Assessment of shear modulus by different seismic wave-based techniques

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Abstract. Using combined setup of bender elements and accelerometers, tests were conducted on Coimbra sand specimens in order to measure and interpret seismic wave velocities to assess initial shear modulus. For these tests both time and frequency domain analyses were performed. Resonant column tests were also performed on the same sand to validate the results obtained with the bender elements and accelerometers setup. As is well known, in the last decades the development of new laboratory techniques to assess soil stiffness through the use of seismic wave-based techniques, has received significant attention due to its simplicity and versatility of the equipment setup. One of these techniques is the bender elements test which have been one of the most widely used, although some limitations concerning its usage. In this context, the combined use of bender elements with other seismic wave-based testing techniques, such as accelerometers or the resonant column, is quite important to compare and validate the testing techniques. Given its miniature size, the installation of accelerometers on the side of the sample is considered feasible without significant disturbance on the other measuring techniques. The resonant column is a widely used and accurate testing technique due to its reliability and repeatability. Finally, the results of this combined tests allow a critical discussion on the advantages and limitations of the use of bender elements and accelerometers, in contrast with the resonant-column for the assessment of the shear modulus in sand.

Keywords. Accelerometers, Bender elements, Resonant-column, Shear modulus

1. Introduction

The importance of soil characterization on the very small to small strains domain (i.e. shear strain from 10^{-6} to 10^{-4}) for engineering design purposes is well established. At this deformation domain, the response of soil can be considered quasi-elastic, being the corresponding shear modulus designated by maximum or initial shear modulus, G_{max} or G_0). This shear modulus is not affected by the nature of the loadings (monotonic or cyclic) since there is no stiffness degradation in load-unload cycles [1]. Previous studies [2] and [3] using different testing techniques showed that the initial shear modulus is not affected by the type of test. The source of excitation frequency and the strain rate do not seem to affect the response of the soil, and the major constraint for

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determining the initial shear modulus is usually attributed to the precision of the measuring instruments.

Considering the relation $G_0=\rho.V_s^2$, where ρ is the soil mass density and V_s is the shear wave velocity, the initial shear modulus can be computed after the determination of the shear wave velocity. The effect of different factors on the shear wave velocity in sands was studied by [4] with resonant column tests and the results were computed by regression curves under the form $V_s=C(B-e)(\sigma_0')^{n/2}$. The research revealed two factors as the most important on the shear wave velocity: the mean effective stress and the void ratio [5]. Replacing the soil mass density given by $\rho=\left(\frac{\gamma_s}{g}\right).\frac{1}{1+e}$ the initial shear modulus can be expressed and simplified by:

$$G_0=A\frac{(B-e)^2}{1+e}(\sigma_0')^n=A.F(e).(\sigma_0')^n \tag{1}$$

where A, B and n are empirical constants experimentally determined, e the void ratio, σ_0' the mean effective stress and $F(e)=(B-e)^2/(1+e)$ is a function of the void ratio. Generally, G_0 increases with σ_0' and decreases with e, which means that it increases with the increase of the relative density of the material $\left[ID=\frac{e_{max}-e}{e_{max}-e_{min}}\right]$. According to data collected by [6], dense sand and gravelly deposits adjust well with Eq. (1) the exponent n varies between 0.38 and 0.85 mostly depending of the grain size distribution and contact conditions between particles and B is equal to 2.17 for the majority of soils. If one divides G_0 by $F(e)$ the ratio gives the normalized value of the G_0 allowing comparing test results of the same material but with different void ratio.

2. Shear modulus assessment

There are two main ways to determine G_0: one through the theory of elasticity, using stress-strain measurements under small cycles, and another through the theory of wave propagation, using the measurement of shear wave velocities. The standard test for G_0 assessment is the resonant column test (RC), which uses the shear wave propagation velocity theory. Bender elements test (BE) is one of the most spread techniques to also assess shear modulus due to its simplicity. Both techniques apply a shear strain level near of 10^{-6} to the material [1].

In a conventional RC test, a cylindrical specimen of soil is subjected to a steady-state harmonic excitation, and the response of the system in terms of vibration is measured. The frequency of the input signal is shifted until resonance is achieved. It is possible to compute the dynamic properties of the soil (stiffness and damping) as derived from the dynamic equilibrium of the specimen [7]. In other hand, in a BE test a voltage signal is applied to a piezoceramic element (transmitter) which transmits a small shearing movement over one end of the cylindrical soil specimen. This disturbance travels across the specimen length until the other end is reached, where a similar piezoceramic element (receiver) receives the mechanical perturbation and generates a voltage. The time interval between the emitted and received signals enables to compute the shear velocity. BE tests have however some limitations which influence its accuracy [7]. **Table 1** lists the main limitations identified and the possible alternatives to overcome these difficulties.

Table 1 - Main limitations of RC and BE tests and the possible alternatives (from [7])

Method	Limitations	Alternative considered
RC test	Only harmonic excitation Shear strain > 10^{-6} Determination of G and ξ only at resonant frequency Time consuming procedure Too many cycles (soil disturbance)	Random noise, ambient noise Random noise, combined methods Sweep sine Controlled source
BE test	Excess of human judgment Uncertainties about the actual behavior of BE transmitter Great amount of data to process and analyze	Frequency domain and statistical methods External controlled source Miniature accelerometers Automation

The use of more than one type of test, complementary between them, can improve the testing reliability. Thus the results of one type of test can be compared with another one and therefore the interpretation can be more consistent.

2.1. Interpretation of Bender Elements and Accelerometers results

Once the interpretation of BE results involves some uncertainty different approaches have been proposed to deal with the interpretation issues and they are usually based on the time or on the frequency domain analysis. Generally the frequency domain method produces an estimate of shear wave velocity, which is lower than that from traditional time domain readings [8].

Known as the most simple, common and usual procedure for interpreting BE measurements, the first direct arrival method consists on the identification of the first instant of arrival of the wave in the output signal, similarly to the techniques used in geophysical tests. The usage of this method is sometimes source of error once some factors can interfere with the wave arrival identification. These factors, reported in [9], can be overcome using some alternatives considered in **Table 1**. The use of piezoelectric accelerometers (AC) takes advantage of the accuracy on using calibrated equipment and the possibility of reading acceleration determining the arrival of the shear wave in a particular direction [10]. It has been demonstrated that accelerometers used in a coupled system with benders work as receivers in a more accurate way [11].

The use of continuous signals is also an alternative to be considered to reduce the error on BE tests. This technique requires the shear wave velocity to be decoded from measurements of relative phase of transmitted and received signals. These called frequency domain (FD) methods have a number of advantages over traditional time-based (TD) measurements, namely the possibility of creating an algorithm to determine travel time by establishing the gradient of a graph of phase difference against frequency [8]. Generally, a continuous harmonic sinusoid is used as input signal, though a generic input signal can also be used to evaluate the phase delay by decomposing the signal into its harmonics, using the Fourier transform. Also in this technique, accelerometers can be used to acquire the response of the system as a receiver.

3. Methods

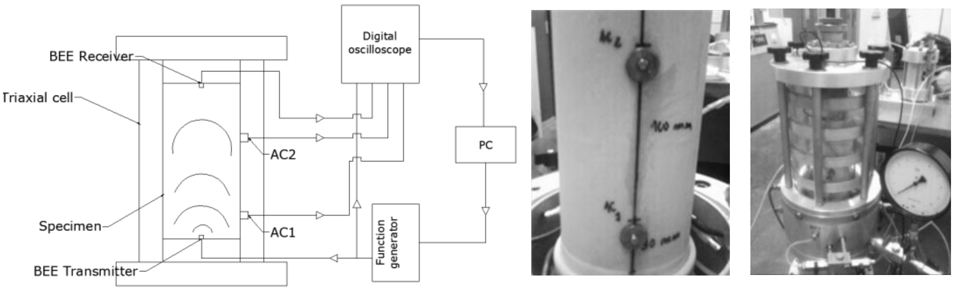
3.1. Equipment and tested material

The RC equipment from the University of Lisbon (**Figure 1**) is a Drnevich-type manufactured by Seiken Inc. in 1992. It consists of three subsystems: pneumatic, electro-mechanic and electronic. The pneumatic subsystem provides the conditions to the control of cell pressure, backpressure and axial force; the electro-mechanical subsystem allows the torsional vibration and the electronic subsystem provides the input signal and measures the response of the system.

a) b) c)

Figure 1 – RC details: a) Prepared sample; b) Pneumatic system control; c) Electronic system control

The system at University of Minho for BE and AC tests is a 100mm Bishop-Wesley stress-path chamber, adapted to accommodate BE. Two AC were applied directly on the side of the sample, in order to validate the BE signals and minimize the subjectivity in interpretation [10] and [11] (**Figure 2**a). The AC used are from Bruel & Kjær, these are piezoelectric sensors (type 4513-001, 100 mV/g sensitivity, ±50 g measuring range, 1 Hz to 10 kHz frequency range, 12.7 mm in diameter, 15.65 mm in height, 9.0 g in weight). In order to ensure adequate coupling and stability during testing, the AC were fixed to the sides of the specimens at specific points, by means of pins screwed to the back of the AC, which involved puncturing the membrane and carefully insulating the hole (**Figure 2**b).



a) b) c)

Figure 2 – a) Schematic view of the 100mm stress-path chamber system integrating the combined use of BE and AC; b) AC pins and its isolation; c) Setup overview.

The first AC (AC1) was placed at 30 mm from the base of the specimen and AC2 was placed 100 mm above the first. The AC axis were placed in the same plane direction of the movement of soil particles. TD and FD techniques were used in combination and a minimum of four input and output signals were recorded in order to eliminate problems such as random noise and to get a clear response signal. Regarding to the FD techniques, a sinusoidal signal of linear sweep of frequencies from 1 to 50 kHz for a total period of 20 ms and amplitude of 20 peak-to-peak voltage (V_{pp}) was used.

A portuguese river sand called Coimbra Sand was used for this study. The samples were prepared using the dry deposition technique, by means of deposing the sand with a funnel to achieve a certain relative density. The tests were performed in dry conditions. The sand used for the tests exhibit a D_{50} = 0.28 mm and a uniformity coefficient (C_u) around 1.22. The particle size distribution obtained for this material [12], is presented in **Figure 3**. The initial physical properties of the samples tested on the RC (RC1 and RC2) and BE/AC equipment (P1) are shown in **Table 2**.

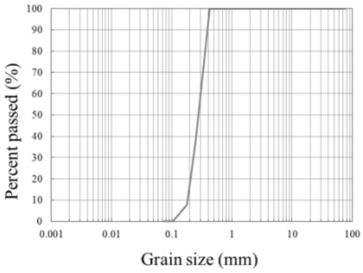


Figure 3 - Grain size distribution of Coimbra sand

Table 2 - Initial physical properties of Coimbra sand specimens

Test	ID (%)	Height (mm)	Diameter (mm)
RC1	83	101.0	69.8
RC2	65	101.0	70.4
P1	61	200,8	98,9

4. Results

Figure 4 shows the initial shear modulus results of the RC tests for the two samples and for different mean effective stresses. The regression equations obtained for each test are also shown in the same figure. **Figure 5** shows the normalized measured initial shear modulus in bi-logarithmic scale.

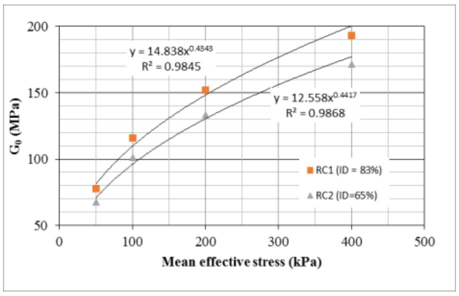


Figure 4 - Initial shear modulus. RC tests

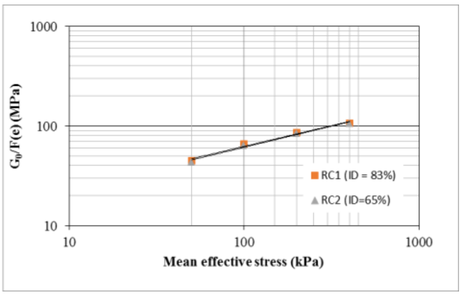


Figure 5- Initial shear modulus normalized by the void ratio function, RC tests

As expected, the initial shear modulus decreases with the decrease of the relative density and the values of G_0 are consistent with the type of tested material [13]. As can be seen in **Figure 5**, the normalized values of the initial shear modulus are very consistent, with A=8.95, B=2.17 and n=0.42 from the Eq. (1). The results are consistent with [6] and the previously referred values of n between 0.38 and 0.85.

Figure 6 shows the overlapping signals (transmitter and receiver) and the identification of the travel time for the P1 BE test, by the first direct arrival method. With this result and using the expression $G_0=\rho.V_s^2$ it was possible to calculate G_0 for the sample.

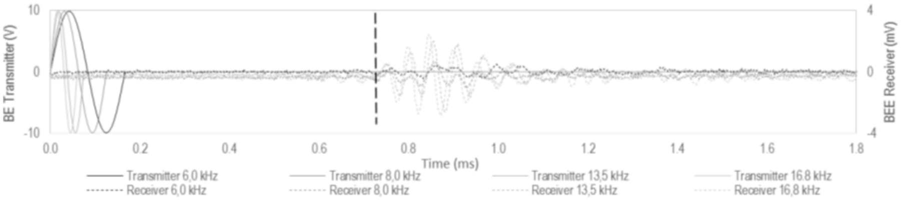
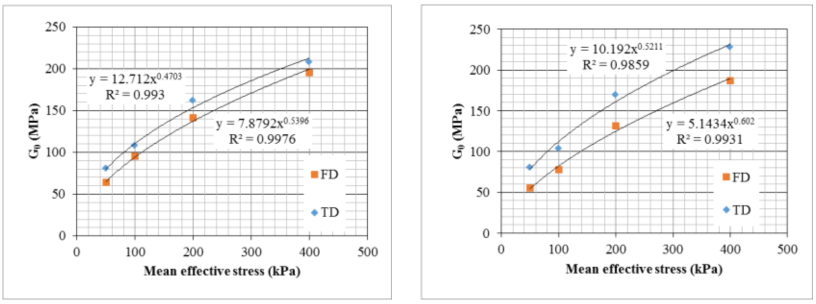


Figure 6 - Example of overlapping signals and identification the travel time for BE test

Figure 7 shows the initial shear modulus measured on the BE setup and on the AC setup, for TD and FD analysis, for the P1 test (ID=61%).



a)

b)

Figure 7 - G_0: a) measured on the BE setup, for TD and FD; b) measured on the AC setup, for TD and FD

As expected, the results obtained by the FD method gave an estimate of the initial shear modulus lower than time domain readings [8]. Comparing the BE results with those from AC, it can be said that the results are similar. However the FD analysis results show lower values on the AC compared to the BE and higher values on TD analysis on the AC for higher mean effective stresses. This differences can be explained by the fact of the AC allow to measure the shear wave velocity between two cross sections that are not the boundaries and are not influenced by the boundary effects and coupling. **Figure 8** compares the results obtained in BE and AC setups, for time and frequency domain, for the P1 test (ID=61%) (simultaneous measurements at the same stress conditions).

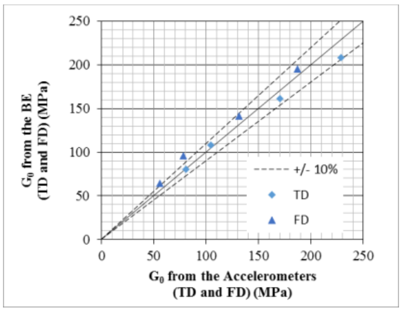
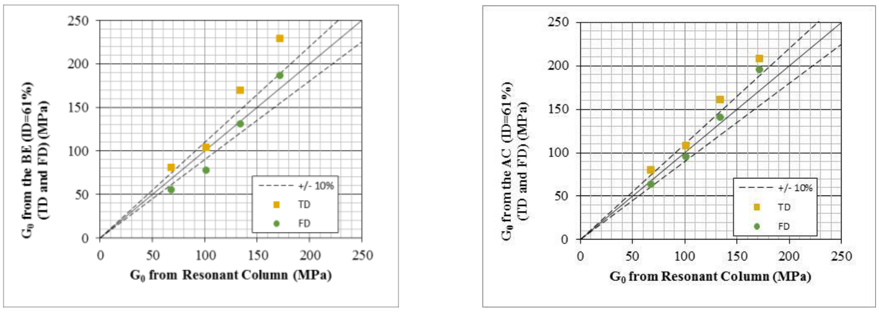


Figure 8 – Relation between initial shear modulus measured on the BE and AC setups, for time and frequency domain

The correlation between the results of the two types of tests indicates a good agreement between them (less than 10% difference) even showing higher differences for the FD analysis, as previously referred. **Figure 9**a shows the comparison between BE (TD and FD) and RC setups for the same stress conditions and relative density. **Figure 9**b compares the results between AC and RC setups, for time and frequency domain. In both figures the values plotted on the horizontal axis are the initial shear modulus G_0 for RC2 test (*ID*=65%) and on the vertical axis are plotted the G_0 results for P1 test (*ID*=60%, BE on **Figure 9**a and AC on **Figure 9**b both for time and frequency domain.



a) b)

Figure 9– Relation between initial shear modulus measured on the: a) BE (P1) and RC (RC2) setups, for time and frequency domain; b) AC (P1) and RC (RC2) setups, for time and frequency domain.

As can be seen the correlation between the results is good since the major part of the results fall inside the +/-10% difference range. Only for higher mean effective stresses a ratio higher than 1.1 has been observed. Besides the results from the AC are closer to the RC than the BE, one can conclude that the referred boundary conditions influence the results and the stress level increase that effect. As shown in **Figure 7**, the difference between frequency and time domain analyses is larger in the BE test than in the AC tests.

5. Conclusions

Using combined setup of BE and AC, tests were conducted on Coimbra sand in order to assess initial shear modulus. RC tests were also performed on the same sand to validate the results obtained with the BE and AC setup.

The results obtained by the frequency domain method gave an estimation of the initial shear modulus lower than time domain readings both in BE and AC tests. The correlation between the results of BE and AC tests indicates a good agreement between them (differences less than 10%) even showing higher differences for the frequency domain analysis.

The BE and AC test results also agrees well with RC. Only for higher mean effective stresses a ratio higher than 1.1 has been observed. The accelerometers reveal a better accuracy than the BE being less influenced by the boundary and stress conditions.

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