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Michele Bisceglia
Roberto Cellini
Luigi Siciliani
Odd Rune Straume

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Universidade do Minho
Escola de Economia e Gestão

Optimal dynamic volume-based price regulation*

Michele Bisceglia[†] Roberto Cellini[‡] Luigi Siciliani[§] Odd Rune Straume[¶]

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Abstract

We consider a model of optimal price regulation in markets where demand is sluggish and asymmetric providers compete on quality. Using a spatial model, which is suitable to investigate the health care and education sector, we investigate within a dynamic set-up the scope for price premiums or penalties on volume. We show that the socially optimal time path of quality provision off the steady state can be replicated by a simple dynamic pricing rule where the dynamic part of the rule is *ex-ante non-discriminatory* in the sense that the price premium or penalty on volume is common across providers, despite their differing production costs. Whether the price schedule involves a penalty or a premium on volume relates to two concerns regarding production costs and consumer benefits, which go in opposite directions. Price adjustments over time occur only through the price penalty or premium, not time directly, which highlights the simplicity and thus applicability of this regulation scheme.

Keywords: Price regulation; Quality; Differential games.

JEL Classification: C73, I11, I14, L13.

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[†]Department of Economics, Management and Quantitative Methods, University of Bergamo; Via Caniana 2 - 24127 Bergamo, Italy; E-mail: michele.bisceglia@unibg.it

[‡]Department of Economics and Business, University of Catania; C.so Italia 55, 95129 Catania, Italy; E-mail: cellini@unict.it

[§]Department of Economics and Related Studies, University of York, Heslington, York YO10 5DD, UK; E-mail: luigi.siciliani@york.ac.uk

[¶]Corresponding author. Department of Economics/NIPE, University of Minho, Campus de Gualtar, 4710-057 Braga, Portugal; and Department of Economics, University of Bergen. E-mail: o.r.straume@eeg.uminho.pt

1 Introduction

In markets such as health care or education, prices are commonly regulated across a range of OECD countries. Providers instead compete for consumers on product quality. But quality is not easily observable to regulators, although one of the key objectives of regulation is to improve quality or to achieve an adequate level of quality. Instead, regulators observe volumes (e.g., patients, pupils, students) and these can be used indirectly to incentivise quality improvements.

In this study, we investigate optimal price regulation under the assumption that demand is *sluggish*, and it is modelled such that only a fraction of consumers respond to quality changes at each point in time. This implies that it will take some time before potential demand is fully realised. Sluggish demand is a plausible assumption in health care and education, due to asymmetry of information between providers and consumers (Arrow, 1963) and related uncertainty and noise in the observed quality, or due to consumer habits, trust or confidence in particular providers.[1]

In markets where demand responds sluggishly to changes in quality, adjustments to any type of shock (e.g., exogenous cost changes or entry of new providers) are likely to take a long time, implying that such markets might seldom be characterised by allocations of qualities and demand that are close to a steady state outcome. This implies, in turn, that a price regulation scheme based on theoretical insights from static equilibrium analyses is unlikely to produce a socially optimal outcome. What is instead called for is a dynamic analysis that allows for a characterisation of the equilibrium (as well as the socially optimal) dynamic path off the steady state.

Finding simple rules for price regulation that induce a more efficient quality provision is a challenge for regulation authorities, even in a static context. Dynamic effects of price regulation on volume and quality, due to demand sluggishness, make the challenge considerably more difficult, and call regulation authorities to face dynamic programming problems. The main contribution of the present study is to characterise the properties of a specific form of dynamic price regulation, in which the unit price paid to the provider is linear in demand (or more precisely is an affine function of demand). We suggest a price regulation scheme that allows for the price to increase with demand, a form of price *premium* on volume, or to decrease with demand, a form of price *penalty* on volume. We argue that this form of price regulation is not only simple and thus highly applicable, since volume is easily observable while quality is not, but it is also potentially highly

[1] See, e.g., Jung et al. (2011) and Raval and Rosenbaum (2018) for empirical evidence of demand sluggishness in hospital markets.

efficient. Under some assumptions, we show that a welfare maximising regulator, by optimally choosing a price rule of this kind, would be able to induce the socially optimal dynamic path of quality provision, thus ensuring a socially optimal outcome at each point in time.[2]

Interestingly, this kind of volume-based price regulation is also in line with some real-world examples of price regulation in health care markets. An early example is what were known as 'cost and volume' contracts when the purchaser-provider split was introduced in England in the early nineties under the internal markets. Hospitals facing higher-than-agreed volumes would face a reduction in the DRG tariff of up to 30% of the of agreed tariff below the first volume threshold, and with the tariff gradually decreasing with higher volume thresholds (Fenn et al., 1994). In recent years, possibly following recent budgetary restrictions, concerns over excess expenditure have again led purchasers (currently known as commissioners) to make increased use of volume caps or reduced tariffs for volumes in excess of expected ones (Allen and Petsoulas, 2016).[3] There are current proposals to introduce 'blended' payments that comprise a fixed amount (linked to expected levels of activity) and a variable volume-related element that reflects actual levels of activity, where the variable element could involve a payment as low as 20% of the regular tariff (NHS Improvement, 2018). Is such a price regulation scheme, with penalties for higher volumes, likely to be socially efficient in a dynamic sense? Or would instead social welfare improve by the adoption of a pricing scheme that *rewards* higher volumes? In the present study we identify some general conditions that can answer these questions.

As a basic building block for our analysis, we use a Hotelling model of quality competition under regulated prices. In case of health care or education markets, a spatial competition framework is particularly appropriate for several reasons. These are markets where demand decisions are very much in line with the unit demand assumption (where consumers demand one medical treatment or one school admission, for example), and where travel distance is a key factor (in addition to quality) in determining consumers' choice of provider.[4] The standard Hotelling assumption of fixed

[2] Although we focus on the health care and education sectors, the form of price regulation investigated in this study may also apply to other regulated sectors such as telecommunication, transport and energy.

[3] There are elements of this form of price regulation also in other countries. For example, in Germany or the Netherlands, the DRG-based payment systems operate within a global budget. If the global budget is reached, hospitals receive a reduced tariff up to no payment. Countries such as France, Poland and Hungary have created Macro-Level Price Control mechanisms, which again assure hospital expenditures remain within a national hospital budget. This is usually achieved by reducing the monetary conversion factor by a certain amount leading to a proportionate reduction of the monetary value for every DRG cost weight or score (Moreno-Serra and Wagstaff, 2010).

[4] There are many empirical studies confirming that travelling distance and quality are key predictors of the choice of hospitals (Gutacker et al., 2016; Kessler and McClellan, 2000; Tay, 2003) and schools (Chumacero et al., 2011; Gibbons et al., 2008; Hastings et al., 2005).

total demand is also a reasonable approximation to markets with non-price competition and small (or even zero) consumer copayments, implying that total demand is highly inelastic.

We consider an infinite-time horizon differential game with two providers offering one product each. Providers are located at the extremes of a unit line. We therefore consider spatial locations as given, as is the case for schools or hospitals. We allow providers to differ in production costs (e.g., due to differences in land, capital and labour costs), and such differences are observable to the regulator. In turn, such differences in costs can affect the optimal regulated price, which we also allow to differ across providers. Given that the model is dynamic and that observed demand for each firm varies over time, the price paid to each provider also varies over time through changes in demand. Our focus is to determine whether the optimal price regulation scheme involves a premium or a penalty on volume.

Before moving to optimal price regulation, we first characterise the optimal provider plans regarding product quality over time and consider both what is known in differential games as (i) the open-loop solution, where firms set their quality plans at the beginning of the game, and quality only depends on time along the optimal path, and (ii) the state-feedback solution, where current choice of quality depends on current demand (the state variable) which is observable by firms.[5] Both solutions are plausible depending on the institutional context. The former solution concept is a more likely representation of a setting where providers can (or must) commit to long-term plans when deciding on quality investments, e.g., providers are public (schools or hospitals) and act within a heavily regulated environment. The latter solution concept is more applicable in deregulated markets under any-willing-provider clauses, where private (for profit and non-profit) providers have more discretion and independence over investment decisions.

This part of our analysis is positive, and we explore how equilibrium quality is affected in the presence of a price premium or penalty on volume. Under both solution concepts, the introduction of a price *premium* on volume increases quality and amplifies differences in quality and demand between providers. Conversely, the introduction of a price *penalty* on volume reduces quality and dampens differences in quality and demand between providers. Moreover, we show that quality is generally lower in the steady state under the state-feedback decision rule, when providers cannot

[5] The different solution concepts (open-loop vs. state-feedback) also correspond to different information sets used by players in each instant of time when the game takes place. Under the open-loop rule, only the initial condition and the time are used; under the state-feedback rule, the current state of the world is considered. For a simple introduction to the different solution concepts of differential games, see Basar et al. (2018) or Lambertini (2018); see also Dockner et al. (2000).

commit to long-term investment plans, and this holds also in the presence of price penalties or premiums on volume. Quality differences across providers with differing production costs are also smaller in the state-feedback solution. Therefore, the more intense dynamic strategic interaction in the state-feedback solution contributes to dampening both quality and quality differences across providers.

We then proceed by taking a social welfare perspective and deriving the optimal price regulation rule. This part of our analysis provides the most novel insights. We show that the socially optimal time path of quality provision off the steady state can be replicated by a dynamic pricing rule where the dynamic part of the rule is *ex-ante non-discriminatory*, in the sense that the price premium or penalty on volume is common across providers, despite their differing production costs. Instead, the fixed price component, which does not vary with volume, differs across providers to reflect different costs, with a higher value for the provider with lower marginal production costs. Price adjustments over time occur only through the price penalty or premium on volume, not time directly, which highlights the simplicity and thus applicability of this regulation scheme.

Whether the price schedule involves a penalty or a premium on volume relates to two concerns regarding production costs and consumer benefits, which go in opposite directions. Under the assumption of decreasing returns to scale, concerns for cost-efficient production dictate that demand should be steered towards the provider with lower demand, which can be achieved by lower quality investments by the high-demand provider. Instead, concerns for consumer welfare dictate that the high-demand provider should invest more in quality, implying that demand is steered away from the low-demand provider. If the former concern for cost efficiency dominates, so that welfare is increased by reducing (increasing) the quality investments of the high-demand (low-demand) provider, this can be achieved by introducing a price penalty on volume, which reduces the price-cost margin, and thus incentives for quality investments, of the high-demand provider relative to the low-demand provider. This is optimal if the convexity of production costs is sufficiently high, which implies that concerns for production cost efficiency are sufficiently strong.

The rest of the study is organised as follows. In Section 2 we provide a brief overview of the relevant literature and specify how our study contributes to this literature. The assumptions of the model are presented in Section 3. In Section 4 we derive the providers' optimal choice of quality both under the open-loop and the state feedback solution concepts. In Section 5 we introduce the welfare analysis by deriving and characterising the first-best solution, and in Section 6 we show

how this solution can be implemented by an optimally chosen dynamic price rule. Some concluding remarks are given in Section 7.

2 Related literature

Our analysis relates to the large body of economic literature analysing different types of regulatory mechanisms to stimulate efficient quality provision.[6] A starting point of this literature is the insight developed by Spence (1975) and Sheshinski (1976), who show that an unregulated monopolist is unlikely to provide a socially optimal level of product quality and that some form of regulation might be called for.[7] A large bulk of the subsequent literature has focused on the effects of direct quality regulation (e.g., in the form of minimum quality standards) in the context of a vertical differentiation framework, with or without competition.[8]

Our study is more closely related to a different strand of this literature, focusing on quality competition in a *spatial* framework. Under the assumption of fixed provider locations, Ma and Burgess (1993) show how socially optimal quality provision can be achieved by price regulation in such a setting. Brekke et al. (2006) extend this analysis to the case where firms' locations are endogenous, which implies that the socially optimal outcome cannot be achieved by simple price regulation. However, Bardey et al. (2012) show that efficient quality provision can be restored if price regulation is optimally combined with provider cost reimbursement. A further extension to this type of analysis is made by Mak (2018), who considers a model with multidimensional quality and a richer set of regulatory tools, including reference pricing and pay-for-performance bonuses.[9]

Our main contribution to the above mentioned literature is that we identify *demand sluggishness* as an independent source of inefficiency, which introduces a dynamic dimension to the optimal price regulation problem. In this sense, our model is most closely related to Brekke et al. (2012), who investigate quality decisions under price regulation with sluggish demand. We expand their analysis in two key dimensions. First, we adopt a more general model where production costs differ across firms rather than being uniform. Production costs are unlikely to be symmetric; they tend to differ

[6] See, e.g., Sappington (2005) for a broad survey of this literature.

[7] Sheshinski (1976) considers both price and quality regulation as alternative means to remedy the problem, whereas Spence (1975) suggests rate-of-return regulation as an attractive alternative due to the informational problems faced by real-world regulators.

[8] See, e.g., Besanko et al. (1987) for the case of a multi-product monopolist, and Ronnen (1991) and Crampes and Hollander (1995) for the case of competition between vertically differentiated duopolists.

[9] Another key contribution to this strand of the literature, using the same type of spatial competition framework, is Wolinsky (1997), who compares the efficiency of two different regulatory schemes, namely managed competition and regulated monopolies.

across providers due to different input costs (labour and capital) reflecting local market conditions. Second, we investigate a plausible form of price regulation, involving a price premium or penalty on volume, which has not been previously considered in the literature. In Brekke et al. (2012) price regulation mostly focuses on the steady state, where volume is fixed, and when price regulation is considered off the steady state, it depends on time, not volume.[10] However, regulators are unlikely to be able to commit to a price rule which depends directly on time. Instead, we argue that they are more likely to be able to commit to price regulation which depends on the volumes observed, without having to specify explicitly a continuum of prices over time (see also Bisceglia et al., 2019b). In our model, the regulator only needs to specify two price related values for each provider.

Our analysis also relates more broadly to the literature on quality competition in a dynamic context (see Brekke et al., 2018, for a review). Brekke et al. (2010) assume that demand adjusts instantaneously to quality, but quality is a stock variable which increases if its investment is higher than its depreciation rate. They show that if prices are regulated and marginal costs are increasing in volume, quality is lower under the feedback solution, which is arguably the solution concept when competition is more intense, than under the open-loop solution, when providers can commit to optimal quality plans at the beginning of the game. An analogous result is obtained by Cellini et al. (2018) when providers also compete on price in addition to quality: competition reduces price, and by doing so, it induces quality to be lower under the feedback solution even when marginal production costs are linear. In a model with regional regulators and asymmetric providers, Bisceglia et al. (2019a) show that the above result does not hold when the price which applies to the extra-regional demand, set by a national authority, is sufficiently high, since in that case, more efficient providers have strong incentives to attract consumers from another region.

3 The model

We consider a market with two competing providers located at either end of a Hotelling line $S = [0, 1]$, populated by a uniform distribution of individuals, with total mass normalised to 1.[11] At time t, each consumer demands one unit of service from one of the providers. Since prices are regulated and paid by a third party, consumer's choice of provider is based on provider quality and

[10] Siciliani et al. (2013) also develop a dynamic model with sluggish demand, when providers are altruistic or intrinsically motivated, and follow a similar approach to Brekke et al. (2012) in terms of optimal price regulation.

[11] This is a widely used framework for analysing quality competition in health care markets; see, e.g., Calem and Rizzo (1995), Beitia (2003), Brekke et al. (2007), Karlsson (2007). A similar framework is used by Del Rey (2001) to study quality competition in education markets.

travelling costs. Let $q_i \geq \underline{q}$ denote the quality offered by Provider i, $i = 1, 2$, and let the marginal cost of travelling be given by τ.[12] The lower bound $\underline{q}$ represents the minimum quality providers are allowed to offer (see, e.g., Cellini and Lamantia, 2015) and is, for simplicity, set to zero. If a consumer located at $x \in [0, 1]$ buys the service offered by Provider i, located at $z_i \in \{0, 1\}$, the utility of this consumer is assumed to be given by

$$U(x, z_i) = v + q_i - \tau \left| x - z_i \right|, \tag{1}$$

where $v > 0$ is the utility of consuming one unit at minimum quality without having to travel. We assume that v is high enough for the market always to be covered (i.e., each consumer prefers to demand one unit from the most preferred provider rather than to forgo consumption).

If each consumer makes a utility-maximising choice of provider, the *potential demand* of Provider i at time t is given by

$$D_i^* (t) = \frac{1}{2} + \frac{q_i (t) - q_j (t)}{2\tau} \tag{2}$$

with $i, j = 1, 2$ and $j \neq i$. However, under the assumption that consumers have *sluggish beliefs* about quality, the actual demand might differ from the potential demand for each provider (see Brekke et al., 2012). Denoting the *actual demand* of Provider i at time $t \in [0, \infty)$ by $D_i (t)$, we assume that this demand evolves according to the following linear ordinary differential equation (henceforth ODE):

$$\dot{D}_i(t) = \gamma(D_i^*(t) - D_i(t)), \tag{3}$$

where $\gamma \in [0, 1]$ is an inverse measure of the degree of demand sluggishness. More specifically, we can interpret γ as the fraction of consumers who, at each point in time, become aware of a previous change in the quality difference between the two providers, and therefore re-optimise their choice of provider. The lower this fraction is, the more sluggishly demand responds to quality changes over time. This formulation of demand sluggishness implies that (actual) demand is a state variable in our model. Given the assumptions of unit demand and full market coverage, the demand for Provider j is given by $D_j (t) = 1 - D_i (t)$, which implies that the dynamic evolution of both providers' demand is described by (3).

Providers are assumed to be profit oriented, with the same (constant) preference discount rate

[12] It is straightforward to show that all the results in the paper would remain unchanged if we assume that travelling costs are quadratic instead of linear in distance. Details are available upon request.

$\rho > 0$. Denoting by $p_i(D_i(t))$ the unit price received by Provider i, its instantaneous profit is given by

$$\pi_i(t) = p_i(D_i(t))D_i(t) - c_i D_i(t) - \frac{\beta}{2}[D_i(t)]^2 - \frac{\theta}{2}[q_i(t)]^2, \tag{4}$$

where $\theta > 0$, $c_i > 0$ and $\beta > 0$. We thus allow the two providers to differ in terms of production costs due to efficiency and other cost factors (such as differences in land, capital and labour costs), measured by the cost parameter c_i. We instead assume that the cost of quality, as captured by the parameter θ, is homogeneous across providers and relates to investment in machines and capital which can be purchased from suppliers at competitive and uniform rates. We also assume that the cost function is strictly convex in output, reflecting the presence of (smooth) capacity constraints, for example in the form of congestion costs that, for a given level of demand, are also similar across providers.[13] To keep the presentation of the model simpler we assume that the marginal cost of output (quality) does not depend on quality (output). However, our main results are robust to the introduction of cost dependence between quality and output.[14]

The unit price received by each provider is regulated and paid by a third-party payer. We propose a specific pricing formula where the regulated price is provider-specific and linked to the provider's demand in the following way:

$$p_i(D_i(t)) = a_i + bD_i(t), \tag{5}$$

where a_i is a fixed price, which is provider *specific*, and b is a price component which is linear in demand and is *common* across providers. The parameter b can be positive or negative. If b is positive, we refer to this parameter as a price *premium* on volume. Instead, if b is negative, we refer to this parameter as a price *penalty* on volume.

In principle, we could also make the parameter related to a possible price premium or penalty specific to each provider (i.e., specifying a parameter b_i). However, in Section 5 we will show that, by an appropriate choice of the parameters a_i and b, a pricing rule of the form given by (5) can induce the socially optimal (first-best) quality provision for each provider at each point in time. Therefore, a provider-specific parameter for the volume-based part of the pricing rule is not required.

[13] Increasing marginal cost of output is an empirically relevant phenomenon in both health care and education markets. See Brekke et al. (2012) for discussion and references.

[14] Details are available upon request.

One sector that closely relates to the suggested payment system is the hospital sector. In most of the OECD countries, hospitals are paid by a fixed price schedule, known as Diagnosis Related Groups (DRGs) system. For a given diagnosis, the hospital receives a fixed price for every patient treated regardless of the costs sustained.[15] Moreover, the price is adjusted across hospitals to account for differences in exogenous cost factors (e.g., related to land, capital and labour).[16] Finally, several variants of the DRG system have been implemented across countries and over time, and, as previously mentioned, these often involve penalties for high volumes, so that the fixed tariff decreases at higher volumes.[17]

In order to ensure that all the optimisation problems in the subsequent analysis are well-behaved, we impose the following parameter restrictions:

A1 $\theta > 1/\tau$.

A2 $\beta/2 > b$.

Assumption A1 states that the marginal cost of quality from machines and other investments is sufficiently high relative to the marginal responsiveness of demand to quality. This assumption is in line with the features of the health sector, where the cost of investing in machines (for magnetic resonance imaging, equipment for surgical operating theaters, etc.) is high, and where a large body of empirical literature suggests that demand is relatively inelastic to quality.[18]

Assumption A2 is always satisfied if the price involves a penalty on volume. If the regulator instead uses a premium on volume (i.e., $b > 0$), then we assume that the premium is small relative to the degree of convexity in production costs.[19]

[15] In most education markets, the payment to publicly funded schools (whether public or private) is also typically based on the number of students.

[16] For example, in England under Payment by Results, hospitals are paid a Health Resource Group (HRG) price (the English version of DRG prices) based on national average costs adjusted by a provider specific index, known as the market factor forces (MFF; Department of Health, 2004). The MFF adjusts the national price for local unavoidable differences in factor prices for staff, land and building costs (see Miraldo et al., 2011, for a theoretical analysis).

[17] In practice, policymakers identify volume thresholds, with the tariff decreasing further when each of the thresholds is reached. Our pricing rule can be interpreted as a continuous approximation of a step-wise function where the price decreases with volume.

[18] See Brekke et al. (2014) for a survey on the empirical evidence on demand responsiveness to quality in hospital markets.

[19] In Section 6 we show that the optimal pricing rule always satisfies A2 if $\beta > \tau(\gamma + \rho)/\gamma$.

4 Equilibrium quality provision

In this section we derive the equilibrium of a dynamic game of quality competition between the two providers under two different game-theoretic solution concepts. First, we derive the *open-loop Nash equilibrium*, in which each provider commits to a complete time profile of quality investments at the beginning of the game and sticks to it thereafter. This solution concept is more likely to represent of a setting where schools or hospitals have to commit to long-term plans when deciding on quality investments within a heavily regulated environment. Second, we derive the arguably more *state feedback Nash equilibrium*, in which each provider can respond to the observed evolution of the state variable, implying that the optimal investment choice at any point in time is a function of contemporaneous demand. This solution concept is more applicable in deregulated markets where providers have discretion and independence over timely investment decisions.

4.1 Open loop Nash equilibrium

Consider the case where the providers use open-loop decision rules. In the open-loop Nash equilibrium (henceforth OLNE) of the game, Provider i takes as given its rival's strategy and solves the following optimal control problem:

$$\max_{q_i(t)} \int_0^\infty \pi_i(t) e^{-\rho t} dt, \tag{6}$$

subject to the dynamic constraint (3) and the initial condition $D_i(0) = D_{i0} > 0$. In Appendix A.1 we show that the OLNE strategies solve the following ODE system:[20]

$$\begin{cases} \dot{q}_i = (\rho + \gamma) q_i - \frac{\gamma}{2\theta\tau} [\sigma_i - (\beta - 2b) D_i] \\ \dot{q}_j = (\rho + \gamma) q_j - \frac{\gamma}{2\theta\tau} [\sigma_j - (\beta - 2b)(1 - D_i)] \\ \dot{D}_i = \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i \right) \end{cases}, \tag{7}$$

where $\sigma_i := a_i - c_i$ is the difference between the provider-specific fixed price and production costs. Given assumption A2, which is required for the second-order conditions to be satisfied, the dynamic relationship between quality and demand for each provider on the equilibrium path to the steady state is as follows. Suppose we start off steady state at a level where the initial demand is high.

[20] To ease notation, we omit the time indicator t in most of the subsequent expressions.

For this provider, we observe a period of decreasing demand and increasing quality. Thus, demand and quality move in opposite directions over time on the equilibrium path to the steady state. The result is similar to the one analysed and discussed in Brekke et al. (2012).

In the *steady state* of the OLNE, where $\dot{q}_i = \dot{q}_j = \dot{D}_i = 0$, the quality and demand for each provider are given by

$$\overline{q}_i^{OL} = \gamma \frac{(\beta - 2b)\left((\sigma_i + \sigma_j)\gamma - 2(\rho+\gamma)\tau^2\theta\right) - \gamma(\beta - 2b)^2 + 4\tau^2\theta(\gamma+\rho)\sigma_i}{4\tau\theta(\gamma+\rho)((\beta-2b)\gamma + 2\tau^2\theta(\gamma+\rho))} \tag{8}$$

and

$$\overline{D}_i^{OL} = \frac{1}{2} + \frac{\gamma(\sigma_i - \sigma_j)}{2\left((\beta-2b)\gamma + 2\tau^2\theta(\gamma+\rho)\right)}, \tag{9}$$

for $i = 1, 2$ and $j \neq i$. In Appendix A.1 we show that the steady state constitutes a saddle point. Furthermore, an interior solution with positive demand for both providers requires that the degree of asymmetry in costs or prices is not too high. More specifically:

$$|\sigma_i - \sigma_j| < \frac{(\beta-2b)\gamma + 2\tau^2\theta(\gamma+\rho)}{\gamma}. \tag{10}$$

In the symmetric case, with $\sigma_i = \sigma_j = \sigma$, steady state quality is given by

$$\overline{q}^{OL} = \gamma \frac{2\sigma - (\beta - 2b)}{4\theta\tau\,(\gamma+\rho)}, \tag{11}$$

while demand is $\overline{D}^{OL} = 1/2$. An interior solution (i.e., $\overline{q}^{OL} > 0$) requires that the price-cost margin is sufficiently high: $\sigma > (\beta/2) - b$. We characterise the steady state, and how it depends on the price parameters a_i and b, as follows:

Proposition 1 *In the steady state of the open-loop Nash equilibrium, (i) Provider i has higher (lower) quality and demand than Provider j if $\sigma_i > (<)\, \sigma_j$, i.e., if the difference between its provider-specific price parameter and production costs is higher (lower). (ii) An increase in the provider-specific price parameter a_i (a_j) increases the quality of both providers, but shifts demand towards Provider i (Provider j). (iii) Compared to the case where the common price parameter is absent and $b = 0$, a premium (penalty) on volume, i.e., a positive (negative) value of b, amplifies (dampens) the quality and demand differences between the providers.*

Proof. Appendix B.1. ∎

All else equal, an increase in a_i implies that Provider i is paid a higher price p_i, which increases the marginal revenue of quality investments and therefore leads to higher steady state quality for this provider. However, an increase in a_i also leads to higher steady state quality for Provider j, even if the price of Provider j is not affected. This effect is caused by strategic complementarity. To see this, consider the best-response function of Provider i in a static version of the game, where demand adjusts instantaneously. We can derive this static best-response function by maximising the instantaneous profit function (4) with respect to quality, yielding

$$q_i = \frac{(\beta - 2b)(q_j - \tau) + 2\tau\sigma_i}{\beta - 2b + 4\theta\tau^2}, \tag{12}$$

from which we derive

$$\frac{\partial q_i}{\partial q_j} = \frac{\beta - 2b}{\beta - 2b + 4\theta\tau^2} > 0. \tag{13}$$

A higher quality level by Provider j implies that demand is shifted away from Provider i, which has two, potentially opposing, effects on the Provider i's incentives for quality investments. Lower demand reduces the marginal production cost, which increases the profitability of quality investments. This effect is reinforced (partly offset) by a higher (lower) price if $b < (>)\, 0$. If $b > 0$, assumption A2 nevertheless ensures that the former effect dominates, making quality investments (static) strategic complements.

The common price parameter b serves to either amplify or dampen quality (and thus demand) differences between the two providers in the steady state, depending on whether b is positive or negative. In the steady state, the provider with higher quality has higher demand. If $b > (<)\, 0$, the provider with higher demand is given a price *premium* (*penalty*) on volume which reinforces (weakens) the incentive for quality investments, thus amplifying (dampening) the quality difference between the providers. If $c_i < c_j$, the most cost-efficient provider has higher quality (and thus higher demand) in the steady state as long as the differences in production costs are larger than differences in the provider-specific price component, $c_j - c_i > a_j - a_i$, which is equivalent to $\sigma_i > \sigma_j$.

4.2 State feedback Nash equilibrium

We now turn to the game with a state feedback information structure. In line with the literature, we restrict attention to stationary linear Markovian strategies, in which the current value of the control variable only depends on the current value of the state variable and the rule is time invariant. We assume that each player takes the rival's strategy as given, implying Nash behaviour, and we look for a set of strategies that constitute a State Feedback Nash Equilibrium (henceforth SFNE).

In Appendix A.2 we show that the linear SFNE strategies are given by

$$q_i^F = \frac{\gamma}{2\tau\theta}\left(\alpha_1^i + \alpha_2 D_i\right), \tag{14}$$

where

$$\alpha_1^i = \frac{(2\varphi+3\rho)\,\sigma_i + \varphi\sigma_j + (2\gamma+\rho)\,(\sigma_i-\sigma_j) + (\gamma+2\rho)\,\gamma\alpha_2 - (\beta-2b)\,(\varphi-(\gamma+\rho))}{\frac{1}{2}\,(\rho+\varphi)\,(4\gamma+5\rho+\varphi)}, \tag{15}$$

$$\alpha_2 = -\frac{2\theta\tau^2\,(\varphi-(2\gamma+\rho))}{3\gamma^2} < 0, \tag{16}$$

for $i, j = 1, 2$, $i \neq j$, and

$$\varphi := \sqrt{(2\gamma+\rho)^2 + \frac{3(\beta-2b)\gamma^2}{\theta\tau^2}}. \tag{17}$$

Since $\alpha_2 < 0$ and $D_i = 1 - D_j$, the SFNE is characterised by *inter-temporal strategic complementarity*, according to the definition provided by Jun and Vives (2004), meaning that the control variable (quality) of each player responds positively to a change in the state (demand) of the other player. Formally, $\partial q_j^F/\partial D_i > 0$ and $\partial q_i^F/\partial D_j > 0$. The intuition behind this dynamic strategic complementarity mirrors the intuition behind the static strategic complementarity in qualities explained in the previous subsection, in relation to the OLNE. Higher demand for Provider i necessarily implies lower demand, and therefore lower marginal production costs, for Provider j, which makes it more profitable to attract demand by investing more in quality. The optimal dynamic response to lower demand (or higher demand for the rival) is therefore to increase quality investments. Notice that these incentives are either reinforced or dampened by the considered pricing scheme, which links the provider-specific price to the demand of each provider through the common price parameter b. A positive value of b (a premium on volume) implies that a demand reduction is accompanied by a price reduction, which weakens the strength of the inter-temporal strategic complementarity. The opposite conclusion holds if b is negative (a penalty on volume).

In the *steady state* of the SFNE, qualities and demand are given by

$$\overline{q}_i^F = \frac{\frac{3\gamma}{\theta\tau^2}\,(\beta-2b)\,(\sigma_i+\sigma_j-(\beta-2b)) + (4\gamma+\varphi+5\rho)\,(2\sigma_i-(\beta-2b))}{\frac{2\theta\tau}{3\gamma^2}\,(\gamma+\varphi-\rho)\,(\varphi+\rho)\,(4\gamma+\varphi+5\rho)}, \tag{18}$$

and

$$\overline{D}_i^F = \frac{1}{2} + \frac{(\sigma_i - \sigma_j)\, 3\gamma^2}{2\theta\tau^2 (\varphi + \rho)(\gamma + \varphi - \rho)}, \tag{19}$$

for $i, j = 1, 2$, $i \neq j$. Demand is positive for both providers if

$$|\sigma_i - \sigma_j| < \frac{\theta\tau^2 (\rho + \varphi)(\varphi + \gamma - \rho)}{3\gamma^2}. \tag{20}$$

Under symmetry, $\sigma_i = \sigma_j = \sigma$, steady state quality is

$$\overline{q}^F = \frac{3\gamma^2 (2\sigma - (\beta - 2b)) \left(3\gamma (\beta - 2b) + \theta\tau^2 (4\gamma + 5\rho + \varphi)\right)}{2\theta^2\tau^3 (\gamma + \varphi - \rho)(\varphi + \rho)(4\gamma + \varphi + 5\rho)}. \tag{21}$$

As in the OLNE, the condition $\sigma > (\beta/2) - b$ is required for an interior solution with $\overline{q}^F > 0$. It is relatively straightforward to show that the steady state of the SFNE has the same properties as those described in Proposition 1 for the OLNE:

Proposition 2 *The results in Proposition 1 under the open-loop Nash equilibrium also apply to the state-feedback Nash equilibrium.*

Proof. Appendix B.2. ■

4.3 Comparison of equilibria

A comparison of steady state qualities and demand across the two solution concepts yields the following insights:

Proposition 3 *(i) In the symmetric case, $\sigma_i = \sigma_j$, steady state quality is higher in the open-loop than in the state-feedback Nash equilibrium. (ii) In the asymmetric case, $\sigma_i \neq \sigma_j$, the steady state quality and demand differences between the two providers is larger in the open-loop than in the state-feedback Nash equilibrium.*

Proof. Appendix B.3. ■

The first part of the proposition confirms the unambiguous ranking of steady state quality levels derived by Brekke et al. (2012) and shows that quality provision is higher with open-loop decision rules also under a pricing scheme where each provider's price is linked to its demand at each point in time. The intuition is related to strategic complementarity between the providers' quality investment decisions, as explained previously. Under feedback decision rules, where each provider can adjust its investment choices at each point in time according to the evolution of demand, there exists an inter-temporal trade-off for each provider in the sense that the instantaneous gain from higher quality must be traded-off against the future losses caused by the positive quality response of the rival provider. This dynamic strategic interaction, which is absent under open-loop decision rules, leads to lower steady state quality in the SFNE than in the OLNE.

The second part of the proposition confirms that the above described dynamic strategic interaction also serves to dampen the quality (and thus demand) differences between the providers caused by differences in production cost efficiency. Thus, state-feedback decision rules generally lead to lower quality levels and lower quality differences between the providers in the steady state.

5 Socially optimal quality provision

Suppose that a social planner can set each provider's quality level at each point in time over an infinite time horizon, taking as given the demand for the two providers and its sluggish adjustment to quality changes. The resulting dynamic quality paths constitute a socially optimal (first-best) solution and are derived from the following optimal control problem:

$$\max_{q_i, q_j} \int_0^{\infty} e^{-\rho t} W(t) dt, \tag{22}$$

subject to the dynamic constraint (3) and the initial condition $D_i(0) = D_{i0} > 0$, where $W(t)$ is the instantaneous social welfare defined as

$$W(t) = \int_0^{D_i} (v + q_i - \tau x) dx + \int_{D_i}^{1} (v + q_j - \tau(1-x)) dx - \frac{\theta}{2}(q_i^2 + q_j^2) - (c_i D_i + c_j(1-D_i)) - \frac{\beta}{2}(D_i^2 + (1-D_i)^2). \tag{23}$$

In Appendix A.3 we show that the first-best quality and demand time paths solve the following ODE system:

$$\begin{cases} \dot{q}_i = (\rho+\gamma)q_i + \frac{\gamma}{\tau\theta}\left(\beta - \frac{\tau}{\gamma}(\rho+\gamma)\right) D_i + \frac{\gamma}{2\tau\theta}(c_i - c_j - \beta) \\ \dot{q}_j = (\rho+\gamma)q_j + \frac{\gamma}{\tau\theta}\left(\beta - \frac{\tau}{\gamma}(\rho+\gamma)\right)(1 - D_i) + \frac{\gamma}{2\tau\theta}(c_j - c_i - \beta) \\ \dot{D}_i = \gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right) \end{cases}, \tag{24}$$

and that the *feedback representation* of the first-best solution is given by

$$q_i^* = \frac{1}{2\theta}\left(\widehat{\alpha}_1^i + \widehat{\alpha}_2 D_i\right), \tag{25}$$

where

$$\widehat{\alpha}_1^i = \frac{2\theta\gamma\left(\beta + c_j - c_i\right) + \left(\theta\tau - 1\right)\left(\theta\tau\left(2\gamma + \rho\right) - \kappa\right)}{\left(\kappa + \theta\tau\rho\right)}, \tag{26}$$

$$\widehat{\alpha}_2 = \frac{\theta\tau\left(2\gamma + \rho\right) - \kappa}{\gamma}, \tag{27}$$

for $i, j = 1, 2$, $i \neq j$, and

$$\kappa := \sqrt{\theta\left(4\beta\gamma^2 + 4\tau\gamma(\rho + \gamma)(\tau\theta - 1) + \rho^2\tau^2\theta\right)}. \tag{28}$$

We can characterise the socially optimal dynamic relationship between quality and demand, for each provider, as follows:

Proposition 4 *On the socially optimal time paths, quality and demand move in the opposite (same) direction for each provider if production cost convexity in volume is sufficiently high (low):* $\beta > (<)\,\tau\left(1 + (\rho/\gamma)\right)$.

Proof. Appendix B.4. ■

The optimal dynamic relationship between quality and demand is similar for both providers, and does not depend on potential cost differences between the providers. Thus, this relationship reflects a welfare trade-off that is unrelated to allocational concerns related to differences in cost-efficiency. If the initial difference in demand is higher than the socially optimal steady state demand difference, the social planner must decide whether to attribute higher or lower quality to the provider with more demand. Because of production cost convexity in volume ($\beta > 0$), cost-efficiency is improved by reallocating demand from the provider with higher demand to the provider with lower demand, which suggests that higher quality should be attributed to the provider with *lower* demand. On the other hand, the marginal benefit of quality investments is larger if these investments are made at the provider with higher demand. The latter consideration dominates if the degree of production cost convexity is sufficiently low. In this case, welfare is maximised by attributing higher quality to the provider with *higher* demand, implying that both demand and quality decreases (increases) for the provider with higher (lower) initial demand along the socially optimal dynamic path towards the steady state.[21]

In the *steady state*, the socially optimal qualities and demand are given by

$$\overline{q}_i^* = \frac{\beta\gamma + \tau(\gamma\theta(c_j - c_i) + (\gamma + \rho)(\tau\theta - 1))}{2\theta(\beta\gamma + \tau(\gamma + \rho)(\tau\theta - 1))} \tag{29}$$

and

$$\overline{D}_i^* = \frac{1}{2} + \frac{\gamma\tau(c_j - c_i)}{2\tau(\beta\gamma + \tau(\gamma + \rho)(\tau\theta - 1))}, \tag{30}$$

for $i, j = 1, 2$, $i \neq j$. In Appendix A.3 we show that this constitutes a saddle point.

In the symmetric case when production costs are the same across providers, $c_i = c_j$, the socially optimal quality provision in the steady state only depends on the marginal benefits and costs of quality investments and is given by $q^* = 1/2\theta$. However, under cost asymmetry across providers, $c_i \neq c_j$, the social planner faces a trade-off between production cost efficiency and travelling cost efficiency. Concerns for production cost efficiency imply that demand should be shifted towards the most efficient provider, but this would increase aggregate travelling costs, which are minimised with equal market shares for both providers. The socially optimal balancing of these two considerations is characterised as follows:

Proposition 5 *If production costs differ across providers, $c_i \neq c_j$, (i) the socially optimal steady state quality provision is higher (lower) for the most (less) cost-efficient provider; (ii) the socially optimal quality difference between the providers is decreasing in the degree of production cost convexity, β, and in the degree of demand sluggishness, γ^{-1}; and (iii) a marginal increase in consumers' travelling costs, τ, increases (decreases) the socially optimal quality difference between the providers if τ and θ are sufficiently low (high).*

Proof. Appendix B.5. ■

The social planner can improve cost efficiency by choosing a higher quality level for the most efficient provider. This will shift demand towards this provider and therefore reduce total production costs. However, because of production cost convexity ($\beta > 0$), total production costs are not minimised by letting the most cost efficient provider have all demand. Thus, the higher the degree of cost convexity, measured by β, the smaller is the socially optimal difference in demand (and thus

[21] If the socially optimal path implies higher (lower) initial quality for the provider with initial demand above (below) the steady state level, so that quality and demand move in the same direction over time for both providers, the initial difference in quality must of course be sufficiently low in order to yield demand reallocation from the high-demand to the low-demand provider, thus ensuring dynamic convergence towards the steady state.

quality) between the two providers. A similar relationship appears between the degree of demand sluggishness and the optimal quality (and demand) difference between the providers. A lower value of γ, which means that demand responds more sluggishly to quality changes, implies that quality investments become a less effective instrument to reallocate demand between the two providers, in the sense that the gains from quality investments, in the form of a more cost-efficient allocation of demand, take longer time to materialise. Consequently, the socially optimal demand and quality difference between the providers is smaller when demand adjusts more sluggishly.

The effect of higher travelling costs is somewhat more complicated. For given qualities, a marginal increase in τ has two different *allocational* welfare effects that are relevant for the socially optimal quality difference between the providers. On the one hand, it affects each consumer's trade-off between quality and travelling distance in a way which reduces the difference in market shares between the two providers. All else equal, this leads to less cost-efficient production, and cost efficiency can only be restored by increasing the quality difference between providers. On the other hand, a marginal increase in τ also increases the importance of travelling cost efficiency in the social planner's welfare trade-off, leading, all else equal, to a *reduction* in the optimal market share difference (and therefore the optimal quality difference) between the two providers. Since aggregate travelling costs are convex in τ, the second effect is stronger the higher τ is to begin with. Furthermore, a higher degree of cost convexity in quality provision, measured by θ, makes it more costly to increase the quality difference in response to the first effect. Thus, a larger quality difference in response to higher travelling costs is socially optimal only if both τ and θ are sufficiently low.

6 Optimal dynamic price regulation

In this section we derive the optimal dynamic price regulation rule, which differs depending on whether the decision rules used by the providers are open loop or state feedback. As before, we consider each of these cases in turn, starting with the case in which the providers use open-loop decision rules.

6.1 Open loop Nash equilibrium

If quality competition between providers takes place in the context of open-loop decision making, optimal dynamic price regulation has the following characteristics:

Proposition 6 *In the open-loop Nash equilibrium, the socially optimal quality provision is replicated, at each point in time, if the regulated price is* $p_i^{OL}(t) = a_i^{OL} + b^{OL} D_i(t)$*, with the common price parameter being set at*

$$b^{OL} = \frac{\tau(\gamma + \rho)}{\gamma} - \frac{\beta}{2} \tag{31}$$

and the provider-specific parameter being set at

$$a_i^{OL} = c_j + \beta, \tag{32}$$

for $i, j = 1, 2$*,* $i \neq j$*, given that the parameter condition* $\beta > \tau(\gamma + \rho)/\gamma$ *holds.*

Proof. Appendix B.6. ■

Perhaps the most striking feature of this result is that optimal quality provision can be achieved by a very simple pricing rule. In the case of equally cost-efficient providers, $c_i = c_j$, equilibrium quality provision is symmetric in the steady state but will generally differ across the two providers off the steady state. Nevertheless, Proposition 6 shows that the socially optimal time path of quality provision can be replicated in equilibrium by each provider through an *ex ante non-discriminatory price regulation scheme* that links each provider's price to its demand in the exact same way, through the premium or penalty parameter b as given by (31).

In the optimal pricing rule, the common parameter b remains unchanged if the providers have different production costs, $c_i \neq c_j$. In this case, the prices just need to be adjusted through the provider-specific parameters a_i and a_j, which are set to reflect the marginal production costs, with the more cost-efficient provider receiving a higher price. This leads to a larger quality difference between the providers and thus cost savings through a reallocation of demand towards the most cost-efficient provider. Notice, however, that the optimal values of a_i and a_j are constant and thus time independent. Price adjustments over time occur only through the common parameter b, which highlights the simplicity, and thus applicability, of this price regulation scheme.[22]

The condition $\beta > \tau(\gamma + \rho)/\gamma$ is needed for assumption A2 to hold with the pricing rule given in Proposition 6, and requires that production costs are sufficiently convex in volume. This condition implies that the first-best solution is achievable through price regulation only for the subset of parameter configurations in which quality and demand move in opposite directions along

[22] Notice that, in case of cost asymmetries, the optimal pricing rule has some characteristics that resemble *yardstick competition* (Shleifer, 1985), in the sense that the time-invariant component of the price (a_i) depends only on the rival's marginal cost and not own marginal cost.

the socially optimal dynamic path towards the steady state, as stated in Proposition 4. Given this parameter condition, the dynamic properties of the socially optimal pricing rule follow directly from Proposition 6:

Corollary 1 *In the open-loop Nash equilibrium, the socially optimal price involves a penalty (premium) on demand of each provider, at each point in time, if the cost convexity parameter* $\beta >(<) \widehat{\beta}^{OL}$*, where*

$$\widehat{\beta}^{OL} := \frac{2\tau(\gamma+\rho)}{\gamma}. \tag{33}$$

The intuition is closely related to the welfare trade-off determining the socially optimal time paths of quality provision given by Proposition 4 for the case in which $\partial q_i^*/\partial D_i < 0$. The premium or penalty common price parameter b works as an instrument to shift demand between the two providers.

Whether the price schedule involves a penalty or a premium relates to two concerns regarding production costs and consumer benefits which go in opposite directions. Concerns for cost-efficient *production* dictate that demand should be steered towards the provider with lower demand, which can be achieved by lower quality investments by the high-demand provider. Instead, concerns for *consumer welfare* dictate that the high-demand provider should invest more in quality, implying that demand is steered away from the low-demand provider.

If the *production* concern dominates, so that welfare is increased by reducing (increasing) the quality investments of the high-demand (low-demand) provider, this can be achieved by introducing a *penalty* on volume (setting $b < 0$) in the optimal pricing rule, which reduces the price-cost margin, and thus incentives for quality investments, of the high-demand provider relative to the low-demand provider. This is optimal if the convexity of production costs β is sufficiently high, which implies that concerns for production cost efficiency are sufficiently strong. The scope for a penalty with $b < 0$ in the optimal pricing rule is also larger if patients' transportation costs and/or the degree of demand sluggishness is lower. All else equal, a reduction in τ or an increase in γ implies that the demand response to a change in quality provision is larger, implying that cost efficiency can be improved (through demand reallocation) at a lower cost (in terms of consumer welfare). An optimal price which involves a penalty on volume with $b < 0$ is also more likely if the discount rate ρ is lower, which implies that the future gains of improved cost-efficiency through demand reallocation towards the low-demand provider are given a larger weight relative to the instantaneous welfare loss of lower quality investments by the high-demand provider.

Alternatively, if the degree of cost convexity β is relatively low, the concern for consumer welfare dominates, implying that social welfare is increased by directing a larger share of quality investments towards the high-demand provider. This can be achieved by imposing a price *premium* on volume ($b > 0$) which makes quality investments relatively more profitable for the high-demand provider, thus amplifying the equilibrium quality difference between providers.

6.2 State feedback Nash equilibrium

If the providers instead use state-feedback decision rules, making quality dynamic competition more intense, price regulation is optimised as follows:

Proposition 7 *In the state-feedback Nash equilibrium, the socially optimal quality provision is replicated, at each point in time, if the regulated price is* $p_i^F(t) = a_i^F + b^F D_i(t)$*, with the common price parameter being set at*

$$b^F = \frac{\tau}{4\gamma^2}\left((2\gamma+\rho)\kappa - 2\gamma(2\tau\theta-3)(\gamma+\rho) - \rho^2\tau\theta\right) - \beta \tag{34}$$

and the provider-specific parameter being set at

$$a_i^F = \frac{\left[\begin{array}{c} 6\theta\gamma\left(\varepsilon\left(\beta-(c_i-c_j)\right)+\kappa c_i+\theta\tau\rho\left(\beta+c_j\right)\right) \\ +\left(\theta\tau-1\right)\left(\theta\tau\left(\varepsilon\left(6\gamma+5\rho\right)+\theta\tau\rho\left(2\gamma+\rho\right)-\kappa\left(4\gamma+5\rho\right)\right)-\kappa\varepsilon\right) \end{array}\right]}{6\theta\gamma\left(\kappa+\theta\tau\rho\right)}, \tag{35}$$

where κ *is given by (28) and*

$$\varepsilon := \sqrt{\theta\left(3\gamma^2\left(\beta-2b\right)+\theta\tau^2\left(2\gamma+\rho\right)^2\right)}, \tag{36}$$

for $i, j = 1, 2$, $i \neq j$, *given that the parameter condition* $\beta > \tau(\gamma+\rho)/\gamma$ *holds.*

Proof. Appendix B.7. ■

Though the mathematical expressions are more complicated, the characteristics and properties of the optimal pricing rule are similar to the equivalent rule in the OLNE. Given the parameter condition $\beta > \tau(\gamma+\rho)/\gamma$, the socially optimal dynamic path of quality provision can be obtained for both providers with a pricing rule that consists of a provider-specific (time independent) component, given by parameter a_i, and a dynamic component that links the price to each provider's demand through the common parameter b. Notice that the parameter conditions in Propositions 6 and 7 are identical and equal to the parameter condition determining the dynamic nature of the socially optimal solution. Thus:

Corollary 2 *Regardless of whether providers use open-loop or state feedback decision rules, the first-best solution can be implemented by a pricing rule of the form given by (5) if and only if quality and demand move in opposite directions on the socially optimal dynamic path towards the steady state.*

Furthermore, it can also be shown that the dynamic properties of the optimal pricing rule are similar under the two solution concepts:

Corollary 3 *In the state-feedback Nash equilibrium, the socially optimal price involves a penalty (premium) linked to the demand of each provider, at each point in time, if the cost convexity parameter* $\beta > (<) \widehat{\beta}^F$*, where*

$$\widehat{\beta}^F := \frac{\tau\left(12\gamma\left(\gamma+\rho\right)-\theta\tau\left(2\gamma+\rho\right)^2+\left(2\gamma+\rho\right)\sqrt{\theta\tau\left(\theta\tau\left(2\gamma+\rho\right)^2+8\gamma\left(\gamma+\rho\right)\right)}\right)}{8\gamma^2}. \quad (37)$$

Proof. Appendix B.8. ■

As in the OLNE, and for the same reasons, the optimal price is adjusted downwards (upwards), involving a penalty (premium) on volume, in response to higher demand if the degree of production cost convexity is sufficiently high (low).

The structure of the optimal pricing rule, which is similar under the two solution concepts, suggests that the different price parameters serve distinctly different purposes in the process of replicating the first-best solution on and off the steady state. The common parameter b, which links the price to demand for each provider at each point in time, is set to induce the socially optimal *slope* of the equilibrium path towards the steady state, which determines the relative magnitudes of quality and demand adjustments for each provider over time. For a given slope, the time independent parameters a_i and a_j are then set to induce the socially optimal *level* of quality for each provider along the equilibrium path, with different parameter values being chosen for each provider under cost asymmetries.

Additional insights about the optimal choice of the common price parameter b can therefore be gained by comparing the slopes of the equilibrium path and the socially optimal path towards the steady state. From (14), the slope of the equilibrium path in the SFNE is negative, since $\alpha_2 < 0$, implying that quality and demand move in opposite directions over time for each provider, which in turn means that the provider with initially higher (lower) demand has quality below (above) the steady state level. By Proposition 4, the slope of the socially optimal path is also negative under the parameter condition $\beta > \tau(\gamma + \rho)/\gamma$, which is a requirement for the optimal pricing rule under both solution concepts (cf. Corollary 2). Using (14) and (25), the *difference* in these slopes is given by

$$\frac{\partial q_i^*}{\partial D_i} - \frac{\partial q_i^F}{\partial D_i} = \frac{1}{2\theta}\left(\frac{\gamma\alpha_2}{\tau} - \widehat{\alpha}_2\right) = \frac{\theta\tau\left(2\left(\gamma+\varphi\right)+\rho\right) - 3\kappa}{6\theta\gamma}. \tag{38}$$

Both slopes, and their difference, depend on b but not on a_i or a_j. For $b = 0$, the slope difference is zero at $\beta = \widehat{\beta}^F$, as given by (37) in Corollary 3. Furthermore, it is fairly straightforward to verify that, for $b = 0$, this difference is monotonically decreasing in β, which implies that[23]

$$\left|\frac{\partial q_i^F}{\partial D_i}\right| > (<) \left|\frac{\partial q_i^*}{\partial D_i}\right| \text{ for } b = 0 \text{ if } \beta < (>)\, \widehat{\beta}^F. \tag{39}$$

Thus, if the degree of production cost convexity is sufficiently low, $\beta < \widehat{\beta}^F$, the equilibrium quality difference between the providers (for $b = 0$) is larger than what is socially optimal at any initial (asymmetric) state of demand. In other words, at the initial state, the high-demand provider chooses too low quality and the low-demand provider chooses too high quality. Since the marginal utility gains of quality investments are higher at the provider with more demand, social welfare would increase if more of the quality investments were channeled towards the provider with initially higher demand. This can be achieved by setting $b > 0$, which stimulates the incentives for quality investments by the high-demand provider, thereby reducing the equilibrium quality difference in the initial state, implying that the equilibrium path to the steady state is characterised by a more gradual reallocation of demand over time.

[23] From (38) we derive

$$\frac{\partial}{\partial\beta}\left(\frac{\partial q_i^*}{\partial D_i} - \frac{\partial q_i^F}{\partial D_i}\right)\Bigg|_{b=0} = -\frac{\gamma\left(2\theta\tau\varphi - \kappa\right)\kappa}{2\theta^2\tau\varphi\left(4\gamma\left(\beta\gamma - \tau\left(\gamma+\rho\right)\right) + \theta\tau^2\left(2\gamma+\rho\right)^2\right)},$$

which is negative if $2\theta\tau\varphi > \kappa$. By squaring both sides of this inequality and collecting terms, we obtain

$$8\beta\gamma^2 + 3\theta\tau^2\rho^2 + 4\tau\gamma\left(\gamma+\rho\right)\left(3\theta\tau+1\right) > 0.$$

On the contrary, if the degree of cost convexity is sufficiently high, $\beta > \widehat{\beta}^F$, the initial equilibrium quality difference is too small, from a welfare point of view. In this case, concerns for cost-efficient production necessitates a faster reallocation of demand towards the provider with lower initial demand. This can be achieved by setting $b < 0$, which dampens the incentives for quality investments by the high-demand provider, thereby increasing the equilibrium quality difference in the initial state.

6.3 Comparison

Comparing the dynamic components of the optimal pricing rule in the two equilibria, it is relatively straightforward to verify that $\widehat{\beta}^{OL} > \widehat{\beta}^F$, which immediately yields the following result:[24]

Corollary 4 *The common price components of first-best pricing rules in the two equilibria involve:*

(i) a price premium, $b^{OL} > 0$ *and* $b^F > 0$, *if* $\beta \in (\frac{\tau(\gamma+\rho)}{\gamma}, \widehat{\beta}^F]$;

(ii) a price premium under OLNE, $b^{OL} > 0$, *and a price penalty under SFNE,* $b^F < 0$, *if* $\beta \in (\widehat{\beta}^F, \widehat{\beta}^{OL}]$;

(iii) a price penalty, $b^{OL} < 0$ *and* $b^F < 0$, *if* $\beta > \widehat{\beta}^{OL}$.

Thus the optimal price regulation always involves a price penalty (premium) on volume if the cost convexity parameter is sufficiently high (low). Instead, it involves a price penalty under the state-feedback solution and a price premium under the open-loop solution for intermediate values of the cost convexity parameter.

The key insight is that the scope for a price *penalty* on volume is larger when the providers choose quality investments based on state-feedback decision rules. In light of the above discussion, this also suggests that the equilibrium dynamic path towards the steady state is flatter in (D_i, q_i) space, with lower quality differences at the initial state, in the SFNE than in the OLNE.

7 Concluding remarks

In regulated markets where providers compete on quality, but where demand responds sluggishly to changes in quality, optimal price regulation is an inherently dynamic problem where the challenge

[24] Using (33) and (37), the inequality $\widehat{\beta}^{OL} > \widehat{\beta}^F$ can be rearranged to

$$4\gamma(\gamma+\rho)(\theta\tau+1) + \theta\tau\rho^2 > (2\gamma+\rho)\sqrt{\theta\tau\left(\theta\tau(2\gamma+\rho)^2 + 8\gamma(\gamma+\rho)\right)}.$$

Squaring both sides and collecting terms, this inequality reduces to $16\gamma^2(\gamma+\rho)^2 > 0$.

is to ensure that the equilibrium quality provision follows a socially optimal dynamic path towards the steady state. Prime examples of such markets are health care and education.

In this study we suggest an attractively simple solution to a complicated dynamic regulation problem. We show that a simple pricing rule that links each provider's regulated price to an easily observable metric, namely the provider's contemporaneous demand, can in principle ensure that the socially optimal (first-best) outcome is realised at each point in time, on and off the steady state. This pricing rule can be interpreted as a price premium or a price penalty on volume. A necessary condition for such a price scheme to work is that the socially optimal dynamic path towards the steady state is characterised by demand and quality moving in opposite direction over time for each provider, which requires, in turn, that the degree of production cost convexity is sufficiently large. This conclusion is based on a differential-game version of a Hotelling duopoly framework where two exogenously located profit-maximising providers face regulated prices and compete in terms of quality to attract consumers.

Given that the first-best dynamic path can be replicated by the use of such a pricing rule, the remaining key design issue is whether the regulated price should be optimally designed with a *penalty* or a *premium* on higher volume. We show that this depends on the welfare trade-off of two opposing concerns. On the one hand, the concern for *consumer welfare* indicates that quality investments should be redirected (off the steady state) towards the provider with higher demand. This can be achieved by a regulated price that depends *positively* on demand, and therefore involves a price premium. On the other hand, the concern for *cost-efficient production* indicates that quality investments should be redirected (again, off the steady state) *away from* the provider with higher demand. This can be achieved by a regulated price that depends *negatively* on demand, and therefore involves a price penalty. As a result of this welfare trade-off, we show that a price schedule designed with a price penalty (premium) on volume is optimal if the degree of production cost convexity is sufficiently high (low).

In terms of policy implications, as discussed in the Introduction, there are several real-world examples of price regulation in hospital markets that resemble the type of pricing scheme that we suggest in this study, where the price received by each provider changes according to some volume thresholds. However, in all the examples of this kind that we are aware of, the price depends negatively on volume, and therefore involve a *penalty* for high volumes. In light of our analysis, such a pricing scheme can be socially optimal only if concerns for cost efficiency are sufficiently important. Such concerns are likely to be more important either in health systems with lower health expenditure per capita that imply tighter capacity constraints (e.g., fewer beds per capita), or within a health system that experiences budgetary restrictions following an economic downturn. For example, recent discussions in England around blended payment which involve lower payments at higher volume appear to be motivated by concerns over excess expenditure. More generally, our analysis suggests that dynamic price regulation with volume penalties is more likely to be optimal in markets where capacity constraints are important, including the education sector, implying rapidly increasing marginal costs of production.

As a final remark, it is worth re-iterating that our analysis is based on a theoretical framework where total demand is fixed. When lower demand for one provider is exactly offset by higher demand for the competing provider, the regulator can induce any desired demand allocation between the providers by the use of a single instrument. This is the reason why the first-best outcome can be implemented by a price regulation scheme where the dynamic part of the pricing rule is ex ante non-discriminatory (i.e., the penalty or premium on volume is common across providers). Our analysis is therefore more applicable to markets where total demand is relatively inelastic with respect to quality.

Appendix A: Supplementary calculations

In this appendix we provide supplementary calculations for the derivation of the open-loop and state feedback equilibrium solutions, as well as the socially optimal solution.

A.1 Open-loop Nash equilibrium

Consider the optimal control problem of Provider i defined by (6), and let $\mu_i(t)$ be the current-value co-state variable associated with the dynamic constraint (3). The current-value Hamiltonian for Provider i is then given by[25]

$$H_i = p_i(D_i)D_i - \frac{\theta}{2}q_i^2 - c_iD_i - \frac{\beta}{2}D_i^2 + \mu_i\gamma\left[\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right]. \tag{A1}$$

The solution must satisfy the following conditions: (i) $\partial H_i/\partial q_i = 0$, (ii) $\dot{\mu}_i = \rho\mu_i - \partial H_i/\partial D_i$, (iii) $\dot{D}_i = \partial H_i/\partial \mu_i$, along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\mu_i(t)D_i(t) = 0$. Condition (i)

[25] To save notation, the time indicator t is omitted in most of the subsequent expressions.

yields

$$\mu_i = \frac{2\theta\tau}{\gamma} q_i, \tag{A2}$$

from which, by taking time derivative, we obtain

$$\dot{\mu}_i = \frac{2\theta\tau}{\gamma} \dot{q}_i. \tag{A3}$$

Condition (ii) is given by

$$\dot{\mu}_i = (\rho + \gamma)\mu_i - \sigma_i + (\beta - 2b)D_i. \tag{A4}$$

By combining (A3) and (A4) we obtain

$$\dot{q}_i = (\rho + \gamma)q_i - \frac{\gamma}{2\theta\tau}[\sigma_i - (\beta - 2b)D_i], \tag{A5}$$

which, together with the equivalent equation for Provider j and the dynamic constraint (3), constitute the ODE system implicitly describing the OLNE strategies.

The second-order conditions are satisfied if the Hamiltonian is concave in the control and state variables, which requires that the matrix

$$\begin{bmatrix} \frac{\partial^2 H_i}{\partial q_i^2} & \frac{\partial^2 H_i}{\partial D_i \partial q_i} \\ \frac{\partial^2 H_i}{\partial D_i \partial q_i} & \frac{\partial^2 H_i}{\partial D_i^2} \end{bmatrix} = \begin{bmatrix} -\theta & 0 \\ 0 & 2b - \beta \end{bmatrix} \tag{A6}$$

is negative semidefinite, which is true under assumption A2.

To obtain the *feedback representation* of the OLNE, denoted by $q_i = \psi_i(D_i)$, we totally differentiate it with respect to time and use (A5) to obtain

$$\dot{q}_i = \frac{d\psi_i}{dD_i}\dot{D}_i = (\rho + \gamma)\psi_i - \frac{\gamma}{2\theta\tau}[\sigma_i - (\beta - 2b)D_i], \tag{A7}$$

and, after substituting the state variable dynamic, we finally obtain

$$\frac{d\psi_i}{dD_i}\gamma\left(\frac{1}{2} + \frac{\psi_i - \psi_j}{2\tau} - D_i\right) = (\rho + \gamma)\psi_i - \frac{\gamma}{2\theta\tau}[\sigma_i + (2b - \beta)D_i] \tag{A8}$$

for $i, j = 1, 2$, $j \neq i$.

The equilibrium point is computed by imposing $\dot{q}_i = \dot{q}_j = \dot{D}_i = 0$ in the system (7), yielding steady state qualities and demand as given by (8)-(9). To determine the stability of the equilibrium point, we compute the eigenvalues of the Jacobian matrix of the linear system (7), which are given by

$$\lambda_1 = \gamma + \rho, \tag{A9}$$

$$\lambda_2 = \frac{1}{2}\left(\rho + \frac{\sqrt{\theta(2(\beta - 2b)\gamma^2 + (2\gamma + \rho)^2\tau^2\theta)}}{\tau\theta}\right), \tag{A10}$$

$$\lambda_3 = \frac{1}{2}\left(\rho - \frac{\sqrt{\theta(2(\beta - 2b)\gamma^2 + (2\gamma + \rho)^2\tau^2\theta)}}{\tau\theta}\right). \tag{A11}$$

Clearly, $\lambda_1 > 0$ and $\lambda_2 > 0$, and it is straightforward to verify that $\lambda_3 < 0$ under assumption A2, which implies that the steady state is a saddle point.

A.2 State feedback Nash equilibrium

Since $D_j = 1 - D_i$, we can define the value function of both providers as a function of D_i. Given the linear-quadratic structure of the model, we specify the value functions as

$$V_i(D_i) = \alpha_0 + \alpha_1 D_i + \frac{\alpha_2}{2} D_i^2, \tag{A12}$$

$$V_j(D_i) = k_0 + k_1 D_i + \frac{k_2}{2} D_i^2. \tag{A13}$$

These value functions have to satisfy the following Hamilton-Jacobi-Bellman (HJB) equations:

$$\rho V_i = \max_{q_i \geq 0}\left[(a_i + bD)D - \frac{\theta}{2}q_i^2 - c_i D_i - \frac{\beta}{2}D_i^2 + \frac{dV_i}{dD_i}\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right)\right], \tag{A14}$$

$$\rho V_j = \max_{q_j \geq 0}\left[(a_j + b(1 - D_i))(1 - D_i) - \frac{\theta}{2}q_j^2 - c_j(1 - D_i) - \frac{\beta}{2}(1 - D_i)^2 + \frac{dV_j}{dD_i}\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right)\right]. \tag{A15}$$

Using (A12)-(A13), the maximisation of the RHS of the HJB equations yields

$$q_i = \phi_i(D) = \frac{\gamma}{2\tau\theta}(\alpha_1 + \alpha_2 D_i), \tag{A16}$$

$$q_j = \phi_j(D) = -\frac{\gamma}{2\tau\theta}(k_1 + k_2 D_i). \tag{A17}$$

After substituting these expressions into the HJB equations, and proceeding by identification, we find that α_2 and k_2 solve the following system:

$$\rho\frac{\alpha_2}{2} = \frac{8\tau^2\theta b - 4\beta\tau^2\theta + \gamma\alpha_2(\gamma(\alpha_2 + 2k_2) - 8\tau^2\theta)}{8\tau^2\theta}, \tag{A18}$$

$$\rho\frac{k_2}{2} = -\frac{4\beta\tau^2\theta - \gamma^2 k_2(2\alpha_2 + k_2) + 8\tau^2\theta(\gamma k_2 - b)}{8\tau^2\theta}, \tag{A19}$$

which admits four pairs of solutions:

$$\alpha_2 = 2\frac{(2\gamma + \rho)\tau^2\theta \pm \sqrt{\tau^2\theta(3(\beta - 2b)\gamma^2 + (2\gamma + \rho)^2\tau^2\theta)}}{3\gamma^2}, \tag{A20}$$

$$k_2 = 2\frac{(2\gamma + \rho)\tau^2\theta \pm \sqrt{\tau^2\theta(3(\beta - 2b)\gamma^2 + (2\gamma + \rho)^2\tau^2\theta)}}{3\gamma^2}. \tag{A21}$$

In order for the value functions to be concave we must have $\alpha_2 < 0$ and $k_2 < 0$, which eliminate the two positive roots. We therefore select

$$\alpha_2 = k_2 = -\frac{2\theta\tau^2}{3\gamma^2}\left(\sqrt{\tau^2\theta(3(\beta - 2b)\gamma^2 + (2\gamma + \rho)^2\tau^2\theta)} - (2\gamma + \rho)\right) < 0. \tag{A22}$$

Analogously, by collecting and equating to zero the terms containing D_i in each of the HJB equations, we obtain the following (linear) system to be solved for α_1 and k_1:

$$\rho\alpha_1 = \frac{\gamma\alpha_2(\gamma k_1 + 2(\gamma\alpha_1 + \tau^2\theta)) + 4\tau^2\theta(\sigma_i - \gamma\alpha_1)}{4\tau^2\theta}, \tag{A23}$$

$$\rho k_1 = \frac{\gamma\alpha_2(\gamma\alpha_1 + 2(\gamma k_1 + \tau^2\theta)) + 4\tau^2\theta(\beta - 2b - \sigma_j - \gamma k_1)}{4\tau^2\theta}. \tag{A24}$$

The solutions are given by

$$\alpha_1 = 2\tau^2\theta\frac{2\gamma\alpha_2((\beta - 2b - 2\sigma_i - \sigma_j)\gamma + 2\tau^2\theta(\gamma + \rho)) + 8\tau^2\theta\sigma_i(\gamma + \rho) - \gamma^3\alpha_2^2}{(4\theta\tau^2(\gamma + \rho) - 3\gamma^2\alpha_2)(4\theta\tau^2(\gamma + \rho) - \gamma^2\alpha_2)}, \tag{A25}$$

$$k_1 = 2\tau^2\theta\frac{2\gamma\alpha_2((2(\sigma_j - (\beta - 2b)) + \sigma_i)\gamma + 2\tau^2\theta(\gamma + \rho)) + 8\tau^2\theta(\gamma + \rho)(\beta - 2b - \sigma_j) - \gamma^3\alpha_2^2}{(4\theta\tau^2(\gamma + \rho) - 3\gamma^2\alpha_2)(4\theta\tau^2(\gamma + \rho) - \gamma^2\alpha_2)}. \tag{A26}$$

By substituting α_2 from (A22) into (A25) and simplifying, (A25) reduces to α_1^i as defined by (15). Using the fact that $k_2 = \alpha_2$, and keeping in mind that $D_i = 1 - D_j$, we can rewrite (A17) as

$$q_j = \phi_j(D) = \frac{\gamma}{2\tau\theta}(\alpha_1^j + \alpha_2 D_j), \tag{A27}$$

where $\alpha_1^j = -k_1 - \alpha_2$ and is given by (15) if replacing i with j.

A.3 Socially optimal quality provision

Consider the optimal control problem of the social planner defined by (22), and let η be the current-value co-state variable associated with the dynamic constraint (3). The current-value Hamiltonian for the social planner problem is then given by

$$H = W + \eta\gamma\left[\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i\right], \tag{A28}$$

where W is given by (23). The solution must satisfy the following conditions: (i) $\partial H/\partial q_i = 0$, (ii) $\partial H/\partial q_j = 0$, (iii) $\dot{\lambda} = \rho\eta - \partial H/\partial D_i$ and (iv) $\dot{D}_i = \partial H/\partial \eta$, along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\eta\,(t)\,D_i\,(t) = 0$.

From (i) and (ii) we derive

$$\eta = \frac{2\tau}{\gamma}\left(\theta q_i - D_i\right), \tag{A29}$$

$$\eta = -\frac{2\tau}{\gamma}\left(\theta q_j - (1 - D_i)\right), \tag{A30}$$

from which we get

$$q_j = \frac{1}{\theta} - q_i. \tag{A31}$$

The adjoint equation (iii) is given by

$$\dot{\eta} = (\rho + \gamma)\eta - (q_i - q_j) + (c_i - c_j) + (\tau + \beta)(2D_i - 1). \tag{A32}$$

From these equations we easily obtain the ODE system for the first-best quality and demand time paths, given by (24).

The second-order conditions are satisfied if the Hamiltonian is concave in the control and state variables, which requires that the matrix

$$\begin{bmatrix} \frac{\partial^2 H_i}{\partial q_i^2} & \frac{\partial^2 H_i}{\partial q_j \partial q_i} & \frac{\partial^2 H_i}{\partial D_i \partial q_i} \\ \frac{\partial^2 H_i}{\partial q_j \partial q_i} & \frac{\partial^2 H_i}{\partial q_j^2} & \frac{\partial^2 H_i}{\partial D \partial q_j} \\ \frac{\partial^2 H_i}{\partial D_i \partial q_i} & \frac{\partial^2 H_i}{\partial D_i \partial q_j} & \frac{\partial^2 H_i}{\partial D_i^2} \end{bmatrix} = \begin{bmatrix} -\theta & 0 & 1 \\ 0 & -\theta & -1 \\ 1 & -1 & -2\beta - 2\tau \end{bmatrix} \tag{A33}$$

is negative semidefinite, which is true under assumption A1.

To obtain the first-best solution in *feedback* form, we define the value function of the social planner as

$$V(D_i) = \alpha_0' + \alpha_1' D_i + \frac{\alpha_2'}{2} D_i^2. \tag{A34}$$

This value function must solve the HJB equation given by

$$\rho V = \max_{q_i, q_j} \left[W + \frac{dV}{dD_i} \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D_i \right) \right], \tag{A35}$$

where W is given by (23). The first-order conditions with respect to q_i and q_j yield, respectively,

$$q_i = \frac{\gamma}{2\tau\theta} \alpha_1' + \left(\frac{1}{\theta} + \frac{\gamma \alpha_2'}{2\tau\theta} \right) D_i, \tag{A36}$$

$$q_j = \frac{1 - D_i}{\theta} - \frac{\gamma}{2\tau\theta} (\alpha_1' + \alpha_2' D_i). \tag{A37}$$

After substituting them into the HJB equation, and proceeding by identification, we find

$$\alpha_1' = \tau \frac{\gamma \alpha_2' (\tau\theta - 1) + 2\tau((\beta + \tau + c_j - c_i)\theta - 1)}{2\tau(\gamma(\tau\theta - 1) + \rho\tau\theta) - \gamma^2 \alpha_2'} \tag{A38}$$

and

$$\alpha_2' = \tau \frac{2\gamma(\tau\theta - 1) + \rho\tau\theta \pm \sqrt{\theta \left(4\beta\gamma^2 + 4\gamma\tau(\rho + \gamma)(\tau\theta - 1) + \rho^2 \tau^2 \theta\right)}}{\gamma^2}. \tag{A39}$$

The condition that the value function must be concave leads us to select the negative root in (A39). We then define

$$\widehat{\alpha}_1^i := \frac{\gamma}{\tau} \alpha_1' \quad \text{and} \quad \widehat{\alpha}_2 := 2 + \frac{\gamma \alpha_2'}{\tau}, \tag{A40}$$

which, after substitution and re-arranging, allows us to express (A36) as (25). Similarly, (A37) is equal to (25) when i is replaced by j in the latter expression.

The equilibrium point is computed by imposing $\dot{q}_i = \dot{q}_j = \dot{D} = 0$ in the ODE system (24), yielding the steady state qualities and demand given by (29)-(30). To determine its stability, we compute the eigenvalues of the Jacobian matrix of (24). These are given by

$$\lambda_1 = \gamma + \rho, \tag{A41}$$

$$\lambda_2 = \frac{1}{2}\left(\rho + \frac{\sqrt{\theta\left(4\beta\gamma^2 + 4\tau\gamma(\rho+\gamma)(\tau\theta - 1) + \rho^2\tau^2\theta\right)}}{\tau\theta}\right), \tag{A42}$$

$$\lambda_3 = \frac{1}{2}\left(\rho - \frac{\sqrt{\theta\left(4\beta\gamma^2 + 4\tau\gamma(\rho+\gamma)(\tau\theta - 1) + \rho^2\tau^2\theta\right)}}{\tau\theta}\right). \tag{A43}$$

Clearly, $\lambda_1 > 0$ and $\lambda_2 > 0$, and it is also straightforward to verify that $\lambda_3 < 0$ under assumption A1. This proves that the steady state is a saddle point.

Appendix B: Proofs

This appendix contains proofs of all the propositions in the paper, as well as a proof of Corollary 3. The proofs of Corollaries 1 and 2 are trivial (given the propositions they are derived from) and therefore omitted.

B.1 Proof of Proposition 1

(i) From (8) we derive

$$\overline{q}_i^{OL} - \overline{q}_j^{OL} = \frac{\tau\gamma\left(\sigma_i - \sigma_j\right)}{(\beta - 2b)\gamma + 2\tau^2\theta(\gamma+\rho)}. \tag{B1}$$

The result in the first part of the proposition follows directly from (B1) and (9).

(ii) $\partial\overline{q}_i^{OL}/\partial a_i > 0$ and $\partial\overline{q}_j^{OL}/\partial a_i > 0$ follow directly from (8). $\partial\overline{D}_i^{OL}/\partial a_i > 0$ follows directly from (9).

(iii) The result follows directly from (B1).

B.2 Proof of Proposition 2

From (18) we derive

$$\overline{q}_i^{F} - \overline{q}_j^{F} = \frac{\left(\sigma_i - \sigma_j\right)3\gamma^2}{\theta\tau\left(\varphi+\rho\right)\left(\gamma+\varphi-\rho\right)}. \tag{B2}$$

The results then follow by simple inspection of (18), (19) and (B2).

B.3 Proof of Proposition 3

(i) Suppose that $\sigma_i > \sigma_j$, implying that Provider i has a larger share of the market in the steady state. From (9) and (19) we derive

$$\overline{D}_i^{OL} - \overline{D}_i^{F} = \frac{\theta\tau^2\gamma^2\left(\varphi - (2\gamma+\rho)\right)(\sigma_i - \sigma_j)}{2\theta\tau^2\left(\rho+\varphi\right)\left(\gamma-\rho+\varphi\right)\left((\beta-2b)\gamma + 2\tau^2\theta(\gamma+\rho)\right)} > 0. \tag{B3}$$

Since the difference in demand between the two providers is uniquely determined by the difference in their quality provision, and since $D_j = 1 - D_i$, it follows that $\overline{D}_i^{OL} > \overline{D}_i^{F}$ implies $\overline{q}_i^{OL} - \overline{q}_j^{OL} > \overline{q}_i^{F} - \overline{q}_j^{F}$.

(ii) From (8) and (18) we derive

$$\overline{q}^{OL} - \overline{q}^{F} = \gamma\frac{\left(2\sigma - (\beta-2b)\right)\left(3\gamma^2\left(\varphi-(\gamma+\rho)\right)(\beta-2b) + 2\theta\tau^2\gamma\left(\gamma+2\rho\right)\left(\varphi-(2\gamma+\rho)\right)\right)}{4\theta^2\tau^3\left(\varphi+\rho\right)\left(\gamma+\rho\right)\left(\varphi+4\gamma+5\rho\right)\left(\varphi+\gamma-\rho\right)} > 0. \tag{B4}$$

B.4 Proof of Proposition 4

The result is directly confirmed by the feedback representation of the socially optimal time quality investment rule, (25), from which we derive

$$\frac{\partial q_i^*}{\partial D_i} = \frac{\widehat{\alpha}_2}{2\theta} > (<)\, 0 \text{ if } \theta\tau\left(2\gamma+\rho\right) > (<) \sqrt{\theta\left(4\beta\gamma^2 + 4\tau\gamma(\rho+\gamma)(\tau\theta-1) + \rho^2\tau^2\theta\right)}, \tag{B5}$$

which is equivalent to

$$\beta < (>)\, \tau\left(1 + \frac{\rho}{\gamma}\right). \tag{B6}$$

B.5 Proof of Proposition 5

(i) The result follows directly from (29).

(ii) The steady state quality difference is given by

$$\Delta\overline{q}^* := \left|\overline{q}_i^* - \overline{q}_j^*\right| = \frac{\gamma\tau\left|c_j - c_i\right|}{\beta\gamma + \tau(\gamma+\rho)(\tau\theta-1))}, \tag{B7}$$

from which it is immediately clear that $\partial\Delta\overline{q}^*/\partial\beta < 0$. Furthermore,

$$\frac{\partial\Delta\overline{q}^*}{\partial\gamma} = \frac{\tau^2\rho\left(\theta\tau-1\right)\left|c_j-c_i\right|}{\left(\beta\gamma+\tau(\gamma+\rho)(\tau\theta-1))\right)^2} > 0 \text{ (under assumption A1).} \tag{B8}$$

(iii) From (B8) we derive

$$\frac{\partial \Delta \overline{q}^*}{\partial \tau} = \frac{\gamma\left(\beta\gamma - \theta\tau^2\left(\gamma+\rho\right)\right)|c_j - c_i|}{\left(\beta\gamma + \tau(\gamma+\rho)(\tau\theta-1)\right)^2} > (<)\, 0 \text{ if } \theta\tau^2 < (>)\, \frac{\beta\gamma}{\gamma+\rho}. \tag{B9}$$

B.6 Proof of Proposition 6

Following Benchekroun and Long (1998), the first-best pricing rule in the OLNE is computed by imposing that the first-best quality levels in feedback form, given by (25), solve equation (A8), which is the feedback representation of the OLNE. The price parameters a_i, a_j and b must thus solve

$$\frac{dq_i^*}{dD_i}\gamma\left(\frac{1}{2} + \frac{q_i^* - q_j^*}{2\tau} - D_i\right) = (\rho+\gamma)q_i^* - \frac{\gamma}{2\theta\tau}[\sigma_i - (\beta-2b)D_i] \tag{B10}$$

and

$$\frac{dq_j^*}{dD_i}\gamma\left(\frac{1}{2} + \frac{q_i^* - q_j^*}{2\tau} - D_i\right) = (\rho+\gamma)q_j^* - \frac{\gamma}{2\theta\tau}[\sigma_j - (\beta-2b)(1-D_i)]. \tag{B11}$$

Using (25) and the fact that $q_j^* = \frac{1}{\theta} - q_i^*$, we can write (B10) and (B11) as, respectively,

$$\frac{\widehat{\alpha}_2\gamma}{2\theta}\left(\frac{1}{2} + \frac{\left(\widehat{\alpha}_1^i + \widehat{\alpha}_2 D_i\right) - 1}{2\theta\tau} - D_i\right) = \frac{(\rho+\gamma)}{2\theta}\left(\widehat{\alpha}_1^i + \widehat{\alpha}_2 D_i\right) - \frac{\gamma\left(\sigma_i - (\beta-2b)D_i\right)}{2\theta\tau} \tag{B12}$$

and

$$-\frac{\widehat{\alpha}_2\gamma}{2\theta}\left(\frac{1}{2} + \frac{\left(\widehat{\alpha}_1^i + \widehat{\alpha}_2 D_i\right) - 1}{2\theta\tau} - D_i\right) = \frac{(\rho+\gamma)}{2\theta}\left(\frac{1}{\theta} - \widehat{\alpha}_1^i - \widehat{\alpha}_2 D_i\right) - \frac{\gamma\left(\sigma_j - (\beta-2b)\left(1-D_i\right)\right)}{2\theta\tau} \tag{B13}$$

By collecting the terms containing D_i, we obtain, from both equations, the following identity:

$$\frac{\widehat{\alpha}_2\gamma}{2\theta}\left(\frac{\widehat{\alpha}_2}{2\theta\tau} - 1\right) = (\rho+\gamma)\frac{\widehat{\alpha}_2}{2\theta} + \frac{\gamma}{2\theta\tau}(\beta-2b). \tag{B14}$$

Solving (B14) for b yields $b = b^{OL}$ as given by (31). Analogously, by collecting the other terms in (B12) and (B13), we obtain two equalities that allow us to solve for the optimal values of the other price parameters a_i^{OL} and a_j^{OL}.

Assumption A2 implies that the optimal solution exists if

$$\beta > 2b^{OL}, \tag{B15}$$

which is equivalent to

$$\beta > \frac{\tau(\gamma+\rho)}{\gamma}. \tag{B16}$$

B.7 Proof of Proposition 7

In order to obtain the first-best pricing rule in the linear SFNE, we only need to equate the providers' equilibrium strategies given by (14) with the feedback representation of the first-best solution given by (25). The optimal price parameters b^F, a_i^F and a_j^F are then found by solving the system

$$\begin{cases} \alpha_2\left(b\right) = \widehat{\alpha}_2 \\ \frac{\gamma}{\tau}\alpha_1^i\left(a_i, a_j, b\right) = \widehat{\alpha}_1^i \\ \frac{\gamma}{\tau}\alpha_1^j\left(a_i, a_j, b\right) = \widehat{\alpha}_1^j \end{cases} \tag{B17}$$

Under the optimal pricing rule, assumption A2 requires $\beta > 2b^F$, which holds if and only if

$$\beta > -\frac{\tau}{6\gamma^2}[2\gamma(2\tau\theta-3)(\gamma+\rho)+\rho^2\tau\theta] \tag{B18}$$

and

$$\beta < -\frac{\tau[\gamma(8\tau\theta-9)(\gamma+\rho)+2\rho^2\tau\theta]}{9\gamma^2} \quad \text{or} \quad \beta > \frac{\tau(\gamma+\rho)}{\gamma}. \tag{B19}$$

Since

$$\frac{\tau(\gamma+\rho)}{\gamma} > -\frac{\tau}{6\gamma^2}[2\gamma(2\tau\theta-3)(\gamma+\rho)+\rho^2\tau\theta] > -\frac{\tau[\gamma(8\tau\theta-9)(\gamma+\rho)+2\rho^2\tau\theta]}{9\gamma^2}, \tag{B20}$$

assumption A2 is satisfied if and only if condition (B16) holds.

B.8 Proof of Corollary 3

The result is found by first computing

$$\frac{\partial b^F}{\partial \beta} < 0 \iff \beta > -\tau\frac{4\gamma(3\tau\theta-4)(\gamma+\rho)+3\rho^2\tau\theta}{16\gamma^2}. \tag{B21}$$

The second inequality in (B21) always holds since

$$\beta > \frac{\tau(\gamma+\rho)}{\gamma} > -\tau\frac{4\gamma(3\tau\theta-4)(\gamma+\rho)+3\rho^2\tau\theta}{16\gamma^2}. \tag{B22}$$

Thus, the supremum over the values of b^F is obtained for $\beta \to \left[\frac{\tau(\gamma+\rho)}{\gamma}\right]^+$, and it is obviously given by $\frac{1}{2}\frac{\tau(\gamma+\rho)}{\gamma} > 0$; and $\lim_{\beta\to\infty} b^F = -\infty$. This implies that there exists a unique threshold $\widehat{\beta}^F$ such that $b^F > (<)0$ if $\beta < (>)\,\widehat{\beta}^F$, and this threshold is given by (37) in Corollary 3.

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