

## Pre-service elementary school teachers’ ideas about fractions

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**Abstract:** This paper focuses on a study carried out to analyse pre-service Primary school teachers’ knowledge about fractions. It addresses three questions: 1) What ideas do Pre-service Primary School teachers have about fractions? 2) How do these teachers understand ordering and equivalence of fractions? And 3) How these teachers understand the representation of fractions? A survey was carried out with 86 pre-service Primary school teachers from several parts of Portugal. The survey comprised 52 questions related to the concept of fraction, the invariants of rational numbers (ordering and equivalence), fraction representations, and the interpretations of fractions. Results revealed that Pre-service Primary school teachers have several difficulties concerning fractions. Several difficulties were identified in these pre-service teachers: some do not recognise the need to divide the whole into equal parts in fractions representations; they find difficult to relate parts of unit to 2 or more units; the property of the density of rational number set is not recognised by students, many believe they can count the number of fractions between 0 and 1; many cannot find the double of a given fraction; and many of them revealed difficulties with the interpretations of fractions, in spite of having to teach them in a close future.

**Résumé:** Cet article se concentre sur une étude réalisée pour analyser les connaissances préalables des enseignants du primaire sur les fractions. L’étude aborde trois questions: 1) Quelles idées les enseignants des écoles primaires ont-ils des fractions? 2) Comment ces enseignants comprennent l’ordre et l’équivalence des fractions? Et 3) Comment ces enseignants comprennent la représentation des fractions? Un sondage a été réalisé avec 86 enseignants du primaire de plusieurs régions du Portugal. L’enquête comprenait 52 questions liées à la notion de fraction, les invariants des nombres rationnels (ordre et équivalence), les représentations de fraction et les interprétations des fractions. Les résultats ont révélé que les enseignants de l’école maternelle ont plusieurs difficultés en ce qui concerne les fractions. Plusieurs difficultés ont été identifiées dans ces enseignants de pré-service: certains ne reconnaissent pas la nécessité de diviser l’ensemble en parties égales dans les représentations de fractions; Ils trouvent difficile de relier des parties de l’unité à 2 unités ou plus; La propriété de la densité de l’ensemble des nombres rationnels n’est pas reconnue par les étudiants, d’autres croient pouvoir compter le nombre de fractions entre 0 et 1; Beaucoup ne peuvent pas trouver le double d’une fraction donnée; Et beaucoup d’entre eux ont révélé des difficultés d’interprétation de fractions, en dépit d’avoir à leur enseigner dans un proche avenir.

### 1. Framework

This study aims to understand pre-service teachers’ knowledge of fractions. For that, it addresses three questions: 1) What ideas do future Primary School teachers have about fractions? 2) How do these teachers understand ordering and equivalence of fractions in different interpretations of fractions? And 3) How these teachers understand the representation of fractions in those interpretations?

The concept of fraction is considered fundamental for a successful and proper development of children’s mathematical thought. Nevertheless, is also one of the most complex concepts that children learn during the

elementary grades. Knowing the concept of fraction demands the understanding of the logical aspects of fractions (ordering and equivalence) and the ability to use distinct modes of representation, in different interpretations of this concept (Behr et al., 1983; Nunes, Bryant, Pretzlik, Wade, Evans & Bell, 2004; Mamede & Nunes, 2008).

Pre-service teachers must be competent in the domain of rational numbers in order to be able to develop fruitful practices with their primary school students. But are future teachers adequately prepared to teach rational numbers to their students? Little research has been developed in order to explore this issue regarding the Portuguese reality.

In the domain of rational numbers, the Portuguese mathematics curriculum for primary school demands the use of mathematical and pedagogical knowledge quite challenging for teachers. They are supposed to be fully acquainted with the representation, ordering and equivalence of fractions, as well as with different interpretations of fractions. Teachers are also supposed to help students to establish the link between different representations of rational numbers (fractions, decimals, percentage), and compute with fractions and decimals.

## **2. Teachers' difficulties with fractions**

Literature suggests that very often teachers have the same difficulties of their students and have the same misconceptions (see Lamon, 2003). Previous research conducted with Portuguese elementary school teachers has revealed that teachers have several difficulties concerning the teaching of rational numbers. These difficulties comprise conceptual and didactical features. Pinto and Ribeiro (2013) analysed 27 pre-service teachers' mathematical knowledge about fractions – interpretations of fractions, representation of fractions, the concept of unit, ordering and equivalence of fractions, and the density of the rational number set. Results indicate that most of the teachers felt comfortable only with part-whole interpretation of fractions; only 50% of them could identify the unit when a part of the whole was given; 43% and 97% of teachers revealed problems with equivalence and ordering of fractions, respectively; and 73% of them possess erroneous ideas concerning the density of the rational number set.

Concerning the didactical issues, Cardoso and Mamede (2013) interviewed two primary school teachers to understand how they explored the concept of fractions with their 3<sup>rd</sup> graders, when quotient interpretation is involved, as recommended by the curriculum guidance. Their results showed that teachers did not possess any lesson plan to teach fractions, because usually they do not teach them. Teachers believe that their students were not capable of learning all the content included in the curricula, nevertheless they had never tried to teach them fractions before. Perhaps the devaluation of students' abilities could be hiding some of the teacher's difficulties with fractions. Indeed, one of the teachers could not identify a pictorial representation of a fraction in a quotient interpretation context. These authors pointed out that the fragility of teachers' knowledge regarding these issues compromises their teaching of rational numbers (see Cardoso & Mamede, 2013; Pinto & Ribeiro, 2013), and consequently, the development of students' rational number sense.

In this scenario, it becomes of utmost importance to have an insight into pre-service primary school teachers' preparation to teach fractions. The identification of pre-service teachers' difficulties is relevant to improve the future teaching and learning of rational numbers, as it interferes with the mathematisation process of the students.

## **3. Methods**

A survey was conducted with 86 pre-service elementary school teachers (mean age: 22 years, 5 months), from several parts of Portugal. The survey comprised 52 questions related to fractions (18 related to the concept and properties of fractions; 13 concerning the invariants of rational numbers - ordering and equivalence; 13 about the representation of fractions and concept of unit; and 8 about the interpretations of fractions).

#### 4. Results

The mean of correct responses was 31,5 (standard deviation of 6,56). Table 1 resumes the means and standard deviation of the proportions of pre-service teachers' correct responses.

Type of questions	Mean (standard deviation)
Concept of fraction	.68 (.17)
Invariants of fractions	.60 (.22)
Representation of fractions and the concept of unit	.58 (.21)
Interpretations of fractions	.62 (.15)

Table 1. Mean (standard deviation) of proportions of correct responses by each Type of questions.

There were 18 questions concerning the concept of fractions, but only 2 teachers (2,3%) could succeed in all of them; only 31,4% of the teachers could succeed in at least 75% of the questions.

In one of the questions regarding the concept of fractions, most of the participants presented an incomplete answer when asked to identify pictures that could represent  $1/3$ , as those of the Figure 1.

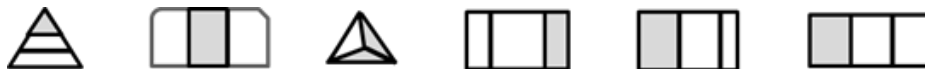


Figure 1. Pre-service teachers were asked to circle the pictures that represent  $1/3$ .

Only 12 teachers (14%) presented a total correct answer; 70 (81,4%) presented an incomplete answer considering only the last picture; and 4 presented responses indicating that the size of the part into which the whole is divided is irrelevant to represent fractions. In another question related to the concept of fraction in which it was asked “How many ninth are there in 3 units?”, 31 participants (36%) answered correctly, 18 teachers (20,9%) presented no answer, 12 participants (14%) referred ‘ $27/9$ ’, and almost 29% gave other incorrect answers. When asked “How many fractions are there between 0 and 1? Justify your answer.”, only 36 teachers (41,9%) answered correctly, 37 presented no answer (43%), and 13 presented an incorrect response (15,1%). Pre-service teachers' justifications reveal that some of those who gave a correct answer could not present a written justification, only 13 gave an explanation based on the property of the density of rational number set (15,1%); 47 presented no justifications at all (54,7%), and the remaining justifications were incorrect.

When asked whether “ $4/8$  is 2 times bigger than  $2/4$ ”, 34 participants (39,5%) believed that that was true, but 52 considered the sentence false; when they were asked whether “ $4/8$  results from multiplying  $2/4$  by 2”, 40 pre-service teachers (46,5%) could not recognise it as a false sentence. This reveals some of the difficulties of prospective teachers with fractions. It is not expected that future elementary school teachers could accept  $4/8$  as the double of  $2/4$ .

Regarding the invariants of fractions, 13 questions were presented to the pre-service teachers concerning ordering and equivalence of fractions, but only 1 teacher (1,2%) could succeed in all of them, and 10 teachers (11,6%) succeeded in 12 questions. Only 9,75% of the teachers could succeed in at least 75% of the questions.

Concerning the invariants of fractions, in the problem of the Figure 2, teachers had to compare fractions in each situation. In the case A) 56 teachers (65,1%) identified the correct fraction, 12 gave a wrong answer (14%) and 18 students (20,9%) could not answer; in the case B) 47 students (54,7%) identified the correct fraction, 14 gave a wrong answer (16,3%) and 25 students (29,1%) could not answer; in the case C) 52 students (60,5%) identified the correct fraction, 9 gave a wrong answer (10,5%), and 25 students (29,1%) could not answer; in the case D) 49 students (57%) identified the correct fraction, 14 gave a wrong answer (16,3%) and 23 students (26,7%) could not answer.

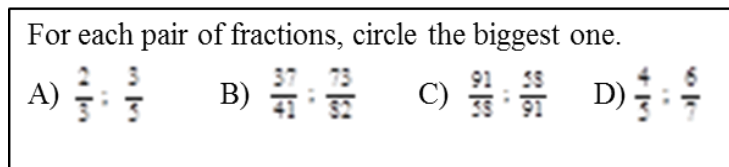


Figure 2. A problem of ordering of fractions presented to students.

In the case A) 30 pre-service teachers were not able to compare correctly  $\frac{2}{3}$  and  $\frac{3}{5}$ , which are fractions that children use in the 3<sup>rd</sup> grade; probably, for the same reason 37 teachers failed in the case D), comparing  $\frac{4}{5}$  and  $\frac{6}{7}$ . In some of the correct answers of these questions, students had to reduce fractions to the same denominator to compare them in order to produce a correct answer. Nevertheless, 4<sup>th</sup> graders cannot use this procedure to compare fractions. In case B), 47 pre-service teachers (54,7%) presented a correct answer, but 39 failed to compare these fractions (45,4%), being unable to recognise, for instance, that one denominator is the double of the other, but the correspondent numerators are not. In the case C), 52 teachers (60,5%) succeeded, but 34 (39,6%) could not realise that one of the fractions was smaller than 1 and the other was bigger. This lack of knowledge when dealing with fractions compromises the implementation of meaningful classes about rational numbers conducted by these pre-service teachers. More results will be presented in the conference regarding pre-service knowledge about fractions.

## 5. Final remarks

Pre-service elementary school teachers need to improve their ideas about fractions. Several difficulties were identified in these pre-service teachers: some do not recognise the need to divide the whole into equal parts in fractions representations; they find difficult to relate parts of unit to 2 or more units; the property of the density of rational number set is not recognised by students, many believe they can count the number of fractions between 0 and 1; many cannot find the double of a given fraction; and many of them revealed difficulties with the interpretations of fractions, in spite of having to teach them in a close future. Discussion and the educational implications of our findings will be presented in the conference and in the version of the paper for the proceedings.

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