Probabilistic Lifetime-Oriented Multiobjective Optimization of Bridge Maintenance: Single Maintenance Type

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Abstract: The main objective of a bridge manager is to find the best maintenance plan for a group of bridges over a prescribed time horizon. The bridge manager usually faces conflicting objectives, as maintenance plans resulting in safer and less deteriorated structures also lead to higher costs. In general, the problem is posed as a deterministic single-objective optimization where cost is minimized keeping performance above predefined thresholds. However, single-objective optimization results in only one optimal solution that does not provide the advantages of considering other objectives and constraints. In addition, the effects of uncertainties are not taken into account or are included in a very simplified way. The bridge manager obtains, in this way, only one deterministic optimum maintenance plan, and not a set of different probabilistic maintenance solutions from which the best, for each particular situation, can be chosen. In this paper, a full probabilistic multiobjective approach to bridge maintenance considering single maintenance types is developed. This approach is based on the latest developments in bridge management by considering probabilistic continuous performance indicators and probabilistically defined objectives and constraints. The problem is solved using multiobjective genetic algorithms and a Latin hypercube sampling technique. Multiobjective applications to existing reinforced concrete bridge components under probabilistic deterioration and probabilistic-defined single maintenance types are presented and discussed.

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Introduction

Over the past two decades, significant efforts have been invested in the development of tools to help in rational decision making on bridge maintenance and management. Currently employed tools define performance in terms of the condition states resulting from visual inspections (Hawk and Small 1998; Thompson 1993). More advanced tools (Frangopol et al. 2001) consider the performance defined by the reliability index. In both cases, single objective optimization is used to select strategies leading to the minimum life-cycle cost, respecting constraints on the condition or reliability indices over a prescribed time horizon.

The use of a condition state as the only indicator of the performance of a structure can be misleading. In fact, structural damages that are not visible or not discovered by visual inspections can be extremely detrimental to the structural safety.

Furthermore, results of visual inspection can be significantly influenced by the experience of the inspectors, accessibility to the structure, and recent repairs that might have hidden existing defects. Consequently, bridge management systems have to also consider the load carrying capacity (or structural reliability) deterioration (Das 1998; Frangopol and Das 1999). Using the reliability index alone does not take advantage of the huge amount of information collected over the last decades on the deterioration of condition state of existing structures.

Significant efforts are in progress (Neves and Frangopol 2005) to include both condition and safety as performance indicators of deteriorating structures. In addition, a deterministic multi-objective optimization approach by considering condition, safety, and cost was recently proposed by Liu and Frangopol (2004, 2005). The limitations of this approach, where only a deterministic performance is optimized, are overcome in the present paper. In fact, a full probabilistic multiobjective optimization approach is considered and solved herein.

In this paper, both the condition index (resulting from visual inspections) and the safety index (resulting from structural analysis) are used as indicators of the performance of deteriorating structures in a probabilistic lifetime-oriented multiobjective optimization framework considering single maintenance types. These indicators, as well as maintenance cost, are fully probabilistically described. The condition index, the safety index, and the present value of cost are considered as objective functions to be balanced.

In this way, the decision maker is provided with the ability to choose the best possible compromise among available funds, necessary safety and condition levels, and acceptable levels of deterioration, depending on the specific situation under consideration, the preferences of the bridge manager, and the maintenance policy in effect. This results, for each problem of interest, in a set

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of Pareto optimal solutions. From this set, a decision maker can choose the solution that best balances the objectives for a specific situation.

Multiobjective optimization is performed by using genetic algorithms (GAs). The condition index, safety index, and present value of cumulative cost are defined in a probabilistic framework as described in Neves and Frangopol (2005). Latin hypercube sampling is used to compute the evolution in time of performance indicators and cost. Application of multiobjective optimization to existing reinforced concrete bridge components under probabilistic deterioration and probabilistic-defined single maintenance types are presented and discussed.

Condition, Safety, and Cost under Uncertainty

The structural deterioration and effects of maintenance actions on condition and safety of a group of similar elements or structures are described using the model proposed in Neves and Frangopol (2005). In this model, the performance under no maintenance is defined by considering that, for a time period after construction, there is no deterioration of condition and safety. After the expiration of this time period, deterioration of conditions and safety are expressed by prescribed (linear or nonlinear) functions. The condition index and safety index profiles under no maintenance are defined by six random variables as follows: (1) initial condition index and safety index, C_0 and S_0 , respectively; (2) times of initiation of deterioration of condition and safety, t_{ic} and t_i , respectively; and (3) deterioration rates of condition index and safety index, α_c and α , respectively.

Each maintenance action can lead to one, several, or all of the following effects: (1) improvement in the condition index and/or safety index immediately after application; (2) suppression of the deterioration in condition index and/or safety index during a time interval after application; and (3) reduction of the deterioration rate of condition index and/or safety index during a time interval after application (Neves and Frangopol 2005).

These effects are modeled through several random variables as follows: (1) increase in condition index and safety index immediately after application, γ_c and γ , respectively; (2) time during which the deterioration processes of condition index and safety index are suppressed, t_{dc} and t_d , respectively; (3) time during which the deterioration rates of condition index and safety index are suppressed or reduced, t_{pdc} and t_{pd} , respectively; and (4) deterioration rate reduction of condition index and safety index, δ_c and δ , respectively. All these random variables are defined in detail in Neves and Frangopol (2005). Alternatively, the reduction in deterioration after application of maintenance can be defined by the deterioration rates of condition and safety during effect of maintenance, θ_c and θ , respectively.

Latin Hypercube Sampling

To compute the probabilistic indicators of the condition index, safety index, and present value of cumulative cost, Latin hypercube sampling is used. This sampling technique allows the reduction of the number of necessary samples to reach a certain level of confidence. This is a fundamental requirement when the probabilistic profiles have to be computed repeatedly, as is the case when optimization is used.

In Latin hypercube sampling (McKay et al. 1979) the sample space of each random variable is divided in m intervals, and one sample is used from each of these intervals. The samples are generated by creating (1) a matrix $\mathbf{P}$ ($n \times k$), in which each of the k columns is a permutation of $1, \dots, n$, where n is the number of samples, and k is the number of random variables, and (2) a matrix $\mathbf{R}$ ($n \times k$) of independent random numbers, uniformly distributed between 0 and 1. These numbers can be generated using a pseudorandom generator, similar to that proposed in Press (1992). The samples are described by matrix $\mathbf{S}$ as

$$\mathbf{S} = \frac{1}{n}(\mathbf{P} - \mathbf{R}) \quad (1)$$

Each element of $\mathbf{S}$ is then transformed to the respective element of the marginal distribution $\mathbf{X}$ as follows:

$$x_{ij} = F_{x_j}^{-1}(s_{ij}) \quad (2)$$

where $F_{x_j}^{-1}$ =inverse cumulative distribution of random variable x_j . Each vector x_i represents one sample. However, the generation of samples using this methodology might result in a set of correlated samples. It has been shown (Olsson et al. 2003) that this correlation can be reduced by modifying the permutation matrix $\mathbf{P}$.

This is achieved by mapping on the Gaussian space the values of matrix $\mathbf{P}$ divided by the number of samples plus one as

$$y_{ij} = \Phi_{0,1}^{-1}\left(\frac{p_{ij}}{n+1}\right) \quad (3)$$

where $\Phi_{0,1}^{-1}$ =inverse Gaussian cumulative distribution function, p_{ij} =elements of matrix $\mathbf{P}$, and n =number of random variables.

The covariance of matrix $\mathbf{Y}$ is computed and Cholesky decomposed as

$$\text{cov}(\mathbf{Y}) = \bar{\mathbf{L}}\bar{\mathbf{L}}^T \quad (4)$$

where $\text{cov}(\mathbf{Y})$ =covariance of matrix $\mathbf{Y}$, and $\bar{\mathbf{L}}$ =lower triangular matrix from the Cholesky decomposition of matrix $\text{cov}(\mathbf{Y})$.

A new matrix $\mathbf{Y}^*$ with a sample covariance equal to the identity is computed as

$$\mathbf{Y}^* = \mathbf{Y}(\bar{\mathbf{L}}^{-1})^T \quad (5)$$

The ranks of matrix $\mathbf{Y}^*$ become the columns of the matrix $\mathbf{P}^*$. This matrix replaces matrix $\mathbf{P}$ in Eq. (1) resulting in a significant reduction of undesired correlation.

If the target correlation matrix is not the identity matrix and correlation exists among the random variables, Eq. (5) is replaced with

$$\mathbf{Y}^* = \mathbf{Y}(\bar{\mathbf{L}}^{-1})^T\mathbf{L}^T \quad (6)$$

where $\mathbf{L}$ =lower triangular matrix from the Cholesky decomposition of the target correlation matrix.

Genetic Algorithms

In this paper, multiobjective optimization is performed using GAs. These algorithms loosely emulate the evolution of species according to Darwin theory, simulating the optimization process as a sequence of generations where each new generation is produced based on the properties of the fittest individual of the previous generation (Goldberg 1989; Gen and Cheng 1997). The use of multiobjective genetic algorithms allows, in a simple manner, the computation of a large number of Pareto optimal solutions, particularly in problems involving discontinuous

objective functions and/or constraints and/or frequent local optimal solutions. GAs do not require the computation of gradients, making them particularly well suited for problems where numerical gradients are imprecise. The main disadvantage of the approach based on GAs is the large number of times the objectives and constraints must be computed. The method used can be shortly described by the following five steps (Deb and Goel 2001):

1. Randomly create an initial population P_0 of size N ;
2. Compute the offspring population Q_i of size N , where i =generation index ($i=0, 1, 2, \dots$);
3. Combine the two populations P_i and Q_i to form the population R_i of size $2N$;
4. Select the N fittest values of R_i composing P_{i+1} ; and
5. Repeat the process from the second step (i.e., computing the offspring population Q_i) using the population selected above.

Unlike traditional GA, this method keeps solutions from the previous generation, if these prove to be more fit than their offsprings. Although computationally more expensive, this use of elitism leads to faster conversion to optimal solutions for small populations.

The offspring population Q_i is obtained from the parent population P_i using the crossover and mutation procedures. The crossover procedure consists of the combination of the properties of two individuals (p_1 and p_2) to obtain two new individuals (c_1 and c_2). The crossover can be performed as indicated by Deb and Goel (2001):

1. Choose a random number u uniformly distributed between 0 and 1;
2. Calculate β_q as

$$\beta_q = \begin{cases} (2u)^{1/(\eta_c+1)} & \text{if } u \leq 0.5 \\ \left[\frac{1}{2(1-u)} \right]^{1/(\eta_c+1)} & \text{otherwise} \end{cases} \quad (7)$$

where η_c =distribution index controlling the spread of offspring solutions usually in the range $[1, 10]$; and

3. Compute two offsprings as

$$c_1 = 0.5[(1 + \beta_q)p_1 + (1 - \beta_q)p_2] \quad (8)$$

$$c_2 = 0.5[(1 - \beta_q)p_1 + (1 + \beta_q)p_2] \quad (9)$$

The mutation mechanism allows the creation of new individuals with new sets of properties, allowing the inclusion of characteristics that were not included in the initial population. Mutations can be created as a perturbation of the solution c as

$$c' = c + (x^{(u)} - x^{(l)})\bar{\delta} \quad (10)$$

where c' =mutated value; $x^{(u)}$ and $x^{(l)}$ =upper and lower bounds of the design variable x , respectively, and

$$\bar{\delta} = \begin{cases} (2r)^{1/(\eta_m+1)} - 1 & \text{if } r < 0.5 \\ 1 - [2(1-r)]^{1/(\eta_m+1)} & \text{if } r \geq 0.5 \end{cases} \quad (11)$$

where r is uniformly distributed between 0 and 1; and η_m =distribution index, which controls the spread of the mutated solution c' from c . Usually this parameter is kept in the range $[10, 100]$ (Deb and Goel 2001).

Now the best N individuals must be selected to form a new generation. In single-objective optimization this can be achieved by simple comparison of the objective and constraints associated with each solution. However, in multiobjective optimization

Fig. 1. Dominated and nondominated solutions

more than one objective exists and direct comparison cannot be performed. Instead, the concept of dominance is used. Assuming that all objective functions are to be minimized, an individual A is considered to be dominated by an individual B if (Deb and Goel 2001):

1. Individual B has at least one objective value lower than the corresponding one for individual A ; and
2. All other values of the objectives associated with individual B are lower than or equal to the corresponding ones associated with individual A .

This concept is illustrated in Fig. 1. In this figure, the objectives associated with both nondominated and dominated solutions are represented.

In most cases, there are not exactly N nondominated solutions. If more than N nondominated solutions exist, those leading to a better distribution of results are selected (Deb and Goel 2001). If less than N dominated solutions exist, solutions are selected by fronts. In the second front, nondominated solutions, disregarding the solutions in the first front, are selected. This process is illustrated in Figs. 2 and 3.

The process described above is relatively simple to implement, requires no use of gradients of objective functions or constraints, but in general is slow (i.e., computationally expensive).

Probabilistic Multiobjective Optimization of RC Bridge Crossheads

Definition of Multiobjective Optimization Problem

The multiobjective algorithm presented was used, along with the Latin hypercube sampling, to optimize the times of application of maintenance actions on existing reinforced concrete (RC) bridge crossheads considering single maintenance types. The data used was compiled in the United Kingdom by Denton (personal communication 2002). This data was discussed in depth in Neves and Frangopol (2005) where all parameters describing the condition index and safety index under no maintenance, as well as the effects of five different maintenance types (silane, replacement of expansion joints, cathodic protection, rebuild, and minor concrete repair) were presented. Of these, only silane and rebuild are considered in this paper. Silane is a time-based preventive maintenance action and rebuild is a safety-based essential maintenance action.

Fig. 2. Selection of N solutions from R_i to be used as new generation P_{i+1}

The design variables considered, using different multiobjective formulations, are as follows:

- Time of first application and time interval between subsequent applications of silane; and
- Safety index threshold at which rebuild is applied.

The objective vector include three performance indicators as follows:

- Condition index;
- Safety index; and
- Present value of cumulative maintenance cost.

Constraints can also be considered for two or all of the above performance indicators. These indicators (i.e., condition index, safety index, and present value of cumulative maintenance cost) can be defined by the corresponding mean values, percentiles, and/or minimum or maximum values. In this paper, mean values are used for defining performance indicators.

Probabilistic Multiobjective Optimization of Mean Performance under Rebuild

If rebuild is considered as the only maintenance type, there is a single design variable (i.e., safety index threshold at which rebuild is applied). As a result, all feasible solutions are nondominated and the only role played by GA is finding a set of solutions as well distributed (i.e., a set of evenly distributed solutions in the objective space) as possible. For this purpose, a population size of 100 with five generations was used. The probabilities of crossover and mutation considered are 0.6 and 0.1, respectively.

The optimization problem is defined as:

Goal

Find the (optimal) safety index threshold S_T that:

- Minimizes present value of mean cumulative maintenance cost at time horizon;
- Minimizes maximum (worst) mean condition index during time horizon; and

Fig. 3. Selection of nondominated solutions

- Maximizes minimum (worst) mean safety index during time horizon.

Subject to

- Maximum mean condition index ≤ 3.0 ; and
- Minimum mean safety index ≥ 0.91 .

Given

- The condition and safety indices under no maintenance shown in Fig. 4;
- The effects of rebuild on the condition index and safety index, as well as the associated cost, shown in Fig. 4.

It is noted that parameters defining the condition index, safety index, and cost are defined by triangular distributed random variables with parameters (i.e., minimum, mode, maximum) indicated in Denton (personal communication 2002). The improvement in the safety index due to rebuild was modified from that described by Denton (personal communication 2002), since the safety index threshold is now the variable of interest. In fact, the value of the safety index of a RC crosshead before rebuild does not influence its safety index after this maintenance action. This was incorporated into the problem by defining the safety index after application as independent of its value before maintenance is applied. The value to which the safety index is improved when rebuild is applied was chosen so that the same improvement as that initially proposed in Denton (personal communication 2002) occurs.

The mean condition index profiles, mean safety index profiles, and present value of mean cumulative cost profiles, considering a discount rate of money of $v=6\%$, are computed using Latin hypercube sampling with 1,000 samples.

Rebuild is, of all maintenance actions considered, the maintenance leading to the most significant improvements in both condition and safety indices, and also the one associated with higher cost. After application of rebuild, the condition and safety indices are improved to the values associated with the initial (i.e., new) state, independently of the performance before application of this maintenance type.

In Fig. 5 the worst (i.e., maximum) mean condition index, the worst (i.e., minimum) mean safety index, and the present value of mean cumulative cost at time horizon, are compared for all solutions. Fig. 5(b) shows that the relation between lowest mean safety index and present value of cumulative cost at time horizon (i.e., 50 years) is almost linear. In terms of the relationships between the maximum condition and the other two indicators (i.e., minimum mean safety index and present value of

Fig. 4. Effects of rebuild on mode values: (a) condition index; (b) safety index; and (c) cumulative cost

mean cumulative cost at 50 years) the feasible set can be divided in three regions: a central region (associated with values of the maximum mean condition between 1.8 and 2.1) where functional relationships are nonlinear and steeper, and the other two regions (i.e., associated with values of the maximum mean condition index below 1.8 or above 2.1) where the interrelationships are almost linear. Also, there is a significant change in the slope of the functions describing these relationships (much steeper in the central region).

In Fig. 6, it is shown that for safety index threshold S_T between 1.19 to 1.52 the worst mean condition index is almost constant. These values are associated with at most one application of rebuild for most RC crossheads. For higher thresholds, rebuild is applied earlier and to higher number of RC crossheads. If maintenance is applied earlier (i.e., to less deteriorated crossheads) its impact on each crosshead is smaller. However, the application to more crossheads generates a higher impact on the mean condition and safety indices. The two conflicting effects lead to an

almost constant worst mean condition index for all values of the safety index threshold S_T between 1.19 and 1.52. This is indicated in Fig. 7(a) where the mean condition index profiles of Solutions A, B, and C, defined in Figs. 5 and 6 are shown. This figure indicates that higher safety index thresholds (e.g., Points C_i in Fig. 6) are associated with earlier application of maintenance, higher probability of maintenance application, better performance, and higher cost. However, as the effect of this action ends, the deterioration rate approximates that observed under no maintenance. If a lower threshold is considered, the probability of application is smaller, and the cost is reduced. However, as rebuild is applied to more deteriorated RC crossheads, the improvement in condition is higher, resulting in similar condition at the end of the 50 year time horizon, for different safety index thresholds.

In Table I the design variables and objective functions associated with multiobjective Solutions A, B, and C in Fig. 7 are

Fig. 5. Rebuild; Relationships among performance indicators: (a) maximum (i.e., worst) mean condition index; (b) minimum (i.e., worst) mean safety index; and (c) present value of mean cumulative cost at time horizon. Points A_i , B_i , and C_i , $i=1,2,3$, are also defined in Fig. 6.

compared. These results show the influence of the safety index threshold S_T on the present value of mean cumulative cost, mean condition index, and mean safety index. As the safety threshold increases from Cases A to C, there is also an improvement in mean safety, a small decrease in mean condition, and a significant increase in mean cumulative cost.

Probabilistic Multiobjective Optimization of Mean Performance under Silane

The times of application of silane are defined by two random variables: time of first application, T_{pi} , and time interval between subsequent applications, T_p . Unlike the previous example,

Fig. 6. Rebuild; Effect of threshold safety index (i.e., design variable): (a) maximum (worst) mean condition index during time horizon; (b) minimum (worst) mean safety index during time horizon; and (c) present value of cumulative cost at time horizon (at 50 years)

Table 1. Rebuild: Comparison of Safety Index Threshold and Performance Indicators for Three Different Pareto Solutions

Solution	Safety index threshold (S_T)	P.V. of mean cumulative cost at time horizon [$(v=6\%) \text{ Cost}_{50} \text{ (k£)}$]	Maximum mean condition index ($\max C$)	Minimum mean safety index ($\min S$)
A	1.30	5150.8	2.01	1.79
B	1.42	6639.0	2.01	1.89
C	1.52	7738.6	1.99	1.97

REBUILD

Fig. 7. Rebuild: (a) mean condition index; (b) mean safety index; and (c) present value of mean cumulative cost profiles. Points A_i , B_i , and C_i , $i=1,2,3$, are also defined in Figs. 5 and 6.

different combinations of the two design variables might lead to similar maximum (worst) mean condition index and minimum (worst) mean safety index, but very different present values of mean cumulative costs. GA was used considering a population size of 100 with 20 generations. The probabilities of crossover and mutation considered were 0.6 and 0.1, respectively. The probabilistic indicators of the performance profiles were computed using Latin hypercube sampling with 1,000 samples. The optimization of the time of application of silane in a multi-objective framework is defined as:

Goal

Find the optimal mean time of first application, $\mu_{T_{pr}}$, and mean time of subsequent applications, μ_{T_p} , that:

- Minimizes present value of mean cumulative maintenance cost at time horizon;
- Minimizes maximum (i.e., worst) mean condition index during time horizon; and
- Maximizes minimum (i.e., worst) mean safety index during time horizon.

Subject to

- Maximum mean condition index ≤ 3.0 , and
- Minimum mean safety index ≥ 0.91 .

Given

- The deterioration of condition and safety indices under no maintenance described in Fig. 8.

Fig. 8. Effects of silane on mode values of: (a) condition index; (b) safety index; and (c) cumulative cost

- The effects on condition and safety indices, cost of application, and times of application of silane given in Fig. 8.

Considering that the objective of the optimization process is to define the time of application of silane, the initial triangular distribution of the time of first application (in years) was modified from (0.0, 7.5, 12.5) (as defined in Denton, personal communication 2002) to (10.0, 12.5, 15.0) as defined in Fig. 8. This corresponds to a reduction in the dispersion of the time of first application of maintenance. All the random variables are characterized by triangular distributions with parameters (i.e., minimum, mode, maximum) defined in Fig. 8.

As already indicated: (1) the condition index profiles, safety index profiles, and present value of mean cumulative cost profiles are computed using Latin hypercube sampling with 1,000 samples; (2) the design variables considered are the mean time of first application and the time interval between subsequent

applications; and (3) the times of application are characterized by triangular probability density distributions (see Fig. 8). The probability density functions used for each solution are defined using the corresponding design variable as mean and keeping the distances (i.e., time intervals) between minimum and mean and between maximum and mean constant during the optimization process.

In general, if the same number of maintenance actions are applied during a specified time horizon, the strategies associated with latter applications will result in lower mean cumulative cost due to the discount rate of money. Considering this, the optimal strategy should be associated with a concentration of application of maintenance at the end of the time horizon. On the other hand, if two or more maintenance actions have effects that are active at the same instant, the deterioration rates of condition index and safety index are the smallest among those associated with

Fig. 9. Pareto and dominated solutions associated with application of silane; three-objective optimization including maximum (worst) mean condition index during time horizon, minimum (worst) mean safety index during time horizon, and present value of mean cumulative cost at time horizon

the effect of each maintenance action considered separately. Consequently, the application of one maintenance action before the end of the effect of the previous one results in a decrease of effectiveness. These two conflicting trends (i.e., increase in the number of applications of maintenance close to the end of the time horizon considered and decrease in the effectiveness of maintenance for two or more maintenance actions) result in optimal solutions associated with maintenance actions applied as late as possible, but avoiding the superposition of effects of maintenance actions.

The first set of results obtained using GA show that the genetic algorithm does not always converge to a distributed set of solutions. In fact, some regions of the design space are often not properly covered and more than one run must be completed to ensure that almost all optimal solutions are found. In the present example, an initial run was performed, with side constraints on the design variables as follows

$$2.5 \text{ years} \leq \mu_{T_{pi}} \leq 60.0 \text{ years} \quad (12)$$

$$3.51 \text{ years} \leq \mu_{T_p} \leq 60.0 \text{ years} \quad (13)$$

where μ_{T_p} and $\mu_{T_{pi}}$ = mean values of T_p and T_{pi} , respectively.

The lower bound of the mean time of first application is defined so that, considering the triangular probabilistic distribution associated (10, 12.5, 15 years), the probability of nonnegative values of the time of first application is one (as it must be), for all admissible values of the mean. The lower bound of the interval between subsequent applications is defined so that no more than one maintenance action can occur in each 1 year interval. The upper values are selected so that solutions without maintenance or with only one maintenance action can be included.

The results obtained using these side constraints allow an overview of the optimization problem and give a good indication of the feasible set. However, most solutions obtained in the first run are in the infeasible space and, for this reason, are of no interest. Therefore, a more restricted optimization problem was performed using the following side constraints

$$2.5 \text{ years} \leq \mu_{T_{pi}} \leq 15.0 \text{ years} \quad (14)$$

$$3.51 \text{ years} \leq \mu_{T_p} \leq 15.0 \text{ years} \quad (15)$$

With these new constraints, a significant part of the solutions are in the feasible space, and more accurate optimal solutions can be found. The analysis of the results obtained with this second analysis showed that the feasible space was correctly considered.

The Pareto optimal (nondominated) and dominated solutions obtained using GA are shown in Fig. 9. From these results it is clear that the GA converges very fast to a set of solutions close to optimal. Projections of the results presented in Fig. 9 in a bidimensional space are presented in Fig. 10. Figs. 10(a and b) show that optimal solutions can be divided into two regions. The first region is composed of optimal solutions associated with a present value of mean cumulative cost at time horizon lower than about 75 k£ (equivalent to about 107,000 US \$ in 2002). These solutions are characterized by a small probability of superposition of effect of maintenance actions. In fact, as indicated in Figs. 11 and 12, the mean time interval between applications is close to 10 years and the time of first application changes between 2.5 and 8.0 years. For the solutions in this group, maximum (worst) mean condition index, minimum (worst) mean safety index, and present value of cumulative cost are almost perfectly correlated.

However, in order to produce solutions associated with better mean condition index and higher mean safety index, it is necessary to reduce the mean time interval between subsequent applications. This reduction of mean time interval between subsequent applications leads to an increase in the probability of superposition of the effect of several maintenance actions and in the duration of this superposition. Consequently, as shown in Fig. 10, small improvements in mean condition and safety indices are associated, for these solutions, with large increases in mean cost.

For all solutions in Fig. 10 the optimal maximum mean condition and the optimal minimum mean safety indices are almost perfectly correlated. This is mostly a consequence of the similarity between the effect of silane on the condition and safety indices.

In Fig. 10 several optimal Solutions (A, B, C, and D) and dominated Solutions (A' and B') are highlighted. The design variables and objective functions associated with Solutions A, A', B, B', C, and D are shown in Table 2.

In Fig. 11, the optimal values of the mean time of first application and mean time interval between subsequent applications are plotted for each value of the objective functions. Each set of three values of the objective functions is associated with a solution, described by two values of the design variables. Projections of these results in a bidimensional space are shown in Fig. 12.

The mean and standard deviation of the performance indicators associated with Solutions A and A' in Fig. 10 are compared in Fig. 13. These results show that, although resulting in similar mean condition and safety indices, Solution A', associated with an earlier application of maintenance (as can be seen by inspecting the mean cost profiles) and larger interval between subsequent applications [resulting in a steeper reduction of condition index as shown in Fig. 13(a)], results in a higher cost.

In Fig. 14 the mean and standard deviation of the condition index, safety index, and present value of cumulative cost associated with Solutions B and B' are compared. As in the previous example, the two solutions have similar worst mean condition index and lowest mean safety index, but significantly different costs. In this case, the main cause of increase in present value of cumulative cost is the smaller interval between subsequent applications of silane in Case B'. The small time interval used (i.e., 4.94 years in Table 2) is associated with a large number of appli-

SILANE

Fig. 10. Silane: Pareto and dominated solutions for three-objective optimization including worst mean condition index during time horizon; worst mean safety index during time horizon; and present value of mean cumulative cost at time horizon

cations of maintenance each with smaller impact on condition and safety indices, due to superposition of the effect of more than one application of silane.

In the last two rows of Table 2 the mean values of the condition index, safety index, and cumulative cost profiles for Solutions C and D are compared. These two solutions are associated with the best and worst possible mean condition and safety indices, respectively. These results show that although the mean cost changes significantly between these two extreme solutions, there is a relatively small variation in the mean condition indices and even smaller in mean safety indices. This is a direct consequence of the fact that all feasible solutions are associated with design variables very close to their lower bound (2.5 and 3.5 years for the mean time of first application and time interval between subsequent applications, respectively).

Solution C is associated with a very early first application of maintenance and an interval between applications that results

Fig. 11. Silane: comparison of design variables and objective functions associated with optimal solutions

SILANE

Fig. 12. Silane: comparison of design variables associated with three-objective optimization (mean time of first application and mean time interval between subsequent applications) with objective functions. (a) worst mean condition index during time horizon; (b) worst mean safety index during time horizon; and (c) present value of mean cumulative cost at time horizon.

Table 2. Silane; Comparison of Design Variables and Performance Indicators at Time Horizon for Six Different Solutions

Solution	Mean time of first application [t_{pi} (years)]	Mean time interval between subsequent applications [t_p (years)]	P.V. of mean cumulative cost at time horizon [$Cost_{50}$ (k£)]	Mean condition index at time horizon (C_{50})	Mean safety index at time horizon (S_{50})
A	6.36	9.07	60.2	2.90	1.20
A'	2.73	11.03	66.3	2.90	1.19
B	4.00	8.85	71.4	2.74	1.22
B'	4.96	4.94	109.2	2.74	1.22
C	2.50	5.37	117.9	2.58	1.24
D	6.96	9.88	54.3	3.00	1.18

SILANE

Fig. 13. Silane: comparison of (a) condition index; (b) safety index; and (c) present value of cumulative cost profiles associated with Pareto Solution A and dominated Solution A'. Points A_i and A'_i , $i=1,2,3$, are also defined in Fig. 10.

in the maintenance effect being almost always active. However, Solution D is associated, as all solutions with a lifetime cumulative cost lower than 75 k£, with mean time interval between applications close to 10 years, and a time of first application as late as possible, satisfying the condition index constraint.

Tables 3 and 4 show the present value of mean cumulative cost associated with optimal solutions if constraints more restrictive than those initially defined (worst mean condition index lower or equal to 3.0 and lower mean safety index higher than 0.91) are chosen by the decision maker. In Table 3 the present value of mean cumulative cost, mean time of first application, and mean time interval between subsequent applications are shown for five different acceptable maximum mean condition indices (including the initially defined value of 3.0), during the time horizon of 50 years. Although the changes in maximum mean condition are

small, there is a significant reduction in the present value of cumulative cost between solutions associated with mean condition indices of 2.6 and 3.0. Furthermore, there is a significant variation in the mean time of first application. A similar trend is present when different acceptable minimum safety indices are considered, as shown in Table 4.

If compared to the rebuild maintenance action, the application of silane is associated with much lower cost, but both the worst (maximum) mean condition index and worst (minimum) mean safety index are very close to the prescribed thresholds. These values can only be obtained by applying silane early and frequently during the lifetime of deteriorating RC crossheads. Although the mean condition and mean safety indices do not violate the prescribed thresholds during the entire time horizon, the probabilities of violating these thresholds are significant for all solutions obtained.

SILANE

Fig. 14. Silane: comparison of (a) condition index; (b) safety index; and (c) present value of cumulative cost profiles associated with Pareto Solution B and dominated Solution B'. Points B_i and B'_i, i=1,2,3, are also defined in Fig. 10.

Table 3. Optimal Solution for Different Maximum Acceptable Mean Condition Indices

Maximum acceptable mean condition index	Maximum mean condition index (max C)	P.V. of mean cumulative cost [Cost ₅₀ (k£)]	Minimum mean safety index (min S)	Mean time of first application [T _{pi} (years)]	Interval between subsequent applications [T _p (years)]
2.60	2.60	86.88	1.24	2.56	7.71
2.70	2.70	74.06	1.22	3.11	9.09
2.80	2.80	66.62	1.21	4.60	9.23
2.90	2.90	59.65	1.20	6.19	9.31
3.00	3.00	54.34	1.18	6.96	9.88

Table 4. Optimal Solution for Different Minimum Acceptable Mean Safety Indices

Minimum acceptable mean safety index	Minimum mean safety index (min S)	P.V. of mean cumulative cost (Cost ₅₀)	Maximum mean condition index (max C)	Mean time of first application [T_{pi} (years)]	Mean time interval between subsequent applications [T_p (years)]
1.19	1.19	57.0	2.95	6.68	9.52
1.20	1.20	61.3	2.88	5.82	9.23
1.21	1.21	66.6	2.80	4.60	9.23
1.22	1.22	72.5	2.72	3.28	9.24
1.23	1.23	78.4	2.65	2.52	8.87

Conclusions

A general probabilistic lifetime-oriented approach to multiobjective optimization of bridge under single maintenance type was proposed in this paper. Based on the proposed approach and results presented the following conclusions are made:

1. The main objective of a bridge manager is to find the best maintenance plan for a group of bridges, or bridge components, over a prescribed time horizon. This objective can only be achieved by probabilistic multiobjective optimization.
2. A powerful approach for solving the probabilistic multiobjective maintenance optimization problem consists in using GAs along with sampling methods. The combination of these algorithms and methods produce Pareto maintenance solutions for deteriorating structures and structural components.
3. The proposed approach proved to be extremely efficient for solving a complex and discontinuous multiobjective lifetime-oriented optimization associated with deteriorating RC cross-heads under uncertainty. The combination of GA and Latin hypercube sampling converged fast to the optimal objectives in all examples presented.
4. The results showed that preventive maintenance actions alone, such as silane, can lead to feasible solutions even if constraints more restrictive than those initially defined on condition or safety thresholds are specified. However, these solutions are associated with a relatively small objective space. This is due to the fact that is impossible, with application of silane alone, to obtain mean condition and mean safety performance levels significantly higher than those associated with prescribed thresholds. In contrast, essential maintenance actions, such as rebuild, allow for a wide range of feasible solutions, varying from those very close to the constraints prescribed to those associated with much higher performance. However, these solutions are also much more expensive.
5. The main limitation of the proposed approach is the fact that a single maintenance type was considered during the entire lifetime of a deteriorating structure or group of structures or components under uncertainty. However, studies are underway to overcome this limitation by considering combinations of different types of maintenance actions (Neves et al. 2006).

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