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Error patterns in Portuguese students’ addition and subtraction calculation tasks: Implications for teaching
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Abstract

Purpose – The purpose of this descriptive study is to investigate why some elementary children have difficulties mastering addition and subtraction calculation tasks.

Design/methodology/approach – The researchers have examined error types in addition and subtraction calculation made by 697 Portuguese students in elementary grades. Each student completed a written assessment of mathematical knowledge. A system code (e.g. FR = failure to regroup) has been used to grade the tests. A reliability check has been performed on 65 per cent randomly selected exams.

Findings – Data frequency analyses reveal that the most common type of error was miscalculation for both addition ($n = 164; 38.6$ per cent) and subtraction ($n = 180; 21.7$ per cent). The second most common error type was related to failure to regroup in addition ($n = 74; 17.5$ per cent) and subtraction ($n = 139; 16.3$ per cent). Frequency of error types by grade level has been provided. Findings from the hierarchical regression analyses indicate that students’ performance differences emerged as a function of error types which indicated students’ types of difficulties.

Research limitations/implications – There are several limitations of this study: the use of a convenient sample; all schools were located in the northern region of Portugal; the limited number of problems; and the time of the year of assessment.

Practical implications – Students’ errors suggested that their performance in calculation tasks is related to conceptual and procedural knowledge and skills. Error analysis allows teachers to better understand the individual performance of a diverse group and to tailor instruction to ensure that all students have an opportunity to succeed in mathematics.

Social implications – Error analysis helps teachers uncover individual students’ difficulties and deliver meaningful instruction to all students.

Originality/value – This paper adds to the international literature on error analysis and reinforces its value in diagnosing students’ type and severity of math difficulties.

Keywords Assessment, Error analysis, Learning, Mathematics calculation, Error patterns, Elementary grades

Paper type Research paper
Mathematics achievement is a subject of international concern. There is compelling evidence that mathematics skills are fundamental to long-term academic success, provide life-long skills and are needed for obtaining employment in today’s competitive market (Judge and Watson, 2011; Lembre et al., 2012). Lack of mathematical literacy can seriously limit an individual’s opportunities to succeed in school and life (Aud et al., 2013; Duncan et al., 2007). However, both in the USA and in Portugal, a significant percentage of K-12 students function below proficiency levels in mathematics (Aud et al., 2013; Serrão et al., 2010). The most recent assessment results from the Program for International Student Assessment from 2015 (OECD, 2016) show that the mathematics average score for Portugal is 492 and for the USA is 470. Similar average scores were shown in the 2011 Trends in International of Mathematics and Science Study. American fourth graders’ average score was 541 and Portuguese fourth graders’ average score was 532 (Provasnik et al., 2012).

Given the fact that American and Portuguese elementary students perform similarly in mathematics, examining the mathematics skills of Portuguese students can contribute to our understanding of typical and atypical development of elementary students’ mathematical skills. Given the growing number of Portuguese students in American schools and the similarities in their mathematical performance with regard to their American counterparts in the two international assessments, we decided to collaborate on an investigation with the purpose:

- to examine why some elementary children have difficulties mastering addition and subtraction calculation tasks;
- to determine the most common types of errors elementary Portuguese students in Grades 1 through 4 make when solving addition and subtraction calculation problems;
- to identify the most predominant errors at each grade level; and
- to investigate what type of errors predicts student performance in addition and subtraction.

Importance of error analysis
Improving mathematics education often is thought as increasing teacher content knowledge or the depth and breadth of knowledge of the mathematical concepts and skills they will be teaching (National Commission on Mathematics and Science Teaching for the 21st Century, 2000; National Mathematics Advisory Panel [NMAP], 2008). There is no doubt that teachers must know well the concepts and skills of the mathematics curriculum they will be teaching to students with diverse instructional needs. However, besides knowledge of content, mathematics teachers must understand the nature of errors students make to provide corrective feedback to students to eliminate those errors (Ashlock, 2010). It is advantageous that teachers identify what is affecting their students’ progress as early as possible and provide explicit instruction that addresses their individual needs.

Diagnostic teaching, which includes the qualitatively analysis of students’ error patterns, provides teachers the opportunity to identify students’ types of errors and to determine the misconceptions and difficulties of their students (Ashlock, 2010; O’Connell, 1999). It is important that mathematics teachers examine students’ erroneous mathematics concepts and/or procedures to provide effective instruction that addresses the diverse needs of individual students. In the elementary grades, many students make systematic and consistent errors when computing addition and subtraction problems. Often, teachers assume that these are careless mistakes or reflect a lack of fact fluency. However, this is not...
always the case. If those error patterns or misconceptions are not corrected early, the error patterns may persist and affect students’ acquisition of higher mathematical skills such as algebra (Ashlock, 2010; Khan and Chishti, 2011). Accordingly, early identification of error patterns by means of error analysis is one way to afford students the equal opportunity to improve the mathematics outcomes of all students and narrow the gap between struggling and achieving students.

The National Council of Teachers of Mathematics (NCTM, 2000) considers computation to be one of the critical skills in the mathematics curriculum. Number and operations is first of the five content standards. This standard includes understanding numbers, knowing the meaning of operations and computing fluently. Several researchers (Riccomini, 2005; Tolar et al., 2009) have asserted that if students do not acquire proficiency in computation skills, they will experience difficulties in the other areas of mathematics. For example, Tolar et al. (2009) found that computational fluency is the strongest influence factor on the algebra performance of undergraduate students. In addition, student computational skills are emphasized in the National Mathematics Advisory Panel’s (NMAP)(2008) final report. The NMAP panel encouraged elementary, middle school and special education teachers to adequately prepare their students for algebra by making sure that students develop conceptual understanding of mathematical operations, computational fluency and problem-solving skills. Thus, it is important that teachers recognize students’ misconceptions and/or lack of conceptual and procedural knowledge on solving computational problems to craft the most effective instructional intervention.

Although computation is an important academic skill, many students have difficulty with this basic mathematics skill, and, often, their error patterns are not acknowledged or addressed. Knowing the meaning of the basic arithmetic operations involves the understanding and the application of the commutative, associative and distributive properties of the operations (National Mathematics Advisory Panel [NMAP], 2008). The same seems to be true for students who have mathematics learning difficulties and/or disabilities (MD). It is well documented that students with MD have deficits in many areas of mathematics, especially computation fluency (Geary, 2004; Jordan et al., 2010; Judge and Watson, 2011; Traff and Samuelson, 2013; Zheng et al., 2012). To better meet students’ needs, teachers should identify students’ errors and analyze those through a systematic examination to pinpoint the cause(s) of the error types (Ashlock, 2010). For example, many students do not understand the concept of regrouping to solve addition or subtraction problems such as 46 + 17 or 46 – 17, and make systematic errors. This suggests that they do not have a good understanding of place value. In this case, intervention should focus on the students’ conceptual knowledge of place value and not on the procedural knowledge of solving addition or subtraction problems. Conceptual understanding often promotes transfer of learning to new problems (Ashlock, 2010; Gilmore and Bryant, 2008).

Analysis of error patterns in computation problems is a valuable assessment tool for teachers. It shows what type of conceptual and/or procedural knowledge the student is having difficulty understanding. By analyzing the errors and identifying the patterns of mistakes students make when solving computation problems, teachers will be better able to plan appropriate and effective interventions (Luneta and Makonye, 2010; Riccomini, 2005) and provide students an equal opportunity to succeed in mathematics.

**Brief review of studies on error analysis**

Studies on student error analysis of computation problems were mostly conducted in the late 1960s, 1970s and 1980s. For example, Roberts (1968) investigated third graders and found that 36 per cent of their errors were because of defective algorithms (e. g. regrouping).
He classified students’ errors according to four categories: wrong operation, obvious computational error, defective algorithm and random responses. Roberts noticed that those students who made careless or fact calculation errors made them consistently throughout the four operations. In a two-year study, Cox (1975a, 1975b) analyzed the errors of second through sixth graders. Results indicated that 5 to 6 per cent of the students made systematic errors in addition, multiplication and division; whereas, 13 per cent made errors in subtraction. Follow-up data showed that 25 per cent of the students either were making the same type of errors or another systematic type of error on the same algorithm. Several years later, Engelhardt (1977) studied the computation errors of third- and sixth-grade students on an 84-item test and extended Roberts’ (1968) classification of errors. He identified eight error types: basic fact, grouping, inappropriate inversion, incorrect operation, defective algorithm, incomplete algorithm, identity and concept of zero. Results showed that basic fact errors were the most common type. Furthermore, over 40 per cent of the total errors were made by students in the lowest quartile. On the basis of the data, Engelhardt (1977) concluded that defective algorithm error type distinguished high performers from low performers. Brown and Burton (1978) reported similar findings in their analysis of procedural “bugs” (errors) of fourth, fifth and sixth graders. Using a computer program to analyze students’ computational errors, they found that approximately 40 per cent of the students consistently made incorrect algorithm type of errors, especially in subtraction calculation problems where the top digit was a zero.

There have not been many studies on error analysis of computation problems in the twenty-first century. McIntosh (2002) described the common errors in mental computation of students in Grades 3-10. Riccomini (2005) identified systematic subtraction error patterns of students with learning disabilities. Raghubar et al. (2009) and colleagues investigated the types of errors in multi-digit computation in 291 third- and fourth-grade students with and without learning disabilities and the role of attention in math performance. They reported that math fact errors were related to the severity of math difficulties. Students with math learning disabilities also made more procedural errors than those without math disabilities. One of the few studies published is the investigation of Traff and Samuelson (2013). They analyzed errors of multi-digit computation and problem-solving in 142 elementary students with MD and compared them to 112 students without MD. They reported that a higher number of students with MD made errors when subtracting the smaller from the larger number as well as in regrouping and failure to regroup than students without MD. Their findings were similar to those reported by Raghubar et al. (2009). Finally, Traff and Samuelson (2013) suggested that the errors of students with MD were because of deficits related to both conceptual and procedural knowledge.

In sum, research on error analysis of students’ basic computation problems revealed that basic fact error type was the most common type of error. One of the most prevalent error patterns was in subtraction problems in which the students subtracted a smaller number from a larger number (e.g. 53 – 28 =). Regrouping and difficulty with the concept of zero also were systematic errors among students.

In the present study, we provide information on the types of errors Portuguese elementary students who had completed grades one through four made when solving addition and subtraction computation problems. We examined students’ addition and subtraction computation errors by grade level and compared them with error patterns of students among grade levels. In this article, our goal is to offer a diagnostic analysis to guide teachers in the evaluation of their students’ error patterns to plan effective interventions that address the specific needs of each student.
Method

Participants

Participants were 697 students from 42 classrooms of seven elementary schools in Northern Portugal. Students had completed the school year as first through fourth graders (Table I). In total 62 percent of participants came from schools where 31 percent of students received free lunch. The other participants (38 percent) were from lower socioeconomic schools, with 35-51 percent of students receiving free lunch. There were 339 (48.6 percent) girls and 358 (51.6 percent) boys aged 6-13 years old (age average was 8.78). All students were receiving instruction in the general education curriculum for their particular grade level. In Portugal, learning disabilities is not a recognized disability. This means that students who may have a learning disability in mathematics have not been identified as having MD although their teachers might have identified them as underachievers. However, some Portuguese schools do provide assistance to those students struggling in mathematics.

Materials

Each student completed a written assessment test of mathematical knowledge developed by Lospes and Bueno (2014). The instrument reflects the Portuguese mathematics curriculum of first through fourth grade. It is divided into three main sections, number knowledge, calculation and problem-solving, and it has a total of 46 items. In this article, we report on the ten items of the second part of the test (i.e. calculation) that involve addition and subtraction computation tasks. A principal component analysis, based on a tetracoric correlation matrix, revealed an initial six-factor solution for the test. A subsequent principal component analysis with an oblique rotation was conducted to test the hypothesis of a second-order factor related to the math general achievement. A 42-item solution representing five primary factors that cluster in a single second-order factor related to general math achievement was found. The Cronbach $\alpha$ for the general factor was 0.94. A psychometric analysis based on Rasch's model of the item response theory showed that the items cover the full extension of the subjects' skills.

Procedures

Before the school year started, two researchers met with the teachers from the seven schools involved in the project. In those meetings, the researchers described the purpose of the project, the development of the assessment tool and how students’ answers would be evaluated. They explained to the teachers that their students were to complete the problems relevant to their particular grade level. Oral and written instructions on how to administer the test were provided and teachers’ questions were answered during the meeting. Each classroom teacher received a package with the tests containing the written directions on how to administer the test to their students. Teachers were asked to instruct their students to complete the problems relevant to their specific grade level, but they could encourage the

<table>
<thead>
<tr>
<th>Grade</th>
<th>No. of participants</th>
<th>Gender</th>
<th>Percentage of participants</th>
<th>Age range of participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>117</td>
<td>63</td>
<td>54</td>
<td>16.8</td>
</tr>
<tr>
<td>2nd</td>
<td>139</td>
<td>73</td>
<td>66</td>
<td>19.9</td>
</tr>
<tr>
<td>3rd</td>
<td>144</td>
<td>71</td>
<td>73</td>
<td>20.7</td>
</tr>
<tr>
<td>4th</td>
<td>297</td>
<td>151</td>
<td>146</td>
<td>42.6</td>
</tr>
<tr>
<td>Total</td>
<td>697</td>
<td>358</td>
<td>339</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table I. Descriptive analysis of participants
students to answer other problems they knew how to solve. Whole-class assessment occurred in the students’ own classrooms during the first two weeks of the school year. Our first graders were just starting second grade, our second graders were starting third grade, etc. The exams were returned to the researchers in their original packages.

Two researchers graded all tests three times. The first time, they identified correct/incorrect items. The second time, they used a code system to identify the types of errors (e.g. MC = miscalculation and FR = failure to regroup). The system code used was based on Engelhardt’s (1977) and Traff and Samuelson’s (2013) work. In addition to those types of errors, we found other errors that did not fit the coding system. We noted all different types of errors found in students’ answers and added them to the original coding system. We re-examined all the problems using the new coding system (Table II). Errors were only counted when the student made the error solving his/her grade-level type of problem. For example, in the problem 347 + 236 = 572, there are two different types of errors: 1) 7 + 6 is 13, not 12 and 2) 4 + 3 is 7, but the student should have added 1 to regroup and the answer should be 8. The first error type is a miscalculation error and the latter is an example of failure to regroup type of error. Thus, in the answer for this addition problem, there are two different types of errors, a miscalculation and a failure to regroup error.

After grading all tests for the third time, a reliability check was performed on 65 per cent (n = 456) of randomly selected exams from different grade levels. Inter-observer agreement was calculated by reporting agreements on occurrences divided by agreements plus disagreement (A / (A + D)). The percentage of agreement was 0.98. The two raters discussed the few disagreements and the disagreements were resolved by one of the other two researchers.

Results

The three most common error types

In this study, we investigated the types of errors Portuguese elementary students make when performing addition and subtraction computation problems. There were 424 addition errors and 854 subtraction errors. Data frequency analyses showed that the most common type of error was miscalculation for both addition (n = 164; 38.6 per cent) and subtraction (n = 180; 21.7 per cent) among the 697 students. In subtraction calculation problems, fourth graders had the highest percentage (35 per cent) of all miscalculation errors. Correlation analysis showed that there is a small significant negative correlation between grade-level and miscalculation errors in addition problems, r(695) = −0.2, p < 0.005. However, no

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miscalculation = MC</td>
<td>Miscalculation = MC</td>
</tr>
<tr>
<td>Wrong operation = WO</td>
<td>Wrong operation = WO</td>
</tr>
<tr>
<td>Failure to regroup = FR</td>
<td>Failure to regroup = FR</td>
</tr>
<tr>
<td>Concept of zero = CZ</td>
<td>Concept of zero = CZ</td>
</tr>
<tr>
<td>Regrouping error = RE</td>
<td>Regrouping error = RE</td>
</tr>
<tr>
<td>Decimal error = Ea</td>
<td>Decimal error = Ra</td>
</tr>
<tr>
<td>Copied number incorrectly = Rb</td>
<td>Copied number incorrectly = Rb</td>
</tr>
<tr>
<td>Incorrect answer w/o explanation = Rc</td>
<td>Incorrect answer w/o explanation = Rc</td>
</tr>
<tr>
<td>Omission (no attempt) = Rd</td>
<td>Omission (no attempt) = Rd</td>
</tr>
<tr>
<td>Procedure error = Re</td>
<td>Procedure error = Re</td>
</tr>
</tbody>
</table>

Table II.

Error types coding system

Subtraction failure smaller from larger = SFL
significant correlation was found between grade and subtraction miscalculation errors. A significant positive correlation was observed between addition and subtraction calculation errors, \( r(695) = 0.15, p < 0.005 \).

The second most common error type among the 697 students was related to failure to regroup in addition \((n = 74; 17.5 \text{ per cent})\) and subtraction \((n = 139; 16.3 \text{ per cent})\). Failure to regroup in addition was positively correlated to the concept of zero error, \( r(695) = 0.22, p < 0.005 \); decimal error, \( r(696) = 0.23, p < 0.005 \); and miscalculation error, \( r(696) = 0.15, p < 0.005 \). Significant positive correlations were found between subtraction failure to regroup error and the concept of zero error, \( r(696) = 0.26, p < 0.005 \) and decimal error, \( r(696) = 0.24, p < 0.005 \). Grade level was positively correlated to failure to regroup error in addition, \( r(696) = 0.09, p < 0.05 \) and failure to regroup in subtraction, \( r(696) = 0.28, p < 0.005 \).

Omission and procedure errors together \((n = 242; 18.9 \text{ per cent})\) were the third most common type of errors among all students. Omission and procedure errors indicate the student did not even attempt to solve the problem (i.e. omission) or they started to align the numbers but did not know how to proceed. We concluded that several students \((n = 179; 25.7 \text{ per cent})\) did not know how to add \((n = 84; 34.7 \text{ per cent})\) and/or subtract \((n = 158; 65.3 \text{ per cent})\) two-digit and multi-digit numbers. Those types of problems were part of the students’ grade-level curriculum. If the problem was not part of the student’s grade-level curriculum, it was not counted as an error of any type. There were 51 \((61 \text{ per cent})\) omission types of errors in addition of two-digit and multi-digit problems and 44 \((27.8 \text{ per cent})\) subtraction of two-digit and multi-digit problems were left blank. We coded them as omission errors. A total of 29 students \((4.2 \text{ per cent})\) started to solve the addition of two-digit and multi-digit problems, but they did not know how to solve them. A total of 70 students \((10 \text{ per cent})\) attempted to answer the subtraction of two-digit and multi-digit problems, but they did not know how to unravel the problems and left them incomplete. We coded those as procedure errors in addition \((n = 33; 3.9 \text{ per cent})\) and subtraction \((n = 114; 72 \text{ per cent})\). Procedure errors in addition had a small significant positive correlation to decimals, \( r(695) = 0.12, p < 0.005 \), and a small significant negative correlation to decimals in subtraction, \( r(695) = -0.9, p < 0.05 \).

**The most predominant errors at each grade level**

We used frequency to depict the number of times each error type occurred at each grade level. Miscalculation in addition was the most frequent type of error in all four grade levels \((\text{first} = 53.3 \text{ per cent}, \text{second} = 47.3 \text{ per cent}, \text{third} = 62.3 \text{ per cent}, \text{and fourth} = 22.7 \text{ per cent})\); adding decimals was the second most predominant type of error in Grade 4 \((22.2 \text{ per cent})\). In subtraction, omission \((36.4 \text{ per cent})\) and miscalculation \((32.5 \text{ per cent})\) were the most frequent types of errors in first grade, subtracting a larger from a smaller number \((32.9 \text{ per cent})\) was the number one type of error among second graders, failure to regroup \((26.5 \text{ per cent})\) occurred more often in Grade 3, and errors involving the concept of zero \((22.8 \text{ per cent})\) and failure to regroup \((21.3 \text{ per cent})\) were repeatedly found among fourth graders. The number and percentage of the most recurrent types of errors in each grade level are displayed in Table III for addition and Table IV for subtraction.

**Predictors of student performance**

A series of hierarchical multiple regression analyses were performed to determine whether each independent variable (i.e. grade level and the most common type of errors) has a role in explaining the variance of student performance in addition and subtraction problems. In each analysis, the grade level of each student was entered into the regression equation, first as a control variable, followed by the six most common types of addition or subtraction...
errors. These models were used to predict the total student performance in addition calculation problems and subtraction calculation problems. Intercorrelations of predictor and outcome variables are presented in Table V. This methodological approach permitted isolation of the relative contributions of the grade level first, followed by the individual contributions of each of the effects of error pattern variables on match calculation measures (Cohen and Cohen, 1983). In each of the analyses, the increments ($I$) in $R^2$ were determined to assess whether the different independent measures accounted for a significant proportion of variance in the dependent measures.

Table VI presents results of the hierarchical regression analysis predicting the addition calculation performance score. As can be seen in Table VI, this regression was significant ($F = 76.07, p < 0.001$). The addition of each predictor variable led to a statistically significant increase in $R^2$. After controlling for the grade-level variable, all five error patterns accounted for a statistically significant 36 per cent of the variance in the addition calculation performance score. Miscalculation error pattern was the strongest statistical predictor for addition calculation performance, $\Delta R^2 = 0.22, \Delta F (1,694) = 491.81, p < 0.001$. In addition, the procedure error pattern accounted for 7 per cent of the variance, $\Delta F (1,693) = 187.86, p < 0.001$. Thus, these two error patterns accounted for 29 per cent of the variance in addition calculation performance.
| Variables                      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   | 13   | 14   | 15   |
|-------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1. Grade                      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 2. Addition miscalculation    | -0.19*** |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 3. Addition failure to regroup| 0.09*** | 0.15*** |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 4. Addition concept of zero   | 0.20*** | 0.01  | 0.22*** |      |      |      |      |      |      |      |      |      |      |      |      |
| 5. Addition decimal           | 0.25*** | -0.01 | 0.23*** | 0.70*** |      |      |      |      |      |      |      |      |      |      |      |
| 6. Addition omission          | -0.10** | -0.02 | -0.01 | 0.04  | 0.12** |      |      |      |      |      |      |      |      |      |      |
| 7. Addition procedure         | -0.10** | 0.07  | -0.06 | -0.05 | -0.06 | 0.01  |      |      |      |      |      |      |      |      |      |
| 8. Subtraction miscalculation | -0.01  | -0.16*** | -0.09* | -0.02 | 0.04  | 0.02  | 0.02  |      |      |      |      |      |      |      |      |
| 9. Subtraction failure to regroup| 0.28*** | -0.04 | 0.10** | 0.13*** | 0.15*** | 0.02  | -0.05 | 0.03  |      |      |      |      |      |      |      |
| 10. Subtraction failure       | -0.03  | 0.09* | 0.10** | 0.02  | 0.02  | -0.05 | -0.04 | -0.00 | -0.07 |      |      |      |      |      |      |
| 11. Concept of zero           | 0.42*** | -0.06 | 0.12** | 0.35*** | 0.32*** | -0.02 | -0.08* | 0.04  | 0.26*** | 0.09* |      |      |      |      |      |
| 12. Subtraction omission      | -0.12** | 0.18** | -0.03 | -0.01 | -0.01 | 0.07  | 0.47*** | -0.01 | -0.09* | -0.06 | -0.11** |      |      |      |      |
| 13. Subtraction procedure     | -0.05  | -0.00 | 0.01  | 0.08* | 0.10** | 0.33*** | 0.04  | 0.03  | 0.01  | -0.02 | 0.07  | -0.03 |      |      |      |
| 14. Addition performance      | 0.68*** | -0.59*** | -0.17** | -0.02 | -0.01 | -0.21*** | -0.35*** | -0.13** | 0.16*** | -0.02 | 0.22*** | -0.27*** | -0.10* |      |      |
| 15. Subtraction performance   | 0.67*** | -0.29*** | -0.00 | 0.06  | 0.07  | -0.13** | -0.25*** | -0.32*** | -0.04 | -0.15*** | 0.06  | -0.40*** | -0.18*** | 0.69*** |      |
| \( M \)                       | 2.89  | 0.24  | 0.11  | 0.05  | 0.06  | 0.05  | 0.07  | 0.26  | 0.20  | 0.20  | 0.15  | 0.16  | 0.06  | 3.85  | 3.12  |
| \( SD \)                      | 1.13  | 0.52  | 0.31  | 0.27  | 0.25  | 0.24  | 0.32  | 0.56  | 0.46  | 0.43  | 0.36  | 0.57  | 0.31  | 1.15  | 1.35  |

**Notes:** *p < 0.05; **p < 0.01; ***p < 0.001
Table VII presents the results of the hierarchical regression analyses for variables predicting the subtraction calculation performance score. Once more, the regression was significant ($F = 50.23, p < 0.001$). Similar to the previous results, all six error patterns accounted for a statistically significant 34 per cent of the variance in the subtraction calculation performance score after controlling for the grade-level variable. However, the omission (i.e. no attempt) error pattern was the strongest statistical predictor for subtraction calculation performance, $\Delta R^2 = 0.11$, $\Delta F (1,694), p < 0.001$, and that miscalculation error, concept of zero error, failure to regroup, subtraction of larger from smaller, and procedure error also contribute to the prediction.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model 1</th>
<th></th>
<th>Model 2</th>
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<tbody>
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<td></td>
<td>$B$</td>
<td>$SEB$</td>
<td>$\beta$</td>
<td>$B$</td>
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<tr>
<td>Step 1</td>
<td></td>
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<tr>
<td>Grade</td>
<td>0.69</td>
<td>0.03</td>
<td>0.68*</td>
<td>0.61</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscalculation</td>
<td>-0.98</td>
<td>0.04</td>
<td>-0.44*</td>
<td></td>
</tr>
<tr>
<td>Procedure</td>
<td>-0.98</td>
<td>0.06</td>
<td>-0.27*</td>
<td></td>
</tr>
<tr>
<td>Decimal</td>
<td>-0.59</td>
<td>0.08</td>
<td>-0.13*</td>
<td></td>
</tr>
<tr>
<td>Failure to regroup</td>
<td>-0.54</td>
<td>0.06</td>
<td>-0.15*</td>
<td></td>
</tr>
<tr>
<td>Omission</td>
<td>-0.69</td>
<td>0.08</td>
<td>-0.14*</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.46</td>
<td></td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.46</td>
<td></td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>F-value</td>
<td>597.28*</td>
<td></td>
<td>76.07*</td>
<td></td>
</tr>
<tr>
<td>$\Delta R^2$</td>
<td></td>
<td></td>
<td>0.36*</td>
<td></td>
</tr>
</tbody>
</table>

Note: * $p < 0.001$
Discussion

Miscalculation in both addition and subtraction was a common type of error in all grade levels (i.e. first through fourth). Miscalculation often is associated with poor retrieval strategy (Geary, 1996; Geary et al., 2000), inattentive behavior (Hecht and Vagi, 2010) or inability to count (Cheng and Chan, 2005). We found that one group of first graders relied heavily on drawing and then counting to arrive at the answers. The drawing-counting strategy seemed to have contributed to the high number of miscalculations in first graders in both addition and subtraction. Some authorities (Cheng, 2012; Murata, 2004) suggest that counting is an immature strategy and may delay the development of other mathematical skills.

Other types of errors found among the participants in both addition and subtraction tasks suggest that some students had problems with the conceptual knowledge of decimals, the base-10 system and place value. Failure to regroup was the second most common type of error which may indicate students’ misunderstanding of place value. Some researchers (Cauley, 1988; Gilmore and Bryant, 2008; Hiebert and Lafevre, 1986) have asserted that conceptual and procedural knowledge should be linked for students to become competent in mathematics. Thanheiser (2012) and Weber (2001) added strategic knowledge to achieving competence in mathematics. They explained that students must know in what situations to apply their conceptual and procedure knowledge. Many students do not recognize the connection between the base-10 system context and the standard algorithms when regrouping, especially when adding and subtracting multi-digit numbers (Thanheiser, 2012). Covariant analyses showed significant correlations between failure to regroup, decimals and the concept of zero errors. Identifying students’ misunderstanding and/or lack of knowledge of a mathematical concept and providing interventions that address those areas may offer struggling students opportunities to grow as their same-age peers without mathematics difficulties.

Difficulty with the conceptual knowledge of addition was observed in the answers for problems in which the sum of the given numbers did not result in a larger number than the digits given. The misunderstanding of the concept of subtraction was revealed in responses in which students subtracted a larger number from a smaller one. The analyses of errors suggest a relationship between conceptual and procedural knowledge (Fuson and Kwon, 1992; Schneider and Stern, 2010; van Lehn, 1982) and strategic knowledge (Thanheiser, 2012). This information is helpful to teachers because they are able to better examine students’ levels of understanding, ability to integrate procedural and conceptual knowledge, and application of strategies used to solve calculation problems. Using the data from students’ performance, educators can make informed decisions about effective instruction that addresses the individual needs of their students (Baroody et al., 2007).

Findings from the hierarchical regression analyses indicate that students’ performance differences emerged as a function of error types which indicate students’ type of difficulties. As shown in Tables VI and VII, error patterns in addition and subtraction calculation problems accounted for significant variance in students’ scores in addition and subtraction calculation problems. These results underscore the need for analysis of students’ type of errors. Certain types of errors (e.g. failure to regroup) indicate students’ difficulties with a particular kind of conceptual and procedure knowledge (e.g. base-10). For that reason, educators must thoroughly assess each student’s strengths and weaknesses before deciding which area or type of instruction a student needs.

Results from this study add to the available literature on mathematics development in several ways. Grade level significantly affected students’ performance, suggesting an association with pre-skills to calculation (e.g. place value, counting strategies). These
findings indicate that early intervention on students’ understanding, procedural and strategic knowledge of pre-skills may be an effective way to eliminate the occurrence of certain type of errors in the upper grades. Some researchers (Hill et al., 2005; Thanheiser, 2009) assert that teachers must have deep and detailed knowledge of mathematics to be able to explain adequate mathematical concepts (e.g. algorithm) to their students. This type of teacher knowledge, mathematical knowledge for teaching, has been shown to predict student success in mathematics (Ball et al., 2005; Hill et al., 2005). Professionals who work to construct curriculum and programs that involve the acquisition and teaching of mathematics knowledge and skill may find this information useful to provide equal opportunities to students whose evidence varied in mathematics knowledge and skills.

Limitations
There are several limitations of this study that should be noted. First, is the use of a convenient sample. We assessed students from schools who welcomed our project. Second, all schools were located in the northern region of Portugal and only two of the seven schools were considered suburban schools. The other five were rural schools. Future research would be enhanced by including participants from different parts of Portugal and students from more diverse settings (e.g. urban). The same is true for the socioeconomic background of students; our participants were from medium to low socioeconomic upbringings.

A third limitation of the study was the administration of only ten calculation problems representing four different grade levels. Another possible limitation was the administration of the assessment instrument during the first two weeks of the beginning of the school year. Our first graders were just starting second grade, our second graders were starting third grade, etc. Although, we wanted to examine the maintenance of skills from one grade to another, students may have increased or decreased their mathematics performance during the summer (two-and-a half-month vacation). Administering the assessment tool at the end of the school year and again at the beginning of the following year would afford researchers the opportunity to measure students’ maintenance of skills over the summer.

Implications for practice
The findings of this study have practical implications beyond statistical significance. First, teachers responsible for instruction of mathematics need to assess students’ mathematical conceptual and procedural knowledge and skills. Error analysis can yield important information about students’ thinking, understanding and misconceptions (Busi and Jacobbe, 2014; Silver et al., 2009; Thanheiser, 2009). Second, misconceptions come from prior knowledge and errors are the result of their naïve concept (Luneta and Makonye, 2010). With students’ possible varying background knowledge and the earlier identified differences, the greater was the opportunity to promote student success in mathematics. Third, error analysis allows teachers to give specific corrective feedback to students and address individual student’s misconceptions (Riccomini, 2005). Errors should be viewed as learning opportunities and used to monitor student progress. Another critical implication for practice is the importance of diagnostic assessment provided by error analysis. Diagnostic assessment of student error patterns gives teachers the opportunity to deliver meaningful instruction to every student (Horn et al., 2015) and promote equality among a diverse population of students.

Conclusion
An international study involving the analysis of mathematics errors in other countries serves to inform classroom assessment and intervention practices. By examining the
addition and subtraction calculation errors of elementary-aged Portuguese students, we identified the specific type of errors students make when adding and subtracting numbers. Without a thorough evaluation of students’ error types, teachers will be unable to develop effective instruction. Assessments of student learning should be translated into diagnostic information to support instructional decision-making and, in turn, student learning (Horn et al., 2015).

In the USA, mathematics instruction has suffered because of the limited amount of information on the development of students’ deep understanding of mathematical concepts (Silver et al., 2009). Learning mathematics with understanding requires teachers’ ability to identify specific students’ type of errors that reflect students’ lack of understanding of conceptual knowledge. Not surprisingly, some authorities (Ball, 2000; Ball et al., 2008; Hill and Ball, 2004) have identified teachers’ knowledge of common student misconceptions and errors as one of the six constructs of mathematics teacher effectiveness to foster student learning. Teachers should be aware of high probability errors at each grade level. Drawing from accumulated research and results of the present study affords the importance of quality instruction for all students to be successful in the important area of mathematics. Quality instruction leads to student learning, equality in education and mathematics success (Kersting et al., 2012).

References


Further reading


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