Universidade do Minho
Escola de Economia e Gestão

João André Cimbron Cabral Mendes Jerónimo

On the interaction between financial constraints and economic growth

Dissertação de Mestrado

Economia Monetária, Bancária e Financeira

Trabalho efetuado sob a orientação de

Professora Maria João Cabral Almeida Ribeiro Thompson

Professor José Assis Ribeiro de Azevedo

outubro 2019

DECLARAÇÃO DE INTEGRIDADE

Declaro ter atuado com integridade na elaboração do presente trabalho académico e confirmo que não recorri à prática de plágio nem a qualquer forma de utilização indevida ou falsificação de informações ou resultados em nenhuma das etapas conducente à sua elaboração. Mais declaro que conheço e que respeitei o Código de Conduta Ética da Universidade do Minho.

Universidade do Minho, 28 de outubro de 2019

Nome completo: João André Pimbreu Cabral Mendes Jerónimo

Assinatura: João Jerónimo

Resumo

Fricções financeiras constituem imperfeições nos mercados financeiros – por exemplo, mercados de crédito e de derivados - e, portanto, desvios dos quadros teóricos vigentes. Estas imperfeições constituem toda e qualquer situação passível de colocar um ou mais agentes económicos numa posição de vantagem comparativa em relação aos demais. Um exemplo recorrente de um tipo de fricções financeiras são as assimetrias de informação (Fu, 1996), que se podem manifestar de várias maneiras e, de acordo com a literatura empírica, ter vários tipos de repercussões. Com vista a contribuir para o corpo de literatura a ser produzido nesta área de investigação, e pela pertinência do tema à data da escrita desta dissertação, pretende-se construir um modelo teórico em que se estabeleça uma relação, na solução de equilíbrio da economia, entre um tipo específico de assimetrias de informação e o crescimento económico de longo prazo. No modelo proposto, o setor de bens de capitais opera num mercado monopolista, em que existem incentivos ao endividamento e posterior aquisição de patentes tecnológicas. Pretende-se ainda verificar a validade empírica do quadro teórico, com auxílio de estimadores Arellano-Bond (1998), aplicados em painéis dinâmicos.

Palavras-chave: fricções financeiras, mercados de crédito, assimetrias, crescimento económico.

Abstract

Financial frictions are imperfections observed in financial markets – e.g., credit and derivatives' markets – and, therefore, deviations from the existing theoretical frameworks. These imperfections comprise every situation capable of putting some economic agent(s) in a position of advantage, relative to another(s). One common example of a type of financial frictions are informational asymmetries (Fu, 1996), that can express themselves in several way and, according to the empirical literature on the subject, have different kinds of consequences. With the purpose of contribute to the body of literature being produced in this research area, and by the relevance of this topic at the time of writing, we want to build a theoretical model in which we establish a relation between a particular kind of informational asymmetries and long term economic growth. In our proposed model, the capital goods' sector operates in monopolistic competition, where there are incentives to indebtedness and consequent patents' acquisition. We finally verify the empirical validity of our proposed theoretical framework with Arellano-Bond estimators (1998) applied to dynamic panels.

Keywords: financial frictions, credit markets, asymmetries, economic growth.

Content

DECLARAÇÃO DE INTEGRIDADE 3
Resumo 4
Abstract 5
1. Introduction 7
2. Literature Review 9
2.1. Romer's model 10
2.2. On the financial sector: some relevant literature 20
2.3. BGG's model 22
2.4. Some final considerations 26
3. Extending Romer's model: an endogenous growth model with financial frictions 28
3.1. On the incompatibilities between the existing frameworks 28
3.2. The model 29
3.2.1. Balanced Growth Path Solution 37
3.3. Interaction between informational asymmetries and economic growth 40
3.4. Some final considerations 43
4. Testing the model: an empirical analysis 44
4.1. Data and Econometric Model 44
4.2. Interpretation and Discussion 51
5. Final Remarks 56
References 58
Appendix A 62
Appendix B 66

1. Introduction

With the latest financial crisis came the realization that finance's impact on the real economy is definitely not innocuous. Not only have financial markets become greatly important for most developed countries, moving trillions of dollars on a daily basis, but they have also come to connect virtually all countries through increasingly complex and sophisticated instruments, thereby increasing the forms of liquidity available in the economy. Although these markets may present themselves as alternative sources of financing for traditionally bank-based economies such as the Eurozone, the ever-growing global interconnectedness carries some concerning drawbacks, such as important resource misallocations during the expansion phase of a financial cycle, and financial hyper-sensitivity due to increased systemic risk.

While one part of the existing literature already considers fluctuations in asset prices, credit and capital flows as some key factors of disturbance upon real macroeconomic variables (Claessens & Kose, 2018), another part does not acknowledge the role of financial frictions in the transmission of shocks through, e.g., asset prices, net worth, interest rates and/or monetary channels (Christiano, Eichenbaum, & Evans, 2005). The inexistence of consensus regarding the role of financial frictions in the real economy makes it hard to grasp and interpret in depth the whole economic reality. Reflecting this "academic and empirical controversy regarding the importance of financial channels" in the real economy (Gerke et al, 2012), most economic growth models do not contemplate financial frictions.

Jokivuolle and Tunaru (2017, pp. 1-7) do point out that every financial-economic crisis seems to be different from its predecessor, with no apparent regularity, although some specific types of asset, such as the real estate, appear to be more sensible to financial fluctuations. Nevertheless, crises can be interpreted, to some extent, as extreme manifestations of the existing relationship between the financial sector and the real economy (Claessens & Kose, 2013), which makes a strong call on economists to endeavor for a better understanding of macro-financial interactions. Plenty academic and policy literature has been produced in recent years on this topic. Even though there are still many questions unanswered, it is also true that some common denominators to financial dynamics have already been found. This claim is supported by the convergence of some key parameters used in policy models from institutions such as the Bundesbank, the European Central Bank, Banca d'Italia, Sveriges Riksbank and the National Bank of Poland (Gerke, et al., 2013).

It follows that Blanchard (2018), Borio (2018), Brunnermeier and Sannikov (2017), among others, regard existing theoretical models as no longer fully able to address current macroeconomic phenomena, specifically, that of the role of finance in the real economy. Notwithstanding, some notable advances have been made in recent years on this topic. Our work heavily relied on some significant studies such as Kiyotaki and More (1997) and, to a larger extent, Bernanke et al (1999). Wishing to contribute to this call for theoretical macroeconomic developments, we propose a growth model that contemplates financial frictions. This study explores the interactions between real growth variables and a particular kind of financial frictions that arises from informational asymmetries, within one economy, over an infinite horizon. Starting with a cost state verification problem, similarly to Morales (2003), we find the existence of a limit at which the existence of financial frictions appears to be irrelevant for economic growth, while the impact outside the neighborhood of this limit can either be negative or positive, depending on the possible marginal relations between an elasticity measure and growth.

We proceed as follows. In Section 2, we explain some of the literature's and our model's underlying roots and mechanics, with special emphasis on Romer's model of endogenous growth (Evans, Honkapohja, & Romer, 1998), and the modelling of some types of financial frictions, with special emphasis on Bernanke, Gertler and Gilchrist (1999), hereafter referred as BGG. In particular, the choice of BGG as one of the underlying models has been made due to a perceived consensus in the literature regarding its importance in introducing and explaining financial frictions in dynamic economic modelling. In Section 3 we build our proposed model and proceed to find a balanced growth path solution and its main predictions. Here, we also focus on the main effects of informational asymmetries between the model's agents on economic growth. Throughout section 4 we conduct an empirical analysis with dynamic panel data estimation methods, finishing with some concluding remarks in Section 5.

2. Literature Review

The matter of the determinants of economic growth and aggregate wealth distribution are key issues of macroeconomic studies and attempts to model these phenomena go far back. Solow (1956) developed a growth model based on physical capital accumulation with an exogenous household's rate of savings, showing how long-run growth depends upon technical changes. Later, the Ramsey-Cass-Koopmans model endogenizes the saving rate, by deriving the evolution of capital stock from the interaction between maximizing firms and infinitely-lived households in competitive markets, thereby providing a check on Solow's model, by showing that it survives once the analyst allows for more complex saving behavior. Both models build upon a one sector economy with a production function that takes as inputs $K(t)$ and $A(t)L(t)$, respectively, functions of capital and effective labor, the latter given by the labor function $L(t)$ and an effectiveness function $A(t)$ that proxies technological progress and, more precisely, a continuum of types of knowledge, ranging from the highly abstract to the highly applied. In both models, however, labor growth and technological progress are exogenous.

One could argue that the models that followed these studies were either based on human capital or technological progress as the main long-term growth determinants[1]. The first example of a human capital-based macroeconomic growth model was developed by Lucas (1988). The author assumed a linear specification for the accumulation of human capital, which then allowed for endogenous growth. Earlier studies such as that of Becker (1964) helped the formulation of such theoretical models by drawing important macroeconomic implications from a microeconomic study of the links between education, output and overall economic performance. Despite the existence of a large body of literature on human capital-based growth models, we've chosen to support the endogenous growth on technological progress.

The case for the importance of technological progress – often measured as investment on research and development (R&D) - on economic growth is overwhelming. Perhaps one of the first relevant papers in doing so was that of Solow (1956), whose empirical work had as an important result the fact that accumulation of physical capital alone cannot account for most of the growth over time in the output per person – a very significant finding for the argument against the classical convergence hypothesis. Following works such as Arrow (1962), Phelps (1966) Evans et al (1998), Shell (2010)

[1] A third group of macroeconomic growth models, albeit less representative, eliminates the assumption of diminishing returns to capital in the aggregate production function.

made strong contributions for the current understanding of the role of technological progress in economic growth, suggesting that increases on the level of resources allocated to the R&D sector and consequent invention of new capital goods leads to a continuously increasing growth, through overall enhancements in terms of costs, output and resource allocation. The following subsection explains Romer's model of endogenous growth in detail.

2.1. Romer's model

The model of endogenous growth developed by George Evans, Seppo Honkapohja and Paul Romer (1998) comprises a set of properties that makes it utterly desirable to study and extend to the financial sector. First, the technology here is, to a large extent, endogenous[2]. This allows one to fully analyze the system's dynamics around the steady-state equilibria, regarding this unambiguously important variable. Second, building upon the rational expectations' hypothesis, i.e., perfect-foresight, the model delivers multiplicity of equilibria in the (g, r) space – common in Arrow-Debreu economies. This feature, and its extension by the introduction of expectational indeterminacy, delivers some convenient properties that characterize sequential economies, a framework often used to study the financial sector.

We start with a three-sector economy[3] – final goods', capital or intermediate goods' and R&D sectors. Labor is assumed to be fixed and positive in any moment in time. Furthermore, denoting $L_A(t)$ as the function of labor devoted to R&D, and $L_Y(t)$ the function of labor devoted to the final goods' sector, we'll have that, in any moment in time, the existence of equilibrium in the labor market implies that

$$L_A(t) + L_Y(t) = L(t) \qquad (2.1)$$

The economy faces a Cobb-Douglas-like production function

[2] The authors design an output function of technology dependent on a parameter that reflects the incentives to R&D, which is formally exogenous.
[3] The model's derivation follows the works of Evans et al (1998), Thompson (2003) and Romer (2012).

(2.2)
$$Y(t) := F\big(L, x(.)\big) = L_Y(t)^{1-\alpha}\left(\int_0^{A(t)} x_i(t)\,di\right)^{\alpha}$$

in which $K(t) = \int_0^{A(t)} x_i(t)\,di$, which represents the capital at any given time, for $\alpha \in]0,1[$, $A(t)$, which is assumed to be continuous, represents the range of differentiated capital goods that have already been designed (proxy for technological progress and new ideas and designs), and $x_i(.)$ represents the number of units of capital of type i in use. The model, therefore, presents constant returns to scale, while $x_i(.)$ presents diminishing returns to scale. However, given that A is productive, technological growth compensates for the latter effect. Furthermore, it is assumed that the same production technology is used in all productive activities, i.e., (1) making consumption goods, (2) designs for new types of machines and (3) physical capital machines for types that have already been designed. Hence, the relative price of each output, given by the slope of the economy's production possibility frontier, is linear.

The model also assumes an infinitely lived representative consumer that maximizes its discounted CRRA-like utility of future consumption $C(t)$

(2.3)
$$\int_0^{\infty} e^{-\rho t} u\big(C(t)\big)dt\,, \qquad u\big(C(t)\big) = \frac{C(t)^{1-\sigma}}{1-\sigma}$$

subject to a budget constraint similar with that of the Ramsey-Crass-Koopmans model, where the present value of the lifetime consumption cannot exceed the consumer's initial wealth plus the present value of his lifetime labor income, such that

(2.4)
$$\int_{t=0}^{\infty} e^{-rt} C(t)dt \le X(0) + \int_{t=0}^{\infty} e^{-rt} w(t)dt$$

from which results, by defining and calculating the Hamiltonian function, that the consumer faced with the constant rate of return r will rationally choose to have his consumption growing at the rate given by the Euler equation

(2.5) $$g_C := \frac{\dot{C(t)}}{C(t)} = \frac{1}{\sigma}(r - \rho)$$

When dealing with time-varying production functions, one needs to address the matter of the inputs' accumulation functions, i.e., the inputs' production functions. Therefore, capital accumulation is given by

(2.6) $$\dot{K(t)} = Y(t) - C(t)$$

where the underlying assumption is that the real sector is always in equilibrium, that is, investment equals the private sector's savings. This is an important observation to make, as it will follow us throughout the paper, being particularly relevant when analyzing the short-run capital structure model for the financial sector from Greenwald and Stiglitz (1993). Furthermore, given that the production function for new ideas is linear in the labor devoted to the R&D sector, and proportional to the existing stock of knowledge, its accumulation function is

(2.7) $$\dot{A(t)} = BL_A(t)A(t)$$

where $B > 0$ represents the efficiency of the research activities, and $A(0) > 0$. The linearity of $\dot{A(t)}$ in $A(t)$ implies the existence of a balanced growth path solution[4]. Knowledge interferes in the production in two ways. First, a new design implies the production of a new capital good, which

[4] This is easily seen by solving the ordinary differential equation (2.7) for $A(t)$, from where we get that

$$A(t) = A(0)e^{\int_0^t L_A(\tau)d\tau}$$

will be, as I will soon explain, rented to the final goods' sector. Furthermore, it increases the existing stock of knowledge, which constitutes a *positive*[5] externality, given the *non-rivalry* property of knowledge. This is the first important characteristic of knowledge that conveys important implications for the model. The other one is its *partial excludability* through patent laws, which is fundamental for the set of a monopolistic competition framework in the capital goods' sector. Second, the higher the existing stock of knowledge $A(t)$, the higher the marginal productivity of researchers. This is a trivial consequence of the production function (2.2).

Unlike the capital goods' sector, the final goods' sector finds itself in a perfect competition environment. The latter rents capital goods at a rent $R(t) = \frac{dY(t)}{dx(t)}$ which it pays to the capital goods' sector which, in turn, faces the following supply curve[6],

(2.8) $$R(t) = \alpha {L_Y}^{1-\alpha} x(t)^{\alpha-1}$$

The problem lies, now, on how the capital goods sector, which operates in monopolistic competition, makes its production decisions. Assuming an unrestricted borrowing capacity at a fixed interest rate r, the monopolistic producers maximize their profits

(2.9) $$\pi(t) = R(t)x(t) - rx(t)$$

from where we, after taking the first order conditions, get the monopoly price

(2.10) $$R = \frac{r}{\alpha}$$

[5] This is true because R&D firms are not required to compensate the researchers for past ideas. However, it is neglected by the authors, hence creating a source of non-optimality of the final general-equilibrium solution, as it will be shown ahead.

[6] The model enjoys the *symmetry property*, which means that $x_i(t) = x(t)$, implying that $R_i(t) = R(t)$ and $\pi_i(t) = \pi(t)$.

The background assumption that the developer of an idea has monopoly rights to the use of the idea implies that the developer can charge above the marginal production cost for the use of the idea. This generates profits that create incentives for R&D. The free-entry condition of the R&D sector implies that the present value of future discounted profits of selling the input embodying a new idea must, at least, equal the cost of creating. That is, the price of the initial investment is

(2.11) $$P_A(t) = \int_t^{\infty} e^{-r(\tau-t)}\pi(\tau)d\tau = \frac{w(t)}{BA(t)}$$

where $w(t)$ represents the wage. By applying the Leibniz rule to its time differentiation, we get

(2.12) $$\dot{P}_A(t) = \int_t^{\infty} re^{-r(\tau-t)}\pi(\tau)d\tau + \left[e^{-r(\tau-t)}\pi(\tau)\right]_t^{\infty} = rP_A(t) - \pi(t)$$

which Thompson (2003) interprets as being the trade-off that the companies face between investing the existing endowments at the risk-free return and investing on a patent and therefore opting for the risky monopoly rent returns, at any given time.

When we analyze the model's long run dynamics, we are interested on a balanced growth path solution that, by (2.7), we know to exist. By observing that, on a balanced growth path, the rate of interest r must be constant, we find, through (2.8), that the capital goods production

(2.13) $$x(t) = L_Y(t)\left[\frac{\alpha^2}{r}\right]^{\frac{1}{1-\alpha}}$$

is also constant, giving that $L_Y(t)$ is fixed. This result consequently yields, by some substitutions in (2.2), that K and Y grow at the same rate as A. Therefore, (2.14) $$g_Y = g_K = g_A$$

From which we can, by dividing (7) by $A(t)$, reach the technological progress growth mechanism

(2.15) $$g_A = BL_A(t)$$

which implies that the growth rate of the technological progress depends on the number of researchers at any moment in time (assuming full employment of the production factor L).

The assumption that the labor market is competitive implies that the wages are the same in every sector. Therefore, the intersection between the following equations

(2.16.1) $$w_Y = \frac{dY(t)}{dL_Y(t)} = (1-\alpha)\left[\frac{Ax}{L_Y}\right]^{\alpha}$$

(2.16.2) $$w_A = \frac{d\dot{A}}{dL_A(t)}P_A = BAP_A$$

yields the equilibrium price of investment in R&D

(2.17) $$P_A = \frac{1-\alpha}{B}\left[\frac{Ax}{L_Y}\right]^{\alpha}$$

which implies that $\dot{P}_A = 0$ and, therefore, by (2.12),

(2.18)
$$r = \frac{\pi}{P_A}$$

substituting in by the corresponding equations for the firms' profits $\pi(t)$, (2.8) and (2.17), we get

(2.19)
$$g_A = B\bar{L} - \frac{r}{\alpha}$$

This important result can be extended, by working on the accumulation functions, hence concluding that the aggregate economy's growth rate, under the perfect-foresight hypothesis, is given by

(2.20)
$$g = \frac{\delta\alpha\bar{L} - \rho}{\alpha + \sigma}$$

The multiple equilibria appear when we introduce *expectational indeterminacy* in the model, under which changes in expectations cause the economy to switch between the high and low growth rate states, and when we combine complementarity effects between capital goods with a non-linear trade-off between investment and consumption. The first step is to define a state variable Z

(2.21)
$$Z = aA(t) + K(t)$$

which comprises both physical capital and the patents. Hence,

(2.22)
$$C(t) = y(t) - \dot{Z}(t)$$

Equation (2.22) characterizes a linear trade-off between consumption and investment. In order to introduce non-linearity (and, consequently, variability of the relative price of sectorial outputs), the authors create a convex cost function $\chi(.)$ such that

(2.23)
$$C(t) = y(t) - Z(t)\chi\left(\frac{Z(t+1) - Z(t) - D(t)}{Z(t)}\right)$$

in which $D(t)$ represents any depreciation of the physical capital stock. In order to later reach the BGP solution, the authors introduce the relative price of general capital Z given by[7]

(2.24)
$$P_Z = \chi'(g_Z)$$

When considering equation (2.2), we realize that it implies that the capital goods are additively separable – therefore, independent. In order to introduce complementarity between capital goods, the authors specify

(2.25)
$$y(t) = L^{1-\alpha}\left(\int_0^{A(t)} x(t)^{\gamma}\, di\right)^{\theta}$$

where $\theta\gamma = \alpha$, and $\theta > 1$. If we assume that one unit of Z is required for the production each physical machine, we'll have it distributed between machinery and new designs' investment

[7] We take the limit, by ignoring the depreciation $D(t)$

$$\frac{Z(t+1) - Z(t)}{Z(t)} \approx \frac{\dot{Z}}{Z} \equiv g_Z$$

(26) $$Z(t) = \int_0^{A(t)} x_i(t)di + \int_0^{A(t)} i^{\varepsilon} di = K(t) + \int_0^{A(t)} i^{\varepsilon} di$$

in which i^{ε} represents the units of Z necessary for the production of the i-ith patent for a given capital good. To force the system to converge for a balanced growth path solution, the following restriction is made

(2.27) $$\varepsilon = \frac{\theta - 1}{1 - \alpha}$$

which the authors have obtained through numerical methods. Similar assumptions of those made for solving the model without capital goods' complementarities nor non-linear trade-off between investment and consumption lead to the multi-equilibria solution. The profit maximizing capital goods' sector sets the monopolistic rent equal to their inputs' marginal productivity

(2.28) $$R(t) = \frac{dy(t)}{dx(t)} = \theta\gamma L^{1-\alpha} x(t)^{\gamma-1} \left(\int_0^{A(t)} x(t)^{\gamma} \, di \right)^{\theta-1}$$

Now, the monopolistic firm maximizes it revenues $R(t)x(t)$, with respect to its expenditures $rP_Z x(t)$. The first order condition yields that

(2.29) $$R = \frac{rP_Z}{\gamma}$$

which enables us to, through the model's symmetry properties, solve the problem for its balanced growth path, from where we get

(2.30)
$$g_Z = g_K = g_Y = (1+\varepsilon)g_A$$

The key to determine g_A is to impose the non-transversality condition that arises from the Fisher equation on $A(t)$, such that

(2.31)
$$P_Z A(t)^{\varepsilon} = \int_t^{\infty} e^{-r(\tau-t)}\pi(\tau)d\tau$$

which, by time-differentiating, with P_Z held constant, gives

(2.32)
$$g_A = \frac{1}{\varepsilon}\left(r - \frac{\pi}{P_Z A(t)^{\varepsilon}}\right)$$

We can reduce (2.32) by one degree of uncertainty by transforming the equation so it becomes dependent on the behavioral parameter α. This equation then delivers the new growth curve, given by a more complex form that generalizes (2.20). In any macroeconomics setting, whenever there are multi-equilibria, we have learned to accept that they have different degrees of stability. The concept of stability relates with the idea that, for a given set of initial values for the state variable, the system's dynamic behavior will make it fluctuate between equilibriums. In particular, (2.32) provides us a multiple equilibrium framework, which will fluctuate between the high and low growth solutions, depending on time-specific phenomena and preferences. Therefore, the authors introduced the *stability under learning* criterium, implying that the agents will learn and react to the variables' values at any given time, according to the following rule

(2.33)
$$r(t+1)^e = r(t)^e + \delta(t)(r(t) - r(t)^e), \qquad \delta(t) = \frac{\delta}{t}$$

hence generating a dynamic interest rate that tells us how the equilibria fluctuate. Under this framework, agents base their consumption/investment choices on expected values for the future interest rate, r^e, which will, in turn, affect the aggregate growth rate. This aggregate growth rate is, in turn, considered on the firms' production decisions, which require a sufficient amount of financial aid, which is dependent on the realization of the interest rate. This generates growth cycles, providing the existence of a dynamic rate of interest.

2.2. On the financial sector: some relevant literature

What the previous section fails to contemplate is the possibility of agent-based disturbance factors. These appear in the literature in many ways, such as intermediary rates and capital flows, or any type of information asymmetries. Their effects on financial markets are, however, controversial. Trivedi (2015) finds, by analyzing financial innovation[8] on the Indian banking system, a positive relation between innovation and enhanced bank performance, measured in terms of profitability and stability of the income. Similar conclusions regarding the beneficial effects of innovation are found in Guimaraes et al (2010), for whom it comprises dimensions such as strategic leadership, competitive intelligence, management of technology and some specific characteristics of the bank or firm's change process used to conduct innovation projects. However, Epure and Lafuente (2015), by analyzing the Costa Rican banking sector, argue that, although the general average bank-specific inefficiency has decreased in the pre-2008 period, characterized by increased innovation practices, no such relation was observed during the post-crisis period, during which the average banking specific inefficiency scores from the authors' technology concept, and the average return on assets and net interest margin all remained relatively unchanged. This second period faced a significant increase on the regulatory pressure, therefore jeopardizing innovative impulses and practices, on a global scale. Therefore, at least at a micro level, it may be reasonable to assume that the effects of financial frictions on economic performance aren't all straightforward.

In a frictionless economy, funds can flow to the most profitable project, and such flows are determined by differences in productivity and risk aversion at the micro level, and differences in

[8] The author studies financial innovation as being foremost, but not exclusively, the search for development of new income sources and consequent increases in portfolio diversification, at the bank-level. These include, e.g., non-loan related fees.

capital and labor at the macro level (Brunnermeier, Eisenbach, & Sannikov, 2013). In an economy with financial frictions, however, there is an inherent instability and the agent dynamics are utterly probabilistic, due to the existence of uncertainty. Hall (2013) suggests that such frictions can be thought of as a spread between the returns earned in businesses from the physical capital – plant and equipment – and the returns earned by savers, or the market cost of capital. This could suggest that the uncertainty sources of an economy are non-negligible for the purposes of economic and policy analysis. The accumulation of such frictions prevents funds from flowing to undercapitalized sectors, hence preventing Pareto-optimality, through resource misallocations, and generating unequal growth and development opportunities.

The dynamics of economies with financial frictions are highly non-linear[9] (Brunnermeier & Sannikov, 2017), which is why small shocks can lead to significant economic dislocations[10]. This was the case of the Lehman Brothers' collapse in September 2008, which triggered a self-sustained sequence of balance-sheet defaults that spread worldwide. Brunnermeier and Sannikov point out that, while the financial markets are self-stabilizing in normal times, economies become quite vulnerable to a crisis after a run up of debt imbalances and credit bubbles. Greenwald and Stiglitz (1993) developed a simple model of economic fluctuations, that comprises some of the key issues that characterize the financial sector's short-run dynamics, based upon the kinds of informational imperfections that are chiefly related to adverse selection and moral hazard.

There is a significant number of other models that assume various financing restrictions. Specifically, we can identify two main approaches of financial frictions in macroeconomics. The first approach is due to Kiyotaki and Moore (1997), which focuses on the idea of the inalienability of human capital. The assumption results in the lenders' demand that the value of the borrowers' durable assets must at least equal the value of the outstanding debt. Hence, these assets play a dual role – they are both means of production and serve as collateral for current loans. The second approach is that of BGG (1999). Despite the similarities between the approaches, such as the need of an optimal contract between the lender and the borrower as a solution to an agency problem and the feedback effects from asset markets, they are quite different. Essentially, BGG build upon a debt contract as a solution to a problem of asymmetric information between the lender and the borrower, which endogenously motivates the existence of an external financing premium. Later

[9] The very nature of these dynamics makes the system's behavior inherently probabilistic.

[10] The most important features of such shocks are their *persistence*, which refers to its duration in time, and *amplification*, which refers to its ability to spread and reproduce its effects along a growing number of agents and economies.

models such as that of Morales (2003) build upon the same agency problem as in BGG, albeit in a fashion different from the standard sticky price DSGE framework. The following subsection explains the partial equilibrium framework of BGG, in the context of the authors' log-linearized stochastic model.

2.3. BGG's model

BGG give us two reasons for the introduction of financial frictions in macroeconomic models. First, doing so seems to enhance the models' capacity of explaining cyclical fluctuations, such as credit supply in the economy. Second, a lot of empirical research on the determinants of aggregate supply and demand attributes an important role to financial frictions. Thus, the authors developed a variant of the dynamic new keynesian framework – i.e. stochastic aggregate growth that incorporates monetary policy, monopolistic competition and nominal price rigidity -, modified to allow for financial accelerator effects on investment. Their main purpose is to assess how credit market frictions influence the monetary policy transmission mechanisms.

The model comprises three kinds of agents: households, retailers and entrepreneurs. Similarly to Romer's model, households are assumed to be infinitely lived. Furthermore, they hold the economy's goods, services and interest bearing assets. Retailers are, together with entrepreneurs, market suppliers, and are assumed to work in monopolistic competition in order to introduce price stickiness in the model. This ensures a linear relation between the demand for capital goods and the net worth of this model's most important agent.

Entrepreneurs have, indeed, a central role in BGG's framework. Each single entrepreneur is finitely lived, with a constant probability $P \in [0,1]$ of surviving until the following period. At any given moment t, each entrepreneur buys physical capital, financed by their current net worth and external funds, combined with labor to generate output in $t+1$. The external funds thus finance the difference between input costs and the entrepreneur's net worth, such that

(2.35) $$B_{t+1}^{j} = Q_t K_{t+1}^{j} - N_{t+1}^{j}$$

where B_{t+1}^{j} represents the debt amount that the j-th entrepreneur must incur in at $t+1$, Q_t the price of capital goods at t, K_{t+1}^{j} the amount of capital goods purchased for use at $t+1$ and N_{t+1}^{j} its net worth at the end of t, going to $t+1$.

Building upon Townsend (1979) the authors introduced a cost state verification problem in order to endogenously motivate the existence of an external finance premium, which will depend on the borrower's financial position at $t+1$. The nature of this problem implies that the entrepreneur holds more information than the lender, regarding its future net worth, to the extent that he can observe its capital returns at $t+1$. This means that there's asymmetric information regarding the borrower's ability to repay its loan at $t+1$, resulting in the need of an optimal contract to mitigate the risk of adverse selection and moral hazard. The optimal contract established between the lender and the borrower is characterized by the minimum requirement rule for external financing

$$\bar{w}^{j}\gamma_{t+1}Q_tK_{t+1} = Z_{t+1}^{j}B_{t+1}^{j} \tag{2.36}$$

where Z_{t+1}^{j} represents a gross non-default rate, $\bar{w}^{j}$ the limit value of the idiosyncratic shock w^{j} and γ_{t+1} represents the aggregate risk of the economy. Both w^{j} and γ_{t+1} are stochastic variables. Upon the borrower's net worth realization at $t+1$, one of two situations may occur. If $w^{j} \geq \bar{w}^{j}$, the borrower is able to repay the loan at the rate Z_{t+1}^{j} to the financial intermediary. If, however, $w^{j} < \bar{w}^{j}$, the borrower goes bankrupt. This means that the intermediary incurs in an auditing cost $hw\gamma_{t+1}Q_tK_{t+1}$ and keeps what he finds, while the defaulting entrepreneur receives nothing. Given the finite cardinality of the relevant set of nature states of w^{j} at any given moment, and assuming that the intermediary is able to diversify its portfolio, the loan contract satisfies

$$\left\{[1-F(\bar{w}^{j})]\bar{w}^{j} + (1-h)\int_{0}^{\bar{w}^{j}} w\,dF(w)\right\}\gamma_{t+1}^{k}Q_tK_{t+1}^{j} = r_{t+1}\left(Q_tK_{t+1}^{j} - N_{t+1}^{j}\right) \tag{2.37}$$

where r_{t+1} represents the riskless rate at $t+1$. This way, we are able the express the lender's expect return as a function of the cutoff value of the idiosyncratic shock $\overline{w}^j$ on the entrepreneur's productivity. Once we've determined the value for $\overline{w}^j$, contingent upon the *ex post* realization of the systemic shock γ_{t+1}^k and the *ex ante* choices of $Q_t K_{t+1}^j$ and B_{t+1}^j, we now address the entrepreneur's optimal choice of K_{t+1}^j. By making expectations regarding the realization of γ_{t+1}^k, the entrepreneur maximizes

(2.38)
$$E\left\{\left[1 - h\int_0^{\overline{w}^j} w dF(w)\right] U_{t+1}^{rk}\right\} E\{\gamma_{t+1}^k\} Q_t K_{t+1}^j - r_{t+1}\left(Q_t K_{t+1}^j - N_{t+1}^j\right)$$

subject to (2.37), where $U_{t+1}^{rk} \equiv \frac{\gamma_{t+1}^k}{E\{\gamma_{t+1}^k\}}$. The external finance premium is thus derived from the monitoring costs, which the entrepreneur internalizes due to the assumption that he is risk neutral. The maximization of (2.38) relative to K_{t+1}^j and $\overline{w}^j$ yields the following relation for optimal capital purchases:

(2.39)
$$Q_t K_{t+1}^j = \varphi(s_t) N_{t+1}^j, \qquad with\ \varphi(1) = 1, \varphi'(.) > 0$$

where $s_t \equiv E\left\{\frac{\gamma_{t+1}^k}{r_{t+1}}\right\}$ represents the expected discounted return to capital. This means that the higher the expected capital returns relative to the riskless rate – i.e. the loan repayment's rate minus the premium – the higher the capital investment and, consequently, the higher the indebtedness. There is also a multiplicative effect of the entrepreneur's net worth on capital acquisition. Therefore, the higher the financial stability, the larger the credit and the capital investment. Furthermore, this implicitly means that the higher the expected discounted return, the lower the expected default probability.

This partial equilibrium is integrated within a broader framework. The authors specify an Romer's-like aggregate production function for any t given by

(2.40) $$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

where Y_t represents the aggregate output of wholesale goods, K_t the aggregate amount of capital purchased by entrepreneurs in period $t-1$, L_t the labor input and A_t is an exogenous technological parameter. The discrete accumulation function for the aggregate capital purchases is given by

(2.41) $$K_{t+1} = \theta\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t, \qquad \theta(0) = 0, \theta'(.) > 0$$

where I_t represents the aggregate investment expenditures and δ the depreciation rate. The inclusion of adjustment costs results in a variable price of capital, making price variability contribute to volatility in entrepreneurial net worth, which the authors define by

(2.42) $$N_{t+1} = PV_t + W_t^e$$

where

(2.43) $$V_t = \gamma_t^k Q_{t-1} K_t - \left(r_t + \frac{h\int_0^{\bar{w}^j} w\gamma_t^k Q_{t-1} K_t \; dF(w)}{Q_{t-1}K_t - N_{t-1}}\right)\left(Q_{t-1}K_t - N_{t-1}\right)$$

represents the entrepreneurial equity and W_t^e denotes the entrepreneurial wage[11]. By further specifying the total labor input as

[11] The authors assume that, besides their net worth and external funds, entrepreneurs finance their current period's production through the inelastic supply of one unit of labor to the labor market.

(2.44) $$L_t = H_t^{\Omega}(H_t^e)^{1-\Omega}$$

where $\Omega \in]0,1[$, the authors obtained the demand curves for household and entrepreneurial labor, through which they deduced the aggregate entrepreneurial net worth

(2.45)
$$N_{t+1} = P\left[\gamma_t^k Q_{t-1}K_t - \left(r_t + \frac{h\int_0^{\overline{w}^j} w\gamma_t^k Q_{t-1}K_t \; dF(w)}{Q_{t-1}K_t - N_{t-1}}\right)(Q_{t-1}K_t - N_{t-1})\right] + (1-\alpha)(1-\Omega)A_t K_t^{\alpha} H_t^{(1-\alpha)\Omega}$$

Equation (2.45) thus describes the endogenous variation in the aggregate net worth, whilst the aggregate form of (2.39) describes how financial strength and expectations regarding future systemic shocks influence the cost of capital. After introducing household, retail and government sectors, the authors obtained the full model through regular Taylor approximations of the stochastic factors. The final set of log-linearized equations allows the researcher to assess the monetary policy transmission mechanisms in a macroeconomic environment with financial frictions.

2.4. Some final considerations

This section comprised some theoretical examples from the fields of economic growth and finance and was meant to explore the possibility of synergy between the two. Although some of the most relevant endogenous growth models don't necessarily comprise agent-based financial frictions, some partial equilibrium frameworks have appeared in the literature to meet this area of research potential. Despite a perceived consensus in the literature regarding the relative importance of studies such as BGG (1999) and Kiyotaki and Moore (1997), others have managed to introduce different financial factors within more conventional frameworks. Greenwald and Stiglitz (1993), from a two-agent labor-dependent economy with risk averse firms, built a general price level which fluctuates according to a marginal bankruptcy cost. Similarly to BGG, firms make decisions according to an equation that reflects their solvency situation. By construction, they show that the representative firm's profit maximizing output is a linear function of their equity. Others, like Pietra and Siconolfi (1996), characterize the economy's equilibrium as the representative household's maximization problem, which depends on expectations about future realizations of stochastic variables – namely, the asset prices according to which their portfolio allocations vary. By representing the economy as a space of real open subsets representing the households' utility functions, the authors represented the equilibrium price solution as a smooth manifold diffeomorphic to the space in which all states of nature are embedded. Hernández and Santos (1996) follow a similar methodology.

Given the in-depth analysis of Romer and BGG's models, the following section will proceed to integrate some of the mechanisms. There are a few factors that motivated the choice of the latter in the proposed expansion. First, in both models we find three productive sectors, with at least one working in monopolistic competition. Second, in both models we find a retailer – albeit with different designations – and an intermediate goods sector whose production inputs are capital goods. These factors hint at a similar architecture base between the two models, which may suggest a possible complementarity between them.

3. Extending Romer's model: an endogenous growth model with financial frictions

In this section, we proceed as follows. First, we address some of the technical incompatibilities between Romer's model and BGG's partial equilibrium framework. The purpose is to help understand the specific challenges that one needs to face if we ought to create a full general equilibrium model with financial frictions. Second, we start building Romer's model with our proposed modifications, whilst explaining why we believe they can be implemented. Third, we deduce the economy's balanced growth path solution. Fourth, we analyze the transition dynamics of the expanded model, followed by some concluding remarks.

3.1. On the incompatibilities between the existing frameworks

We start by addressing the perfect-foresight hypothesis. Under BGG (1999), this does not hold, given that the entrepreneur's net worth at $t + 1$ is uncertain, subject to a decomposed risk measure which captures both the aggregate and the idiosyncratic risk. The uncertainty around the net worth's realization (which we proxy by the capital firms' profit functions) gives rise to an informational asymmetry between the lender and the borrower which, in turn, gives an endogenously motivated reason for the existence of an external finance premium within BGG, whose existence is meant to compensate the lender for a problem of adverse selection and moral hazard[12]. The asymmetry constitutes a financial friction that can lead to more pronounced macroeconomic fluctuations, through BGG's financial accelerator (Claessens & Kose, 2018, pp. 75-79). We would then argue that Romer's deterministic setting constitutes a problem for the introduction of these financial imperfections.

Unlike the infinitely lived agents in Romer, the entrepreneurs in BGG are allowed to go bankrupt, should the cut-off value of the following period's capital returns be below the outstanding debt value multiplied by the risk-free rate of the economy. This is the external finance rule that borrowers must meet at each moment in time, in order to get external financing at t and continue the production,

[12] The existence of uncertainty around the net worth's realization and the drawing of an optimal contract as a solution to an agency problem is also present in KM and other studies like Carlstrom and Fuerst (1997), who integrates collateral constraints on the firm's side by assuming that labor employment must be partially financed through loans.

at least, until $t + 1$. The idiosyncratic nature of the entrepreneurs' bankruptcies holds the authors from assuming symmetry within their model, hence studying the partial equilibrium in terms of individual agents at arbitrary moments in time, in contrast with what happens in Romer's framework. Given that macro-financial linkages originate at the microeconomic level, one would expect for any model with financial frictions to shed light over agent-based phenomena. However, given the very nature of the field, we ought to analyze aggregate dynamics and patterns. Therefore, under the integration of BGG's partial equilibrium within a R&D-based endogenous growth framework, the agents' finite horizons may be a concern.

So far, we've identified three incompatibilities between BGG's partial equilibrium and Romer's model: the latter is fundamentally deterministic, while BGG has a probabilistic setting; in the two models, agents present different time horizons; and, while Romer's framework is derived in continuous time, BGG is written in discrete time, as suggested by the above mentioned one-period gap between the economic decision and the stochastic net worth's realization. Our main purpose is to expand Romer's model, so as to encompass BGG's partial equilibrium. In the following section, we propose ways to surpass these incompatibilities, in order to build a complete general equilibrium model of endogenous growth with financial frictions.

3.2. The model

Let's take the partial equilibrium setting of external finance dynamics in BGG, where the authors start by stating that

$$B_{t+1}^{j} = Q_t K_{t+1}^{j} - N_{t+1}^{j} \tag{3.1}$$

where K_{t+1}^{j} represents the capital bought at t by the j-th entrepreneur, $Q_t K_{t+1}^{j}$ the payoff realized at $t + 1$, N_{t+1}^{j} the net worth of the j-th entrepreneur, realized at $t + 1$, and B_{t+1}^{j} is the debt amount needed by the j-th entrepreneur at $t + 1$, borrowed from a financial intermediary (e.g. a bank). This means that, at each period, each entrepreneur acquires physical capital through his personal net worth and loans, which is combined with labor through some production technology to generate output in the following period.

In order for the entrepreneur's net worth to influence the loan terms, the borrowers need to face *finite time horizons*. The rationale is pretty straight forward: the assumption of a Cost State Verification (CSV) problem in BGG gives an endogenously motivated reason for the existence of an external finance premium, i.e., an opportunity cost for internal funding, meant to compensate the lender for a problem of adverse selection and moral hazard. This premium will depend on the borrower's ability to repay its loan at $t+1$ – i.e., its net worth –, which will be dependent on a default probability. For such a probability to exist, the entrepreneur must be allowed to go bankrupt, hence the purpose of the finite horizon, which captures the phenomenon of the continuous "birth" and "death" of firms, within a given economy.

We want the entrepreneurial sector in BGG to be analogous to Romer's capital sector. There are two difficulties that we need to surpass. The first one is the extension of the partial equilibrium to a continuous framework, which implies reducing the one-period gap to an arbitrarily small gap between the agents' decisions and the variables' realization. Second, the entrepreneur's finite horizon is incompatible with Romer's framework, in which firms don't go bankrupt. This constitutes a problem if we ought to integrate the financial accelerator mechanism. The problem of expanding Romer's model such as to encompass BGG's partial financial equilibrium is, we believe, intimately related with the solution of the two above mentioned incompatibilities[13].

By looking at equation (3.1), one could argue that a continuous approximation would be fairly straight forward, given that B_{t+1}^{j} is linear both in $Q_t K_{t+1}^{j}$ and in N_{t+1}^{j}. However, despite the mathematical validity, this approximation by time-differentiation wouldn't be economically reasonable. To understand why, let us rewrite (3.1)

$$B_{t+h}^{j} = Q_t K_{t+h}^{j} - N_{t+h}^{j} \tag{3.2}$$

[13] The original authors introduced monopolistic competition in the retail sector so as to obtain nominal rigidity, intrinsic to the Dynamic New Keynesian approach, suited for their purpose of monetary policy analysis. A monopolistic entrepreneurial sector would imply a non-linear relation between the entrepreneur's net worth and his demand for capital. We believe that, by proxying the capital sector's net worth by the profit function, the problem of a non-linear relation does not arise here.

with respect to an arbitrarily small time amount h. When $h \to 0$, (3.2) implies that the borrower both receives and repays the external funds almost instantaneously, which not only is unintuitive, as it does not represent the reality of credit flows at the firm level.

We know that the free entry condition in Romer's capital market implies that the investor needs to make an initial investment $P_A(t)$ to participate. If we assume that this initial cost is financed through a loan, in a similar fashion of Carlstrom and Fuerst (1997) with the labor employment, we can introduce a model of *debt management* to replace the current loan dynamics present in BGG. We do this by stating that, in any given period, the firm pays some percentage of the outstanding debt, which corresponds to the initial investment cost $P_A(t)$.[14] The rationale is that some net losses are expected at the beginning of the firm's creation, which may turn to profits once it becomes fully developed. In order to preserve the aggregate capital accumulation, we introduce the following transversality condition

(3.3)
$$\int_0^t (P_A(\tau) - \beta(\tau)P_A(\tau))d\tau = 0$$

where $\beta(\tau) > 0$ represents a variable portion of outstanding debt paid at each moment in time. The condition implies the full repayment of outstanding debt, in the long run.

This is an important step in the model's architecture. The optimal contract established between the lender and the borrower in BGG is characterized by the minimum requirement rule for external financing

(3.4)
$$\overline{w}^j \gamma_{t+1} Q_t K_{t+1} = Z_{t+1}^j B_{t+1}^j$$

where Z_{t+1}^j represents a gross non-default rate, $\overline{w}^j$ the limit value of the idiosyncratic shock given by the stochastic variable w^j, which fluctuates in the interval $[0,1]$, such that $w^j \geq \overline{w}^j$ implies that the lender is able to repay the loan at the rate Z_{t+1}^j, and γ_{t+1} represents the aggregate risk

[14] Further in this study, we show how this introduction of indebtedness comes at the cost of the complementarities between capital goods assumption.

of the economy. If, on the other hand, $w^j < \bar{w}^j$, then the firm is unable to repay its loan and goes bankrupt. Under the proposed model of continuous debt management, we can adapt (3.4) to our capital sector

$$\bar{w}_j \gamma R_j(t) x_j(t) = r x_j(t) + Z_j(t) \beta_j(t) P_{A,j}(t) \tag{3.5}$$

where $R_j(t)$ represents the j-th firm's selling price of its capital goods $x_j(t)$, and r represents the risk-free rate. We omit the expected value operators so as to facilitate the exposition. Equation (3.5) implies that the capital firm must guarantee, at each moment, sufficiently large sales so as to meet both the required production inputs and debt obligations. This represents the minimum requirement to be met by the j-th capital firm and gives us a reinterpretation of the profit function in Evans et al (1998), expanded in order to include a decomposed risk measure. Hence, the profit of the j-th capital firm is given by

$$\pi_j(t) \equiv w_j \gamma R_j(t) x_j(t) - r x_j(t) - Z_j(t) \beta_j(t) P_{A,j}(t) \tag{3.6}$$

where the optimal output level guarantees that $\pi_j(t) > 0$. Hence, liquidity and wealth distribution begin to matter. Levered capital firms become susceptible to adverse shocks, in the advent of which they may see a large fraction of their net worth wiped off which, in turn, may lead to systemic persistence and amplification of the financial shocks.

The households allocate their wealth between the riskless asset at the risk-free rate and loans to the capital firms, according to their risk aversion. Omitting the role of a financial intermediary[15], the households are able to diversify the idiosyncratic risk component through a sufficiently big number of granted loans, which implies that their opportunity cost is the riskless rate. Therefore, the applicable finance rule is

$$\bar{w}_j \gamma R_j(t) x_j(t) = r x_j(t) + r \beta_j(t) P_{A,j}(t) \tag{3.7}$$

[15] We make this simplification because, at any given moment, the intermediary's net worth is set to zero.

We still need to find the external finance premium paid by the j-th capital firm. To do so, we need to introduce the monitoring cost $hw\gamma(t)R_j(t)x_j(t)$, where $h \in [0,1[$, in line with BGG's CSV problem. In order to do so, we need to observe that there are two possible outcomes in any given period. If $w_j \geq \bar{w}_j$, the lender pays $rx_j(t) + rP_{A,j}(t)$ and keeps the difference $w_j\gamma R_j(t)x_j(t) - rx_j(t) - rP_{A,j}(t)$. If, on the other hand, $w_j < \bar{w}_j$, the borrower goes bankrupt, whilst receiving nothing. The lender pays the monitoring cost $hw\gamma R_j(t)x_j(t)$ and keeps what he finds. Following (3.7), we modify the rule so as to include the default probability $F(\bar{w}_j) = \Pr[w_j < \bar{w}_j]$ and, by extension, the external finance premium, such that

(3.8)
$$\left\{[1 - F(\bar{w}_j)]\bar{w}_j + (1-h)\int_0^{\bar{w}_j} wdF(w)\right\}\gamma R_j(t)x_j(t) = r\big(x_j(t) + \beta_j(t)P_{A,j}(t)\big)$$

Similarly to what happens under the original framework, we maintain the risk neutrality assumption in our borrowers. For that reason, they are willing to absorb the monitoring costs faced by the lenders, thus facing the external finance premium $h\int_0^{\bar{w}_j} wdF(w)$.

We are now capable of writing our optimization problem for the j-th firm. Naturally, the firm will ought to maximize its profits, subject to the finance constraint imposed by (3.8). The result is the following generalization of Romer's original optimization problem

(3.9)
$$\max_{x_j(t),\bar{w}_j} \big(1 - \Gamma(\bar{w}_j)\big) sR_j(t)x_j(t)$$

subject to

(3.10)
$$[\Gamma(\bar{w}_j) - h\Phi(\bar{w}_j)]sR_j(t)x_j(t) = x_j(t) + \beta_j(t)P_{A,j}(t)$$

where $\Gamma(\bar{w}_j) \equiv \int_0^{\bar{w}_j} wf(w)dw + \bar{w}_j \int_{\bar{w}_j}^{+\infty} f(w)dw$ is the expected gross share of profits of the lender, $h\Phi(\bar{w}_j) \equiv h\int_0^{\bar{w}_j} wf(w)dw$ are the expected monitoring costs and $s = \frac{\gamma}{r}$. Moreover, f represents a density function, which means $f(w) = F'(w)$. While $\Gamma(\bar{w}_j)$ and $h\Phi(\bar{w}_j)$ represent real numbers, s remains variable in the steady state. Therefore, in order to obtain a balanced growth path solution, we define the following concept:

***Def**: we call a steady state solution to the vector $R \in \mathbb{R}^{\mathcal{F}_t}$ that solves the optimization problem (3.9) such that $E\{s\} = \frac{1}{r}$, where $\mathcal{F}_t$ represents the cardinality of the set of capital firms in existence at each moment t in time.*

By defining the steady state solution as we did and using it in order to find a balanced growth rate for the economy, we are stating that, in the equilibrium, we would expect for the absence of aggregate shocks, for any probability distribution of the variable γ. After defining a new function and using Lagrange multipliers, the first order conditions for problem (3.9) are[16]

(3.11)
$$\left[1 - \Gamma(\bar{w}_j) - \lambda\left(\Gamma(\bar{w}_j) - h\Phi(\bar{w}_j)\right)\right] sR_j(t) - \lambda = \lambda\alpha\beta_j(t)\frac{1-\alpha}{\delta} L_{Y,j}^{-\alpha} x_j^{\alpha-1}$$

(3.12)
$$\Gamma'(\bar{w}_j) sR_j(t) = \lambda\left(\Gamma'(\bar{w}_j) - h\Phi'(\bar{w}_j)\right) sR_j(t)$$

where λ is the Lagrange multiplier. This is known to be true because the solutions are interior. From (3.12), it follows that

(3.13)
$$\lambda = \frac{\Gamma'(\bar{w}_j)}{\Gamma'(\bar{w}_j) - h\Phi'(\bar{w}_j)}$$

[16] The conditions follow from the existence of a perfectly competitive labor market under Romer's framework, together with the accumulation of intellectual stock $\dot{A} = \delta L_A(t)A(t)$.

which is always a positive number. We can see this by assuming, like it was made in BGG, that $\left(\frac{\bar{w}_j f(\bar{w}_j)}{1-F(\bar{w}_j)}\right)' > 0$. When solving (3.13), we get $\frac{f(\bar{w}_j)(1-F(\bar{w}_j)-\bar{w}_j f(\bar{w}_j))}{\left(1-F(\bar{w}_j)\right)^2} > 0$, which is the same as saying that $\Gamma'(\bar{w}_j) > \Phi'(\bar{w}_j)$ Now, let

(3.14)
$$\rho(\bar{w}_j) = \frac{\lambda}{1 - \Gamma(\bar{w}_j) - \lambda\left(\Gamma(\bar{w}_j) - h\Phi(\bar{w}_j)\right)}$$

then, we find that the optimal rent level of the j-th capital firm is given by

(3.15)
$$R_j = \frac{\rho(\bar{w}_j)(1-\alpha)}{s\alpha\left[1 - \rho(\bar{w}_j)\left(\Gamma(\bar{w}_j) - h\Phi(\bar{w}_j)\right)\right]}$$

where $\rho(\bar{w}_j)$ is a measure of the elasticity between the lender's expected return and that of the borrower, relative to the total contractual expected return.

Let us denote by i the generic element of the set of final goods' firms in existence at any given moment in time. We know from Evans et al (1998) that this sector's optimization problem yields the following conditions:

(3.16.1)
$$w_{Y,j}(t) = (1-\alpha)\frac{Y_j(t)}{L_{Y,j}(t)}$$

(3.16.2)
$$R(t) = \alpha\left(\frac{L_{Y,j}(t)}{x_j(t)}\right)^{1-\alpha}$$

By intersecting the final goods' demand with the equilibrium rent, we find that the equilibrium capital consumption of a non-symmetric economy with financial frictions is given by

(3.17)
$$x_j(t) = L_Y(t)\left[\frac{s\alpha^2\left(1-\rho(\overline{w}_j)\left(\Gamma(\overline{w}_j)-h\Phi(\overline{w}_j)\right)\right)}{\rho(\overline{w}_j)(1-\alpha)}\right]^{\frac{1}{1-\alpha}}$$

Our next goal is to integrate the possibility of bankruptcy within our capital sector, such that it is possible to evaluate long term dynamics – agents' optimal choices, balanced growth paths - of a given economy, over an infinite horizon.

Let $\mathcal{F}_t \subset \mathbb{N}$ be the non-empty number of capital firms in existence at any given moment $t \in T$. Although individual firms can disappear, by guaranteeing that there will always be at least one firm in existence at any given moment, one could study the optimal behavior of those in existence when $t \to \infty$. Hence, in order to study the aggregate dynamics of some economy through the optimal behavior of capital firms, we need to guarantee both intra-temporal and intertemporal *symmetry*, i.e., to guarantee the existence of a representative firm at any given moment, whose optimal behavior is invariant across time. Following Acemoglu's theorem of the representative firm (2009, pp. 229-231), let $X_t = \left\{\sum_{j\in\mathcal{F}} x_j : x_j \in X_j \; for \; each \; j \in \mathcal{F}_t\right\}$ be the economy's set of production possibilities and $\hat{X}(\hat{R}) \subset X$ the set of profit maximizing net supplies. Let $\hat{x} = \sum_{j\in\mathcal{F}} \widehat{x_j}$ be the optimal production decision of a representative firm, for the optimal capital goods' price vector $\hat{R} \in \mathbb{R}^F$, which corresponds to (3.15). Let's assume that $\hat{x} \notin \hat{X}(\hat{R})$. This implies the existence of x' such that $\hat{R}x' > \hat{R}\hat{x}$. By definition of X, there exists $\{x_j\}_{j\in\mathcal{F}}$ with $x_j \in X_j$ such that

(3.18)
$$\hat{R}\left(\sum_{j\in\mathcal{F}} x_j\right) > \hat{R}\left(\sum_{j\in\mathcal{F}} \widehat{x_j}\right)$$

such that there is at least one $j' \in \mathcal{F}$ such that $\hat{R}x_{j'} > \hat{R}\hat{x}_{j'}$, which contradicts the hypothesis that $\hat{x}_j \in \hat{X}_j(\hat{R})$. Hence, we are in sufficient conditions to guarantee the existence of a representative capital firm, for which $\hat{x} = \sum_{j\in\mathcal{F}} \widehat{x_j}$. Maintaining the symmetry Romer *et al* created on the final goods' sector, we can deduce symmetry over the individual rents, in order to find the following optimal equations for the representative capital firm

(3.19)
$$R = \frac{\rho(\overline{w})(1-\alpha)}{s\alpha\Big(1-\rho(\overline{w})\big(\Gamma(\overline{w})-h\Phi(\overline{w})\big)\Big)}$$

(3.20)
$$x(t) = L_Y(t)\left[\frac{s\alpha^2\Big(1-\rho(\overline{w})\big(\Gamma(\overline{w})-h\Phi(\overline{w})\big)\Big)}{\rho(\overline{w})(1-\alpha)}\right]^{\frac{1}{1-\alpha}}$$

that aggregate the dynamics of the individual firms. In appendix A, we prove that equations (3.19) and (3.20) are consistent with the accumulation of physical capital within Romer's framework. From Acemoglu's theorem and proof of consistency, $\rho(\overline{w})$ in equations (3.19) and (3.20) represents a sum of the individual $\rho(\overline{w}_j)$, over the set $\mathcal{F}_t \subset \mathbb{N}$. Given that $\mathcal{F}_t$ is non-empty, while individual capital firms can go bankrupt – meaning, in the steady state, $\rho(\overline{w}_j) = 0$ – the representative agent has an infinite horizon. We can thus proceed to find the balanced growth path of the economy.

3.2.1. Balanced Growth Path Solution

From Romer's representative household's optimization problem

(3.21)
$$\max_{C(t)} \int_0^\infty e^{-\rho t} u\big(C(t)\big)dt \;\; s.t.$$
$$\dot{B(t)} = rB(t) + r\beta(t)P_A(t)A(t) + w(t)L(t) - C(t) - \int_0^t (P_A(\tau) - \beta(\tau)P_A(\tau))d\tau$$

we get the Euler equality

(3.22)
$$g_c = \frac{1}{\sigma}(r - \rho)$$

The consumer side of the economy in our model is equivalent to the one in Romer's model, hence the use of (3.22) and the assumption of a constant real interest rate up until now. Taking our definition of the steady state solution and applying it in order to calculate the growth rate of the economy, we find that the amount of capital goods given by (3.20) is constant. Therefore, the patent price

(3.23)
$$P_A = \frac{1-\alpha}{\delta} L_Y^{-\alpha} x^{\alpha}$$

is also constant. This holds true because, by (3.20), the ratio $\frac{x(t)}{L_Y(t)}$ is constant. In the appendix A, we deduct the Bellman-Hamilton-Jacobi equation for the patent price, in terms of the capital firm's profit. By showing that (3.23) is constant, we find that

(3.24)
$$r = \frac{\delta\alpha}{(1+\beta)(1-\alpha)} L_Y \left(1 - \frac{\alpha l(\overline{w})}{1-\alpha}\right)$$

where

(3.25)
$$l(\overline{w}) \equiv \frac{\left(1 - \rho(\overline{w})\left(\Gamma(\overline{w}) - h\Phi(\overline{w})\right)\right)}{\rho(\overline{w})}$$

and, because the economy's production function is Cobb-Douglas, we are able to build the system

(3.26)

$$\begin{cases} g_c = \frac{1}{\sigma}(r-\rho) \\ g_Y = \delta\bar{L} - \frac{r(1+\beta)(1-\alpha)}{\alpha\left(1-\frac{\alpha l(\bar{w})}{1-\alpha}\right)} \\ \dot{K(t)} = Y(t) - C(t) - \int_0^t (P_A(\tau) - \beta(\tau)P_A(\tau))d\tau \end{cases}$$

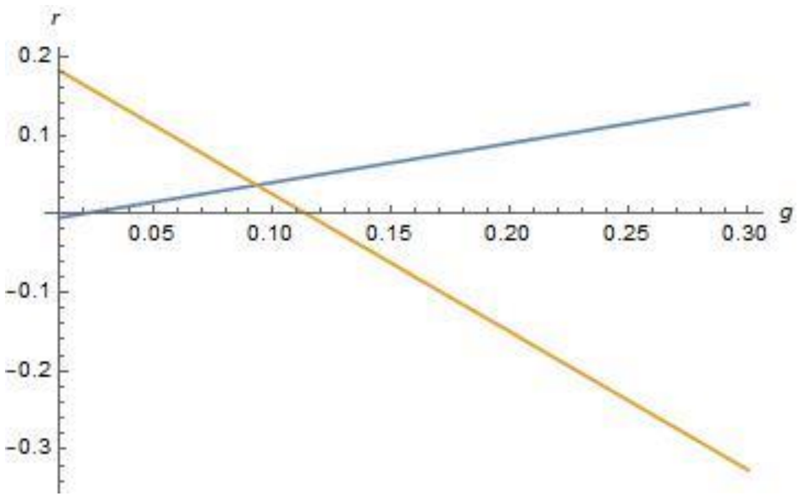
which, because of the transversality condition (3.3), and because the model is to hold over an infinite horizon, gives us the balanced growth path solution

(3.27)

$$g = \frac{\delta\bar{L}\alpha\left(1-\frac{\alpha l(\bar{w})}{1-\alpha}\right) - \rho(1+\beta)(1-\alpha)}{\alpha\left(1-\frac{\alpha l(\bar{w})}{1-\alpha}\right) + \sigma(1+\beta)(1-\alpha)}$$

which is a generalization of the original framework that encompasses the effects of financial frictions in the economy, through $l(\bar{w})$. Further details are available in appendix A. What we've done to prove equality between the growth rates was to take the limit of $\frac{\dot{K(t)}}{K(t)}$ when $t \to \infty$. Because of the transversality condition, the third term disappears, rendering the capital accumulation equivalent to that of Romer's original framework. Figure 1 represents a simulation ran for the general equilibrium solution.

Fig. 1: General Equilibrium Solution



The plot displayed in figure 2 was obtained with the following parameter values:

$$\sigma = 2; \quad \rho = 0.02; \quad \alpha = 0.4$$
$$\bar{L} = 1; \quad \beta = 1; \quad \delta = 0.1$$

which, in turn, were chosen considering the previous simulation by Thompson (2008). The negatively sloped plot represents the consumer's preferences, while the positively sloped plot represents the technology curve. The intersection occurs at the point where the interest rate equals 0.0931998 and the economy's growth rate equals 0.0365999. Under the assumption of a normal distribution of the contractual risks and, therefore, the existence of well-defined cumulative distribution functions, we chose the value -1.0667 for our parameter $l(\bar{w})$.

Equation (3.27) thus represents the equilibrium long term growth rate of an economy with agent-based financial frictions and generalizes (2.33). The following subsection explores the properties of the equilibrium regarding fluctuations on $l(\bar{w})$.

3.3. Interaction between informational asymmetries and economic growth

The function l evaluated in $\bar{w}$, formally, represents the following relation between the contractual returns

(3.28)
$$l(\bar{w}) = -\varepsilon_{NL,GB}\big(1 - \Gamma(\bar{w})\big) - 2\big(\Gamma(\bar{w}) - h\Phi(\bar{w})\big)$$

where $\varepsilon_{NL,GB}$ represents a ratio of derivatives – sort of an elasticity measure between the lender's net expected returns and the borrower's gross expected returns. Then, from (3.27) we obtain

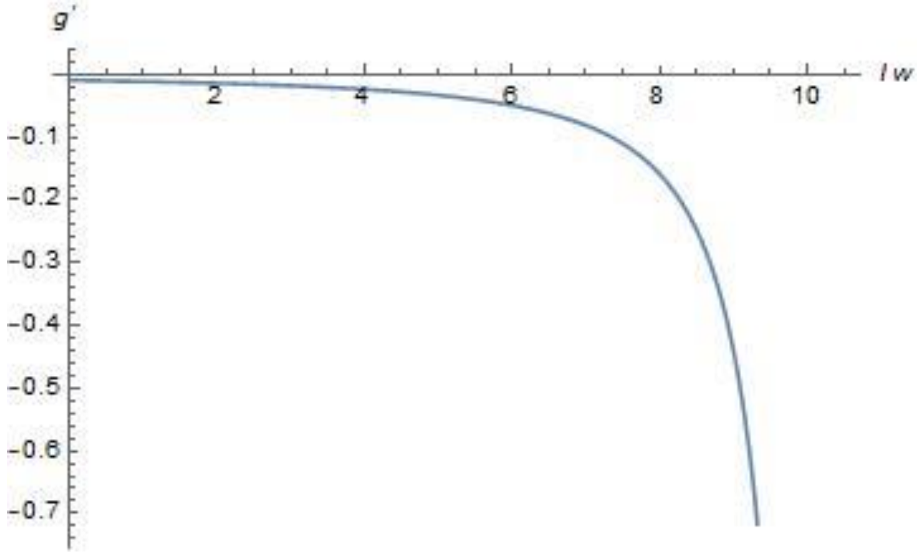
(3.29)
$$g'\big(l(\bar{w})\big) = -\frac{\alpha^2(1+\beta)(\delta\bar{L}\sigma + \rho)}{\left[\alpha\left(1 - \frac{\alpha x}{1-\alpha}\right) + \sigma(1+\beta)(1-\alpha)\right]^2}$$

and

(3.30)
$$g''(l(\overline{w})) = -\frac{2\alpha^4(1+\beta)(\delta\overline{L}\sigma+\rho)}{(1-\alpha)\left[\alpha\left(1-\frac{\alpha x}{1-\alpha}\right)+\sigma(1+\beta)(1-\alpha)\right]^3}$$

This means that the effects of these asymmetries on economic growth are always negative. Figure 2 represents a simulation plot of (3.29). The parameters' values remain the same as the ones used in the previous simulation.

Fig. 2: Plot of the derivative of g, regarding the financial parameter x



For the general case, the applicable domain where the solution doesn't explode is $\left\{l(\overline{w}) \in [0,1]:\ l(\overline{w}) < \frac{\sigma(1+\beta)(1-\alpha)^2+\alpha(1-\alpha)}{\alpha^2}\right\}$ which maps to $\{y \in \mathbb{R}:\ \ y < 0\}$. The solution explodes when $l(\overline{w})$ tends to $\frac{\sigma(1+\beta)(1-\alpha)^2+\alpha(1-\alpha)}{\alpha^2}$. Here, increases on the elasticity $\varepsilon_{NL,GB}$ further decrease economic growth at a marginally increasing rate, while increases of the external finance premium are beneficial to economic growth. If one were to assume an inverse relation between risk aversion and our elasticity measure, this could imply a positive effect of the risk aversion coefficient σ on growth, as suggested by Davidsson (2012). Furthermore, giving that

$g' \to 0$ when $l(\bar{w}) \to 0$, the effects of these informational asymmetries on growth are asymptotically null.

The overall negative effect of the asymmetries on growth is in line with studies such as Fu (1996), who argues that asymmetric information affects investment which, in turn, directly affects economic growth. The marginal contribution of this study to the large body of literature on informational asymmetries, growth and capital accumulation is to shed some light at the channel through which such effects may take place. The overall increase on the elasticity measure results from a decrease on the economy's monitoring costs. Assuming that the monitoring costs h are an increasing function of the representative firm's revenue (Jain, 2001), this decrease in h means lower revenues and, therefore, lower aggregate growth. In our simulated example, this happens in the neighborhood of $\frac{21}{2}$. If the elasticity drops low enough, its effects on growth tend to zero.

As it is in the original partial equilibrium framework, the effects of isolated systemic shocks in some given period also move across time. To help visualize the accelerator mechanism, let's take, between t and $t+1$, the aggregate shock $\gamma - \Delta$, where $\Delta > 0$ represents any unforeseen effect that affects the representative capital firm's output, relative to the previous period. We approximate $\dot{K(t)} \approx K(t+1) - K(t)$, which follows from the households' intertemporal constraint in (3.21), because of the linearity of $\dot{K(t)}$ in its terms. We also assume that fluctuations in the capital accumulation are very small, which is a plausible assumption, given that the capital's growth rate $\frac{\dot{K(t)}}{K(t)}$ is constant. In face with the negative shock, the firm's profits are given by

(3.31)
$$\pi_\Delta(t) := w(\gamma - \Delta)\frac{(1-\alpha)}{s\alpha l(\bar{w})}x(t) - rx(t) - x\beta(t)P_A(t)$$

which, in order to isolate the quantitative effects of the aggregate shock, can be represented as

(3.32)
$$\pi_\Delta(t) = \pi(t) - \frac{\Delta w(1-\alpha)}{s\alpha l(\bar{w})}x(t)$$

which, given the proposed approximation, when replaced in the households' intertemporal constraint, will decrease the following period's aggregate capital by $\frac{\Delta w(1-\alpha)}{s\alpha l(\bar{w})} x(t)$. Appendix A offers technical details on the relation between aggregate capital accumulation and the capital firms' profits. Without loss of generality, we attach this loss on the representative household's balance sheet to the returns from loans to capital firms. Holding the remaining terms constant, this implies a decrease of the number of firms entering the capital market at $t+1$, given the decrease of the external financing needed to meet the capital firms' free entry condition. Under the premise of the representative capital firm, this implies a lower aggregate output on the following period, hence perpetuating and amplifying the isolated shock Δ at moment t.

3.4. Some final considerations

In its original model, Evans et al went further, by assuming the existence of complementarities between capital goods, and obtaining a multiple equilibrium balanced growth path solution. In appendix A, we show how the same assumption, under our proposed framework, leads to a contradiction, and cannot be made. The introduction of financial frictions therefore sacrifices the introduction of imperfect substitutability between capital goods, hence falling short of the original model.

We have created an extension of Romer's model that encompasses financial frictions, through the introduction of uncertainty in the capital sector's profit function. This created endogenously motivated informational asymmetries between lenders and borrowers, which turned out to have an impact on the monopolistic capital market and on long-term growth, through a function l evaluated in $\bar{w}$. We were then able to reach a balanced growth path solution in which the sensitivity of the representative parties expected contractual returns to one another negatively influences economic growth. While this feature may add complexity to the original framework and potentially make it more representative of the economic reality, the introduction of indebtedness appears to unable the assumption of complementarities between capital goods, hence falling short of the original model. A solution for this incompatibility may be a direction for future research.

4. Testing the model: an empirical analysis

This section's main purpose is to test the empirical validity of our expanded version of Romer's model, through appropriate econometric mechanisms. Section 4.1. briefly discusses the dynamic nature of economic growth and presents the data, as well as the econometric model of interest. The results are then presented and discussed in Section 4.2., followed by some concluding remarks in Section 4.3.

4.1. Data and Econometric Model

There is a large body of empirical research on the determinants of economic growth. Although there are many variables that, historically, have contributed to explain growth, the importance of initial conditions is well established in the literature, and the need of initial income as a regressor in any empirical model is perhaps indisputable. The claim can be supported by countless empirical studies – to name a few, take Barro (1991), Mankiw et al (1992), Borensztein et al (1998), Alfaro et al (2004) and Moral-Benito (2012). In econometric terms, this idea is materialized through the analysis of dynamic panels.

The use of dynamic panel data models, namely for aggregate growth analysis, is far from new, and one of its advantages relative to simpler data structures is that it allows for a better understanding of the dynamics of adjustment (see e.g. Islam (1995)). Therefore, the technical challenges that arise under this framework are well known. While some of them can be fixed under more conventional procedures such as least squares estimation – e.g. fixed effects (FE) and random effects (RE) commonly used on static panels – others may require more specialized methods. Furthermore, the researcher may find additional challenges in panels with a small number of individuals N and large time period T – typical macro panels.

Dynamic panel models are characterized by the presence of the lagged dependent variable among the regressors, i.e.,

$$y_{it} = \rho y_{i,t-1} + x_{it}'\beta + u_{it}, \qquad i = 1, \dots, N; t = 1, \dots, T \tag{4.1}$$

where ρ is a scalar, x'_{it} is the $1 \times K$ matrix of the regressors (excluding, of course, the initial income) and β is the $K \times 1$ matrix of coefficients. The error component u_{it} is assumed to consist of a time-invariant individual effect and a white-noise effect, such that

$$u_{it} = \mu_i + v_{it} \tag{4.2}$$

where $\mu_i \sim IID(0, \sigma_\mu^2)$ and $v_{it} \sim IID(0, \sigma_v^2)$ are independent of each other and among themselves. This regression has two sources of persistence over time. Autocorrelation, due to the presence of the lagged dependent variable, and the unobserved heterogeneity μ_i. Throughout this chapter we will show that the most appropriate procedure for the model's estimation is based on the work of Blundell and Bond (1998), and it's known as the difference generalized method of moments (GMM) estimator.

Our panel contains observations for 27 member states of the European Union[17] during the period 2008-2016 (i.e. $N = 27$ and $T = 9$). Although, following Durusu-Ciftci et al (2017), we could've additionally singled out Finland and the Netherlands from the analysis, we believe that the choice would've only increased the possibility of finite-sample biases, without a necessarily proportional precision increase on the estimation of the effects of our parameter of interest on growth. The baseline empirical growth model is the augmented Solow model, which means that the selected variables comprise measures of initial income, rates of human and physical capital accumulation and population growth. In addition to these variables, we've also built a variable that proxies the previous chapter model's key parameter, $l(\overline{w})$, and we've included some variables considered by Moral-Benito (2012), according to their Bayesian posterior probabilities. Table 1 lists both the regressors and the dependent variable.

The variable lw_{it} was built according to definition (3.28) and can thus be separated in two components. First, the elasticity of the lenders' liquid returns relative to the borrowers' gross returns was approximated by calculating the ratio between the countries' banks return on assets (after tax) and the representative firms' gross profits, both as percentage variations. The data for the banks' return on assets was obtained through the Global Financial Development digital database, from The World Bank. The data for each country's representative firms' gross profits

[17] Due to data unavailability, Denmark was not included in the analysis.

Table 1: Variable Sources and Definitions

Variable	Source	Definition
Lgdp (Dependent Variable)	OECD	Logarithm of real gross domestic product (GDP) per capita
Invshare	PWT 6.2	Capital investment as a share of GDP
Lfp15	WDI	Labor force participation rate as a percentage of the population with ages 15+ (national estimates)
Lhsp	OECD	Logarithm of household spending in millions of USD
Lp	WDI	Logarithm of total population
Lfwe1	WDI	Percentage of total labor force with basic education
Lfwe2	WDI	Percentage of total labor force with intermediate education
Lfwe3	WDI	Percentage of total labor force with advanced education
Lw	Amadeus + GFD	Financial parameter from the extended version of Romer's model that proxies the behavior of $l(\overline{w})$
Popg	WDI	Population growth
Up	WDI	Logarithm of the total urban population
Yp	WDI	Logarithm of the population below 15 years old

Note: OECD refers to the Organization for Economic Cooperation and Development's digital database; PWT 6.2 refers to Penn World Table 6.2; WDI refers to World Development Indicators from The World Bank; GFD refers to Global Financial Development from The World Bank; and Amadeus refers to a European database with information about approximately 21 million firms, including financial reports, accounting and administrative data.

was obtained through Amadeus: for each country in the panel, we've filtered the active firms with a maximum current ratio of 1, therefore limiting our analysis to companies with at least as much liabilities as assets. From most of the samples of filtered firms meeting these requirements, we've randomly selected between 10-50% of the lists[18]. We then used Acemoglu's theorem to average each sample at each period, thus obtaining, for each country, a time-series of the representative firms' gross profits. The second component of (3.28) was obtained in a similar fashion, by averaging the firms' annual operational revenues. The results are displayed on table 2. The apparently downward trend in the returns may be related with the global financial crisis of 2008, the beginning of our panel period.

Table 2: Representative firms' annual gross profits

Country	2008	2009	2010	2011	2012	2013	2014	2015	2016
Austria	15,8934	2,5071	14,0623	9,3237	6,8007	7,1015	5,7311	6,4757	6,9184
Belgium	2,7613	2,0394	3,0228	2,9467	2,4705	2,3244	3,4579	3,9311	4,1002
Bulgaria	4,3271	2,2881	2,0247	3,0101	3,5613	2,7851	2,0914	2,6788	2,7381
Croatia	1,2307	-0,123	-0,9994	-0,344	-0,246	0,0564	0,7377	1,1329	1,5194
Cyprus	5,8502	1,4311	3,5972	-3,052	-3,520	-2,077	-0,442	-0,669	0,8720
Czech Republic	1,1309	-0,297	0,8941	0,4978	0,5041	0,0012	1,1199	1,5965	1,4637
Estonia	3,7929	1,5300	5,5690	6,9686	7,1330	7,2877	5,6555	5,3026	4,6495
Finland	3,3524	2,4016	3,2178	3,0552	2,5296	2,3288	2,0038	2,3627	2,0590
France	1,8926	1,4124	1,8805	1,4422	0,8615	0,9879	0,7552	0,6996	0,4622
Germany	2,8411	2,2328	3,4572	3,4317	3,1931	2,5515	2,6906	2,8731	4,1284
Greece	-0,8516	-1,881	-4,5887	-6,736	-8,449	-4,979	-4,242	-5,180	-6,143
Hungary	1,7682	1,2349	1,1090	1,0661	3,4142	2,0791	2,1567	2,8979	3,0244
Ireland	1,3444	0,0672	0,6369	1,4918	2,4393	2,6005	2,6125	1,9949	2,7072
Italy	0,6383	0,0747	1,0393	0,8316	-0,013	0,2429	0,2293	0,4303	0,7848
Latvia	1,5207	-3,200	0,1259	1,8468	2,7897	2,6281	1,7087	1,7065	-0,131
Lithuania	0,9303	-2,177	-0,7672	1,3946	0,9462	1,1788	0,6656	-0,144	0,4176
Luxembourg	0,4900	-1,041	-0,8275	-1,662	-0,131	-1,027	-0,011	0,2255	0,7586
Malta	4,1240	4,2746	4,9135	5,4723	4,5590	5,4247	6,0420	6,6598	8,8822
Netherlands	3,6467	4,6337	5,2386	3,5021	2,5986	2,0551	2,6176	2,9039	5,2646
Poland	2,0617	2,2347	2,3659	1,0499	1,7944	1,4843	1,4454	1,6763	1,3984
Portugal	-0,2714	-0,224	2,3308	0,9804	-0,009	1,2428	2,4577	3,2998	3,8283
Romania	1,9450	0,6198	1,1797	1,2557	0,5702	1,0806	1,7154	2,8969	3,1705
Slovakia	2,8477	-0,554	0,9780	0,4840	0,0802	-0,249	0,8414	0,8675	0,9763
Slovenia	1,5290	-0,440	1,2109	1,2948	1,1682	1,5314	2,1383	2,4330	3,4420
Spain	1,2979	0,5549	1,1240	0,6810	0,3272	0,6977	1,4277	2,2427	3,0194
Sweden	4,1088	4,8248	5,6970	5,1429	4,2716	4,4532	5,2556	5,9407	5,6917
United Kingdom	1,7111	-2,591	3,7323	3,4145	3,4083	3,7896	3,3749	2,9919	1,4848

[18] For Austria, Cyprus, Greece, Lithuania and Luxembourg, we've selected the entirety of the output lists, due to the relative shortage of companies meeting the needed requirements.

Due to data shortage, we could not evaluate our variables before 2008, in order to compare the pre and post crisis GDP's response to variations in the regressors. Table 3 contains the descriptive statistics of our variables.

The first period of the panel was marked by a low average lw_{it}, with high cross sectional variability. Given that the variable reflects the symmetric form of (3.28), it can suggest both a low average elasticity of the lenders' returns relative to the borrowers' returns and/or low representative firms' returns. By the end of the analyzed period, this variable's coefficient of variation decreased to approximately 1/10 of its initial value.

The population growth decreased between 2008 and 2016, which reflects on only slight variations on both the percentages of urban and young populations in these countries. This is not surprising, since it reflects a global trend of decreasing birth rates in developed countries (see, e.g., Grant et al (2004)).

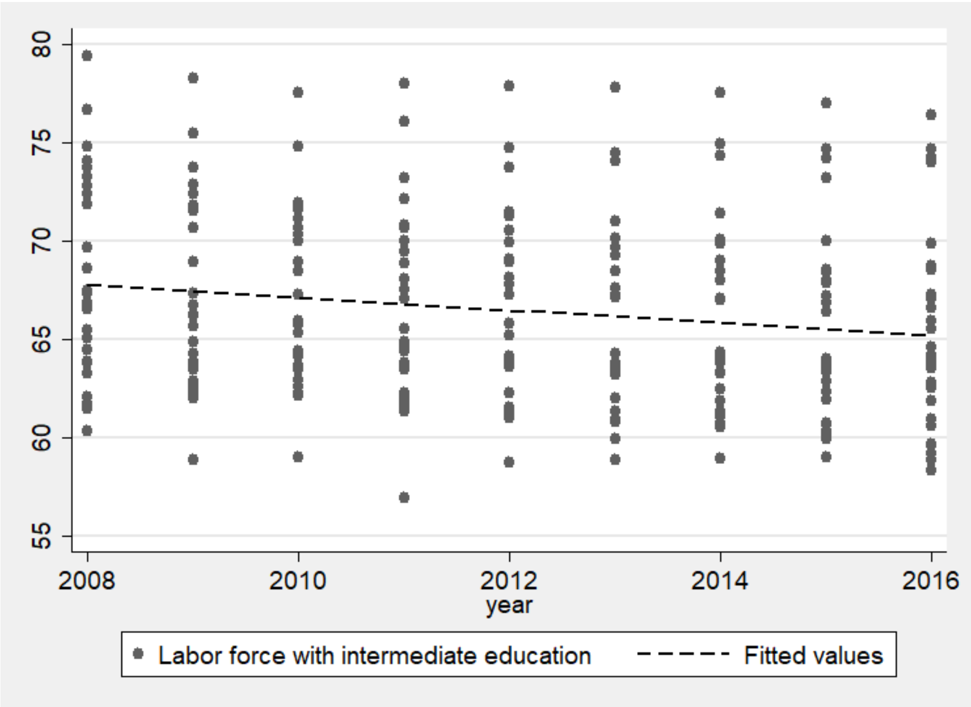
The overall trend amongst the different levels of education in each country's labor force, reflecting human capital accumulation, was a substantial decrease between 2008 and 2016. The cross sectional average of the labor force engaging in secondary education dropped around 3 percentage points, from approximately 32% to 29% of the labor force. The cross sectional (percentage) average of the labor force with a college degree registered a slight increase due to the overall drop of the cross sectional averages. The main effect was therefore registered at the level of intermediate education, Lfwe2. Hence, one would expect this variable to have a more significant impact on growth, during this period, than its counterparts. Figure 4 helps us to visualize the variable's evolution. This downward trend may be unsurprising if we consider its possible relation with the global phenomenon of slowing innovation and technological diffusion (Andrews, Criscuolo, & Gal, 2015). This effect, combined with a cross sectional average decrease of approximately 10 percentage points in capital investment, despite an average increase on household spending, may explain the slow aggregate growth registered between 2008 and 2016; this appears to be consistent with the findings of a growing body of literature on the great productivity slowdown (see, e.g. Duval et al (2017)).

Table 3: Descriptive statistics

	N		Mean		Standard dev.		p25		p50		min		max	
VARIABLES	2008	2016	2008	2016	2008	2016	2008	2016	2008	2016	2008	2016	2008	2016
Lgdp	27	27	10.30	10.52	0.389	0.360	9.944	10.22	10.34	10.47	9.553	9.878	11.37	11.56
Invshare	27	27	0.250	0.146	0.0519	0.0593	0.214	0.0958	0.248	0.131	0.165	0.0711	0.343	0.320
Lfp15	27	27	57.97	58.35	4.909	4.750	53.81	55.13	59.24	58.61	49.10	49.50	66.55	72.09
Lhsp	27	27	3,362	4,098	4,805	5,910	429.8	554.4	1,351	1,728	61.44	78.46	16,822	21,105
Lp	27	27	15.89	15.90	1.448	1.437	14.98	14.87	16.04	16.10	12.92	13.03	18.22	18.23
Lfwe1	27	27	36.66	33.99	11.56	10.87	28.34	26.05	36.72	33.86	14.92	15.30	63.58	58.56
Lfwe2	27	27	68.07	65.31	5.135	5.126	63.88	60.95	66.89	64.21	60.32	58.33	79.37	76.38
Lfwe3	27	27	80.03	78.62	3.608	3.946	76.44	74.80	80.31	78.82	73.41	71.52	85.86	84.25
Lw	27	27	29.14	146.4	348.0	174.4	14.90	26.09	88.43	142.8	-1,017	-345.4	684.7	521.4
Popg	27	27	0.328	0.261	0.869	0.851	-0.175	-0.315	0.313	0.129	-1.666	-1.271	2.039	2.289
Up	27	27	0.714	0.728	0.125	0.129	0.611	0.627	0.684	0.708	0.522	0.538	0.976	0.979
Yp	27	27	0.159	0.156	0.0174	0.0180	0.145	0.144	0.155	0.152	0.133	0.131	0.204	0.217

Note: elaborated by the author, with Stata 14 software. The displayed data refers both to the first and last observed years of the panel.

Fig. 4: Evolution of the percentage of labor force with an intermediate level of education



The following section will be devoted to the estimation of model (4.1) through several methods, from which the most efficient one will be singled out. There will be a discussion and explanation of the efficiency and drawbacks present at each estimation procedure. Two main goals can be identified. First, we want to assess the overall significance of lw_{it}, which may validate our proposal for the extension of Romer's model to incorporate informational asymmetries. However, due to data unavailability, we will certainly have to consider the differences between the asymptotic properties of the estimators and their finite sample behaviors. Although some small sample corrections may be applied in some cases, the results should definitely be retested in future research. Second, assuming the statistical significance of lw_{it} in explaining aggregate growth, we want to assess its signal. As predicted, the effect of lw_{it} on growth should be negative. A proximity to zero should indicate either a very high or low elasticity.

4.2. Interpretation and Discussion

Table 4 displays the estimation results of model (4.1), through ordinary least squares (OLS), fixed effects (FE) and random effects (RE) estimation procedures. All estimates are corrected for heteroskedasticity and random patterns of autocorrelation among countries. Furthermore, those estimates were calculated with year dummy variables, to make the assumption of no correlation across individuals in the idiosyncratic disturbances made by the robust estimates of the coefficient standard errors more likely to hold. The choices made regarding the explanatory variables were influenced by the descriptive statistics - the major effect, regarding human capital accumulation, appears to lie with the intermediate level of education, which is supported by the literature (Moral-Benito, 2012) – and by issues of collinearity arising due to the percentage of young population and the logarithm of household spending. Furthermore, we've identified an adjustment process in the capital investment share of GDP, hence the use of its first lag.

The OLS estimation of model (4.1) presents highly statistically significant estimates, including for the variable lw_{it}. However, it faces one major problem, given that, by construction, $y_{i,t-1}$ is endogenous to the error term u_{it}: because $y_{i,t}$ is a function of the unobserved heterogeneity μ_i, it follows that $y_{i,t-1}$ is also a function of μ_i. This is called "dynamic panel bias". The existing positive correlation between the lagged dependent variable and the error term makes the OLS estimator upward biased and inconsistent. The RE GLS estimation is also biased, given that the demeaned transform renders the new lagged variable endogenous to the error term.

The FE estimation, despite eliminating μ_i, does not eliminate dynamic panel bias[19]. Under the Within Groups transformation, $y_{i,t-1}$ becomes $y^*_{i,t-1} = y_{i,t-1} - \sum_{t=2}^{T} y_{i,t-1}/(T-1)$ and the error becomes $u^*_{i,t-1} = u_{i,t-1} - \sum_{t=2}^{T} u_{i,t-1}/(T-1)$, which correlates negatively with $y_{i,t-1}$, by construction. This, therefore, renders the Within estimator of ρ downward biased and inconsistent. However, the estimator will be consistent when $T \to \infty$. For this reason, some authors argue that, when analyzing macro panels, which typically cover a small number of countries N over a large period T, the bias of the Within estimator will not be that large for moderate T (Baltagi, 2005, pp. 135-136). However, that is not the case here. Furthermore, given

[19] An Hausman test on the RE and FE estimates indicated that the cross-sectional differences are systematic, hence justifying the need to worry about dynamic panel bias and eliminating the fixed effect.

Table 4: Estimation Results of the Least Squares Methods

	OLS	Random Effects	Fixed Effects
Log. GDPpc(t-1)	1.0106***	0.9995***	0.6704***
	(0.0118)	(0.0155)	(0.0532)
Investment share(t-1)	-0.3223***	-0.3301***	-0.0424
	(0.0753)	(0.1087)	(0.3212)
Lw	-0.0000**	-0.0000	-0.0000
	(0.0000)	(0.0000)	(0.0000)
Labor Education 2	-0.0001	0.0001	-0.0016
	(0.0005)	(0.0006)	(0.0019)
Population Growth	-0.0070		
	(0.0052)		
Urban Population		-0.0048	-0.5038
		(0.0555)	(0.3233)
R^2	0.99	.	0.93
RMSE	0.03	0.03	0.03
N	148	148	148

* p<0.1; ** p<0.05; *** p<0.01

that the OLS estimates are upward biased and the FE estimates are downward biased, it follows that the "true" value of the parameter ρ must belong to the interval $]0.6704, 1.0106[$. This gives us a useful check on the results from the other estimators.

While the dynamic panel bias problem could, in principle, be solved by instrumental variables estimation (2SLS), the estimators would be biased if we faced ourselves with weak instruments. Furthermore, following Baum et al (2003), we've performed a White test in order to exclude homoskedasticity, rendering the GMM-based methods preferable to IV, *ceteris paribus*. The difference GMM (dGMM) estimator introduces lagged levels of the endogenous variables, rendering them predetermined instead. The optimal estimators are obtained by solving a minimization problem in a system of moment conditions, where too many instruments can lead to overidentification of the system, with too many algebraic solutions and, ultimately, inefficient estimates. It then uses the first-differences transform to purge the fixed effects. Table 5 displays the GMM's estimation results. The results suggest high statistical significance, in particular, for the capital investment share from the previous period, which presents a negative relation with current period's logarithm of real GDP per capita. We've predicted the residuals of the first stage regression of the capital investment share on its first lag and used them on the full OLS equation. A t-test on the coefficient of the residuals resulted in a rejection of the null hypothesis, thereby rendering the investment share endogenous, along with the lagged dependent variable. The sign of the lagged investment share stays unchanged across the estimations. Assuming that the aggregate capital investment is financed by credit growth in the economy – a necessary assumption for the validity of our proposed extension of Romer's model –, the negative sign of the estimate may be understood in light of Leitão (2012), whose results suggest that credit growth weakens the banking system, hence weakening the overall economy. The benefits of intermediate education of the labor force on economic growth might then be conditioned by the amount of credit in the banking system. Relating with our proposed growth theory, this could yield some useful information on the nature of the different marginal relations between asymmetries and growth discussed in section 3.3. Nonetheless, the coefficient of the labor force with an intermediate level of education holds no statistical significance across the different estimations. However, given the underlying panel's dimension and Moral-Benito's (2012) rule of thumb for inference validity, one should, at the very least, be skeptic about generalizing these relations to periods other than those following some great financial distress. Furthermore, both the results for population growth and the initial income appear to be consistent, in direction, with other results such as the FE LSDV model for the unrestricted model of Islam (1995).

The efficiency of the difference GMM estimator increases with T. However, it has been shown to perform poorly on persistent series with small T (Baltagi, 2005, pp. 147-148). The system GMM (sGMM) estimator, on the other hand, has high efficiency gains relative to dGMM, as $\rho \to 1$. However, the estimate obtained for ρ does not lie within the credible range – values above 1 suggest an unstable dynamic, with accelerating divergence away from equilibrium values. The dGMM estimate, however, not only lies within the credible range, but is actually in pair with the estimates of Islam (1995) for the OECD sample. Both estimates were subject to a small-sample correction to the covariance matrix estimate, resulting in t instead of z test statistics for the coefficients and an F instead of Wald χ^2 test for overall fit, which tends to over-reject the null as a result of small sample sizes (Roodman, 2009). We've instrumented

Table 5: Estimation Results of the Generalized Method of Moments' Methods

	sGMM	dGMM
Log. GDPpc(t-1)	1.0737***	0.9553***
	(0.0207)	(0.0775)
Investment share(t-1)	-0.6098***	-0.8980***
	(0.0800)	(0.1412)
Lw	-0.0001***	-0.0001***
	(0.0000)	(0.0000)
Urban Population	0.0763**	-0.8255
	(0.0352)	(0.5218)
Labor Education 2	0.0013**	0.0025
	(0.0006)	(0.0029)
Population Growth	-0.0372***	-0.0251
	(0.0118)	(0.0287)
Sargan test	0.00	0.41
N	148	129

* p<0.1; ** p<0.05; *** p<0.01

both $lgdp_{it}$ and the first lag of $invshare_{it}$ with second order levels, which are exogenous by construction. This means that Stata created 7 instruments for $lgdp_{it}$ and 6 instruments for $invshare_{it}$. The remaining regressors were considered exogenous, along with 7 time dummies – the first two rows of each country are eliminated in the equation in levels – making a total of 23 instruments in the dGMM estimation. Here, the Sargan test for overidentifying restrictions did not reject the null hypothesis, with $Prob > Chi2 = 0.155$, which attests for the validity of these instruments. We've rejected the null hypothesis in the sGMM estimates, hence rendering the instruments invalid. Furthermore, the Arellano-Bond test for second-order autocorrelation – which is the one of interest, given the levels used as instruments for the endogenous regressors – did not reject the null hypothesis of inexistence of autocorrelation, with $Prob > z = 0.990$. As expected, in both models we've rejected the null of first order autocorrelation, which is built in the model by default. Finally, we've applied the forward orthogonal deviations transform instead of first differencing to eliminate the country-specific effect, following the work of Hayakawa (2009) Theoretically, we believe that the results render the estimates obtained through dGMM technically superior. As expected, $\hat{\rho} \in]0.6704, 1.0106[$.

The estimate for the coefficient of lw_{it} renders the informational asymmetries between lenders and borrowers irrelevant – i.e., close to null - for aggregate growth, with a high degree of statistical significance, during the analyzed period. In light of our proposal for the expansion of Romer's model, we would then expect the sign of the coefficient to be negative. The results from table 5 suggest a high statistically significance for lw_{it} and, as expected, a negative effect, which may suggest the empirical validity of the proposed financial parameter in explaining economic growth.

We've started the analysis by identifying the two main goals. First, we wanted to assess the overall significance of lw_{it}, in order to validate our proposal for the extension of Romer's model to incorporate information asymmetries. Second, we wanted to assess the direction of the effect of these financial asymmetries on growth. The results were, in part, conclusive.

The estimates present somewhat unanimous results regarding the impact of lw_{it} on growth. Our theoretically superior estimator dGMM finds high statistical significance for lw_{it}. The impact of lw_{it} on economic growth is negative, albeit close to null. The results hence indicate that, not only is the financial parameter statistically significant, as it also has the predicted sign. This suggests that the theoretical framework may, at least, be close to the truth.

These results should not, however, stand without scrutiny. Data limitations forced the analysis to be limited to the post-crisis period, with a limited panel dimension. The asymptotic properties of the GMM estimators have been shown to suffer from finite-sample biases. Despite the careful choice of the appropriate estimation methods and the small-sample correction to the covariance matrix estimate, there may be efficiency gains in expanding the analysis in order to cover a greater number of countries over a greater period of time. Ideally, future data sets will be large enough to analyze pre and post crises relations between informational asymmetries and aggregate growth. Furthermore, the employment of Bayesian analysis – and, quite specifically, Bayesian Model Averaging (BMA) – could potentially enrich the obtained results. Looking at Moral-Benito (2012), one could apply the BACE-SDM approach to the dynamic panel in order to obtain the parameters' posterior probabilities, after obtaining the models' posterior probabilities through Markov Chain Monte Carlo simulations. However, this approach must necessarily be based on least squares estimation of the dynamic panel data model (3.29) – for efficiency purposes, through the FE model – in order to obtain a value for each model's sum of squared errors. As we stand, the analysis would be inefficient, given the small T defining our panel and, very possibly, the lack of studies to base our $l(\overline{w})$'s prior assumption on. We address this thought to future research.

5. Final Remarks

The main purpose of this research was to contribute to a growing body of literature on the effects of financial frictions on the real economy. Having built a theoretical model, we then wanted to test it empirically, ideally giving it validity. To the best of our knowledge, this study is the first one expanding Romer's model such as to include financial imperfections, and the first one presenting an equilibrium long term growth rate as a function of such imperfections.

To address the matter of building a theoretical model, we've analyzed several research studies from several authors. Romer's model is widely viewed as the state of art of R&D-based endogenous growth models. Although there are some DSGE and business cycle models in the literature that might have had some potential to address this particular study, we felt, from the analysis of studies such as Morales (2003), that the outline of Romer's model would be more appropriate and easy to work with. Through a process of analyzing various models with financial markets and studying their differences and similarities with our baseline growth model, BGG's study became our choice for the proposed expansion, due to the its importance in the literature and to the overall perceived feasibility.

The full expanded model sheds some light over the interaction between agent-based frictions and aggregate economic growth and shows how the marginal relation changes with fluctuations on the periodical payments of outstanding debt, aggregate capital amount and/or the discount rate of the economy. To the best of our knowledge, the main contribute of this study is to shed some light on the channels through which these particular financial frictions influence economic growth. Notwithstanding, the financial idiosyncratic shocks remain exogenous to the model which is, in this author's opinion, a downside of the proposed expansion. Further research should address the challenge of endogenizing these financial shocks. Studies such as that of Gerke et al (2013) may suggest some potential in the introduction of monetary policy to address this gap, as well as to enable the debate on policy implications, in the context of our expanded version of Romer's model.

From the econometric analysis, we were able to prove the statistical significance of the financial parameter in the BGP growth solution. The regression results were similar to the simulation results, i.e., to what one would expect under the assumption of a normal distribution of the contractual risks. The sign of the coefficient's estimate was in line with the predictions made in subsection 3.3. The estimations were optimized through GMM methodology, which are sensitive to small sample biases. Future empirical research should be made with a wider data set, in order to strengthen the results' robustness. The analysis of pre and post financial crisis data should yield some important information on the mechanics of the proposed inflection point, and on possibly hidden effects that may help endogenizing financial shocks, in future theoretical models.

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth.* Massachusets: Princeton University Press.

Aghion, P., Howitt, P., & Mayer-Foulkes, D. (2005). The Effect of Financial Development on Convergence: Theory and Evidence. *The Quarterly Journal of Economics, 120(1)*, pp. 173-222.

Alfaro, L., Chanda, A., Kalemli-Ozcan, S., & Sayek, S. (2004). FDI and economic growth: the role of local financial markets. *Journal of International Economics, 61(1)*, pp. 89-112.

Andrews, D., Criscuolo, C., & Gal, P. (2015). Frontier Firms, Technology Diffusion and Public Policy: Micro Evidence from OECD Countries. *OECD Productivity Working Papers, No. 2*.

Arrow, K. J. (1962). The Economic Implications of Learning by Doing. *The Review of Economic Studies, 29(3)*, pp. 155-173.

Baltagi, B. (2005). *Econometric Analysis of Panel Data.* England: John Wiley & Sons, Ltd.

Barro, R. (1991). Economic Growth in a Cross Section of Countries. *The Quarterly Journal of Economics, 106(2)*, pp. 407-443.

Baum, C., Schaffer, M., & Stillman, S. (2003). Instrumental Variables and GMM: Estimation and testing. *The Stata Journal 3(1)*, pp. 1-31.

Becker, G. (1964). *Human Capital.* Chicago: University of Chicago Press.

Bernanke, B., Gertler, M., & Gilchrist, S. (1999). The Financial Accelerator in a Quantitative Business Cycle Framework. *Handbook of Macroeconomics, 1(1)*, pp. 1341-1393.

Blanchard, O. (2018). On the future of macroeconomic models. *Oxford Review of Economic Policy*, pp. 43-54.

Blundell, R., & Bond, S. (1998). Initial conditions and moment restrictions in dynamic panel data models. *Journal of Econometrics, 87(1)*, pp. 115-143.

Borensztein, E., De Gregorio, J., & Lee, J.-W. (1998). How does foreign direct investment affect economic growth? *Journal of International Economics, 45(1)*, pp. 115-135.

Borio, C. (2018). A blind spot in today's macroeconomics? *Weak Productivity: the role of financial factors and policies* (pp. 1-14). Paris: Bank of International Settlements.

Brunnermeier, M., & Sannikov, Y. (2017). Macro, Money and Finance: A ContinuousTime Approach. *Handbook of Macroeconomics*, pp. 1497-1546.

Brunnermeier, M., Eisenbach, T., & Sannikov, Y. (2013). Macroeconomics with Financial Frictions: A Survey. In D. Acemoglu, M. Arellano, & E. Dekel, *Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society* (pp. 4-94). New York: Cambridge University Press.

Carlstrom, C., & Fuerst, T. (1997). Agency Costs, Net Worth, and Business: A Computable General Equilibrium Analysis. *American Economic Review, 87(5)*, pp. 893-910.

Christiano, L., Eichenbaum, M., & Evans, C. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy, 113(1)*, pp. 1-45.

Claessens, S., & Kose, M. (2018). Frontiers of macrofinancial linkages. *BIS Papers*, Bank for International Settlements, number 95, July.

Claessens, S., & Kose, M. A. (2013). Financial Crises: Explanations, Types and Implications. *IMF Working Papers 13/28, International Monetary Fund*.

Davidsson, M. (2012). Economic Growth and Risk Aversion. *European Journal of Social Sciences, 28(1)*, pp. 92-100.

Durusu-Ciftci, D., Ispir, M., & Yetkiner, H. (2017). Financial development and economic growth: Some theory and more evidence. *Journal of Policy Modeling, 39(2)*, pp. 290-306.

Duval, R., Hong, G., & Timmer, Y. (2017). Financial Frictions and the Great Productivity Slowdown. *IMF Working Papers 17/129*, International Monetary Fund.

Ehnts, D. (2017). *Modern Monetary Theory and European Macroeconomics.* New York: Routledge.

Epure, M., & Lafuente, E. (2015). Monitoring bank performance in the presence of risk. *Journal of Productivity Analysis, 44(3)*, pp. 265-281.

Evans, G., Honkapohja, S., & Romer, P. (1998). Growth Cycles. *American Economic Review, 88(3)*, pp. 495-515.

Fu, J. (1996). The Effects of Asymmetric Information on Economic Growth. *Southern Economic Journal, 63(2)*, pp. 312-326.

Gerke, R., Jonsson, M., Kliem, M., Kolasa, M., Lafourcade, P., Locarno, A., . . . McAdam, P. (2013). Assessing macro-financial linkages: A model comparison exercise. *Economic Modelling, 31*, pp. 253-264.

Grant, J., Hoorens, S., Sivadasan, S., van het Loo, M., DaVanzo, J., Hale, L., . . . Butz, W. (2004). *Low Fertility and Population Ageing.* Santa Monica: RAND Corporation.

Greenwald, B. C., & Stiglitz, J. E. (1993). Financial Market Imperfections and Business Cycles. *The Quarterly Journal of Economics, 108(1)*, pp. 77-114.

Guimaraes, T., Brandon, B., & Guimaraes, E. (2010). Empirically Testing Some Major Factors for Bank Innovation Success. *Journal of Performance Management, 23(2)*, pp. 34-46.

Hall, R. E. (2013). Financial Frictions. *New Frameworks for Monetary Policy Analysis in an Era of Crises* (pp. 155-163). Chile: International Journal of Central Banking.

Hayakawa, K. (2009). First Difference or Forward Orthogonal Deviation- Which Transformation Should be Used in Dynamic Panel Data Models?: A Simulation Study. *Economics Bulletin 29(3)*, pp. 2008-2017.

Hernández, A., & Santos, M. S. (1996). Competitive Equilibria for Infinite-Horizon Economies with Incomplete Markets. *Journal of Economic Theory 71(1)*, pp. 102-130.

Islam, N. (1995, November). Growth Empirics: A Panel Data Approach. *The Quarterly Journal of Economics, 110(4)*, pp. 1127-1170.

Jain, N. (2001). Monitoring costs and Trade Credit. *The Quarterly Review of Economics and Finance, 41(1)*, pp. 89-110.

Jokivuolle, E. (2017). *Preparing for the Next Financial Crisis.* Cambridge: Cambridge University Press.

Kiyotaki, N., & Moore, J. (1997). Credit Cycles. *Journal of Political Economy 105(2)*, pp. 211-248.

Leitão, N. (2012). Bank Credit and Economic Growth: A Dynamic Panel Data Analysis. *The Economic Research Guardian, 2(2)*, pp. 256-267.

Lucas, R. (1988). On the Mechanics of Economic Development. *Journal of Monetary Economics 22(1)*, pp. 3-42.

Mankiw, N., Romer, D., & Weil, D. (1992). A Contribution to the Empirics of Economic Growth. *The Quarterly Journal of Economics, 107(2)*, pp. 407-437.

Meier, A., & Müller, G. (2006). Fleshing out the monetary transmission mechanism - output composition and the role of financial frictions. *Journal of Money, Credit and Banking 38 (8)*, pp. 2099-2133.

Moral-Benito, E. (2012). Determinants of Economic Growth: A Bayesian Panel Data Approach. *Review of Economics and Statistics, 94(2)*, pp. 566-579.

Morales, M. (2003). Financial Intermediation in a Model of Growth Through Creative Destruction. *Macroeconomic Dynamics 7(3)*, pp. 363-393.

Noe, T., & Vulkan, N. (2017). The Role of Personality in Financial Decisions and Financial Crises. In E. Jokivuolle, *Preparing for the Next Financial Crisis* (pp. 139-156). Cambridge: Cambridge University Press.

Patinkin, D. (1989). Walras's Law. In J. Eatwell, M. Milgate, & P. Newman, *General Equilibrium* (pp. 328-339). London: Macmillan Press Limited.

Phelps, E. S. (1966). Models of Technical Progress and the Golden Rule of Research. *The Review of Economic Studies 33(2)*, pp. 133-145.

Pietra, T., & Siconolfi, P. (1996). Equilibrium in Economies with Financial Markets: Uniqueness of Expectations and Indeterminacy. *Journal of Economic Theory 71(1)*, pp. 183-208.

Romer, D. (2012). *Advanced Macroeconomics.* New York: McGraw-Hill.

Roodman, D. (2009). How to do xtabond2: An introduction to difference and system GMM in Stata. *Stata Journal, 9(1)*, pp. 86-136.

Shell, K. (2010). A Model of Inventive Activity and Capital Accumulation. *Levine's Working Paper Archive*, David K. Levine.

Solow, R. (1956). A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics 70(1)*, pp. 65-94.

Thompson, M. (2008). Journal of Economics. *Complementarities and costly investment in a growth model 94(3)*, pp. 231-240.

Thompson, M. J. (2003). Endogenous Growth: Theoretical Investigations and Developments (Doctoral thesis). *University of Warwick*.

Townsend, R. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory, 21(2)*, pp. 265-293.

Trivedi, S. R. (2015). Banking Innovations and New Income Streams: Impact on Banks' Performance. *SAGE Publications 40(1)*, pp. 28-41.

Villanaci, A., Carosi, L., Benevieri, P., & Battinelli, A. (2002). *Differential Topology and General Equilibrium with Complete and Incomplete Markets.* Boston: Springer Science+Business Media Dordrecht.

Appendix A

In order to solve system (3.26) with the accumulation of capital over an infinite horizon, we need to study the wealth allocations within the economy. Let's start by analyzing the following equation

(A.1) $$B(t) = K(t) + P_A(t)A(t)$$

where $B(t)$ represents the assets from the representative consumer (lender), $K(t)$ the stock of physical capital and $P_A(t)A(t)$ the holding of shares on capital goods. The corresponding law of motion of the consumer's assets is given by

(A.2) $$\dot{B(t)} = rB(t) + r\beta(t)P_A(t)A(t) + w(t)L(t) - C(t) - \int_0^t (P_A(\tau) \ - \beta(\tau)P_A(\tau))d\tau$$

where $w(t)$ represents the consumer's wage, $L(t)$ the working time and $C(t)$ the consumption. In each moment in time, consumers build a well-diversified portfolio with firm loans, which is why their opportunity cost is the risk-free rate. The functional form is in line with Durusu-Ciftci et al (2017), given the separation of the households' funds in two kinds of investment. Because of the finance rule built in the borrower's optimization problem, equation (A.2) is consistent with firms' behavior.

Substituting (A.1) in (A.2) and solving for the evolution of the stock of physical capital, we get

(A.3) $$\dot{K(t)} = rK(t) + \left(rP_A(t) - \dot{P_A(t)}\right)A(t) + r\beta(t)P_A(t)A(t) - P_A(t)\dot{A(t)} + w(t)L(t) - C(t) - \int_0^t (P_A(\tau) \ - \beta(\tau)P_A(\tau))d\tau$$

If we take equation (3.16.2), replace $s = \frac{R^k}{r}$ and solve it for r, we get the following

(A.4)
$$r(t) = \frac{\gamma \alpha^2 l(\overline{w})}{(1-\alpha)} \frac{Y(t)}{K(t)}$$

In equilibrium, final goods producers have zero profits, unlike the capital goods producers. In order to participate in the capital goods' market, one has to acquire a patent for the value of $P_A(t)$. After this initial investment, the producer has property rights over its time horizon. Notwithstanding, our representative producer in existence at each moment in time will enjoy property rights over an infinite horizon. The patent price is given by

(A.5)
$$P_A(t) = \int_t^{\infty} \pi_u e^{-\int_t^u r_v dv} du$$

because future cash flows are discounted at a rate that matches the cost of obtaining the necessary funds to finance those cash flows. Therefore,

(A.6)
$$\dot{P_A}(t) = -\left[\pi_u e^{-\int_t^u r_v dv}\right]_t^{\infty} + \int_t^{\infty} \pi_u \left[r(t)\left(e^{-\int_t^u r_v dv}\right)\right] du$$

which is equivalent to the Hamilton-Jacobi-Bellman

(A.7)
$$\pi(t) = r(t)P_A(t) - \dot{P_A}(t)$$

Taking (A.4) and the fact that $K(t) = A(t)x(t)$, by solving for the amount of capital goods we get

(A.8)
$$x(t) = \frac{\gamma\alpha^2 l(\overline{w})}{r(t)(1-\alpha)}\frac{Y(t)}{A(t)}$$

which we can now use in the capital firms' expanded profit function under optimal behavior, yielding

(A.9)
$$\pi(t) = \alpha\frac{Y(t)}{A(t)} - \frac{\gamma\alpha^2 l(\overline{w})}{(1-\alpha)}\frac{Y(t)}{A(t)} - r(t)\beta(t)P_A(t)$$

implying that

(A.10)
$$\dot{K(t)} = rK(t) + \pi(t)A(t) + r\beta(t)P_A(t)A(t) - P_A(t)\dot{A(t)} + w(t)L(t) - C(t) - \int_0^t (P_A(\tau) \ - \beta(\tau)P_A(\tau))d\tau$$

Recalling (3.16.1), that the assumption of a competitive labor market implies $w_Y(t) = w(t) = w_A(t)$ and that the free entry condition in the capital goods market is equivalent to $P_A(t)\dot{A(t)} = w_A(t)L_A(t)$ and, therefore, $P_A(t)\dot{A(t)} = (1-\alpha)Y(t)\frac{L_A(t)}{L_Y(t)}$, we have that

(A.11)
$$\dot{K(t)} = rK(t) + \alpha Y(t) - \frac{\gamma\alpha^2 l(\overline{w})}{(1-\alpha)}Y(t) - P_A(t)\dot{A(t)} + w(t)L(t) - C(t) - \int_0^t (P_A(\tau) \ - \beta(\tau)P_A(\tau))d\tau$$

which is equivalent to

(A.12)

$$\dot{K(t)} = Y(t) - C(t) - \int_0^t (P_A(\tau) \ - \beta(\tau)P_A(\tau))d\tau$$

which allows us to relate the BGP growth rates in (3.36).

Appendix B

Building upon Evans et al (1998), we would introduce complementarities between capital goods by defining the following production function

(B.1)
$$Y(t) = \left(\int_0^A x_j^{\varphi} dj\right)^{\theta}, \qquad \theta > 1, \qquad \theta\varphi = \alpha$$

where the assumption $\theta > 1$ is made so that an increase in the quantity of one good increases the marginal productivity of the other good (Thompson M. , 2008). Furthermore, in order to solve the model for a constant growth rate, we impose the following:

(B.2)
$$\varepsilon = \frac{\theta - 1}{1 - \alpha}$$

From the assumption of a perfectly competitive labor market we can verify that (1.26) continues to hold within this framework. However, because of the new functional form (B.1) for the production function, the rental price for the capital goods is now given by

(B.3)
$$R_j(t) = \frac{\partial Y}{\partial x_j} = \alpha L_Y^{1-\alpha} A_j^{\theta-1} x_j^{\alpha-1}$$

which, rearranged and with symmetry, is equivalent to

(B.4)
$$x(t) = L_Y(t) A^{\varepsilon}(t) \left[\frac{\alpha}{R(t)}\right]^{\frac{1}{1-\alpha}}$$

However, considering equations (1.22) and (1.23), we have that

(B.5)

$$x(t) = L_Y(t)\left[\frac{\alpha}{R(t)}\right]^{\frac{1}{1-\alpha}}$$

which implies that $A^{\varepsilon}(t) = 1$ and, therefore[20], that $\theta = 1$. This contradicts the initial assumption that $\theta > 1$, invalidating the assumption of complementarities between capital goods.

[20] We're excluding the trivial case where $A(t) = 1$.