# Prospective Primary School Teachers' Knowledge of the Ratio Concept

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**Abstract**
Prospective primary school teachers (PPST) learn about some mathematics concepts in several courses besides the mathematics ones. This happens with the ratio concept which is a cross subject and instrumental concept. Research has shown that this concept is quite hard to master even though it is often used in school as well as in everyday life. This research aims at investigating PPST's knowledge on the ratio concept, namely with regard to their ability to interpret and compare ratios in two different contexts. Data were collected from 81 PPST attending a Portuguese university by means of a questionnaire. Participants were asked to answer to two questions that involve the ratio concept: one of them deals with a pizza division and requires a comparison of homogeneous quantities; the other one deals with the speed concept and involves a comparison of heterogeneous quantities. Both questions require information from a graph to be picked up. Data analysis showed that, in the pizza question, participants in the study tend to use numerical representations under the format of a fraction, which led them to do correct comparison between two ratios. In the case of the speed question, PPST showed more difficulties which seem to have been caused by the physical meaning of speed. Thus, the results suggest that most of these PPST hold a limited and rigid knowledge of the ratio concept that may be due to learning process based on numerical representations and carried out within mathematics courses. An implication of this is that teacher educators need to find ways of developing PPST's cross subject knowledge of the ratio concept so that they can be better prepared to teach this concept to young children embedded into cross disciplinary everyday life contexts.

**Keywords:** Prospective primary school teachers, ratio comparisons, ratio representations

## Introduction

After the implementation of the Bologna process, primary school teacher education in Portugal is a two-step process, including a 180 ECTS three yearlong undergraduate programme (*Licenciatura* in Basic Education) followed by 120 ECTS two yearlong master programmes. These master's curricula depend on whether prospective teacher qualify to teach up to the 4th grade or up to the 6th grade (with a specialization either on science and mathematics or on Portuguese language, history and geography). The undergraduate programme provides training on the diverse subjects that teacher candidates will teach in the future (Portuguese, mathematics, natural and social sciences, arts and physical education) as well as on education. The master programmes provide further training on education and on the school subjects that prospective teachers are preparing to teach, but they concentrate especially on subject specific methods courses and on teaching practice.

Thus, prospective primary school teachers (PPST) learn mathematics (at least 25 ECTS, as dictated by the Portuguese Law) in their undergraduate programme. This encompasses all the mathematics knowledge they would formally acquire within the scope of their training to teach the mathematics component to 1st to 4th graders. As matter of fact, only those PPST that choose a science and mathematics master's specialization (that would enable them to teach Science and Mathematics to grades 5 and 6) will learn some more mathematics (10 ECTS).

Taken together, the undergraduate programme and the master's programme cover all the knowledge components that Shulman (1986) has highlighted as being necessary for a teacher to teach effectively. These components are: content knowledge; pedagogical content knowledge; curricular knowledge and pedagogical knowledge.

Ball and others (2008) differentiate between two types of mathematics content knowledge: the common one and the specialized one. The former has to do with mathematics knowledge that anyone with a formal background in mathematics holds; the latter has to do with the understanding of the procedures and language relative to a given mathematics concept. This type of content knowledge distinguishes the mathematics teacher from another person with a mathematics background and it enables teachers to use and teach appropriate representations of mathematics concepts. This paper focuses on content knowledge, more specifically on mathematics knowledge.

**Objective of the Research**

The ratio concept is a multifaceted mathematics concept that relates to other mathematics concepts and that is used in other subjects, namely in science. The ratio concept dependency on other concepts and the variety of conceptions and representations that may be associated with it may require teachers to hold a good level of specialized content knowledge (Ball, Thames, & Phelps, 2008) if they are expected to appropriately teach this concept.

Thus, the aim of this piece of research was find out how PPST deal with problem situations involving ratio representations and comparisons. It draws on and adds to previous research dealing with PPST's understanding of ratio as it compares how PPST perform in different problem situations which is an issue that seems to have not yet been tackled.

**Theoretical Background**

***Representations of Mathematics Concepts***

The representations of mathematical concepts have concentrated researchers' attention for a long time. Lesh, Post and Behr (1987) state that representations are related to the internal assimilation of mathematics ideas, and therefore they have to do with the mental reproduction of a concept, and to the representation of images, symbols and signals associated with it. Thus, representations may be pictures and diagrams, spoken and written language, manipulative models, and real world situations (Lesh, Post, & Behr 1987). Thus, a representation "is a configuration that can *represent* something else in some manner" (Goldin 2002).

Several authors (e.g., Dreyfus 2002; Goldin, & Kaput 1996) differentiate internal from external representations. The external representations are ways of personalization of ideas and concepts that use written or spoken language and that aim at making the communication about the concept easier (Dreyfus 2002). Maps, tables, graphs, diagrams, models, and formal symbol systems are examples of external representations. Friedlander and Tabach (2001) distinguish four ways of doing external representations that they believe are at the core of mathematics: (i) verbal; (ii) numeric; (iii) graphic; (iv) algebraic. The internal representations are the cognitive constructions that are formed in an individual's mind (Goldin, & Kaput 1996). They are often named as mental images and have to do with internal schemes or frameworks through which a person interacts with the external world.

The use of several representations of a given concept, object or situation facilitates the transition from a concrete and limited understanding to a more flexible and abstract one (Dreyfus 2002). Lesh, Post and Behr (1987) stated that the understanding of a mathematics idea depends partly on: "(1) the capacity to recognize it when it is absorbed in a variety of different representational systems; (2) the capacity to manipulate the idea in a flexible way through representational systems; and (3) the translation of that idea from one system to the other" (p. 36).

In order to take most profit from the different representations, Dreyfus (2002) argued for the complementarity of the processes of abstraction and representation and added that they should develop in four stages as follows: (i) use of a single representation; (ii) use of more than one representation; (iii) establishment of connexions between representations; and (iv) integration of representations and establishment of flexible relationships among them. This way of conceiving the formation of mathematics concepts from multiple representations aims at facilitating the attainment of an ever increasing abstract conception of a mathematics concepts. The process that underpins the links among representations is translation which indicates that a change from a formulation of a mathematics relationship to another one is taking place.

***The Ratio Concept***

The ratio concept, taken as a comparison between quantities, is a multifaceted concept that is related with several other mathematics concepts, including rational number, proportionality and similarity. There are several conceptions of ratio and consequently there is no consensual way of defining it.

The ratio concept has connections with the rational number concept. This is why Lamon (2007) includes ratio in one of his constructs related to rational numbers. On one hand, the part-whole interpretation of rational numbers has to do with measure, and operator. Going through this type of interpretation processes facilitates the development of an understanding of measurement units and equivalent fractions. On the other hand, the quotient interpretation of rational numbers has to do with ratio and rates which are necessary to compare and to sum and subtract fractions. Besides, the operator interpretation has to do with multiplication and division of fractions which is an appropriate context for introducing these operations.

Lamon (2007) assumes that a ratio is a comparison between two quantities and he distinguishes between internal or homogeneous ratios and external or heterogeneous ratios. The former involve comparison between quantities of the same magnitude (within the variable); the latter involve comparisons between quantities of different magnitudes (between variables). Besides, according to Suggate, Davis and Goulding (2006), there are three types of ratio comparisons: part-part (e.g., Joseph eats two parts of the cake and Maria eats three parts of it); part-whole (e.g., Joseph has eaten two of the three parts of a cake); and whole-whole (e.g.; 1m in the map corresponds to 1 000 000m in reality). However, Viana and Miranda (2016) found out that when students are asked to compare ratios, they only use the equivalence of fractions (by reducing to the same denominator) or the transformation of a ratio into a decimal number or a percentage.

Besides, there is some empirical evidence that textbooks are sparse to support teaching of ratio in the primary school (Stafford, Oldham, & O'Dowd 2015). These results may at least in part be responsible for the fact that primary school teachers show a limited domain of mathematics content knowledge and do not perceive the difference between fraction and ratio and proportional reasoning (Livy, & Vale 2011). However, it is worth noting that this lack of ratio content knowledge may also be due to the fact that initial teacher education does not include the formal study of the ratio concept (Stafford, Oldham, & O'Dowd 2015) in some countries, including Portugal.

Berenson and others (2013) found out that prospective American, Irish and Portuguese mathematics and science teachers defined ratio as a comparison/relationship or as a fraction/percentage/proportion/division. As far as the representations of ratio through mathematical symbols are concerned, they used column (:), either isolated or integrated in expressions like: X:Y or 3:2, and fractions. In what concerns representations about how fractions are used, they made drawings, diagrams or other pictorial representations, showing comparisons or numerical representations (usually in everyday settings) or geometric properties (e.g., similarity) or statistical representations (e.g., bar graphs); very few participants in the study showed drawings (e.g., map scales in architecture or design settings) to illustrate applications of ratio.

Price (2014) also collected data with American elementary and secondary school teachers. She concluded that even though the results were similar to those reported by Berenson and others (2013), the elementary school teachers showed more diversity in their answers that the secondary school teachers did probably because the latter's training focused on a narrower content knowledge area and therefore they had more limited choices for their answers.

Data collected from prospective kindergarten teachers and PPST teachers (Fernandes, & Leite 2015) through the same questionnaire used by Berenson and others (2013) showed that subjects conceptualize ratio as a comparison or a relationship between magnitudes or as a mathematical operation but they do not make explicit the type of comparison involved in a ratio. Besides, they hardly relate it to rational numbers. Also, even though they used mathematical symbols to represent ratios, the majority used operations with letters or with constants or even the operation signals only. However, when they were asked to describe how they would explain ratios, they still used mathematical symbols but diagrams and graphs were the most used types of representations.

A follow up study, with an improved version of the questionnaire, mainly for language issues, was carried out with Irish prospective mathematics teachers (Oldham, & Shuilleabhain 2014). Even though the authors got more extensive responses, the results were also similar to the ones obtained by Berenson and others (2013). Afterwards, another research study (Oldham, Stafford, & O'Dowd 2015) focusing on primary school Irish prospective teachers highlighted their lack of readily available mathematics content knowledge to answer to the ratio questions, probably because ratio has not given enough attention in the teacher education curricula.

Ilany, Keret and Bem-Chaim (2004) concluded that the use of investigative activities that include tasks focused on settings that are familiar to prospective mathematics teachers and that require the use and the relationship of concepts that are relevant for a good understanding of the ratio concept may promote prospective teachers content knowledge as well as pedagogic content knowledge.

Oldham and Shuilleabhain (2014) have pointed out that ratio has been largely accepted as intuitively understood by students and teachers alike. However, the results of the research reviewed show that this is not the case. Rather, research involving PPST reveals conceptual problems that emerge independently of curricular and pedagogical traditions, while also pointing to approaches reflected in responses from one country that may be helpful to another (Oldham, Stafford, & O'Dowd 2015). Thus, further understanding of prospective teachers' conceptual difficulties with the ratio concept is necessary if action is to be taken by teacher educators to improve prospective teachers' content knowledge on this concept.

**Methodology**

To attain the objective of this study, 81 PPST attending the 6$^{th}$ (that is the last) semester of a three yearlong undergraduate studies programme (*Licenciatura* in Basic education) in a University of the north of Portugal answered to a questionnaire on ratio. The subjects' age range was 20 to 34 years old with an average of 22,0 years old. All of them had studied a few mathematics courses at university. However, some of them had taken mathematics in secondary school while other had not. It is worth mentioning that they had not studied the ratio concept at university and had no experience of explaining the ratio concept to someone else.

Questionnaires on ratio (e.g., Berenson et al 2013; Oldham, & Shuilleabhain 2014) were not appropriate for the purpose of this study as they were not focused on subjects' performance in different ratio situations. Thus, two problem situations dealing with representations and comparisons of ratio were designed: the pizza question, involving part-whole comparisons and homogeneous magnitudes; the speed question, dealing with part-part comparisons and heterogeneous magnitudes. The problem situations were reviewed by science and mathematics education specialists. The versions of the problems that were used in this study are given in figures 1 and 2, after translation from Portuguese to English.

PPST were invited by one of the authors, in a face to face basis, to participate in the study during a class time. They were informed about the objective of the study and the anonymous character of the questionnaire and that they would be allowed to withdraw at any time even after initiating the process of answering to the questionnaire. All of them volunteered to participate. Data were collected under exam conditions by one of the authors who was teaching them the final mathematics course of their undergraduate studies. This fact may have led PPST to engage deeply into the questionnaire problem-solving process.

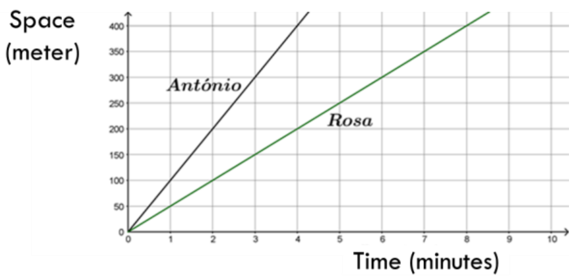
João and Maria bought a certain amount of pizza each one, as shown in the picture. The two pizzas are equal and so are the parts in which each pizza is divided.

Give other representations (as much as you can) of the amounts of pizza that João and Maria have bought. Explain those representations.

Who bought the largest amount of pizza, João or Maria? Explain your answer.

*Figure 1: The pizza problem*

António and Rosa went out for a walk. Their movement is shown in the following graph:




Who moved faster, António or Rosa? Why?

*Figure 2: The speed problem*

Data were content analysed based on a set of categories emerging from answers obtained in each question. Afterwards, absolute frequencies and percentages were computed for each category. To promote data reliability, content analysis was done by two of the authors, separately and discrepant cases were discussed with a third author.

## Findings

### *Results Relative to the Pizza Problem*

PPST were asked to give other representations (different from the one given in figure 1) of the amounts of pizza that João and Maria have bought and to explain those representations. Table 1 shows that they gave three general types of representations (diagrammatic, numeric, and numeric line representations), with some specific subtypes of representations in the diagrammatic and the numeric types. The types of representations obtained include two of the four types of external representations that Friedlander and Tabach (2001) consider as essential in mathematics.

The numeric representation was the one given by larger number of PPST. Almost all students (94%) used the fraction representation subtype and some of them also used the decimal (30%) and the percentage (10%) ways of representing a ratio. Most of the students that used decimal and percentage subtypes of representation started by doing a fraction representation and afterwards they transformed the fraction into the other subtypes of representation, through operative processes.

*Table 1: Types of other representations given by the PPST (N=81)*

| **Types of representation of Ratio** | | **f** | **%** |
|---|---|---|---|
| Diagrammatic representation | Continuous model | 61 | 75 |
| | Discrete model | 6 | 7 |
| Numerical representation | Fraction | 76 | 94 |
| | Decimal | 24 | 30 |
| | Percentage | 8 | 10 |
| Numerical line | | 3 | 4 |
| Do not answer | | 1 | 1 |

According to Lesh, Post and Behr (1987), the ability to translate an idea from one system of representation to another provides evidence of understanding of the idea that is at stake. In fact, the fraction could be obtained directly from reading the representation provided in the problem but the others needed to be computed and this requires understanding. Figure 3 shows an example of an answer that uses these three subtypes of numeric representation and even adds a diagrammatic representation which is different from the one given in the pizza problem (see figure 1). However, attention was given to the number of parts only and not to the rectangles that represent the unit (which are different in the two cases).

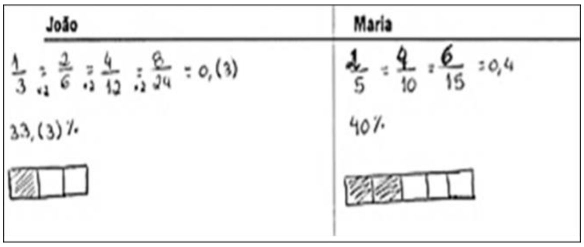


*Figure 3: representations of ratio (S14)*

As shown in table 1, large percentages of students gave diagrammatic representations, being the continuous (75%) subtype of representation much more frequent than the discrete one (7%). The larger use of the continuous representations may be due to the fact that it is usually used in classes to work with fractions and therefore it was probably very familiar to PPST. Figure 4 shows an example of a numerical (fraction) and a discrete representation.

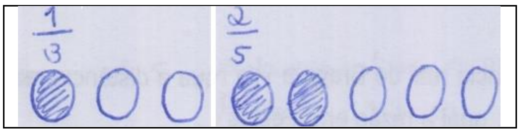


*Figure 4: representations of ratio (S11)*

The numeric line representations were the least frequent (4%) and they divided the unit in three and five parts respectively. Figure 5 shows an example of this type of representation.

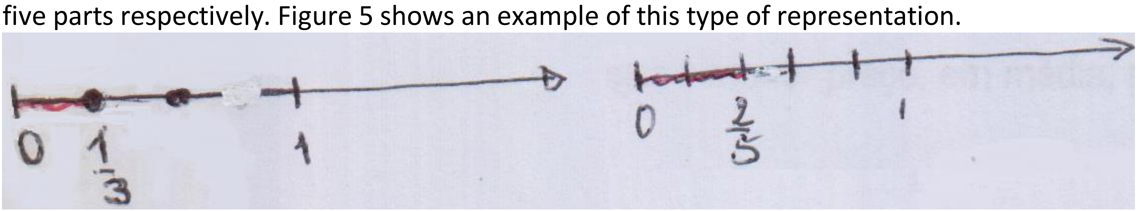


*Figure 5: Numeric line representation of ratio (S33)*

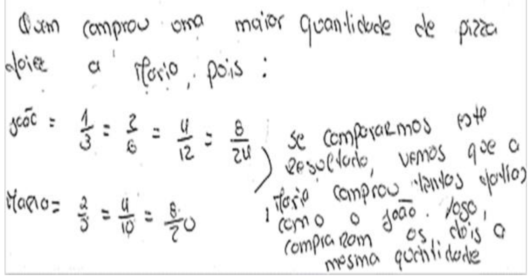
Thus, PPST were able to do part-whole representations of ratio that differ from the one given in the problem. Besides, as the values of the percentages obtained for the most frequent subtypes of representations indicate, several PPST gave more than one type of representation. In fact, they have combined numeric representations (mainly of the fraction and/or decimal subtypes) with diagrammatic representations (mainly of the continuous subtype). These types of representations had been found in previous studies focusing on PPST (e.g., Berenson et al 2013; Stafford, Oldham, & O'Dowd 2015) even though Fernandes and Leite (2015) found that the diagrammatic representations were more popular to explain ratio to someone else than to just represent ratios.

Afterwards, PPST were asked to compare the two ratios and to identify the biggest one. To do so, they would need to select and use a type of representation. Table 2 shows that about one third (31%) of them did not explain how they reached that answer or why it is the correct answer. The remaining 68% used numerical representations to do the required ratio comparison.

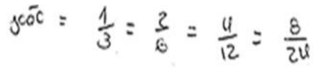
*Table 2: Types of comparisons of ratio done by the students (N=81)*

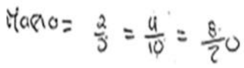
| **Types of comparisons** | | **f** | **%** |
|---|---|---|---|
| Fractions comparison | Reducing to same numerator/denominator | 21 | 26 |
| | Intuition | 12 | 15 |
| Percentage comparison | | 4 | 5 |
| Decimal comparison | | 18 | 22 |
| No justification | | 25 | 31 |
| Do not answer | | 1 | 1 |

Most of the PPST that used fractions computed equivalent fractions by reducing fractions to the same denominator or to de same numerator, that is they used a strategy previously described by Viana and Miranda (2016). However, some of the students that have opted for reducing fraction to the same numerator found some difficulties that are illustrated in figure 6.



Who bought the largest amount of pizza was Maria, because:



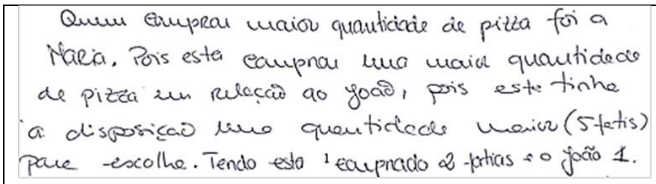


If we compare these results, we will note that Maria has bought as many slices as João. Then, they bought the same amount.

*Figure 6: Fractions comparison based on reduction to the same numerator (S77 original answer | translated answer)*

In fact, by focusing only on the numerator, they forgot about the denominators (which were different) and drew a wrong conclusion. Besides, in some cases, they gave answers containing internal contradictions (which the PPST seem to have not perceived). It is the case of the answer given in figure 6 that started by stating that Maria had bought the largest amount of pizza and that ended by concluding that both João and Maria bought the same amount of pizza.

PPST that made intuitive comparisons did an intuitive comparison of the areas of pizza bought by João and Maria based on the number of parts of pizza bought by each of them. They compared the fractions 1/3 and 2/5 and concluded that 2/5 would represent the largest amount as it corresponds to two parts while the 1/3 corresponds to only one part. They just ignored that the areas of each slice were different in the two cases and drew a conclusion without taking it into account. This reasoning is illustrated in figure 7. Even though 2/5 is larger than 1/3 and the result is correct, this type of reasoning (that ignores the denominators) does not give any systematic guarantee of reaching a correct answer through it.



Who bought the largest amount of pizza was Maria. Because she bought more pizza compared with João, as she had a larger amount of pizza (5 slices) to choose from. And she bought 2 slices and João bought only 1.

*Figure 7: Intuitive comparison based on the numerator only (S03 original answer | translated answer)*

Thus, a considerable amount of PPST used decimals (22%) but only a few used percentages (5%) to compare the parts of pizza that João and Maria have bought. Most of them did correct answers showing ability to do what Lamon (2007) and Suggate, Davis and Goulding (2006) call part-whole comparisons.

***Results Relative to the Speed Problem***

Table 3 shows the types of strategies that PPST used to find out who moved faster (António or Rosa). It should be emphasised that to succeed in doing so they were required to calculate ratios (speed values) and to do part-part comparisons.

*Table 3: Types of strategies used to obtain the highest speed (N=81)*

| Type of strategy | f | % |
|---|---|---|
| Compute speeds to choose the highest | 28 | 35 |
| Compare times needed to follow a certain path | 42 | 52 |
| Compare spaces followed in a given time interval | 7 | 9 |
| Do not answer | 4 | 5 |

PPST that computed the highest speed started by calculating António's and Rosa's speeds, afterwards they compared them and finally they identified the highest speed. This was the case shown in figure 8. It should be noted that as no information was provided on the units that should be used for speed, most PPST used m/min, as it was in the graph but a few used the international units' system speed unit (m/s), as S36 did. A few PPST reached a wrong result because they took wrong values from the graph or they did not pay attention to units that were used in the graph.

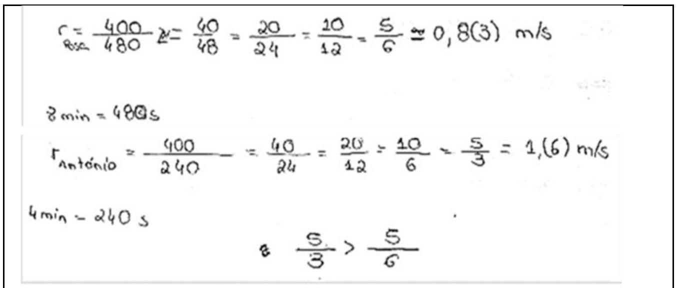


*Figure 8: Computation of the highest speed through speeds comparison (S36)*

As far as the comparison of times needed to cover a certain path is concerned, PPST compared the times used by António and Rosa to cover the path and concluded that the highest speed belongs to the person hat used less time to do it. Most of these students took as reference 400m that is the highest path length value given in the graph (figure 9). It would be interesting to understand the reasoning underpinning this choice. Half of the other PPST used other path values and the remaining just did a qualitative comparison without mentioning any specific path length value.

| | |
|---|---|
| O António, pois percorreu 400 m em 4 minutos e a Rosa percorreu 400 m em 8 minutos, ou seja, demorou o dobro do tempo a percorrer a mesma distância do António. | António because he walked 400 m in 4 minutes and Rosa walked 400 m in 8 minutes that is, she doubled the time used by António to walk the same distance. |

*Figure 9: Computation of the highest speed through times comparison (S13 original answer | translated answer)*

PPST that opted for comparing the path covered in a given time by António and Rosa, they took the values from the graph to conclude that the person that walked faster was the one that walked more meter. This time, they concentrated on 1 min or on 2 min that are the lower times explicitly shown in the graph. However, one did it for several time instants. These types of strategies should be further investigated as they may be due to speed alternative conceptions or to lack of graphicacy.

Hence, more than half of the PPST succeeded on solving the problem by comparing times or path lengths for the same space or the same time, instead of calculating and comparing heterogeneous ratios based on the speed mathematics formula and doing part-part comparisons.

## Conclusions and Implications for Teacher Education

Results indicate that most PPST that participated in the study were able to solve problems that require operating with ratio in two different contexts: a context with homogeneous magnitudes and part whole comparisons and a context with heterogeneous magnitudes and part-part comparisons. However, nearly two thirds of them were not able to give more than one or two types of representation, about one third was not able to explain ratio comparisons, and only about one third were able to compare path covered/time ratios.

Thus, the results of this study suggest that a considerable number of PPST that participated in the study reported in this paper may lack content knowledge on the ratio concept. They also suggest that initial primary school teacher education needs to pay more attention to the ratio concept, by formally integrating it in the teacher education curriculum and by approaching it explicitly. Besides, the complex nature of the concept (Lamon 2007; Ilany, Keret, & Bem-Chaim 2004) together with the differences between the results obtained in the two problem situations suggest that this concept should be approached in several contexts (and not only in the mathematics one) from a conceptual and a representational points of view. This means that ratio content knowledge and ratio pedagogical content knowledge should be approached in an integrated manner to trigger each other. Finally, this may require mathematics' teacher educators and mathematics educators to work together to foster the development of PPST content knowledge of ratio.

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## References

Ball, D., Thames, M., & Phelps, G. 2008. Content knowledge for teaching: what makes it special? Journal of Teacher Education 59, nº 5: 389-407.

Berenson, S. et al. 2013. Investigating representations of ratio among prospective mathematics and science teachers: an international study. In Proceedings of the 37th ATEE Conference, ed. E. Agaoglu. 78-92. Brussels: ATEE.

Dreyfus, T. 2002. Advanced mathematical thinking processes. In Advanced mathematical thinking. ed. D. Tall, 25-41. Dordrecht. Kluwer Academic Publisher.

Fernandes, J., & Leite, L. 2015. Compreensão do conceito de razão por futuros educadores e professores dos primeiros anos de escolaridade. Bolema (Rio-Claro, SP) 29, nº 52: 241-262.

Friedlander, A., & Tabach, M. 2001. Promoting multiple representations in algebra. In The roles of representation in school mathematics, ed. A. Cuoco, & F. Curcio, 173-185. Reston, VA: NCTM.

Goldin, G. 2002. Representation in mathematical learning and problem-solving. In Handbook of international research in mathematics education, ed. L. English, 197-278. Mahwah, NJ: LEA.

Goldin, G., & Kaput, J. 1996. A joint perspective on the idea of representation in learning and doing mathematics. In Theories of mathematical learning, ed. L. Steffe, et al., 397-429. Mahwah, NJ: LEA.

Ilany, B., Keret, Y., & Bem-Chaim, D. 2004. Implementation of a model using authentic investigative activities for teaching ratio & proportion in pre-service teacher education. In Proceedings of the 28th Conference of the International Group for the PME, ed. M. J. Hoines, & A. B. Fuglestad, 81-88. Bergen: PME.

Lamon, S. 2007. Rational numbers and proportional reasoning: toward a theoretical framework for research. In Second handbook of research on mathematics teaching and learning, ed. F. Lester, 629-667. Charlotte, NC: Information Age Publishing,

Lesh, R., Post, T., & Behr, M. 1987. Representations and translations among representations in mathematics learning and problem-solving. In Problems of representation in the teaching and learning of mathematics, ed. C. Janvier, 33-40. Hillsdale, NY: LEA.

Livy, S., & Vale, C. 2011. First year pre-service teachers' mathematical content knowledge: methods of solution for a ratio question. Mathematics Teacher Education and Development 13, nº 2: 22-43.

Oldham, E., & Shuilleabhain, A. 2014. Investigating representations of ratio among prospective mathematics teachers: a study of student-teachers and students use of mathematics in an Irish university. In Proceeding of the 38th ATEE Conference, ed. E. Arntzen, 298-321. Brussels: ATEE.

Oldham, E., Stafford, P., & O'Dowd, V. 2015. Investigating representations of ratio among prospective primary teachers in Ireland: implications for teacher education. In Proceedings of the 39th ATEE Conference, ed. L. Leite et al., 329-340. Brussels: ATEE/CIEd.

Price, E. (2014). Teachers' use of ratio. In Educating for the future: Proceeding of the ATEE 38th Annual Conference 2013, ed. E. Arntzen, 322-340. Brussels: ATEE.

Shulman, L. 1986. Those who understand: knowledge growth in teaching. Educational Researcher 15, nº 2: 4-14.

Stafford, P., Oldham, E. & O'Dowd, V. 2015. Ratio and proportional thinking: a study in the Irish context. In Proceedings of the British Society for Research into Learning Mathematics-35/1, ed. G. Adams, 78-83. Reading: BSRLM.

Suggate, J., Davis, A., & Goulding. M. 2006. Primary mathematical knowledge for primary teachers (3rd ed.). Londres: David Fulton Publishers.

Viana, O., & Miranda, J. 2016. O raciocínio proporcional e as estratégias de resolução de problemas de valor omisso e de comparação. REVEMAT 11, nº 1: 194-213.

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