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A comprehensive survey of the analytical, numerical and experimental methodologies for dynamics of multibody mechanical systems with clearance or imperfect joints

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Abstract

A comprehensive survey of the literature of the most relevant analytical, numerical, and experimental approaches for the kinematic and dynamic analyses of multibody mechanical systems with clearance joints is presented in this review. Both dry and lubricated clearance joints are addressed here, and an effort is made to include a large number of research works in this particular field, which have been published since the 1960's. First, the most frequently utilized methods for modeling planar and spatial multibody mechanical systems with clearance joints are analyzed, and compared. Other important phenomena commonly associated with clearance joint models, such as wear, non-smooth behavior, optimization and control, chaos, and uncertainty and links' flexibility, are then discussed. The main assumptions procedures and conclusions for the different methodologies are also examined and compared. Finally, future developments and new applications of clearance joint modeling and analysis are highlighted.

Keywords: Clearance joints, Dry models, Lubrication models, Analytical formulations, Numerical approaches, Experimental investigations

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1. Introduction

Mechanisms, as multibody systems, are made of several mechanical components and inter connections, which can be classified into two major groups, namely links; i.e., bodies with a convenient geometry; and kinematic joints, which introduce some kinematic constraints or restrictions on the relative motion between adjacent links [1]. Usually, the links are modeled as rigid or flexible bodies [2], while joints are mathematically represented by a set of kinematic constraints [3]. The functionality of a mechanical joint relies upon the relative motion allowed between the connected links. The existence of a gap, that is, a clearance between the mating parts in a physical system leads to several complex dynamics phenomena such as surface contact, shock transmission and the development of different regimes of friction, lubrication and wear. No matter how small that clearance is, it can lead to vibration and fatigue phenomena, lack of precision, or even chaotic overall behavior. Therefore, in order to achieve the required performances of the mechanisms with some tolerances or clearance joints, it is quite important to quantify the effects of the joint clearances on these systems' dynamic responses [4].

The traditional analysis of mechanisms has inherent limitations since the kinematic joints are formulated without taking into account their physical characteristics, but instead as ideal kinematic constraints. Therefore, factors such as tolerance [5], clearance [6], friction [7], local elastic deformation [8], lubrication [9], and wear [10] are not considered. In reality, mechanisms are connected by joints, in which some clearance is always present. That clearance is indispensable to permit a correct functioning of the pair elements. The clearances cause collision between the elements that compose the clearance joints, and therefore, contact-impact forces are developed and transmitted throughout the system. It can be stated that joint clearances, dry or lubricated, do not impose any kinematic constraint, but instead they impose force constraints. From the modeling point of view, joints with clearance can be defined as force-joint elements instead of kinematic joints [11].

The problem of modeling and simulating joints with clearance in mechanical systems is a quite fertile research subject in different fields that has attracted the attention of many authors over the last decades, such as vehicle steering suspensions and bushing joints [12, 13], robotic and parallel manipulators [14], space deployable systems [15], ball bearings [16, 17] and human joints biomechanics [18, 19]. This significant interest led to the development of relevant work and even to the publication of a good number of dissertations entirely devoted to this subject, namely those by Seneviratne [20], Soong [21], Deck [22], Gu [23], Ravn [24], Schwab [25], Flores [26], Pedersen [27], Koshy [28], Mukras [29], Malça [30], Baiceanu [31], Renani [32].

From the manufacturing and operating points of view, the existence of a gap in the joints is necessary and unavoidable, because of manufacturing and assembly tolerances, and more importantly, to allow some flexibility and permit the relative motion between adjacent parts [33-51]. If there is no lubricant in the mechanical joints, direct collisions take place in the mechanical systems causing vibration and fatigue phenomena [22, 25, 52-58]. In some applications, the joints are designed to run with some fluid lubricant, with the purpose of reducing friction, wear and to provide load capacity to keep the joint elements apart [9, 44, 59-66].

Mechanisms with rigid and flexible components and with non-ideal joints have been treated in the past [67-71]. Studies have considered joint compliance and friction but without clearances [72-77]. Methods for modeling joint connections and external impacts using the coefficient of restitution and momentum-balance/impulsive approaches have also been proposed in the literature [78, 79]. While such methods offer the advantage of relatively low computational effort over techniques that explicitly model the joint impact, they are not valid for continuous contact and do not explicitly provide values of these forces, which are important from the design point of view.

The degradation of the performance of mechanical systems with contact-impact events due to the existence of clearance joints has been recognized for several decades. Goodman [80] and Kobrinskiy et al. [81] have been pioneers in investigating mechanical systems with clearances joints. Ever since, many researchers have devoted their efforts to the problem of modeling and simulating mechanisms with clearance joints. Some of the fundamental mathematical and experimental approaches of clearance joints for mechanism dynamics have also been summarized in several works published over the last decades, such as those by Haines [82], Flores et al., [83], Liu and Yu [84], Muvengei et al., [85], and more recently Wang and Liu [86]. Gummer and Sauer [87] presented an overview of several models for simulating the planar slider-crank mechanism with a revolute joint with clearance using the commercial software RecurDyn.

In the present review work, the methodologies based on the collection of approaches available in the literature for the dynamic analysis of multibody systems considering realistic joint characteristics, namely, the joints with clearance and lubrication, are presented. For the case of the joints with clearance modeled as a dry contact, the technique using a continuous approach for the evaluation of the contact force can be applied, in which the energy dissipation is in form of hysteresis damping. The nature of contact phenomenon is its inelastic nature, and not all the kinetic energy is recovered after impact. In general, some energy is dissipated in the process of contact/impact, and hence a hysteresis is formed representing the progression of the contact force during the contact period. The hysteresis form of the contact force represents the nature of energy dissipation during the contact period. At relatively low impact velocities, for which the impact speeds are much smaller that the speed of the propagation of the elastic waves across the bodies in contact, the energy is dissipated internally by the bodies in the form of heat, represented by internal damping. The term "hysteresis damping" refers to the form of energy dissipation for which the trend of the contact forces during the compression and that of restitution is different, and no relative permanent deformation or indentation is left on the bodies after separation. In turn, the friction forces are calculated using a Coulomb-based friction law or other friction models. For the lubricated joint case, the hydrodynamic theory for dynamically loaded journal-bearings is used to compute the forces generated by lubrication action. In a simple way, the forces built up by the lubricant fluid are evaluated from the state variables of the system and included into the equations of motion of the multibody system. Both squeeze-film and wedge-film hydrodynamic effects are included in the dynamically loaded journal-bearings models. The transition models, which combine the squeeze action and the dry contact model, are also analyzed in this paper. This model considers the existence of the lubrication during the free flight trajectory of the journal, prior to contact, and the possibility for dry contact under some conditions, seems to be well suited to describe the physics of the revolute joints with clearances in mechanical systems. In summary, in multibody mechanical systems, a clearance joint or lubrication joint does not produce any kinematic constraint like the ideal joint. Instead, it acts in a similar way to a force element producing time-dependent forces. These forces are evaluated from the state variables of the joint elements and included into the equations of motion of the multibody system.

The topic of joint clearances and related studies has seen a tremendous growth in the last couple of decades, as there seems to be a number of advances every year in this research topic. Thus, this review includes the state-of-the-art on the subject of clearance joint modeling in multibody mechanical systems. This paper is divided into six main sections. While Section 1 presents a general and brief introduction of mechanisms with clearance or imperfect joints, Sections 2 and 3 deal with the main issues related to the kinematic and dynamic analysis of dry and lubricated joints for planar and spatial systems. A comprehensive review of the relevant works on the different types of clearance joints is also presented in section 2. In Section 4, the some of the most important experimental investigations on mechanical systems with clearance joints are presented and discussed. Section 5 presents and analyzes other relevant issues associated with clearance joints, such as chaos, system's control, and wear. Finally, the work is summarized in the last section and the future directions for research in this area are outlined.

2. Models for mechanisms with dry clearance joints

2.1. Planar revolute joint with clearance

In 1967, Chace [88] highlighted the lack of investigation on the performance of mechanisms with clearance joints. Bagci [89] presented a study on the friction and damping at joints for dynamic analysis, but without clearances at the joints, in which a four bar linkage and a slider-crank mechanism were analyzed. Lee and Wang [90] also demonstrated the importance of proper modeling the contact-impact events in clearance joints. Dubowsky and Morris [91] and Thompson [92] discussed the consequences of the intra-joint contact-impact forces in terms of noise and durability of the parts. Garrett and Hall [33] described a mathematical technique for optimizing a four-bar linkage with revolute joint clearances. The effects of the clearances and tolerances were presented in terms of mobility bands. In this investigation, the dynamic effects were neglected [34]. Lee and his co-authors [36, 37] presented a method for allocating tolerances on the link lengths and clearances in dynamic planar linkages. The method included a general approach for sensitivity analysis. Kolhatkar and Yajnik [93] analyzed a four-bar mechanism with revolute clearance joints with special emphasis on the output errors. It was demonstrated that the maximum output error took place when the vectors representing the non-ideal revolute joints were parallel to the coupler. Mansour and Townsend [94] considered the four-bar mechanism to investigate the effects of clearance size, driving speed and friction coefficient on the impact spectra in high-speed mechanisms. Design guidelines for analyzing wear, components durability and noise in mechanisms with dry revolute joints with clearance were proposed by Townsend and Mansour [95]. Dubowsky and Gardner [96] also studied the effects of clearances and flexibility of links on the level of stresses in joints of high-speed linkages. These authors demonstrated that the links flexibility tends to reduce the joint stresses. Winfrey et al. [97] were the pioneers in investigating mechanisms with flexible links and clearances. A cam-follower valve train mechanism was utilized as an example of application. More recently, Erkaya and Uzmay [98] studied the influence of joint clearances on the levels of vibration and noise in slider crank mechanism with two non-ideal joints.

A joint with clearance can be included in a multibody mechanical system much like a revolute joint. Figure 1 shows a typical connection with revolute clearance joints found in planar multibody systems, where the clearance size is exaggerated for the illustration purpose only. When there is no lubricant in the joint, contact-impacts events can take place and the corresponding impulsive forces are transmitted throughout the mechanism parts. These impacts and the eventual continuous contact can be characterized by a force model, which accounts for the geometric and material characteristics of the joint components.

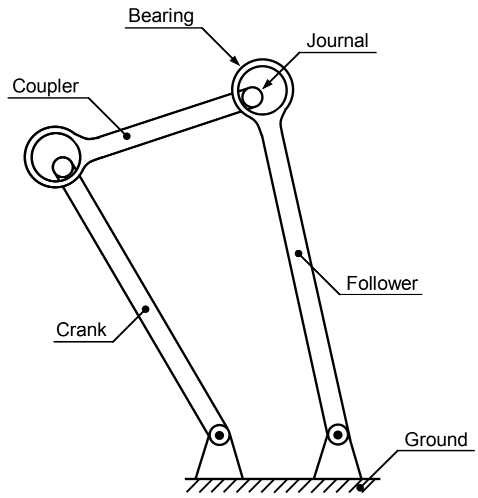

Fig. 1 Typical connection with planar revolute clearance joints found in a four bar mechanism [11].

Flores and his co-workers [99, 100] described three different ways for modeling revolute joints with clearance, namely, massless link approach, spring-damper approach, and momentum exchange approach. Figure 2 shows these three modeling approaches. Earles and Wu [101, 102] considered the massless link approach to predict the occurrence of contact loss in planar revolute joints. The massless link has a length equal to clearance size, as the dynamic simulations are restricted to the range motion that starts when the contact between the joint parts is terminated. More recently, based on neural network and genetic algorithms, Erkaya and Uzmay [103, 104] investigated the influence of joint clearances on the mechanism path generation and transmission angle. In this case, the clearance joints have been modeled as massless virtual links. Furuhashi and his co-authors [105-108] also investigated the problem of clearance joints using the concept of massless link. Dubowsky [109] studied the dynamic response of clearance joints in planar mechanical systems in which linear springs and dampers are used to model the elasticity of the contacting surfaces. Ravn [24] used the colliding approach to model the journal and bearing parts of a planar revolute clearance joint as two colliding elements. Claro and Flores [110] investigated the performance of a planar slider-crank mechanism with a revolute clearance joint between the connecting rod and slider.

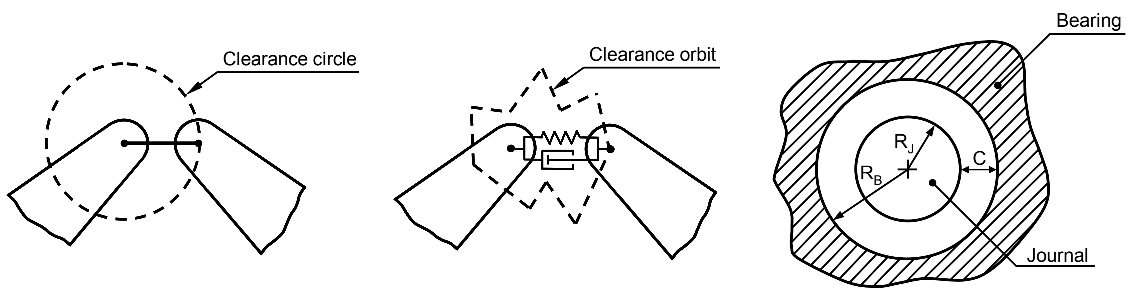

Fig. 2 Three different approaches for modeling revolute joints with clearance: (a) Massless link method; (b) Spring-damper method; (c) Colliding model formulation [26].

Dubowsky and Freudenstein [111, 112] proposed an impact pair model to predict the dynamic response of an elastic mechanical joint with clearance. In their model, springs and dashpots were arranged as Kelvin-Voigt model [78, 79]. This approach, valid for frictionless and unlubricated joints, was subsequently extended by Dubowsky and his co-workers [113-117]. Dubowsky [118] demonstrated that a simple impact could significantly affect the dynamic characteristics of mechanical systems with clearances. Moreover, Dubowsky [118] showed how clearances could interact dynamically with machine control systems to destabilize and produce undesirable performance. Dubowsky et al. [114] applied the perturbation method, treating the angular motion caused by clearances and elastic deformations as small quantities and neglecting the high order and high-frequency responses of the small variables. This method simplified the dynamics equations, but was not suitable for the conditions of large elastic deformations or large clearances. Dubowsky and Moening [119] obtained a reduction in the impact force level and noise in mechanisms with joint clearances by introducing the flexibility of the links. Kakizaki et al. [120] presented a model for the dynamics of robotic manipulators with flexible links and joint clearances, where the effect of the clearance was considered to control the robotic system.

In 1980, Earles and Kilicay [121] presented a design criterion for maintaining contact at plain bearings. The method was successful applied to four-bar and five-bar mechanisms having two revolute joints with clearances. In their work, the authors improved the methodology previously presented by Wu and Earles [102] to increase the computational efficiency and its generality. Bahgat and his co-authors [122-124] developed a general approach for the dynamic analysis of planar mechanisms with multiple clearance joints. The method proposed was effective in predicting the occurrence of contact loss between journal and bearing surfaces. The planar four-bar linkage with three clearance joints and the quick-return mechanism with seven clearance joints were considered as demonstrative application examples. Megahed and Haroun [125] studied the kinematic and dynamic behavioral performance of a planar slider-crank mechanism with one and two clearance revolute joints when working in vertical and in horizontal planes. From the simulation results, they observed that for the two-clearance-joint mechanism, the maximum contact force always appeared at the clearance joint close to the input link, and the number of actuating torque peaks would increase with the crank speed especially in horizontal plane motion, which indicated that the service life of mechanism joints and actuators could be extended if the mechanism worked in a vertical plane. Shin and Kwak [126] presented a general method for preventing the contact loss in revolute clearance joints. The planar slider crank mechanism was utilized to demonstrate the effectiveness of the analytical method developed. Seneviratne and his co-workers [20, 53, 127, 128] investigated the chaotic behavior exhibited during contact loss in a revolute clearance joint of a planar four-bar mechanism. For this purpose, the massless link approach was utilized to model the clearance joints. Dubowsky and Gardner [96] pointed out that the inclusion of links flexibility did not influence the contact loss in clearance joints, but reduced in a significant manner the level of impact forces produced. Bengisu et al. [129] proposed an accurate method for predicting the contact loss or separation in joint, which was based on the ideal and frictionless joint analysis.

In 1988, Rhyu and Kwak [130] proposed an optimization procedure for the design of mechanisms considering both tolerances on the link lengths and clearances in the joints. The multi-objective problem included two objective functions, namely the tolerance widths and the clearance sizes. The methodology was developed and implemented for a planar four-bar mechanism several revolute clearance joints. Muvengei et al. [131] investigated the effects of clearance size, contact models and the input crank speed on the dynamic response of a planar rigid slider-crank mechanism with a frictionless revolute clearance joint. They found that changing the driving speed of a mechanism could change the dynamic response of the mechanism from either periodic to chaotic, and the responses of the joint between the slider and connecting rod was more sensitive to the clearance size than the joint between the crank and the connecting rod. Muvengei et al. [132] further studied the influence of the friction on the dynamic responses of the same slider-crank mechanism by using the LuGre friction force model, which could capture experimentally observed phenomena such as the sliding displacement and the stick-slip motion associated with the Stribeck effect. Again, through simulating the mechanism with two revolute clearance joints, these authors [133] found that compared to the connection clearance joint far from the driving crank, the revolute clearance joint close to the driving crank would generate the larger contact forces, which would further lead to large wear rate of the revolute clearance joint close to the driving crank. Yang et al. [134] extended the vector form intrinsic finite element method proposed by Ting et al. [135, 136] and Shih et al. [137] into the dynamic analysis of a planar rigid four-bar mechanism with multiple revolute clearance joints.

Zhao et al. [138] utilized the ADAMS software to perform a study on the dynamic behavior of reciprocating compressor system with clearance joints. In this research work, the effects of cylinder pressure load, clearance size, driving speed and flexibility of the links were investigated. Feng et al. [139] developed an optimization method to control the change of inertia forces by optimizing the mass distribution of moving links in planar linkages with clearances at joints. Innocenti [140] presented a method for analysis of mechanical structures with revolute clearance joints. Orden [141] presented a methodology for the study of typical smooth joint clearances in multibody systems. This proposed approach took advantage of the analytical definition of the material surfaces defining the clearance, resulting in a formulation where the gap did not play a central role, as in standard contact models. This approach was demonstrated as an effective and efficient method in solving the equations of motion. The thermal effect of high temperature on mechanisms having clearance joints was studied by Bing and Ye [142]. They demonstrated that by increasing the combined clearance of the revolute joint, a favorable reduction of the friction torque between the bush and shaft could be obtained for normal operation conditions of a reheat-stop-valve mechanism. However, if the steam temperature reaches to 650 ºC, the friction torque between the bush and shaft elements becomes larger and the sticking phenomenon occurred.

Figure 3 shows the typical configuration of a revolute joint with clearance. The joint elements are the bearing and journal, with radii are R_B and R_J, respectively. The difference between the bearing and journal radii is the radial clearance, c. The existence of the clearance in the revolute joints allows two extra degrees-of-freedom; that is, the horizontal and vertical displacements and, consequently, the journal and bearing can freely move relative to each other. Figure 3 also shows the relative penetration or indentation between the journal and bearing when the two bodies contact-impact one another [4].

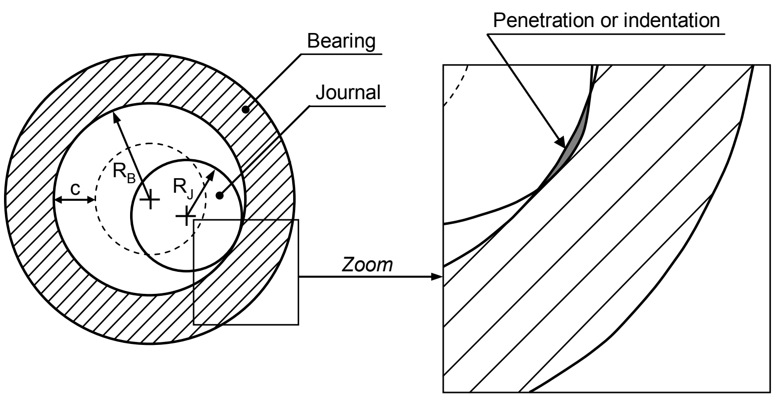

Fig. 3 Revolute joint with clearance, in which the clearance size is exaggerated for clarity [4].

Three different modes of journal motion inside the bearing can be considered, namely the contact or following mode, the free flight mode and the impact mode [11, 26, 94, 100, 143], as it is depicted in Fig. 4. In the contact or following mode, the journal and the bearing are in permanent contact and a relative sliding motion is assumed to exist. In this mode, the relative penetration varies along the circumference of the bearing. In practice, this mode ends when the journal and bearing separate from each other and the journal enters in free flight mode. In the free flight mode, the journal can move freely inside the bearing boundaries, that is, the journal and the bearing are not in contact and no reaction force is developed at the joint. In the impact mode, which occurs at the termination of the free flight mode, impact forces are applied and removed in the system. This mode is characterized by a discontinuity in the kinematic and dynamic characteristics and a significant exchange of momentum occurs between the two impacting bodies. At the termination of the impact mode, the journal can enter either in free flight or in follower mode. During the dynamic simulation of a revolute joint with clearance, if the path of the journal center is plotted for each instant, these different modes of motion can be easily identified [11]. Townsend and Mansour [95] modeled a four-bar crank-rocker mechanism with clearance as two sets of compound pendulums. Nonetheless they ignored the motion in contact mode entirely, a close succession of small pseudo-impacts was assumed for the simulation. Subsequently, Miedema and Mansour [143] extended their previous two-mode model, for the free flight and impact modes, to a three mode model in which a following mode was proposed. In their numerical simulations the following mode was always assumed to occur immediately after the impact mode, however, this is not frequently observed in practice [25, 26].

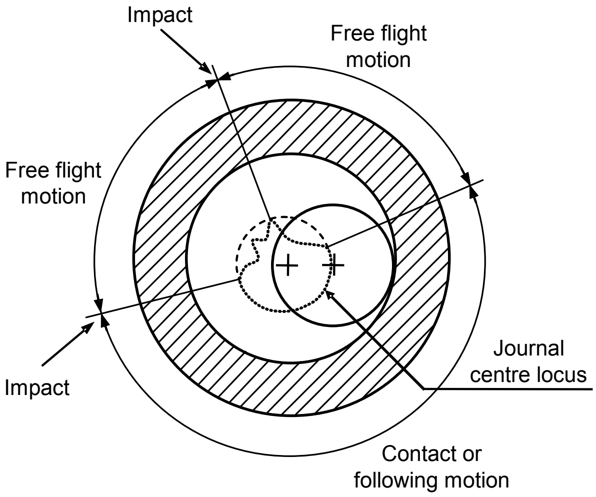

Fig. 4 Types of journal motion inside the bearing boundaries in a planar revolute clearance joint [11].

Due to the existence of the clearance in the joint, contact forces are generated at the joints when the journal reaches the bearing surface. In a simple manner, the joint behavior consists of periods of free flight motion followed by hard-surface to hard surface contact, typically metal-to-metal, when the journal and bearing collide to each other. In the first situation, there is no reaction force at the joint. In the second case, a contact force between the journal and bearing is developed and the direction of this joint reaction force is parallel to the relative position of the centers of the bearing and the journal, as shown in Fig. 5. The joint force magnitude upon the contact can be obtained from the Hertzian contact theory [144], which is based on the assumption that the dimension of the contact region between the journal and bearing is much smaller than the radius of each body. The Hertz contact theory is restricted to frictionless surfaces and perfectly elastic bodies. The contact force model proposed by Lankarani and Nikravesh [145] can account for both the elastic and damping effects. The damping effect is associated with the energy dissipated during the impact process, together with the dissipative effect associated with the Coulomb friction on the contact surface [146, 147].

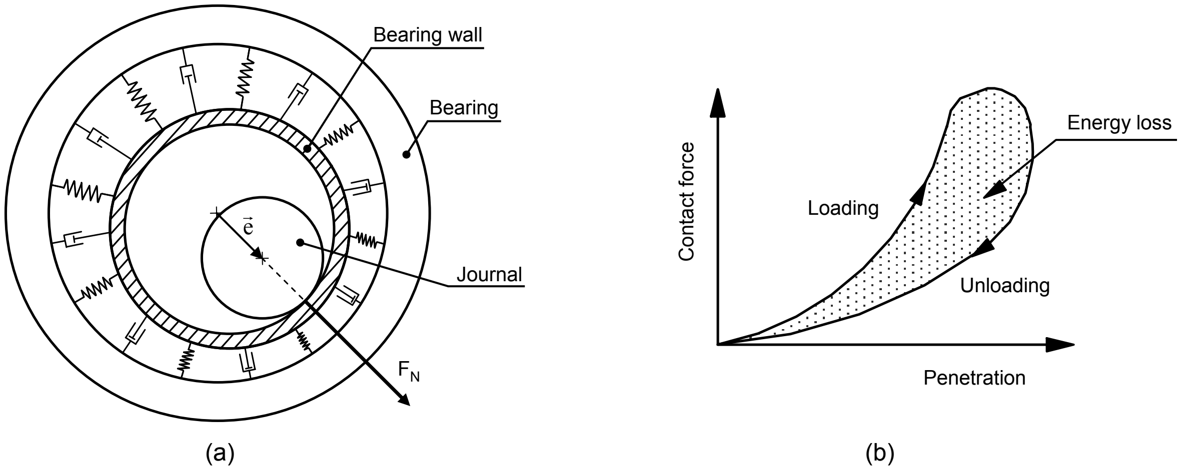

Fig. 5 (a) Contact between the journal and bearing represented by a nonlinear viscous-elastic force model; (b) Contact force versus penetration depth [146].

The relative penetration, δ, between the journal and bearing can be written as

$$\delta = e - c\,, \tag{1}$$

where e is the absolute eccentricity and c is the radial clearance.

The eccentricity is given by

$$e = \sqrt{\Delta X^2 + \Delta Y^2}\,, \tag{2}$$

in which ΔX and ΔY represent the horizontal and vertical displacements of the journal inside the bearing. These relative displacements are obtained from the global position vectors of the bearing and journal centers, respectively [83].

The journal is considered in free flight motion relative to the bearing if the relative penetration formulated by Eq. (1) is less than zero. Ideally, when $\delta = e - c = 0$, the bearing and the journal are in contact to each other. However, due to the computation round-off errors accumulation, a tolerance is introduced in order to accommodate for inaccuracies in the numerical results.

Since the most suitable and sophisticated contact-impact force models are dependent on the contact velocities, it is important to evaluate these velocities in order to account for the dissipative effects during the contact-impact process [145]. In particular, in the continuous contact force model it is necessary to calculate the relative velocity between impacting surfaces. The relative velocity vector between bearing and journal can be expressed

$$\mathbf{v} = \frac{d\left(\left\|\mathbf{r}_B^C - \mathbf{r}_J^C\right\|\right)}{dt}\mathbf{n} + \left(\left\|\mathbf{r}_B^C - \mathbf{r}_J^C\right\|\right)\left(\omega_B - \omega_J\right)\mathbf{t}\,, \tag{3}$$

where $\mathbf{r}_B^C$ and $\mathbf{r}_J^C$ are the position vectors of the contact points in the bearing and journal, $\mathbf{n}$ and $\mathbf{t}$ are the normal and tangential vectors, and ω_B and ω_J are the bearing and journal angular velocities [146].

The relative velocity between the contact points is projected onto the plane of collision and onto the normal plane of collision, yielding a relative tangential velocity, v_T, and a relative normal velocity, v_N, as follows

$$v_N = \mathbf{v}^{\mathrm{T}}\mathbf{n}\,, \tag{4}$$

$$v_T = \mathbf{v}^{\mathrm{T}}\mathbf{t}\,. \tag{5}$$

The dynamics of a dry journal bearing is characterized by two different situations. Firstly, when the journal and bearing are not in contact with each other, there are no contact forces associated with the journal-bearing. Secondly, when the contact between the two bodies occurs the contact-impact forces are modeled according to a nonlinear Hertz contact force law together with the Coulomb's friction law [148]. These conditions are

$$\begin{aligned} \mathbf{F} &= \mathbf{0} \quad && if \quad \delta < 0 \\ \mathbf{F} &= \mathbf{F}_N + \mathbf{F}_T \quad && if \quad \delta \geq 0 \end{aligned} , \tag{6}$$

in which, $\mathbf{F}_N$ and $\mathbf{F}_T$ are normal and tangential force components and $\mathbf{F}$ is the sum of contact-impact forces.

Various types of constitutive laws have been suggested in the literature, and one of the more prominent was proposed by Hertz [144]. This law is purely elastic in nature and cannot explain the energy loss during the impact process. Lankarani and Nikravesh [145] overcame this difficulty by separating the contact force into elastic and dissipative components as

$$F_N = K\delta^n + D\dot{\delta} , \tag{7}$$

where the first term represents the elastic force and the second term accounts for the energy dissipation. In Eq. (7), K is the generalized stiffness parameter [144, 149], D is a hysteresis damping coefficient and $\dot{\delta}$ is the relative impact velocity [145]. The contact force model proposed by Lankarani and Nikravesh is frequently written in the following form

$$F_N = K\delta^n \left[1 + \frac{3(1-c_e^2)}{4} \frac{\dot{\delta}}{\dot{\delta}^{(-)}} \right], \tag{8}$$

where c_e denotes the coefficient of restitution and $\dot{\delta}^{(-)}$ is initial impact velocity [145]. This contact force model that accounts for the energy dissipation is found to be satisfactory for general mechanical contacts. Shivaswamy [150] demonstrated experimentally that at low impact velocities, the energy dissipation due to the internal damping is the main contributor to energy loss. As stated earlier, at low impact velocities, impact energy is mostly dissipated in the form of internal damping or heat. At higher impact velocities, for which the impact velocity is not much smaller the speed of propagation of the elastic waves across the bodies in contact, the impact energy is mostly dissipated in the form of "permanent indentation" or "plastic deformation", as demonstrated experimentally by Shivaswamy [150]. It is true that some of the impact energy is dissipated in the form of stress wave propagation, as in the study by Wu et al. [169], but this dissipated energy is much smaller that the energy dissipated in the form of plastic deformation. In essence, studies by both Shivaswamy [150] and Wu et al. [169] mentioned the same phenomenon, and the terms "stress wave propagation" and "internal damping" represented the same form of energy dissipation. The force model given by Eq. (8) is only valid for low impact velocities, that is, speeds that are at the most one order of magnitude lower than the speed of elastic wave propagation [151].

It must be stated that there are other candidate contact force models to be utilized in multibody system contact problems. In particular, the interested reader can find relevant information on the impact between spheres in the publications by Yigit et al. [152, 153], Thornton [154], Gugan [155], Falcon et al. [156], Rigaud and Perret-Liaudet [157], Kuwabara and Kono [158], Minamoto and Kawamura [159], Kagami et al. [160], Pust and Peterka [161], Vu-Quoc et al. [162, 163], Burgin and Aspden [164], Bordbar and Hyppänen [165], Yoshioka [166], Villaggio [167], Ramírez et al. [168], Wu et al. [169, 170], Li et al. [171]

Shi and Polycarpou [172], Tatara and Moriwaki [173], Tatara [174], Gonthier et al. [175], Qin and Lu [176], Lankarani and Nikravesh [177], and Garib and Hurmuzlu [178]. In some models, the dissipative term is dependent on empirical parameters that characterize the contact zone, namely in the models proposed by Kuwabara and Kono [158], Tsuji et al. [179], and Bordbar and Hyppänen [165]. Other approaches exhibit discontinuities in the contact force evolution [180], and problems of consistency of units [181]. There are, however, a set of contact force models where the hysteresis damping factor is expressed in terms of the local contact properties, contact geometry and contact kinematics, making them appropriate for multibody dynamics simulations [90, 145, 182-190].

The contact force model proposed by Lankarani and Nikravesh (L-N) has extensively been utilized to study the dynamics of mechanisms with planar revolute clearance joints. For instance, Flores [146] considered the L-N force model to perform a study on the quantification of the influence of the system parameters, such as the clearance size, the input crank speed, and the number of clearance joints, on the nonlinear dynamics of a planar rigid crank-slider mechanism with multiple revolute clearance joints. It was found that with a small change in one of these parameters the response of the system could shift from chaotic to periodic and vice versa. Also using the L-N continuous contact force model and a modified Coulomb law by Ambrósio [147], Flores et al. [148] studied the influence of the clearance size and the friction coefficient on the dynamic response of planar rigid multi-body systems with revolute clearance joints. Through the use of Poincaré maps, they observed that the simulated system could show a periodic motion at certain clearance sizes and friction coefficients. In turn, Xu [191] analyzed the impact dynamics between a sliding-crank mechanism containing a rolling ball bearing joint and a target rigid body with an elastic support. In this work, the ball rolling, sliding and friction in the rolling path were all neglected. Xu et al. [192] proposed a method to simulate the planar rigid slider-crank mechanism with noncircularity clearance revolute joint between the crank and the connecting rod. According to this investigation, some potential contact points were used to approximate the noncircularity bearing inner boundary, and the contact force between these discrete points and the journal were evaluated according to the L-N contact model. However, the number of discrete points of the bearing inner boundary affected the result accuracy and computation efficiency. Bai et al. [193], based on the L-N contact force model studied the effects of the joint clearance on the dynamics of the dual-axis positioning mechanism of a satellite antenna. By a comprehensive study, Flores et al. [194] concluded that compared to the linear Kelvin-Voigt contact model and the Hertz model, the results obtained by using the L-N continuous contact force model were more accurate. More recently, Li et al. [195] established a planar rigid-flexible dynamics model of the spacecraft with large deployable solar arrays and multiple clearance revolute joints. The L-N continuous contact force model was also used to evaluate the normal contact force. They found that the joint clearance intensified the satellite yaw due to the impact at clearance joint and lagged the vibration of spacecraft solar arrays, especially when the panel flexibility was considered.

One difficulty associated with the L-N model is that it can lead to contact forces much lower (especially for lower clearance values [196]) than those predicted by the Johnson model [197], which has been experimentally validated. Flores et al. [186] also found that the L-N contact force model was only valid for high values of the coefficient of restitution (close to unit). By deriving a new hysteresis-damping factor for the Hunt and Crossley contact model [181], Flores and his co-authors [186] developed a new continuous contact force model applicable for both soft and hard contacts. Based on the Hertz contact equations, Hu and Guo [198] proposed a dissipative sphere to sphere contact force model, in which the energy loss difference between the compression phase and restitution phase of the contact process was considered. The numerical results indicated that the model was effective for the entire range of the coefficient of restitution (0-1). Bai and Zhao [199, 200] presented a new hybrid contact force model, which combined the L-N continuous contact model and an improved Winkler elastic foundation model. Using this model they studied the dynamic characteristics of mechanism with revolute clearance joint. Recently, Wang et al. [201] proposed a new nonlinear contact force model, which was a combination of a nonlinear contact law with nonlinear contact stiffness, and accounts for the axial dimension of bearing. The numerical results obtained by the proposed model were validated by the experimental results obtained by Flores et al. [202]. It was observed that the new nonlinear contact force model could effectively describe the contact force of planar clearance revolute joints for both lower and higher coefficient of clearance size, restitution coefficient, and initial velocity. Further, Wang et. [203] studied the effects of the restitution coefficient and material characteristics on the dynamic response of a planar rigid slider-crank mechanism with a revolute clearance joint between the connecting rod and slider. Xu et al. [204] studied the dynamic characteristics and motion consistency of a planar rigid parallel mechanism with two dry friction revolute clearance joints.

Ma et al. [205] presented a hybrid contact force model based on the L-N contact force model and the elastic foundation model. The discrete element theory and Gaussian quadrature were utilized to analyze and simulate the contact process. Numerical results for the slider-crank mechanism were presented and compared with the experimental ones. They observed that compared to the L-N contact force model, the application domain of the hybrid model was wider, which could provide better results, especially for small clearances and low restitution coefficients. Using the proposed model, Ma and Qian [206] studied the dynamic behaviors of a planar rigid slider-crank mechanism with multiple revolute clearance joints. They observed that the motion type or mode in one clearance joint would have a non-trivial effect on the motion of the other joint, and therefore changed the overall responses of the mechanical system. Also the clearance joint nearer to the input link suffered more serious contact effects and required more input torque. Tan et al. [207] proposed a modified model of the L–N contact force model, and simulated a planar rigid slider-crank with multiple revolute clearance joints. They found that dynamic response of a mechanism with two clearance joints was not a simple superposition of that in a mechanism with one clearance joint. They also observed that with the increase in the number of clearance joints, the input crank torque was larger. Using the co-simulation of Hypermesh and ADAMS softwares, Zhang et al. [208] studied the dynamics of a sewing machine mechanism with multi-clearance joints. They found that when the clearance size increased, the needle's velocity and acceleration would increase dramatically, and that the number of clearance joints would also affected on the needle's responses significantly. Ambrósio et al. [209] studied the contact dynamics of a planar chain drive mechanisms with different initial pretension forces, in which the connection between each pair of links was modeled as a revolute clearance joint. The normal contact forces in the clearance were evaluated by using an enhanced cylindrical contact model proposed by Pereira et al. [210]. Numerical results indicated that suitable pretension force could enforce the continuous contact with lower oscillations on the contact forces, which helped to achieve a higher smoothness of the chain drive dynamic responses.

The above penalty approaches based on different contact force models has been widely utilized to perform the dynamic analysis of the multibody systems with clearance joints due to their simplicity and efficiency. However, a drawback of such an approach is that the numerical stabilization of contact forces and accelerations during the continuous contact phases is not an easy task, and spurious oscillations may appear. An alternative method to treat the contact-impact problems in multibody systems is to use the non-smooth dynamics approach, namely the Linear/Nonlinear Complementarity Problem (LCP/NCP) [211] and Differential Variational Inequality (DVI) [212, 213]. The nonsmooth models, represented as differential inclusions, complementarity systems or variational inequalities, are used in order to model the unilateral nature of the constraints, usually modeled by the Signorini condition. One of the main features of unilateral constraints is the impenetrability, which means that candidate points for contact must not cross the boundaries of antagonist bodies. The DVI has been recognized to be a powerful tool to deal with multiple contact problems in multibody dynamics. This approach has the advantage that it does not need the use of small time steps as in the case of penalty approaches. However, the algorithmic computation procedures that result from the DVI approach are of great complexity. Based on the nonsmooth dynamical approach Akhadkar et al. [214] proposed a methodology for modeling and simulation of planar four-bar mechanisms with multiple revolute clearance joints. In their study, the combined projected Moreau-Jean event-capturing (time-stepping) scheme derived by Acary [215] was used to solve the contact-impact problem. Their numerical results indicated that this methodology could significantly improve the drift issue at the position level and avoid contact force and acceleration spurious oscillations. Wang et al. [216] proposed an impulse-based differential approach based on the Stronge's improved model for restitution to study impact dynamics of a planar slider-crank mechanism with a revolute clearance joint. For the permanent contact state, the contact force was obtained by solving the constraint reaction force. Because of the non-penetration assumption, the velocity and acceleration curves obtained by the proposed method were quite smooth. Thümmel and Funk [217] used the complementarity approach to model impact and friction in a slider-crank mechanism with both revolute and translational clearance joints. Again, Krinner and Thümmel [218] studied the non-smooth behavior of a planar 6-bar linkage mechanism with revolute clearance joints by the methods of unilateral contacts.

In a multibody system, the friction force is likely to appear in joints when contacting surfaces have a relative sliding motion. The Coulomb law [219, 220] of sliding friction can represent the most fundamental and simplest model of friction between dry contacting surfaces. Nevertheless, the implementation of the standard Coulomb friction model in a general-purpose program can lead to numerical difficulties. In order to avoid these numerical difficulties, a modified Coulomb law can be used [147]

$$F_T = -c_f c_d F_N \frac{\mathbf{v}_T}{v_T}, \tag{9}$$

in which c_f is the friction coefficient, F_N is the normal contact force, v_T is the relative tangential velocity and c_d is a dynamic correction coefficient, which is expressed as [147]

$$c_d = \begin{cases} 0 & if \quad v_T \le v_0 \\ \dfrac{v_T - v_0}{v_1 - v_0} & if \quad v_0 \le v_T \le v_1 \\ 1 & if \quad v_T \ge v_1 \end{cases}, \tag{10}$$

where v_0 and v_1 are given tolerances for the tangential velocity. The dynamic correction factor c_d prevents the friction force to change direction for almost null values of the tangential velocity, which is perceived by the integration algorithm as a dynamic response with high frequency contents, forcing to reduce the time step size [221]. The friction model represented by Eq. (9) does not account for other tribological phenomena like the adherence between the sliding contact surfaces [7, 222].

From Eq. (9), it can be observed that when the relative tangential velocity v_T is approaching zero, a falsely infinite large friction force will be achieved. A solution for this problem was found in the model proposed by Karnopp, which was developed to overcome the problems with zero velocity and to avoid switching between different state equations for sticking and sliding [223]. The drawback with this model is that it is so strongly coupled with the rest of system. The external force is an input to the model and this force is not always explicitly given. Variations of the Karnopp model are widely used since they allow efficient simulations, such as the modified Karnopp model by Centea et al. [224] and the reset integrator model by Haessig and Friedland [225]. In fact, the presence of friction in the contact surfaces makes the contact problem more complicated as the friction may lead to different modes, such as sticking or sliding. For instance, when the relative tangential velocity of two impacting bodies approaches zero, stiction occurs. Indeed, as pointed out by Ahmed et al. [226], the friction model must be capable of detecting sliding, sticking and reverse sliding to avoid energy gains during impact. This work was developed for the treatment of impact problems in jointed open-loop multibody systems. Lankarani [227] extended the formulation by Ahmed et al. [226] to analyze the frictional in any general open and closed loop multibody system.

In recent years, there has been much interest on the subject of friction [228-230]. A number of papers have been devoted to this issue, most of which utilize the Coulomb friction model with some modification in order to avoid the discontinuity at zero relative tangential velocity and to obtain a continuous friction force [7, 72, 89, 222, 231-234]. Although the friction coefficient utilized in the Coulomb law is assumed to be constant, experimental data have demonstrated that the friction coefficient is a function of the relative velocity [235]. Therefore, the speed dependent friction models have therefore been proposed to take the velocity dependence of friction into account. Moreover, most of contacts are lubricated and the friction force depends on the friction-speed regimes. Stribeck suggested an approach, known as the Stribeck model, which can convey the friction behavior in the different four friction regions [236-238].

2.2. Spatial revolute joint with clearance

The problem of the dynamic modeling of multibody systems with planar revolute clearance joints has been extensively investigated over the last decades. However, there exist many systems mounted on spatial revolute joints, such as the engine rotor and the robot arm, that exhibit out-of-plane motion. There are also many planar systems with misaligned joint axes, and they operate only because of the clearance joints and/or their compliance. Therefore, these systems should be also considered as spatial systems. For these spatial cases, the available methodologies for planar revolute joints with clearance joints are no longer valid. Therefore, the development of the spatial revolute clearance joint models to assess the influence of the clearance on the systems' response is significantly important. Figure 6 shows a typical configuration of a spatial revolute joint with clearance. The pair of elements in this type of mechanical joint is the bearing and the journal, where the difference in radius between the bearing and journal defines the radial clearance. In most of the cases, the clearance size in actual connections is much smaller than the nominal radius of the journal and bearing elements. Typical clearance sizes in revolute joints are 0.01-0.0001 times of the journal diameter, while typical values for the length of the journal are 1-10 times its diameters. A spatial revolute clearance joint does not constrain any degree-of-freedom from a multibody system, but imposes some restrictions in terms of contact forces on the journal motion inside the bearing. The dynamics of the spatial revolute joint is governed by contact forces when the journal and bearing surfaces are in contact with each other [26, 83].

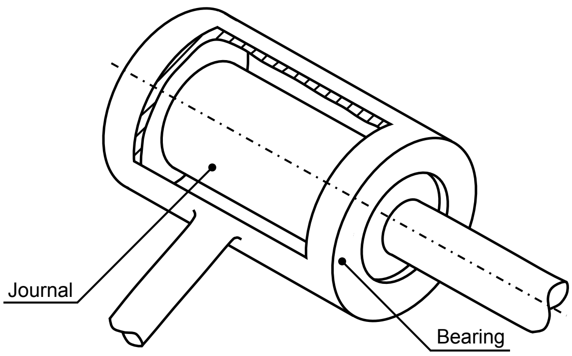

Fig. 6 Typical representation of a spatial revolute joint with clearance [83].

The models for spatial revolute joints with clearance are much more complex than the planar case, since there are several different possible configurations between the bearing and journal surfaces, namely: (*i*) nominal position or free flight motion where there is no contact between the two elements; (*ii*) the journal contacts with the bearing wall at a point; (*iii*) the journal and bearing contact with each other at a line; (*iv*) two contact points between the journal and bearing wall, but in opposite sides [26, 83, 239, 240]. These four possibilities are illustrated in Fig. 7. The dynamic response of the joint is a function of these four scenarios, which depend on the system configuration. In a non-contact situation, no forces are introduced into the system, because the journal moves freely inside the bearing boundaries until it reaches the bearing wall. When the journal and bearing are in contact with each other, local deformations take place at the contact area and, consequently, contact-impact forces characterize the interaction between the bodies.

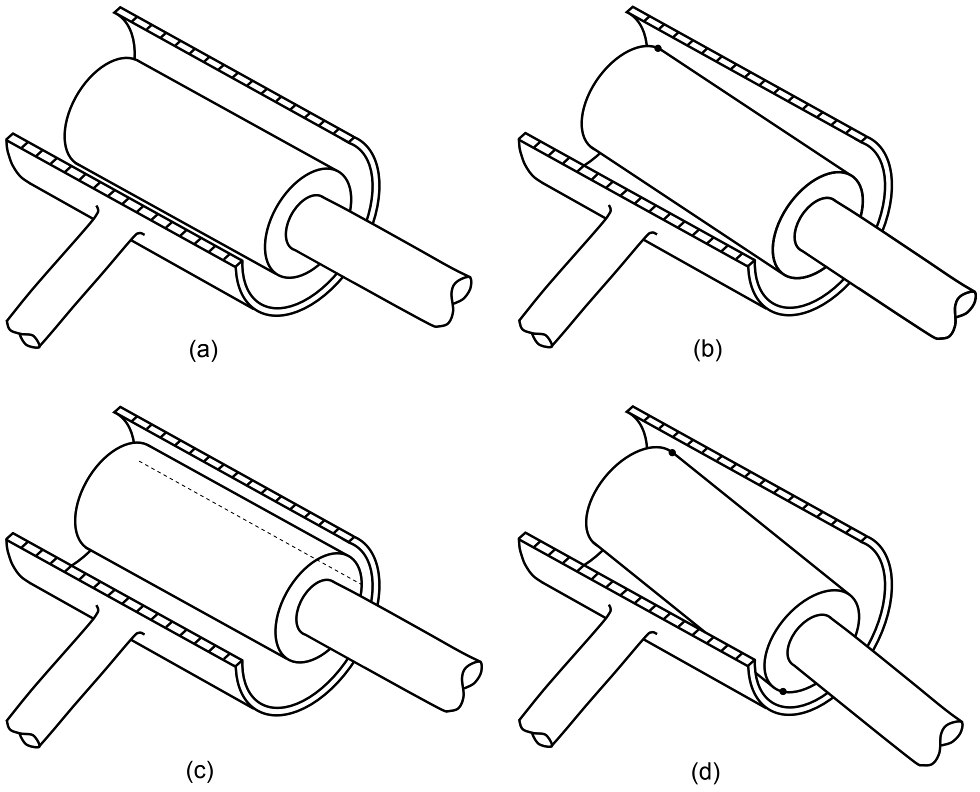

Fig. 7 Four different possible scenarios for the journal motion relative to the bearing: (a) no contact; (b) one point contact; (c) line contact; (d) two contact points in opposite sides [239].

One of the most common ways to model spatial revolute clearance joints is to consider two impact rings located at the top and bottom of the journal [240-242]. These two impact rings can be analyzed as two planar revolute clearance joints, which simplify the modeling process. Figure 8 shows the situation in which the journal contacts with the bearing wall into two points. The relative contact velocities are also represented in Fig. 8. A detailed analysis of this type of clearance joint can be found in references [26, 83].

Dhande and Chakraborty [243] developed an analytical procedure to analyze the output error of function-generating spatial mechanisms considering random clearances in different mechanical joints. These authors were the pioneers in investigating spatial joints with clearance, being the methodology proposed illustrated with RSSR and RRSS spatial mechanisms. In this model, the wobbling of the pin was not taken into account. Dubowsky et al. [114] were also the pioneers on the investigation of the spatial revolute joints with clearance. In this investigation, these authors presented a formulation for three-dimensional dynamics of spatial machine systems taking into account both the vibrations in the links, supporting structures and impacts at the clearance joints using a finite element approach. From this work, it was possible to establish design guideline for high-performance machine systems and minimization of vibrations and noise. Later on, Kakizaki et al. [120] presented a model for the spatial dynamics of robotic manipulators with flexible links and joint clearances, where the effect of the clearance was taken to control the robotic system. From the results obtained, it was observed that the combined actions of clearance, flexibility of links and control system characteristics considerably affected the systems performance. Deck and Dubowsky [113] theoretically and experimentally investigated the dynamic behavior of mechanical systems with flexible links and clearance joints. Also, the flexibility of the links was responsible for the reduction of the level of vibrations and impacts. Hahn [244] suggested a method for modeling spatial joint clearances, which was based on a maximal coordinate representation and differential-algebraic equations of motion. Zakhariev [245] considered a triple pendulum system with spatial revolute clearance joints, in which frictional effects were taken into account.

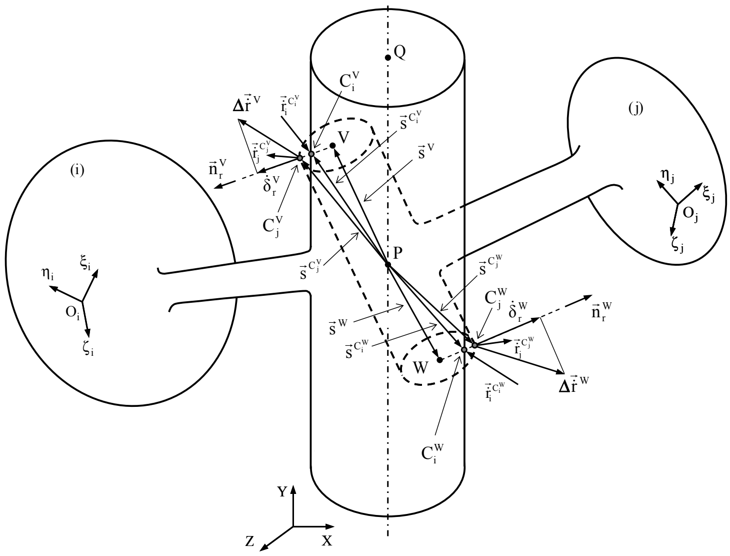

Fig. 8 Location of the contact points and representation of impact velocities at two contact points between journal and bearing elements in a spatial revolute joint with clearance [26].

More recently, Brutti and his co-workers [246] proposed a new formulation for spatial revolute joints with clearance, in which the bases of the journal and bearing could contact with each other, as the contact forces were modeled with nonlinear spring-damper elements. In this investigation, four local reference frames were also fixed to the four centers of the journal and bearing end faces to describe the journal and bearing attitude. The stiffness coefficients of the contact force model were obtained from finite element analysis. The friction effects were not considered in this investigation. This investigation allowed for the improvement of the accuracy in the evaluation of the dynamic performance of a spatial slider-crank mechanism [246]. Figure

9 shows the schematic representation of the spatial revolute clearance joint model presented by Liu et al. [247], where the natural coordinate formulation was utilized [248, 249]. Different from the models by Brutti et al. [246] and by Flores et al. [240], both the rigid journal and bearing were considered as independent bodies described by the natural coordinate formulation, and four potential contacting points were considered, namely the points P, Q, N and M represented in Fig. 9. The influence of the clearance on the trajectory tracking error of an actuated space flexible robot with six spatial revolute joints with clearance was investigated. By assuming that the joint clearance was infinitely small so that the joint constraint violations and collisions were negligible, Qi et al. [250] derived a recursive formulation for simulating multi-rigid body systems with frictional cylindrical joints with tiny clearances. According to their method, both the normal and frictional forces could be determined by using corresponding perfect joint reaction forces. Yan et al. [251] proposed a comprehensive model for spatial revolute joints with clearance in multibody systems. For this, an experimental investigation on a single journal-bearing system was performed with the purpose to reveal the main characteristics of the journal motion inside the bearing element. Zhou and Guan [252] also investigated the dynamics analysis of space deployable systems with spatial revolute joints with clearance and links flexibility. Both analytical and experimental models were considered. These authors showed that both the radial and axial clearance played an important role in the system's behavior, and evidenced that the flexibility of the links reduced the level of impact forces due to the energy dissipation. Bauchau and Rodriguez [253] also investigated on the effects of clearance and flexibility of bodies in multibody systems and demonstrated the importance of proper modeling of elasticity of the links and on the correct information on the dissipative forces.

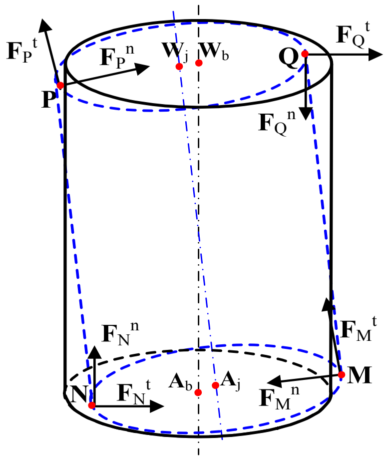

Fig. 9 The spatial revolute joint and its contact forces at the four potential contact points [247].

Flores et al. [240] proposed an approach for spatial revolute joints with clearance taking into account the journal misalignment. Based on the assumption that the clearance was much smaller than the journal length, the analytical formulations of evaluating the global positions of potential contact points were deducted. The Lankarani and Nikravesh [145] contact force model was utilized to evaluate the normal contact forces. However, the computation of the contact forces required a good number of coordinate transformation operations. In this work, the friction at the clearance joints was neglected. Virlez et al. [254] developed a spatial revolute clearance joint model, which accounted for the clearance, the misalignment and the friction effects. This new joint approach was embedded into an automotive TORSEN differential multibody dynamics model. They found that the continuous contact models required very small time steps to achieve the convergence of the integration algorithm, and then suggested that the nonsmooth dynamics algorithm could be an alternative for faster simulations. Erkaya [255] provided an investigation of joint clearance effects on spatial welding robot manipulators with a revolute joint with clearance. In this work, a continuous contact force model and the classic Coulomb friction force model were respectively utilized to evaluate the normal contact force and the friction force in the clearance joint. The author observed that the higher size of joint clearance caused the increase in amplitudes of accelerations and joint forces, while the lower size of joint clearance led to higher kinematic and dynamic response frequencies. In addition, discontinuities could also appear on the actual trajectory of weld seam. However, in this study, the revolute joint journal misalignment was not considered, and the effect of the number of clearance joints on the system's responses was also not addressed. Magalhães et al. [256] studied the dynamics of a spatial light rail vehicle with cylindrical clearance joints in operation in a mountainous track according to the standard EN 14363 [257]. For the clearance model, if the journal and bearing were not aligned, the L-N [145] continuous contact force model was used to calculate the normal contact force, or the enhanced cylindrical contact model proposed by Pereira et al. [258] was adopted. The significant different results were observed for the railway vehicle modeled with cylindrical clearance joints or with equivalent force elements for the joints. The use of clearance joints relaxed the modeling assumptions on the equivalent force elements with limited physical meaning. Venanzi and Parenti-Castelli [259, 260] presented a technique to study the accuracy of planar and spatial mechanisms with revolute clearance joints and applied their approach on a pure translational parallel manipulator. In contrast with other deterministic approaches, knowledge of the forces acting on a mechanism was not needed in their technique; but, the maximum pose error of the link of interest was directly determined instead. Bu et al. [261] proposed a methodology based on the trajectory planning to avoid the detachment of clearance joints parts in robotic manipulators with spatial revolute joints with clearance.

The contact force model considered to compute the intra-joint contact forces in spatial revolute clearance joints plays a key role in the system's behavior. The contact model must include the information on the impact velocity, as well as on the geometrical, material and physical properties of the contacting surfaces. Moreover, the contact force model should be suitable for the stable integration of system's equations of motion. Most of the works described above utilized the well known L-N contact force model [145]. However, this approach has some limitations due the nature of the contact associated with length of the journal. In turn, most of the analytical research works performed in the past decades considered that the contacting surfaces were modeled with linear or nonlinear spring-damper elements [101, 109, 111, 112, 116, 118, 262]. Veluswami et al. [263] experimentally demonstrated that the dynamic response and the contact duration were of more complex nature, and dependent on the impact velocity and clearance size. Liu et al. [264] presented a compliant force model for cylindrical joints with clearances. This approach was compared and validated with results obtained with FEM analysis [265]. These authors demonstrated that the Hertzian contact theory was not valid for large clearances, but instead the Persson approach permitted more accurate results. Gummer and Sauer [266] also investigated the influence of the contact geometry in clearance joints on the contact stiffness and energy loss. In turn, Wang and Liu [267] presented an improved contact force model for revolute clearance joints taking into account the axial length of the journal, the nonlinearity of the contacting materials and energy dissipated during the contact process. Based on the Johnson contact model [197] and Lankarani and Nikravesh approach [145], Pereira et al. [210, 258] developed an enhanced contact force model to describe the interaction between a journal and bearing. These authors demonstrated that the enhanced contact force model has better performance than the Johnson approach, both in terms of range of application and simplicity. More recently, Isaac et al. [268] performed a comparative study on several different contact force models applied to spatial revolute joints with clearance. This investigation was validated with data produced via FEM analysis.

It is known that the normal contact force model proposed by Lankarani and Nikravesh [145] given be Eq. (8) can only de applied in colliding situations with ellipsoidal contact area [194]. For the case of cylindrical contact surfaces, such as in spatial revolute joints with clearance, other researchers proposed to utilize the force-penetration relation expressed by Eq. (8), but with the nonlinear exponent n ranging between 1.0 and 1.5 [181]. Dietl et al. [269] considered the pure elastic hertzian contact force model with a nonlinear exponent equal to 1.08 to model and simulate the contact between journal and bearing elements. This approach was also validated with experimental data.

Radzimovsky [270] presented a formula to evaluate the normal contact force developed between two cylinders. The expression that relates the normal contact force and penetration in spatial revolute joints with clearance is given by

$$\delta = \frac{F_N}{\pi E^*}\left\{\frac{2}{3} + \ln\left(\frac{8\left(R_i - R_j\right)}{m_c^2 F_N}\right)\right\}, \tag{11}$$

where E^* is the composite modulus of the cylinders in contact, and R_i and R_j respectively denote the bearing and journal radii [196]. The parameter m_c depends on the material property in contact, assuming the value of 2.15 for similar materials and 1.60 for different materials. It must be highlighted that Eq. (11) represents a nonlinear algebraic equation that requires an iterative numerical procedure to solve it for the contact force, F_N, such as the Newton-Raphson scheme. Furthermore, Eq. (11) represents a pure elastic approach and, consequently, does not accommodate any energy dissipation during the contact process.

Based on the Hertzian contact theory [144], Dubowsky and Freudenstein [111] proposed an alternative expression that relates the penetration and normal contact force as

$$\delta = F_N\left(\frac{\sigma_i + \sigma_j}{L}\right)\left[\ln\left(\frac{L^b (R_i - R_j)}{F_N R_i R_j (\sigma_i + \sigma_j)}\right) + 1\right], \tag{12}$$

in which σ_i and σ_j are the contact material parameters [1], L denotes the length of the contacting elements, and R_i and R_j represent the radii of the bearing and journal, respectively. The exponent b is equal to 3 for this particular model. It should be noted that Eq. (12) is a nonlinear implicit function for the normal contact force F_N.

A similar expression to Eq. (12) was proposed by Goldsmith [149], but with the value of exponent b equal to 1, which leads to a problem of consistency of the units in the expression. For this model, the relation between the contact force and penetration is almost linear [194].

The ESDU 78035 Tribology Series [271] established several different expressions for contact mechanics analysis suitable for engineers' applications. For a circular contact area the ESDU 78035 model is the same as the pure Hertz law [144]. For rectangular contact, such as, a pin inside a cylinder, the expression is given by

$$\delta = F_N\left(\frac{\sigma_i + \sigma_j}{L}\right)\left[\ln\left(\frac{4L\left(R_i - R_j\right)}{F_N\left(\sigma_i + \sigma_j\right)}\right) + 1\right], \tag{13}$$

where all quantities have been defined above.

Taking into consideration the Hertzian contact theory [144], Johnson [197] presented an expression that permits to compute the contact force for collision between two cylinders. For the case of a spatial revolute joint with clearance, the Johnson expression can be written as

$$\delta = \frac{F_N}{\pi E^*}\left\{ \ln\left(\frac{4\pi E^*(R_i - R_j)}{F_N} \right) - 1 \right\}, \tag{14}$$

in which all quantities have been defined above.

Alternative normal contact force models for spatial revolute joints with clearance can be found in the works by Ciavarella and Decuzzi [272, 273], Greenwood et al. [274], Liu et al. [265] and Pereira and her coauthors [210]. Critical analysis and comparative investigations on the different contact force models for clearance joints have been performed by Flores et al. [194], Pereira et al. [196] and, more recently by Isaac and his coworkers [268].

2.3. Spherical joint with clearance

The fundamental aspects on the modeling of multibody systems with spherical clearance joints are revisited here. In a simple way, a spherical joint with clearance can be understood as extension of the planar revolute clearance joint to the 3D domain. In standard multibody models, it is assumed that the connecting points of two bodies, linked by an ideal spherical joint, are coincident. The introduction of the clearance in a spherical joint separates these two points and the bodies became free to move relative to one another [114]. A spherical joint with clearance does not constrain any rotational degree-of-freedom from the system like an ideal spherical joint. In a spherical clearance joint, the dynamics of the joint is controlled by contact-impact forces, which result from the collision between the connected bodies. This type of joint can be called as force-interaction-joint, since it deals with force effects rather than the kinematic constraints. Figure 10 depicts two bodies i and j connected by a spherical clearance joint. A spherical part of body j, the ball, is inside of a spherical part of body i, the socket. The radii of socket and ball are R_i and R_j, respectively. The difference in radius between the socket and the ball defines the radial clearance. When the clearance is included in a spherical joint, the ball and socket can move relative to each other [83].

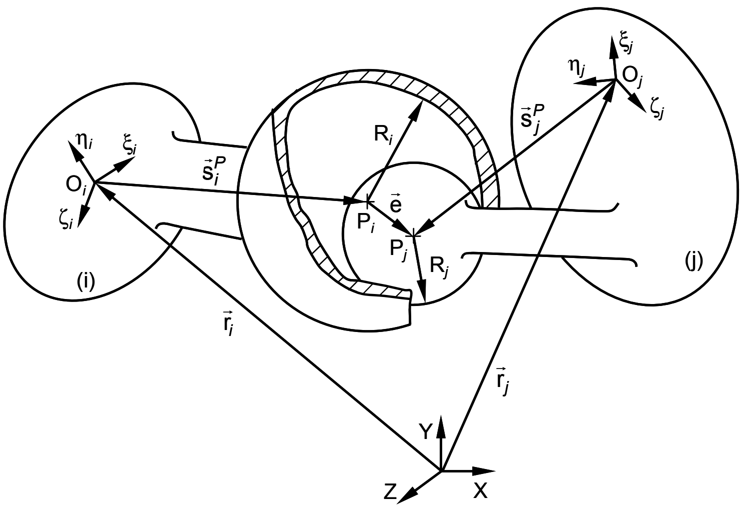

Fig. 10 The rigid spherical clearance joint by Flores et al. [83]

Figure 11 illustrates the three motion modes of the rigid ball inside the socket; namely: the permanent contact or following mode, the free flight mode, and the impact mode. In the contact or following mode, the ball and the socket are in permanent contact and a sliding motion relative to each other exists. This mode ends when the ball and socket separate from each other and the ball enters in free flight mode. In the free flight motion, the ball moves freely inside the socket boundaries, that is, the ball and the socket are not in contact; hence there is no joint reaction force. In the impact mode, which occurs at the termination of the free flight motion, impact forces are applied to the system. This mode is characterized by an abrupt discontinuity in the kinematic and dynamic responses, and a significant exchange of momentum occurs between the two impacting bodies is observed. At the termination of the impact mode, the ball can enter either in free flight or in the following mode [83].

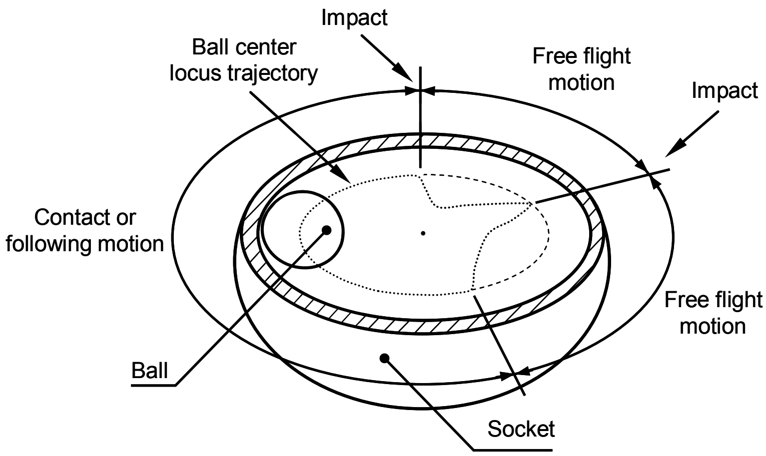

Fig. 11 Modes of the ball motion inside the socket [83]

One of the first investigations that focused on the topic of spherical clearance joints in mechanical systems was conducted by Dhande and Chakraborty [243], who presented a complete and comprehensive approach. They proposed a synthesis procedure to incorporate the tolerances and clearances in spatial linkages with the purpose to study the mechanical error in terms of linkages outputs. Two spatial linkages of the RRSS and RSSR type were considered as application examples. Dubowsky et al. [114] were pioneers to model and simulate mechanisms with spherical clearance joints. These researchers modeled this type of clearance joint as two impacting elements, namely the ball inside a socket, in which some amount of clearance existed. When the ball and socket elements were not in contact, the contact forces and moments were null. In turn, for the case of contact between the ball and socket surfaces, the intra-joint forces were determined using a linear contact force model that includes both compliance and damping terms. Also, a simple friction force model based on the Coulomb's law was utilized to include the friction effects at the clearance joints. An impact beam model and slider-crank mechanism were considered as demonstrative examples of application.

The number of published works especially devoted to spherical clearance joints is quite small, and most of them are focused on systems in which only one joint is modeled as a realistic joint. Bauchau and Rodriguez [253], based on the finite element dynamic analysis of nonlinear flexible multibody systems, proposed a formulation for mechanisms with revolute and spherical joints with clearance. The effects of structural damping and driving speed were investigated. Orden [141] presented a methodology for the study of typical smooth joint clearances in multibody systems. The proposed approach took advantage of an analytical definition of the material surfaces defining the clearance, resulting in a formulation where the gap did not play a central role, as it happened in standard contact models. This approach has been demonstrated as an effective and efficient method in solving the equations of motion. Liu et al. [275] developed a contact force formulation of the spherical clearance joints in multibody mechanical systems, using the distributed elastic forces to model the compliance of the surfaces in contact. Flores et al. [276] presented an analytical methodology to assess the influence of the spherical joint clearances in spatial multibody mechanical systems. This approach is only valid for the case of dry frictionless contact between the socket and ball.

Tian and his co-authors [277] presented a novel computational formulation for dynamic analysis of three-dimensional flexible multibody systems described by the absolute nodal coordinate formulation (ANCF), considering the effects of the clearances and lubrication for the case spherical joints. The frictional effect at the contact was simply accounted using the Coulomb's friction law. The effectiveness of the methodology proposed was demonstrated by different numerical examples.

More recently, Wang et al. [278] investigated on the wear phenomena in spherical joints with clearance. For this purpose, the well-known Archard's wear model was considered and incorporated into the dynamics equations of motion for constrained multibody mechanical systems. This approach was validated using the finite element method. Wang and Liu [279] studied the response of a 4-SPS/CU parallel mechanism, which included a spherical joint clearance. In this investigation, an enhanced contact force models has been proposed to deal with the normal contact-impact analysis between the joint elements. Besides its limitations, the Coulomb's law was considered to account for the friction action at the joint clearance. Zhang et al. [15] analyzed the nonlinear dynamics behavior of spherical joints with clearance. In their work, according to the classical Winkler contact approach, a new formulation was used to define the contact stiffness when the radial clearance was small. The proposed formulation was validated against the response from a finite element model. Zheng et al. [280] investigated the dynamics of rigid-flexible spatial multibody systems with spherical joints with clearance, with particular emphasis on the response of ultra-precision presses.

Flores et al. [276] initially studied the effects of dynamics of a rigid spatial four-bar mechanism, which included a spherical clearance joint between the coupler and rocker. In their work, the attitudes and displacements of the rigid bodies were represented by Euler parameters and Cartesian position coordinates, respectively. According to the obtained generalized coordinates and velocities, the L-N continuous model was further used to calculate the contact forces. Their results indicated that the introduction of spherical clearance joints significantly affect the prediction of components' position and drastically increased the peaks in acceleration and reaction moments at the joints. However, the friction force was not included in their dynamics model. Marques et al. [281] studied the influence of the Coulomb's law based friction force models on the dynamic responses of a spatial four-bar mechanism with a spherical clearance joint between its rocker and coupler. It was observed that the presence of friction helped to stabilize the system's response and made it less chaotic, while the friction force models utilized did not significantly affect the dynamic response of the mechanism. In the biomechanical field, the human hip joint is also an example of a spherical joint. Askari et al. [282, 283] developed a planar and a spatial multi-rigid body hip joint model with spherical clearance for studying the friction-induced joint vibration. They found that the cause of hip squeaking was friction-induced vibration owing to different phenomena such as stick–slip friction, negative-sloping friction and contact force changes. It was found from the FFT analysis of the audible sounds from CoC hip acceleration that the decrease of hip implant size increased both hip squeaking frequencies and penetration depth, while the initial conditions did not have significant influence on squeaking frequencies. Askari et al. [284] also made a comprehensive survey on the efforts dedicated to understand the mechanics of the hip squeaking, the readers can refer this work to gain a deep insight into factors contributing to sound emission from CoC hip articulation. However, their developed model was not a general multibody model undergone large overall spatial displacement and rotation. Quental et al. [285] also utilized a spherical clearance joint with clearance to propose a new shoulder model to perform inverse dynamic analysis to estimate the muscle and joint reaction forces of the upper limb and glenohumral articulation.

Weiss et al. [286] performed a study on the friction-induced dynamics of spherical joints with an emphasis on the joint instability and bifurcation behavior. Using the pseudo-rigid-body model (PRBM) technique, Erkaya et al. [287] investigated on the dynamics of a partly compliant mechanism with spherical joint with clearance. These authors considered a spatial slider-crank mechanism as a demonstrative example of application. The results showed that clearance in the joint led to chaotic behavior of the system. Furthermore, due to the flexibility effect of small pivot and the relation of force-closed kinematic pair between the ball and socket in a clearance joint, compliant mechanism was a good solution to decrease the chaotic phenomenon arising from clearance in mechanism's outputs.

With reference to Fig. 10, the penetration related to the contact between the ball and the socket surfaces can be evaluated as

$$\delta = e - c\,, \tag{15}$$

where e is the absolute eccentricity and c is the radial clearance [66]. The penetration rate can be determined in a similar way as for the case of planar revolute joint with clearance.

The intra-joint contact force due the collision between ball and socket can be evaluated using any Hertz-type contact force model available in the literature [188], such as the one proposed by Lankarani and Nikravesh and given by Eq. (8) [145]. For the case of spherical clearance joints one critical aspect deals with the determination of the contact stiffness due to the surface nature of the contact. In fact, one limitation associated with the Hertz's force model is the evaluation of the contact stiffness, particularly when the bodies contact in a line or surface instead of a point [196, 210, 258]. For the case of large contact areas such as in the spherical clearance joints demands the reevaluation of the contact stiffness. Goodman and Keer [288] demonstrated that conformal contacts can be up to 25 percent stiffer in compression than the one predicted by the classical Hertzian contact theory. This conclusion has been corroborated by Pereira et al. [196]. In turn, Bai and Zhao [200] presented an enhanced contact force model that is given as

$$F_N = K_{\mathrm{mod}}\delta^n + D_{\mathrm{mod}}\dot{\delta}, \tag{16}$$

where the stiffness and damping parameters are expressed as follows

$$K_{\mathrm{mod}} = \frac{1}{8}\pi E^* \sqrt{\frac{2\delta[3(R_i - R_j) + 2\delta]^2}{(R_i - R_j + \delta)^3}}\,, \tag{17}$$

$$D_{\mathrm{mod}} = \frac{3K_{\mathrm{mod}}(1-c_e^2)e^{2(1-c_e)}\delta^n}{4\dot{\delta}^{(-)}}, \tag{18}$$

in which the geometrical and material variables have the same meaning as described above.

Based on the Steuermann theory [289], Liu et al. [275] developed a pure elastic contact for model for spherical joints with clearance that extended the Hertz contact law. This new formulation can be expressed as

$$F_N = \frac{4q}{2q+1}\frac{E^* R_i \delta \sqrt{2(R_i - R_j)\delta + \delta^2}}{R_i - R_j + \delta}\,, \tag{19}$$

where q denotes the index of polynomial of the degree approximation to represent the profiles of the contacting surfaces, while the remaining parameters have the same meaning as described earlier. This formulation has been extensively analyzed and compared with the classical Hertz contact force model and validated with FEM data.

Luo and Nahon [290] proposed a new approach that extended the Hertz contact approach for polyhedral contacting bodies, namely for line and face contacting objects. They explicitly considered the distinction between true contact geometry and interference geometry. This new approach was accompanied with both FEM and experimental discussions. Another way to overcome the difficulties of the Hertz's law, when the contact area cannot be represented as a single contact point, is to consider the elastic foundation model [291]. The approach proposed by Luo and Nahon [290] was based on representation of the body surfaces by polygon meshes and contact force determination by the elastic foundation model. This approach allowed for the modeling of contact between complex geometries and scenarios where the contact area is relatively large, having good computational efficiency when compared with the FEM analysis. Bei and Fregly [292] proposed a computationally efficient methodology for combining multibody dynamic simulation method with a deformable contact knee model. In their study, the contact between knee surfaces was modeled through the use of the elastic foundation approach for both natural and artificial knee articulations. Pérez-González et al. [293] developed a modified elastic foundation approach for application in three-dimensional models of the prosthetic knees, in which both contacting bodies were considered to be deformable solids with their own elastic properties. Mukras et al. [294] also used the elastic foundation method to evaluate the contact forces for wear modeling and analysis in the framework of multibody systems formulations. The results obtained for a planar slider-crank mechanism with a dry clearance joint were compared and validated with those produced via FEM.

2.4. Translational joint with clearance

In this section, the most relevant works on the modeling of planar translational joints with clearance are summarized and examined. Figure 12 shows a typical configuration of a planar translational joint with clearance. For this particular case, the clearance *c* can be defined as the distance between the guide and slider surfaces. Other important geometric characteristics of the translational clearance joint are the length of the slider *L*, the slider width *W*, and the distance between the guide surfaces *H*. In the context of the multibody systems formulations, the slider and the guide elements that constitute a translational clearance joint can be modeled as two colliding bodies and the dynamics of the joint is governed by contact-impact forces [26].

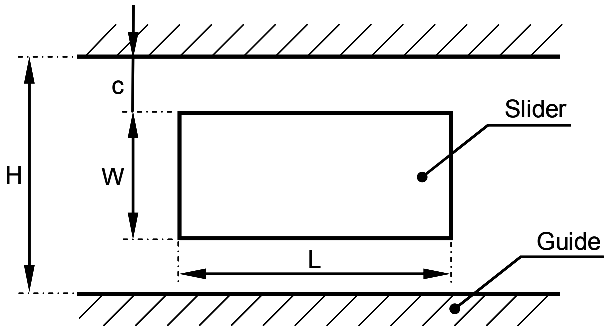

Fig. 12 Translational joint with clearance, that is, the slider and its guide [83]

In an ideal or perfect planar translational joint, the two mechanical parts that compose it, that is, the slider and the guide, translate with respect to each other along the line of translation, so that there is neither rotation between the bodies nor relative translational motion in the direction perpendicular to the axis of the joint [1-3]. Therefore, an ideal planar translational joint reduces the number of degrees-of-freedom of the system by two. The existence of a clearance in a translational joint removes the two kinematic constraints and introduces two extra degrees-of-freedom. Consequently, the slider can move freely inside the guide limits. When the slider reaches the guide surfaces an impact occurs and contact forces govern the dynamic response of the joint. These contact forces can be evaluated according to the continuous contact force proposed together with the dissipative friction force model selected [145]. Although a translational clearance joint does not constraint any degree-of-freedom from the multibody mechanical system, as an ideal joint does, it imposes some restrictions on the slider motion inside the guide. Thus, while the motion of a perfect joint in a multibody system is addressed by the kinematic constraints, a clearance joint is accounted for by contact forces [24].

In sharp contrast to the revolute and spherical clearance joints, not much work has been done to model translational joints with clearance in the measure that in this case several different configurations between the joints elements can take place [83]. In fact, as shown in Fig. 13, the contact configurations of slider and guide include: (*i*) no contact between the two the slider and guide parts; (*ii*) one corner of the slider is in contact with the guide surface; (*iii*) two adjacent slider corners are in contact with the guide surface, which corresponds to have a face of the slider in contact with the guide surface; (*iv*) two opposite slider corners are in contact with the guide surface [295-297]. Moreover, each contact point may be in stick or in slip phase, which greatly enlarges the number of contact configurations. The conditions for switching between different cases depend on the system's dynamics and can be a quite demanding task both from geometric and dynamic points of view [221].

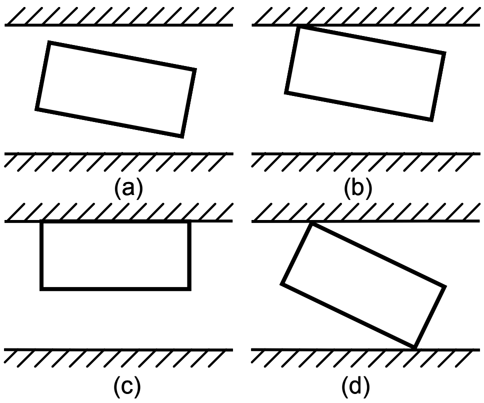

Fig. 13 Different scenarios for the slider motion inside the guide: (a) no contact; (b) one corner in contact with the guide; (c) two adjacent corners in contact with guide; (d) two opposite corners in contact with guide [83]

In a simple and general manner, the contact detection between the slider and guide parts in a translation joint with clearance involves the determination of the distance between each slider corner and guide surface. The contact detection for all the four corners of the slider is quite similar, and in what follows only for the slider corner denoted by A is considered, as it is shown in Fig. 14. In this work, a noncontact situation and a case where penetration between the slider corner A and guide surface are illustrated. The vectors $\boldsymbol{\delta}$ and $\mathbf{n}$ are parallel to each other and oriented in opposite directions. Thus, the condition for penetration between the slider and guide is expressed as [83]

$$\mathbf{n}^T\boldsymbol{\delta} < 0. \tag{20}$$

The magnitude of the penetration depth for point A is evaluated as

$$\delta = \sqrt{\boldsymbol{\delta}^T\boldsymbol{\delta}}\,. \tag{21}$$

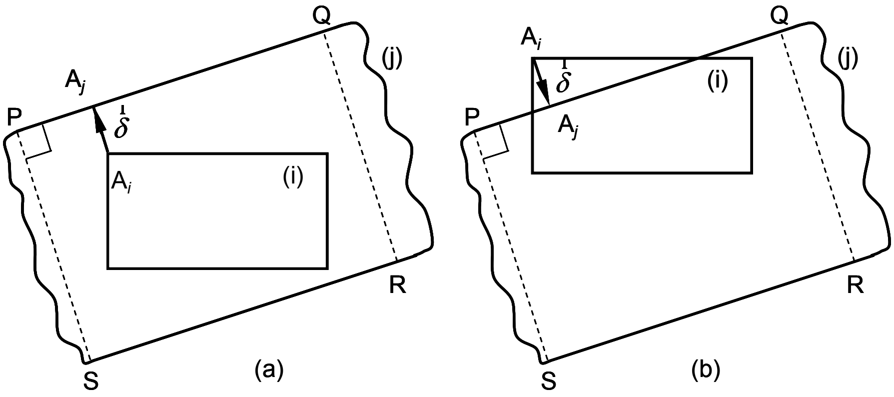

Fig. 14 (a) Non contact situation; (b) penetration between the slider corner A and the guide surface [83]

When the contact between the slider and the guide surfaces takes place, normal and tangential forces appear in the contact points. In dealing with translational clearance joints, it is essential to define how the slider and guide surfaces contact each other, and consequently the selection of most appropriate contact force model. Lankarani [298] presented a linear model for contact between two square plane surfaces as

$$F_N = K\delta, \tag{22}$$

where the stiffness parameter K is given by

$$K = \frac{a}{0.475(\sigma_i + \sigma_j)}, \tag{23}$$

having a contact area a length of $2a$, and quantities σ_i and σ_j are local material properties as before.

When two adjacent slider corners contact with the guide surface, the resulting contact force is applied at the geometric center of the penetration area, denoted as *GC* in Fig. 15a, and the contact force model given by Eq. (22) is used. Otherwise, when one or two opposite slider corners contact the guide surface the contact is assumed to be between a spherical surface and a plane surface, allowing for the contact model given by Hertz law with hysteretic damping factor expressed by Eq. (8) to be applied. In order to evaluate the equivalent stiffness a small curvature radius R_c is assumed on the contact corner, represented in Fig. 15b. The unified contact model is obtained by using the pseudo-stiffness expressed by Eq. (23) in the continuous force model represented by Eq. (8). It must be stated that this particular aspect of modeling the contact between non-Hertzian contacts is still an open problem in the context of multibody dynamics.

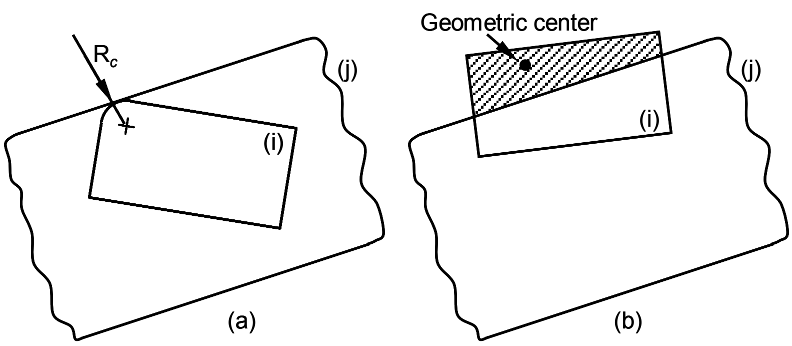

Fig. 15 (a) Contact between a spherical surface and a plane; (b) contact between two plane surfaces [83]

In what follows, some of the most relevant works dealing with planar translation joints with clearance are analyzed. Wilson [299] was pioneer in studying mechanism dynamics with translational joints with dry clearance. Wilson, in 1971, presented a detailed description of both analytical and experimental approaches for a planar slider-crank mechanism, which allowed for the investigation of the influence of the clearance at the slider joint. It was found that the theoretical and experimental results agreed quite well. Wilson and Fawcett [300] derived the equations of motion for all different possible configurations of the slider motion inside the guide, resulting in a total of 40 equations. These authors also showed how the slider motion in a translational clearance joint depends on the geometry, speed and mass distribution. Farahanchi and Shaw [301] studied the dynamic response of a planar slider-crank mechanism with slider clearance. In fact, they investigated the influence of the clearance size, friction, impact parameters and input crank speed on the dynamic behavior of a planar slider-crank mechanism having clearance at the translational joint. They demonstrated how complex the system's response was, which could be chaotic or periodic. In turn, Thümmel and Funk [302] used the complementary approach to model impact and friction in a slider-crank mechanism with both revolute and translational clearance joints. Klepp [303], using a multibody formulation, derived the dynamic equations of motion for sliding joints with friction phenomena. However, the clearance, as well as the contact-impact events, was not taken into account, being the main focus of the investigation on the existence and uniqueness of solutions.

More recently, Flores et al. [297] studied the dynamics of a planar rigid slider-crank mechanism with a translational clearance between its slider and guide. The contact between the slider four corners and the guide inner surfaces were controlled by the geometry of the joint elements and system dynamics. A continuous contact force model and the modified Coulomb law were utilized to compute the normal contact-impact force and friction force, respectively. They observed that when the clearance size was reduced the system response became closer to the system with ideal joints, and the system's dynamic behavior tended to be periodic. Flores et al. [304] also studied the nonsmooth dynamics of a planar rigid slider-crank mechanism with a translational clearance joint, which was modeled with multiple frictional unilateral constraints. The resulting contact-impact problem was formulated and solved as a linear complementary problem, which was embedded in the Moreau time-stepping method. Using a similar way, they also simulated a cam-follower mechanism of a cutting file machine [305]. The fundamental property of the Moreau-Jean scheme is that the unilateral constraints are imposed at velocity level. However, in the Moreau-Jean scheme, the unilateral constraints at position level are only used to support the decision to activate the constraint but they are not exactly satisfied. The consequence is that some penetration can be observed in the numerical solution, which may not be physically acceptable.

By assuming that the clearance size is infinitely small that the joint constraint violations and collisions are negligible, Qi et al. [250] derived a recursive formulation for simulating multi-rigid body systems with frictional joints with tiny clearances. According to their method both the normal and frictional forces could be determined by using corresponding perfect joint reaction forces. The authors [306] also studied frictional dynamics of rigid mechanisms with planar and spatial prismatic joints. In their work, based on the tiny clearance assumption the joint, reaction forces were formulated in terms of resultant frictional forces under the condition of the kinematical constraints of the joint being kept at the acceleration level. Similar to the work by Qi et al. [306], still under the tiny clearance size assumption, Zhuang and Wang [307, 308] studied the nonsmooth dynamics of a planar rigid multibody system with frictional translational joints. In their works, the geometric constraints of the translational joints were treated as bilateral constraints, and the transitions of the stick-slip of the sliders was formulated as a horizontal linear complementarity problem (HLCP), which was solved by using the Lemke's algorithm. Zhang and Wang [309] further studied the nonsmooth dynamics of a frictional translational joint with a flexible slider and clearance. In their work, fictitious springs were appended between nodes and guide, and the geometric constraints of the translational joints were treated as multi-unilateral constraints. However, their method was based on the small slider deformation assumption and cannot be used to simulate the slider undergone large deformation, such as the rubber slider. Chen et al. [310] proposed a nonsmooth generalized-α algorithm to solve the nonsmooth dynamic equations of the rigid-slider mechanism with a translational clearance joint simulated by Flores [294]. Numerical results showed that in the presence of holonomic bilateral constraints, the nonsmooth generalized-α method had less drifting phenomena compared with the ones with the Moreau-Jean method. However, the proposed nonsmooth generalized-α algorithm had just first order accuracy. Furthermore, by introducing the Gear-Gupta-Leimkuhler (GGL) approach into the equations of motion for nonsmooth multibody systems, Brüls et al. [311] developed a nonsmooth-α GGL scheme. The algorithm can guarantee the exact satisfaction of the complementarity condition at position level, which means that no penetration is allowed, and at the velocity level. Recently, using the nonsmooth generalized-α method proposed by Brüls et al. [311], Cavalieri and Cardona [312] studied the nonsmooth dynamics of a spatial rigid crank-slider mechanism with a spatial revolute joint. Luo et al. [313] investigated the non-colliding contact in planar translational joints, where the main focus was the friction effect. This investigation was developed under the framework of multibody systems formulations, as the friction was incorporated in the form of the dry Coulomb's law. The proposed approach was also compared and validated, in term of accuracy and efficiency, with the outcomes produced using the ADAMS software. Ting et al. [314] presented a new approach to quantify the effects of the clearance on the nominal mechanisms response in terms of output link position. These authors studied both revolute and translational joints with clearance. For that, they used the well-established Ting's rotatability laws. Different types of slider-crank mechanisms were considered as demonstrative application examples. Recently, Wu et al. [315] studied a planar rigid mechanism with two driving links and a prismatic (similar to a translational joint) clearance joint. They concluded that the largest Lyapunov exponents were dependent on the clearance size and the input speed.

3. Models for mechanisms with lubricated joints

3.1. Planar revolute joint with lubrication

It is known that in most engineering applications, the mechanical joints are designed to operate with some lubricant fluid, as in the case of the well-established journal-bearings. The high pressures generated in the lubricant fluid act to keep the joint elements apart. In fact, lubricated joints are designed so that even when the maximum load is applied, the journal and bearing surfaces do not come in contact. The thin film formed by lubricant reduces friction and wear, provides load capacity, and adds damping to dissipate undesirable mechanical vibrations [316-318]. Therefore, proper modeling of lubricated revolute joints in machines and mechanisms is required to achieve a better understanding of the dynamic performance of mechanical systems. This aspect gains paramount importance due to the demand for the proper design of the journal-bearings in many industrial applications [24, 27, 30, 319-324].

In general, machines and mechanisms demand journal-bearings in which the load varies in both magnitude and direction, which results in dynamically loaded journal-bearings. Figure 16 shows the cross section of a smooth dynamically loaded journal-bearing. When the load acting on the journal-bearing is not constant in direction and/or magnitude, the journal center describes an orbit within the bearing boundaries. Typical examples of dynamically loaded journal-bearings include the crankshaft bearings in combustion engines, and high-speed turbines bearings supporting dynamic loads caused by unbalanced rotors [325-329].

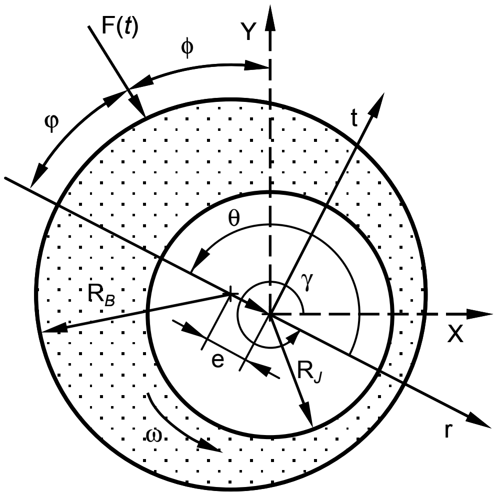

Fig. 16 Cross section of a smooth dynamically loaded journal-bearing [83].

When the space between the journal and the bearing is filled with a fluid lubricant, a hydrodynamic resistance forces developed in opposition to the journal's motion. This force is caused by both squeeze film and wedge film actions [316-318]. In the squeeze film effect, the pressure generation is due solely to the relative radial velocity of the journal bearing surfaces; i.e., the bodies move towards each other. Whereas the wedge film effect deals with the relative rotational velocity; i.e., the fluid is dragged due to the relative angular velocity between the two elements. The hydrodynamic forces that act on the journal-bearing depend on the fluid properties and on the journal motion relative to bearing [330-332]. In a similar manner to the dry revolute clearance joint case, the dynamic analysis of the system with lubrication is performed by evaluating and adding the hydrodynamic forces to the system's equations of motion.

The hydrodynamic (HD) lubrication force is often expressed by its force components along the eccentricity direction (F_r) and its perpendicular direction (F_t). These force components can be obtained by integrating the pressure field either around the entire surface 2π, or around a half-surface π. These boundary conditions are respectively associated with the pressure field corresponding to the Sommerfeld's and Gümbel's boundary conditions, respectively (see Fig. 17). The Sommerfeld's condition is not realistic in many applications due to the lubricant inability to maintain sufficient subambient pressures [333]. Although the Gümbel's conditions results in more realistic predictions of the load capacity, it leads to a violation of the continuity of flow at the outlet [334]. According to Sommerfeld's or Gümbel's condition, the HD lubrication force components can be established analytically for both the infinitely-short and infinitely-long journal-bearings.

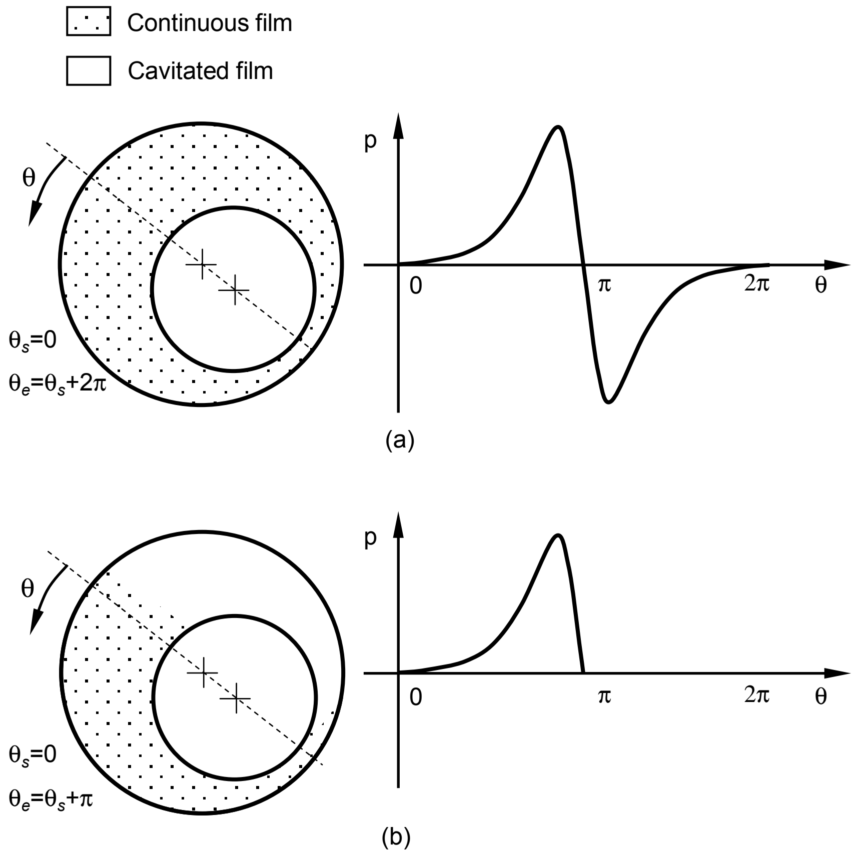

Fig. 17 (a) Sommerfeld's boundary conditions; (b) Gümbel's boundary conditions [83].

The Reynolds' equation can be used to determine the hydrodynamic forces developed by the fluid pressure field in a journal-bearing. The isothermal Reynolds' equation, for a dynamically loaded journal bearing for which the fluid is incompressible and the journal and bearing do not experience any elastic deformation, can be written as [316-318]

$$\frac{\partial}{\partial X}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial X}\right)+\frac{\partial}{\partial Z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial Z}\right)=6U\frac{\partial h}{\partial X}+12\frac{dh}{dt}, \tag{24}$$

where h is the fluid film thickness, p denotes the fluid pressure, μ represents the absolute fluid viscosity, U is the relative tangential velocity, and X and Z are the radial and axial directions, respectively. The two terms on the right hand side of Eq. (24) represent the two different effects of pressure generation on the lubricant film; i.e., the "wedge" and the "squeeze" film actions, respectively. Equation (24) is a nonhomogeneous partial differential of elliptical type. This equation can be solved by approximations when the first or second term on the left hand side is considered negligible and set to zero. These solutions correspond to those for an infinitely-short and an infinitely-long journal-bearing.

For an infinitely-long journal bearing, a constant fluid pressure and a negligible leakage in the Z direction are assumed. In many cases, it is possible to treat a journal-bearing as infinitely-long and consider only its middle point. This solution, firstly presented by Sommerfeld [333], is valid for length to diameter ratios L/D greater than 2. Considering the case of an infinitely long journal-bearing, the Reynolds' equation is simplified as [318]

$$\frac{\partial}{\partial X}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial X}\right)=6U\frac{\partial h}{\partial X}+12\frac{dh}{dt}. \tag{25}$$

By integrating Eq. (25), the pressure filed in the journal-bearing can be expressed as

$$p = 6\mu\left(\frac{R_J}{c}\right)^2 \left\{\frac{(\omega - 2\dot{\gamma})(2+\varepsilon\cos\theta)\varepsilon\sin\theta}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2} + \frac{\dot{\varepsilon}}{\varepsilon}\left[\frac{1}{(1+\varepsilon\cos\theta)^2} - \frac{1}{(1+\varepsilon)^2}\right]\right\}, \tag{26}$$

where c is the radial clearance, θ denotes the angular coordinate, ε is the eccentricity ratio, $\dot{\varepsilon}$ is the time rate of change of eccentricity ratio, μ is the dynamic lubricant viscosity and R_J is the journal radius [318]. Equation (26) enables the calculation of the pressure distribution in a hydrodynamic infinitely-long loaded journal-bearing as a function of the dynamic journal-bearing parameters and geometry. It is convenient to determine the force components of the resultant pressure field in the directions aligned and perpendicular to the line of centers of the journal and bearing. These force components can be obtained by integration of the pressure field around half domain π; i.e., the pressure distribution is integrated only over the positive region by setting the pressure in the remaining portion equal to zero [334-338]. These are known as the Gümbel's boundary conditions. Thus, the component forces along eccentricity direction and its perpendicular direction are, for $\dot{\varepsilon} > 0$, given by [316]

$$F_r = -\frac{\mu L R_J^3}{c^2}\frac{6\dot{\varepsilon}}{(2+\varepsilon^2)(1-\varepsilon^2)^{3/2}}\left[4k\varepsilon^2 + (2+\varepsilon^2)\pi\frac{k+3}{k+3/2}\right], \tag{27}$$

$$F_t = \frac{\mu L R_J^3}{c^2}\frac{6\pi\varepsilon(\omega - 2\dot{\gamma})}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}\frac{k+3}{k+3/2}, \tag{28}$$

and for $\dot{\varepsilon} < 0$

$$F_r = -\frac{\mu L R_J^3}{c^2}\frac{6\dot{\varepsilon}}{(2+\varepsilon^2)(1-\varepsilon^2)^{3/2}}\left[4k\varepsilon^2 - (2+\varepsilon^2)\pi\frac{k}{k+3/2}\right], \tag{29}$$

$$F_t = \frac{\mu L R_J^3}{c^2}\frac{6\pi\varepsilon(\omega - 2\dot{\gamma})}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}\frac{k}{k+3/2}, \tag{30}$$

in which the parameter k is given by

$$k^2 = (1-\varepsilon^2)\left[\left(\frac{\omega - 2\dot{\gamma}}{2\dot{\varepsilon}}\right)^2 + \frac{1}{\varepsilon^2}\right], \tag{31}$$

in which L is the journal-bearing length, c is the radial clearance, ω is the relative angular velocity between the journal and the bearing and γ is the angle between the eccentricity direction and the X-axis [314]

Equations (27) through (31), for infinitely-long journal-bearings, present the relationship between the journal center motion and the fluid reaction force on the journal. The solution of these equations should be straightforward since the journal center motion is known from the dynamic analysis of the mechanical system. In traditional tribology analysis of journal-bearings, the external forces are known and the motion of the journal center inside the bearing boundaries is evaluated by solving the differential equations for the time dependent variables. However, in the present context, instead of knowing the applied loads, the relative journal-bearing motion characteristics are known. Some studies have utilized this approach over the last two decades, such as Ravn and his co-authors [339], Alshaer and Lankarani [340], Alshaer et al. [62], Flores et al. [4, 341], Tian et al. [64]. The readers can also find the corresponding lubricant force formulations for the infinitely-short journal-bearings in the work by Flores et al. [342]. In addition, if the effect of lubricant cavitation is considered in these HD lubrication force models, according to the work by Ravn et al. [339], the eccentricity ratio can be calculated by

$$
\begin{cases} \varepsilon = \dfrac{2c+\delta}{2c} & \text{if } \dot{\varepsilon} > 0 \\ \varepsilon = \dfrac{-\delta}{2c} & \text{if } \dot{\varepsilon} < 0 \end{cases} . \tag{32}
$$

With these eccentricity ratio evaluations, the components of forces along eccentricity and perpendicular directions are determined in the same way as in Eqs. (21)-(31), and components are added to the system's equations of motion.

It has been recognized by many authors that the main difficulty in obtaining hydrodynamic forces lies on adequately defining the boundary conditions of the Reynolds equation [83]. In dynamically loaded journal bearings, the evaluation of the force components obtained from the integration of the Reynolds equation only over the positive pressure regions, assuming the negative pressure as null, involves finding the zero points; i.e., the angle for which a positive pressure begins and the angle for which the pressure is null. For a dynamically loaded journal bearing, these angles are time-dependent, and the evaluation of the force components involves a good deal of mathematical manipulation. The details in the treatment of these angles are described in the work by Pinkus and Sternlicht for infinitely-long journal bearings [316]. For infinitely-short bearings, Daniel et al. [9] presented a simple and effective approach to determine the lubricant pressure integration boundaries, namely the angular position in the bearing reference system for which the pressure is null. A planar rigid slider-crank mechanism with a lubricated joint was utilized as a demonstrative application example. The authors observed that the results obtained by using different HD lubrication models were not able to replicate the realistic numerical solutions of Reynolds equation. However, the results obtained by using their proposed approach for the infinitely-short bearings were in a good agreement with numerical data. Theses observations were particularly evident for the plots of fluid film thickness and journal's center trajectory.

Roger and Andrews [60] developed a two-dimensional mathematical model for the journal-bearing elements, which takes into account the effect of clearance, surface compliance, and lubrication. However, their lubrication model only accounted for the squeeze-film effect. These authors showed that the radial oscillations are about 15% of the size of the circumferential oscillations that are related to the compliance of the bearing surface. Choi et al. [44] utilized the mobility method to investigate the tolerance optimization of planar multibody systems with lubricated revolute joints. A four-bar mechanism and a slider-crank mechanism were considered as two demonstrative application examples. The outcomes presented evidenced the validity of the proposed approach. Schwab et al. [343], based on the work by Moes et al. [344], applied the impedance method to model the lubricated revolute joints in a slider-crank mechanism. Flores et al. [100] investigated the dynamics of a planar rigid slider-crank mechanism with a lubricated revolute joint between the connecting rod and slider. The numerical results for the hydrodynamic lubrication model matched quite well with those obtained with ideal joints, which indicated that the use of lubricant at the machine joints was an effective way to ensure better performance. Nonetheless, numerical difficulties were also observed if either the fluid viscosity was very low or the radial clearance of the journal-bearing was too large. More recently, Machado et al. [65] comparatively studied the effects of three HD lubrication models, clearance sizes, and crank speeds and lubricant viscosity on the dynamic responses of a planar rigid slider-crank mechanism with a lubricated revolute joint between the connecting-rod and slider. They found that the two joint reaction force peaks observed during each crank rotation would increase when the viscosity decreased (because the journal and bearing surfaces were moving too close, when the clearance size was small) and that when the clearance size was small, the response of slider-crank mechanism would be closer to the ideal response. They also showed that when the clearance size was too large or the fluid viscosity was too low, some numerical difficulties would be observed. Bannwart et al. [345] developed a HD lubrication model, which was able to consider the alternating rotational motion in the journal bearing of the connecting rod-slider joint and the motion of the bearing fixed in the slider. The lubricant mass and momentum equations were integrated to achieve the velocity field, pressure distribution and hydrodynamic forces under the infinitely-long journal-bearing assumption. However, the lubricant cavitation effects were neglected in this model. In the biomechanical field, the human hip joint can also be simplified as a type of revolute joint. Thus, under the framework of multi-rigid body dynamics, Costa et al. [346] studied the dynamics of a rigid human gait model with a HD lubricated planar revolute joint as its hip joint. However, the geometry of the revolute joint differs from the practical shape of the human hip joint [347, 348]. Recently, Ebrahimi et al. [349] used a multiple scales method to perform a nonlinear vibration analysis of a planar rigid sliding pendulum with dry and lubricated clearance joint between the slider and the guide. According to Flores' work [26], they developed a linear spring and a nonlinear damper lubrication model to describe the lubricant behavior. They observed that the steady state response frequency for both dry and lubricated cases depended on the linear natural frequency corresponding to the pendulum oscillation. They also found that only the lubricant fluid stiffness had influence on the amplitude of the steady state response, and the fluid did not make any effect on the response frequencies after the transient response vanished.

With the purpose of investigating the influence of flexibility of the bodies, Tian et al. [64] presented an approach to account for both link's flexibility and HD lubrication in revolute clearance joints. The effects of the squeeze lubrication and the HD lubrication for the infinitely-short journal-bearing with Sommerfeld's boundary condition on the system dynamic responses were compared and analyzed. The flexibility of the links were meshed by the locking-free shear deformable beam element of absolute nodal coordinate formulation (ANCF), which was originally proposed by Shabana [350], and has been considered as an important development in the flexible multibody dynamics [351]. If lubricant cavitation occurs, the load bearing capacity of the bearing is reduced. To address this problem, in their work, the eccentricity ratio in the squeeze lubrication force model was calculated according to a modification procedure proposed by Ravn et al. [339]. Their simulation results indicated that the contact forces for the relative flexible system model were much smaller than the contact forces for the stiff system model. Their numerical results also indicated that for the mechanism with high joint rotational velocity, the utilized pure squeeze lubrication model was not effective. Zheng et al. [280] studied the dynamics of a spatial press muli-link transmission mechanism by using the co-simulation of ANSYS and ADAMS. In their work, the lubrication force for infinitely-long bearings, the ADAMS nonlinear spring-damper norm contact force and the LuGre frictional force between the lubricated journal and bearing were calculated by using the ADAMS user-defined subroutines. Their results demonstrated that the dynamic responses of flexible multi-link mechanism with lubricated clearance joint model agreed more closely with the experimental data than those with dry clearance model. Based on the floating reference frame, Liu and Lin [61] established the multibody dynamics model of a planar slider-crank mechanism with a spring-damper connected to its slider. A lubricated clearance revolute joint was used to connect the slider and the finite strain flexible connecting rod. To consider the lubricant wedging action, the lubricant Reynolds' equation was solved by using the finite difference method. The work by Meng et al. [352] indicated that the computational cost of directly solving the lubricant Reynolds' equation could be reduced by using the radial basis function neural network (RBFNN) to approximate the mapping between the oil-film force and the inputs training data such as the lubricant viscosity, the composite roughness and the crank angular speed. With the aim at investigating the bearing inner surface roughness, Zhao et al. [353, 354] adopted the average Reynolds' equation presented by Patir and Cheng [355] to study the dynamics of a planar rigid slider-crank mechanism with a lubricated revolute joint between the crank and the connecting rod. According to the Galerkin theory the dimensionless lubricant partial differential equation was transformed into an integral form, and then solved by using the finite element method (FEM). The simulated results show that within a given range, the smaller clearance was suitable for the dynamics and lubrication performances of the mechanism, and would decrease the friction power loss. By a similar way, Zhao et al. [356] also studied the dynamics of a planar rigid piston-rod-crank mechanism with a lubricated clearance joint between the piston skirt and liner. Their simulation results revealed that the clearance could provide a small secondary motion space, and thus helped to reduce the slap noise and wear. However, the clearance would generate high oil film shear stress and more surface asperities contact. Zhao et al. [357] also studied the dynamic behaviors of a planar rigid slider crank mechanism with lubricated translational and revolute joints at the piston and the big-end bearing of the connecting rod, respectively. The lubrication models were deducted according to the average Reynolds equation, and solved with finite element method to derive the hydrodynamic forces according to the motion of the system. They found that the lubricated skirt-liner system was conducive to improve the lubrication performances of the revolute joint at the big-end bearing of the connecting rod.

3.2. Spatial revolute joint with lubrication

In this section, the fundamental aspects related to the modeling of spatial revolute joints with lubrication are revisited, namely for the hydrodynamic (HD) and elastohydrodynamic (EHD) cases. These formulations are particularly of interest since the aforementioned studies on the dynamic performance of machines and mechanisms with lubricated joints have mostly focused on the planar revolute joints only. Nevertheless, a good number of mechanical systems has spatial or out of plane motions, such as in the case of vehicle models [256], car suspension systems [13], and robotic manipulators [117].

Figure 18 shows the generic configuration of a lubricated spatial revolute joint and its cross section with lubricant [358]. In particular, the journal misalignment is represented in Fig. 18a, which is a quite relevant characteristic in spatial mechanisms. In a similar way to the case of dry contact situation, the bearing center of mass is denoted by point P, and the journal-bearing length represented by B. The local coordinate system is denoted by $\xi\eta\zeta$. Figure 18b depicts an arbitrary journal cross section along ζ axis, which is used to describe the general methodology for determination of the hydrodynamic forces.

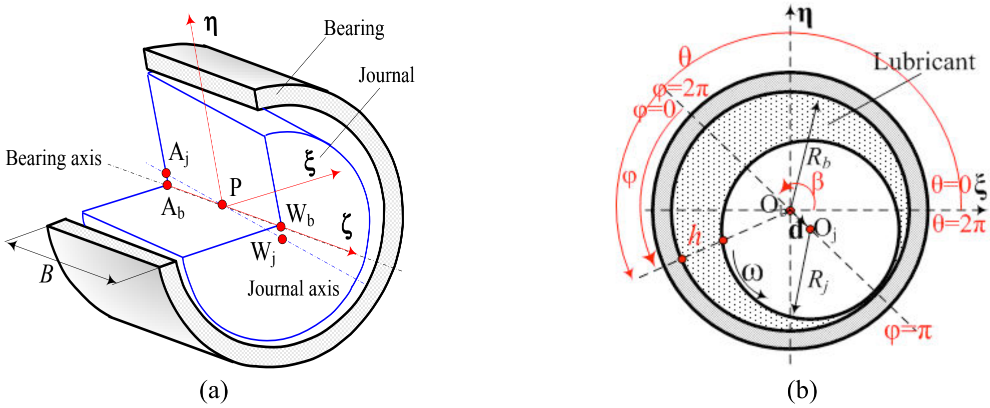

Fig. 18 The generic configuration of a lubricated spatial revolute joint: (a) The lubricated spatial revolute joint with journal misalignment (b) The cross section of the lubricated spatial revolute joint [358].

With regards to Fig. 18, the general form of the isothermal Reynolds' equation can be written in the following form [359]

$$\frac{1}{R_j^2}\frac{\partial}{\partial\varphi}\left(h^3\frac{\partial p}{\partial\varphi}\right)+\frac{\partial}{\partial\zeta}\left(h^3\frac{\partial p}{\partial\zeta}\right)=6\mu\omega\frac{\partial h}{\partial\varphi}+12\mu\frac{\partial h}{\partial t}, \tag{33}$$

where p denotes the lubricant pressure, μ is the dynamic lubricant viscosity, and the lubricant film thickness h is given by

$$h=c+d\cos\varphi=c+d\cos(\theta-\beta), \tag{34}$$

in which c is the radial clearance and d represents the journal-bearing eccentricity.

When the journal misalignment is taken into account, the coordinates and the time derivate of an arbitrary journal cross section along the axis ζ are expressed as functions of the local coordinates of points $A_j(\xi_1, \eta_1, -B/2)$ and $W_j(\xi_2, \eta_2, B/2)$ as follows

$$\begin{aligned}\xi_j&=\xi_2+\frac{(\xi_2-\xi_1)(\zeta-B/2)}{B}, \eta_j=\eta_2+\frac{(\eta_2-\eta_1)(\zeta-B/2)}{B}\\ \dot{\xi}_j&=\dot{\xi}_2+\frac{(\dot{\xi}_2-\dot{\xi}_1)(\zeta-B/2)}{B}, \dot{\eta}_j=\dot{\eta}_2+\frac{(\dot{\eta}_2-\dot{\eta}_1)(\zeta-B/2)}{B}\end{aligned}. \tag{35}$$

For an infinitely-short journal-bearing, it is convenient to deduct the force components of the resultant pressure that act at the journal center of mass P. These force components can be expressed as [358]

$$F_\xi=-\int_{-B/2}^{B/2}\int_0^{2\pi}p(\theta,\zeta)R_j\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\zeta, \tag{36}$$

$$F_\eta=-\int_{-B/2}^{B/2}\int_0^{2\pi}p(\theta,\zeta)R_j\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\zeta. \tag{37}$$

In a similar way, the resulting moment that act at the journal center of mass P can be obtained and expressed as

$$M_\xi=-\int_{-B/2}^{B/2}\int_0^{2\pi}p(\theta,\zeta)R_j\zeta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\zeta, \tag{38}$$

$$M_\eta=\int_{-B/2}^{B/2}\int_0^{2\pi}p(\theta,\zeta)R_j\zeta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\zeta. \tag{39}$$

Finally, the resulting forces and moments are obtained by numerical integration of Eqs. (36)-(39). The pressure field can only be considered over the positive part by setting the pressure in the remaining portion equal to zero. This boundary condition, associated with the pressure field, corresponds to Gümbel's boundary conditions [316, 317].

Stefanelli et al. [360] investigated the dynamic behavior of the rigid rotor system supported by HD lubricated spatial revolute joints, in which the journal misalignment was taken into account. The HD lubrication pressure model for the infinitely-short journal-bearing with Gümbel's boundary condition was used. In their work, the rigid body attitude and displacement were described by using Euler parameters and Cartesian position coordinates, respectively. Based on the work by Stefanelli et al. [360], Tian et al. [358] further proposed a new HD lubricated spatial revolute joint model described by the natural coordinate formulation (NCF), in which the journal misalignment was considered. Equations (36)-(39) were used to evaluate the force components of the resultant pressure.

For a finite-length journal-bearing, the solution of Reynolds' equation can not be simplified as in the cases of infinitely-short and infinitely-long journal bearings, requiring the use of numerical computations. Kumar et al. [361] studied the steady and transient responses of a spatial HD lubricated revolute joint without considering the journal misalignment. The

HD lubrication pressure was obtained by solving the Reynolds' finite difference equation for the incompressible lubricant with constant viscosity. However in this work, both the rigid journal and bearing were just treated as the lubrication boundaries not the rigid bodies with inertia effect. Harish and Manish [362] presented an analytical formulation to describe the lubricant pressure in elastomeric bearings for a marine propeller. Sun and Gui [363] proposed a finite difference approach to solve the lubricant pressure, with the journal misalignment being taken into account. The results showed that there were obvious changes in film pressure distribution, the highest film pressure, film thickness distribution, the least film thickness, and the misalignment moment when misalignment took place. Nonetheless, similar to the work by Kumar et al. [361], this work also only focused on the system steady responses, and both the rigid journal and bearing inertias were not considered.

Several investigations in the tribology field have revealed that joint outer bearing deformation would alter the journal bearing's fluid film profile, modify the lubricant pressure distribution, and affect performance, especially for lubricated systems subjected to heavy loads and high operating speeds [364, 365]. The elastohydrodynamic (EHD) lubrication is present in lubricated contra formal contact areas, where the elastic deformation of the lubricated surface has a substantial influence on the thickness of the lubricating film [365]. The EHD analysis of lubricated joints has attracted the attention of numerous researchers in the field of tribology [364, 365]. Liu et al. [366] and Attia et al. [367] are among the very few authors who performed the steady-state response analysis of the EHD lubricated spatial revolute joint by using the Fluid-Structure Interaction (FSI) analysis softwares. Their studies clearly demonstrated that the bearing deformations affected the pressure field in the clearance and increased the minimal film thickness. However, these works [366, 367] was only focused on the system steady state response of isolated journal bearing systems, and the rigid journal was just considered as a moving boundary not a rigid body with inertia effect. In addition, the journal misalignment was also not taken into account. Slim et al. [368] comparatively studied the acoustic and vibration behaviors of oil lubricated journal bearings by using the HD and EHD approaches. They showed that the sound level of the bearing was significantly influenced by the flexibility of bearing liner, and that the bearing noise decreased due to the EHD lubrication since the film thickness was larger than that of HD lubrication.

Although the transient responses were obtained through different softwares co-simulation, the journal was also just acted as a moving boundary, and its inertia effect was still not considered. Again in this work, the journal misalignment effect was also not studied. In recent years, there has also been extensive analysis of EHD lubrication for lubricated artificial human hip joints, which can also be considered EHD lubricated spherical joints. This interest is due, in part, to the growing number of artificial hip joint implant surgeries [347, 369] being performed worldwide. Jin et al. [370] found that when the joint cup was assumed rigid, the minimum film thickness predicted was at least three orders of magnitude less than the thickness predicted when the flexibility of the joint cup was considered.

It can be observed that most of the before-mentioned studies from the field of tribology have mainly focused on isolated EHD lubricated joints. The coupled dynamic behaviors of the multibody system with EHD lubricated joints have not yet been rigorously considered. Choi et al. [371] developed an EHD lubricated cylindrical joint model. Using the commercial software RecurDyn, the dynamics of a general multibody system with EHD lubricated cylindrical joints was simulated. As shown in Fig. 19, Tian et al. [372] developed a new EHD lubricated spatial revolute joint model, in which the rigid journal was described by the NCF and the flexible bearing was meshed by a new 20-node isoparametric hexahedral element proposed according to the continuum mechanics theory.

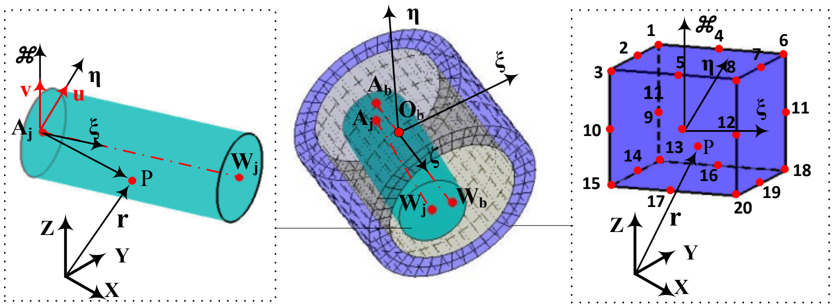

Fig. 19 EHD lubricated spatial revolute joint

For the EHD lubricated spatial revolute joint model, the general form of the isothermal Reynolds' equation is the same as Eq. (33). However, when the elastic deformation of bearing is considered, the lubricant film thickness should be expressed as

$$h = c - \xi_j \cos\theta - \eta_j \sin\theta + \delta, \tag{40}$$

where δ denotes the elastic deformation of bearing.

As shown in Fig. 20, an EHD lubricated cylindrical joint can be unfolded along with the circumferential direction (φ). Then, the lubricant pressure field can be evaluated by imposing Eq. (33) to each grid point of a finite-difference method [373].

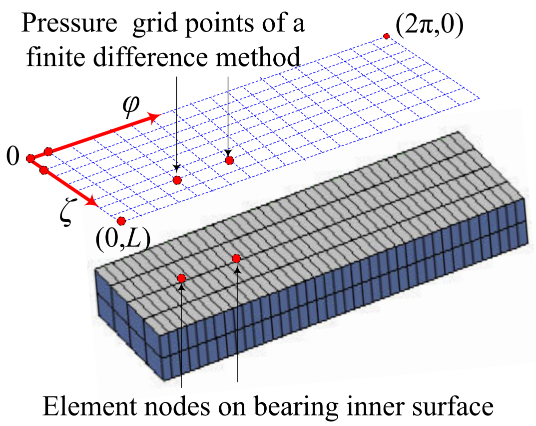

Fig. 20 EHD model of an unfolded lubricated spatial revolute joint [373]

Tian et al. [372] investigated elastohydrodynamic lubrication in mechanical joints taking into account the journal misalignment, as the lubricant pressure obtained by solving the Reynolds' finite difference equations using the successive over relaxation (SOR) algorithm. The numerical results showed that the bearing flexibility affected the system behavior in a significant manner because the bearing flexible extended the lubricant distribution space, and then reduced the lubricant pressure. The reported outcomes reported were compared and verified with the data produced with commercial code ADINA. The ADINA results were based on the Sommerfeld's conditions, complete or full film conditions, which did not take into account the cavitation phenomenon nor allowed for the existence of negative pressure. This may not be realistic, because in many applications, the fluid is unable to sustain significant vacuum pressures. Therefore, ADINA negative lubricant pressures can be ignored. In a subsequent work, Tian et al. [374] investigated the coupling dynamics of a very complex rigid-flexible quick-return mechanism driven by a geared rotor supported by four EHD lubricated spatial revolute joints (I, II, III and IV), as Fig. 21 illustrates. The geared multibody system was described by using the Absolute-Coordinate-Based (ACB) method that combines the Natural Coordinate Formulation (NCF) describing rigid bodies and the Absolute Nodal Coordinate Formulation (ANCF) characterizing the flexible bodies. Based on the ACB method, the mass matrix of the equations of motion remains constant in the numerical iteration process, resulting in efficient computation. The normal contact forces of gear pair along the Line Of Action (LOA) was calculated by using a spring-damper model with a time-varying mesh stiffness, a mesh damping and a Static Transmission Error (STE). The friction force along the Off-Line-Of-Action (OLOA) were computed by using the Coulomb friction models with a time-varying coefficient of friction under the EHD lubrication condition of gear teeth. From their numerical simulations, the dynamic behavior of the mechanism was fully analyzed taking into account the lubrication effects. Thus, for instance, it can be observed that the directions of the lubrication pressure for the cylindrical joints I and II are close to the direction of gear pair LOA.

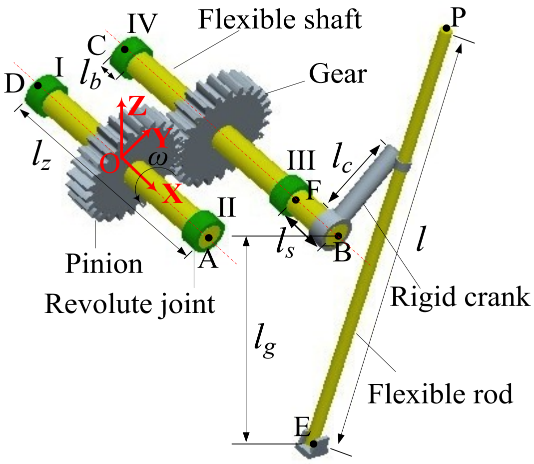

Fig. 21 A geared quick-return mechanism supported by four EHD lubricated spatial revolute joints [374]

More recently, Krinner and Rixen [375] described and compared three reduction approaches for elastic structures with lubricated interfaces, namely the classical Craig Bampton, the two-step Craig Bampton reduction and load dependent formulation. These different approaches were compared considering two demonstrative application examples. Nonetheless, most existing works [366, 367, 372-375] on the EHD lubricated clearance joint analysis were based on the staggered algorithms in which the solid phase and the fluid phase were solved alternatively between iterations. Based on the mortar discretization of interface fields, Yang and Laursen [376] proposed a novel general monolithic scheme to simultaneously solve the EHD lubricated contact problems. According to their work, without the need for iterative or staggered approaches between the fluid and solid phases, the fluid film thickness, directly related to the deformation of the solid phase, can be computed from a least squares projection based on dual basis functions. This work has initiated a new way to study and analyze the EHD lubricated joints.

3.3. Spherical joint with lubrication

In contrast with the planar revolute joint, there are very few works for modeling and simulating spherical joint with lubrication in the context of multibody system dynamics. Wang et al. [377] investigated the tribological aspects of spherical bearings with complex spherical-based geometry with the purpose of modeling and simulating friction and lubrication at this type of mechanical joint. These authors also studied the wear in spherical joints. Based on the work developed by Goenka [378], Tian et al. [277] proposed a HD lubricated spherical joint for flexible multibody systems, which is schematically represented in Fig. 22 where the Reynolds' equation is represented in spherical coordinate system r-θ-ϕ.

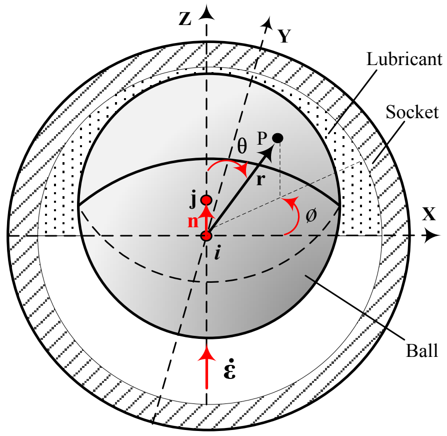

Fig. 22 Schematic representation of a lubricated spherical joint [277]

For the case of lubricated spherical joints, the full general form of the isothermal Reynolds' equation can be stated as [316, 378]

$$\begin{gathered}\frac{1}{R_s^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\theta}\left(\frac{h^3}{12\mu}\sin\theta\frac{\partial p}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial}{\phi}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial\phi}\right)\right]\\=\frac{1}{R_s\sin\theta}\left(h\cos\theta\bar{U}^\theta+h\sin\theta\frac{\partial\bar{U}^\theta}{\partial\theta}+h\frac{\partial\bar{U}^\phi}{\partial\phi}+\sin\theta\bar{U}^\theta\frac{\partial h}{\partial\theta}+\bar{U}^\phi\frac{\partial h}{\partial\phi}\right)+\frac{\partial h}{\partial t}\end{gathered}, \tag{41}$$

from which, it is possible to obtain the following expressions

$$2\bar{U}^\theta=-R_s\omega^x\sin\phi+R_s\omega^y\cos\phi, \tag{42}$$

$$2\bar{U}^\phi=-R_s\omega^x\cos\phi\cos\theta-R_s\omega^y\sin\phi\cos\theta+R_s\omega^z\sin\theta, \tag{43}$$

$$h=c(1-\varepsilon^x\sin\theta\cos\phi-\varepsilon^y\sin\theta\sin\phi-\varepsilon^z\cos\theta), \tag{44}$$

where p denotes the lubricant pressure, r, θ and ϕ are the spherical coordinates, h is the lubricant thickness, μ is the dynamic lubricant viscosity, ε^x, ε^y and ε^z are the components of the eccentricity vector in the X, Y and Z directions, and ω^x, ω^y and ω^z represent the angular velocity components in the X, Y and Z directions. Under the assumption of the pure squeeze, the analytical formulation [277] of the squeeze lubrication force acting at the ball center can be expressed as

$$F_{squeeze}=\begin{cases}\dfrac{6\pi\mu R_s^4\&}{c^2}\left[\dfrac{1}{\varepsilon^3}\ln(1-\varepsilon)+\dfrac{1}{\varepsilon^2(1-\varepsilon)}-\dfrac{1}{2\varepsilon}\right], & if\ \varepsilon\neq 0\\ \dfrac{4\pi\mu R_s^4\&}{c^2}, & f\ \varepsilon=0\end{cases}, \tag{45}$$

where μ is the dynamic lubricant viscosity, and the remaining parameters associated with the joint kinematics are the same.

In the work by Tian et al. [277], the flexible bodies were meshed by the fully parameterized beam elements of ANCF. The numerical results evidenced that the dry contact forces obtained by using the relative flexible multibody system model are smaller than those obtained with rigid system model, and the performance of the lubricated multibody system was closer to the perfect system. Similarly, Flores and Lankarani [379] also investigated the dynamics of spatial four-bar mechanism with a HD lubricated spherical joint between the coupler and rocker. However, in their work when the eccentricity ratio was close to zero the pressure was neglected. More recently, Tian et al. [380], using unified global coordinate system frame, proposed a new EHD lubricated spherical joint model for rigid-flexible multibody dynamics. This model is illustrated in Fig. 23, where the rigid ball was described by the ANCF-RN, and the flexible socket was meshed by the 20-node isoparametric hexahedral element proposed in their work [372] and a new proposed fifteen-node isoparametric finite element with global nodal position coordinates. For the multibody system with EHD lubricated spherical joints, the relative positions of the ball, the lubricant, and the socket will change as a function of movements. To solve the non-conformal problem existing between the lubricant grid and the socket inner surface node grid, they also proposed a lubricant finite-difference grid rotation scheme.

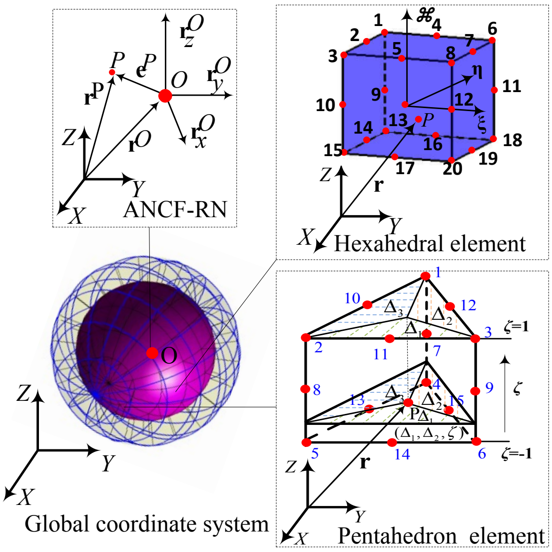

Fig. 23 The finite element grid of the spherical joint [380]

3.4. Hybrid/transition joints models

This section provides a description of hybrid formulations that allow the study of the transition between the dry and the lubricated joint models. In fact, when the eccentricity is very high (thin fluid film thickness), the pressure will cause elastic deformation of the bearing surfaces, which can be of the same order as the lubricant film thickness. Consequently, the lubrication force may approach infinity. To smooth the force transition between the lubricated and dry contact, a force transition model from hydrodynamic lubrication forces to dry contact forces was proposed by Flores et al. [100], which can ensure continuity in the joint reaction force. This model that considers the existence of the lubrication during the free flight trajectory of the journal, prior to contact, and the possibility for dry contact under some conditions, seems to be well fitted to describe revolute joints with clearances in mechanical systems. Figure 24 shows a partial view of a mechanical system representing a revolute clearance joint with lubrication effect, where both the journal and the bearing can have planar motion. The parallel spring-damper element represented by a continuous line refers to the solid-to-solid contact between the journal and the bearing wall, whereas the damper represented by a dashed line refers to the lubricated model. If there is no lubricant between the journal and the bearing, the journal can freely move inside the bearing boundaries. When the gap between the two elements is filled with a fluid lubricant, a viscous resistance force exists and opposes the journal motion. Since the radial clearance is specified, the journal and bearing can work in two different modes. In the mode 1, the journal and the bearing wall are not in contact with each other and they have a relative radial motion. For the journal-bearing model without lubricant, when $e<c$, the journal is in free flight motion and the forces associated with the journal-bearing are set to be zero. For lubricated journal-bearing model, the lubricant transmits a force, which must be evaluated from the state variables of the mechanical system using one of the models described in the previous section. In the mode 2, the journal and bearing wall are in contact, thus the contact force between the journal and the bearing is modeled with the continuous contact force model represented by Eq. (8).

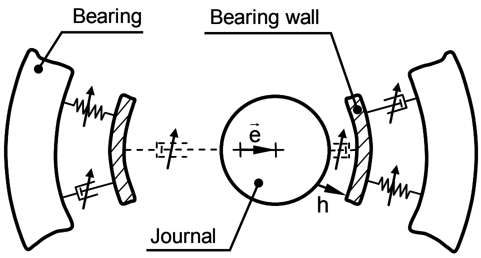

Fig. 24 Mechanical system representing a revolute joint with lubrication effect [100]

In order to avoid numerical instabilities and to ensure a smooth transition from pure squeeze model to dry contact model, a weighted average is used. When the journal reaches the boundary layer, for which the hydrodynamic theory is no longer valid, the squeeze force model is replaced by the dry contact force model, as represented in Figures 25a and 25b. This approach ensures continuity in the joint reaction forces when the squeeze force model is switched to dry contact force model. This transition force model is expressed by [100]

$$F=\begin{cases} F_{squeeze} & if\ e\leq c \\ \dfrac{(c+e_{t0})-e}{e_{t0}}F_{squeeze}+\dfrac{e-c}{e_{t0}}F_{dry} & if\ c\leq e\leq c+e_{t0}\,, \\ F_{dry} & f\ e\geq c+e_{t0} \end{cases} \tag{46}$$

where e_{t0} is a given tolerance for the eccentricity. It should be noted that the clearance used for the pure squeeze force model is not c but it is $c+e_{t1}$ instead. The values of the e_{t0} and e_{t1} parameters must be chosen carefully, since they depend on the clearance size.

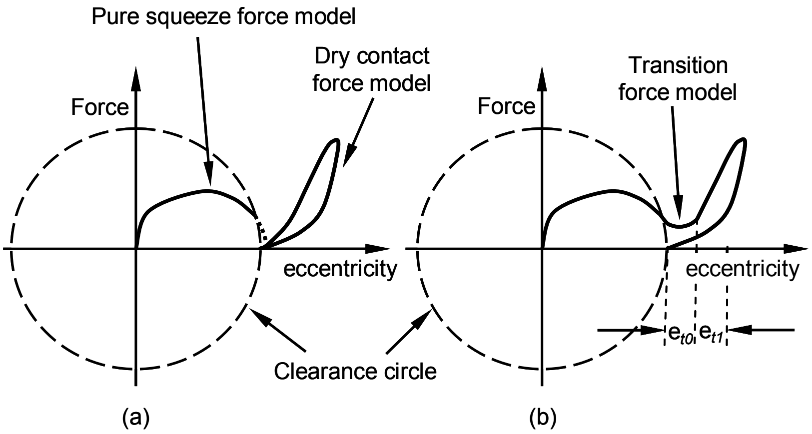

Fig. 25 (a) Pure squeeze and dry contact force models; (b) Transition force model between lubricated and dry contact situations [100]

Daniel and Cavalca [381] proposed a simple lubrication transition model to simulate a planar rigid slider-crank mechanism with a lubricated revolute joint connecting its rod and slider. According to their model, if the slider pin eccentricity is less than 0.9, the HD lubrication model proposed by Bannwart et al. [345] is introduced into the system's equations of motion to evaluate the lubrication forces. Otherwise, it is assumed that the slider pin and the joint bearing are in a continuous contact state, and the corresponding perfect mechanism is simulated. Subsequently, Reis et al. [66] comparatively studied the influence of the HD lubrication model proposed by Bannwart et al. [345] and the Pinkus and Sternlicht's lubrication force model [316] on the dynamic behaviors of the same planar rigid slider-crank mechanism. Due to the difficulty in choosing the preselected parameters, the Flores's transition model [100] lacks general applicability in mechanisms with different clearance sizes. According to the lubrication status transition criteria defined by Luo and Yan [382], Li et al. [383] proposed another transition model for the lubricated clearance joint to tackle with the transition among the contact statuses: namely the hydrodynamic lubrication, the EHD lubrication, the partial lubrication and the dry contact. With the relationship between the oil film thickness, and the lubrication status, the mathematical expression of the proposed transition model can be written as

$$F=\begin{cases} F_{HDL} & if\ h_{\min}>4\bar{R} \\ F_{EHL} & if\ 0.4\bar{R}\le h_{\min}\le 4\bar{R}, \\ F_{Dry}+F_{PL} & if\ h_{\min}\le 0.4\bar{R} \end{cases} \tag{47}$$

in which F_{HDL} is the force transmitted by hydrodynamic lubrication, F_{EHL} the elastohydrodynamic lubrication, and F_{PL} the partial lubrication, F_{Dry} the metal-to-metal contact force, and h_{min} the minimum film thickness. $\bar{R}$ depends on the roughness of the contact surfaces. Different from the Flores' model of contact status, in the proposed model the journal is assumed to be supported by both pure contact force and partial lubrication pressure. Compared to the Flores' transition model [100], the transition model proposed by Li et al. [383] is more accurate. Figure 26 gives a schematic view of their transition model.

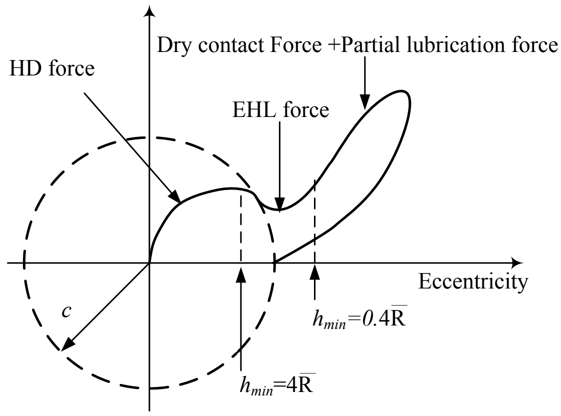

Fig. 26 The transition model proposed by Li et al. [383].

4. Experimental investigations on clearance joints

4.1. Simple journal-bearing systems

The subject of the representation of real physical mechanical joints has attracted the attention of significant number of researchers, and a large number of theoretical and experimental works on the dynamics of multibody mechanical systems with clearance joints has been published [118]. In this section, some of the most relevant experimental investigations on mostly simple journal-bearing are examined. Mahrus [384] conducted an experimental investigation into journal-bearing performance. A test machine was developed on which steady as well as varying unidirectional or full two-component dynamic load could be applied to the test journal-bearing. The corresponding journal center path was measured simultaneously with the load to show the effect of the load diagram on hydrodynamic lubrication. Haines [385] derived equations of motion that described the contributions at a journal-bearing system with clearance but with no lubrication present. In a continuation of the study, Haines [386] carried out an experimental investigation on the dynamic behavior of revolute joints with varying degrees of clearance. Under static loads, the deflection associated with contact elasticity in the dry journal-bearing was found to be much larger and less linear than predicted. In this investigation, the contact loss between the journal and bearing was predicted using proximity transducers. Norton et al. [387] discussed the bearing forces as a function of mechanical stiffness and the vibration isolation in an experimental setup consisting of a four bar mechanism. The test rig was instrumented with piezoelectric accelerometers and force transducers. As expected, the clearances in the physical model created significantly larger dynamic accelerations, torques and forces on the bearings than was predicted theoretically by the rigid body dynamic model.

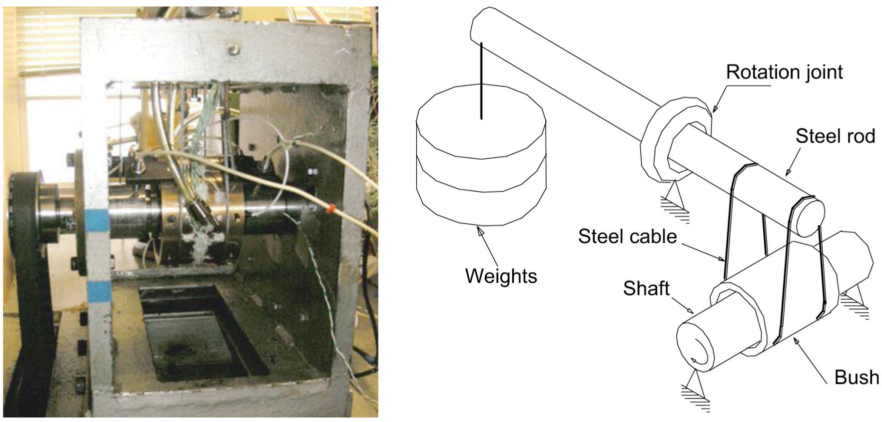

Fig. 27 Test rig utilized at the University of Minho to investigate simple journal-bearings [390].

The performance of journal-bearings considering lubricant supply conditions has been studied theoretically and experimentally by Miranda [388], Claro [389], Costa [390] and Brito [391]. Figure 27 shows a general view of the test rig utilized at the University of Minho to investigate on the performance of journal-bearings for different scenarios and conditions [392]. Bouyer and Fillon [393] designed and built an experiment test rig to study the journal misalignment effects on hydrodynamic plain journal bearing performances. Figure 28 depicts a general view of this test rig. The authors found that the maximum pressure in the mid-plane decreased by 20 percent for the largest misalignment torque, while the minimum film thickness was reduced by 80 percent. The misalignment caused more significant changes in bearing performance when the rotational speed or load was low, and the hydrodynamic effects were then relatively small and the bearing offered less resistance to the misalignment. Sun et al. [394] also developed a special test bench for studying the lubrication performance of cylindrical journal bearings considering journal misalignment. The results indicated that the journal misalignment led to the obvious changes at distribution and oil film pressure, oil film thickness and, oil temperature of journal bearing, and the higher the load on the shaft. They also observed that the larger the journal misalignment resulted from shaft deformation, the more obvious effect was seen on lubrication performance of journal-bearing.

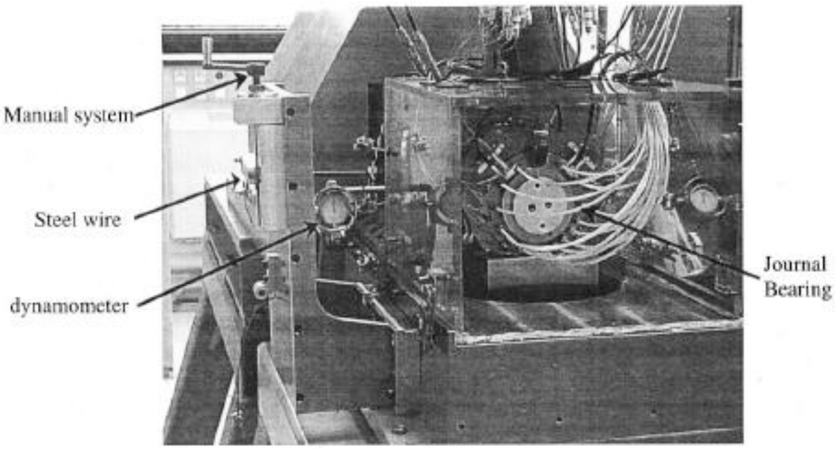

Fig. 28 Experimental rig designed by Bouyer and Fillon [393].

More recently, Yan et al. [251] developed a spatial revolute joint modeling and a test setup considering the journal misalignment, in which a group of contact force models were employed to describe different contact-impact phenomena. Figure 29 illustrates an overview of the test rig apparatus presented by Yan et al. [251]. It must be highlighted that the study for the first time established an experiment test rig of a spatial revolute joint with radial and axial clearances to reveal the characteristics of the relative motion between the journal and bearing. Yan and his co-authors demonstrated that the trajectories of the geometric centers of journal front and back end surfaces did not coincide, which meant the journal misalignment occurred. Their simulation results also indicated that the initial misalignment of the journal and the bearing could lead to the rotation of the bearing, which would further cause the system's out-of-plane motion.

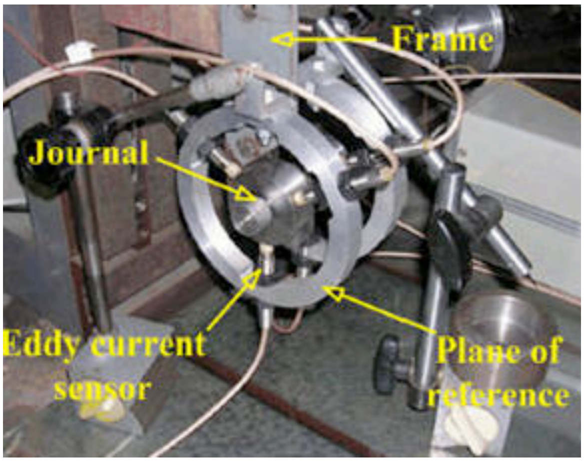

Fig. 29 Experimental setup for the revolute joint with radial and axial clearances [251].

4.2. Mechanisms with clearance joints

Over the last decades, a large number of works have been proposed based on various methodologies for modeling mechanisms with clearance joints, as described in detail in Section 3. Most of these studies, however deal with analytical and numerical models only. Thus, in the subsection, some of the most relevant experimental investigations on mechanisms with clearance joints are examined. In 1971, Wilson [299] presented a detailed description of both analytical and experimental approaches for a slider-crank mechanism, which allowed for the investigation of the influence of the clearance at the slider joint. It was found that the theoretical and experimental results agreed quite well [299]. Fawcett and Burdess [395] quantified the contact forces developed at clearance joints in a four-bar mechanism. These authors showed that the system's response tended to be smooth after certain period of analysis; i.e., the oscillations tended to disappear. Shimojima et al. [396] measured the local motion of the revolute joint with clearance using a stroboscopic approach. The correlation between the theoretical and experimental results did not agree too well, because the clearance size was too large (1 mm) and the friction and inertia forces were of the same magnitude. Earles and Kilicay [397] also experimentally investigated the dynamic response of mechanisms with clearance joints. The authors considered a spatial linkage with two clearance joints with distinct behaviors. Morita and his co-authors [106, 107] investigated the fluctuations in the driving moment in a four-bar mechanism with clearance joints. At that time, the outcomes produced were quite promising in terms of contact loss prediction between the journal and bearing elements. Dubowsky and Moening [119] were among the pioneers to experimentally study the interactions between clearance joints, using a Scotch-Yoke mechanism. In turn, Grant and Fawcett [398] investigated the effects of clearance size, lubrication, and material properties on the contact loss in a four-bar linkage with one clearance joint. Their experimental results confirmed the validity of the theoretical approach proposed, but only for a limited class of systems, but could not overcome the lack of universality of the method [82]. Deck and Dubowsky [113] reported some experimental results obtained from a spatial slider crank mechanism. Li et al. [399] presented a method to predict the occurrence of the contact loss in the clearance joint. The calculation did not have the need to solve the differential equations of system. They also made an experiment test rig to validate their numerical results, as shown in Fig. 30. The bearing between the coupler and rocker had an adjustable clearance. They found that the contact loss phenomenon occurred once almost always at a specific crank angle position (360°) during a rotation circle.

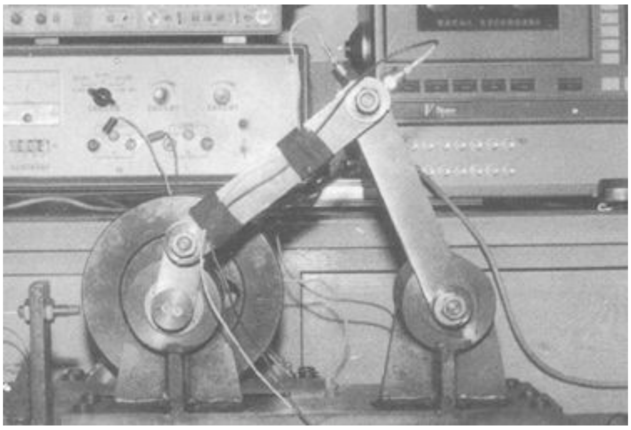

Fig. 30 Test rig of a four-bar linkage with a clearance at the coupler-rocker joint [399].

The problem of impacts associated with clearances has also been investigated experimentally by Pfeiffer [400]. Stammers and Ghazavi [401] showed aperiodic response in terms of accelerations for systems with clearances. For that, a four-bar mechanism a clearance joint between the coupler and follower links was considered. The contact loss between the journal and bearing elements was detected by using an electrical circuit. The theoretical and experimental data correlated well. Bengisu et al. [129] also studied the contact loss in revolute clearance joints of a four-bar mechanism, and the relative motion between the journal and bearing was measured by employing optical methods. The authors developed a separation parameter for a four-bar linkage, which was based on a zero-clearance analysis. The theoretical results were compared with the experimental ones and showed good qualitative agreement. They also predicted contact loss in a mechanism with multiple clearance joints. More recently, Tasora and his co-authors [402] also used an optical approach to analyze and visualize the local motion at revolute clearance joints in a four-bar mechanism test setup, as shown in Fig. 31. This experimental system was utilized to verify and validate the numerical method developed by these authors. Soong [21] presented an analytical and experimental investigation of the elastodynamic response characteristics of a planar linkage with bearing clearances. Soong and Thompson [403] presented a theoretical and experimental investigation of the dynamic response of a slider-crank mechanism with a revolute clearance joint where the slider and the connecting rod accelerations were quantified by using accelerometers. The correlation between analytical and experimental results was quite impressive. Jia and his co-authors [404] theoretically and experimentally investigated the dynamic response of a slider-crank mechanism with a clearance joint and showed the effect of clearance size and input speed on the behavior of the system.

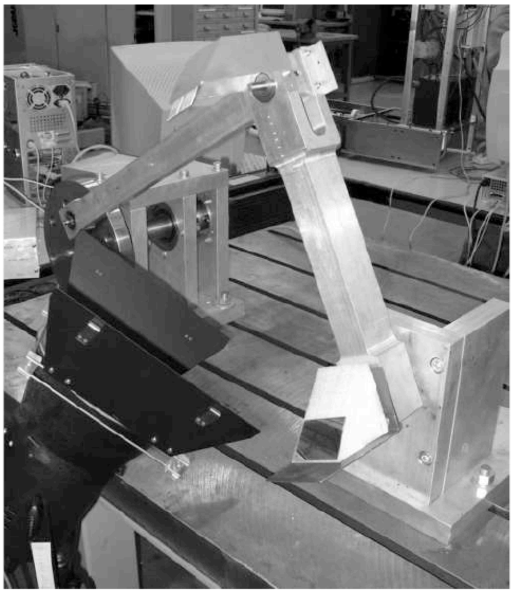

Fig. 31 The four-bar linkage test setup, with the mirrors mounted on the rocker by Tasora et al. [402].

As shown in Fig. 32, to verify the Lankarani and Nikravesh (L-N) continuous contact model, Ravn [319, 405] performed an interesting experimental investigation at Wichita State University on a double pendulum with a revolute clearance joint between the top pendulum and the impact pendulum. The motion of the pendulum was record by using a high-speed camera (1000 frames/s). This author found if the impact plate was made of steel, the simulation results of the numerical model were in a good agreement with the experiment data, while if the impact plate was made of rubber, deviation between the numerical results and the experiment data was observed. Furthermore, Ravn demonstrated that the selection of the contact parameters at the clearance joints had a significant effect on the system's response. The relative position of the pendulum arms at the instant of impact played a crucial role in the impact behavior [319]. This setup was further developed and used to validate the frictional impact analysis of multibody systems using a non-smooth methodology [406].

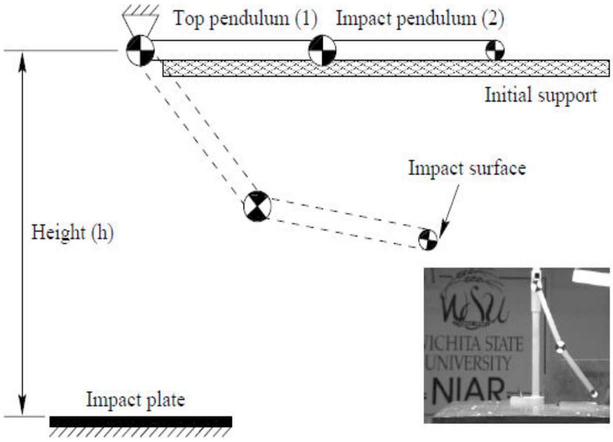

Fig. 32 The test rig of a double pendulum with/without a clearance joint [319].

Erkaya and Uzmay [98] presented an experimental investigation of the vibration and noise characteristics of a planar slider-crank mechanism having two revolute joints with clearance. Under the assumption of the continuous contact mode between journal and bearing, they also performed kinematic and dynamic theoretical analysis by employing the VML approach. The contact forces were obtained by evaluating the joint reaction force in the equations of motion, which was established by using the force equilibrium conditions. In their work however, the interaction of two clearance joints was not studied, and the VML approach was not applicable if the clearance size was large. To test the effectiveness of numerical simulation results, as shown in Fig. 33, Flores et al. [202] designed and constructed a slider-crank mechanism at Wichita State University. The radial clearance of the revolute joint between the connecting rod and slider was adjustable. The experimental data and the numerical results for different simulation and test scenarios were in a very good agreement with one another. Through the FFT analysis, it was found that the number and magnitude of the contribution of the dominant frequencies increased with the increase in crank speed and the clearance size. Haroun and Megahed [407] also theoretically and experimentally investigated the effects of clearances in a slider-crank mechanism. In this work, the system was modeled with a single clearance joint. The experimental data were shown to be close to those predicted by the analytical approach.

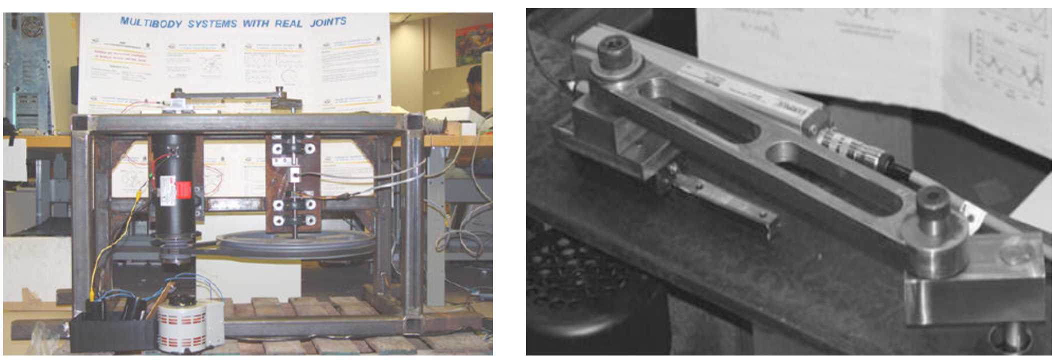

Fig. 33 Experimental test rig and its slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod [202].

More recently, Lai et al. [408] proposed an efficient procedure to evaluate revolute clearance joint wear of planar mechanism at low velocities. To verify their results, as shown in Fig. 34, a test-bed was designed to perform the wear tests. Their results indicated that when the increments of the wear depth of revolute joints were not large, the method could provide high prediction accuracy. However, as the predicted wear depth increased, the prediction error increased as well.

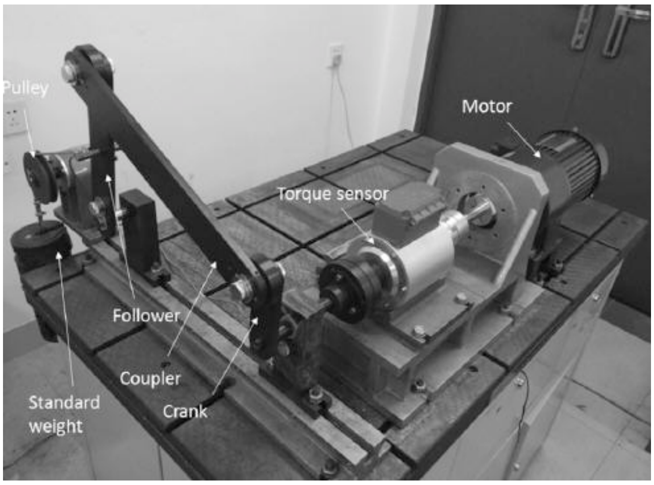

Fig. 34 Wear test-bed of the four-bar mechanism [408].

4.3. Mechanisms with flexible links

In this subsection, some of the works that combine the influence of clearance joints and flexibility of the links are revisited. In general, only few studies have included the influence of the bodies' flexibility in the dynamic performance of mechanisms besides the existence of gaps in the joints [97, 253]. Dubowsky and Young [409] supported the idea that the flexibility of the links could reduce the level of impact when mechanisms included clearance joints. This conclusion helps to the flexible links design for mechanisms' better dynamic performance. Moreover, these authors demonstrated the importance of clearances in machine joints on the amplification of connection forces, taking into account the friction at joints and the flexibility of the links. In a subsequent work, Dubowsky and Moening [119] demonstrated a reduction in the impact force level by introducing flexibility of the bodies. They also observed a significant reduction of the acoustic noise produced by the impacts when the system incorporates flexible bodies.

Through the co-simulation of ADAMS and ABAQUS, Khemili and Romdhane [410] showed that in the presence of clearance, the flexible rod of a slider-crank mechanism played a role of suspension in the mechanism by smoothening the dynamic responses and reducing the peak values, and that the dynamic behavior was more periodic as compared to rigid link case. To validate simulation results, they also designed and tested an experimental set-up, which consisted of a slider-crank mechanism with a rigid crank and a slider, as shown in Fig. 35. The authors also proposed an approach to account of flexibility of the links on the mechanisms' response when incorporating clearance joints. The numerical model was compared and validated with the experimental ones. The authors identified the existence of the three different types of motion between the journal and bearing elements in a revolute joint clearance.

Fig. 35 Slider–crank mechanism with a flexible connecting rod [410].

In turn, Erkaya and Uzmay [411] also designed a test rig for the slider-crank mechanism with two clearance revolute joints. They investigated how the negative effects of joint clearances on mechanisms could be decreased to some degree, by using both link flexibility and balancing. Figure 36 shows the experimental apparatus of the systems designed by these authors. The experimental results showed that joint clearance led to sudden changes in motion characteristics of the mechanism. The flexibility of the mechanism link had a crucial role in decreasing the negative vibration arising from joint clearance. Also, the undesired effects of clearance could be reduced to some degree by utilizing the balancing of the system.

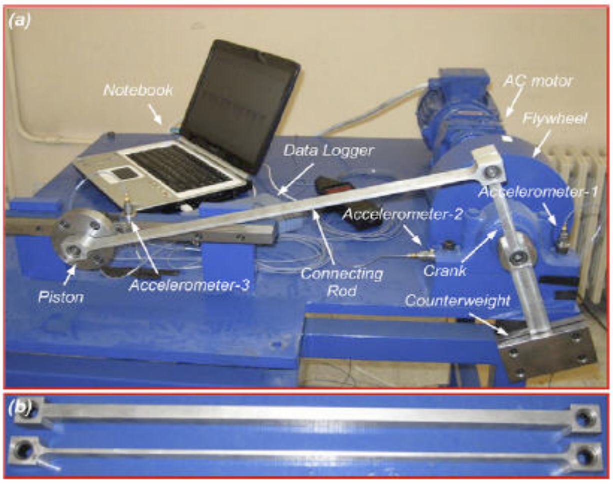

Fig. 36 Representation of experimental test rig designed by Erkaya and Uzmay [411].

More recently, Erkaya et al. [412] presented both numerical and experimental investigation to analyze the effects of joint clearance on partly compliant slider-crank mechanism with a small flexural pivot. Fig. 37 shows the experiment test rig of the studied mechanism. The pseudo-rigid-body model of the partly slider-crank mechanism was established and simulated by using ADAMS software. Their results showed that the flexibility of flexural pivot had a clear suspension effect that minimized the undesired outputs of joint clearance on mechanisms. Also, the small-length flexural pivot was an important tool to prevent the separation between journal and bearing by constituting a force-closed kinematic pair in a joint with clearance.

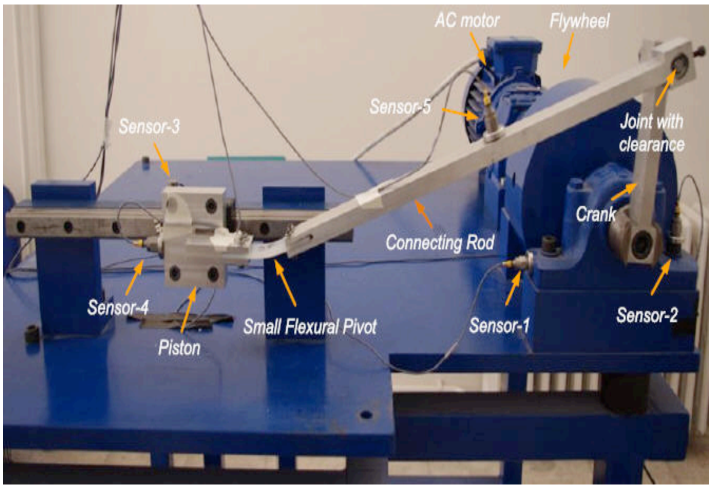

Fig. 37 The experiment test rig of a partly compliant slider-crank mechanism with a small flexural pivot [412].

Using commercial software co-simulation techniques, Zheng et al. [280] studied the dynamics of the flexible multi-link mechanism with clearance and lubrication for the ultra-precision presses. As shown in Fig. 38, to validate their numerical results, a test platform for the studied multi-link mechanism was established. The authors found that the numerical results for the flexible multi-link mechanism with lubricated clearance joint model agreed more closely with the experimental data than those associated with dry clearance model.

Fig. 38 The test platform of multi-link mechanism for high-speed press system [280].

More recently, Ahmedalbashir [413, 414] numerically and experimentally studied a four-bar mechanism with clearance joints and flexibility of the links. With the purpose of reducing the level of impact, a spring element was used between the moving links that were connected by a revolute joint with clearance. Figure 39 shows the experimental apparatus of the actual mechanical system developed. This author demonstrated that the flexibility of the links in a mechanism with clearance joints played a crucial role in its dynamic behavior. The correlation obtained between analytical and experimental data was quite well.

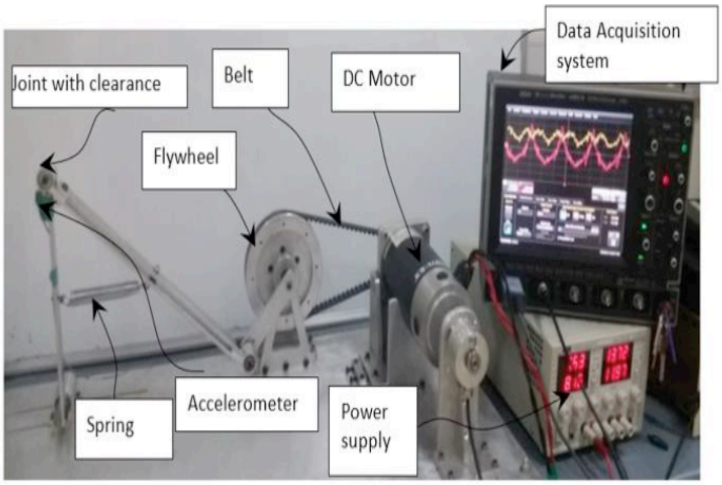

Fig. 39 Experimental apparatus of a four-bar mechanism with a spring element and clearance joint [413].

4.4. Other experiments with clearance joints

Over the last years, some experimental works on clearance joints that involve other aspects have been published. Dupac and Beale [415] investigated a planar slider-crank mechanism with translational clearance joints in which several cracks at the connecting rod were also considered. For this, the connecting rod was modeled with lumped masses. These authors utilized the kinematic Newton's impact law to model and analyze the contact-impact events developed at the clearance joints. Using the Lyapunov exponents, it was demonstrated that the clearance and imperfect links had a significant influence on the system's behavior. This study highlighted the importance of considering clearance in the dynamic design of mechanisms. In turn, Mukras et al. [416] conducted a numerical and experimental study to model and analyze some tribological phenomena at clearance joints, such as contacts, impacts, friction, and wear. The numerical tool developed by these authors was based on the multibody systems formulation, which incorporated the above tribological aspects. Numerical and experimental data were analyzed for ideal and non-ideal joint models for a slider-crank mechanism. Liu and co-authors [417] designed and built an experimental apparatus to compare and validate different contact force models for the case of revolute joints with clearance. Both conformal and nonconformal cases were investigated by these authors. Because the analytical and experimental approaches did not agree, the authors proposed a new formulation that permitted more accurate prediction of the local deformations and contact forces evolution for three-dimensional revolute joints with clearance. The outcomes produced were compared with the classical Hertzian contact theory and also with experimental data obtained with the test rig developed. The benefits of the new approach when compared with simple and traditional contact force models were demonstrated. More recently, Koshy et al. [418] utilized an experimental slider-crank mechanism with a revolute clearance joint to investigate and compare different contact force models, as illustrated in Fig. 40. The intra-joint contact forces developed during the contact-impact phases were evaluated by employing several different elastic and dissipative force models described in this work. An ad hoc approach was used to evaluate the system's combined damping from the stiffness values from the ESDU tribology-based model and the Hertzian-based Lankarani-Nikravesh (L-N) model. Based on the general results obtained from the computational and experimental analysis it was observed that the ESDU tribology-based contact force model with the ad hoc damping provided results that reasonably matched experimental test results. Overall, it was concluded that selection of the appropriate force model together with the dissipative term played a crucial role in the dynamic behavior of multibody systems involving contact events. This issue was particularly relevant when the systems operated at low or moderate impact velocities and low coefficient of restitution.

Fig. 40 Experimental slider-crank mechanism utilized by Koshy et al. [418].

Meng et al. [419] investigated on high-speed and heavy-load mechanisms with multiple clearance joints. Results for 15 clearance joints were the cause for hysteresis effects on the velocity and acceleration levels. Furthermore, the authors have also experimentally demonstrated the existence of three different modes of motion within a clearance revolute joint, namely the free-flight, the continuous or permanent contact motion, and the impact mode. The severity of the contact between the journal and bearing surfaces at a clearance joint was evident by the severity of the local elastic and plastic deformations. Akhadkar et al. [420] presented a complete model for three-dimensional revolute joints with dry clearance, which was applied to an industrial circuit breaker system. Moreover, some experimental work was conducted to verify and validate the contact forces developed at joints. The theoretical and experimental results agreed quite well in terms of contact forces evolution during the contact process. More recently, Skrinjar et al. [421] developed a validated model for a pin-slot clearance joint. Based on the pin position in the slot, different contact models were used to evaluate the contact forces. The numerical results were also validated by the experiment test setup, depicted in Fig. 41. The external force was introduced using a stinger mounted on an electro-dynamical shaker. The applied force was measured using a force sensor. Additionally, a force sensor was used to acquire the contact forces on body 1. They concluded that when compared to the experimental observations, the Lankarani-Nikravesh (L-N) contact force model was found to provide the best correlation.

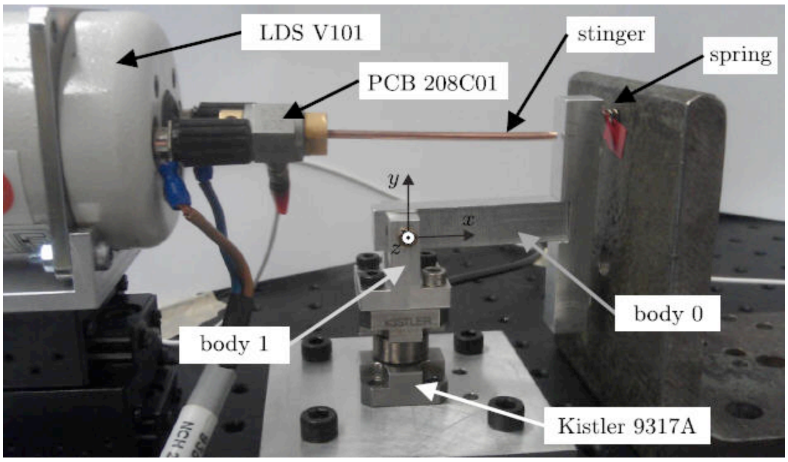

Fig. 41 Experimental multibody system with a pin-slot clearance joint developed by Skrinjar et al. [421].

5. Other issues related to clearance joints

5.1. Wear in mechanisms with clearance joints

According to the standard DIN 50320, wear can be defined as "the progressive loss of material from the surface of a solid body due to mechanical action; i.e., the contact and relative motion against a solid, liquid or gaseous counter body" [422]. The work by Meng and Ludema [423] shows that there are more than 300 studies for modeling the wear and friction phenomena. In a broad sense, there are two main wear models commonly used in tribology field, namely the Reye's model [424], and the Archard's model [425]. The two models correlate the wear volume with some physical and geometrical properties of the bodies in contact, such as applied load, sliding distance and hardness, among others. It has been shown that the wear behavior is the prominent characteristics of dynamic analysis of mechanisms with clearance joints [26, 29, 32, 416].

Strömberg [426] proposed a method for planar structural dynamics problems with friction and wear. The wear effects were simulated by using a wear gap variable in the nonsmooth contact dynamics method proposed by Moreau [427], and a generalization of the Archard's wear law was presented. The results indicated that for solving contact problems with friction and wear, the proposed method could avoid the singularity problem of the classic quasi-static assumption. Using the LCP algorithm, Slavič and Boltežaer [428] modeled and simulated a planar multi-rigid-body system with rough contacting surfaces. In this work, a wear model based on the local loss of mechanical energy was proposed to reshape the rough contacting surfaces under dynamical loads. Under the framework of multibody systems formulation, Flores [429] studied the wear of a planar rigid four-bar mechanism with a revolute clearance joint. In order to correctly compute the area covered by sliding distance, the joint surface was then divided into several sectors before starting the simulation, and the wear depth in each sector was obtained by integrating the first differential form of the Archard's wear law. At the end of each motion cycle, the journal and bearing radii were updated according the total amount of wear depth. Figure 42 shows a schematic representation of this wear model approach. This work revealed that the wear depth along the joint surface was nonuniform. The computational efficiency was a major limiting factor for performing wear prediction of clearance joints. Much of this limitation arose from extensive geometry calculations required by contact analyses. Lin et al. [430] proposed a Kriging surrogate modeling approach to improve the computation efficiency of the wear prediction in the total knee replacement process. According to their work [430], the computational speed of wear simulations was reduced from an estimated 230 hours to 4 hours per analysis.

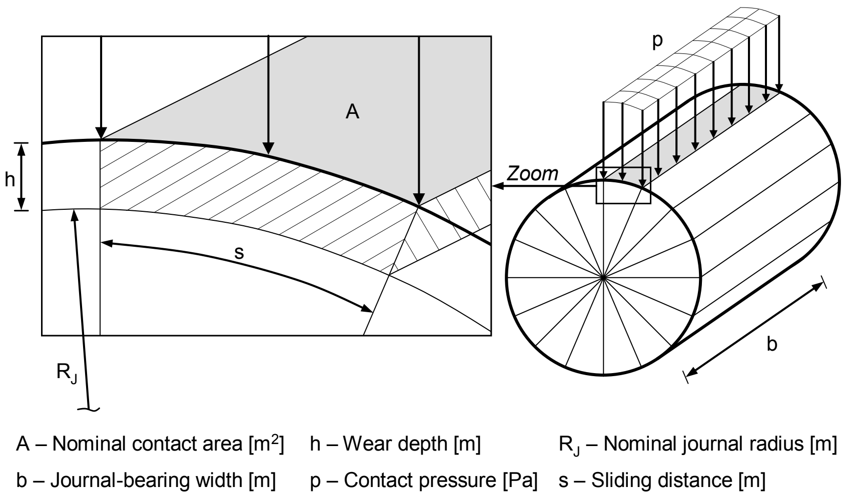

Fig. 42 Journal surface divided into several sectors to conduct wear analysis [429].

Mukras and his co-workers [431] investigated the wear of a planar clearance pin-and-pivot joint with oscillatory contacts. In their work, the wear depth was obtained by integrating the differential form of the Archard's wear law and using the forward Euler formulation. The contact pressure was computed using the commercial software, ANSYS. To avoid the remeshing of the contact interface geometry at the end of each step and to improve computation efficiency, they also proposed an adaptive extrapolation scheme and an intermediate cycle-update parallel scheme. However, the friction effect on the wear was not considered, and the wear of the classic multibody system with large displacement and relative rotation was also not addressed. The wear prediction on the components of a mechanical system without considering the system as a whole will, in most cases, lead to inaccurate predictions. Mukras et al. [416] further studied the joint wear of a planar rigid slider-crank mechanism. According to their work, the contact force at each time increment is firstly calculated using the L-N contact model [145] and then used to evaluate the contact pressure through a finite element analysis. The incremental wear depth could then be computed, and the geometry could be updated. The results were also validated against those from actual experiments.

Using the VML method to describe the clearance, Su et al. [432] proposed a numerical approach of wear prediction integrating with system kinematics. In their work, the differential form of the Archard's wear law [425] was used to calculate the wear of the clearance flexible bushing, which was modeled by using the commercial software ANSYS. The Archard's wear law is known to be based on the unchanged contact stress and surface assumption, which was proven to be incorrect [433]. Therefore, to predict the accurately wear, the flexible bushing model in ANSYS was remeshed frequently during the computation process, which significantly increased the computation cost. The assumption of continuous contact was ensured by the tension force of the spring attached to a slider. Their simulation results were also validated by with experimental results. Using the hybrid contact force model proposed by Bai and Zhao [200] and the Archard's wear model, Bai et al. [434, 435] also predicted the wear of a planar revolute clearance joint of a rigid four-bar mechanism. Also using the differential form of the Archard's wear law Zhao et al. [436-438] studied the wear problem of a planar flexible slider-crank mechanism with a revolute clearance between the connecting rod and the slider. In their work, the flexible crankshaft or connecting rod were meshed by using the locking-free shear beam element of ANCF, and the effects of the flexibility of the crankshaft or connecting rod and the clearance size on the joint wear were comparatively studied. In order to determine the wear coefficient at different contact pressures, the radial basis function neural network (RBFNN) technique [437] is introduced. As shown in Fig. 43, the flexible bushing in the clearance joint was modeled by using the commercial software ABAQUS [438]; however, its motion was not considered. They found that [438] both the maximum wear depth and the total wear volume in the case of flexible rod were less than those in the case of rod, and that the maximum wear depth increased with the increase of the clearance size.

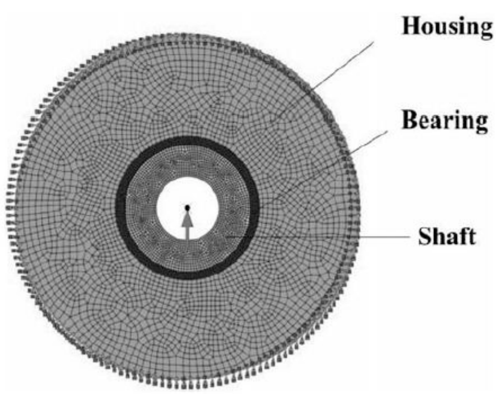

Fig. 43 ABAQUS bearing finite element model [438].

More recently, Askari and his co-authors [439] predicted a linear and volumetric wear of a human rigid hip joint model by also using the Archard's wear model [425]. As shown in Fig. 44, in order to determine the exact area covered by the sliding distance, the joint surface was divided into several subdomains before starting the dynamic simulation. They found that the friction-induced vibration significantly increased predicted wear rates in ceramic-on-ceramic hip prostheses. However, the lubrication condition was not considered in their study [439]. Based on the Archard's wear model, Wang et al. [440] predicted the 3D wear depth of the spherical clearance joint socket in a 4-SPS/CU parallel mechanism. The simulation results showed that the wear depth in different directions along the socket surface was nonuniform, in every direction. They also found that the 3D main wear areas were closer to the boundary of the socket because the kinematic trajectory of parallel mechanism determined the kinematic scope of spherical joint.

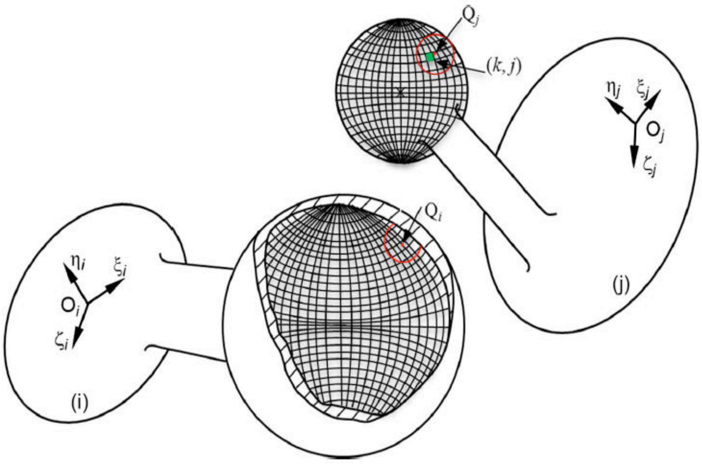

Fig. 44 A representation of subdomains on the cup and head surface [439].

Similar to the work by Flores [429], Xiang et al. [441] studied the influence of clearance size and driving power (speed) on the clearance wear of a planar rigid slider-crank mechanism with one revolute clearance joint. According to the Archard's wear model, contact pressure and sliding distance are the two main factors to determine the wear rate. However, the contributions of contact pressure and sliding distance to the wear are generally different for different contact states and different joints. In the work by Xiang et al. [441], the index of concordance was introduced to reveal the implicit relations among the contact pressure, sliding distance, and wear rate. The cosine matching function (CMF) was employed in their study to evaluate the index of concordance. Li et al. [10] studied the influence of the clearance sizes on the wear of a planar rigid slider-crank mechanism with two revolute clearance joints. They found that appropriate relationships between two clearance sizes would significantly decrease the wear of two clearance joins, which was quite important for the performance of mechanical system, specially its service life. Li et al. [442] and Zhu et al. [443, 444] proposed an integrated wear prediction model to perform wear prediction of planar revolute clearance joints for crank-slider mechanisms. Similar to the VLM method, the clearance joint reaction force was calculated by a stationary clearance link algorithm (SCLA). An efficient unsymmetrical Winkler foundation model was also proposed to calculate the contact pressure. The Archard's wear model was adopted to calculate the wear amount of contact surface. The comparison of the prediction data and the experiment tests (Fig. 45) showed that the integrated wear prediction model was accurate and computationally efficient. Wang and Liu [445] modeled and simulated wear in a planar five-bar linkage considering clearance joints and flexibility of the links. For that, the absolute nodal coordinate formulation (ANCF) was used to obtain the equations of motion. The intra-joint contact forces were calculated using approach proposed by Flores el al. [187], while the wear at the clearance joints were computed by employing the Archard's model. Nikolic et al. [446] proposed a new algorithm to predict the wear of internal combustion engine crankshaft main bearings for design purposes. The algorithm was applied to a six-cylinder diesel engine crankshaft.

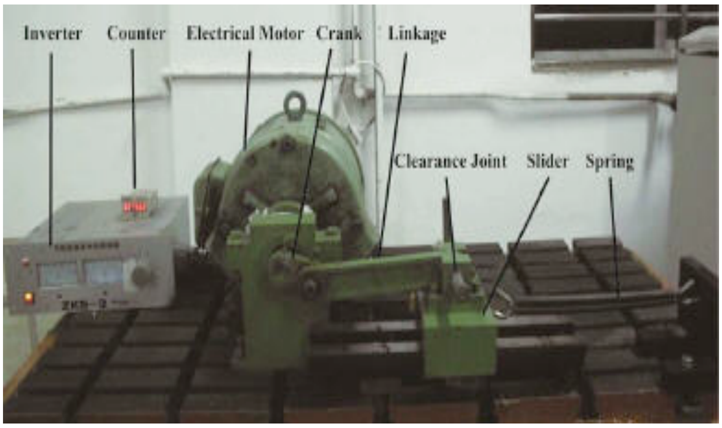

Fig. 45 The wear experimental device of a crank slider mechanism with a clearance joint [443].

5.2. Optimization and control of mechanisms with clearance joints

The dynamic behavior of the mechanisms with clearance joints is in general quite sensitive to small changes of parameters, namely the clearance size and the friction coefficient. In fact, the system response can change from periodic to chaotic with a very small variation of the initial parameters. Seneviratne and Earles [53] used a massless link model to study the behavior of a four bar mechanism with a clearance joint. Farahanchi and Shaw [301] studied the dynamic performance of the slider-crank mechanism with a slider clearance. This research showed that both periodic and chaotic behavior depends on the values of friction coefficient and crank velocity. Moreover, different types of motion between the clearance joint elements have been identified, namely: "free-flight" mode, "impact" mode, and "following" or "continuous" mode. Stammers and Ghazavi [401] demonstrated the occurrence of aperiodic behavior in terms of accelerations for systems with clearances in the case of a four-bar linkage with clearance joints. Rhee and Akay [447] used a discontinuous contact force model to analyze a four bar mechanism with one revolute clearance joint. They showed that, depending on the friction coefficient and clearance size, the system could exhibit periodic or chaotic behavior. More recently, Ravn [319] and Flores [148] demonstrated, through the use of Poincaré maps, that the dynamic response of mechanical systems with clearance joints can be periodic, in some situations but also chaotic, for other conditions. Chunmei and co-workers [54] showed that mechanisms with dry clearance joints have less stable behaviors when compared with those modeled with ideal or perfect joints only. The same conclusion was also observed by Farahan et al. [448], Xiang et al. [449] and Varedi et al. [450].

Sun and Xu [451] investigated the output motion error of an offset slider-crank mechanism with clearance joints by using the ADAMS software. By optimizing the design parameters of the mechanism, the transmission angle error decreases by 8.44%。 An optimization procedure was then implemented to obtain the design parameters that ensured a minimum position error. In turn, Jawale and Thorat [452] utilized a serial chain two revolute joint planar manipulator to study the position error of the end effector in workspace, where the clearance influence was taken as a major factor. The authors showed that the maximum error in the end effector position could be effectively predicted when clearances were considered in the analysis. Chaker et al. [14] also investigate the position errors of a spherical parallel manipulator with dry clearance joints and manufacturing tolerances. Their results indicated that the position error was dependent on configuration and its maximum values occurred near singular configurations.

With the purpose of minimizing the manufacturing costs, Choi et al. [44] proposed an analytical approach for the tolerance optimization of the planar mechanisms with lubricated joints based on mechanical error analysis. The mobility method was applied to consider the lubrication effects, and the clearance vector was used to describe the clearance revolute joint. In the optimization process, the uncertain link lengths and radial clearances were selected as random design variables. Their results showed that the optimum tolerance allocations not only could satisfy all the kinematic constraints but also benefit the reduction of the manufacturing cost values. Under on the assumption that the pin was always in contact with the joint socket, Feng et al. [139] proposed an optimization method to reduce the oscillations or changes in clearance joint contact force by re-distribution of masses of the moving links of a planar four-bar mechanism with three clearance joints. The clearances were described by using the virtual mass-less link (VML) method. In this method, the massless imaginary link length was equal to the clearance size. However, this assumption was not always valid, especially for large clearances, because the journal and bearing cannot be in contact at all times.

Zhao et al. [453] presented an investigation on the random behavior of a slider-crank mechanism with clearance joint. The dynamics equations of motion were derived using the well-established Lagrange's equation. The approximation function that related the system random parameters and system's dynamics was obtained by employing a neural network. The study presented results for the dynamic system's characteristics taking into account the physical and geometrical parameters of the slider-crank mechanism. Erkaya and Uzmay [103] proposed a neural-genetic scheme for modeling the joint characteristics and minimizing the undesired effects due to clearances. The joint clearance was also modeled by using the VML method and its kinematics equations with respect to the input variable were characterized using the Neural Network. The transmission angle was optimized by using the Genetic Algorithm (GA) approach. However, this work was only focused on the kinematic motion optimization process, and the dynamic impact effects were not considered. Erkaya and Uzmay [454] further presented an investigation of joint clearance influences on the mechanism path generation and transmission angle. The GA approach was also used to determine the direction of the joint clearance described by the VML and also to minimize the error between desired and actual paths. Erkaya [455] again proposed a methodology for modeling and optimization of a planar rigid Jansen's mechanism used in a walking machine. Two clearance revolute joints were considered in their study, and the Lankarani-Nikravesh continuous contact model [145] was used to evaluate the normal contact forces in the clearance joints. The joint clearance characteristics such as magnitudes of clearance vectors and their angular variations were achieved by using the adaptive network-based fuzzy inference system (ANFIS). Their optimum values of design variables were shown to minimize the trajectory errors. Cammarata [456] described a novel method to find the nodal displacements and rotations of overconstrained mechanisms due to clearance-affected joints. The proposed method required a single elastostatic analysis based on the Principle of Virtual Work. The results were also validated by using the ADAMS software.

By minimizing the system's potential energy by means of the sequential quadratic programming method, Younes et al. [457] optimized the relative positions of spatial mechanical elements connected by parallel clearance joints. The kinematics relations of the clearance joint elements were introduced as the constraint conditions of the objective function (potential energy) into the optimization process. The flexibility of the mechanical elements was not considered in this study, and consequently, the strain energy was not included into the system potential energy. Zhang et al. [458] proposed a design-of-experiment (DOE)-based method for optimizing the performance a slider-crank mechanism with a revolute joint clearance between the connecting rod and slider. To reduce computation cost, a Kriging surrogate model was established and using the sample points, which were selected from the ADAMS simulation results by using the Latin hypercube sampling (LHS) method. The GA algorithm was used to obtain optimal results on the design parameter. Their numerical results indicated that the dynamic response of the mechanism did not significantly change with a change in the contact stiffness coefficient, and in general, the reduction in the input crank speed and clearance size would reduce the contact force between the bearing and the journal. Sardashti et al. [459] performed an optimal path generation synthesis of a planar rigid four-bar mechanism with different number of clearance revolute joints and 20 specified trajectory points. The clearance joint was described by using the VML method, and the Particle Swarm Optimization (PSO) based algorithm was used to solve this highly nonlinear constrained optimization problem. Also, using the PSO based optimization algorithm, Varedi et al [450, 460] successfully reduced the impact forces in the clearance joint by optimizing the mass distribution of the links for a planar rigid slider-crank mechanism with one or two clearance joints. In this work, the normal impact force was calculated by using the Lankarani-Nikravesh contact model [145], and the friction force was evaluated based on the modified Coulomb's friction law proposed by Ambrósio [147]. By incorporating the continuous contact constraint condition into the equations of motion, Yaqubi et al. [461] proposed an input driving torque control scheme to maintain continuous contact at clearance joints of a planar rigid slider-crank mechanism. The system's nonlinear dynamic behaviors were comparatively analyzed by using the Poincaré maps and bifurcation diagrams. The results showed that, for maintaining continuous contact in a mechanism with clearance joints, in some cases larger amounts of the input torque may be needed. Again, Yaqubi et al. [462, 463] introduced a continuous contact constraint equation into the dynamics equations of a planar rigid-slider mechanism with two clearance joints, and derived the required crank control torque to maintain the continuous contact state in the clearance joint. Their numerical results indicated that the continuous contact state could be established. However, using one actuator mounted at the crank joint for the control scheme might cause the actuator to reach its saturation limitation. Therefore, to control a slider-crank mechanism with link flexibility, they further introduced an actuator mounted at the joint connecting the crank and the connection rod. By using the enhanced control scheme with two actuators, not only the continuous contact could be established, but also the journal and the bearing in clearance joint would remain in contact in exactly one point. Thus, the journal would no longer rotate around clearance circle. Moghadasi et al. [464] performed a topology optimization of a planar slider-crank mechanism with a rigid crank and a flexible connecting rod. An efficient modified approach based on the classic Hertzian contact law was presented to model the revolute joint connected to the sliding mass and the crank. The solid isotropic material with penalization (SIMP) approach based the topology optimization was used to optimize the flexible connecting rod, and the goal was to minimize the deviation of the flexible system's responses from corresponding rigid system. They found that the proposed modified approach could enhance the accuracy of bearing domains in topology optimization of flexible multibody systems, and that it was more efficient than the nonlinear finite element-based approach.

Using the Poincaré portrait, Rahmanian and Ghazavi [465] analyzed the chaotic and bifurcation behavior of a planar rigid slider-crank mechanism with revolute clearance at connecting rod-slider connection. They observed that for some clearance sizes the system response was periodic, while in most cases it exhibited chaotic behavior. In their work, the bifurcation diagrams of the system responses versus clearance size provided a useful technique for understanding the effect of clearance size on the system nonlinear dynamics behavior. According to their work, however, the runtime simulation for plotting each bifurcation diagram took approximately 40 hours. To avoid the chaotic motion and to maintain continuous contact of clearance joint, and according to the Pyragas method introduced by Pyragas in 1992 [466], Olyaei and Ghazavi [467] introduced an extended delayed feedback control (EDFC) unit into a planar rigid slider-crank mechanism with a revolute clearance joint between the slider and the connecting rod, as shown in Fig. 47. The numerical results demonstrated that the controlled system achieved a stable and periodic service performance.

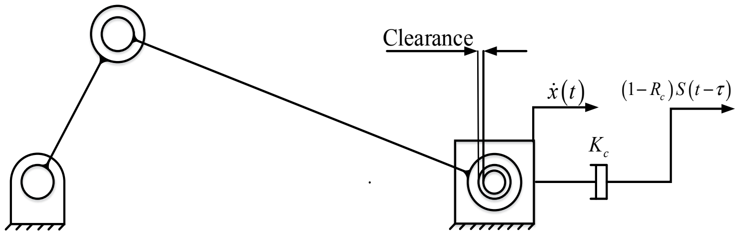

Fig. 47 The EDFC controlled planar rigid slider-crank mechanism with a revolute clearance joint between the slider and the connecting rod [467].

Systems with joint clearance occupy a particular place within the set of controlled Lagrangian systems with unilateral constraints and impacts. The major issues for the control of such systems are the under-actuation. More recently, Brogliato [468] studied the problem of feedback control of mechanisms with joint clearance, and three important feedback controllers, mainly the impactless trajectories with persistent contact, control through collisions, the stabilization of equilibrium points, and the trajectory tracking control, were comparatively analyzed. Zhao and his co-authors [469] presented a methodology to improve the accuracy prediction of dynamic behavior of planar mechanism with dry clearance joints, with particular application to a reciprocating compressor. For this, the authors used the ADAMS software to model and solve the equations of motion that incorporated the normal and tangential contact forces developed at the clearance joints. The results showed a good correlation between the computational results and experimental data.

5.3. Uncertainty of mechanisms with clearance joints

Large majority of previous studies on the dynamic modeling and analysis of mechanisms with clearance joints are based on the dynamics equations with deterministic system parameters. However, a practical mechanical system usually contains many uncertain parameters, which cannot be exactly measured, such as the rigid body inertia, the material Young's modulus, and the joint clearance size. These uncertain parameters may lead to a quite different response compared to those of the corresponding deterministic system. Thus, to capture the real dynamic response of a physical practical multibody system, the parameter uncertainty should be considered in the dynamics model. There are two main types of methods to describe the uncertain parameters [470-472], namely the probabilistic methods and the non-probabilistic methods. The probabilistic methods are usually considered to describe the random parameters with known probability density functions, while the non-probabilistic methods are mainly utilized to solve the uncertain problems with uncertain, but bounded parameters.

Pang et al. [473] proposed a modeling approach that allowed for the analysis of accuracy reliability and sensitivity of planar mechanisms with multi-factors such as clearances, manufacturing and assemble tolerances among others. The authors demonstrated the effectiveness of the proposed method to an aircraft door retractable mechanism. Zhang and Du [474] used a hybrid method combining the Hybrid Dimension Reduction Method (HDRM) [475] and the envelope method (originally for interval reliability without clearances) to conduct a time-dependent (interval) kinematic reliability analysis for a function generation mechanisms with random joint clearances. Pandey and Zhang [476] presented an accurate and efficient method for evaluating the reliability of robotic manipulators with random clearance joints. The principle of maximum entropy was used to compute the extreme value distribution of the position error.

Yan and Guo [477] studied the dynamics of a planar flexible four-bar mechanism with three revolute clearance joints. In their work, the dynamic equations of the flexible mechanisms were established using the impact-pair model of clearance connection and the kineto-elastodynamics (KED) method, and the link lengths, pin radius, and radial clearance size were assumed to be the random parameters with normal distribution. The dynamic equations were solved using the Newmark algorithm, and the Monte Carlo method was employed to analyze the effects of parameter uncertainty on the system's motion accuracy. Using the probability methods, Zhu and Ting [47] established a general approach to derive the probability density function of the endpoints of planar and spatial robots. This approach can provide a convenient tool to obtain the probability value for a robot to position its end effector within a desired tolerance zone. Ting et al. also [38] presented an approach to identify the worst position and direction errors due to the joint clearance in linkages. The uncertain link orientation and point position caused by joint clearance in linkages were determined by using the N-bar rotatability laws. They modeled joint clearance as a small virtual link at the joints. Chen et al. [478] presented a unified approach to predict the kinetics accuracy of the general planar parallel manipulators (PPM) both due to the actuated joint input uncertainties and the joint clearance. In their work, utilizing the generalized kinematic mapping of constrained plane motions, the manipulator end-effector's exact output error bound for a specified configuration were obtained as an accurate and complete description for the manipulator's accuracy performance. The readers interested in the influence of the joint clearance on the mechanism's kinematic responses are also referred to the work by Parenti-Castelli and Venanzi [260], the work by Chaker et al. [14], and the work by Wu and Rao [479]. However, in these works [14, 38, 47, 60, 479], the mechanisms' dynamics effects were not considered.

Using the Archard's wear law, Sun et al. [480] studied the wear of a slider-crank mechanism with clearance joint driven by a harmonic drive involving random and epistemic uncertainty. The mechanisms with rigid/flexible connecting rod were comparatively studied, the random uncertainty was represented with probability, and the interval analysis was used to represent epistemic uncertainty. Also, the confidence region method (CRM) was used to quantify the uncertainty effect, and the Kriging surrogate model was established to improve the computation efficiency. Their results indicated that the suspension effect of collisions for clearance with application of harmonic drive was superior to that by using the flexible connecting rod. The study also showed that when both random and epistemic uncertainties were considered, the wear volume boundary would be wider than that when only the random uncertainty was considered. Sun and Chen [481] simulated a planar rigid slider-crank mechanism with a clearance joint between the connecting rod and the slider. The bearing radius and link length were epistemic uncertain variables and represented by interval parameters. To improve computation efficiency, the surrogate models for the uncertain system equations of motion were constructed using the neural network approach. They concluded that comparing with probability theory the results obtained by the proposed method was more reliable when there was parameter uncertainty with incomplete knowledge. Furthermore, Sun [482] studied the effects of the interval clearance size on the tip trajectory of a two-link flexible manipulator with lubricated revolute joint. A fuzzy self-tuning proportion integration differentiation (PID) control was applied to track the desired tip trajectory of the manipulator. The manipulator flexible links were meshed using the two-dimensional shear deformable beam element of ANCF. Numerical results showed that the uncertainty of the manipulator would reduce the control accuracy.

Wang et al. [483] studied the dynamics of a planar slider-crank mechanism with an interval clearance revolute joint between the flexible connecting rod and rigid slider (similar to the example shown in Fig. 3). The uncertain joint clearance size was considered as an interval parameter [472]. The mechanism was meshed by using one unified mesh of absolute nodal coordinate formulation (ANCF). The flexible connecting rod was meshed using the finite elements of the ANCF, while the rigid parts were described via the ANCF Reference Nodes (ANCF-RN) [484]. By using the Chebyshev tensor product (CTP) sampling method [485], a computational efficient surrogate model of the interval differential algebraic equations (IDAEs) was established. To ensure a continuous contact of the clearance joint, a spring element was introduced to the original EDFC controller. The results indicated that the modified EDFC controller could be utilized to reduce the chaotic motion of the planar flexible mechanisms with interval clearance revolute joint. In the work by Wang et al. [483], the OpenMP directives were used to parallelize the computation processes, which improved the computation efficiency significantly. Luo et al. [486] proposed a robust formulation for synthesis of mechanisms with truncated dimension variables and interval clearance variables, which was applied to a four-bar linkage of function generating type. For that, a quality loss function was defined with the mean and standard deviation of the maximal motion error with respect to interval clearance variables, which allowed for the reduction of the motion error.

For a specific contact problem, it is difficult to determine the parameters of the contact force models. These parameters can also be considered the uncertain parameters. The uncertain parameter estimation methods have attracted many researchers' attention. Ma et al. [487] proposed a modified linear estimation method to identify the model parameters of the L-N contact force model according to the Taylor series expansion and exponentially weighted recursive least squares (EWRLS) estimation methods. Their numerical example results indicated that the estimated results and actual ones were in a good agreement under a wider range of initial conditions. Haddadi and Zaad [488] proposed a new method for online parameter estimation of the Hunt-Crossley contact force model and provided a mild set of conditions for guaranteed unbiased estimation. The readers can gain an insight on this topic in other works by Ma et al. [489, 490], by Haddadi and Zaad [491], and by Shin et al. [492].

5.4. Link flexibility in mechanisms with clearance joints

Some works on the effect of flexibility of the links on the performance of mechanisms with clearance joints has also been mentioned in the previous sections. Nevertheless, due to its relevance and importance, this issue deserves to be treated and highlighted here as an independent section. As it was presented earlier, there are some authors who experimentally investigated the effect of flexibility of the bodies in mechanisms with clearance joints [119, 280, 410-413]. In fact, many researchers have studied the influence of the link flexibility of a mechanism on the dynamic performance of multibody systems in presence of clearance joints. Dubowsky and Gardner [96] were pioneers in investigating the effects of clearances and flexibility of links on the level of stresses in joints of high-speed linkages.

They demonstrated that the links flexibility tends to reduce the clearance joint stresses. Winfrey et al. [97] also studied mechanisms with flexible links and clearances. A cam-follower valve train mechanism was utilized as an application example. Soong [21] presented an analytical and experimental investigation of the elastodynamic response characteristics of a planar linkage with bearing clearances. Dubowsky and Moening [119] observed a significant reduction of the acoustical noise produced by the impacts, when the system incorporates flexible bodies. They also observed a significant reduction of the acoustic noise produced by the impacts, when the system incorporated flexible bodies. Kakizaki et al. [120] presented a model for the spatial dynamics of robotic manipulators with flexible links and joint clearances, where the effect of the clearance was assessed in the control of the robotic system. Deck and Dubowsky [113] theoretically and experimentally investigated the dynamic behavior of mechanical systems with flexible links and clearance joints. The flexibility of the links was shown in this study to be responsible for the reduction of the level of vibrations and impacts. Ravn [24] investigated the influence of joint clearance on a planar multibody system including the effects of lubrication and link flexibility. Chunmei et al. [54] also showed that the level of impact forces was reduced with flexibility of bodies considered in the formulation. Schwab and his co-authors [343] investigated the dynamic response of mechanisms with clearance joints and link flexibility as well.

Erkaya et al. [287] investigated the dynamics of a partly compliant mechanism with a spherical joint with clearance. It was observed that the flexibility of the small pivot helped to reduce the mechanism chaotic responses arising from clearance joints. Erkaya et al. [411] also presented both numerical and experimental investigations to analyze the effects of joint clearance on partly compliant slider-crank mechanism with a small flexural pivot. The dynamic responses of a planar four-bar mechanism having two revolute clearance joints and coupler link flexibility were studied by Erkaya and Uzmay [493]. The normal contact force in the joints with clearance was evaluated using a nonlinear spring-damper model, and the friction effect was considered using the Coulomb friction model. They observed that the value and number of contact force peaks for the rigid mechanism were larger than that of the flexible mechanism. The link's flexibility was shown to provide a damping effect for the mechanism. Erkaya and Doğan [494] studied a planar compliant slider–crank mechanism having joints with clearance, and compared the effects of joint clearance. A pseudo-rigid body model of the flexible connecting rod was constituted. They found that in case of lower running speed and higher clearance size, the small-length flexural pivot in compliant mechanism exhibited a good suspension effect for the peak values arising from clearance. The readers can find more interesting works by Erkaya and his co-authors in the references [495, 496].

Shiau et al. [57] studied the effect of links' flexibility on a 3-PRS mechanism with clearance joints. They demonstrated that the clearance joints affected the mode shapes by which the rotational motions were dominated. They showed that in some cases, the natural frequencies decreased as the clearance size increased. In addition, the authors also confirmed that the intra-joint contact force would sharply increase with the increase of the clearance size and the friction coefficient. Dupac and Beale [415] investigated the dynamic response and stability of a planar slider-crank mechanism with a flexible connecting rod and a clearance joint. The authors showed that both links' flexibility and clearance size affected the system behavior. Zhao et al., [138] utilized the ADAMS software to perform a study on the dynamic behavior of a reciprocating compressor system with clearance joints. In this research work, the effects of cylinder pressure load, clearance size, driving speed, and flexibility of the links were investigated. Zhou and Guan [252] also investigated the dynamics of space deployable systems with spatial revolute joints with clearances and links flexibility. Both analytical and experimental models were considered. The authors showed that both the radial and axial clearance played an important role in the system's behavior, and showed that the flexibility of the links reduced the level of impact forces due to the energy dissipation.

With the purpose of investigating the influence of flexibility of the bodies, Tian et al. [64] presented a new approach to account for both link's flexibility and HD lubrication in revolute clearance joints. The effects of the squeeze lubrication and the HD lubrication on the system dynamics response were analyzed and compared for the infinitely-short journal-bearing with Sommerfeld's boundary condition. The flexibility of the links was considered by meshing with the locking-free shear deformable beam element, and the absolute nodal coordinate formulation (ANCF), originally proposed by Shabana [350], and considered as an important development in the flexible multibody dynamics [351]. Tian and his co-authors [372] investigated the elastohydrodynamic lubrication in mechanical joints taking into account the journal misalignment, and the lubricant pressure obtained by solving the Reynolds' finite difference equations using the successive over relaxation (SOR) algorithm.

The numerical results showed that the bearing flexibility affected the system behavior in a significant manner, because the bearing flexible extended the lubricant distribution space, and then reduced the lubricant pressure. Li et al. [6] presented a comprehensive study on a planar slide-crank mechanism with flexible components and two revolute clearance joints. For this investigation, an optimization procedure was proposed. Wang and Liu [445] modeled and simulated wear in a planar five-bar linkage considering clearance joints and flexibility of the links. For that, the absolute nodal coordinate formulation (ANCF) was also used to obtain the equations of motion. The intra-joint contact forces were calculated using the approach proposed by Flores el al. [187], while the wear at the clearance joints were computed by employing the Archard's model. Sun and his co-authors [477] also studied mechanisms with clearance joints and flexible elements, such as the connecting rod in a slider-crank mechanism. Aginada et al. [497] presented a numerical procedure for the pose error calculation of parallel manipulators due to clearances and elastic deformations. The proposed formulation uses a simple way to included clearance effects and elastic deformations in the same loop equations. Li et al. [498] studied the combined effects of damping, friction, gravity and flexibility on the dynamic response of a deployable mechanism that included clearance joints. The commercial software ADAMS was utilized in this study. The results showed that the deployable system had nonlinear dynamic and chaotic response. Zhao et al. [499] presented an investigation on the wear prediction corresponding to revolute clearance joints in mechanisms with flexible parts. The flexibility of the links was also modeled by using the absolute nodal coordinate formulation (ANCF), while the contact force developed at the clearance joint was evaluated employing a continuous contact force model. The planar slider-crank mechanism was used as application example. In general, the results showed that the flexibility of the links tended to minimize the negative impacts of the clearance joints, and the corresponding joint wear were smaller when compared with the one corresponding to rigid links. Using ANCF, Song et al. [500] presented a modular dynamic modeling approach to study the dynamics of a planar flexible 3-RRR planar parallel robot with multiple revolute clearance joints. They observed that when the flexibility of the links was considered, the moving accuracy was decreased, and that the kinematic stability was better compared with the case of having only the clearance joint effect was considered.

6. Concluding remarks and future challenges

The main objective of the presented work was to provide a general and comprehensive overview of the methodologies dealing with the modeling and analysis of realistic mechanical systems or mechanisms, which include imperfect or clearance joints; i.e., joints in which the effects of clearance, friction, and lubrication are taken into account. First, the contextualization of the problem and the main aspects associated with clearance joints was characterized. The most relevant approaches and methodologies for modeling and analyzing planar and spatial dry clearance joints of different configuration were examined. In this process, the planar and spatial revolute joints, spherical joints, and translational joints were each addressed individually. A full description of the methodologies related to the lubrication effects within the context of clearance joints in mechanisms was then provided. For that, several tribological aspects were revisited with the purpose of incorporating them under the framework of dynamic modeling and simulation of multibody mechanical systems that incorporate lubricated clearance joints. Subsequently, experimental investigations on the mechanisms with clearance joints, which have been proposed over the last decades, were discussed. Finally, other issues related to the clearance joints were presented. These topics included wear phenomena in clearance joints, optimization and control of mechanism that incorporate clearance joints, uncertainty of mechanical systems with clearance joints, and effect of link flexibility.

From the comprehensive survey, it can be clearly observed that a large majority of the existing works have only been focused on the modeling and numerical algorithms for the simple mechanisms with clearance joints. There are many complex mechanisms with enormous number of clearance joints (maybe hundreds even more), such as the space deployable structures, systems with general ball and roller bearings, biomechanical joints, magnetic clearance joints used to insulate vibration, just to mention a few. The extension and application of the current methodologies on joint clearances to those complex application examples is to be considered. Also, the clearance joint dynamic behavior is quite sensitive to the thermal and temperature variations. There are also the solid or semi-solid lubricants used in the mechanism joints. However, the dynamics of the mechanisms with joints lubricated by the solid or semi-solid lubricants still needs further investigation.

There are many practical examples such as the automobile suspension system in vehicles the existence of clearance joints and compliance such as bushing are needed for proper operation of the system. Accurate dynamic analysis of mechanisms with clearance joints plays a very important role for the design of these joints. The numerical modeling and simulation of mechanisms with clearance joints is still quite a challenge faced by the researchers in this field. For additional future work in the field of modeling and analysis of mechanical systems with imperfect joints, the following challenge subjects are still in need of further investigation:

(1) Modeling mechanisms with multiple clearance joints, and with a variety of joints such as prismatic and universal joints, by proposing efficient numerical algorithms to simulate the complex dynamical systems;

(2) Implementation of the methodologies presented in this work to more complex mechanical systems, such as automotive systems where the compliance between contacting surfaces may be described using different contact force laws. The effect of clearance size is also worthy of further investigation, especially the coupling effects of clearance size, flexibility and friction;

(3) Flexibility of components, joint clearance, and uncertainty generally exist in multibody systems simultaneously. Dynamic analyses and optimization of complex flexible multibody systems with clearance joints and uncertainty is still a challenge subject which need to be further studied;

(4) Including of the effect of roughness and geometric imperfections of the contacting surfaces, and friction laws, including stick-slip conditions;

(5) Extending the models for the flexible and soft bodies or joints themselves in order to obtain a more accurate picture of the local body or joint deformations, impact forces, and accelerations experienced during the contact periods, and examining the intra-relation between link flexibility and joint clearance on the dynamic response a system;

(6) Carrying out an extended experimental investigation on multibody systems with lubricated joints and flexible bodies, in order to better characterize the existing models and help in identifying parameters and coming up with new models;

(7) Development of new types of joints, models, and formulations based on contact between complex shape surfaces, which are of fundamental importance in describing many real physical models such as the human skeletal biomechanical systems;

(8) Investigating new monolithic schemes to simulate dynamics of clearance joints exposed to multi-domain loads, such as temperature, heat flux, and electromagnetic force.

Acknowledgements

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References

1. J.J. Uicker, G.R. Pennock, J.E. Shigley, Theory of Machines and Mechanisms, Oxford University Press, New York, 2003.
2. A.A. Shabana, Dynamics of Multibody Systems, John Wiley & Sons, New York, 1989.
3. P.E. Nikravesh, Newtonian-based methodologies in multi-body dynamics, Roc. IMechE Part K: J. Multibody Dyn. 222 (4) (2008) 277-288.
4. P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani, C.S. Koshy, A study on dynamics of mechanical systems including joints with clearance and lubrication, Mech. Mach. Theory 41 (3) (2006) 247-261.
5. X.Y. Geng, X.J. Wang, L. Wang, R.X. Wang, Non-probabilistic time-dependent kinematic reliability assessment for function generation mechanisms with joint clearances, Mech. Mach. Theory 104 (2016) 202-221.
6. Y.Y. Li, G.P. Chen, D.Y. Sun, Y. Gao, K. Wang, Dynamic analysis and optimization design of a planar slider-crank mechanism with flexible components and two clearance joints, Mech. Mach. Theory 99 (2016) 37-57.
7. F. Marques, P. Flores, J.C.P. Claro, H.M. Lankarani, A survey and comparison of several friction force models for dynamic analysis of multibody mechanical systems, Nonlinear Dyn. 86 (3) (2016) 1407-1443.
8. Y. Chen, Y. Sun, C. Chen, Dynamic analysis of a planar slider-crank mechanism with clearance for a high speed and heavy load press system, Mech. Mach. Theory 98 (2016) 81-100.
9. G.B. Daniel, T.H. Machado, K.L. Cavalca, Investigation on the influence of the cavitation boundaries on the dynamic behavior of planar mechanical systems with hydrodynamic bearings, Mech. Mach. Theory 99 (2016) 19-36.
10. P. Li, W. Chen, D.S. Li, R.F. Yu, W.J. Zhang, Wear analysis of two revolute joints with clearance in multibody systems, J. Comput. Nonlinear Dyn. 11 (1) (2016) 011009-7.
11. P. Flores, H.M. Lankarani, Dynamic response of multibody systems with multiple clearance joints, J. Comput. Nonlinear Dyn. 7 (3) (2012) 031003-13.
12. A.R. Crowther, R. Singh, N. Zhang, C. Chapman, Impulsive response of an automatic transmission system with multiple clearances: Formulation, simulation and experiment, J. Sound Vib. 306 (3) (2007) 444-466.
13. J. Ambrósio, P. Verissimo, Improved bushing models for general multibody systems and vehicle dynamics, Multibody Sys. Dyn. 22 (4) (2009) 341-365.
14. A. Chaker, A. Mlika, M.A. Laribi, L. Romdhane, S. Zeghloul, Clearance and manufacturing errors' effects on the accuracy of the 3-RCC spherical parallel manipulator, Eur. J. Mech. A/Solid 37 (2013) 86-95.
15. J. Zhang, H.W. Guo, R.Q. Liu, Z.Q. Deng, Nonlinear characteristic of spherical joints with clearance, Journal of Aerospace and Technology Management, J. Aerosp. Technol. Manage. 7 (2) (2015) 179-184.
16. L.X. Xu, Y.G. Li, Modeling of a deep-groove ball bearing with waviness defects in planar multibody system, Multibody Sys. Dyn. 33 (3) (2015) 229-258.
17. Z.H. Qi, G. Wang, Z.G. Zhang, Contact analysis of deep groove ball bearings in multibody systems, Multibody Sys. Dyn. 33 (2) (2015) 115-141.
18. C. Quental, J. Folgado, J. Ambrósio, J. Monteiro, Multibody system of the upper limb including a reverse shoulder prosthesis, J. Biomech. Eng. 135 (11) (2013) 111005.
19. M. Machado, P. Flores, J. Ambrósio, A. Completo, Influence of the contact model on the dynamic response of the human knee joint, Proc. IMechE Part K: J. Multibody Dyn. 225(4) (2011) 344-358.
20. L.D. Seneviratne, A design criterion for maintaining contact at a revolute joint with clearance, University of London, 1985 Ph.D. thesis.
21. K. Soong, An analytical and experimental study of the elastodynamic response characteristics of planar linkage mechanism with bearing clearances, Michigan State University, 1988 Ph.D. thesis.

22. J.F. Deck, The dynamics of spatial elastic mechanisms with clearances and support structures, Massachusetts Institute of Technology, 1992 Ph.D. thesis.
23. P.Y. Gu, Chaotic vibration in machine systems and its implications for design, Massachusetts Institute of Technology, 1994 Ph.D. thesis.
24. P. Ravn, Analysis and synthesis of planar mechanical systems including flexibility, contact and joint clearance, Technical University of Denmark, 1998 Ph.D. thesis.
25. A.L. Schwab, Dynamics of flexible multibody systems, small vibrations superimposed on a general rigid body motion, Delft University of Technology, 2002 Ph.D. thesis.
26. P. Flores, Dynamic analysis of mechanical systems with imperfect kinematic joints, University of Minho, 2004 Ph.D. thesis.
27. S.L. Pedersen, Simulation and analysis of roller chain drive systems, Technical University of Denmark, 2004 Ph.D. thesis.
28. C.S. Koshy, Characterization of mechanical systems with real joints and flexible links, Wichita State University, 2006 Ph.D. thesis.
29. S. Mukras, Analysis and design of planar multibody systems with revolute joint wear, University of Florida, 2009 Ph.D. thesis.
30. C. Malça, Dynamics of chain drives using a multibody approach, Technical University of Lisbon, 2009 Ph.D. thesis.
31. M. Baiceanu, Dynamic modeling of mechanical systems, Technical University “Gheorghe Asachi”, 2012 Ph.D. thesis.
32. E.A. Renani, Tribology of artificial hip joints, Australian School of Advanced Medicine, 2015 Ph.D. thesis.
33. R.E. Garrett, A.S. Hall, Effect of tolerance and clearance in linkage design, J. Eng. Ind. 91 (1969) 198-202.
34. S.G. Dhande, J. Chakraborty, Analysis and synthesis of mechanical error in linkages – A stochastic approach, J. Eng. Ind. 95 (3) (1973) 672-676.
35. O.M.A. Sharfi, M.R. Smith, A simple method for the allocation of appropriate tolerances and clearances in linkage mechanisms, Mech. Mach. Theory 18 (2) (1983) 123-129.
36. S.J. Lee, B.J. Gilmore, The determination of the probabilistic properties of velocities and accelerations in kinematic chains with uncertainty, J. Mech. Des. 113 (1991) 84-90.
37. S.J. Lee, B.J. Gilmore, B.B. Ogot, Dimensional tolerance allocation of stochastic dynamic mechanical systems through performance and sensitivity analysis, J. Mech. Des. 115 (3) (1993) 392-402.
38. K.L. Ting, J.M. Zhu, D. Watkins, The effects of joint clearance on position and orientation deviation of linkages and manipulators, Mech. Mach. Theory 35 (3) (2000) 391-401.
39. W.J. Lee, T.C. Woo, Tolerances: Their analysis and synthesis, J. Eng. Ind. 112 (1990) 113-121.
40. K.W. Chase, A.R. Parkinson, A survey of research in the application of tolerance analysis to the design of mechanical assemblies, Research in Engineering Design, 3 (1991) 23-37.
41. K.L. Ting, Y. Long, Performance quality and tolerance sensitivity of mechanisms, J. Mech. Des., 118 (1) (1996) 144-150.
42. L. Joskowicz, E. Sacks, V. Srinivasan, Kinematic tolerance analysis, Computer-Aided Design, 29 (2) (1997) 147-157.
43. Z. Shi, Synthesis of mechanical error in spatial linkages based on reliability concept, Mech. Mach. Theory 32 (2) (1997) 255-259.
44. J.H. Choi, S.J. Lee, D.H. Choi, Tolerance optimization for mechanisms with lubricated joints, Multibody Sys. Dyn. 2 (2) (1998) 145-168.
45. A.A. Fogarasy, M.R. Smith, The influence of manufacturing tolerances on the kinematic performance of mechanisms, Proc. IMechE Part C: J. Mech. Eng. Sci. 212 (1998) 35-47.
46. S.R. Hummel, C. Chassapis, Configuration design and optimization of universal joints with manufacturing tolerances, Mech. Mach. Theory 35 (2000) 463-476.
47. J.M. Zhu, K.L. Ting, Uncertainty analysis of planar and spatial robots with joint clearances, Mech. Mach. Theory 35 (9) (2000) 1239-1256.
48. M. Schulz, H. Brauchli, Two methods of sensitivity analysis for multibody systems with collisions. Mech. Mach. Theory 35 (10) (2000) 1345-1365.
49. X. Dong, J. Ye, Performance analysis of the reheat-stop-valve mechanism under dimensional tolerance, misalignment and thermal impact, J. Mech. Eng. 55 (9) (2009) 507-520.
50. X. Huang, Y. Zhang, Robust tolerance design for function generation mechanisms with joint clearances, Mech. Mach. Theory 45 (9) (2010) 1286-1297.

51. P. Flores, A methodology for quantifying the position errors due to manufacturing and assemble tolerances, J. Mech. Eng. 57 (6) (2011) 457-467.
52. M.A.B. Abdallah, I. Khemili, N. Aifaoui, Numerical investigation of a flexible slider–crank mechanism with multijoints with clearance, Multibody Sys. Dyn. 38 (2) (2016) 173-199.
53. L.D. Seneviratne, S.W.E. Earles, Chaotic behaviour exhibited during contact loss in a clearance joint in a four-bar mechanism, Mech. Mach. Theory 27 (3) (1992) 307-321.
54. J. Chunmei, Q. Yang, F. Ling, Z. Ling, The nonlinear dynamic behavior of an elastic linkage mechanism with clearances, J. Sound Vib. 249 (2) (2002) 213-226.
55. E. Salahshoor, S. Ebrahimi, M. Maasoomi, Nonlinear vibration analysis of mechanical systems with multiple joint clearances using the method of multiple scales, Mech. Mach. Theory 105 (2016) 495-509.
56. P. Barthels, J. Wauer, Modeling and dynamic analysis of telescopic systems of structural members with clearance, Nonlinear Dyn. 42 (2005) 371-382.
57. T.N. Shiau, Y.J. Tsai, M.S. Tsai, Nonlinear dynamic analysis of a parallel mechanism with consideration of joint effects, Mech. Mach. Theory 43(4) (2008) 491-505.
58. Y.A. Khulief, Modeling of impact in multibody systems: An overview, J. Comput. Nonlinear Dyn. 8 (2013) 021012-15.
59. Y. Huang, Z.X. Tian, R.C. Chen, H.Y. Cao, A simpler method to calculate instability threshold speed of hydrodynamic journal bearings, Mech. Mach. Theory 108 (2017) 209-216.
60. R.J. Roger, G.C. Andrews, Dynamic simulation of planar mechanical systems with lubricated bearing clearances using vector-network methods, J. Eng. Ind. 99 (1) (1977) 131-137.
61. T.S. Liu, Y.S. Lin, Dynamic analysis of flexible linkages with lubricated joints. J. Sound Vib. 141 (2) (1990) 193-205.
62. B.J. Alshaer, H. Nagarajan, H.K. Beheshti., H.M. Lankarani, S. Shivaswamy, Dynamics of a multibody mechanical system with lubricated long journal bearings, J. Mech. Des. 127 (2005) 493-498.
63. F. Nonato, K.L. Cavalca, On the non-linear dynamic behavior of elastohydrodynamic lubricated point contact, J. Sound Vib. 329 (22) (2010) 4656-4671.
64. Q. Tian, Y.Q. Zhang, L.P. Chen, J.J. Yang, Simulation of planar flexible multibody systems with clearance and lubricated revolute joints, Nonlinear Dyn. 60 (2010) 489-511.
65. M. Machado, J. Costa, E. Seabra, P. Flores, The effect of the lubricated revolute joint parameters and hydrodynamic force models on the dynamic response of planar multibody systems, Nonlinear Dyn. 69 (2012) 635-654.
66. V.L. Reis, G.B. Daniel, K.L. Cavalca, Dynamic analysis of a lubricated planar slider-crank mechanism considering friction and Hertz contact effects, Mech. Mach. Theory 74 (2014) 257-273.
67. E. Askari, P. Flores, D. Dabirrahmani, R. Appleyard, Study of the friction-induced vibration and contact mechanics of artificial hip joints, Tribol. Int. 70 (2014) 1-10.
68. A.A. Shabana, Flexible multibody dynamics: Review of past and recent developments, Multibody Sys. Dyn. 1 (2) (1997) 189-222.
69. P.R.G. Kurka, J.H. Izuka, K.L.G. Paulino, Dynamic loads of reciprocating compressors with flexible bearings, Mech. Mach. Theory 52 (2012) 130-143.
70. Y. Ling, A.G. Song, W. Lu, A novel rigid–flexible combined sampler for lunar exploration, Mech. Mach. Theory 77 (2014) 25-34.
71. Q. Zhang, J.K. Mills, W.L. Cleghorn, J. Jin, C. Zhao, Trajectory tracking and vibration suppression of a 3-PRR parallel manipulator with flexible links, Multibody Sys. Dyn. 33 (2015) 27-60.
72. G.T. Rooney, P. Deravi, Coulomb friction in mechanism sliding joints, Mech. Mach. Theory 17 (3) (1982) 207-211.
73. E.J. Haug, S.C. Wu, S.M. Yang, Dynamics of mechanical systems with coulomb friction, stiction, impact and constraint addition deletion - I Theory, Mech. Mach. Theory 21 (1986) 401-406.
74. J.R. Cannon, L.L. Howell, A compliant contact-aided revolute joint, Mech. Mach. Theory 40 (11) (2005) 1273-1293.
75. J. Frączek, M. Wojtyra, On the unique solvability of a direct dynamics problem for mechanisms with redundant constraints and Coulomb friction in joints, Mech. Mach. Theory 46 (3) (2011) 312-334.
76. J. Guo, K.M. Lee, Compliant joint design and flexure finger dynamic analysis using an equivalent pin model, Mech. Mach. Theory 70 (2013) 338-353.
77. N.N. Xu, W.C. Tang, Y.J. Chen, D.F. Bao, Y.J. Guo, Modeling analysis and experimental study for the friction of a ball screw, Mech. Mach. Theory 87 (2015) 57-69.

78. Y.A. Khulief, A.A. Shabana, Dynamic analysis of constrained system of rigid and flexible bodies with intermittent motion, J. Mech. Transm. Autom. Des. 108 (1986) 38-45.
79. Y.A. Khulief, A.A. Shabana, A continuous force model for the impact analysis of flexible multibody systems, Mech. Mach. Theory 22 (1987) 213-224.
80. T.P. Goodman, Dynamic effects of backlash, Mach. Des. 35 (12) (1963) 150-157.
81. A.E. Kobrinsky, V.I. Babistsky, Periodic movements of two mass vibration systems in a cavity. (in Russian), TruD. Mash. MeKh. 103 (1964) 56-70.
82. R.S. Haines, Survey: 2-dimensional motion and impact at revolute joints, Mech. Mach. Theory 15 (1980) 361-370.
83. P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani, Kinematics and Dynamics of Multibody Systems with Imperfect Joints: Models and Case Studies, Springer Science & Business Media, 2008.
84. Y.C. Liu, Y.Q. Yu, A survey of mechanism and robot with clearances, Mech. Sci. Tech. 23 (4) (2004) 454-457.
85. O. Muvengei, J. Kihiu, B. Ikua, Dynamic analysis of multi-body mechanical systems with imperfect kinematic joints: A literature survey and review, Sustainable Research & Innovation Proceedings, 3 (2011) 61-76.
86. G.X. Wang, H. Liu, Research progress of joint effects model in multibody system dynamics, Chinese J. Theor. App. Mech. 47 (1) (2015) 31-50.
87. A. Gummer, B. Sauer, Modeling planar slider-crank mechanisms with clearance joints in RecurDyn, Multibody Sys. Dyn. 31 (2014) 127-145.
88. M.A. Chace, Analysis of the time-dependence of multi-freedom mechanical systems in relative coordinates, J. Eng. Ind. 89 (1) (1967) 119-125.
89. C. Bagci, Dynamic motion analysis of plane mechanisms with coulomb and viscous damping via the joint force analysis, J. Eng. Ind. 97 (1975) 551-560.
90. T.W. Lee, A.C. Wang, On the dynamics of intermittent-motion mechanisms. Part I. Dynamic model and response, J. Mech. Transm. Autom. Des. 105 (1983) 534-540.
91. S. Dubowsky, T.L. Morris An analytical and experimental study of the acoustical noise produced by machine links, Journal of Vibration, Acoustics, Stress, and Reliability in Design, 105 (3) (1983) 393-401.
92. B.S. Thompson, A variational formulation for the finite element analysis of the vibratory and acoustical response of high speed machinery, J. Sound Vib. 89 (1) (1983) 7-15.
93. S.A. Kolhatkar, K.S. Yajnik, The effects of play in the joints of a function-generating mechanism, J. Mech. 5 (4) (1970) 521-532.
94. W.M. Mansour, M.A Townsend, Impact spectra and intensities for high-speed mechanisms, J. Eng. Ind. 97 (1975) 347-353.
95. M.A. Townsend, W.M. Mansour, A pendulating model for mechanisms with clearances in the revolutes, J. Eng. Ind. 97 (1975) 354-358.
96. S. Dubowsky, T.N. Gardner, Dynamic interactions of link elasticity and clearance connections in planar mechanical systems, J. Eng. Ind. 97 (2) (1975) 652-661.
97. R.C. Winfrey, R.V. Anderson, C.W. Gnilka, Analysis of elastic machinery with clearances, J. Eng. Ind. 95 (1973) 695-703.
98. S. Erkaya, I. Uzmay, Experimental investigation of joint clearance effects on the dynamics of a slider-crank mechanism, Multibody Sys. Dyn. 24 (1) (2010) 81-102
99. P. Flores, J. Ambrósio, Revolute joints with clearance in multibody systems, Comput. Struct. 82 (17-19) (2004) 1359-1369.
100. P. Flores, J. Ambrósio, J.P. Claro, Dynamic analysis for planar multibody mechanical systems with lubricated joints, Multibody Sys. Dyn. 12 (2004) 47-74
101. S.W.E. Earles, C.L.S. Wu, Motion analysis of a rigid link mechanism with clearance at a bearing using Lagrangian mechanics and digital computation, J. Mech. (1973) 83-89
102. C.L.S. Wu, S.W.E. Earles, A determination of contact-loss at a bearing of a linkage mechanism, J. Eng. Ind. 99 (2) (1977) 375-379.
103. S. Erkaya, I. Uzmay, Optimization of transmission angle for slider-crank mechanism with joint clearances, Struct. Multidiscipl. Opt. 37 (2009) 493-508.
104. S. Erkaya, I. Uzmay, A neural-genetic (NN-GA) approach for optimising mechanisms having joints with clearance, Multibody Sys. Dyn. 20 (2008) 69-83.
105. T. Furuhashi, N. Morita, M. Matsuura, Research on dynamics of four-bar linkage with clearances at turning pairs (1st Report, General theory of continuous contact model), Bulletin JSME 21 (1978) 518-523.

106. N. Morita, T. Furuhashi, M. Matsuura, Research on dynamics of four-bar linkage with clearances at turning pairs (2nd Report, Analysis of crank-level mechanism with clearance at joint of crank and coupler using continuous contact model), Bulletin JSME 21 (1978) 1284-1291.
107. N. Morita, T. Furuhashi, M. Matsuura, Research on dynamics of four-bar linkage with clearances at turning pairs (3rd Report, Analysis of crank-level mechanism with clearance at joint of coupler and lever using continuous contact model), Bulletin JSME 21 (1978) 1292-1298.
108. T. Furuhashi, N. Morita, M. Matsuura, Research on dynamics of four-bar linkage with clearances at turning pairs (4th Report, Force acting at joints of crank-level mechanism), Bulletin JSME 21 (1978) 1299-1305.
109. S. Dubowsky, On predicting the dynamic effects of clearances in planar mechanisms, J. Eng. Ind. 96 (1974) 317-323.
110. J.C.P. Claro, P. Flores, Influência da modelização das juntas na análise do desempenho de um mecanismo. 8as Jornadas Portuguesas de Tribologia, Aveiro (2002) 215-219.
111. S. Dubowsky, F. Freudenstein, Dynamic analysis of mechanical systems with clearances, Part 1. Formulation of dynamic model, J. Eng. Ind. 93 (1) (1971) 305-309.
112. S. Dubowsky, F. Freudenstein, Dynamic analysis of mechanical systems with clearances, Part 2. Dynamic response, J. Eng. Ind. 93 (1) (1971) 310-316.
113. J.F. Deck, S. Dubowsky, On the limitations of predictions of the dynamic response of machines with clearance connections, J. Mech. Des. 116 (1994) 833-841.
114. S. Dubowsky, J.F. Deck, H. Costello, The dynamic modeling of flexible spatial machine systems with clearance connections, J. Mech. Transm. Autom. Des. 109 (1987) 87-94.
115. S. Dubowsky, T.N. Gardner, Design and analysis of multilink flexible mechanisms with multiple clearance connections, J. Eng. Ind. 99 (1) (1977) 88-96.
116. S. Dubowsky, M. Norris, E. Aloni, A. Tamir, An analytical and experimental study of the prediction of impacts in planar mechanical systems with clearances, J. Mech. Transm. Autom. Des. 106 (1984) 444-451.
117. S. Dubowsky, P.Y. Gu, J.F. Deck, The dynamic analysis of flexibility in mobile robotic manipulator systems, in: Proceedings of VIII World Congress on the Theory of Machines and Mechanisms, 1911, pp. 26-31.
118. S. Dubowsky, On predicting the dynamic effects of clearances in one-dimensional closed loop systems, J. Eng. Ind. 96 (1974) 324-329.
119. S. Dubowsky, M.F. Moening, An experimental and analytical study of impact forces in elastic mechanical systems with clearances, Mech. Mach. Theory 13 (1978) 451-465.
120. T. Kakizaki, J.F. Deck, S. Dubowsky, Modeling the spatial dynamics of robotic manipulators with flexible links and joint clearances, J. Mech. Des. 115 (1993) 839-847.
121. S.W.E. Earles, O. Kilicay, A design criterion for maintaining contact at plain bearings, Proc. IMechE Part C: J. Mech. Eng. Sci. 194 (1) (1980) 249-258.
122. B.M. Bahgat, M.O.M. Osman, T.S. Sankar, On the effect of bearing clearances in the dynamic analysis of planar mechanisms, J. Mech. Eng. Sci. 21 (1979) 429-437.
123. M.O.M. Osman, B.M. Bahgat, T.S. Sankar, On the dynamic analysis of planar mechanisms with multiple clearances, Proc. IMechE Part C: J. Mech. Eng. Sci. 197 (1983) 89-95.
124. B.M. Bahgat, M.O.M. Osman, T.S. Sankar, An approach for dynamic analysis of mechanical systems with multiple clearance using Lagrangian mechanics, Proc. IMechE Part C: J. Mech. Eng. Sci. 197 (1983) 17-23.
125. S.M. Megahed, A.F. Haroun, Analysis of the dynamic behavioral performance of mechanical systems with multi-clearance joints, J. Comput. Nonlinear Dyn. 7 (1) (2012) 011002.
126. J.K. Shin, B.M. Kwak, A design method for reducing the effects of clearances at revolute joints, Proc. IMechE Part C: J. Mech. Eng. Sci. 199 (1985) 153-158.
127. S.W.E. Earles, L.D. Seneviratne, Design guidelines for predicting contact loss in revolute joints of planar linkage mechanisms, Proc. IMechE Part C: J. Mech. Eng. Sci. 204 (1990) 9-18.
128. L.D. Seneviratne, S.W.E. Earles, D.N. Fenner, Analysis of a four-bar mechanism with a radially compliant clearance joint, Proc. IMechE Part C: J. Mech. Eng. Sci. 210 (1996) 215-223.
129. M.T. Bengisu, T. Hidayetoglu, A. Akay, A theoretical and experimental investigation of contact loss in the clearances of a four-bar mechanism, J. Mech. Transm. Autom. Des. 108 (1986) 237-244.
130. J.H. Rhyu, B.M. Kwak, Optimal stochastic design of four-bar mechanisms for tolerance and clearance, J. Mech. Transm. Autom. Des. 110 (1988) 255-262.
131. O. Muvengei, J. Kihiu, B. Ikua, Numerical study of parametric effects on the dynamic response of planar multi-body systems with differently located frictionless revolute clearance joints, Mech. Mach. Theory, 53 (2012) 30-49.

132. O. Muvengei, J. Kihiu, B. Ikua, Dynamic analysis of planar multi-body systems with LuGre friction at differently located revolute clearance joints, Multibody Sys. Dyn. 28(4) (2012) 369-393.
133. O. Muvengei, J. Kihiu, B. Ikua, Dynamic analysis of planar rigid-body mechanical systems with two-clearance revolute joints, Nonlinear Dyn. 73 (2013) 259-273.
134. Y.L. Yang, J.J.R. Cheng, T.Q. Zhang, Vector form intrinsic finite element method for planar multibody systems with multiple clearance joints, Nonlinear Dyn. 86 (2016) 421-440.
135. E.C. Ting, C. Shih, Y.K. Wang, Fundamentals of a vector form intrinsic finite element: Part I. Basic procedure and a plane frame element, J. Mech. 20 (2) (2004) 113-122.
136. E.C. Ting, C. Shih, Y.K. Wang, Fundamentals of a vector form intrinsic finite element: Part II. Plane solid elements, J. Mech. 20(2) (2004) 123-132.
137. C. Shih, Y.K. Wang, E.C. Ting, Fundamentals of a vector form intrinsic finite element: Part III. Convected material frame and examples, J. Mech. 20 (2) (2004) 133-143.
138. H. Zhao, M. Xu, J. Wang, G. Chen, A dynamic analysis of reciprocating compressor transmission mechanism with joint clearance, Applied Mechanics and Materials 226-228 (2012) 641-645.
139. B. Feng, N. Morita, T. Torri, A new optimization method for dynamic design of planar linkage with clearances at joints - Optimizing the mass distribution of links to reduce the change of joint forces, J. Mech. Des. 124 (2002) 68-73.
140. C. Innocenti, Kinematic clearance sensitivity analysis of spatial structures with revolute joints, J. Mech. Des. 124 (2002) 52-57.
141. J.C.G. Orden, Analysis of joint clearances in multibody systems, Multibody Sys. Dyn. 13 (2005) 401-420.
142. S. Bing, J. Ye, Dynamic analysis of the reheat-stop-valve mechanism with revolute clearance joint in consideration of thermal effect, Mech. Mach. Theory 43 (2008) 1625-1638.
143. B. Miedema, W.M. Mansour, Mechanical joints with clearance: A three mode model, J. Eng. Ind. 98 (1976) 1319-1323.
144. H. Hertz, On the contact of solids, 1881. On the contact of rigid elastic solids and on hardness, 1882. Translated by D.E. Jones and G.A. Schott, Miscellaneous Papers, MacMillan and Co. Ltd., London, (1896) 146-183.
145. H.M. Lankarani, P.E. Nikravesh, A contact force model with hysteresis damping for impact analysis of multibody systems, J. Mech. Des. 112 (1990) 369-376.
146. P. Flores, A parametric study on the dynamic response of planar multibody systems with multiple clearance joints, Nonlinear Dyn. 61 (2010) 633-653.
147. J.A.C. Ambrósio, Impact of Rigid and Flexible Multibody Systems: Deformation Description and Contact Models, Springer Netherlands, (2003) 57-81.
148. P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani, Dynamic behavior of planar rigid multibody systems including revolute joints with clearance, Proc. IMechE Part K: J. Multibody Dyn. 221 (2) (2007) 161-174.
149. W. Goldsmith, Impact - The Theory and Physical Behaviour of Colliding Solids, Edward Arnold Ltd, London, England, 1960.
150. S. Shivaswamy, Modeling Contact Forces and Energy Dissipation During Impact in Multibody Mechanical Systems, Wichita State University, 1997 Ph.D. thesis.
151. A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity. Fourth edition, Dover Publications, New York, 1944.
152. A.S. Yigit, A.G. Ulsoy, R.A. Scott, Spring-dashpot models for the dynamics of a radially rotating beam with impact, J. Sound Vib. 142 (1990) 515-525.
153. A.S. Yigit, A.P. Christoforou, M.A. Majeed, A nonlinear visco-elastoplastic impact model and the coefficient of restitution, Nonlinear Dyn. 66 (2011) 509-521.
154. C. Thornton, Coefficient of restitution for collinear collisions of elastic-perfectly plastic spheres, J. Appl. Mech. 64 (1997) 383-386.
155. D. Gugan, Inelastic collision and the Hertz theory of impact, Am. J. Phys. 68 (2000) 920-924.
156. E. Falcon, C. Laroche, S. Fauve, C. Coste, Behaviour of one elastic ball bouncing repeatedly off the ground, The European Physical Journal B 3 (1998) 45-57.
157. E. Rigaud, J. Perret-Liaudet, Experiments and numerical results on non-linear vibrations of an impacting Hertzian contact. Part 1: harmonic excitation, J. Sound Vib. 265 (2003) 289-307.
158. G. Kuwabara, K. Kono, Restitution coefficient in a collision between two spheres, Japanese J. Appl. Phys. 26 (1987) 1230-1233.
159. H. Minamoto, S. Kawamura, Effects of material strain rate sensitivity in low speed impact between two identical spheres, Int. J. Impact Eng. 36 (2009) 680-686.

160. J. Kagami, K. Yamada, T. Hatazawa, Contact between a sphere and a rough plate, Wear 87 (1983) 93-105.
161. L. Pust, F. Peterka, Impact oscillator with Hertz's model of contact, Meccanica 38 (2003) 99-116.
162. L. Vu-Quoc, X. Zhang, L. Lesburg, A normal force-displacement model for contacting spheres accounting for plastic deformation: force-driven formulation, J. Appl. Mech. 67 (2000) 363-371.
163. L. Vu-Quoc, X. Zhang, L. Lesburg, Normal and tangential force-displacement relations for frictional elasto-plastic contact of spheres, Int. J. Solids Struct. 38 (2001) 6455-6490.
164. L.V. Burgin, R.M. Aspden, Impact testing to determine the mechanical properties of articular cartilage in isolation and on bone, J. Mater. Sci. Mater. Med. 19 (2008) 703-711.
165. M.H. Bordbar, T. Hyppänen, Modeling of binary collision between multisize viscoelastic spheres, Journal of Numerical Analysis, Industrial and Applied Mathematics 2 (2007) 115-128.
166. N. Yoshioka, A review of the micromechanical approach to the physics of contacting surfaces, Tectonophysics 277 (1997) 29-40.
167. P. Villaggio, The Rebound of an elastic sphere against a rigid wall, J. Appl. Mech. 63 (1996) 259-263.
168. R. Ramírez, T. Pöschel, N.V. Brilliantov, T. Schwager, Coefficient of restitution of colliding viscoelastic spheres, Phys. Rev. E 60 (1999) 4465-4472.
169. C.Y. Wu, L.Y. Li, C. Thornton, Energy dissipation during normal impact of elastic and elastic-plastic spheres, Int. J. Impact Eng. 32 (2005) 593-604.
170. C.Y. Wu, L.Y. Li, C. Thornton, Rebound behaviour of spheres for plastic impacts. Int. J. Impact Eng. 28 (2003) 929-946.
171. L.Y. Li, C. Thornton, C.Y. Wu, Impact behavior of elastoplastic spheres with a rigid wall, Proc. IMechE Part C: J. Mech. Eng. Sci. 214 (8) (2000) 1107-1114.
172. X. Shi, A.A. Polycarpou, Measurement and modeling of normal contact stiffness and contact damping at the meso scale, J. Vibr. Acoust. 127 (2005) 52-60.
173. Y. Tatara, N. Moriwaki, Study on impact of equivalent two bodies: coefficients of restitution of spheres of brass, lead, glass, porcelain and agate, and the material properties, Bulletin of Japan Society of Mechanical Engineers 25 (1982) 631-637.
174. Y. Tatara, Extensive theory of force-approach relations of elastic spheres in compression and in impact. Journal of Engineering Materials and Technology, 111 (1989) 163-168.
175. Y. Gonthier, J. McPhee, C. Lange, J.C. Piedboeuf, A regularized contact model with asymmetric damping and dwell-time dependent friction, Multibody Sys. Dyn. 11 (2004) 209-233.
176. Z.Y. Qin, Q.S. Lu, Analysis of impact process based on restitution coefficient, J. Dyn. Control. 4 (2006) 294-298.
177. H.M. Lankarani, P.E. Nikravesh, Continuous contact force models for impact analysis in Multibody Systems, Nonlinear Dyn. 5 (1994) 193-207.
178. M. Gharib, Y. Hurmuzlu, A new contact force model for low coefficient of restitution impact, J. Appl. Mech. 79 (6) (2012) 4506.
179. Y. Tsuji, T. Tanaka, T. Ishida, Lagrangian numerical simulation of plug flow of cohesionless particles in a horizontal pipe, Powder Technology 71 (1992) 239-250.
180. R. Jankowski, Analytical expression between the impact damping ratio and the coefficient of restitution in the non-linear viscoelastic model of structural pounding, Earthquake Engineering & Structural Dynamics, 35 (2006) 517-524.
181. T. Schwager, T. Pöschel, Coefficient of normal restitution of viscous particles and cooling rate of granular gases, Phys. Rev. E 57 (1998) 650-654.
182. K.H. Hunt, F.R.E. Crossley, Coefficient of restitution interpreted as damping in vibroimpact, J. Appl. Mech. 42 (2) (1975) 440-445.
183. R.G. Herbert, D.C. McWhannell, Shape and frequency composition of pulses from an impact pair, J. Eng. Ind. 99 (1977) 513-518.
184. D.W. Marhefka, D.E. Orin, A compliant contact model with nonlinear damping for simulation of robotic systems, IEEE T. Syst. Man. Cy. A 29 (1999) 566-572.
185. Y.N. Zhang, I. Sharf, Compliant force modeling for impact analysis, in: Proceedings of the 2004 ASME International Design Technical Conferences, Salt Lake City, Utah, USA, 2004, pp. 595-601.
186. Y.N. Zhang, I. Sharf, Validation of nonlinear viscoelastic contact force models for low speed impact, J. Appl. Mech. 76 (5) (2009) 911-914.
187. P. Flores, M. Machado, M.T. Silva, J.M. Martins, On the continuous contact force models for soft materials in multibody dynamics, Multibody Sys. Dyn. 25 (3) (2011) 357-375.

188. J. Alves, N. Peixinho, M.T.D. Silva, P. Flores, H.M. Lankarani, A comparative study on the viscoelastic constitutive laws for frictionless contact interfaces in multibody dynamics, Mech. Mach. Theory 85 (2015) 172-188.
189. A. Banerjee, A. Chanda, R. Das, Historical origin and recent development on normal directional impact models for rigid body contact simulation: A critical review, Arch. Comput. Methods E. 24 (2) (2017) 397-422.
190. R. Masoudi, J. McPhee, A novel micromechanical model of nonlinear compression hysteresis in compliant interfaces of multibody systems, Multibody Sys. Dyn. 37 (3) (2016) 325-343.
191. L.X. Xu, A general method for impact dynamic analysis of a planar multi-body system with a rolling ball bearing joint, Nonlinear Dyn. 78 (2) (2014) 857-879.
192. L.X. Xu, Y.C. Han, A method for contact analysis of revolute joints with noncircular clearance in a planar multibody system, Proc. IMechE Part K: J. Multibody Dyn. 230 (4) (2016) 589-605.
193. Z.F. Bai, Y.Q. Liu, Y. Sun, Investigation on dynamic responses of dual-axis positioning mechanism for satellite antenna considering joint clearance, J. Mech. Sci. Technol. 29 (2) (2015) 453-460.
194. P. Flores, J.A.C. Ambrósio, J.C.P. Claro, H.M. Lankarani, Influence of the contact-impact force model on the dynamic response of multibody systems, Proc. IMechE Part K: J. Multibody Dyn. 220 (1) (2006) 21-34.
195. Y.Y. Li, Z.L. Wang, C. Wang, W.H. Huang, Planar rigid-flexible coupling spacecraft modeling and control considering solar array deployment and joint clearance. Acta Astronautica, (2017) Online, DOI:10.1016/j.actaastro.2017.10.008
196. C.M. Pereira, A.L. Ramalho, J.A. Ambrósio, A critical overview of internal and external cylinder contact force models, Nonlinear Dyn. 63 (4) (2011) 681-697.
197. K.L. Johnson, Contact Mechanics, Cambridge University Press, Cambridge, England, 1985.
198. S.W. Hu, X.L. Guo, A dissipative contact force model for impact analysis in multibody dynamics, Multibody Sys. Dyn. 35 (2) (2015) 131-151.
199. Z.F. Bai, Y. Zhao, Dynamic behaviour analysis of planar mechanical systems with clearance in revolute joints using a new hybrid contact force model, Int. J. Mech. Sci. 54 (1) (2012) 190-205.
200. Z.F. Bai, Y. Zhao, A hybrid contact force model of revolute joint with clearance for planar, Int. J. Nonlinear Mech. 48 (2013) 15-36.
201. X.P. Wang, G. Liu, S.J. Ma, Dynamic analysis of planar mechanical systems with clearance joints using a new nonlinear contact force model, J. Mech. Sci. Technol. 30 (4) (2016) 1537-1545.
202. P. Flores, C.S. Koshy, H.M. Lankarani, J. Ambrósio, J.C.P. Claro, Numerical and experimental investigation on multibody systems with revolute clearance joints, Nonlinear Dyn. 65 (4) (2011) 383-398.
203. X. Wang, G. Liu, S. Ma, R. Tong, Effects of restitution coefficient and material characteristics on dynamic response of planar multi-body systems with revolute clearance joint, Journal of Mechanical Science and Technology 31 (2017) 587–597.
204. B.C. Xu, X.P. Wang, X.M. Ji, R.T. Tong, Y.M. Xue, Dynamic and motion consistency analysis for a planar parallel mechanism with revolute dry clearance joints, Journal of Mechanical Science and Technology, 31(7) (2017) 3199-3209.
205. J. Ma, L.F. Qian, G.S. Chen, M. Li, Dynamic analysis of mechanical systems with planar revolute joints with clearance, Mech. Mach. Theory 94 (2015) 148–164.
206. J. Ma, L.F. Qian, Modeling and simulation of planar multibody systems considering multiple revolute clearance joints, Nonlinear Dyn. 90 (2017) 1907–1940.
207. H.Y. Tan, Y.J. Hu, L. Li, A continuous analysis method of planar rigid-body mechanical systems with two revolute clearance joints, Multibody Sys. Dyn. 40 (2017) 347-373.
208. G.H. Zhang, Q.Q. Xie, S.W. Zhu, Y.Q. Zhang, Flexible multibody dynamics of sewing machine with multi-clearance joints, SAE Technical Paper (2017). 2017-01-0422
209. J. Ambrósio, C. Malça, A. Ramalho, Planar roller chain drive dynamics using a cylindrical contact force model, Mech. Based Des. Struct. Mach.44 (1-2) (2016) 109-122.
210. C.M. Pereira, A.L. Ramalho, J.A. Ambrósio, An enhanced cylindrical contact force model, Multibody Sys. Dyn. 35 (3) (2015) 277-298.
211. F. Pfeiffer, C. Glocker, Multibody Dynamics with Unilateral Contacts, Wiley, New York, 1996.
212. D.E. Stewart, J.S. Pang, Differential variational inequalities, Math. Program. 113 (2) (2008) 345-424.
213. A. Tasora, M. Anitescu, A matrix-free cone complementarity approach for solving large-scale, nonsmooth, rigid body dynamics, Comput. Methods Appl. Mech. Eng. 200 (2011) 439-453.
214. N. Akhadkar, V. Acary, B. Brogliato, Analysis of collocated feedback controllers for four-bar planar mechanisms with joint clearances, Multibody Sys. Dyn. 38 (2) (2016) 101-136.

215. V Acary, Projected event-capturing time-stepping schemes for nonsmooth mechanical systems with unilateral contact and Coulomb's friction, Comput. Methods Appl. Mech. Eng. 256 (2013) 224-250.
216. G. Wang, Z.H. Qi, J. Wang, A differential approach for modeling revolute clearance joints in planar rigid multibody systems, Multibody Sys. Dyn. 39 (4) (2017) 311-335.
217. T. Thümmel, K. Funk, Multibody modelling of linkage mechanisms including friction, clearance and impact. In: Proceedings of the 10th World Congress on the Theory of Machines and Mechanisms in Oulu, June 20–24, vol. 4, pp. 1387–1392. Oulu University Press, Finland (1999)
218. A. Krinner, T. Thümmel, Non-smooth behaviour of a linkage mechanism with revolute clearance joints, New Advances in Mechanisms, Transmissions and Applications (2014) 233–241.
219. H. Dankowicz, On the modeling of dynamic friction phenomena, ZAMM J. Math. Mech. 79 (6) (1999) 399-409.
220. L. Gaul, R. Nitsche, The role of friction in mechanical joints, Appl. Mech. Rev. 54 (2) (2001) 93-106.
221. P. Flores, J. Ambrósio, On the contact detection for contact-impact analysis in multibody systems, Multibody Sys. Dyn. 24 (1) (2010) 103-122.
222. E. Pennestrì, V. Rossi, P. Salvini, Valentini PPReview and comparison of dry friction force models, Nonlinear Dyn. 83 (4) (2016) 1785-1801.
223. D. Karnopp, Computer simulation of stick-slip friction in mechanical dynamic systems, J. Dyn. Syst. Meas. Control 107 (1) (1985) 100-103.
224. D. Centea, H. Rahnejat, M.T. Menday, Non-linear multi-body dynamic analysis for the study of clutch torsional vibrations (judder), Appl. Math. Model. 25 (3) (2001) 177-192.
225. D.A. Haessig, B. Friedland, On the modelling and simulation of friction, J. Dyn. Syst. Meas. Control 113 (3) (1991) 354-362.
226. S. Ahmed, H.M. Lankarani, M.F.O.S. Pereira, Frictional impact analysis in open loop multibody mechanical system, J. Mech. Des. 121 (1999) 119-127.
227. H.M. Lankarani, A Poisson based formulation for frictional impact analysis of multibody mechanical systems with open or closed kinematic chains, J. Mech. Des. 115 (2000) 489-497.
228. J.B. Keller, Impact with friction, J. Appl. Mech. 53 (1) (1986) 1-4.
229. Y. Wang, M.T. Manson, Two dimensional rigid-body collisions with friction, J. Appl. Mech. 59 (3) (1992) 635-642.
230. I. Han, B.J. Gilmore, Multi body impact motion with friction analysis, simulation, and validation, J. Mech. Des. 115 (3) (1993) 412-422.
231. D.C. Threlfall, The inclusion of Coulomb friction in mechanisms programs with particular reference to DRAM, Mech. Mach. Theory 13 (4) (1978) 475-483.
232. S.C. Wu, S.M. Yang, E.J. Haug, Dynamics of mechanical systems with Coulomb friction, stiction, impact and constraint addition deletion – II Planar systems, Mech. Mach. Theory 21 (5) (1986) 407-416.
233. S.C. Wu, S.M. Yang, E.J. Haug, Dynamics of mechanical systems with Coulomb friction, stiction, impact and constraint addition deletion – III Spatial systems, Mech. Mach. Theory 21 (5) (1986) 417-425.
234. S. Andersson, A. Söderberg, B. jörklund, Friction models for sliding dry, boundary and mixed lubricated contacts, Tribol. Int. 40 (4) (2007) 580-587.
235. Y.G. Panovko, I.I. Gubanova, Stability and oscillations of elastic systems, paradoxes, fallacies, and new concepts, Consultants Bureau Enterprises, New York, 1965.
236. B. Armstrong-Helouvry, Control of Machines with Friction, Kluwer Academic Publishers, Dordrecht, 1991.
237. R. Stribeck, The key qualities of sliding and roller bearing, Zeitschrift des Vereines Seutscher Jngenieure, 46 (1902) 1342-1348.
238. F.T. Barwell, Tribology: A systems approach, Tribol. Int. 17 (1984) 299-307.
239. H.H.S. Wang, B. Roth, Position errors due to clearances in journal bearings, J. Mech. Transm.-T ASME, 111 (3) (1989) 315-320.
240. P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani, Spatial revolute joints with clearances for dynamic analysis of multibody systems, Proc. IMechE Part K: J. Multibody Dyn. 220 (4) (2006) 257-271.
241. J. Ambrósio, P. Verissimo, Sensitivity of a vehicle ride to the suspension bushing characteristics, J. Mech. Sci. Technol. 23 (4) (2009) 1075-1082.
242. S. Dubowsky, J.M. Prentis, R.A. Valero, On the development of criteria for the prediction of impact in the design of high speed systems with clearances, in: Proceedings of the Fifth World Congress of Machines and Mechanisms, Montreal, Canada, 1979, pp. 968-971.

243. S.G. Dhande, J. Chakraborty, Mechanical error analysis of spatial linkages, J. Mech. Des. 100 (4) (1978) 732-738.
244. H. Hahnm, Mathematical modelling and computer simulation of rigid body systems including spatial joints with clearance, in: Control Engineering and Systems Theory Group, Department of Mechanical Engineering, University of Kassel, Germany, 1994, pp. 261-267.
245. E. Zakhariev, Dynamics of rigid multibody systems with clearances in the joints, Mech. Struct. Mach. 27 (1) (1999) 63-87.
246. C. Brutti, G. Coglitore, P.P. Valentini, Modeling 3D revolute joint with clearance and contact stiffness, Nonlinear Dyn. 66 (4) (2011) 531-548.
247. C. Liu, Q. Tian, H.Y. Hu, Dynamics and control of a spatial rigid-flexible multibody system with multiple cylindrical clearance joints, Mech. Mach. Theory 52 (2012) 106-129.
248. J.G.D. Jalón, E. Bayo, Kinematic and Dynamic Simulation of Multibody Systems the Real-Time Challenge, Springer, New York, 1994.
249. J.G.D. Jalón, Twenty-five years of natural coordinates, Multibody Sys. Dyn. 18 (1) (2007) 15-33.
250. Z.H. Qi, Y.S. Xu, X.M. Luo, S.J. Yao, Recursive formulations for multibody systems with frictional joints based on the interaction between bodies, Multibody Sys. Dyn. 24 (2010) 133-166.
251. S.Z. Yan, W.W.K Xiang, L. Zhang, A comprehensive model for 3D revolute joints with clearances in mechanical systems, Nonlinear Dyn. 80 (1) (2015) 309-328.
252. Y.J. Zhou, F.L. Guan, Dynamics analysis of mechanisms with elastic bars and three-dimensional clearance revolute joints, J. Harbin Inst. Technol. 44 (10) (2012) 122-127.
253. O.A. Bauchau, J. Rodriguez, Modeling of joints with clearance in flexible multi-body systems, Int. J. Solids Struct. 39 (1) (2002) 41-63.
254. G. Virlez, O. Brüls, E. Tromme, P. Duysinx, Modeling joints with clearance and friction in multibody dynamic simulation of automotive differentials, Theor. Appl. Mech. Lett. 3 (1) (2013) 013003.
255. S. Erkaya, Investigation of joint clearance effects on welding robot manipulators, Robot. Com-Int. Manuf. 28 (4) (2012) 449-457.
256. H. Magalhães, J. Ambrósio, J. Pombo, Railway vehicle modelling for the vehicle-track interaction compatibility analysis, Proc. IMechE Part K: J. Multibody Dyn. 230 (3) (2016) 251-267.
257. EN 14363, Railway applications-testing for the acceptance of running characteristics of railway vehicles-Testing of running behaviour and stationary tests, CENELEC European Committee for Electrotechnical Standardization, Brussels, Belgium, 2005.
258. C. Pereira, A. Ramalho, J. Ambrosio, Applicability domain of internal cylindrical contact force models, Mech. Mach. Theory 78 (2014) 141-157.
259. S. Venanzi, V. Parenti-Castelli, A new technique for clearance influence analysis in spatial mechanisms, J. Mech. Des. 127 (3) (2005) 446-455.
260. V. Parenti-Castelli, S. Venanzi, Clearance influence analysis on mechanisms, Mech. Mach. Theory 40 (12) (2005) 1316-1329.
261. W.H. Bu, Z.Y. Liu, J.R. Tan, S.M. Gao, Detachment avoidance of joint elements of a robotic manipulator with clearances based on trajectory planning, Mech. Mach. Theory 45 (6) (2010) 925-940.
262. K.M. Yudin, On the problem of the dynamics of mechanisms with clearance, in: Proceeding of the Third World Congress of IFToMM, 1971, pp. Vol. A,197-298.
263. M.A. Veluswami, F.R.E. Crossley, G. Horvay, Multiple impacts of a ball between two plates - Part 2: Mathematical modeling, J. Eng. Ind. 97 (3) (1975) 828-835.
264. C.S. Liu, K. Zhang, L. Yang, The compliance contact model of cylindrical joints with clearances, Acta Mech. Sinica 21 (2005) 451-458.
265. C.S. Liu, K. Zhang, R. Yang, The FEM analysis and approximate model for cylindrical joints with clearances, Mech. Mach. Theory 42 (2007) 183-197.
266. A. Gummer, B. Sauer, Influence of contact geometry on local friction energy and stiffness of revolute joints, J. Tribol. 134 (2) (2012) 021402.
267. X.P. Wang, G. Liu, Study on impact dynamics of development for solar panel with cylindrical clearance joint, in: 2014 Asia-Pacific International Symposium on Aerospace Technology, APISAT, Procedia Engineering, Shanghai, China, 2014, pp. 1345-1357.
268. F. Isaac, F. Marques, N. Dourado, P. Flores, Recent developments on cylindrical contact force models with realistic properties, New Trends in Mechanism and Machine Science, Vol. 43 of the series Mechanisms and Machine Science, Springer International Publishing, 2017.

269. P. Dietl, J. Wensing, G.C. van Nijen, Rolling bearing damping for dynamic analysis of multi-body systems -experimental and theoretical results, Proc. IMechE Part K: J. Multibody Dyn. 214 (1) (2000) 33-43.
270. E.I. Radzimovsky Stress, distribution and strength conditions of two rolling cylinders pressed together, University Of Illinois Engineering Experiment Station, Bulletin Series No. 408, 1953.
271. ESDU 78035 Tribology Series, Contact Phenomena. I: stresses, deflections and contact dimensions for normally loaded unlubricated elastic components, Engineering Sciences Data Unit, London, 1978.
272. M. Ciavarella, P. Decuzzi, The state of stress induced by the plane frictionless cylindrical contact 1: the case of elastic similarity, Int. J. Solids Struct. 38 (2001) 4507-4523.
273. M. Ciavarella, P. Decuzzi, The state of stress induced by the plane frictionless cylindrical contact 2: the general case (elastic dissimilarity), Int. J. Solids Struct. 38 (2001) 4523-4533.
274. J.A. Greenwood, C. Putignano, M. Ciavarella, A Greenwood & Williamson theory for line contact, Wear 270 (3-4) (2011) 332-334.
275. C.S. Liu, K. Zhang, L. Yang, Normal force-displacement relationship of spherical joints with clearances, J. Comput. Nonlinear Dyn. 1 (2) (2006) 160-167.
276. P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani, Dynamics of multibody systems with spherical clearance joints, J. Comput. Nonlinear Dyn. 1 (3) (2006) 240-247.
277. Q. Tian, Y.Q. Zhang, L.P. Chen, P. Flores, Dynamics of spatial flexible multibody systems with clearance and lubricated spherical joints, Comput. Struct. 87 (13-14) (2009) 913-929.
278. G.X. Wang, H.Z. Liu, P.S. Deng, Dynamics analysis of spatial multibody system with spherical joint wear, J. Tribol. 137 (2) (2015) 021605.
279. G.X. Wang, H.Z. Liu, Dynamics Analysis of 4-SPS/CU Parallel Mechanism with Spherical Joint Clearance, J. Mech. Eng. 51 (1) (2015) 43-51.
280. E.L. Zheng, R. Zhu, S.H. Zhu, X.J. Lu, A study on dynamics of flexible multi-link mechanism including joints with clearance and lubrication for ultra-precision presses, Nonlinear Dyn. 83 (1) (2016) 137-159.
281. F. Marques, F. Isaac, N. Dourado, A.P. Souto, P. Flores, H.M. Lankarani, A Study on the Dynamics of Spatial Mechanisms With Frictional Spherical Clearance Joints, J. Comput. Nonlinear Dyn. 12 (5) (2017) 051013.
282. E. Askari, P. Flores, D Dabirrahmani, R Appleyard, Nonlinear vibration and dynamics of ceramic on ceramic artificial hip joints: a spatial multibody modeling, Nonlinear Dyn. 76 (2014) 1365-1377.
283. E. Askari, P. Flores, D. Dabirrahmani, R. Appleyard, A computational analysis of squeaking hip prostheses, J. Comput. Nonlinear Dyn. 10 (2) (2015) 024502.
284. E. Askari, P. Flores, D. Dabirrahmani, R. Appleyard, A review of squeaking in ceramic total hip prostheses, Tribol. Int. 93 (2016) 239-256.
285. C. Quental, J. Folgado, J. Ambrósio, J. Monteiro, A new shoulder model with a biologically inspired glenohumeral joint, Med. Eng. Phys. 38 (9) (2016) 969-977.
286. C. Weiss, M.M. Morlock, N.P. Hoffmanna, Friction induced dynamics of ball joints: Instability and post bifurcation behavior, Eur. J. Mech. A-Solid. 45 (2014) 161-173.
287. S. Erkaya, S. Dogan, E. Şefkatlıoglu, Analysis of the joint clearance effects on a compliant spatial mechanism, Mech. Mach. Theory 104 (2016) 255-273.
288. L.E. Goodman, L.M. Keer, The contact stress problem for an elastic sphere indenting an elastic cavity, Int. J. Solids Struct. 1 (1965) 407-415.
289. B. Noble, M.A. Hussain, Exact solution of certain dual series for indentation and inclusion problems, Int. J. Eng. Sci. 7 (11) (1969) 1149-1161.
290. L.Z. Luo, M. Nahon, Development and validation of geometry-based compliant contact models, J. Comput. Nonlinear Dyn. 6 (1) (2011) 011004.
291. G. Hippmann, An algorithm for compliant contact between complexly shaped bodies, Multibody Sys. Dyn. 12 (2004) 345-362.
292. Y.H. Bei, B.J. Fregly, Multibody dynamic simulation of knee contact mechanics, Med. Eng. Phys. 26 (2004) 777-789.
293. A. Pérez-González, C. Fenollosa-Esteve, J.L. Sancho-Bru, F.T. Sánchez-Marín, M. Vergara, P.J. Rodríguez-Cervantes, A modified elastic foundation contact model for application in 3D models of the prosthetic knee, Med. Eng. Phys. 30 (3) (2008) 387-398.
294. S.M. Mukras, A. Mauntler, N.H. Kim, T.L. Schmitz, W.G. Sawyer, Evaluation of contact force and elastic foundation models for wear analysis of multibody systems, in: Proceedings of the ASME 2010 International Design Engineering Technology Conferences, Montreal, Quebec, Canada, 2010, pp. 1565-1577.

295. J. Awrejcewicz, G. Kudra, C.H. Lamarque, Dynamics Investigation of Three Coupled Rods with Horizontal Barrier, Meccanica, 38 (2003) 687-698.
296. E.D. Stoenescu, D.B. Marghitu, Dynamic analysis of a planar rigid-link mechanism with rotating slider joint and clearance, J. Sound Vib. 266 (2003) 394-404.
297. P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani, Translational joints with clearance in rigid multibody systems, J. Comput. Nonlinear Dyn. 3 (2008) 011007.
298. H.M. Lankarani, Canonical equations of motion and estimation of parameters in the analysis of impact problems, University of Arizona, 1988 Ph.D. thesis.
299. R. Wilson, The dynamics of the slider crank mechanism with clearance in the sliding bearing, University of Newcastle upon Tyne, 1971 Ph.D. thesis.
300. R. Wilson, J.N. Fawcett, Dynamics of slider-crank mechanism with clearance in the sliding bearing, Mech. Mach. Theory 9 (1974) 61-80.
301. F. Farahanchi, S.W. Shaw, Chaotic and periodic dynamics of a slider-crank mechanism with slider clearance, J. Sound Vib. 177 (3) (1994) 307-324.
302. T. Thümmel, K. Funk, Multibody Modeling of Linkage Mechanisms Including Friction, Clearance and Impact, in: Proceedings of Tenth World Congress on TMM, Oulu University Press,Oulu, Finland, 1999, pp. 1375-1386.
303. H.J. Klepp, Modes of contact and uniqueness of solutions for systems with friction-affected sliders, J. Sound Vib. 254 (5) (2002) 987-996.
304. P. Flores, R. Leine, C. Glocker, Modeling and analysis of planar rigid multibody systems with translational clearance joints based on the non-smooth dynamics approach, Multibody Sys. Dyn. 23 (2010) 165-190.
305. P. Flores, R. Leine, C. Glocker, Application of the nonsmooth dynamics approach to model and analysis of the contact-impact events in cam-follower systems, Nonlinear Dyn. 69 (4) (2012) 2117-2133.
306. Z.H. Qi, X.M. Luo, Z.H. Huang, Frictional contact analysis of spatial prismatic joints in multibody systems, Multibody Sys. Dyn. 26 (2011) 441-468.
307. F.F. Zhuang, Q. Wang, Modeling and simulation of the nonsmooth planar rigid multibody systems with frictional translational joints, Multibody Sys. Dyn. 29 (2013) 403-423.
308. F.F. Zhuang, Q. Wang, Modeling and analysis of rigid multibody systems with driving constraints and frictional translation joints, Acta Mech. Sinica 30 (3) (2014) 437-446.
309. J. Zhang, Q. Wang, Modeling and simulation of a frictional translational joint with a flexible slider and clearance, Multibody Sys. Dyn. 38 (4) (2016) 367-389.
310. Q.Z. Chen, V. Acary, G. Virlez, O. Brüls, A nonsmooth generalized-α scheme for flexible multibody systems with unilateral constraints, Int. J. Numer. Meth. Eng. 96 (2013) 487-511.
311. O. Brüls, V. Acary, A. Cardona, Simultaneous enforcement of constraints at position and velocity levels in the nonsmooth generalized-α scheme, Comput. Methods Appl. Mech. Engrg. 281 (2014) 131-161.
312. F.J. Cavalieri, A. Cardona, Non-smooth model of a frictionless and dry three-dimensional revolute joint with clearance for multibody system dynamics, Mechanism and Machine Theory, 121 (2018) 335-354.
313. X.M. Luo, Z.H. Qi, X.C. Kong, Non-colliding contact analysis with friction in the planar prismatic joint, Jisuan Lixue Xuebao(Chin. J. Comput. Mech.)(China) 29 (3) (2012) 387-392.
314. K.L. Ting, K.L. Hsu, Z. Yu, J. Wang, Clearance-induced output position uncertainty of planar linkages with revolute and prismatic joints, Mech. Mach. Theory 111 (2017) 66-75.
315. L.J. Wu, D.B. Marghitu, J. Zhao, Nonlinear dynamics response of a planar mechanism with two driving links and prismatic pair clearance, Mathematical Problems in Engineering, 2017 (2017) 1-12.
316. O. Pinkus, S.A. Sternlicht, Theory of Hydrodynamic Lubrication, McGraw Hill, New York, 1961.
317. B.J. Hamrock, Fundamentals of Fluid Film Lubrication, McGraw Hill, New York ,1994.
318. J. Frêne, D. Nicolas, B. Degneurce, D. Berthe, M. Godet, Hydrodynamic Lubrication - Bearings and Thrust Bearings, Elsevier, Amsterdam, 1997.
319. P. Ravn, A continuous analysis method for planar multibody systems with joint clearance, Multibody Sys. Dyn. 2 (1) (1998) 1-24.
320. L.J. Wu, A.P. Li., X.M. Liu, Modeling and simulation of a slider-crank mechanism with clearance and lubricant joint, Adv. Mat. Res. 510 (2012) 458-466.
321. J. Chen, R. Randall, N.S. Feng, B. Peeters, H. V. D. Auweraer, Modelling and diagnosis of big-end bearing knock fault in internal combustion engines, Proc. IMechE Part C: J. Mech. Eng. Sci. 228 (16) (2014) 2973-2984.
322. J. Chen, R.B. Randall, Intelligent diagnosis of bearing knock faults in internal combustion engines using vibration simulation, Mech. Mach. Theory 104 (2016) 161-176.

323. F. Marques, F. Isaac, N. Dourado, P. Flores, An enhanced formulation to model spatial revolute joints with radial and axial clearances. Mech. Mach. Theory 116: (2017) 123-144.
324. N. Akhadkar, V. Acary, B. Brogliato, 3D Revolute Joint with Clearance in Multibody Systems, The Mechanisms and Machine Science 50 (2018) 11-18.
325. A. Vencl, A. Rac, Diesel engine crankshaft journal bearings failures: Case study, Eng. Fail. Anal. 44 (2014) 217-228.
326. A. Dziedzic, J. Lubas, W. Bochnowski, S. Adamiak, The mechanisms of tribological wear in lubricated sliding bearings composed of soft bearing alloys with borided steel, Proc. IMechE Part J: J. Eng. Tribol. 230 (3) (2016) 350-363.
327. D. Yang, H. Li, Y.G. Hu, Zhao J, H.W. Xiao, Y.S. Lan, Vibration condition monitoring system for wind turbine bearings based on noise suppression with multi-point data fusion, Renew. Energ. 92 (2016) 104-116.
328. X.L. Zhang, B.C. Wen, C.Y. Zhao, Vibratory synchronization and coupling dynamic characteristics of multiple unbalanced rotors on a mass-spring rigid base, Int. J. Nonlinear Mech. 60 (2014) 1-8.
329. Y.J. Hou, P. Fang, Synchronization and Stability of Two Unbalanced Rotors with Fast Antirotation considering Energy Balance, Math. Probl. Eng. (2015) 694145.
330. J.F. Booker, Dynamically Loaded Journal Bearings: Mobility Method of Solution, J. Basic Eng-T ASME 87 (3) (1965) 537-546.
331. P.K. Goenka, Analytical Curve Fits for Solution Parameters of Dynamically Loaded Journal Bearings, J. Tribol. 106 (4) (1984) 421-428.
332. T.A. Stolarsky, Tribology in Machine Design, Butterworth-Heinemann, Oxford, 1990.
333. A. Sommerfeld, Zur Hydrodynamischen Theorie der Schmiermittelreibung, Z. Angew. Math. Phys. 50 (1904) 97-155.
334. D. Dowson, C.M. Taylor, Cavitation in Bearings, Annual Review of Fluid Mechanics 11 (1) (1979) 35-66.
335. H.G. Elrod, A Cavitation Algorithm, J. Lubr. Technol. 103 (1981) 350-354.
336. D.E. Brewe, B.J. Hamrock, B.O. Jacobson, Theoretical and Experimental Comparison of Vapor Cavitation in Dynamically Loaded Journal Bearings, in: NASA Technical Memorandum 87121, USAAVSCOM, Technical Report 85-C-19, 1986.
337. C.M. Woods, D.E. Brewe, The Solution of the Elrod Algorithm for a Dynamically Loaded Journal Bearing Using Multigrid Techniques, J. Tribol. 111 (1989) 302-308.
338. K. Mistry, S. Biswas, K. Athre, A New Theoretical Model for Analysis of the Fluid Film in the Cavitation Zone of a Journal Bearing, J. Tribol. 119 (1997) 741-746.
339. P. Ravn, S. Shivaswamy, B.J. Alshaer, H.M. Lankarani, Joint clearances with lubricated long bearings in multibody mechanical systems, J. Mech. Des. 122 (2000) 484-488.
340. B.J. Alshaer, H.M. Lankarani, Formulation of dynamic loads generated by lubricated journal bearings, in: Proceedings of DETC'01, ASME 2001 Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Pittsburg, Pennsylvania, 2001.
341. P. Flores, H.M. Lankarani, J. Ambrósio, J.C.P. Claro, Modeling lubricated revolute joints in multibody mechanical systems, Proc. IMechE Part K: J. Multibody Dyn. 218 (4) (2004) 183-190.
342. P. Flores, J. Ambrósio, J.C.P. Claro, H.M. Lankarani, C.S. Koshy, Lubricated revolute joints in rigid multibody systems, Nonlinear Dyn. 56 (2009) 277-295.
343. A.L. Schwab, J.P. Meijaard, P. Meijers, A comparison of revolute joint clearance model in the dynamic analysis of rigid and elastic mechanical systems, Mech. Mach. Theory 37 (9) (2002) 895-913.
344. H. Moes, E.G. Sikkes, R. Bosma, Mobility and impedance tensor methods for full and partial-arc journal bearings, J. Tribol. 108 (1986) 612-620.
345. A.C. Bannwart, K.L. Cavalca, G.B. Daniel, Hydrodynamic bearings modeling with alternate motion, Mech. Res. Commun. 37 (2010) 590-597.
346. J. Costa, J. Peixoto, P. Moreira, A.P. Souto, P. Flores, H.M. Lankarani, Influence of the hip joint modeling approaches on the kinematics of human gait, J. Tribol. 138 (2016) 031201.
347. L. Mattei, D.F. Puccio, B. Piccigallo, E. Ciulli, Lubrication and wear modelling of artificial hip joints: A review, Tribol. Int. 44 (2011) 532-549.
348. D.S. Lopes, R.R. Neptune, A.A. Gonçalves, J.A. Ambrósio, M.T. Silva, Shape analysis of the femoral head: a comparative study between spherical, (super) ellipsoidal, and (super) ovoidal shapes, Journal of Biomechanical Engineering 137(11) (2015) 114504.

349. S. Ebrahimi, E. Salahshoor, M. Maasoomi, Application of the method of multiple scales for nonlinear vibration analysis of mechanical systems with dry and lubricated clearance joints, Journal of Theoretical and Applied Vibration and Acoustics 3 (2017) 41-60.
350. A.A. Shabana, An absolute nodal coordinates formulation for the large rotation and deformation analysis of flexible bodies, in: Report. No. MBS96-1-UIC, University of Illinois at Chicago, 1996.
351. J. Gerstmayr, H. Sugiyama, A. Mikkola, Review on the absolute nodal coordinate formulation for large deformation analysis of multibody systems, J. Comput. Nonlinear Dyn. 4 (2013) 031016.
352. F.M. Meng, Y.Z. Hu, H. Wang, Y.Y. Zhang, Analysis of the dynamic performances of a piston-crankshaft system considering oil-film forces reconstructed by a neural network, Proc. IMechE Part D: J. Aut. Eng. 221 (2007) 171-180.
353. B. Zhao, Z.N. Zhang, C.C. Fang, X.D. Dai, Y.B. Xie, Modeling and analysis of planar multibody system with mixed lubricated revolute joint, Tribol. Int. 98 (2016) 229-241.
354. B. Zhao, K. Zhou, Y.B. Xie, A new numerical method for planar multibody system with mixed lubricated revolute joint, Int. J. Mech. Sci. 113 (2016) 105-119.
355. N. Patir, H. Cheng, An average flow model for determining effects of three dimensional roughness on partial hydrodynamic lubrication, J. Tribol. 100 (1978) 12-17.
356. B. Zhao, X.D. Dai, Z.N. Zhang, Y.B. Xie, A new numerical method for piston dynamics and lubrication analysis, Tribol. Int. 94 (2016) 395-408.
357. B. Zhao, Y. Cui, Y.B Xie, K. Zhou, Dynamics and lubrication analyses of a planar multibody system with multiple lubricated joints, Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology (2017) Online, DOI: 10.1177/1350650117713891
358. Q. Tian, C. Liu, M. Machado, P. Flores, A new model for dry and lubricated cylindrical joints with clearance in spatial flexible multibody system, Nonlinear Dyn. 64 (2011) 25-47.
359. Y. Hori, Hydrodynamic Lubrication, Springer, Tokyo, 2006.
360. R. Stefanelli, P.P. Valentini, L. Vita, Modeling of hydrodynamic journal bearing in spatial multibody systems (DETC2005-84858), in: ASME International Design Engineering Technical Conference, California, USA, 2006.
361. M.S. Kumar, P.R. Thyla, E. Anbarasu, Numerical analysis of hydrodynamic journal bearing under transient dynamic conditions, Mechanika 2 (82) (2010) 37-42.
362. H. Harish, V. Manish, Tribological study of elastomeric bearings for marine propeller shaft system, Tribol. Int. l (2009) 378-390.
363. J. Sun, C.L. Gui, Hydrodynamic lubrication analysis of journal bearing considering misalignment caused by shaft deformation, Tribol. Int. 37 (2004) 841-848.
364. D. Dowson, P. Ehret, Past, present and future studies in elastohydrodynamics, Proc. IMechE Part J: J. Eng. Tri. 213 (1999) 317-333.
365. P.M. Lugt, G.E. Morles-Espejel, A review of elasto-hydrodynamic lubrication theory, Tribol. T. 54 (2011) 470-496.
366. H.P. Liu, H. Xu, P.J. Ellision, Z.M. Jin, Application of computational fluid dynamics and Fluid-Structure Interaction method to the lubrication study of a rotor bearing System, Tribol. Lett. 38 (3) (2010) 325-336.
367. H.M. Attia, S. Bouziz, M. Maatar, T. Fakhfakh, M. Haddar, Hydrodynamic and elastohydro dynamic studies of a cylindrical journal bearing, J. Hydrodyn. 22 (2) (2010) 155-163.
368. B. Slim, F. Tahar, H. Mohamed, Acoustic analysis of hydrodynamic and elasto-hydrodynamic oil lubricated journal bearings, J. Hydrodyn. 24 (2) (2012) 250-256.
369. S.S. Brown, I.C. Clarke, A review of lubrication conditions for wear simulation in artificial hip replacements, Tribol. T. 49 (1) (2006) 72-78.
370. Z.M. Jin, D. Dowson, J. Fisher, Analysis of fluid film lubrication in artificial hip joint replacements with surfaces of high elastic modulus, Proc. IMechE Part H: J. Eng. Med. 211 (1997) 247-256.
371. J. Choi, S.S. Kim, S.S. Rhim, J.H. Choi, Numerical modeling of journal bearing considering both elastohydrodynamic lubrication, Int. J. Automot. Techn. 13 (2012) 255-261.
372. Q. Tian, Y. Sun, C. Liu, H.Y. Hu, P. Flores, Elastohydrodynamic lubricated cylindrical joints for rigid–flexible multibody dynamics, Comput. Struct. 114 (2013) 106-120.
373. P. Huang, Numerical Calculation of Elastohydrodynamic Lubrication: Methods and Programs, John Wiley & Sons Singapore Pte Ltd, Singapore, 2015.
374. Q. Tian, Q. Xiao, Y. Sun, H.Y. Hu, P. Flores, Coupling dynamics of a geared multibody system supported by Elastohydrodynamic lubricated cylindrical joints, Multibody Sys. Dyn. 33 (2015) 259-284.

375. A. Krinner, D.J. Rixen, Interface reduction methods for mechanical systems with elastohydrodynamic lubricated revolute joints, Multibody Sys. Dyn. 7 (2017) 1-18.
376. B. Yang, T.A. Laursen, A mortar-finite element approach to lubricated contact problems, Comput. Methods Appl. Mech. Engrg. 198 (2009) 3656-3669.
377. F.C. Wang, L.S. Wang, M.L. Sun, Tribological modelling of spherical bearings with complex spherical-based geometry and motion, WIT Transactions on Engineering Sciences 66 (2010) 3-15.
378. P.K. Goenka, Effect of Surface Ellipticity on Dynamically Loaded Spherical and Cylindrical Joints and Bearings, Cornell University, 1980 Ph.D. thesis.
379. P. Flores, H.M. Lankarani, Spatial rigid-multi-body systems with lubricated spherical clearance joints: modeling and simulation, Nonlinear Dyn. 60(1-2) (2010) 99-114.
380. Q. Tian, J. Lou, A. Mikkola, A new elastohydrodynamic lubricated spherical joint model for rigid-flexible multibody dynamics, Mech. Mach. Theory 107 (2017) 210-228.
381. G.B. Daniel, K.L. Cavalca, Analysis of the dynamics of a slider-crank mechanism with hydrodynamic lubrication in the connecting rod-slider joint, Mech. Mach. Theory 74 (2011) 1434-1452.
382. J.B. Luo, C.N. Yan, On the fuzzy view in the lubrication theory, Lubr. Eng. 4 (1989) 1-4 (in Chinese).
383. P. Li, W. Chen, D.S. Li, R.F. Yu, A novel transition model for lubricated revolute joints in planar multibody systems, Multibody Sys. Dyn. 36 (2016) 279-294.
384. D. Mahrus, Experimental investigation into journal bearing performance, Revista Brasileira de Tecnologia 5(3-4) (1974) 139-152.
385. R.S. Haines, A theory of contact loss at resolute joints with clearance, Journal of Mechanical Engineering Science 22 (3) (1980) 129-136.
386. R.S. Haines, An experimental investigation into the dynamic behaviour of revolute joints with varying degrees of clearance, Mech. Mach. Theory 20 (3) (1985) 221-231.
387. R.L. Norton, H.K. Ault, J. Wiley, T. Parks, R. Calawa, M. Wickstrand, Bearing forces as function of mechanical stiffness and vibration isolation in a four bar linkage, Astm Special Technical Publication 1247 (1995) 11.
388. A.A.S. Miranda, Oil flow, cavitation and film reformulation in journal bearings including interactive computer-aided design study, Department of Mechanical Engineering, University of Leeds, 1983 Ph.D. thesis.
389. J.C.P. Claro, Reformulação de método de cálculo de chumaceiras radiais hi-drodinâmicas - análise do desempenho considerando condições de alimentação, University of Minho, 1994 Ph.D. thesis.
390. L.A.M. Costa, Análise do desempenho de chumaceiras radiais hidrodinâmicas considerando efeitos térmicos, University of Minho, 2000 Ph.D. thesis.
391. F.P. Brito, Thermohydrodynamic performance of twin groove journal bearings considering realistic lubricant supply conditions: an experimental and theoretical study, University of Minho, 2009 Ph.D. thesis.
392. F.P. Brito, A.S. Miranda, J.C.P. Claro, M. Fillon, Experimental comparison of the performance of a journal bearing with a single and a twin axial groove configuration, Tribol. Int. 54 (2012) 1-8.
393. J. Bouyer, M. Fillon, An experimental analysis of misalignment effects on hydrodynamic plain journal bearing performances, J. Tribol. 124 (2002) 313-319.
394. J. Sun, C.L. Gui, Z.Y. Li, An experimental study of journal bearing lubrication effected by journal misalignment as a result of shaft deformation under load, J. Tribol. 127 (2005) 813-819.
395. J.N. Fawcett, J.S. Burdess, Effects of bearing clearance in a four-bar linkage, in: Proceedings of the Third World Congress of IFToMM, Kurapi, Yugoslavia, 1971, pp. 111-126.
396. H. Shimojima, K. Ogawa, K. Matsumoto, Dynamic characteristics of planar mechanisms with clearance, Bulletin JSME, 21 (1978) 303-308.
397. S.W.E. Earles, O. Kilicay, Predicting impact conditions due to bearing clearances in linkage mechanisms, in: Proceedings of Fifth World Congress IFToMM, Montreal, Canada, 1979, pp. 1078-1081.
398. S.J. Grant, J.N. Fawcett, Effects of clearance at the coupler-rocker bearing of a 4-bar linkage, Mech. Mach. Theory 14 (1979) 99-110.
399. Z. Li, L. Li, S.X. Bai, A new method of predicting the occurrence of contact loss between pairing elements in planar linkages with clearances, Mech. Mach. Theory 27 (1992) 295-301.
400. F. Pfeiffer, Stick-slip vibration in machine dynamics, Machine Vibration 1 (1992) 20-28.
401. C.W. Stammers, M. Ghazavi, Theoretical and experimental study of the dynamics of a four bar chain with bearing clearance, pin motion, contact loss and impacts, J. Sound Vib. 50 (2) (1991) 301-315.

402. A. Tasora, E. Prati, M. Silvestri, Experimental investigation of clearance effects in a revolute joint, in: Proceedings of the 2004 AIMETA International Tribology Conference, Rome, Italy, September, 2004, pp. 14-17.
403. K. Soong, B.S. Thompson, A theoretical and experimental investigation of the dynamic response of a slider-crank mechanism with radial clearance in the gudgeon-pin joint, J. Mech. Des. 112 (1990) 183-189.
404. X. Jia, D. Jin, L. Ji, J. Zhang, Investigation on the dynamic performance of the tripod-ball sliding joint with clearance in a crank-slider mechanism. Part 1. Theoretical and experimental results, J. Sound Vib. 252 (5) (2002) 919-933.
405. P. Ravn, S. Shivaswamy, H.M. Lankarani, Modeling Joint Clearances in Multibody Mechanical Systems, in: International Conference on Dynamics and Control (DYCONS99), Ottawa, Canada, August 1999.
406. H.M. Lankarani, S. Shivaswamy, Methodologies for Treatment of Impact in Multibody Mechanical Systems, Chapter 4 in the book, Impact with Friction of Solids, Structures and Machines, A. Guran (Ed.), Birkhauser, Boston, 2003.
407. A.F. Haroun, S.M. Megahed, Simulation and experimentation of multibody mechanical systems with clearance revolute joints, International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering 6 (3) (2012) 658-667.
408. X. Lai, H. He, Q. Lai, C. Wang, J. Yang, Y. Zhang, H. Fang, S. Liao, Computational prediction and experimental validation of revolute joint clearance wear in the low-velocity planar mechanism, Mech. Syst. Signal Pr. 85 (2017) 963-976.
409. S. Dubowsky, S.C. Young, An experimental and analytical study of connection forces in high-speed mechanisms, J. Eng. Ind. 97B (1975) 1166-1174.
410. I. Khemili, L. Romdhane, Dynamic analysis of a flexible slider-crank mechanism with clearance, Eur. J. Mech. A/Solid 27 (2008) 882-898.
411. S. Erkaya. İ. Uzmay, Effects of balancing and link flexibility on dynamics of a planar mechanism having joint clearance, Scientia Iranica 19 (2012) 483-490.
412. S. Erkaya, S. Doğan, S. Ulus, Effects of joint clearance on the dynamics of a partly compliant mechanism: Numerical and experimental studies, Mech. Mach. Theory 88 (2015) 125-140.
413. M. Ahmedalbashir, Dynamic Analysis of Flexible Mechanisms with Clearance, MSc Dissertation, American University of Sharjah, United Arab Emirates, 2016.
414. M. Ahmedalbashir, L. Romdhane, J. Lee, Dynamics of a four-bar mechanism with clearance and springs-modeling and experimental analysis, Journal of Mechanical Science and Technology, 31(3) (2017) 1023-1033.
415. M. Dupac, D.G. Beale, Dynamic analysis of a flexible linkage mechanism with cracks and clearance, Mech. Mach. Theory 45 (12) (2010) 1909-1923.
416. S. Mukras, N.H. Kim, N.A. Mauntler, T.L. Schmitz, W.G. Sawyer, Analysis of planar multibody systems with revolute joint wear, Wear 268 (2010) 643-652.
417. R. Liu, J. Zhang, H. Guo, Z. Deng, Nonlinear characteristics of revolute joints with clearance, J. Aerosp. Technol. Manag. 5 (3) (2013) 341-348.
418. C.S. Koshy, P. Flores, H.M. Lankarani, Study of the effect of contact force model on the dynamic response of mechanical systems with dry clearance joints: computational and experimental approaches, Nonlinear Dyn. 73 (2013) 325-338.
419. F. Meng, S. Wu, F. Zhang, L. Liang, Numerical modeling and experimental verification for high-speed and heavy-load planar mechanism with multiple clearances, Mathematical Problems in Engineering, 2015 (3) (2015) 1-11.
420. N. Akhadkar, V. Acary, B. Brogliato, Multibody systems with 3D revolute joints with clearances: an industrial case study with an experimental validation, Multibody Sys. Dyn. 2 (2017) 1-34.
421. L. Skrinjar, J. Slavič, M. Boltežar, A validated model for a pin-slot clearance joint, Nonlinear Dyn. 88 (2017) 131-143.
422. Z. Gahr, Karl-Heinz, Microstructure and Wear of Materials. Elsevier, Amsterdam, 1987.
423. H.C. Meng, K.C. Ludema, Wear models and predictive equations: their form and content, Wear 181 (95) (1995) 443-457.
424. T. Reye, Zur theorie der zapfengreibund, Der Civilingenieur 4 (1860) 235-255.
425. J.F. Archard, Contact and rubbing of flat surfaces, J. Appl. Phys. 24(8) (1953) 981-988.
426. N. Strömberg, A method for structural dynamic contact problems with friction and wear, Int. J. Numer. Meth. Eng. 58(15) (2003) 2371-2385.

427. J.J. Moreau, Some numerical methods in multibody dynamics: application to granular materials, Eur. J. Mech. A/Solid 13 (1994) 93-114.
428. J. Slavič, M. Boltežaer, Simulating multibody dynamics with rough contact surfaces and run-in wear, Nonlinear Dyn. 45 (2006) 353-365.
429. P. Flores, Modeling and simulation of wear in revolute clearance joints in multibody systems, Mech. Mach. Theory 44 (2009) 1211-1222.
430. Y. Lin, R.T. Haftka, N.V. Queipo, B.J. Fregly, Two-dimensional surrogate contact modeling for computationally efficient dynamic simulation of total knee replacements, J. Biomech. Eng. 131 (4) (2009) 041010.
431. S. Mukras, N.H. Kim, W.G. Sawyer, D.B. Jackson, L.W. Bergquist, Numerical integration schemes and parallel computation for wear prediction using finite element method, Wear 266 (2009) 822-832.
432. Y. Su, W. Chen, Y. Tong, Y. Xie, Wear prediction of clearance joint by integrating multi-body kinematics with finite-element method, Proc. IMechE Part J: J. Eng. Tri. 224 (2010) 815-823.
433. W.G. Sawyer, Wear predictions for a simple-cam including the coupled evolution of wear and load, Lubr. Eng. 57 (9) (2001) 31-36.
434. Z.F. Bai, Y. Zhao, J. Chen, Dynamics analysis of planar mechanical system considering revolute clearance joint wear, Tribol. Int. 64 (2013) 85-95.
435. Z.F. Bai, H.B. Zhang, Y. Sun, Wear prediction for dry revolute joint with clearance in multibody system by integrating dynamics model and wear model, Lat. Am. J. Solids Stru. 11 (14) (2014) 2624-2647.
436. B. Zhao, X.D. Dai, Z.N. Zhang, S.H. Wu, Y.B. Xie, Numerical study of parametric effects on joint wear in the flexible multibody systems with different flexibilities and clearance sizes, Proc. IMechE Part J: J. Eng. Tribol. 228 (8) (2014) 819-835.
437. B. Zhao, Z.N. Zhang, X.D. Dai, Modeling and prediction of wear at revolute clearance joints in flexible multibody systems, Proc. IMechE Part J: J. Eng. Tribol. 228(2) (2014) 317-329.
438. B. Zhao, X.D. Dai, Z.N. Zhang, Y.B. Xie, Numerical study of the effects on clearance joint wear in flexible multibody mechanical system, Tribol. T. 58 (3) (2015) 385-396.
439. E. Askari, P. Flores, D. Dabirrahmani, R. Appleyard, Dynamic modeling and analysis of wear in spatial hard-on-hard couple hip replacements using multibody systems methodologies, Nonlinear Dyn. 82 (2015) 1039-1058.
440. G. Wang, H. Liu, P. Deng, K. Yin, G. Zhang, Dynamic analysis of 4-SPS/CU parallel mechanism considering three-dimensional wear of spherical joint with clearance, J. Tribol 139 (2017) 021608.
441. W.W.K. Xiang, S.Z. Yan, J.N. Wu, A comprehensive method for joint wear prediction in planar mechanical systems with clearances considering complex contact conditions, Science China Technological Sciences, 58 (2015) 86-96.
442. P. Li, W. Chen, A.B. Zhu, An improved practical model for wear prediction of revolute clearance joints in crank slider mechanisms, Science China Technological Sciences, 56 (2013) 2953-2963.
443. A.B. Zhu, S.L. He, C. Zou, W. Chen, The effect analysis of contact stiffness on wear of clearance joint, J. Tribol. 139 (2017) 031403.
444. A.B. Zhu, S.L. He, J.W. Zhao, W.C. Luo, A nonlinear contact pressure distribution model for wear calculation of planar revolute joint with clearance, Nonlinear Dyn. 88 (2017) 315-328.
445. G. Wang, H. Liu, Dynamic analysis and wear prediction of planar five-bar mechanism considering multiflexible links and multiclearance joints, J. Tribololgy 139 (5) (2017) 051606.
446. N. Nikolic, T. Torovic, Z. Antonic, A procedure for constructing a theoretical wear diagram of IC engine crankshaft main bearings, Mech. Mach. Theory 58 (2012) 120-136.
447. J. Rhee, A. Akay, Dynamic response of a revolute joint with clearance, Mech. Mach. Theory 31 (1) (1996) 121-124.
448. S.B. Farahan, M.R. Ghazavi, S. Rahmanian, Bifurcation in a planar four-bar mechanism with revolute clearance joint, Nonlinear Dyn. 87 (2) (2017) 955-973.
449. W. Xiang, S. Yan, J. Wu, R.X. Gao, Complexity evaluation of nonlinear dynamic behavior of mechanisms with clearance joints by using the fractal method, Proc. IMechE Part C: J. Mech. Eng. Sci. 228 (18) (2014) 3482-3495.
450. S.M. Varedi, H.M. Daniali, M. Dardel, A. Fathi, Optimal dynamic design of a planar slider-crank mechanism with a joint clearance, Mech. Mach. Theory 86 (2015) 191-200.
451. J. Sun, G. Xu, Deviation analysis and optimization of offset slider-crank mechanism based on the simulation, Information Technology Journal. 12 (12) (2013) 2390-2397.

452. H.P. Jawale, H.T. Thorat, Positional error estimation in serial link manipulator under joint clearances and backlash, J. Mech. Robot. 5 (2) (2013) 021003.
453. K. Zhao, J.J. Chen, H.J. Cao, Y.H. Yun, Randomness analysis of dynamic responses of a slider-crank system with clearances, Zhendong yu Chongji/Journal of Vibration and Shock, 33 (11) (2014) 130-135.
454. S. Erkaya, İ. Uzmay, Determining link parameters using genetic algorithm in mechanisms with joint clearance, Mech. Mach. Theory 44 (2009) 222-234.
455. S. Erkaya, Trajectory optimization of a walking mechanism having revolute joints with clearance using ANFIS approach, Nonlinear Dyn. 71 (2013) 75-91.
456. A. Cammarata, A novel method to determine position and orientation errors in clearance-affected overconstrained mechanisms, Mech. Mach. Theory 118 (2017) 247-264.
457. M. Younes, P.D. Santo, A. Potiron, Three-dimensional study for the relative positioning of mechanical elements in mechanisms constituted by parallel joints with clearances, Mechanical Engineering: An international Journal, 1 (1) (2014) 1-22.
458. Z. Zhang, L. Xu, P. Flores, H.M. Lankarani, A kriging model for dynamics of mechanical systems with revolute joint clearances, J. Comput. Nonlinear Dyn. 9 (2014) 031013.
459. A. Sardashti, H.M. Daniali, S.M. Varedi, Optimal free-defect synthesis of four-bar linkage with joint clearance using PSO algorithm, Meccanica 48 (2013) 1681-1693.
460. S.M. Varedi, H.M. Daniali, M. Dardel, Dynamic synthesis of a planar slider-crank mechanism with clearances, Nonlinear Dyn. 79 (2015) 1587-1600
461. S. Yaqubi, M. Dardel, H.M. Daniali, M.H. Ghasemi, Modeling and control of crank–slider mechanism with multiple clearance joints, Multibody Sys. Dyn. 36 (2) (2016) 143-167.
462. S. Yaqubi, M. Dardel, H.M. Daniali, M.H. Ghasemi, Nonlinear dynamics and control of crank-slider mechanism with multiple clearance joints, Journal of Theoretical and Applied Vibration and Acoustics, 1 (2015) 21-31.
463. S. Yaqubi, M. Dardel, H.M. Daniali, Nonlinear dynamics and control of crank–slider mechanism with link flexibility and joint clearance, Proc. IMechE Part C: J. Mech. Eng. Sci. 230 (5) (2016) 737-755.
464. A. Moghadasi, A. Held, R. Seifried, Modeling of revolute joints in topology optimization of flexible multibody Systems, J. Comput. Nonlinear Dyn. 12 (2016) 011015.
465. S. Rahmanian, M.R. Ghazavi, Bifurcation in planar slider–crank mechanism with revolute clearance joint, Mech. Mach. Theory 91 (2015) 86-101
466. K. Pyragas (1992) Continuous control of chaos by self-controlling feedback, Phys. Lett. A 170 (1992) 421-428
467. A.A. Olyaei, M.R. Ghazavi, Stabilizing slider crank mechanism with clearance joints, Mech. Mach. Theory 44 (2012) 1211-1222.
468. B. Brogliato, Feedback control of multibody systems with joint clearance and dynamic backlash: a tutorial, Multibody Sys. Dyn. 1 (2017) 1-33.
469. H.Y. Zhao, M.Q. Xu, J.D. Wang, Y.B. Li, A parameters optimization method for planar joint clearance model and its application for dynamics simulation of reciprocating compressor, J. Sound Vib. 344 (2015) 416-433.
470. G. Quaranta, Finite element analysis with uncertain probabilities, Comput. Methods Appl. Mech. Eng. 200 (2011) 114-129.
471. D. Moens, M. Hanss, Non-probabilistic finite element analysis for parametric uncertainty treatment in applied mechanics: Recent advances, Finite Elem. Anal. Des. 47 (2011) 4-16.
472. R.E. Moore, Interval Analysis, Prentice-Hall, Englewood Cliffs, New Jersey, 1966.
473. H. Pang, T. Yu, B. Song, Analyses of kinematic accuracy reliability and sensitivity for a planar linkage mechanism, Zhongguo Jixie Gongcheng/China Mechanical Engineering, 25(18) (2014) 2415-2421
474. J.F. Zhang, X.P. Du, Time-dependent reliability analysis for function generation mechanisms with random joint clearances, Mech. Mach. Theory 92 (2015) 184-99.
475. J.G. Wang, J.F. Zhang, X.P. Du, Hybrid dimension reduction for mechanism reliability analysis with random joint clearances, Mech. Mach. Theory 46 (2011) 1396-1410.
476. M.D. Pandey, X. Zhang, System reliability analysis of the robotic manipulator with random joint clearances, Mech. Mach. Theory 58 (2012) 137-152.
477. S. Yan, P. Guo, Kinematic accuracy analysis of flexible mechanisms with uncertain link lengths and joint clearances, Proc. IMechE Part C: J. Mech. Eng. Sci. 225 (2011) 1973-1983.
478. G.L. Chen, H. Wang, Z.Q. Lin, A unified approach to the accuracy analysis of planar parallel manipulators both with input uncertainties and joint clearance, Mech. Mach. Theory 64 (2013) 1-17

479. W. Wu, S.S. Rao, Uncertainty analysis and allocation of joint tolerances in robot manipulators based on interval analysis, Reliability Engineering and System Safety 92 (2007) 54-64.
480. D.Y. Sun, G.P. Chen, T.C. Wang, Wear prediction of a mechanism with joint clearance involving aleatory and epistemic Uncertainty, J. Tribol. 136 (4) (2014) 041101.
481. D.Y. Sun, G.P. Chen, Kinematic accuracy analysis of planar mechanisms with clearance involving random and epistemic uncertainty, Eur. J. Mech. A-Solid, 58 (2016) 256-261.
482. D.Y. Sun, Tracking accuracy analysis of a planar flexible manipulator with lubricated joint and interval uncertainty, J. Comput. Nonlinear Dyn. 11 (2016) 051024.
483. Z. Wang, Q. Tian, H.Y. Hu, P. Flores, Nonlinear dynamics and chaotic control of planar flexible multibody systems with interval clearance size joint, Nonlinear Dyn. 86 (3) (2016) 1571-1597.
484. A.A. Shabana, ANCF reference node for multibody system analysis, Proc. IMechE Part K: J. Multibody Dyn. 229 (1) (2014) 109-112.
485. J.L. Wu, Z. Luo, N. Zhang, Y.Q. Zhang, A new interval uncertain optimization method for structures using Chebyshev surrogate models, Comput. Struct. 146 (2015) 185-196.
486. K. Luo, J. Wang, X. Du, Robust mechanism synthesis with truncated dimension variables and interval clearance variables, Mech. Mach. Theory 57 (2012) 71-83.
487. J. Ma, L.F. Qian, G.S. Chen, Parameter Estimation of the Lankarani-Nikravesh Contact Force Model Using a New Modified Linear Method, in: IEEE International Conference on Advanced Intelligent Mechatronics, Busan, Korea, (2015) 494-499
488. A. Haddadi, K.H. Zaad, A new method for online parameter estimation of Hunt-Crossley environment dynamic models, in: IEEE/RSJ International Conference on Intelligent Robotics and Systems, Nice, France (2008) 981- 986.
489. J. Ma, L.F. Qian, G.S. Chen, Online parameter estimation of the Lankarani–Nikravesh contact force model using two different methods, International Journal of Computational Methods 13 (2016) 1641017.
490. J. Ma, L.F. Qian, G.S. Chen, S. Dong, Unscented kalman filter for parameter estimation of the lankarani-Nikravesh Contact Force Model, in: 14th World Congress in Mechanism and Machine Science, Taipei, Taiwan, (2015) 25-30.
491. A. Haddadi, K.H. Zaad, Real-time identification of Hunt-Crossley dynamic models of contact environments, IEEE Transactions on Robotics 28 (2012) 555-566.
492. J. Shin, Y.M. Zhong, J. Smith, C.G. Gu, A new parameter estimation method for online soft tissue characterization, Journal of Mechanics in Medicine and Biology 16 (2016) 1640019
493. S. Erkaya, İ. Uzmay, Modeling and simulation of joint clearance effects on mechanisms having rigid and flexible links, Journal of Mechanical Science and Technology 28 (2014) 2979-2986.
494. S. Erkaya, S. Doğan, A comparative analysis of joint clearance effects on articulated and partly compliant mechanisms, Nonlinear Dyn. 81 (2015) 323-341.
495. S. Erkaya, Prediction of vibration characteristics of a planar mechanism having imperfect joints using neural network, J. Mech. Sci. Technol. 26 (2012) 1419-1430.
496. S. Erkaya, İ. Uzmay, Investigation on effect of joint clearance on dynamics of four-bar mechanism, Nonlinear Dyn. 58 (2009) 179-198.
497. J. Aginaga, O. Altuzarra, E. Macho, X. Iriarte, Assessing position error due to clearances and deformations of links in parallel manipulators. Journal of Mechanical Design, 135 (4) (2013) art. no. 041006
498. J. Li, S. Yan, F. Guo, P. Guo, Effects of damping, friction, gravity, and flexibility on the dynamic performance of a deployable mechanism with clearance. Proc. IMechE Part C: J. Mech. Eng. Sci. 227 (8) (2013) 1791-1803.
499. B. Zhao, X.-D. Dai, Z.-N. Zhang, Y.-B. Xie, Prediction of wear in revolute clearance joints in flexible multibody systems. Mocaxue Xuebao/Tribology, 33 (6) (2013) 638-644.
500. Z. Song, X. Yang, B. Li, W. Xu, H. Hu, Modular dynamic modeling and analysis of planar closed-loop mechanisms with clearance joints and flexible links, Proc. IMechE Part C: J. Mech. Eng. Sci. 231 (2015) 1989-1996.