta/\partial k^3 > 0$ if $k > 3\beta^2/2t$. This implies that $\partial^2\Theta/\partial k^2$ is monotonically increasing in k. Evaluated at the lower bound $k = 3\beta^2/2t$, we derive

$$\left.\frac{\partial^2 \Theta}{\partial k^2}\right|_{k=\frac{3\beta^2}{2t}} = 112t^2\beta^4 \left(114s + 272s^2 + 245\right) > 0. \tag{A10}$$

Thus, Θ is strictly convex for $k > 3\beta^2/2t$. Furthermore,

$$\left.\frac{\partial \Theta}{\partial k}\right|_{k=\frac{3\beta^2}{2t}} = t\beta^6 \left(6944s + 10\,336s^2 + 8869\right) > 0 \tag{A11}$$

and

$$\Theta|_{k=\frac{3\beta^2}{2t}} = \frac{75}{2}\beta^8 \left(56s + 64s^2 + 49\right) > 0. \tag{A12}$$

Since Θ is positive and increasing in k at $k = 3\beta^2/2t$, and since Θ is strictly convex for all $k > 3\beta^2/2t$, it follows that Θ is positive also for all $k > 3\beta^2/2t$. Thus, the second-order condition (A6) is satisfied if

$$k > \underline{k} := \frac{3\beta^2}{2t}, \tag{A13}$$

and this condition ensures that the critical conditions in the quality subgame, (A1)-(A3), are also satisfied.

Public funding of all private providers

In the quality subgame, the second-order conditions are satisfied for Provider 2 and 3:

$$\frac{\partial^2 \pi_2}{\partial q_2^2} = \frac{\partial^2 \pi_3}{\partial q_3^2} = -k < 0. \tag{A14}$$

The problem of the welfare-maxmising provider is well-behaved if

$$\frac{\partial^2 W}{\partial q_1^2} = -\frac{\left(kt - \beta^2\right)}{t} < 0, \tag{A15}$$

which is true for $k > \underline{k}$. Furthermore, equilibrium stability requires that the Jacobian is negative definite, which is true if

$$\frac{\partial^2 W}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2}-\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 W}{\partial q_1 \partial q_2}=\frac{\left(kt-\beta^2\right)k}{t}>0 \tag{A16}$$

and

$$\begin{vmatrix} \frac{\partial^2 W}{\partial q_1^2} & \frac{\partial^2 W}{\partial q_1 \partial q_2} & \frac{\partial^2 W}{\partial q_1 \partial q_3} \\ \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_2}{\partial q_2^2} & \frac{\partial^2 \pi_2}{\partial q_2 \partial q_3} \\ \frac{\partial^2 \pi_3}{\partial q_3 \partial q_1} & \frac{\partial^2 \pi_3}{\partial q_3 \partial q_2} & \frac{\partial^2 \pi_3}{\partial q_3^2} \end{vmatrix}=-\frac{\left(kt-\beta^2\right)k^2}{t}<0. \tag{A17}$$

Both conditions hold if $k \geq \underline{k}$.

No public funding for the private providers

In the pricing subgame, the second order conditions are satisfied,

$$\frac{\partial^2 \pi_2}{\partial p_2^2}=\frac{\partial^2 \pi_3}{\partial p_3^2}=-\frac{2}{t}<0, \tag{A18}$$

and equilibrium stability requires that the Jacobian is negative definite, which is easily verified:

$$\frac{\partial^2 \pi_2}{\partial p_2^2}\frac{\partial^2 \pi_3}{\partial p_3^2}-\frac{\partial^2 \pi_2}{\partial p_2 \partial p_3}\frac{\partial^2 \pi_3}{\partial p_2 \partial p_3}=\frac{15}{4t^2}>0. \tag{A19}$$

In the quality subgame, there are two sets of conditions that do not trivially hold. First, the problem of each profit maximising provider is well-behaved if

$$\frac{\partial^2 \pi_2}{\partial q_2^2}=\frac{\partial^2 \pi_3}{\partial q_3^2}=\frac{1}{225t}\left(98\beta^2-225kt\right)<0, \tag{A20}$$

and the problem of the welfare-maxmising provider is well-behaved if

$$\frac{\partial^2 W}{\partial q_1^2}=\frac{1}{9t}\left(8\beta^2-9kt\right)<0. \tag{A21}$$

Second, the Nash equilibrium is locally stable if the Jacobian is negative definite, which requires

$$\frac{\partial^2 W}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2}-\frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 W}{\partial q_1 \partial q_2}=\frac{kt\left(225kt-298\beta^2\right)+56\beta^4}{225t^2}>0, \tag{A22}$$

and

$$\begin{vmatrix} \frac{\partial^2 W}{\partial q_1^2} & \frac{\partial^2 W}{\partial q_1 \partial q_2} & \frac{\partial^2 W}{\partial q_1 \partial q_3} \\ \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_2}{\partial q_2^2} & \frac{\partial^2 \pi_2}{\partial q_2 \partial q_3} \\ \frac{\partial^2 \pi_3}{\partial q_3 \partial q_1} & \frac{\partial^2 \pi_3}{\partial q_3 \partial q_2} & \frac{\partial^2 \pi_3}{\partial q_3^2} \end{vmatrix} = -\frac{k\left(5kt-6\beta^2\right)\left(25kt-14\beta^2\right)}{125t^2} < 0, \tag{A23}$$

All the above conditions are satisfied if $k \geq \underline{k}$.

B. Proofs

Proof of Proposition 1

From (27)-(29) we derive

$$\frac{\partial q_1^*}{\partial \overline{p}} = -\beta \frac{16kst\left(\beta^2+kt\right)+21\beta^2\left(3kt-2\beta^2\right)}{8kt\left(6\beta^4+\left(16kt-23\beta^2\right)kt\right)}, \tag{B1}$$

$$\frac{\partial q_2^*}{\partial \overline{p}} = \frac{7\beta}{8kt}, \tag{B2}$$

$$\frac{\partial q_3^*}{\partial \overline{p}} = \beta \frac{4kst\left(8kt-7\beta^2\right)-7\beta^2\left(2kt-3\beta^2\right)}{4kt\left(6\beta^4+\left(16kt-23\beta^2\right)kt\right)}. \tag{B3}$$

(i) The numerator in (B1) is monotonically increasing in k and positive for all $k > \underline{k}$. Thus, $\partial q_1^*/\partial \overline{p} < 0$ for all $k > \underline{k}$.

(ii) The positive sign of (B2) is trivial.

(iii) The numerator in (B3) is monotonically increasing in s. Setting the numerator equal to zero and solving for s, we derive

$$\frac{\partial q_3^*}{\partial \overline{p}} < (>)\, 0 \text{ if } s < (>)\, \overline{s} := \frac{7\beta^2\left(2kt-3\beta^2\right)}{4kt\left(8kt-7\beta^2\right)}. \tag{B4}$$

Notice that $\overline{s} \in (0,1)$, since

$$1-\overline{s} = \frac{21\beta^4+2kt\left(16kt-21\beta^2\right)}{4kt\left(8kt-7\beta^2\right)} > 0 \text{ for } k \geq \underline{k}. \tag{B5}$$

Q.E.D.

Proof of Proposition 2

Using (31), we derive

$$\frac{\partial \overline{q}}{\partial \overline{p}} = \frac{\beta\left(\theta\overline{p}+\varpi\right)}{96kt\left(6\beta^4+\left(16kt-23\beta^2\right)kt\right)^2}, \tag{B6}$$

where

$$\theta := 3k\left(\begin{array}{c} 16s\left(12s\left(\beta^6+4kt\left(4\beta^4+3kt\left(2kt-3\beta^2\right)\right)\right)-7\left(39\beta^6+2k^2t^2\left(16kt-41\beta^2\right)\right)\right) \\ +49\beta^2\left(219\beta^4+16kt\left(14kt-27\beta^2\right)\right) \end{array}\right) \tag{B7}$$

and

$$\begin{aligned} \varpi \;:\; &= 56\left(54\beta^8+kt\left(2kt\left(519\beta^4+kt\left(80kt-389\beta^2\right)\right)-495\beta^6\right)\right) \\ &+21ck\left(16kt\left(16kt-27\beta^2\right)\left(kt-7\beta^2\right)-1221\beta^6\right) \\ &+32kst\left(18\beta^6+kt\left(147\beta^4+2kt\left(8kt-115\beta^2\right)\right)\right) \\ &+24cks\left(249\beta^6-2kt\left(192\beta^4+kt\left(176kt-145\beta^2\right)\right)\right). \end{aligned} \tag{B8}$$

(i) Define $A := \theta\overline{p}+\varpi$. It follows immediately from (B6) that the sign of $\partial\overline{q}/\partial\overline{p}$ is equal to the sign of A. Consider first the case of $s=0$. In this case, we derive

$$\frac{\partial^4 A}{\partial k^4} = 43\,008t^3\left(3c+5t\right) > 0, \tag{B9}$$

which implies that $\partial^3 A/\partial k^3$ is monotonically increasing in k. Evaluating at the lower bound of k, we derive

$$\left.\frac{\partial^3 A}{\partial k^3}\right|_{k=\underline{k}} = 672t^2\beta^2\left(294\overline{p}-129c+91t\right) > 0. \tag{B10}$$

Thus, we conclude that $\partial^3 A/\partial k^3 > 0$ for all $k \geq \underline{k}$, which implies that $\partial^2 A/\partial k^2$ is monotonically increasing in k. Repeating the above logic, we derive

$$\left.\frac{\partial^2 A}{\partial k^2}\right|_{k=\underline{k}} = 336t\beta^4\left(504\overline{p}-441c-101t\right), \tag{B11}$$

$$\left.\frac{\partial A}{\partial k}\right|_{k=\underline{k}} = 315\beta^6\left(203\overline{p}-247c-84t\right) \tag{B12}$$

and

$$A|_{k=\underline{k}} = \frac{4725\beta^8 (7\overline{p} - 11c - 4t)}{2t}. \tag{B13}$$

It follows that $A > 0$ for all $k \geq \underline{k}$ if the three expressions in (B11), (B12) and (B13) are all positive. This is true if

$$t < \frac{7}{4}\overline{p} - \frac{11}{4}c. \tag{B14}$$

Thus, if t is sufficiently low relative to $(\overline{p} - c)$, then $\partial\overline{q}/\partial\overline{p} > 0$ for all $k > \underline{k}$. This has been shown for $s = 0$ but, by continuity, the result also holds for s sufficiently close to zero.

Consider next the case of $s = 1$. Following the same logic as for the case of $s = 0$, we derive

$$\frac{\partial^4 A}{\partial k^4} = 6144t^3 (12(\overline{p} - c) + 37t) > 0, \tag{B15}$$

$$\left.\frac{\partial^3 A}{\partial k^3}\right|_{k=\underline{k}} = 864t^2\beta^2 (404(\overline{p} - c) + 41t) > 0, \tag{B16}$$

$$\left.\frac{\partial^2 A}{\partial k^2}\right|_{k=\underline{k}} = 48t\beta^4 (6918(\overline{p} - c) - 1603t), \tag{B17}$$

$$\left.\frac{\partial A}{\partial k}\right|_{k=\underline{k}} = 45\beta^6 (3701(\overline{p} - c) - 1212t) \tag{B18}$$

and

$$A|_{k=\underline{k}} = \frac{675\beta^8 (169(\overline{p} - c) - 60t)}{2t}. \tag{B19}$$

It follows that $A > 0$ for all $k \geq \underline{k}$ if the expressions in (B17), (B18) and (B19) are all positive. This is true if

$$t < \frac{169}{60}(\overline{p} - c). \tag{B20}$$

Thus, if t is sufficiently low relative to $(\overline{p} - c)$, then $\partial\overline{q}/\partial\overline{p} > 0$ for all $k > \underline{k}$. This has been shown for $s = 1$ but, by continuity, the result also holds for s sufficiently close to one.

(ii) Evaluating A at the lower bound $k = \underline{k}$ yields

$$A|_{k=\underline{k}} = \frac{675\beta^8 (49\overline{p} - (77 + 92s)c + 4(8s + 7)(2\overline{p}s - t))}{2t}. \tag{B21}$$

The sign of this expression depends on the sign of the numerator, which is monotonically decreasing in t. Since t is unbounded from above, the numerator is negative if t is sufficiently large. Thus, at $k = \underline{k}$, $\partial\overline{q}/\partial\overline{p} < 0$ for all $s \in [0, 1]$ if t is sufficiently high. By continuity, this result holds also for

k sufficiently close to $\underline{k}$. Furthermore, we also see that the numerator in (B21) is monotonically increasing in $\overline{p}$ and monotonically decreasing in c. Setting $\overline{p}$ at the lower bound, i.e., $\overline{p} = c$, we derive

$$A|_{k=\underline{k};\overline{p}=c} = -\frac{1350\beta^8\left[(16s+7)(1-s)c+(7+8s)t\right]}{t} < 0. \tag{B22}$$

Thus, at $k = \underline{k}$, $\partial\overline{q}/\partial\overline{p} < 0$ for all $s \in [0,1]$ if $(\overline{p} - c)$ is sufficiently small. Again, by continuity, this result holds also for k sufficiently close to $\underline{k}$. *Q.E.D.*

Proof of Proposition 3

From (27)-(29), we derive

$$\frac{\partial q_1^*}{\partial s} = -\frac{2\overline{p}\beta\left(\beta^2+kt\right)}{6\beta^4+kt\left(16kt-23\beta^2\right)} < 0, \tag{B23}$$

$$\frac{\partial q_2^*}{\partial s} = 0, \tag{B24}$$

$$\frac{\partial q_3^*}{\partial s} = \overline{p}\beta\frac{8kt-7\beta^2}{6\beta^4+kt\left(16kt-23\beta^2\right)} > 0, \tag{B25}$$

It is straightforward to verify the unambiguous signs of these expressions for all $k \geq \underline{k}$. *Q.E.D.*

Proof of Proposition 4

Using (32), we derive

$$\frac{\partial\overline{q}}{\partial s} = \frac{\beta\overline{p}\left(3\overline{p}\Upsilon+\chi\right)}{12t\left(6\beta^4+kt\left(16kt-23\beta^2\right)\right)^2}, \tag{B26}$$

where

$$\Upsilon := -7\left(2k^2t^2\left(16kt-41\beta^2\right)+39\beta^6\right)+24s\left(4kt\left(4\beta^4+\left(2kt-3\beta^2\right)3kt\right)+\beta^6\right) \tag{B27}$$

and

$$\chi := 4t\left(18\beta^6+kt\left(147\beta^4+\left(8kt-115\beta^2\right)2kt\right)\right)-3c\left(3\beta^4\left(128kt-83\beta^2\right)+2k^2t^2\left(176kt-145\beta^2\right)\right) \tag{B28}$$

Define $E := 3\overline{p}\Upsilon + \chi$. From (B26) it is clear that the sign of $\partial\overline{q}/\partial s$ is given by the sign of E. Notice that E is monotonically increasing in s for all $k > \underline{k}$, which implies that $\overline{q}$ is a convex function of

s. Consider first the case of $s = 0$. In this case, we derive

$$\frac{\partial^3 E}{\partial k^3} = -192t^3\left(21p + 33c - 2t\right). \tag{B29}$$

We see that $\partial^3 E/\partial k^3 < 0$ if t is sufficiently low, which in turn implies that $\partial^2 E/\partial k^2$ is monotonically decreasing in k. Evaluating at the lower bound of k, we derive

$$\frac{\partial^2 E}{\partial k^2}|_{k=\underline{k}} = -4t^2\beta^2\left(1941c + 651\overline{p} + 316t\right) < 0, \tag{B30}$$

which implies that $\partial E/\partial k$ is decreasing in k. Furthermore,

$$\frac{\partial E}{\partial k}|_{k=\underline{k}} = -30t\beta^4\left(189c - 21\overline{p} + 58t\right), \tag{B31}$$

and

$$E|_{k=\underline{k}} = -\frac{225}{2}\beta^6\left(23c - 7\overline{p} + 8t\right). \tag{B32}$$

It follows that $E < 0$, and thus $\partial\overline{q}/\partial s < 0$, for all $k > \underline{k}$, if the expressions in (B29), (B31) and (B32) are all negative. This is true if

$$\overline{p} < \frac{23}{7}c + \frac{8}{7}t. \tag{B33}$$

Consider next the case of $s = 1$. Following the same logic as for the case of $s = 0$, we derive

$$\frac{\partial^3 E}{\partial k^3} = 192t^3\left(33(\overline{p} - c) + 2t\right) > 0, \tag{B34}$$

$$\frac{\partial^2 E}{\partial k^2}|_{k=\underline{k}} = -4t^2\beta^2\left(1941c - 1941p + 316t\right), \tag{B35}$$

$$\frac{\partial E}{\partial k}|_{k=\underline{k}} = -30t\beta^4\left(189c - 189\overline{p} + 58t\right) \tag{B36}$$

and

$$E|_{k=\underline{k}} = -\frac{225}{2}\beta^6 \left(23c - 23\overline{p} + 8t\right) \tag{B37}$$

It follows that $E > 0$, and thus $\partial\overline{q}/\partial s > 0$, for all $k > \underline{k}$, if the expressions in (B35)-(B37) are all positive. This is true if

$$\overline{p} > c + \frac{8}{23}t. \tag{B38}$$

Since E is monotonically increasing in s, we can conclude that, if $\overline{p}$ is neither very low nor very high, or more precisely, if

$$c + \frac{8}{23}t < \overline{p} < \frac{23}{7}c + \frac{8}{7}t, \tag{B39}$$

there exists a threshold value of s which lies strictly between 0 and 1, such that $\partial\overline{q}/\partial s < (>)\, 0$ if s is below (above) this threshold value. *Q.E.D.*

Proof of Proposition 6

From (27)-(29), we derive

$$\frac{\partial q_1^*}{\partial t} = -\frac{\beta\Phi}{8kt^2\left(6\beta^4 + \left(16kt - 23\beta^2\right)kt\right)^2}, \tag{B40}$$

where

$$\begin{aligned}\Phi \;:\; &= -\overline{p}\left(21\beta^2\left(3k^2t^2\left(32kt - 55\beta^2\right) + 4\beta^4\left(23kt - 3\beta^2\right)\right) + 16k^2st^2\left(16k^2t^2 + \beta^2\left(32kt - 29\beta^2\right)\right)\right) \\ &+c\left(256k^4t^4 + 84\beta^6\left(23kt - 3\beta^2\right) + k^2t^2\beta^2\left(2528kt - 3929\beta^2\right)\right) \\ &+16kt^2\beta^2\left(kt\left(37kt - 36\beta^2\right) + 12\beta^4\right),\end{aligned} \tag{B41}$$

$$\frac{\partial q_2^*}{\partial t} = -\frac{7\beta\left(\overline{p} - c\right)}{8kt^2} < 0, \tag{B42}$$

$$\frac{\partial q_3^*}{\partial t} = \frac{\beta\Psi}{6kt^2\left(6\beta^4 + \left(16kt - 23\beta^2\right)kt\right)^2}, \tag{B43}$$

where

$$\Psi \;:\; = -3\overline{p}\left(63\beta^8 + 665k^2t^2\beta^4 - 224k^3t^3\beta^2 - 483kt\beta^6 + 256k^4st^4 + 226k^2st^2\beta^4 - 448k^3st^3\beta^2\right)$$
$$+3c\left(k^2t^2\left(891\beta^4 + 32kt\left(8kt - 21\beta^2\right)\right) - 21\beta^6\left(23kt - 3\beta^2\right)\right) \tag{B44}$$
$$+16kt^2\beta^2\left(\left(12\beta^2 + kt\right)kt - 9\beta^4\right)$$

(i) From (B40) we see that the sign of $\partial q_1^*/\partial t$ is the opposite of the sign of Φ. It is easily verified that Φ is monotonically decreasing in $\overline{p}$. Evaluating Φ at the lower bound of the regulated price, $\overline{p} = c$, yields

$$\Phi|_{\overline{p}=c} = 16kt^2\left(\beta^2\left(12\beta^4 + kt\left(37kt - 36\beta^2\right)\right) + ck\left(1-s\right)\left(\beta^2\left(32kt - 29\beta^2\right) + 16k^2t^2\right)\right) > 0. \tag{B45}$$

Since $\overline{p}$ is unbounded from above and Φ is monotonically decreasing in $\overline{p}$, it follows that Φ changes sign from positive to negative if $\overline{p}$ exceeds some threshold level. Thus, $\partial q_1^*/\partial t < (>)\, 0$ if $\overline{p}$ is sufficiently low (high).

(ii) The negative sign of (B42) is trivial.

(iii) From (B43) we see that the sign of $\partial q_3^*/\partial t$ is given by the sign of Ψ. It is also easy to verify that Ψ is monotonically decreasing in s:

$$\frac{\partial \Psi}{\partial s} = -6k^2\overline{p}t^2\left(113\beta^4 + \left(4kt - 7\beta^2\right)32kt\right) < 0. \tag{B46}$$

Setting s at the lower bound, $s = 0$, we derive

$$\frac{\partial^4 \Psi}{\partial k^4} = 18\,432ct^4 > 0, \tag{B47}$$

implying that $\partial^3\Psi/\partial k^3$ is monotonically increasing in k. Evaluated at the lower bound of k, we have

$$\left.\frac{\partial^3 \Psi}{\partial k^3}\right|_{k=\underline{k}} = 96t^3\beta^2\left(42\overline{p} + 162c + t\right) > 0. \tag{B48}$$

Following the same logic, we also derive

$$\left.\frac{\partial^2 \Psi}{\partial k^2}\right|_{k=\underline{k}} = 6t^2\beta^4\left(343\overline{p} + 1323c + 88t\right) > 0 \tag{B49}$$

and

$$\left.\frac{\partial \Psi}{\partial k}\right|_{k=\underline{k}} = 90t\beta^6 (37c + 6t) > 0, \tag{B50}$$

implying that Ψ is monotonically increasing in k. Evaluating Ψ at the lower bound of k yields

$$\Psi|_{k=\underline{k}} = \frac{135}{4}\beta^8 (33c - 7\overline{p} + 8t) > 0 \text{ if } \overline{p} < \frac{33}{7}c + \frac{8}{7}t. \tag{B51}$$

Thus, if $s = 0$ and $\overline{p} < (33c + 8t)/7$, $\Psi > 0$ for all $k \geq \underline{k}$.

Now setting s at the upper bound, $s = 1$, and using the same logic as for the case of $s = 0$, we derive

$$\frac{\partial^4 \Psi}{\partial k^4} = -18\,432t^4 (\overline{p} - c) < 0, \tag{B52}$$

$$\left.\frac{\partial^3 \Psi}{\partial k^3}\right|_{k=\underline{k}} = -96t^3\beta^2 (162 (\overline{p} - c) - t), \tag{B53}$$

$$\left.\frac{\partial^2 \Psi}{\partial k^2}\right|_{k=\underline{k}} = -6t^2\beta^4 (1323 (\overline{p} - c) - 88t), \tag{B54}$$

$$\left.\frac{\partial \Psi}{\partial k}\right|_{k=\underline{k}} = -90t\beta^6 (37 (\overline{p} - c) - 6t), \tag{B55}$$

$$\Psi|_{k=\underline{k}} = -\frac{135}{4}\beta^8 (33 (\overline{p} - c) - 8t) \tag{B56}$$

It follows that $\Psi < 0$ for $s = 1$ and all $k \geq \underline{k}$ if the expressions in (B53)-(B56) are all negative. This is true if

$$\overline{p} > c + \frac{8}{33}t. \tag{B57}$$

Since Ψ is monotonically decreasing in s, we can conclude that, if $\overline{p}$ is neither very low nor very high, or more precisely, if

$$c + \frac{8}{33}t < \overline{p} < \frac{33}{7}c + \frac{8}{7}t, \tag{B58}$$

there exists a threshold value of s which lies strictly between 0 and 1, such that $\partial q_3^*/\partial t > (<)\, 0$ if s is below (above) this threshold value. *Q.E.D.*

Proof of Proposition 7

From (36), we derive

$$\frac{\partial \overline{p}(s)}{\partial s}=\frac{8kt\left(\beta^{2}+16kt\right)\left(kt-\beta^{2}\right)\varsigma}{3\Delta^{2}}, \tag{B59}$$

where Δ is defined by (38) and where

$$\begin{aligned}\varsigma \ : \ &= 3136t\beta^{2}\left(kt-\beta^{2}\right)\left(2kt-3\beta^{2}\right)\left(3\beta^{4}+\left(16kt-15\beta^{2}\right)kt\right)\\ &-128st\left(2kt-\beta^{2}\right)\begin{pmatrix} 8k^{2}st^{2}\left(kt-\beta^{2}\right)\left(\beta^{2}+16kt\right)\\ +84\beta^{6}\left(17kt-3\beta^{2}\right)+7k^{2}t^{2}\beta^{2}\left(144kt-311\beta^{2}\right)\end{pmatrix}\\ &-3c\begin{pmatrix} 49\beta^{2}\left(8k^{2}t^{2}\beta^{2}\left(864kt-995\beta^{2}\right)+5\beta^{6}\left(727kt-108\beta^{2}\right)-2048k^{4}t^{4}\right)\\ +16s\left(2kt-\beta^{2}\right)\begin{pmatrix} 7\beta^{2}\left(16k^{2}t^{2}\left(64kt-137\beta^{2}\right)+\beta^{4}\left(1427kt-252\beta^{2}\right)\right)\\ +4kst\left(kt-\beta^{2}\right)\left(16kt-\beta^{2}\right)\left(\beta^{2}+16kt\right)\end{pmatrix}\end{pmatrix}\end{aligned} \tag{B60}$$

The sign of $\partial\overline{p}(s)/\partial s$ depends on the sign of ς. Taking the fourth-order derivative of ς with respect to k yields

$$\frac{\partial^{4}\varsigma}{\partial k^{4}}=-49\,152t^{4}\left(80ks^{2}t\left(3c+t\right)-t\beta^{2}\left(49-126s+23s^{2}\right)-3c\beta^{2}\left(49-112s+24s^{2}\right)\right). \tag{B61}$$

It is easy to verify that this expression is positive (negative) if s is sufficiently low (high). Let us first consider that case of $s=0$, which implies that $\partial^{4}\varsigma/\partial k^{4}>0$, thus implying that $\partial^{3}\varsigma/\partial k^{3}$ is monotonically increasing in k. By evaluating the subsequent expressions at the lower bound of k, we derive

$$\frac{\partial^{3}\varsigma}{\partial k^{3}}|_{k=\underline{k},s=0}=37\,632t^{3}\beta^{4}\left(126c+41t\right)>0, \tag{B62}$$

$$\frac{\partial^{2}\varsigma}{\partial k^{2}}|_{k=\underline{k},s=0}=2352t^{2}\beta^{6}\left(563c+176t\right)>0, \tag{B63}$$

$$\frac{\partial\varsigma}{\partial k}|_{k=\underline{k},s=0}=147t\beta^{8}\left(1237c+352t\right)>0, \tag{B64}$$

$$\varsigma|_{k=\underline{k},s=0}=\frac{11025c\beta^{10}}{2}>0. \tag{B65}$$

Thus, we conclude that $\varsigma>0$, and thus, $\partial\overline{p}(s)/\partial s>0$, for all $k>\underline{k}$, if $s=0$. By continuity, this result also applies for values of s sufficiently close to zero.

Next, consider the case of $s=1$, which implies that $\partial^{4}\varsigma/\partial k^{4}<0$, thus implying that $\partial^{3}\varsigma/\partial k^{3}$ is monotonically decreasing in k. By evaluating the subsequent expressions at the lower bound of

k, we derive

$$\frac{\partial^3 \varsigma}{\partial k^3}|_{k=\underline{k},s=1} = -768t^3\beta^4 (22\,635c + 8381t) < 0, \tag{B66}$$

$$\frac{\partial^2 \varsigma}{\partial k^2}|_{k=\underline{k},s=1} = -48t^2\beta^6 (270\,901c + 100\,112t) < 0, \tag{B67}$$

$$\frac{\partial \varsigma}{\partial k}|_{k=\underline{k},s=1} = -3t\beta^8 (702\,827c + 256\,992t) < 0, \tag{B68}$$

$$\varsigma|_{k=\underline{k},s=1} = -\frac{225}{2}\beta^{10} (6351c + 1408t) < 0. \tag{B69}$$

Thus, we conclude that $\varsigma < 0$, and thus, $\partial\overline{p}(s)/\partial s < 0$, for all $k > \underline{k}$, if $s = 1$. By continuity, this result also applies for values of s sufficiently close to one. *Q.E.D*

Proof of Proposition 10

A comparison of the equilibrium expressions in (42)-(43) with the corresponding expressions in (27)-(29) yields:

$$q_1^{PF} - q_1^* = \frac{\beta\Xi}{24kt\left(kt-\beta^2\right)\left(kt\left(16kt-23\beta^2\right)+6\beta^4\right)}, \tag{B70}$$

where

$$\begin{aligned}\Xi \quad : \quad &= 3kt\beta^4(79\overline{p}-63c) - 48kt\left(k^2t^2(c-s\overline{p}) + \beta^4(t+s\overline{p})\right) \\ &-3\beta^2\left(6\beta^4+65k^2t^2\right)(\overline{p}-c) - 8k^2t^3\left(2kt-7\beta^2\right),\end{aligned} \tag{B71}$$

$$q_2^{PF} - q_2^* = \frac{\beta(\overline{p}-c)}{8kt} > 0, \tag{B72}$$

$$q_3^{PF} - q_3^* = \frac{\beta\rho}{12kt\left(kt\left(16kt-23\beta^2\right)+6\beta^4\right)}, \tag{B73}$$

where

$$\begin{aligned}\rho \quad = \quad &: 3\overline{p}\left(3\beta^4 + \left(32kt-39\beta^2\right)2kt\right) - 16kt^2\left(2kt-3\beta^2\right) \\ &-3c\left(2kt-3\beta^2\right)\left(16kt-\beta^2\right) - 12k\overline{p}st\left(8kt-7\beta^2\right).\end{aligned} \tag{B74}$$

(i) The sign of (B70) depends on the sign of Ξ . Taking the third-order derivative of Ξ with respect to k yields

$$\frac{\partial^3 \Xi}{\partial k^3} = -96t^3 \left(3\left(c - s\overline{p}\right) + t\right). \tag{B75}$$

This expression is negative if s is sufficiently low, which in turn implies that $\partial^2\Xi/\partial k^2$ is monotonically decreasing in k. By evaluating the subsequent expressions at the lower bound of k, we derive

$$\frac{\partial^2 \Xi}{\partial k^2}|_{k=\underline{k}} = -2t^2\beta^2 \left(3\overline{p}\left(65 - 72s\right) + 21c + 16t\right), \tag{B76}$$

$$\frac{\partial \Xi}{\partial k}|_{k=\underline{k}} = -12t\beta^4 \left(\overline{p}\left(29 - 23s\right) - 6c - t\right), \tag{B77}$$

$$\Xi|_{k=\underline{k}} = -\frac{45}{4}\beta^6 \left(\overline{p}\left(9 - 8s\right) - c\right). \tag{B78}$$

It follows that $\Xi < 0$, and thus $q_1^{PF} < q_1^*$, for all $k > \underline{k}$ if the expressions in (B76)-(B78) are all negative. It is straightforward to verify that (B78) is negative for all $s \in (0,1)$ while (B76) is negative if s is sufficiently low. For (B77) to be negative, we need the additional condition that $\overline{p} - c$ is sufficiently high relative to t.

(ii) The positive sign of (B72) is trivial.

(iii) The sign of (B73) is given by the sign of ρ. Taking the second-order derivative of ρ with respect to k yields

$$\frac{\partial^2 \rho}{\partial k^2} = 64t^2 \left(3\overline{p}\left(2 - s\right) - 3c - t\right). \tag{B79}$$

It is easy to verify that the expression in (B79) is positive if $\overline{p} - c$ is sufficiently high relative to t, which in turn implies that $\partial\rho/\partial k$ is monotonically increasing in k. By evaluating the subsequent expressions at the lower bound of k, we derive

$$\frac{\partial \rho}{\partial k}|_{k=\underline{k}} = 6t\beta^2 \left(57\overline{p} - 34s\overline{p} - 23c - 8t\right), \tag{B80}$$

$$\rho|_{k=\underline{k}} = 90\overline{p}\beta^4 \left(1 - s\right). \tag{B81}$$

Both of these expressions are positive, implying that $\rho > 0$ and thus $q_3^{PF} > q_3^*$ for all $k > \underline{k}$, if $\overline{p} - c$ is sufficiently high relative to t. *Q.E.D*

Proof of Proposition 11

A comparison of (44)-(45) with (27)-(29) yields:

$$q_1^{NFP} - q_1^* = \frac{\beta\xi}{216kt\left(5kt-6\beta^2\right)\left(6\beta^4+\left(16kt-23\beta^2\right)kt\right)}, \tag{B82}$$

where

$$\begin{aligned}
\xi \quad = \quad & : 567\overline{p}\beta^2\left(3kt-2\beta^2\right)\left(5kt-6\beta^2\right) \\
& -16s\overline{p}\left(\left(105kt-94\beta^2\right)k^2t^2-\left(5kt-6\beta^2\right)14\beta^4\right) \\
& +8t\left(2kt-3\beta^2\right)\left(84\beta^4+\left(7kt-32\beta^2\right)5kt\right) \\
& +c\left(\left(527kt-195\beta^2\right)28\beta^4+\left(1680kt-10\,009\beta^2\right)k^2t^2\right),
\end{aligned} \tag{B83}$$

$$q_2^{NPF} - q_2^* = 7\beta\frac{16t\left(2kt-3\beta^2\right)+16\overline{p}s\left(3kt-2\beta^2\right)-27\overline{p}\left(5kt-6\beta^2\right)-c\left(130\beta^2-87kt\right)}{216kt\left(5kt-6\beta^2\right)}, \tag{B84}$$

$$q_3^{NPF} - q_3^* = \frac{\beta\varrho}{108kt\left(5kt-6\beta^2\right)\left(6\beta^4+\left(16kt-23\beta^2\right)kt\right)}, \tag{B85}$$

where

$$\begin{aligned}
\varrho \quad = \quad & : 189\overline{p}\beta^2\left(5kt-6\beta^2\right)\left(2kt-3\beta^2\right)+8t\left(2kt-3\beta^2\right)\left(42\beta^4+\left(22kt-53\beta^2\right)kt\right) \\
& +c\left(\left(173kt-78\beta^2\right)35\beta^4+\left(816kt-2599\beta^2\right)2k^2t^2\right) \\
& -4\overline{p}s\left(\left(12\beta^2+17kt\right)14\beta^4+\left(408kt-827\beta^2\right)k^2t^2\right).
\end{aligned} \tag{B86}$$

(i) The sign of (B82) depends on the sign of ξ . Taking the third-order derivative of ξ with respect to k yields

$$\frac{\partial^3\xi}{\partial k^3} = 3360t^3\left(3(c-s\overline{p})+t\right). \tag{B87}$$

This expression if positive, implying that $\partial^2\xi/\partial k^2$ is monotonically increasing in k, if s is sufficiently low. Evaluating at the lower bound k, we derive

$$\frac{\partial^2\xi}{\partial k^2}|_{k=\underline{k}} = 2t^2\beta^2\left(\overline{p}\left(8505-6056s\right)-2449c-880t\right), \tag{B88}$$

$$\frac{\partial\xi}{\partial k}|_{k=\underline{k}} = t\beta^4\left(\overline{p}\left(9639-5708s\right)-3931c-1236t\right), \tag{B89}$$

$$\xi|_{k=\underline{k}} = \frac{15}{4}\beta^6 \left(\overline{p}\left(567 - 520s\right) - 47c\right). \quad \text{(B90)}$$

The signs of (B88)-(B90) are all positive if $\overline{p} - c$ is sufficiently large relative to t. It follows that $\xi > 0$ and thus $q_1^{NFP} > q_1^*$, for all $k > \underline{k}$, if s is sufficiently low and $\overline{p} - c$ is sufficiently large relative to t.

(ii) The sign of (B84) depends on the sign of the numerator, which we define as

$$F := -27\overline{p}\left(5kt - 6\beta^2\right) - c\left(130\beta^2 - 87kt\right) + 16t\left(2kt - 3\beta^2\right) + 16\overline{p}s\left(3kt - 2\beta^2\right), \quad \text{(B91)}$$

and from which we derive

$$\frac{\partial F}{\partial k} = -t\left(3\overline{p}\left(45 - 16s\right) - 87c - 32t\right). \quad \text{(B92)}$$

It is easily confirmed that the sign of (B92) is negative if s is sufficiently low and $\overline{p} - c$ is sufficiently large relative to t, implying that F is monotonically decreasing in k. Evaluating F at the lower bound of k yields

$$F|_{k=\underline{k}} = -\frac{1}{2}\beta^2\left(\overline{p}\left(81 - 80s\right) - c\right) < 0. \quad \text{(B93)}$$

It follows that $F < 0$ and thus $q_2^{NPF} < q_2^*$, for all $k > \underline{k}$, if s is sufficiently low and $\overline{p} - c$ is sufficiently large relative to t.

(iii) The sign of (B85) depends on the sign of ϱ. Taking the third-order derivative of ϱ with respect to k yields

$$\frac{\partial^3 \varrho}{\partial k^3} = 192t^3\left(51\left(c - s\overline{p}\right) + 11t\right). \quad \text{(B94)}$$

For a sufficiently low value of s, this expression is positive, which implies that $\partial^2\varrho/\partial k^2$ is monotonically increasing in k. Evaluating the subsequent expressions at the lower bound of k yields

$$\frac{\partial^2 \varrho}{\partial k^2}|_{k=\underline{k}} = 4t^2\beta^2\left(1073c + 945p + 104t - 2018s\overline{p}\right), \quad \text{(B95)}$$

$$\frac{\partial \varrho}{\partial k}|_{k=\underline{k}} = t\beta^4\left(1477c + 567p + 192t - 2044s\overline{p}\right), \quad \text{(B96)}$$

$$\varrho|_{k=\underline{k}} = 165\beta^6\left(c - s\overline{p}\right). \quad \text{(B97)}$$

It is straightforward to see that (B95)-(B97) are all positive if s is sufficiently low. In this case, it follows that $\varrho > 0$ and thus $q_3^{NPF} > q_3^*$ for all $k > \underline{k}$. *Q.E.D*

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