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Dynamic Hospital Competition Under Rationing by Waiting Times*

Luís Sá† Luigi Siciliani‡ Odd Rune Straume§

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Abstract

We develop a dynamic model of hospital competition where (i) waiting times increase if demand exceeds supply; (ii) patients differ in their evaluation of health benefits and choose a hospital based on waiting times; and (iii) there are penalties for providers with long waits. We show that, if penalties are linear in waiting times, a more competitive dynamic environment does not affect waiting times. If penalties are instead non-linear, we find that waiting times are longer under the more competitive environment. The latter result is derived by calibrating the model with waiting times and elasticities observed in the English NHS for a common treatment (cataract surgery), which also shows that the difference between waiting times under the two solution concepts is quantitatively small. Policies that facilitate patient choice, an alternative measure of competition, also lead to higher steady-state waiting times, and tougher penalties exacerbate the negative effect of choice policies.

Keywords: Hospital competition; waiting times; differential games.

JEL Classification: C73; H42; I11; I18; L42.

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1 Introduction

Waiting times for non-emergency (elective) treatments are a key health policy concern across OECD countries, such as Australia, Canada, Ireland, Finland, Norway, Portugal, and the United Kingdom. Mean waiting times range between 50 and 150 days across countries for common procedures such as cataract surgery, hip and knee replacement, hernia, hysterectomy, and prostatectomy (Siciliani et al., 2014). Although some countries like Finland and the UK have had successes in 2000-2005 in reducing waiting times from high levels (e.g., more than 150 days on average for hip and knee replacement), waiting times have stalled in most countries since the financial crisis and have slowly started to rise again in some countries. In countries like Chile, Poland, and Estonia, waiting times for hip and knee procedures are still above one year (OECD, 2017).

Waiting times are a major source of dissatisfaction for patients since they postpone health benefits, may worsen symptoms, deteriorate patients' conditions, and lead to worse clinical outcomes. In response to the dissatisfaction that they generate, governments have taken a variety of measures to reduce waiting times. Many OECD countries have adopted some form of maximum waiting time guarantees (Siciliani, Moran, and Borowitz, 2013). However, the design and implementation of these guarantees can differ significantly across countries.

Two common approaches are to link maximum wait guarantees either to penalties or to competition (and patient choice) policies. The first approach was followed by Finland and England, which combined maximum waiting times with sanctions for failure to fulfil the guarantee. Targets with penalties were introduced in England in 2000-05 with political oversight from the Prime Ministerial Delivery Unit and the Health Care Commission. Senior health administrators risked losing their jobs if targets were not met. As a result, the proportion of patients waiting over six months was reduced by 6-9 percentage points (Propper et al., 2010). In 2010, maximum wait guarantees became a patient entitlement codified into the NHS Constitution, establishing a patient right to a maximum of 18 weeks from GP referral to treatment. In Finland, waiting time guarantees were combined with targets as part of the Health Care Guarantee in 2005, subsequently included in the 2010 Health Care Act. A National Supervisory Agency supervised the implementation of the guarantee through targets and penalised municipalities failing to comply. The number of patients waiting over six months was reduced from 12.6 per $1,000$ population in 2002 to 6.6 per $1,000$ in 2005 (Siciliani, Moran, and Borowitz, 2013).

The second approach involves combining maximum waiting time guarantees with patient choice and competition policies. For example, in Denmark, if the hospital foresees that the maximum waiting time guarantee will not be fulfilled, the patient can choose another public or private hospital. In Portugal, when a patient on the waiting list reaches 75% of the maximum guaranteed time, a voucher that allows the patient to seek treatment at any other provider, including private sector providers, is issued. In several countries, like England and Norway, patients are free to choose any provider within the country (Siciliani et al., 2017).

From an economics perspective, waiting times act as a non-price rationing device to bring into equilibrium the demand for and the supply of health care in publicly-funded health systems. Many countries with a National Health Service or public health insurance combine the absence of co-payments with the presence of capacity constraints. As a result, an excess demand arises, which translates into a waiting list. One way to bring the demand for and the supply of treatments into equilibrium is to rely on waiting times. As argued by Lindsay and Feigenbaum (1984), Martin and Smith (1999), and Iversen (1993, 1997), waiting times tend to discourage demand if patients give up the treatment or opt for treatment in the private sector. Waiting times may also influence positively the supply of health services if altruistic providers exert greater effort and treat more patients when waiting times are higher.

In the present study, we investigate whether competition and patient choice policies play a useful role in reducing waiting times, and the extent to which such a role is altered in the presence of penalties for providers with long waits. Our model is dynamic to capture a key feature of the waiting time phenomenon. Waiting times tend to increase when demand for treatment is higher than the supply of treatment so that new patients are added to the waiting list. Similarly, waiting times tend to reduce when more patients are removed from the waiting list than those added. A second feature of our model is that hospitals compete for patients, with hospitals with lower waiting times attracting more patients.

The combination of a dynamic approach with strategic interactions across providers calls for a differential-game approach. As is customary under this approach (Dockner et al., 2000), we use two solution concepts. First, we derive the open-loop solution, where each hospital commits to an optimal supply plan of treatments before the game starts. This solution is plausible in institutional settings where supply is subject to rigidities (such as investment regulations), implying that hospitals must commit to long-term supply plans. If hospitals are restricted to follow their plans and cannot adjust supply, this case is equivalent to one wherein hospitals cannot observe and react to waiting times once the game starts. Second, we derive the feedback (closed-loop) solution, where hospitals can observe waiting times at each point in time, not only at the beginning of the game, and react to such information. In contrast to the open-loop solution, hospitals are allowed to revise their supply decisions based on the evolution of waiting times. Thus, the key difference between the two solution concepts is the degree of commitment.

To model the demand for healthcare faced by each provider, we use a Hotelling approach with two hospitals located at each endpoint of the unit line segment. We adopt a general specification, which allows for two types of patients who differ in the valuation of their outside option (e.g., to seek treatment in the private sector or to forego treatment altogether), which in turn implies different net benefits, high and low, from hospital treatment. Hospitals compete on the segment of demand with high benefit, while they are local monopolists on the demand segment with low benefit.

We look at two aspects of competition. The first relates to the solution concept. The feedback solution is commonly interpreted as a more competitive solution in the sense that hospitals can, at each point in time, change their treatment plans in response to the dynamics of the waiting times. We therefore compare waiting times under the two solution concepts. Second, under each solution concept, we investigate the effect of policies that facilitate *patient choice*, commonly interpreted as policies that stimulate competition.

We obtain several findings with policy implications. First, we show that the design of penalties has a critical role in predicting the effect of competition on waiting times. If the penalties that providers face are linear in waiting times, then the open-loop and the closed-loop solutions coincide. Therefore, a more competitive dynamic environment does not affect the equilibrium waiting times. If penalties are instead non-linear so that the marginal penalty increases with waiting times, we find that waiting times are *longer* under the more competitive environment (the closed-loop solution). In this case, the optimal closed-loop strategies are characterised by dynamic strategic substitutability in supply. The intuition for this key result is that lower treatment supply by one hospital will be optimally met by increased supply by the competing hospital, which dampens the initial increase in waiting time caused by the supply reduction. This strategic substitutability gives each hospital an incentive to reduce its supply in order to 'free-ride' on the subsequent supply increase by the other hospital.

Our results for the closed-loop solution are numerically derived, since a closed-form solution cannot be obtained in this scenario. To make our results more salient, we calibrate our model based on waiting times observed in the English NHS. The calibration is also informed by demand elasticities which have been estimated in the empirical literature (Martin and Smith, 1999; Sivey, 2012). Our results suggest that the difference between steady-state waiting times under the two solution concepts increases with the degree of convexity in the waiting time penalty structure. The policy implication is that waiting time penalties are likely to be more effective in reducing waiting times if they are designed with a linear penalty structure. However, another key result from the calibration is that, although waiting times are higher under the closed-loop solution, the differences in waiting times between the two solution concepts are very small (less than 1% in all our calibrations with different demand elasticities).

Regarding the effects of patient choice, we show (in the open-loop solution) that policies to facilitate patient choice lead to *higher* steady-state waiting times (regardless of whether penalties are linear or not) and have an ambiguous effect on steady-state treatment supply (though the effect is negative if penalties are linear). This is because patient choice, as captured by lower transportation costs in a Hotelling set-up, makes demand more responsive to changes in waiting times. In turn, this reduces the effectiveness of treatment supply in reducing waiting times, since a reduction in waiting attracts more patients from the other hospital, offseting the potential effect of the initial increase in supply.

Our calibration of the closed-loop equilibrium shows that the above described effects of increased patient choice are qualitatively and quantitatively very similar across the two solution concepts. In both cases, more patient choice leads to higher steady-state waiting times, and the magnitude of the effect depends positively on the convexity of the waiting time penalty. This result has implications for the optimal choice of policy to achieve an overall reduction in hospital waiting times. Consider two different policy options: imposing provider penalties or stimulating patient choice. In our model, not only is the former the only policy that is effective in reducing waiting times, but such a policy also makes choice policies counterproductive, and more so the tougher the penalties. The larger the waiting time penalties, the larger the increase in steady-state waiting times as a result of more patient choice.

The rest of the study is organised as follows. In the next section, we present a brief overview of the literature and explain how we contribute to this literature. In Section 3, we present the model. The open-loop and closed-loop solutions of the model are analysed in Sections 4 and 5, respectively. Section 6 provides concluding remarks.

2 Related Literature

Our study brings together two different strands of the theoretical literature. The first is the literature that investigates the role of waiting times in the health sector. As mentioned above, the idea that waiting times may help bringing the supply and the demand for healthcare into equilibrium goes back to Lindsay and Feigenbaum (1984) and Iversen (1993). Iversen (1997) also investigates whether allowing patients to be treated in the private sector will reduce waiting times in the public sector and shows that the answer depends on the demand elasticity for public treatment with respect to waiting time. Demand and supply responsiveness to waiting times are estimated by Martin and Smith (1999) using English data, and they find that demand is generally inelastic (with an elasticity of about -0.1).

There are also normative analyses in this strand of the literature. Hoel and Sæther (2003) show that concerns for equity can make it optimal to have a mixed system of public and private provision with a positive waiting time in the public sector, though March and Schroyen (2005) find, through a calibration exercise, that the welfare gains of a mixed system might be quite low. Gravelle and Siciliani (2008a, 2008c) investigate the scope for waiting time prioritisation policies across and within treatments and find that prioritisation is generally welfare improving even in a setting where the provider can only observe some dimensions of patient benefit. Gravelle and Siciliani (2008b) also show that rationing by copay tends to be welfare improving relative to rationing by waiting. All the above studies use a static approach assuming that demand and supply adjust instantaneously to reach equilibrium. One exception is Siciliani (2006) who investigates the behaviour of a monopolist in a dynamic set-up. We model waiting time dynamics in a similar way but critically allow for strategic interactions across providers to investigate the role of patient choice and competition.

The second strand of the literature relates to hospital competition with fixed prices. Though most of this literature consists of studies using a static framework, there is a limited but growing literature that models hospital competition in a dynamic framework. It focus, however, on incentives for quality provision rather than on waiting times.[1] Brekke et al. (2010, 2012) find that, if quality is modelled as a stock variable which increases if quality investments are higher than its depreciation, or, if demand is sluggish so that an increase in quality only partially translates into an increase in demand, then quality is higher under the open-loop solution if hospitals face increasing marginal treatment costs. Equilibrium quality instead coincide under the two solution concepts if

[1] See Brekke et al. (2014) for a review of the theoretical literature on hospital competition under regulated prices.

marginal treatment costs are constant. Siciliani, Straume, and Cellini (2013) suggest that these results can be overturned in the presence of altruistic preferences so that quality is higher under the closed-loop solution.

Our modelling of waiting times differs analytically from these previous contributions because the state variable (i.e., waiting time) of the rival enters the dynamic constraint of the maximisation problem of each provider. This is not the case when quality is modelled as a stock (as in Brekke et al. (2010)) because neither the state nor control variable of the rival provider enters the quality stock function, or when demand is modelled as sluggish (as in Brekke et al. (2012)) because demand depends on the control variable of the rival, not the state variable. Thus, because of these fundamental differences in the dynamic nature of the problems, the results from models of dynamic quality competition do not automatically carry over to the case of waiting times. In other words, if we want to study the effects of patient choice and competition on waiting times in a dynamic context, we cannot simply interpret waiting time as 'negative quality' and apply the results from the above mentioned studies of dynamic quality competition.

As previously mentioned, in the main bulk of the theoretical literature on hospital competition, the theoretical framework is a static one. To our knowledge, the only study among these that deals with waiting times is Brekke et al. (2008). Similarly to the present study, they identify a potentially positive relationship between patient choice and equilibrium waiting times. However, the underlying mechanisms are very different. In the static model (Brekke et al., 2008), hospitals choose waiting times to influence demand and in turn revenues. Increased competition (patient choice) makes demand more responsive to changes in waiting time, which then becomes a more effective tool for each hospital to steer demand in the desired direction. If hospitals are semi-altruistic, the equilibrium is such that price is below marginal cost (for the marginal treatment). Hospitals might therefore have an incentive to *reduce* demand, and waiting times become a more powerful tool to achieve this when patient choice increases, paving the way for a positive relationship between patient choice and equilibrium waiting times.

In the present dynamic approach, more competition also makes demand more responsive to waiting times, but then the similarities end. Hospitals choose treatment supply but cannot directly control waiting times. The supply decision is instead used as an instrument to affect waiting times, and this instrument becomes less effective with increased patient choice. This is why more competition leads to higher waiting times in our dynamic setting, and the underlying mechanism is not related to price being below marginal cost in equilibrium, although this feature is also present here. Thus, the present study is not just a dynamic version of Brekke et al. (2008), in the sense that the results rely on the same mechanisms placed in a dynamic context. Rather, placing the analysis in a dynamic framework allows us to uncover new mechanisms that are uniquely related to the dynamic process that generates changes in waiting times. In this sense, the present dynamic analysis complements and reinforces the previous results based on a static framework.

3 The Model

Consider a duopolistic health care market in which hospitals, indexed by i and j, are located at each endpoint of the unit line segment $[0, 1]$. There are N potential patients uniformly distributed on the line segment. In every period t, each of these patients may benefit from treatment at either of the two hospitals. In order to consume one unit of treatment, patients bear no out-of-pocket expenditures at the hospital but face expenses (or disutility) in the form of travelling costs. Furthermore, patients are required to join a waiting list and therefore suffer a disutility of waiting.

There are two types of patients, differing with respect to the value of their outside option (i.e., the utility of not being treated by either of the two hospitals). Whereas a share β of the patients are assumed to have no valuable outside option, the remaining share $(1 - \beta)$ have a strictly positive outside option $k > 0$. For simplicity, we assume that these shares are constant along the line segment. The difference between these two patient types can be attributed either to a difference in illness severity, which creates a difference in the utility of being untreated, or to a difference in the ability to seek treatment elsewhere (e.g., in a private market or abroad), for example, due to differences in income or wealth.

Both types of patients make utility-maximising treatment consumption decisions, taking into account travelling costs as well as the length of time between the moment they join the waiting list and that when treatment is supplied (i.e., the waiting time). The utility in period t of a patient with no valuable outside option, who is located at $x \in [0, 1]$ and chooses Hospital i, located at z_i, is given by

$$u(x, z_i, t) = v - w_i(t) - \tau|x - z_i|, \tag{1}$$

where v is the gross valuation of treatment, $w_i(t)$ is the waiting time at Hospital i in period t, and τ is the marginal disutility of travelling. The marginal disutility of waiting is normalised to one, which allows τ to be interpreted as the marginal disutility of travelling relative to waiting. The equivalent utility in period t of a patient with a strictly positive outside option is

$$u(x, z_i, t) = v - k - w_i(t) - \tau|x - z_i|. \tag{2}$$

For patients with a positive outside option, we assume that k is sufficiently high such that some of these patients will strictly prefer the outside option to being treated by any of the two hospitals in the market. This implies that the relevant choice for each of these patients is between seeking treatment at the most preferred hospital or exercising the outside option. We will refer to this as the *monopolistic segment* of the market. For all the patients without a valuable outside option, we assume that utility is maximised by seeking treatment at one of the hospitals. These patients therefore constitute the *competitive segment* of the market. By concentrating on cases where the competitive segment is fully covered, whereas the monopolistic segment is only partially covered, we ensure that total demand is elastic with respect to waiting times, implying that waiting times have a rationing effect on demand.

3.1 Demand for Hospital Treatment

In the *competitive* segment, the patient who is indifferent between seeking treatment at Hospital i and Hospital j is located at $x_C(t)$, implicitly given by

$$v - w_i(t) - \tau x_C = v - w_j(t) - \tau(1 - x_C), \tag{3}$$

yielding

$$x_C(t) = \frac{1}{2} + \frac{w_j(t) - w_i(t)}{2\tau}. \tag{4}$$

In the *monopolistic* segment, the patient who is indifferent between demanding treatment at Hospital i and consuming his or her outside option is located at $x_M^i(t)$, implicitly given by

$$v - w_i(t) - \tau x_M^i = k, \tag{5}$$

yielding

$$x_M^i(t) = \frac{v - k - w_i(t)}{\tau}. \tag{6}$$

A similar expression can be obtained for Hospital j: $x_M^j(t) = (v - k - w_j(t))/\tau$.

With a total mass N of patients in the market, demand faced by Hospitals i and j is a weighted sum of demand from the competitive and the monopolistic segments and is respectively given by

$$D_i(w_i(t), w_j(t)) = N[\beta x_C(t) + (1-\beta)x_M^i(t)] \tag{7}$$

and

$$D_j(w_i(t), w_j(t)) = N[\beta(1 - x_C(t)) + (1-\beta)x_M^j(t)]. \tag{8}$$

3.2 Hospital Objectives and Treatment Supply

In each period t, Hospital i treats $S_i(t)$ patients. Hospitals are financed by a third-payer (e.g., a regulator or insurer) that offers a prospective payment p for each unit of treatment supplied and a lump-sum transfer T. The instantaneous objective function of Hospital i is assumed to be

$$\Pi_i(t) = T + pS_i(t) - C(S_i(t)) - \Phi(w_i(t)). \tag{9}$$

The cost of supplying hospital treatments is given by an increasing and strictly convex cost function $C(S_i(t)) = \frac{\gamma}{2}S_i(t)^2$, with $\gamma > 0$. The convexity of the cost function captures an important feature in the context of waiting times, namely that hospitals face capacity constraints.[2] The function $\Phi(w_i(t))$ captures the disutility of having positive waiting times. The disutility of waiting time is monetary if the hospital faces penalties levied by the regulator or reductions in funding. Alternatively, it is non-monetary if the hospital takes into the account the reputational damage of reporting long waiting times, or if the hospital is subject to a more stringent monitoring regime by the regulator. We assume that the disutility of waiting time takes the linear-quadratic form

$$\Phi(w_i(t)) = \alpha_1 w_i(t) + \frac{\alpha_2}{2} w_i(t)^2, \tag{10}$$

with $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$. Whether waiting times penalties have a linear or non-linear effect on hospital utility depends on the institutional context. In settings where hospital managers can lose their jobs when waiting times become very long, penalties are arguably non-linear. This may also be the case in health systems where health regulators have mechanisms that escalate from warning messages to agreeing and monitoring action plans with the providers. Other health systems may instead gradually penalise hospitals with longer wait through a proportionate reduction in revenues.

[2] A strictly convex treatment cost function captures the case of *smooth* capacity constraints, where capacity can be increased, but only at an increasing marginal cost.

Waiting times evolve dynamically over time according to

$$\frac{dw_i(t)}{dt} = \dot{w}_i(t) = \theta[D_i(w_i(t), w_j(t)) - S_i(t)] \tag{11}$$

and

$$\frac{dw_j(t)}{dt} = \dot{w}_j(t) = \theta[D_j(w_i(t), w_j(t)) - S_j(t)], \tag{12}$$

where $\theta > 0$ relates changes in waiting times to the difference between the demand faced by each hospital and its activity (i.e., changes in the waiting list). Under this formulation, waiting times increase when current demand exceeds current supply and vice versa, and the speed at which waiting times respond to changes in demand or supply is given by θ.

We are implicitly assuming that the waiting time at each hospital is positive in every period. The hospital objective function depends on the hospital's supply decision, which is given by the number of treatments performed by Hospital i in period t, $S_i(t)$. The objective function does not instead depend directly on demand, which is given by the number of patients added to Hospital i's waiting list in period t, $D_i(w_i(t), w_j(t))$. If $S_i(t) < D_i(w_i(t), w_j(t))$, there is a net increase in the waiting list and the (expected or average) waiting time increases. On the other hand, if $S_i(t) > D_i(w_i(t), w_j(t))$, there is a net reduction in the waiting list and the waiting time therefore falls. In either case, as long as the waiting list is not emptied, the number of treatments performed in period t is given by the hospital's supply of treatments. Demand for treatments only affects the actual number of treatments indirectly through waiting times, which in turn affect each hospital's optimal supply decisions, as we will show later.

We assume that the hospitals maximise their payoffs over an infinite time horizon and have a common constant discount rate, ρ. Formally, the maximisation problem of Hospital i is given by

$$\begin{aligned}
\max_{S_i(t)\in\mathbb{R}_0^+} \quad & \int_0^\infty e^{-\rho t}\Pi_i(t)dt \\
\text{subject to} \quad & \dot{w}_i(t) = \theta[D_i(w_i(t), w_j(t)) - S_i(t)], \\
& \dot{w}_j(t) = \theta[D_j(w_i(t), w_j(t)) - S_j(t)], \\
& w_i(0) = w_{i0} > 0, \\
& w_j(0) = w_{j0} > 0.
\end{aligned}$$

Although, in reality, hospitals do not plan their activity over an infinite time horizon, we argue that this is a reasonable approximation if hospitals are regarded as lasting institutions. Managerial and medical structures are periodically replaced, but the hospital's *mission*—to provide care given its production technology and the regulatory scheme it faces—is likely to remain the same over long periods of time. This is likely if hierarchies are substituted by others with similar objective functions.

3.3 Solution Concepts

We follow Dockner et al. (2000) and use two different solution concepts to solve the differential game describing hospital interaction. Under the *open-loop* solution, hospitals either compute their optimal supply paths at the beginning of the game and are restricted to follow such plans thereafter, or they may observe the state of the world (i.e., waiting times) only at $t = 0$ and cannot therefore condition their actions (i.e., supply) on these observations thereafter. In both cases, strategies are time-profiles that specify the supply to be provided at each point in time.

If, besides current time, hospitals observe waiting times in every period and factor them in their decision making, a *closed-loop* solution arises. Under this solution concept, Hospital i's supply is a function of the contemporaneous waiting times in each t. While the closed-loop solution is informationally more demanding, it involves weaker commitment since hospitals are allowed to adjust supply as waiting times evolve.

The appropriateness of each solution concept depends on the assumptions regarding the players' information set as well as commitment requirements. The open-loop solution implies that hospitals have no information concerning waiting times once the game starts or are committed to the supply plans computed at the beginning of the game, which might be considered an excessively stringent assumption. Due to regulatory requirements, hospitals periodically collect and report data on waiting times, upon which their activity may be conditioned.[3] Moreover, a setting in which hospitals adjust activity according to waiting times is more realistic and relevant for policy-making.[4] Although the closed-loop is arguably the more appropriate concept to solve the game presented above, its full analytical derivation is possible only if waiting time penalties are linear. If penalties

[3] See Siciliani, Moran, and Borowitz (2013) for a description of waiting times regulatory arrangements and policies across OECD countries.

[4] This need not be the case of other analyses of hospital behaviour. The case of quality competition as analysed in, for example, Brekke et al. (2010) provide a setting in which the open-loop solution might be, at least, as appropriate. If hospitals devise investment plans that ought to be followed for long periods of time, meaning that their discretion is strongly restricted, their actions (investment decisions) are *as if* they are not conditional on the state of the world (the stock of quality).

are quadratic in waiting times, only a numerical solution can be derived from a calibrated model. We therefore start by computing the open-loop solution (Section 4) and use it as a benchmark to compare with the closed-loop solutions under constant (Section 5.1) and increasing (Sections 5.2 and 5.3) marginal disutility of waiting.

4 Open-Loop Solution

Let $\mu_i(t)$ and $\lambda_i(t)$ denote, respectively, the costate variables associated with the dynamic equations of $w_i(t)$ and $w_j(t)$ for Hospital i. That is, $\mu_i(t)$ is associated with Hospital i's waiting time and $\lambda_i(t)$ with that of the rival. The current-value Hamiltonian is

$$\begin{aligned} H_i = T + pS_i(t) - \frac{\gamma}{2}S_i(t)^2 - \alpha_1 w_i(t) - \frac{\alpha_2}{2}w_i(t)^2 \\ + \mu_i(t)\theta[D_i(w_i(t), w_j(t)) - S_i(t)] + \lambda_i(t)\theta[D_j(w_i(t), w_j(t)) - S_j(t)]. \end{aligned} \tag{13}$$

Candidates for optimal supply path $S_i(t)$ and costate trajectories $\mu_i(t)$ and $\lambda_i(t)$ must satisfy $\partial H_i/\partial S_i(t) = 0$, $\dot{\mu}_i(t) = \rho\mu_i(t) - \partial H_i/\partial w_i(t)$, and $\dot{\lambda}_i(t) = \rho\lambda_i(t) - \partial H_i/\partial w_j(t)$. More extensively:

$$p - \gamma S_i(t) = \theta\mu_i(t), \tag{14}$$

$$\dot{\mu}_i(t) = \left[\rho + \frac{\theta(2-\beta)N}{2\tau}\right]\mu_i(t) - \frac{\theta\beta N}{2\tau}\lambda_i(t) + \alpha_1 + \alpha_2 w_i(t), \tag{15}$$

and

$$\dot{\lambda}_i(t) = \left[\rho + \frac{\theta(2-\beta)N}{2\tau}\right]\lambda_i(t) - \frac{\theta\beta N}{2\tau}\mu_i(t). \tag{16}$$

The solution must also satisfy the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t}\mu_i(t)w_i(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} e^{-\rho t}\lambda_i(t)w_j(t) = 0. \tag{17}$$

Optimality is established by concavity of the current-value Hamiltonian with respect to $S_i(t)$ and $w_i(t)$.[5] Inserting the definition of demand (7) and the optimality condition for supply using (14) into the dynamic constraint (11) yields

$$\dot{w}_i(t) = \theta N\left[\beta\left(\frac{1}{2} + \frac{w_j(t) - w_i(t)}{2\tau}\right) + (1-\beta)\left(\frac{v - k - w_i(t)}{\tau}\right)\right] - \theta\left(\frac{p - \theta\mu_i(t)}{\gamma}\right). \tag{18}$$

We focus on the symmetric open-loop equilibrium with non-negative waiting times and a partially covered monopolistic segment.

[5] The sufficient conditions are satisfied since $\frac{\partial^2 H_i}{\partial S_i^2} = -\gamma < 0$, $\frac{\partial^2 H_i}{\partial w_i^2} = -\alpha_2 < 0$, $\frac{\partial^2 H_i}{\partial S_i^2}\frac{\partial^2 H_i}{\partial w_i^2} - \frac{\partial^2 H_i}{\partial S_i \partial w_i} = \gamma\alpha_2 > 0$.

4.1 The Steady-State

Let the superscript OL denote the open-loop steady-state in which $w_i(t) = w_j(t) = w^{OL}$, $\mu_i(t) = \mu_j(t) = \mu^{OL}$, and $S_i(t) = S_j(t) = S^{OL}$. Setting $\dot{w}(t) = \dot{\mu}(t) = \dot{\lambda}(t) = 0$ in equations (15), (16), and (18) and solving for the steady-state waiting time and costate variable gives

$$w^{OL} = \frac{\gamma\phi\tau}{(1-\beta)\gamma\phi N + 2\theta\tau^2\alpha_2}\left\{N\left[\frac{\beta}{2} + (1-\beta)\left(\frac{v-k}{\tau}\right)\right] - \frac{p}{\gamma} - \frac{2\theta\tau\alpha_1}{\gamma\phi}\right\} \tag{19}$$

and

$$\mu^{OL} = -\frac{2\tau}{\phi}(\alpha_1 + \alpha_2 w^{OL}), \tag{20}$$

where

$$\phi = \theta(2-\beta)N + 2\tau\rho - \frac{(\theta\beta N)^2}{\theta(2-\beta)N + 2\tau\rho} \in (0,1). \tag{21}$$

Inserting (20) into (14) yields

$$p + \frac{2\theta\tau}{\phi}(\alpha_1 + \alpha_2 w^{OL}) = \gamma S^{OL}. \tag{22}$$

On the one hand, a marginal increase in supply (i) generates more revenues and (ii) reduces the waiting time and its associated disutility (left-hand side of (22)). On the other hand, increasing supply is costly (right-hand side of (22)). In the steady-state, each hospital offers a per-period supply of treatments such that the marginal benefit is exactly offset by the marginal cost. This trade-off is key to understanding the main intuition behind most of our subsequently derived results.[6]

The steady-state supply is $S^{OL} = p/\gamma + 2\theta\tau(\alpha_1 + \alpha_2 w^{OL})/\gamma\phi$. In Appendix A.1, we show that a sufficiently large γ ensures that the steady-state is indeed characterised by non-negative waiting times and a partially covered monopolistic segment. In Appendix A.2, we show that the symmetric equilibrium is stable in the saddle sense.

It follows immediately from equation (20) that a positive steady-state waiting time implies a negative μ^{OL}. This is consistent with the interpretation of μ^{OL} as the *shadow price* of waiting time. Given the disutility of waiting time, a hospital would have to be compensated—hence, a negative

[6] The condition in (22) holds in the steady-state, but it can be shown that a qualitatively similar condition holds at every point in time along the equilibrium path in the open-loop solution. Moreover, an analogous condition holds for all waiting times in the closed-loop solution as well. In other words, hospitals always balance production costs against revenues and waiting time reductions in equilibrium, not only in the steady-state, and regardless of whether strategies are time- or state-profiles. Details are available upon request.

shadow price—in order to accept a marginal increase in its waiting time.[7] Moreover, a negative μ^{OL} implies, together with (14), that the payment-cost margin is negative in the steady-state. The marginal patient is hence unprofitable to treat in the open-loop steady-state.[8]

As expected, a higher price p makes the marginal patient more profitable (or less unprofitable) to treat, which increases supply and reduces the waiting time. An increase in waiting time penalties, as proxied by an increase in α_1 or α_2, increases the disutility of waiting time therefore inducing the hospital to increase supply and reduce waiting times.

4.2 Patient Choice and Waiting Times

How does the degree of patient choice affect steady-state supply and waiting times? Our model includes two parameters that are related to patient choice, namely τ and β. A reduction in τ makes demand more responsive to changes in waiting times, thus reflecting a higher degree of patient choice. Similarly, an increase in β implies directly that a larger share of the patients make choices between the two hospitals in the market.

Consider first a reduction in τ, which is a standard way to measure increased patient choice, or increased intensity of competition, in the hospital competition literature that is based on models of spatial competition. The effect of a marginal change in τ on the steady-state waiting time can be expressed as

$$\frac{\partial w^{OL}}{\partial \tau} = -\frac{(1-\beta)x_M^{OL} + \frac{\tau}{N}\frac{\partial S^{OL}}{\partial \tau}}{1-\beta+\frac{2\theta\tau^2\alpha_2}{\gamma\phi N}} < 0, \tag{23}$$

where

$$\frac{\partial S^{OL}}{\partial \tau} = N\theta^2(\alpha_1+\alpha_2 w^{OL})\frac{(1-\beta)[N\theta(2-\beta)+4\tau\rho]\theta N + (2-\beta)(\tau\rho)^2}{2\gamma(N\theta+\tau\rho)^2[N(1-\beta)\theta+\tau\rho]^2} > 0 \tag{24}$$

is the marginal effect of τ on steady-state supply for a *given* waiting time.

There are two effects that work in the same direction. First, there is a direct demand effect. A reduction in τ increases total demand (and therefore demand for each hospital) since a larger number of patients in the monopolistic segment chooses to opt for treatment (at the nearest hospital). A higher demand directly increases the waiting time at each hospital. This effect is given by the first term in the numerator of (23), and the size of this effect depends on the relative size of the

[7] From equation (16) we see that this result applies to $w_j(t)$ as well, even though the associated shadow price is lower in absolute value.

[8] Notice that, when treatment costs are strictly convex, a negative payment-cost margin for the *marginal* patient does not imply that the payment-cost margin is negative for the *average* patient.

monopolistic segment. If $\beta \to 1$, which implies that total demand is completely inelastic, this effect vanishes.

Second, there is an indirect effect through changes in each hospital's incentive to affect waiting times through its treatment supply decision. Each hospital can lower its waiting time by increasing the supply of treatments, and the effect of a unilateral increase in treatment supply on the waiting time is given by a direct and an indirect (feedback) effect. For a given demand, an increase in treatment supply will reduce the waiting time. However, a lower waiting time will increase demand and therefore dampen the initial reduction in the waiting time. Crucially, the strength of this feedback effect depends on how strongly demand responds to waiting time changes. A lower τ makes demand more responsive to changes in waiting times, which increases the feedback effect and therefore makes treatment supply a less effective instrument to reduce waiting times. Consequently, this *reduces* the marginal benefit of treatment supply and gives each hospital an incentive to reduce the supply of treatments. This effect is captured by the second term in the numerator of (23).

The magnitude of the second effect, which is present for any value of $\beta \in [0, 1]$, is increasing in the waiting time disutility parameters α_1 and α_2 and vanishes if $\alpha_1 = \alpha_2 = 0$. This property has potentially interesting policy implications. Suppose that policy makers aim at reducing hospital waiting times. Two commonly suggested policy options is to either directly target the perceived problem by introducing (or increasing) waiting time penalties, or to stimulate patient choice (e.g., by public reporting of waiting times) with the aim of achieving lower waiting times through increased intensity of competition between the hospitals. In our model, only the former policy works, whereas the latter policy is counterproductive. Moreover, the former policy makes the latter policy more counterproductive. All else equal, the larger the waiting time penalties, the larger is the increase in steady-state waiting times as a result of more patient choice.

Although a reduction in τ unambiguously leads to higher waiting times, the effect on steady-state treatment supply turns out to be ambiguous. From (22), this effect is given by

$$\frac{dS^{OL}}{d\tau} = \frac{\partial S^{OL}}{\partial \tau} + \frac{2\theta\tau\alpha_2}{\gamma\phi}\frac{\partial w^{OL}}{\partial \tau}, \tag{25}$$

which, using (23), can be expressed as

$$\frac{dS^{OL}}{d\tau} = \frac{(1-\beta)N}{(1-\beta)\gamma\phi N + 2\theta\tau^2\alpha_2}\left[\gamma\phi\frac{\partial S^{OL}}{\partial \tau} - 2\theta\tau\alpha_2 x_M^{OL}\right] \lesseqgtr 0. \tag{26}$$

A marginal reduction in τ has two counteracting effects on steady-state supply. On the one hand, a lower τ makes treatment supply a less effective instrument to reduce waiting times, as explained above, which gives each hospital an incentive to reduce their supply. This effect is captured by the first term in the square brackets of (26). On the other hand, a lower τ also increases demand, which—all else equal—leads to higher waiting times. If the disutility of waiting time is strictly convex (i.e., if $\alpha_2 > 0$), such increase in waiting time increases the marginal disutility of waiting time and therefore increases the marginal benefit of supply. This effect is captured by the second term in the square brackets of (26).

Consider next an increase in the relative size of the competitive segment, β. The effect on steady-state waiting times is given by

$$\frac{\partial w^{OL}}{\partial \beta} = \frac{N\left(\frac{1}{2} - x_M^{OL}\right) - \frac{\partial S^{OL}}{\partial \beta}}{\frac{(1-\beta)N}{\tau} + \frac{2\theta\tau\alpha_2}{\gamma\phi}} \lesseqgtr 0, \tag{27}$$

where

$$\frac{\partial S^{OL}}{\partial \beta} = \frac{N\theta^2\tau(\alpha_1 + \alpha_2 w^{OL})}{2\gamma[N\theta(1-\beta) + \tau\rho]^2} > 0 \tag{28}$$

is the marginal effect of β on steady-state treatment supply for a *given* waiting time. Once more, there are two effects that now go in opposite directions. First, there is a direct and positive demand effect, since a larger competitive segment implies that a larger proportion of the potential patients demand treatment at one of the two hospitals. All else equal, this effect leads to higher waiting times and is captured by the first term in the numerator of (27). Notice that it is qualitatively similar to the demand effect of lower travelling costs. However, the second effect, which is captured by the second term in the numerator, goes in the opposite direction. An increase in β also makes overall demand facing each hospital *less* responsive to waiting times, since demand is less responsive to waiting times in the competitive segment.[9] As previously explained, lower demand responsiveness makes treatment supply a *more* effective instrument for each hospital to reduce waiting times. In turn, this leads to higher supply and lower waiting times in the steady-state.

From (22), the effect of a larger competitive segment on steady-state supply is given by

$$\frac{dS^{OL}}{d\beta} = \frac{\partial S^{OL}}{\partial \beta} + \frac{2\theta\tau\alpha_2}{\gamma\phi}\frac{\partial w^{OL}}{\partial \beta}, \tag{29}$$

and consists of two effects, given by the two terms in (29). While the first term is positive, the sign

[9] From (7), the elasticity of demand with respect to waiting time in the competitive and monopolistic segment is respectively given by $\frac{\partial(N\beta x_C)}{\partial w_i}\frac{w_i}{N\beta x_C} = -\frac{w_i}{2\tau x_C}$ and $\frac{\partial[N(1-\beta)x_M]}{\partial w_i}\frac{w_i}{N(1-\beta)x_M} = -\frac{w_i}{\tau x_M}$. Since $x_M < x_C$, it follows that $\left|-\frac{w_i}{2\tau x_C}\right| < \left|-\frac{w_i}{2\tau x_M}\right|$.

of the second term depends on the sign of $\partial w^{OL}/\partial\beta$. However, using (27), we can re-write (29) as

$$\frac{dS^{OL}}{d\beta} = \frac{(1-\beta)\gamma\phi\frac{\partial S^{OL}}{\partial\beta} + \theta\tau^2\alpha_2(1-2x_M^{OL})}{(1-\beta)\gamma\phi N + 2\theta\tau^2\alpha_2} N > 0. \tag{30}$$

Therefore, even if a larger competitive segment reduces waiting times, which weakens provider incentives to increase supply, the direct demand effect always dominates. Thus, an increase in the size of the competitive segment unambiguously increases steady-state supply.

The above analysis is summarised by the following proposition:

Proposition 1. *In the open-loop solution, (i) a reduction in patients' travelling costs leads to higher steady-state waiting times but has an ambiguous effect on steady-state treatment supply, whereas (ii) an increase in the relative size of the competitive segment leads to higher steady-state treatment supply but has an ambiguous effect on steady-state waiting times.*

5 Closed-Loop Solution

We now turn to the case in which hospitals are able to observe the evolution of waiting times. In this section, we derive the closed-loop solution, in which supply decisions depend on current waiting times. Although strongly time-consistent, closed-loop solutions are computationally more involved.

We distinguish two cases: (i) *constant* marginal disutility of waiting time and (ii) *increasing* marginal disutility of waiting time. As mentioned above, which case is more plausible depends on the institutional context and this may differ across countries or even within a country at different points in time. For example, one could argue that in England in 2000-05 the marginal disutility was increasing in waiting times when senior health administrators risked losing their jobs if targets were not met. This would be the case if small deviations from the target would only lead to additional monitoring from the regulator, but a large deviation from the target would culminate into the hospital CEO being dismissed. In contrast, the marginal disutility of waiting time could be linear if deviations from a target lead to a proportionate reduction in hospital income, which was implemented later in England. Therefore, both scenarios are important from a policy perspective. We discuss them in turn.

5.1 Constant Disutility of Waiting Time

This scenario is obtained by setting $\alpha_2 = 0$. In this case, the differential game belongs to the class of the so-called linear-state games, which is characterised by the coincidence between the time path of controls and states under the open- and closed-loop solution concepts. We obtain the following result:[10]

Proposition 2. *If the marginal disutility of waiting time is constant, the open-loop and closed-loop solutions coincide, and the equilibrium is characterised by constant supply of treatment over time.*

The coincidence of the two solution concepts is explained by the lack of strategic interaction between the hospitals. A unilateral increase in supply by Hospital i leads to an initial reduction in waiting times at this hospital. This will shift demand from the rival hospital and therefore will also reduce the waiting time at Hospital j. However, if $\alpha_2 = 0$, the reduction in waiting time at Hospital j does not affect the hospital's marginal disutility of waiting time, so that the hospital will not respond by changing its supply. Thus, when the marginal disutility of waiting time is constant, the optimal supply rule is independent of waiting times, which implies that hospital activity is constant over time at its steady-state level.

Proposition 2 implies that, for constant marginal disutility of waiting time, the results of increased patient choice, given by Proposition 1, also apply to the closed-loop solution, with one exception. With constant marginal disutility of waiting time, a reduction in patients' travelling costs leads unambiguously to lower steady-state supply.

5.2 Increasing Marginal Disutility of Waiting Time

In this scenario, a closed-form solution of supply and waiting times cannot be obtained. Our game belongs to the class of linear-quadratic differential games, wherein the state variables enter the objective function quadratically, while they enter the dynamic constraints linearly. Although the closed-loop solution of linear-quadratic games may generally be computed analytically, this is not always assured. This is the case of our model whose particular structure features both state variables entering the dynamic constraints and has algebraic properties that limit the tractability of its closed-loop solution. We are, however, able to solve for the solution numerically. To make the analysis more salient and policy relevant, we take this constraint as an opportunity to calibrate the model based on real data and available empirical evidence.

The rest of this subsection characterises some general features of the solution, and the next one provides the calibration of the closed-loop solution.

[10] The closed-loop solution is derived in Appendix B.1, and the proof of Proposition 2 is given in Appendix B.2.

Proposition 3. *If the marginal disutility of waiting time is increasing, the optimal closed-loop supply rule for Hospital i is given by:*

$$S_i(w_i, w_j, t) = \frac{p - \theta(\omega_1 + \omega_3 w_i(t) + \omega_5 w_j(t))}{\gamma}, \tag{31}$$

where $\omega_3 < 0$ is required by the concavity of the value function and $\omega_5 \in \Omega$.

See Appendix B.1 for the definition Ω and proof of Proposition 3. $\omega_3 < 0$ implies that an increase in the waiting time of Hospital i increases the hospital's optimal treatment supply. The reason is that a longer waiting time increases the hospital's marginal disutility of waiting time and therefore increases the marginal benefit of supply. In Appendix B.1, we also show that ω_5 is generally different from zero. This suggests that, unlike the case with constant marginal disutility of waiting time, a dynamic strategic interaction is present when the marginal disutility is increasing. The optimal supply rule for Hospital i now depends, at each point in time, on the waiting time of Hospital j. Although we show in Appendix B.1 that ω_5 can in principle be positive or negative, our calibration results provided in the next section show that ω_5 is negative for all the parameter configurations considered.

If ω_5 is negative, then hospitals' supply decisions are characterised by strategic *substitutability*, $\partial S_i(w_i, w_j)/\partial w_j > 0$, for which we provide the following intuition. Consider a unilateral increase in supply by Hospital i. This leads to lower waiting times at Hospital i, which in turn shifts demand from Hospital j to Hospital i, causing a reduction in waiting times also at Hospital j. A lower waiting time at Hospital j reduces its marginal disutility of waiting time, and thus its marginal benefit of supply. Hospital j will therefore optimally respond by reducing its supply of treatments. In other words, a supply increase by Hospital i triggers a supply decrease by Hospital j.

The above described strategic interaction has important implications for the supply incentives of each hospital. Consider once more a unilateral increase in supply by Hospital i, which leads to an immediate reduction in waiting time at this hospital. However, because of strategic substitutability, Hospital j will respond by reducing its supply, as explained above. The subsequent increase in waiting time at Hospital j shifts some demand towards Hospital i, thereby dampening the initial reduction in the waiting time caused by the supply increase of Hospital i. Thus, dynamic strategic substitutability lowers the marginal benefit of treatment supply, giving each hospital an incentive to reduce its own supply in order to 'free ride' on the subsequent supply increase of the rival hospital.

In Appendix B.3, we also show that, if the initial waiting times are the same in both hospitals or if the average initial waiting time equals the steady-state waiting time, waiting times, supply, and demand in both segments of the market converge *monotonically* to the steady-state. In this case, if the condition $|\omega_3| > |\omega_5|$ holds, the equilibrium path to the steady-state is characterised by periods of increasing (decreasing) hospital activity and increasing (decreasing) waiting time, which is in line with Siciliani (2006) in a monopoly setting. Notice that $|\omega_3| > |\omega_5|$ implies that the own waiting time effect on hospital activity is larger than the effect of the waiting time of the competing hospital, which is both intuitive and confirmed by our calibration exercise below.[11]

However, *non-monotonic* convergence may also arise. In Appendix B.4, we show that, if the average initial waiting time is above (below) the steady-state waiting time, the hospital with the shortest (longest) initial waiting time might experience a non-monotonic convergence along the equilibrium path, with the waiting time first increasing (decreasing) before decreasing (increasing) towards the steady-state. One policy implication is that short-run provider performance on waiting times may not be representative of its long-run one.

5.3 Calibration

We calibrate the model using data from the English NHS on cataract surgery, which is a common non-emergency procedure across OECD countries (Siciliani et al., 2014). Our two key variables in the model are the steady-state waiting time and supply.

Waiting time data for cataract surgery is obtained from the Hospital Episode Statistics published by NHS Digital. In the financial year 2016-17, the mean waiting time for a cataract procedure provided either by NHS hospitals or the independent sector (private hospitals treating publicly-funded patients) was 70 days.[12] According to the National Schedule of Reference Costs from NHS Improvement, 234 NHS providers performed $286,596$ cataract procedures in the same year.[13] This gives a monthly average of approximately 100 procedures per provider.

In recent years, the government mandate to the main health regulator (NHS England) does not specify performance standards for non-emergency care (The King's Fund, 2017). We interpret this as a regime where no significant penalties are imposed on providers with longer waits. Within our

[11] Additionally, it follows from equations (B.15) and (B.16) in Appendix B.1 that $|\omega_3| > |\omega_5|$ is a sufficient (but not necessary) condition for convergence to be verified.

[12] Healthcare Resource Group (HRG) code BZ02Z, *Phacoemulsification Cataract Extraction and Lens Implant*, in the HRG4 classification system.

[13] The National Schedule of Reference Costs is detailed according to the HRG4+ classification system, which presents a more thorough description of cataract episodes than the HRG4. Focusing on *Phacoemulsification Cataract Extraction and Lens Implant,* the HRGs considered are BZ34A, BZ34B, and BZ34C in HRG4+.

model this corresponds to the special case when there is no waiting time disutility ($\alpha_1 = \alpha_2 = 0$), and the open-loop and closed-loop solutions coincide. We denote this scenario by superscript s. We express waiting times in months giving $w^s = 2.3$. The figures provided above also imply that monthly supply in the steady-state is about 100 cataract procedures, so that $S^s = D^s = 100$.

On the *supply* side, two key parameters are the tariff for a cataract surgery (the DRG-type price) and the marginal cost of treatment. From the National Schedule of Reference Costs, the national tariff in 2016-17 for a cataract procedure was 731£. We therefore set $p = 731$. Given that the first-order condition $S^s = p/\gamma$ has to hold (when $\alpha_1 = \alpha_2 = 0$), we recover the parameter related to the marginal cost of treatment, $\gamma = 7.31$.

On the *demand* side, the key parameters are the potential demand, the size of the competitive segment, the demand responsiveness, the gross valuation of treatment, and the value of the outside option. These parameters are less easy to obtain but we infer them in the following way. According to OECD (2018), 10.5% of the UK population was covered by private health insurance in 2015. We assume that patients with private insurance opt for private treatment and that publicly-funded cataract procedures account for about 90% of the market.[14] Given that the steady-state supply in each hospital is $S^s = 100$, potential demand across the two hospitals is then given by $N = 222$.

Sivey (2012) estimates a demand elasticity for cataract surgery across NHS providers that is approximately -0.1. The waiting time elasticity of demand evaluated at the steady-state values and $N = 222$ gives

$$\frac{\partial D_i(w_i(t), w_j(t))}{\partial w_i(t)} \frac{w^s}{D^s} = -\frac{N(2-\beta)}{2\tau} \frac{w^s}{D^s} = -\frac{222(2-\beta)}{2\tau} \frac{2.3}{100} = -0.1. \tag{32}$$

We do not know how large is the competitive segment β. We therefore conduct the analysis for three different values, $\beta = \{0.2, 0.5, 0.8\}$. We start by assuming $\beta = 0.2$, so that the competitive segment accounts for 20% of potential demand and is therefore relatively small, and then check how the results differ when it is 50% and 80% (relatively large).

If $\beta = 0.2$, then, from (32), the demand elasticity implies that $\tau = 45.954$. Moreover, from the demand equation evaluated at the steady-state,

$$D^s = N\left[\frac{\beta}{2} + (1-\beta)\left(\frac{v - k - w^s}{\tau}\right)\right], \tag{33}$$

we can recover the difference between the gross valuation of treatment and the value of the outside option: $v - k = 22.4308$. If $\beta = 0.5$, then, from (32), we obtain $\tau = 38.295$ and, from (33), we

[14] This is an approximation since some patients without private insurance may also obtain private care if they pay out of pocket and some with private insurance may not seek private care if they face co-payments.

obtain $v - k = 17.653$. If $\beta = 0.8$, then, from (32), we obtain $\tau = 30.636$ and, from (33), we obtain $v - k = 10.028$. We have thus recovered the demand-side parameters for $\beta = \{0.2, 0.5, 0.8\}$.

We adopt a discount factor of 0.95 per year and take each period t as one month. The monthly discount rate is therefore $\rho = 0.004$ (computed from $e^{-12\rho} = 0.95$).

In the steady-state, it takes one month for Hospital i to treat 100 patients. This implies that, if 10 additional patients are added to the list, the waiting time will increase by 0.1 months (about 3 days). More formally, from the dynamic constraint, $\Delta w^s \approx \theta \Delta(D^s - S^s)$, which gives $\theta = \frac{\Delta w^s}{\Delta(D^s - S^s)} = \frac{0.1}{10} = 0.01$ in the neighbourhood of the steady-state.

We are interested in understanding provider behaviour in the presence of penalties. We therefore need to identify plausible values for α_1 and α_2 under a penalty regime. Propper et al. (2008) find that the introduction of waiting time penalties in the English NHS in 2000-05 reduced the mean waiting time by 13 days (i.e., 0.43 months). Although this estimate refers to an earlier period, it provides us with a plausible order of magnitude if such penalties were re-introduced in 2016-17. We then use this figure to compute the difference between the steady-state waiting time in the model with no disutility of waiting time and the open-loop steady-state waiting time, which is given by

$$w^s - w^{OL} = 2.3 - \frac{\gamma\phi\tau}{(1-\beta)\gamma\phi N + 2\theta\tau^2\alpha_2} \left\{ N \left[\frac{\beta}{2} + (1-\beta)\left(\frac{v-k}{\tau}\right) \right] - \frac{p}{\gamma} - \frac{2\theta\tau\alpha_1}{\gamma\phi} \right\} = 0.43. \quad (34)$$

Inserting the above described parameter values when $\beta = 0.2$, the solution to (34) has one degree of freedom and is given by

$$\alpha_2 = 30.5274 - 0.53486\alpha_1. \quad (35)$$

All α_1 and α_2 that satisfy (35) yield a reduction of 0.43 months in the open-loop steady-state waiting time compared to the case with no disutility of waiting time. We consider three disutility structures: (i) linear disutility ($\alpha_2 = 0$), yielding $\alpha_1 = 57.0826$; (ii) quadratic disutility ($\alpha_1 = 0$), yielding $\alpha_2 = 30.5274$; and (iii) an intermediate case in which $\alpha_1 = \frac{57.0826}{2}$ and $\alpha_2 = \frac{30.5274}{2}$.

Inserting all parameter values into equations (19) and (22) gives the open-loop steady-state waiting time and supply. For the closed-loop solution we insert all parameter values and solve the system (B.6)-(B.8) in Appendix B.1 to yield ω_1, ω_3, and ω_5, which are plugged into (B.26) in Appendix B.3 to obtain the closed-loop steady-state waiting time. With ω_1, ω_3, ω_5, and w^{CL}, we use (31) to retrieve the closed-loop steady-state supply.

The same steps were then repeated for $\beta = 0.5$ and $\beta = 0.8$. The results are summarised in Table 1.

Table 1: Calibration results for a waiting time elasticity of demand of -0.1

β	α_1	α_2	w^{OL}	w^{CL}	S^{OL}	S^{CL}	ω_3	ω_5
	0	0	2.3	2.3	100	100	0	0
0.2	57.0862	0	1.8700	1.8700	101.6620	101.6620	0	0
0.2	28.5431	15.2637	1.8700	1.8703	101.6620	101.6609	-164.6061	-8.3753
0.2	0	30.5274	1.8700	1.8705	101.6620	101.6600	-321.6537	-15.3715
0.5	39.2269	0	1.8700	1.8700	101.2464	101.2464	0	0
0.5	19.6143	10.4885	1.8700	1.8720	101.2464	101.2402	-119.1899	-19.2189
0.5	0	20.9769	1.8700	1.8734	101.2464	101.2353	-233.5920	-36.3039
0.8	13.5675	0	1.8700	1.8700	100.6232	100.6232	0	0
0.8	6.7837	3.6277	1.8700	1.8755	100.6232	100.6147	-52.3298	-20.3702
0.8	0	7.2553	1.8700	1.8795	100.6232	100.6077	-102.5480	-38.9404

Our calibration results confirm that the open-loop and closed-loop steady-states coincide when penalties are linear, $\alpha_2 = 0$. Instead, waiting times are longer under the closed-loop solution when penalties are non-linear, $\alpha_2 > 0$. This is because $\omega_5 < 0$ and supply across hospitals are dynamic strategic substitutes, which leads to lower supply and longer waiting times in the closed-loop steady-state when compared to the open-loop solution.

Moreover, as the waiting time disutility becomes more convex (i.e., more weight is placed on the quadratic term), the longer is the waiting time and the lower is supply in the closed-loop steady-state. The reason is simply that a more convex disutility function increases the magnitude of each hospital's supply response to changes in the waiting time, which reinforces each hospital's incentive to reduce supply in order to provoke a supply increase by the rival hospital, which in turn benefits the former hospital in the form of a lower waiting time.

The difference in steady-state outcomes between the two solution concepts is larger for higher values of the competitive segment, β. This is intuitive, since the strategic substitutability relies on the existence of a competitive segment, wherein changes in the waiting time at one hospital affect demand faced by the rival hospital. Thus, a larger relative size of the competitive segment will magnify the effects of strategic substitutability.

Although waiting times are longer under the closed-loop solution, a key insight from Table 1 is that the difference in waiting times under the open- and closed-loop solutions is very small (less than 1%). This suggests that, even with non-linear penalties, the less computationally demanding open-loop solution offers a close approximation of the closed-loop one. One may worry that these results are due to the low demand elasticity. We therefore extend the analysis under the assumption that the waiting time elasticity is higher. We consider two additional cases. First, we assume that the elasticity is -0.2, twice as large, which is the highest that has been reported in studies for England (see Iversen and Siciliani (2011) for an overview). Second, we assume that the elasticity is -1. This is an upper bound. There is only one study from Australia which provides such a large estimate (Stavrunova and Yerokhin, 2011), and this is consistent with the features of the Australian health system where more than half of the population is treated privately.

Tables 2 and 3 provide the results for waiting time elasticities of demand of -0.2 and -1, respectively. They are derived following the steps detailed above. The key insight is that although the difference in steady-state waiting times between closed- and open-loop widens, the difference remains small.

Table 2: Calibration results for a waiting time elasticity of demand of -0.2

β	α_1	α_2	w^{OL}	w^{CL}	S^{OL}	S^{CL}	ω_3	ω_5
	0	0	2.3	2.3	100	100	0	0
0.2	218.4948	0	1.8700	1.8700	103.3237	103.3237	0	0
0.2	109.2474	58.4211	1.8700	1.8703	103.3237	103.3212	-322.1649	-16.7563
0.2	0	116.8421	1.8700	1.8705	103.3237	103.3193	-629.5163	-31.3129
0.5	148.9097	0	1.8700	1.8700	102.4928	102.4928	0	0
0.5	74.4548	39.8154	1.8700	1.8722	102.4928	102.4791	-231.9189	-38.3288
0.5	0	79.6308	1.8700	1.8738	102.4928	102.4683	-454.4837	-72.3946
0.8	49.207	0	1.8700	1.8700	101.2464	101.2464	0	0
0.8	24.6037	13.1571	1.8700	1.8762	101.2464	101.2272	-98.9201	-39.8422
0.8	0	26.3142	1.8700	1.8807	101.2464	101.2115	-193.7462	-76.1410

Finally, in line with Section 4.2 for the open-loop solution, we conduct comparative statics with respect to the patient choice parameters as measured by τ and β. These are reported in Tables 4 and 5 respectively. Table 4 shows the effects of a 10% reduction in τ, whereas Table 5 shows the effect of a 10% increase in β. All other parameters are kept unchanged, which implies that the results displayed in Table 1 serve as a reference point of comparison. In the last four columns in

Table 3: Calibration results for a waiting time elasticity of demand of -1

β	α_1	α_2	w^{OL}	w^{CL}	S^{OL}	S^{CL}	ω_3	ω_5
	0	0	2.3	2.3	100	100	0	0
0.2	5265.7273	0	1.8700	1.8700	116.6184	116.6184	0	0
0.2	2632.8636	1407.9485	1.8700	1.8703	116.6184	116.6049	-1581.6013	-83.7851
0.2	0	2815.8969	1.8700	1.8706	116.6184	116.5947	-3090.4383	-156.5665
0.5	3561.6215	0	1.8700	1.8700	112.4638	112.4638	0	0
0.5	1788.8108	952.3052	1.8700	1.8724	112.4638	112.3893	-1132.2781	-190.9560
0.5	0	1904.6104	1.8700	1.8741	112.4638	112.3307	-2218.7774	-360.6664
0.8	1126.6109	0	1.8700	1.8700	106.2319	106.2319	0	0
0.8	563.3055	301.2329	1.8700	1.8769	106.2319	106.1257	-469.2457	-194.4604
0.8	0	602.4657	1.8700	1.8818	106.2319	106.0397	-918.7196	-371.5998

Tables 4 and 5, we report the percentage changes in steady-state waiting time and supply under each of the two solution concepts.

Table 4: Steady-state effects of a 10% reduction in τ

β	α_1	α_2	$\Delta\% w^{OL}$	$\Delta\% w^{CL}$	$\Delta\% S^{OL}$	$\Delta\% S^{CL}$
0.2	57.0862	0	111.86	111.86	-0.15	-0.15
0.2	28.5431	15.2637	102.25	102.24	0.61	0.61
0.2	0	30.5274	94.17	94.15	1.25	1.25
0.5	39.2269	0	86.27	86.27	-0.11	-0.11
0.5	19.6143	10.4885	78.85	78.76	0.33	0.33
0.5	0	20.9769	72.60	77.52	0.70	0.70
0.8	13.5675	0	45.34	45.34	-0.05	-0.05
0.8	6.7837	3.6277	41.42	41.25	0.07	0.07
0.8	0	7.2553	38.11	37.93	0.17	0.17

In qualitative terms, the effects of increased patient choice on steady-state waiting times and supply, as shown in Tables 4 and 5, confirm the analytical results from the analysis in Section 4.2. A reduction in τ leads to higher steady-state waiting times, and the effect is larger if the relative size of the competitive segment is smaller. The supply of treatments also increases, unless the disutility

Table 5: Steady-state effects of a 10% increase in β

β	α_1	α_2	$\Delta\% w^{OL}$	$\Delta\% w^{CL}$	$\Delta\% S^{OL}$	$\Delta\% S^{CL}$
0.2	57.0862	0	3.01	3.01	0.02	0.02
0.2	28.5431	15.2637	2.69	2.69	0.04	0.04
0.2	0	30.5274	2.43	2.44	0.06	0.06
0.5	39.2269	0	18.39	18.39	0.08	0.08
0.5	19.6143	10.4885	16.19	16.26	0.18	0.18
0.5	0	20.9769	14.46	14.59	0.27	0.26
0.8	13.5675	0	241.17	241.17	0.23	0.23
0.8	6.7837	3.6277	191.09	193.02	1.04	1.00
0.8	0	7.2553	158.23	162.48	1.57	1.49

of waiting times is linear (i.e., $\alpha_2 = 0$). This is consistent with (26) in Section 4.2, showing that the sign of $dS^{OL}/d\tau$ is generally ambiguous but strictly positive if $\alpha_2 = 0$.

An increase in β also leads to higher steady-state waiting times in all cases considered, although this effect is theoretically ambiguous, and also leads to higher treatment supply, which is consistent with Proposition 1. Thus, Tables 4 and 5 indicate that increased patient choice tend to increase waiting times.

6 Concluding Remarks

We have investigated whether a more competitive environment and patient choice policies play a useful role in reducing waiting times and the extent to which such a role is altered in the presence of penalties for providers with long waits. Overall, our results suggest that competition, however measured, is not helpful in reducing waiting times. In a few scenarios competition can actually increase waiting times and this is more likely to be the case in the presence of provider penalties.

If the penalties that providers face are linear in waiting times, a more competitive dynamic environment does not affect the equilibrium waiting times. If penalties are instead non-linear so that the marginal penalty increases with waiting times, we find that waiting times are *longer* under the more competitive environment (the closed-loop solution). This arises because optimal closed-loop strategies are dynamic strategic substitutes, implying that each hospital reduces supply in order to 'free-ride' on the subsequent supply increase by the other hospital.

Although waiting times are higher under the closed-loop solution, our calibrated model based on cataract surgery in the English NHS suggests that the difference in waiting times under the two solution concepts is quantitatively small (less than 1% in all our calibrations with different demand elasticities). This suggests that although non-linear penalties introduce a negative externality where each provider has an incentive to reduce activity, their negative impact is likely to be quantitatively small.

When we investigate the effect of competition as measured by policies that facilitate patient choice, we find again that these lead to higher steady-state waiting times (regardless of whether penalties are linear or not) and have an ambiguous effect on steady-state treatment supply (though the effect is negative if penalties are linear and increased patients choice is proxied by a reduction in travelling costs). This is because patient choice makes demand more responsive to waiting times, which in turn reduces the effectiveness of supply in reducing waiting times. These effects are exacerbated in the presence of provider penalties.

In summary, our model suggests that although policies based on provider penalties will have the intended effect in reducing waiting times, policies which stimulate competition will not.

References

Brekke, K. R., Cellini, R., Siciliani, L., and Straume, O. R. (2010). Competition and quality in health care markets: A differential-game approach. *Journal of Health Economics*, *29*(4), 508–523. https://doi.org/10.1016/j.jhealeco.2010.05.004

Brekke, K. R., Cellini, R., Siciliani, L., and Straume, O. R. (2012). Competition in regulated markets with sluggish beliefs about quality. *Journal of Economics & Management Strategy*, *21*(1), 131–178. https://doi.org/10.1111/j.1530-9134.2011.00319.x

Brekke, K. R., Gravelle, H., Siciliani, L., and Straume, O. R. (2014). Patient choice, mobility and competition among health care providers. In R. Levaggi and M. Montefiori (Eds.), *Health care provision and patient mobility. Developments in health economics and public policy* (Vol. 12). Milano: Springer. https://doi.org/10.1007/978-88-470-5480-6_1

Brekke, K. R., Siciliani, L., and Straume, O. R. (2008). Competition and waiting times in hospital markets. *Journal of Public Economics*, *92*(7), 1607–1628. https://doi.org/10.1016/j.jpubeco.2008.02.003

Dockner, E. J., Jorgensen, S., Long, N. V., and Sorger, G. (2000). *Differential games in economics and management science.* Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511805127

Gravelle, H., and Siciliani, L. (2008a). Is waiting-time prioritisation welfare improving? *Health Economics*, *17*(2), 167–184. https://doi.org/10.1002/hec.1262

Gravelle, H., and Siciliani, L. (2008b). Optimal quality, waits and charges in health insurance. *Journal of Health Economics*, *27*(3), 663–674. https://doi.org/10.1016/j.jhealeco.2007.08.004

Gravelle, H., and Siciliani, L. (2008c). Ramsey waits: Allocating public health service resources when there is rationing by waiting. *Journal of Health Economics*, *27*(5), 1143–1154. https://doi.org/10.1016/j.jhealeco.2008.03.004

Hoel, M., and Sæther, E. M. (2003). Public health care with waiting time: The role of supplementary private health care. *Journal of Health Economics*, *22*(4), 599–616. https://doi.org/10.1016/S0167-6296(03)00007-9

Iversen, T. (1993). A theory of hospital waiting lists. *Journal of Health Economics*, *12*(1), 55–71. https://doi.org/10.1016/0167-6296(93)90040-L

Iversen, T. (1997). The effect of a private sector on the waiting time in a national health service. *Journal of Health Economics*, *16*(4), 381–396. https://doi.org/10.1016/S0167-6296(96)00518-8

Iversen, T., and Siciliani, L. (2011). Non-price rationing and waiting times. In S. Glied and P. C. Smith (Eds.), *The Oxford handbook of health economics.* Oxford: Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199238828.013.0027

Lindsay, C. M., and Feigenbaum, B. (1984). Rationing by waiting lists. *The American Economic Review*, *74*(3), 404–417.

March, M., and Schroyen, F. (2005). Can a mixed health care system be desirable on equity grounds? *The Scandinavian Journal of Economics*, *107*(1), 1–23. https://doi.org/10.1111/j.1467-9442.2005.00392.x

Martin, S., and Smith, P. C. (1999). Rationing by waiting lists: An empirical investigation. *Journal of Public Economics*, *71*(1), 141–164. https://doi.org/10.1016/S0047-2727(98)00067-X

OECD. (2017). *Health at a glance 2017: OECD indicators.* Paris: OECD Publishing. https://doi.org/10.1787/health_glance-2017-en

OECD. (2018). *OECD health statistics 2018* [Data set]. Retrieved from https://stats.oecd.org/index.aspx?DataSetCode=HEALTH_STAT

Propper, C., Sutton, M., Whitnall, C., and Windmeijer, F. (2008). Did 'targets and terror' reduce waiting times in England for hospital care? *The B.E. Journal of Economic Analysis & Policy*, *8*(2). https://doi.org/10.2202/1935-1682.1863

Propper, C., Sutton, M., Whitnall, C., and Windmeijer, F. (2010). Incentives and targets in hospital care: Evidence from a natural experiment. *Journal of Public Economics*, *94*(3), 318–335. https://doi.org/10.1016/j.jpubeco.2010.01.002

Siciliani, L. (2006). A dynamic model of supply of elective surgery in the presence of waiting times and waiting lists. *Journal of Health Economics*, *25*(5), 891–907. https://doi.org/10.1016/j.jhealeco.2005.12.002

Siciliani, L., Chalkley, M., and Gravelle, H. (2017). Policies towards hospital and GP competition in five European countries. *Health Policy*, *121*(2), 103–110. https://doi.org/10.1016/j.healthpol.2016.11.011

Siciliani, L., Moran, V., and Borowitz, M. (Eds.). (2013). *Waiting time policies in the health sector: What works?* Paris: OECD Health Policy Studies, OECD Publishing. https://doi.org/10.1787/9789264179080-en

Siciliani, L., Moran, V., and Borowitz, M. (2014). Measuring and comparing health care waiting times in OECD countries. *Health Policy*, *118*(3), 292–303. https://doi.org/10.1016/j.healthpol.2014.08.011

Siciliani, L., Straume, O. R., and Cellini, R. (2013). Quality competition with motivated providers and sluggish demand. *Journal of Economic Dynamics and Control*, *37*(10), 2041–2061. https://doi.org/10.1016/j.jedc.2013.05.002

Sivey, P. (2012). The effect of waiting time and distance on hospital choice for English cataract patients. *Health Economics*, *21*(4), 444–456. https://doi.org/10.1002/hec.1720

Stavrunova, O., and Yerokhin, O. (2011). An equilibrium model of waiting times for elective surgery in NSW public hospitals. *Economic Record*, *87*(278), 384–398. https://doi.org/10.1111/j.1475-4932.2011.00726.x

The King's Fund. (2017). *What is happening to waiting times in the NHS?* [Article]. Retrieved from https://www.kingsfund.org.uk/publications/articles/nhs-waiting-times

Appendix A The Open-Loop Solution

A.1 Conditions for an Interior-Solution Open-Loop Steady-State

The open-loop steady-state is characterised by a positive waiting time and a partially covered monopolistic segment if $p \in \mathcal{P} = (\max\{0, \underline{p}\}, \min\{\overline{p}_1, \overline{p}_2\})$, where

$$\underline{p} = \frac{\beta}{2}\gamma N - \frac{2\theta\tau}{\phi}\left[\alpha_1 + \alpha_2(v-k)\right], \tag{A.1}$$

$$\overline{p}_1 = \gamma N \left[\frac{\beta}{2} + (1-\beta)\left(\frac{v-k}{\tau}\right)\right] - \frac{2\theta\tau\alpha_1}{\phi}, \tag{A.2}$$

and

$$\overline{p}_2 = \frac{\gamma N}{2} - \frac{2\theta\tau}{\phi}\left[\alpha_1 + \alpha_2\left(v - k - \frac{\tau}{2}\right)\right]. \tag{A.3}$$

From (19), the waiting time is positive if and only if $p \leq \overline{p}_1$. Then, in order to have a partially covered monopolistic segment in the steady-state, the following condition must be satisfied:

$$0 < \frac{v - k - w^{OL}}{\tau} < \frac{1}{2}. \tag{A.4}$$

The lower bound is satisfied if $p > \underline{p}$, as defined by (A.1). Note that $\underline{p}$ may be negative, but $p \in \mathbb{R}^+$ must hold. Thus, $p > \max\{0, \underline{p}\}$.

The upper bound, in turn, is satisfied if $p < \overline{p}_2$. Since $\overline{p}_1 > 0 \wedge \overline{p}_1 > \underline{p}$, $\mathcal{P}$ is non-empty when $\overline{p}_1 < \overline{p}_2$. Conversely, $\overline{p}_2$ only verifies $\overline{p}_2 > \underline{p}$, as it may be negative. If $\overline{p}_2 < 0$, parameters are such that $\underline{p} < \overline{p}_2 < 0 < \overline{p}_1$. Then, $\mathcal{P}$ is non-empty when $\overline{p}_2 < \overline{p}_1$ if and only if $\overline{p}_2 > 0$, which holds for a sufficiently large γ.

A.2 Open-Loop Stability

The Jacobian matrix of the symmetric system of equations (15), (16), and (18) is:

$$J^{OL} = \begin{bmatrix} -\frac{\theta(1-\beta)N}{\tau} & \frac{\theta^2}{\gamma} & 0 \\ \alpha_2 & \rho + \frac{\theta(2-\beta)N}{2\tau} & -\frac{\theta\beta N}{2\tau} \\ 0 & -\frac{\theta\beta N}{2\tau} & \rho + \frac{\theta(2-\beta)N}{2\tau} \end{bmatrix} \tag{A.5}$$

and has characteristic polynomial

$$P(s) = -s^3 + \left(2\rho + \frac{\theta N}{\tau}\right) s^2 + \left(\frac{\alpha_2\theta^2}{\gamma} + \left(\frac{\theta\beta N}{2\tau}\right)^2 - \left[\rho + \frac{\theta(2-\beta)N}{2\tau}\right]\left[\rho - \frac{\theta(2-3\beta)N}{2\tau}\right]\right) s$$
$$- \frac{\theta(1-\beta)N}{\tau}\left[\rho + \frac{\theta N}{\tau}\right]\left[\rho + \frac{\theta(1-\beta)N}{\tau}\right] - \frac{\alpha_2\theta^2}{\gamma}\left[\rho + \frac{\theta(2-\beta)N}{2\tau}\right]. \quad \text{(A.6)}$$

Since $P(s)$ is a third-degree polynomial whose factorisation is unfeasible, solving analytically for its roots yields little insight into the nature of the eigenvalues. According to the fundamental theorem of algebra, $P(s)$ has exactly three roots (real or complex). The coefficients of the cubic term and constant term are negative, while the coefficient of the quadratic term is positive. Although the sign of the coefficient of the linear term is ambiguous, it still follows that $P(-s)$ has a single change of sign—either between the second the the first powers or between the latter and the constant term. Thus, by Descartes' Rule of Signs, $P(s)$ has a single real negative root, which implies that the steady-state is a saddle point.

Appendix B Closed-Loop Solution

B.1 Increasing Marginal Waiting Time Disutility

The Hamilton-Jacobi-Bellman (HJB) equation for hospital i is[15]

$$\rho V^i(w_i, w_j) = \max\left\{T + pS_i - \frac{\gamma}{2}S_i^2 - \alpha_1 w_i - \frac{\alpha_2}{2}w_i^2 + \theta\frac{\partial V^i}{\partial w_i}(D_i - S_i) + \theta\frac{\partial V^i}{\partial w_j}(D_j - S_j)\right\}. \quad \text{(B.1)}$$

Given the linear-quadratic structure of the game, we conjecture that the value function for hospital i takes the form:

$$V^i(w_i, w_j) = \omega_0 + \omega_1 w_i + \omega_2 w_j + \frac{\omega_3}{2}w_i^2 + \frac{\omega_4}{2}w_j^2 + \omega_5 w_i w_j. \quad \text{(B.2)}$$

Maximisation of the right-hand side of the HJB equations yields:

$$S_i(w_i, w_j) = \frac{p - \theta(\omega_1 + \omega_3 w_i + \omega_5 w_j)}{\gamma}. \quad \text{(B.3)}$$

Substituting Hospital i's supply rule and the analogous to Hospital j into the HJB equation,

[15] To save notation, we omit the time index t in all subsequent expressions.

together with (7)-(8), we obtain:

$$
\begin{aligned}
\rho V^i(w_i,w_j) &= T + p\left[\frac{p-\theta(\omega_1+\omega_3 w_i+\omega_5 w_j)}{\gamma}\right] - \frac{\gamma}{2}\left[\frac{p-\theta(\omega_1+\omega_3 w_i+\omega_5 w_j)}{\gamma}\right]^2 - \alpha_1 w_i - \frac{\alpha}{2}w_i^2 \\
&+\theta(\omega_1+\omega_3 w_i+\omega_5 w_j)\left[\beta\left(\frac{1}{2}+\frac{w_j-w_i}{2\tau}\right)N + (1-\beta)\left(\frac{v-k-w_i}{\tau}\right)N - \frac{p-\theta(\omega_1+\omega_3 w_i+\omega_5 w_j)}{\gamma}\right] \\
&+\theta(\omega_2+\omega_4 w_j+\omega_5 w_i)\left[\beta\left(\frac{1}{2}+\frac{w_i-w_j}{2\tau}\right)N + (1-\beta)\left(\frac{v-k-w_j}{\tau}\right)N - \frac{p-\theta(\omega_1+\omega_3 w_j+\omega_5 w_i)}{\gamma}\right],
\end{aligned}
\tag{B.4}
$$

which can be rewritten as

$$
\begin{aligned}
& T+\frac{p^2}{2\gamma}+\sigma(\omega_1+\omega_2)+\frac{\theta^2}{2\gamma}\omega_1^2+\frac{\theta^2}{\gamma}\omega_1\omega_2-\rho\omega_0 \\
&+ w_i\left\{-\left[\rho+\frac{\theta(2-\beta)N}{2\tau}\right]\omega_1+\frac{\theta\beta N}{2\tau}\omega_2+\sigma(\omega_3+\omega_5)+\frac{\theta^2}{\gamma}\omega_1\omega_3+\frac{\theta^2}{\gamma}\omega_1\omega_5+\frac{\theta^2}{\gamma}\omega_2\omega_5-\alpha_1\right\} \\
&+ w_j\left\{\frac{\theta\beta N}{2\tau}\omega_1-\left[\rho+\frac{\theta(2-\beta)N}{2\tau}\right]\omega_2+\sigma(\omega_4+\omega_5)+\frac{\theta^2}{\gamma}\omega_1\omega_4+\frac{\theta^2}{\gamma}\omega_1\omega_5+\frac{\theta^2}{\gamma}\omega_2\omega_3\right\} \\
&+ w_i^2\left\{-\left[\frac{\rho}{2}+\frac{\theta(2-\beta)N}{2\tau}\right]\omega_3+\frac{\theta^2}{2\gamma}\omega_3^2+\frac{\theta\beta N}{2\tau}\omega_5+\frac{\theta^2}{\gamma}\omega_5^2-\frac{\alpha_2}{2}\right\} \\
&+ w_j^2\left\{-\left[\frac{\rho}{2}+\frac{\theta(2-\beta)N}{2\tau}\right]\omega_4+\frac{\theta^2}{\gamma}\omega_3\omega_4+\frac{\theta\beta N}{2\tau}\omega_5+\frac{\theta^2}{2\gamma}\omega_5^2\right\} \\
&+ w_i w_j\left\{\frac{\theta\beta N}{2\tau}(\omega_3+\omega_4)-\left[\rho+\frac{\theta(2-\beta)N}{\tau}\right]\omega_5+\frac{2\theta^2}{\gamma}\omega_3\omega_5+\frac{\theta^2}{\gamma}\omega_4\omega_5\right\}=0,
\end{aligned}
\tag{B.5}
$$

where $\sigma=\frac{\theta\beta N}{2}+\theta(1-\beta)\left(\frac{v-k}{\tau}\right)N-\frac{\theta}{\gamma}p$.

For the equality to hold, the terms in brackets in the above equation have to be equal to zero. Since the last three terms depend only on ω_3, ω_4, and ω_5, we focus on the system of three equations in three unknowns given by:

$$
-\left[\frac{\rho}{2}+\frac{\theta(2-\beta)N}{2\tau}\right]\omega_3+\frac{\theta^2}{2\gamma}\omega_3^2+\frac{\theta\beta N}{2\tau}\omega_5+\frac{\theta^2}{\gamma}\omega_5^2-\frac{\alpha_2}{2}=0, \tag{B.6}
$$

$$
-\left[\frac{\rho}{2}+\frac{\theta(2-\beta)N}{2\tau}\right]\omega_4+\frac{\theta^2}{\gamma}\omega_3\omega_4+\frac{\theta\beta N}{2\tau}\omega_5+\frac{\theta^2}{2\gamma}\omega_5^2=0, \tag{B.7}
$$

$$
\frac{\theta\beta N}{2\tau}(\omega_3+\omega_4)-\left[\rho+\frac{\theta(2-\beta)N}{\tau}\right]\omega_5+\frac{2\theta^2}{\gamma}\omega_3\omega_5+\frac{\theta^2}{\gamma}\omega_4\omega_5=0. \tag{B.8}
$$

It turns out that the solution to the system depends on the root of sixth degree polynomial, precluding the computation of an analytical solution. Assume, for now, that a solution to (B.6)-(B.8) exists and that it is such that (31) constitutes a Markov Perfect Nash Equilibrium.

From (B.6), two candidate solutions for ω_3 (as functions of ω_5) ensue:

$$\omega_3 = \frac{\gamma}{\theta^2}\left\{\left[\frac{\rho}{2} + \frac{\theta(2-\beta)N}{2\tau}\right] \pm \sqrt{\left[\frac{\rho}{2} + \frac{\theta(2-\beta)N}{2\tau}\right]^2 - \frac{2\theta^2}{\gamma}\left[\frac{\theta^2}{\gamma}\omega_5^2 + \frac{\theta\beta N}{2\tau}\omega_5 - \frac{\alpha_2}{2}\right]}\right\}. \tag{B.9}$$

A solution to Hospital i's maximisation problem is attained if the value function is concave with respect to w_i, which requires $\omega_3 < 0$. The greater root (unambiguously positive) is therefore ruled out. For the smaller root to be negative, the second term under the square-root must be positive, which is true for $\omega_5 \in (\underline{\omega_5}, \overline{\omega_5})$, with

$$\underline{\omega_5} = -\frac{\gamma}{2\theta^2}\left[\frac{\theta\beta N}{2\tau} + \sqrt{\left(\frac{\theta\beta N}{2\tau}\right)^2 + \frac{2\theta^2\alpha_2}{\gamma}}\right] < 0, \tag{B.10}$$

$$\overline{\omega_5} = -\frac{\gamma}{2\theta^2}\left[\frac{\theta\beta N}{2\tau} - \sqrt{\left(\frac{\theta\beta N}{2\tau}\right)^2 + \frac{2\theta^2\alpha_2}{\gamma}}\right] > 0. \tag{B.11}$$

Additionally, in order for (31) to be a Markov Perfect Nash Equilibrium, the value function must be bounded from above. A necessary and sufficient condition for this requirement to hold is that waiting times converge in equilibrium. Inserting (7), (8), (31), and the analogous supply rule for Hospital j into (11)-(12) yields the following system of differential equations:

$$\frac{\dot{w}_i}{\theta} = \left[-\frac{(2-\beta)N}{2\tau} + \frac{\theta}{\gamma}\omega_3\right] w_i + \left[\frac{\beta N}{2\tau} + \frac{\theta}{\gamma}\omega_5\right] w_j + N\left[\frac{\beta}{2} + (1-\beta)\left(\frac{v-k}{\tau}\right)\right] - \left(\frac{p-\theta\omega_1}{\gamma}\right), \tag{B.12}$$

$$\frac{\dot{w}_j}{\theta} = \left[\frac{\beta N}{2\tau} + \frac{\theta}{\gamma}\omega_5\right] w_i + \left[-\frac{(2-\beta)N}{2\tau} + \frac{\theta}{\gamma}\omega_3\right] w_j + N\left[\frac{\beta}{2} + (1-\beta)\left(\frac{v-k}{\tau}\right)\right] - \left(\frac{p-\theta\omega_1}{\gamma}\right). \tag{B.13}$$

The Jacobian of (B.12)-(B.13) is:

$$J^{CL} = \theta\begin{bmatrix} -\frac{(2-\beta)N}{2\tau} + \frac{\theta}{\gamma}\omega_3 & \frac{\beta N}{2\tau} + \frac{\theta}{\gamma}\omega_5 \\ \frac{\beta N}{2\tau} + \frac{\theta}{\gamma}\omega_5 & -\frac{(2-\beta)N}{2\tau} + \frac{\theta}{\gamma}\omega_3 \end{bmatrix} \tag{B.14}$$

and its eigenvalues are

$$s_1 = \theta\left[-\frac{N}{\tau} + \frac{\theta}{\gamma}(\omega_3 - \omega_5)\right], \tag{B.15}$$

$$s_2 = \theta\left[-\frac{(1-\beta)N}{\tau} + \frac{\theta}{\gamma}(\omega_3 + \omega_5)\right]. \tag{B.16}$$

A sufficient condition for waiting times to converge is that both eigenvalues are negative. Then, $s_1 < 0$ if $\omega_5 > -\frac{\gamma N}{\theta\tau} + \omega_3$ and $s_2 < 0$ if $\omega_5 < \frac{\gamma(1-\beta)N}{\theta\tau} - \omega_3$.

Using the expression for ω_3 as a function of ω_5 (B.9), the necessary condition $s_1 < 0 \wedge s_2 < 0 \wedge \omega_3 < 0$ is satisfied if $\omega_5 \in \Omega = \left(\max\{\underline{\omega_5}, \underline{\omega_5}'\}, \min\{\overline{\omega_5}, \overline{\omega_5}'\}\right)$, where

$$\underline{\omega_5}' = \frac{\gamma}{6\theta^2}\left[\rho - \frac{2\theta\beta N}{\tau} - \sqrt{\left(\rho - \frac{2\theta\beta N}{\tau}\right)^2 + \frac{12\theta^2}{\gamma}\left[\frac{\gamma N}{\theta\tau}\left(\rho + \frac{\theta(1-\beta)N}{\tau}\right) + \alpha_2\right]}\right] < 0, \quad \text{(B.17)}$$

$$\overline{\omega_5}' = \frac{\gamma}{6\theta^2}\left[-\left(\rho + \frac{2\theta\beta N}{\tau}\right) + \sqrt{\left(\rho + \frac{2\theta\beta N}{\tau}\right)^2 + \frac{12\theta^2}{\gamma}\left[\frac{\gamma(1-\beta)N}{\theta\tau}\left(\rho + \frac{\theta N}{\tau}\right) + \alpha_2\right]}\right] > 0. \quad \text{(B.18)}$$

Thus, provided that a solution to (B.6)-(B.8) exists, it constitutes a Markov Perfect Nash Equilibrium (or closed-loop equilibrium) if $\omega_5 \in \Omega$. Finally, an equilibrium with $\omega_5 = 0$ is ruled out by inspection of (B.6)-(B.8).

B.2 Constant Marginal Waiting Time Disutility

Here we show that, under constant marginal waiting time disutility, the open-loop and closed-loop solutions coincide and that the optimal supply function $S(t)$ is constant over time.

Setting $\alpha_2 = 0$ in equations (19) and (22), the steady-state waiting time and supply in the open-loop solution are given by

$$\overline{w} = \frac{\tau}{(1-\beta)N}\left\{N\left[\frac{\beta}{2} + (1-\beta)\left(\frac{v-k}{\tau}\right)\right] - \frac{p}{\gamma} - \frac{2\theta\tau\alpha_1}{\gamma\phi}\right\} \quad \text{(B.19)}$$

and

$$\overline{S} = \frac{p}{\gamma} + \frac{2\theta\tau\alpha_1}{\gamma\phi}. \quad \text{(B.20)}$$

Optimality is ensured also for $\alpha_2 = 0$ due to the concavity of the current-value Hamiltonian with respect to $S_i(t)$ and $w_i(t)$.

Let us now proceed to show that $S(t)$ is constant over time. When $\alpha_2 = 0$, the Jacobian of (15), (16), and (18) ((A.5) in Appendix A.2) has a single negative eigenvalue, given by $\hat{s} = -\frac{\theta(1-\beta)N}{\tau}$. A solution to system of differential equations that satisfies the transversality conditions, by ensuring convergence to the steady-state, requires that the arbitrary constants associated with the non-negative eigenvalues are set equal to zero. Note that this result implies that the open-loop equilibrium, like in the game with increasing marginal waiting time disutility, is stable in saddle sense. Such solution takes the form: $w(t) = A\nu_{11}e^{\hat{s}t} + \overline{w}$, $\mu(t) = A\nu_{12}e^{\hat{s}t} + \overline{\mu}$, and $\lambda(t) = A\nu_{13}e^{\hat{s}t} + \overline{\lambda}$, where A is an arbitrary constant, and $\nu = [\nu_{11}\ \nu_{12}\ \nu_{13}]^T$ denotes the eigenvector associated with $\hat{s}$.

It turns out that $\nu = \left[\begin{smallmatrix} c & 0 & 0 \end{smallmatrix}\right]^T$, with $c \in \mathbb{R}$. Then, $\mu(t) = \overline{\mu}$, which from (14) implies that $S(t) = \overline{S} \forall t$.

Consider, now, the closed-loop solution under constant marginal waiting time disutility. When $\alpha_2 = 0$, the system of equations (B.6)-(B.8) has a single candidate solution for which the value function is not convex with respect to w_i. The remaining five candidates have $\omega_3 > 0$ and cannot therefore constitute a solution the hospital's maximisation problem. The solution that yields a linear—hence, concave—value function with respect to w_i is $\omega_3 = \omega_4 = \omega_5 = 0$. This linearity of the value function with respect to waiting times is not surprising given the linear structure of the game when $\alpha_2 = 0$. With $\omega_3 = \omega_5 = 0$, hospital i's optimal supply rule becomes:

$$S_i(w_i, w_j) = \frac{p - \theta\omega_1}{\gamma}. \tag{B.21}$$

Under closed-loop rules, players strategies are, by construction, a function of the state variables—here, waiting times—rather than time-profiles as is the case of open-loop rules. If the marginal waiting time disutility is constant, the optimal supply rule is independent, in each t, of waiting times. Thus, supply is constant over time as in the open-loop solution derived above.

With $\omega_3 = \omega_4 = \omega_5 = 0$, (B.5) simplifies to:

$$\begin{aligned}\left\{T + \frac{p^2}{2\gamma} + \sigma(\omega_1 + \omega_2) + \frac{\theta^2}{2\gamma}\omega_1^2 + \frac{\theta^2}{\gamma}\omega_1\omega_2 - \rho\omega_0\right\}& \\ + w_i\left\{-\left[\rho + \frac{\theta(2-\beta)N}{2\tau}\right]\omega_1 + \frac{\theta\beta N}{2\tau}\omega_2 - \alpha_1\right\}& \\ + w_j\left\{\frac{\theta\beta N}{2\tau}\omega_1 - \left[\rho + \frac{\theta(2-\beta)N}{2\tau}\right]\omega_2\right\}& = 0.\end{aligned} \tag{B.22}$$

Since the last two terms depend only on ω_1 and ω_2, we focus on the 2×2 system and solve for ω_1. The solution is given by

$$\omega_1 = -\frac{\tau\alpha_1\left[2\rho\tau + \theta(2-\beta)N\right]}{2\left[\rho\tau + \theta(1-\beta)N\right]\left[\rho\tau + \theta N\right]} = -\frac{2\tau\alpha_1}{\phi}. \tag{B.23}$$

Inserting the expression for ω_1 into the optimal supply rule for hospitals i and j yields $S_i = S_j = \overline{S}$. Using this result, the closed-loop steady-state waiting time is derived from the equations of motion (11)-(12), with $\dot{w}_i(t) = \dot{w}_j(t) = 0$. Simple algebra shows that $w_i = w_j = \overline{w}$.

Finally, the supply rule constitutes a Markov Perfect Nash Equilibrium if the value function is bounded from above. It is straightforward to see from (B.15) and (B.16) that $s_1 < 0$ and $s_2 < 0$ when $\omega_3 = \omega_5 = 0$.

B.3 Transitional Dynamics

In order to analyse the convergence to the steady-state in the closed-loop solution, we turn to its open-loop representation. That is, we derive time-profiles of the waiting time, supply, and demand from the optimal closed-loop supply rule. Let the superscript CL denote the closed-loop steady-state. The eigenvalues governing the system of differential equations (B.12)-(B.13), s_1 and s_2, are respectively associated with the eigenvectors $\nu_1 = c_1 \left[\, 1 \; -1 \,\right]^T$ and $\nu_2 = c_2 \left[\, 1 \; 1 \,\right]^T$, with $c_1, c_2 \in \mathbb{R}$. Setting $c_1 = c_2 = 1$, the solution of the system of differential equations (B.12)-(B.13) takes the form:

$$w_i(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + w^{CL}, \tag{B.24}$$

$$w_j(t) = -C_1 e^{s_1 t} + C_2 e^{s_2 t} + w^{CL}, \tag{B.25}$$

where C_1 and C_2 are arbitrary constants. The closed-loop steady-state waiting time w^{CL} is retrieved by setting $\dot{w}_i = \dot{w}_j = 0$ in (B.12)-(B.13) and solving for w_i and w_j. This yields:

$$w^{CL} = \frac{N\left[\frac{\beta}{2} + (1-\beta)\left(\frac{v-k}{\tau}\right)\right] - \left(\frac{p-\theta\omega_1}{\gamma}\right)}{\frac{(1-\beta)N}{\tau} - \frac{\theta}{\gamma}(\omega_3 + \omega_5)}. \tag{B.26}$$

Inserting the initial conditions $w_i(0) = w_{0i}$ and $w_j(0) = w_{0j}$ into (B.24)-(B.25) and solving for C_1 and C_2 gives $C_1 = \frac{w_{0i} - w_{0j}}{2}$ and $C_2 = \frac{w_{0i} + w_{0j}}{2} - w^{CL}$. Then, waiting times at Hospital i converge to the steady-state according to:

$$w_i(t) = \left(\frac{w_{0i} - w_{0j}}{2}\right) e^{s_1 t} + \left(\frac{w_{0i} + w_{0j}}{2} - w^{CL}\right) e^{s_2 t} + w^{CL}. \tag{B.27}$$

Consider, now, the dynamics of supply and demand. Inserting (B.27) and the analogous equation for $w_j(t)$ into (31) yields:

$$S_i(t) = \frac{\theta}{\gamma}\left[(\omega_5 - \omega_3)\left(\frac{w_{0i} - w_{0j}}{2}\right) e^{s_1 t} - (\omega_3 + \omega_5)\left(\frac{w_{0i} + w_{0j}}{2} - w^{CL}\right) e^{s_2 t}\right] + \frac{p - \theta[\omega_1 + (\omega_3 + \omega_5) w^{CL}]}{\gamma}. \tag{B.28}$$

Using (7), (B.27), and the analogous equation for $w_j(t)$, the dynamics of demand faced by Hospital i in the competitive and monopolistic segments are respectively given by

$$D_C^i(t) = \beta N\left[\frac{1}{2} + \left(\frac{w_{0j} - w_{0i}}{2\tau}\right) e^{s_1 t}\right] \tag{B.29}$$

and

$$D_M^i(t) = \frac{(1-\beta)N}{\tau}\left[v - k - w^{CL} + \left(\frac{w_{0j} - w_{0i}}{2}\right)e^{s_1 t} + \left(w^{CL} - \frac{w_{0i} + w_{0j}}{2}\right)e^{s_2 t}\right]. \tag{B.30}$$

If $w_{0i} = w_{0j}$, it follows from equations (B.27)-(B.30) that the dynamics of waiting times, supply, and demand are uniquely governed by s_2, and convergence is thus monotonic. By the same token, convergence is monotonic as well if the initial waiting times differ but their average equals the steady-state waiting time w^{CL}. Note, additionally, that demand in the competitive segment always converges monotonically to $\beta N/2$.

For the transitional dynamics in the closed-loop solution under constant marginal waiting time disutility, simply set $\omega_3 = \omega_5 = 0$ in equations (B.27)-(B.30). Constant hospital activity over time for $\alpha_2 = 0$ is then confirmed by (B.28).

B.4 Non-Monotonic Convergence

Equations (B.27)-(B.30) show that convergence to the steady-state depends on two, possible opposing, forces. It depends on whether a hospital's initial waiting time is longer than that of the rival, and whether the average initial waiting time in the market differs from the steady-state waiting time. When these two conditions hold, the possibility of non-monotonic convergence arises. To see why non-monotonic convergence might occur, consider the equilibrium dynamics of waiting times described in (B.27). If the average initial waiting time is above (below) the steady-state, the first two terms have opposite signs for the hospital with the shorter (longer) waiting time. In both cases, whether or not non-monotonic convergence emerges depends on the relative size and speed of convergence (to zero) of each of those terms.

Differentiating (B.27) with respect to time and equating to zero yields a single critical point given by

$$t^* = \left(\frac{1}{s_1 - s_2}\right) ln\left[-\frac{s_2}{s_1}\left(\frac{w_{0i} + w_{0j} - 2w^{CL}}{w_{0i} - w_{0j}}\right)\right], \tag{B.31}$$

where s_1 and s_2 are given by (B.15) and (B.16), respectively. Convergence is non-monotonic for Hospital i if and only if $t^* \in \mathbb{R}^+$. With $s_1, s_2 < 0$, the first factor in (B.31) is negative if $|s_1| > |s_2|$. Thus, $t^* \in \mathbb{R}^+$ if and only if the second factor in (B.31) is defined and is negative, which requires that the expression in the square brackets lies between 0 and 1. It is possible to derive some easily interpretable conditions for this expression to be positive. Since $-\frac{s_2}{s_1} < 0$, we must have $\frac{w_{0i}+w_{0j}-2w^{CL}}{w_{0i}-w_{0j}} < 0$. Two cases then arise:

1. If the average initial waiting time is above the steady-state waiting time, the numerator is positive, and $\frac{w_{0i}+w_{0j}-2w^{CL}}{w_{0i}-w_{0j}}$ is negative only if Hospital i has an initial waiting time below that of Hospital j.

2. If the average initial waiting time is below the steady-state waiting time, the numerator is negative, and $\frac{w_{0i}+w_{0j}-2w^{CL}}{w_{0i}-w_{0j}}$ is negative only if Hospital i has an initial waiting time above that of Hospital j.

Therefore, when he average initial waiting time is above (below) the steady-state waiting time, it is the hospital with the shortest (longest) waiting time that exhibits non-monotonic convergence, provided that $|s_1| > |s_2|$ and $-\frac{s_2}{s_1}\left(\frac{w_{0i}+w_{0j}-2w^{CL}}{w_{0i}-w_{0j}}\right) \in (0, 1)$.

To conclude the proof, we consider the shape of (B.27). Evaluating its second-order derivative with respect to t at t^* yields the following results:

1. If $(w_{0i} + w_{0j} > 2w^{CL}) \wedge (w_{0i} < w_{0j})$, then $w_i^{''}(t^*) < 0$ simplifies to:

$$\left(\frac{s_1}{s_2}\right)^2 e^{(s_1-s_2)t^*}(w_{0i} - w_{0j}) < -(w_{0i} + w_{0j} - 2w^{CL}). \tag{B.32}$$

   Diving both sides by $(w_{0i}-w_{0j})$ reverses the inequality sign. Then, using (B.31), the inequality becomes $\frac{s_1}{s_2} > 1$, which is true.

2. If $(w_{0i} + w_{0j} < 2w^{CL}) \wedge (w_{0i} > w_{0j})$, then $w_i^{''}(t^*) > 0$ simplifies to:

$$\left(\frac{s_1}{s_2}\right)^2 e^{(s_1-s_2)t^*}(w_{0i} - w_{0j}) > -(w_{0i} + w_{0j} - 2w^{CL}). \tag{B.33}$$

   Diving both sides by $(w_{0i} - w_{0j})$ does not reverse the inequality sign. Then, using (B.31), the inequality becomes $\frac{s_1}{s_2} > 1$, which is true.

Hence, if $|s_1| > |s_2|$, $-\frac{s_2}{s_1}\left(\frac{w_{0i}+w_{0j}-2w^{CL}}{w_{0i}-w_{0j}}\right) \in (0, 1)$, and the average initial waiting time is above (below) the steady-state waiting time, the dynamics of the waiting time at the hospital with the shortest (longest) initial wait has a unique maximum (minimum). This implies that the waiting time at the hospital with the shortest (longest) initial wait first increases (decreases) before decreasing (increasing) towards the steady-state.

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