NEW SHEAR STRENGTHENING CONFIGURATIONS OF NSM CFRP LAMINATES FOR RC BEAMS

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ABSTRACT

The effectiveness of the near surface mounted (NSM) technique with carbon fiber reinforced polymer (CFRP) laminates for the shear strengthening of reinforced concrete (RC) beams has been demonstrated during the last decade. Analytical and numerical research indicate that this technique can be more effective if the CFRP laminates are installed deeper into the slits. However, the depth of the slits is conditioned by the thickness of concrete cover. Furthermore, relatively deep slits can only be executed if the height of the beam's cross section is relatively large for assuring an effective bond transfer length for the CFRP laminates. Therefore, an experimental program composed of a series of RC beams of relatively high T cross section web was carried out to assess the effect of installing CFRP laminates deeper into the slits on the shear strengthening effectiveness. The experimental program is described, and the main results are presented and discussed. An analytical model is proposed for determining the contribution of the NSM CFRP laminates on the shear strengthening of RC beams considering the effect of introducing laminates deeper into slits. Furthermore, a 3D multi-directional fixed smeared crack model, which incorporates a constitutive model for simulating the softening behavior of the opening and sliding concrete fracture components, is used to simulate the tested beams. By performing a parametric study with this model, the influence of the geometric and mechanical properties of FRP laminates on the shear strengthening effectiveness of RC beams is assessed.

Keywords: NSM CFRP laminates; Shear strengthening; RC beams; Experimental results, Analytical formulation; FEM analysis.

INTRODUCTION

Near surface mounted (NSM) technique by using carbon fiber reinforced polymer (CFRP) laminates introduced into thin slits open on the side surfaces of reinforced concrete (RC) beams has been proved to be very effective for the shear strengthening of this type of structural elements¹⁻⁶. The laminates are bonded to the surrounding concrete by using epoxy adhesive, and the depth of the slits is limited to the concrete cover of the beam's side faces, which, in general, does not exceed 30 mm (1.18 in). Dias and Barros⁷ concluded that NSM technique is more effective than Externally Bonded Reinforcement (EBR) technique, since NSM provided a larger increase of load carrying capacity after shear crack formation. Utilisation of NSM better mobilizes the tensile capacity of the CFRP reinforcement since higher tensile strains were measured in these composites when this technique was used. An analytical model has demonstrated that the deeper the laminate is installed into the core of the beam's web, the more effective is the NSM technique with CFRP laminates for the shear strengthening⁸. This model assumes that the possible failure modes governing the strengthening effectiveness of a NSM laminate are comprised from loss of bond (debonding), concrete semi-conical tensile fracture, mixed shallow-semi-cone-plus-debonding, and laminate tensile fracture⁹. The semi-conical geometry for the concrete tensile fracture failure mode is assumed to be formed when the axis of the laminate is positioned at the surface of the beam's side face. According to the theoretical fundamentals of this model, if the laminate is positioned deeper into the core of the beam's web, the concrete failure surface has a higher fracture area than the one corresponding to the semi-cone, resulting a higher resisting concrete fracture force and, consequently, higher shear strengthening effectiveness for the NSM technique. Pull-out tests with NSM systems have also demonstrated that the deeper the laminate is installed into the slit, the larger is the maximum pull-out force¹⁰.

Taking into account these experimental and analytical results on the benefits of installing CFRP laminates as deep as possible, an experimental program with T cross section RC beams was carried out, where the depth of the beam's web is high enough to allow the installation of CFRP laminates

1 into relatively deep slits executed above the longitudinal tensile bars, and with enough bond length
2 for the laminates to avoid premature debonding failure mode⁹. This paper describes the
3 experimental program and discusses the relevant results. The performance of the tested CFRP shear
4 strengthening configurations is compared to the one determined in several series of NSM shear
5 strengthened RC beams where the laminates were applied into slits positioned at the beam's web
6 surface. An analytical approach is proposed to estimate the contribution of the NSM CFRP
7 laminates on the shear strengthening of this type of RC beams. By simulating numerically the tested
8 beams, it is demonstrated that a 3D multi-directional fixed smeared crack model is capable of
9 capturing the relevant behavioral aspects registered in the experimental tests, as long as the
10 softening nature of the opening and sliding concrete fracture components are conveniently modeled.
11 Finally, a parametric study was executed to highlight the influence of geometric and mechanical
12 properties of FRP laminates on the shear strengthening effectiveness of RC beams.

13 RESEARCH SIGNIFICANCE

14 An experimental program with relatively high T cross section RC beams, which are quite current
15 in buildings and bridges, was executed to demonstrate the effectiveness of the NSM technique on
16 the shear strengthening of this type of elements increases when the CFRP laminates are deeper
17 installed into the slits. An analytical model was developed for estimating the shear strengthening
18 contribution of NSM-CFRP laminates by considering the influence of the CFRP position into the
19 cross section of beams. A multi-directional smeared crack model capable of simulating
20 appropriately the concrete fracture modes I and II was also used to illustrate that the increase in the
21 tensile strength of the FRP has a more favorable effect in terms of shear strengthening effectiveness
22 than the increase in its elasticity modulus.

23 EXPERIMENTAL PROGRAM

24 Beam prototypes

25 The experimental program is composed of a reference beam (**Fig. 1a**) and four NSM shear
26 strengthened beams (**Fig. 1b**). **Fig. 1a** shows the T cross section geometry and the steel

1 reinforcement details for the series of beams, as well as the loading configuration and support
 2 conditions. The adopted reinforcement was designed to assure shear failure mode on all the tested
 3 beams. To localize the shear failure in only one of the beam shear spans, a three point loading
 4 configuration, with a different length for the beam shear spans was selected, as shown in **Fig. 1a**.
 5 The monitored beam span (a) is 2.5 times the effective depth of the beam ($a/d=2.5$), since according
 6 to the available research¹¹, this is the minimum value that assures a negligible arch effect. The
 7 beams had a percentage of longitudinal tensile steel bars (ρ_{sl}) of 2%. To avoid shear failure in the b
 8 beam span, steel stirrups of 6 mm (0.24 in) diameter at a spacing of 112 mm (4.41 in) ($\phi 6@112$ mm
 9 [$\phi 6@4.41$ in]) were applied in this span. To prevent brittle spalling of the concrete cover at the
 10 supports, the beam ends were strengthened by confining the concrete with a two-directional cage of
 11 $\phi 6@65$ mm ($\phi 6@2.56$ in) horizontal stirrups and $\phi 10@50$ mm ($\phi 10@1.97$ in) vertical stirrups (**Fig.**
 12 **1a**). To overcome the difficulties of bending the $\phi 32$ mm (1.26 in) longitudinal tensile bars, their
 13 ends were welded to a steel plate. The differences between the tested beams are restricted to the
 14 CFRP shear reinforcement in the a beam span. The reference beam is designated as “Ref” (three
 15 steel stirrups in the a shear span leading a steel shear reinforcing ratio, ρ_{sw} , of 0.09%), while the
 16 following NSM strengthening configurations were adopted for the other four beams that also
 17 include three steel stirrups in the a shear span (**Fig. 1b** and **Table 1**):
 18 4LI-S2 - four CFRP laminates of type 2 (with a cross section of 1.4×20 mm² [0.05×0.79 in²]) per
 19 face, inclined at 52 degrees with respect to the longitudinal axis of the beam ($\theta_f = 52^\circ$ - the same
 20 orientation of the line AB present in Fig. 1b), installed from the bottom surface of the flange to the
 21 bottom tensile surface of the beam’s web, i.e., bridging the total side surfaces of the beam’s web;
 22 each CFRP laminate was installed in the outer part of a slit of a depth of 21 mm (0.83 in) executed
 23 on the beam’s web side surfaces. The length of each laminate was 634 mm (24.96 in);
 24 4LI-P2 - four CFRP laminates of type 2 per face, inclined at 52° with respect to the longitudinal
 25 axis of the beam ($\theta_f = 52^\circ$), installed from the bottom surface of the flange up to 10 mm (0.39 in)
 26 above the top surface of the longitudinal tensile steel reinforcement. Each CFRP laminate was

installed in the deeper part of a slit of a depth of 35 mm (1.38 in) from the surface of the beam's web side surfaces. The length of each laminate was 527 mm (20.75 in);

4LI4LI-SP1 - eight CFRP laminates (two per slit) of type 1 (with a cross section of $1.4 \times 10 \text{ mm}^2$ [$0.05 \times 0.39 \text{ in}^2$]) per face, inclined at 52° with respect to the longitudinal axis of the beam ($\theta_f = 52^\circ$).

The configuration of the slits executed in this beam combines the configurations of the 4LI-P2 and 4LI-S2 beams: one laminate was installed as deep as possible into a slit of a depth of 35 mm (1.38 in) and one laminate was installed as superficial as possible;

4LI4LV-SP1 - eight CFRP laminates of type 1 per face, four of them inclined at 52° with respect to the longitudinal axis of the beam ($\theta_f = 52^\circ$) and bridging the total side surfaces of the beam's web (the length of each inclined laminate was 634 mm [24.96 in]), while the other four laminates were installed in vertical slits executed from the bottom surface of the web up to 10 mm (0.39 in) above the top surface of the longitudinal tensile steel reinforcement (the length of each vertical laminate was 432 mm [7.00 in]). The vertical laminates were installed as deep as possible into a slit of a depth of 35 mm (1.38 in) from the surface of the beam's web side surfaces. The inclined laminates were installed as superficial as possible into a slit of a depth of 15 mm (0.59 in) executed on the beam's web side surfaces.

Laminates of $1.4 \times 10 \text{ mm}^2$ ($0.05 \times 0.39 \text{ in}^2$) cross section were used in the strengthening configurations of the 4LI4LI-SP1 and 4LI4LV-SP1 beams in order to assure a shear strengthening ratio for these beams ($\rho_f = 0.113\%$ and $\rho_f = 0.101\%$, respectively) similar to the ones applied in the 4LI-S2 and 4LI-P2 beams ($\rho_f = 0.113\%$), where laminates of $1.4 \times 20 \text{ mm}^2$ ($0.05 \times 0.79 \text{ in}^2$) cross section were applied (**Table 1**). The configuration adopted for the CFRP laminates avoided any type of injury in the steel reinforcement during the execution of the slits (since a minimum distance of 10 mm (0.39 in) was imposed between steel reinforcement and slits).

Material properties

All the NSM shear strengthened beams were constructed with concrete that at the testing age of the beams (371 days) had an average compressive strength (f_{cm}) of 40.1 MPa (5.8 ksi), measured in

1 cylinders of 150 mm (5.9 in) diameter and 300 mm (11.8 in) height according to EN 206-1¹². At
 2 the testing age of the Ref beam (120 days), the value of f_{cm} was 36.4 MPa (5.3 ksi). The values of
 3 the tensile properties of the high bond steel bars used in the beams were obtained from uniaxial
 4 tensile tests performed according to the recommendations of EN 10002-1¹³. The average value of
 5 the yield stress of the steel bars of 6, 12, 16, and 32 mm (0.24, 0.47, 0.63, and 1.26 in) diameter was
 6 565, 566, 560 and 654 MPa (82.0, 82.1, 81.2, and 94.8 ksi), respectively, while the average value of
 7 the tensile strength for these corresponding bars was: 682, 661, 675 and 782 MPa (98.9, 95.9, 97.9,
 8 and 113.4 ksi). CFK 150/2000 S&P laminates were used in the present experimental research. The
 9 tensile properties of both types of laminates (with cross section of $1.4 \times 10 \text{ mm}^2$ [$0.05 \times 0.39 \text{ in}^2$], type
 10 1, and $1.4 \times 20 \text{ mm}^2$ [$0.05 \times 0.79 \text{ in}^2$], type 2) were evaluated following the recommendations of ISO
 11 527-5 (1997)¹⁴. The average value of the tensile strength (f_{fu}), elasticity modulus (E_f), and ultimate
 12 strain (ε_{fu}) of type 1 CFRP laminate was 3009 MPa (436.4 ksi), 169 GPa (24.5 Msi) and 17.8%,
 13 while type 2 CFRP laminate presented the following average values for the corresponding
 14 parameters: 2577 MPa (373.8 ksi), 166 GPa (24.1 Msi) and 15.5%. S&P Resin 220 epoxy adhesive
 15 was used to bond the CFRP laminates to the concrete substrate. The instantaneous and long term
 16 tensile behavior of this adhesive was investigated by Costa and Barros¹⁵. At 3 days, at which the
 17 elasticity modulus ($E_{0.5 \sim 2.5\%}$) has attained a stabilized value, the tensile strength and the $E_{0.5 \sim 2.5\%}$,
 18 determined according to the ISO 527-2 recommendations¹⁶, was about 20 MPa (2.9 ksi) and 7 GPa
 19 (1.0 Msi), respectively.

20 **Monitoring system**

21 The three point beam bending tests (**Fig. 1a**) were carried out using a servo closed-loop control
 22 equipment, taking the signal read in the displacement transducer (LVDT), placed at the loaded
 23 section, to control the test at a deflection rate of 0.01 mm/second (0.039 in/second). With the
 24 purpose of obtaining the strain variation along two laminates (beams 4LI-S2 and 4LI-P2) or two
 25 groups of two laminates (beams 4LI4LI-SP1 and 4LI4LV-SP1) that have the highest probability of
 26 providing the largest contribution on the shear strengthening of the RC beam, four strain gauges

(SG_L) were bonded in each inclined laminate, and three SG_L were bonded in each vertical laminate according to the arrangement presented in **Fig. 2a**. Adopting the same principle, one steel stirrup was monitored with three strain gauges (SG_S) installed according to the configuration presented in **Fig. 2b**. The location of the monitored laminates and stirrups in the NSM shear strengthened beams is indicated in **Fig. 1b**.

Results

Load carrying capacity of the tested beams

The relationship between the applied force (F) and the deflection at the loaded section (u_{LS}) of the beams is illustrated in **Fig. 3a**. In this figure, ΔF represents the increase of the load provided by the shear strengthening system, while F^{Ref} is the load capacity of the reference beam. The $\Delta F/F^{Ref}$ ratio was evaluated for deflections greater than the corresponding to the formation of the first shear crack in the Ref beam, and the $\Delta F/F^{Ref}$ vs. u_{LS} relationship is illustrated in **Fig. 3b**.

Assuming that $\Delta F_{max} = F_{max} - F_{max}^{Ref}$, being F_{max}^{Ref} and F_{max} the load carrying capacity (maximum force) of the Ref beam and of the CFRP shear strengthened beam, respectively, the $\Delta F_{max}/F_{max}^{Ref}$ ratio was also evaluated. The values for F_{max} , $\Delta F_{max}/F_{max}^{Ref}$, and the deflection at loaded section corresponding to F_{max} ($u_{F_{max}}$) are presented in **Table 2**. **Figure 3** and the results included in **Table 2** show that, for deflections higher than the one corresponding to the formation of the first shear crack in the Ref beam, the adopted CFRP configurations provided an increase in the beam's load carrying capacity. This reveals that the CFRP laminates bridging the surfaces of the shear crack offer resistance, mainly to crack opening, resulting to a smaller degradation of the shear stress transfer between the faces of the crack, due to aggregate interlock effect. Therefore, for deflections above the one corresponding to the formation of the shear crack in the Ref beam, an increase of the beam's stiffness is observed in the shear strengthened beams. The crack opening resisting mechanisms provided by the laminates bridging the shear crack also contribute to increase the load at which stirrups enter in their plastic phase. The CFRP shear strengthening configurations provided

an increase in terms of stiffness (after shear crack formation), maximum load (F_{max}), and deflection at the loaded section ($u_{F_{max}}$) in correspondence to F_{max} . It is verified that $\Delta F_{max}/F_{max}^{Ref}$ has varied between 66% and 81%, which is higher than the load carrying increase levels registered in previous experimental programs with NSM CFRP shear strengthening configurations applied in T cross section RC beams⁶. The deflection at maximum load, $u_{F_{max}}$, in all the strengthened beams was also larger than in the Ref beam (between 50% and 90%). The maximum value of the $\Delta F_{max}/F_{max}^{Ref}$ ratio (81%) was recorded in the 4LI-P2 beam, which evidences the benefits of applying the laminates as deep as possible into the beam's web. However, this configuration was not the most suitable for seismic retrofitting, since the adopted inclination of the laminates is not effective in some of the possible directions of shear cracks formed in RC elements susceptible to shear failure in a seismic event. The shear configuration applied in the 4LI4LV-SP1 beam is more suitable for seismic retrofitting (vertical laminates can bridge the shear cracks that form during the reversal cyclic loading of a seismic event), and a significant increase of load carrying capacity was also obtained for this strengthening arrangement (71%). The obtained values of $\Delta F_{max}/F_{max}^{Ref}$ for 4LI-S2 (68%) and 4LI4LI-SP1 (66%) beams were similar to the ones of 4LI4LV-SP1 beam.

Failure modes

Figure 4 represent the crack patterns at failure of the beams. As expected, all the beams failed in shear. The maximum load of the Ref beam was attained when one stirrup crossing the shear failure crack has ruptured. In the beams strengthened with laminates type 2 of a cross section depth of 20 mm (0.79 in) (4LI-S2 and 4LI-P2), the failure was quite violent, since the laminate crossed by the shear failure crack has ruptured (laminates 2 in **Fig. 4**). In the beams strengthened with laminates type 1, of a cross section depth of 10 mm (0.39 in) (beams 4LI4LI-SP1 and 4LI4LV-SP1), the laminates have not ruptured, which has contributed to a pseudo-ductility in the post-peak phase of the response of these beams, due to the progressive debonding of these laminates (**Fig. 3a**). In fact, at a deflection of about 27 mm (1.06 in), the 4LI4LI-SP1 beam presented a load carrying capacity

of 468 kN (105 kips) (**Fig. 3a**), which is 85% of the maximum load (F_{max}). This deflection level (27 mm) was 33% higher than the deflection at F_{max} (20.3 mm [0.8 in]). For a deflection of about 22.2 mm (0.87 in), the 4LI4LV-SP1 beam presented a load carrying capacity of 486 kN (109.3 kips) (**Fig. 3a**), which is 85% of the maximum load (F_{max}). This deflection level (22.2 mm) was 39% higher than the deflection at F_{max} (16 mm [0.66 in]) of this beam.

Strains in the CFRP laminates and steel stirrups

Table 2 includes the strains measured in the monitored laminates up to F_{max} , $\varepsilon_{CFRP}^{SG-Li}$ ($i=1$ to 4 for the inclined laminates, and $i=1$ to 3 for vertical laminates, see **Fig. 2a**), and the maximum strain in these laminates up to F_{max} , ε_{CFRP}^{max} . The ε_{CFRP}^{max} has varied between 12.2‰ and 16.3‰, which is higher than 69% of the ultimate strain of the CFRP laminates, justifying the high strengthening effectiveness provided by the adopted configurations. Comparing the strains in the laminates at different depth, a clear tendency for an increase of the strains with the deeper position of the laminate is not observable, since the strains are more dependent on the relative distance between the SG and the shear failure crack crossing the laminate. However, the maximum average value of the $\varepsilon_{CFRP}^{SG-Li}$ was recorded in the laminate 3D (see **Fig. 4**) of 4LI4LI-SP1 beam (11.3‰). **Fig. 5** illustrates the variation of the strains on the monitored laminates during the loading process of the beam up to its maximum load, where two stages can be distinguished. In the initial stage of loading, while the CFRP was not crossed by a diagonal crack, the contribution of the laminate for the beam's load-carrying capacity is marginal. The second stage initiates when the CFRP was crossed by a diagonal crack. The gradient of strains registered in the SGs was as higher as closer was the SG from the crack plane. This behavior was similar in the monitored stirrups (**Fig. 6**). The evolution of the strains in the steel stirrups indicates that when the strengthened beams failed, the steel stirrups had already developed a significant plastic deformation.

Comparison of the shear strengthening effectiveness

The shear capacity contributed by the CFRP (V_f) was obtained by subtracting the shear resistance of the reference beam ($V_{ref.}$) from the shear resistance of the strengthened beam (V_t):

$$V_t = V_{ref.} + V_f \quad (1)$$

In this approach it is assumed that the steel stirrups give the same contribution in the strengthened and in the corresponding reference beam. By comparing the V_f values of the beams of the present experimental program (ranging between 145.8 kN and 178.1 kN [32.8 and 40.0 kips]), to those determined in several series of NSM shear strengthened RC beams where the laminates were applied in the shear span a (the test setup was similar of the beams of the present experimental program) into slits positioned at the beam's web surface⁶ (**Fig. 7a**), it can be concluded that laminates are, in fact, more effective if installed deeper inside the beam's web.

ANALYTICAL MODEL

From the V_f values, the effective strain in the laminates, ε_{fe} , can be obtained from the equation (2)⁶:

$$\varepsilon_{fe} = V_f / \left(h_w \times \frac{A_{fv}}{s_f} \times E_f \times (\cot g \alpha + \cot g \theta_f) \times \sin \theta_f \right) \quad (2)$$

where h_w is the height of the beam's web, A_{fv} is the cross sectional area of a "laminate stirrup" formed by two side laminates, s_f is the spacing of laminates, E_f is the elastic modulus of the laminate, α is the orientation of the shear failure crack, and θ_f is the inclination of the laminates with respect to the beam axis. In **Fig. 7b**, a comparison of the ε_{fe} values obtained in the present experimental program (ranging between 8.0‰ and 11.5‰) with those obtained in Dias and Barros⁶, where the benefits of placing the laminates as deep as possible into the beam's web, are illustrated. Based on the data obtained from the extensive experimental program carried out by Dias and Barros⁶, the equation to evaluate the value of the parameter ε_{fe} for the possible distinct NSM shear strengthening configurations was derived by adopting the following strategy: on each inclination of the CFRP (θ_f) that was investigated in the experimental program (45°, 60° and 90°), the equation

1 that relates ε_{fe} with the parameter $(E_f \rho_f + E_s \rho_{sw}) / (f_{cm}^{2/3})$ was determined. Thus, according to the
2 experimental results, four parameters with high influence on the behavior of a RC beam shear
3 strengthened with NSM CFRP laminates were considered in this analytical formulation: the
4 percentage (ρ_f) and the orientation (θ_f) of the CFRP laminates; the concrete strength by considering
5 the average concrete compressive strength in cylinders (f_{cm}); and the percentage of steel stirrups
6 (ρ_{sw}). Each of the previously mentioned derived equations corresponds to the best fit of the values
7 of ε_{fe} obtained using equation (2), considering the experimental results of V_f and 45° for the
8 orientation of the shear failure crack (α). In the parameter $(E_f \rho_f + E_s \rho_{sw}) / (f_{cm}^{2/3})$ the elasticity
9 modulus of the steel stirrups (E_s) was assumed to be 200 GPa (28.6 Msi). In **Fig. 8**, the relationship
10 between ε_{fe} and $(E_f \rho_f + E_s \rho_{sw}) / (f_{cm}^{2/3})$ parameter according to the obtained equations⁶ for the
11 above mentioned three CFRP inclinations, is shown. In addition, a comparison between the ε_{fe}
12 values obtained in the present experimental program (ranging between 8.0‰ and 11.5‰) with those
13 obtained in Dias and Barros⁶ is also shown, where the benefits of placing the laminates as deep as
14 possible into the beam's web are evident. The above mentioned equations proposed by Dias and
15 Barros⁶ have the following general format:

$$\varepsilon_{fe} = C_1 \times \left[(E_f \rho_f + E_s \rho_{sw}) / (f_{cm}^{2/3}) \right]^{-C_2} \quad (3)$$

16 where C_1 and C_2 were obtained as a function of the CFRP orientation (θ_f). In fact, considering the
17 values of C_1 and C_2 for the case of θ_f equal to 45° , 60° and 90° , it was possible to define equation
18 (4) and (5) for the parameter C_1 and C_2 , respectively, both as function of the θ_f , and with $R^2=1$:

$$C_1 = 3.76888 \times e^{(-0.1160261\theta_f + 0.0010437\theta_f^2)} \quad (4)$$

$$C_2 = 0.460679 \times e^{(0.0351199\theta_f - 0.0003431\theta_f^2)} \quad (5)$$

19 Considering equations (3), (4) and (5), the analytical formulation that determines the value of the ε_{fe}
20 parameter from the relevant data of the RC beam to be strengthened and the governing
21 characteristics of the NSM CFRP laminates is:

$$\varepsilon_{fe} = 3.76888 \times e^{(-0.1160261\theta_f + 0.0010437\theta_f^2)} \times \left[\left(E_f \rho_f + E_s \rho_{sw} \right) / \left(f_{cm}^{2/3} \right) \right]^{-0.460679 \times e^{(0.0351199\theta_f - 0.0003431\theta_f^2)}} \quad (6)$$

1 Since equation (6) does not attend to all effects that influence the effectiveness of the CFRP
 2 laminates, an uncertainty factor, designated γ_f , was proposed by Dias and Barros⁶[equation (7)],
 3 whose value was determined in order that in 95% of the analyzed RC beams the ratio between the
 4 experimental and analytical results of V_f equal or greater to the unity. Following this criterion, a
 5 value of 1.3 was obtained for the γ_f .

$$\varepsilon_{fe} = \left[3.76888 \times e^{(-0.1160261\theta_f + 0.0010437\theta_f^2)} \times \left[\left(E_f \rho_f + E_s \rho_{sw} \right) / \left(f_{cm}^{2/3} \right) \right]^{-0.460679 \times e^{(0.0351199\theta_f - 0.0003431\theta_f^2)}} \right] / \gamma_f \quad (7)$$

6 The obtained results in the NSM beams of the present study demonstrated that the deeper the
 7 laminate is installed into the core of the beam's web, the more effective the NSM technique is, with
 8 CFRP laminates on the shear strengthening. In order to take into account this evidence, the equation
 9 (7) proposed by Dias and Barros⁶ for the evaluation of the CFRP effective strain is modified by
 10 introducing a coefficient R:

$$\varepsilon_{fe}^* = R \varepsilon_{fe} \quad (8)$$

11 where ε_{fe}^* is the CFRP effective strain considering the effect of installing CFRP laminates deeper
 12 into the slits. To evaluate the R coefficient the following equation is proposed:

$$R = \left[\left(\frac{\varepsilon_{fu}}{\varepsilon_{fe}} - 1 \right) / \left(\frac{b_w}{2} - 0.005 \right) \right] \times z + 1 \quad (9)$$

13 where ε_{fu} is the CFRP ultimate strain, ε_{fe} is the CFRP effective strain obtained from equation (7),
 14 b_w is the width of the beam's web, and z is the distance from the centroid of the cross section of the
 15 laminate, to the centroid of a 10 mm (0.39 in) cross section depth laminate positioned near to the
 16 beam's side surface (**Fig. 9**), since equation (7) was calibrated considering the values of ε_{fe} for
 17 NSM CFRP laminates at this position. According to equation (9), $R=1$ for the case of $z=0$ and
 18 $R=\varepsilon_{fu}/\varepsilon_{fe}$ for the case of $z=b_w/2-0.005$ (with b_w in meters). In this last case, $\varepsilon_{fe}^* = \varepsilon_{fu}$, which

means that CFRP laminate attains its tensile rupture if positioned in the center of the cross section, obviously a condition that technically is not compatible with the execution procedures of the NSM technique, since the installation of FRPs systems in the interior of the cross section should follow the procedures of embedded through-section technique (ETS)¹⁷. However, since no more data are available to develop a more refined equation to predict the R parameter, the simple equation (9) is proposed to be used. **Table 3** includes for the NSM beams of the present study the value of the CFRP effective strain obtained using equations (2), (7) and (8). The average value of the ratio between CFRP effective strain obtained using equations (2) and (7) is 1.45, while the average value of the ratio between CFRP effective strain obtained using equations (2) and (8) is 1.17. These values confirm that equation (7) is very conservative when the NSM CFRP laminates are installed deeper into the slits. For this kind of NSM CFRP shear strengthening configuration equation (8) provides more realistic predictions of the effective strain that can be considered for the laminates.

NUMERICAL SIMULATIONS OF THE TESTED BEAMS

In the present section, a multi-directional fixed smeared 3D crack model, under the nonlinear FEM framework, is proposed to simulate the behavior of concrete structures¹⁸. This model is suitable to be used with 3D solid elements available in the FEMIX computer code¹⁹. The 3D model is fully described elsewhere¹⁸, and its suitability to capture the essential behavioral aspects of T cross section RC beams shear strengthened with NSM laminates positioned at different depth into the cross section of the beam is assessed by simulating the tested beams. Subsequently a parametric study is executed to evidence the influence of geometric and mechanical properties of FRP laminates on the NSM shear strengthening effectiveness of the RC beams.

Predictive performance of the model

To simulate the crack initiation and the fracture mode I propagation of reinforced concrete, the tri-linear tension-softening diagram represented in **Fig. 10a** was adopted. The values that define this diagram are indicated in **Table 4**. This table also includes the data necessary to define the shear crack-softening diagram represented in **Fig. 10b**. Since no available experimental results exist to

1 characterize the crack shear softening diagram, the adopted values were obtained by inverse
 2 analysis fitting the experimental results of the reference beam as best as possible. The finite element
 3 mesh adopted for the reference beam is illustrated in **Fig. 11**. Only half of the full size beam was
 4 modeled, taking advantage of the symmetry of the beams in order to reduce the computational time
 5 of the numerical simulations. Serendipity 8 nodes solid elements with $2 \times 2 \times 2$ Gauss-Legendre
 6 integration scheme were used for the concrete (three degrees-of-freedom per node). The steel
 7 stirrups, longitudinal steel bars and CFRP laminates were modeled with 3D embedded cables of 2
 8 nodes (one degree-of-freedom per node), by using a 2 Gauss-Legendre integration scheme. In the
 9 simulations of the strengthened beams the adhesive was not materially simulated, since assuming
 10 perfect bond between NSM CFRP laminates and surrounding concrete was already proved
 11 acceptable if the global behavior of the beams (force-deflection and failure modes) is the main goal
 12 of the simulations²⁰. Perfect bond was also assumed between steel reinforcement and surrounding
 13 concrete. To model the behavior of this reinforcement, the stress-strain relationship of **Fig. 12** was
 14 adopted. The curve (under compressive or tensile loading) is defined by the points $PT1 = (\varepsilon_{sy}, \sigma_{sy})$,
 15 $PT2 = (\varepsilon_{sh}, \sigma_{sh})$ and $PT3 = (\varepsilon_{su}, \sigma_{su})$, and the parameter P which defines the shape of the last
 16 branch of the curve. Unloading and reloading linear branches with slope $E_s = \sigma_{sy} / \varepsilon_{sy}$ are assumed
 17 in the present approach. The values of the parameters for the constitutive model of the steel are
 18 given in **Table 4**. A linear elastic stress-strain relation up to the ultimate tensile strength was
 19 adopted to simulate the tensile behavior of the CFRP laminates. After the CFRP ultimate tensile
 20 strain, ε_{fu} , the contribution of the laminates was neglected. The relationship between the applied
 21 load and the deflection at the loaded section obtained experimentally and in the numerical
 22 simulations for the tested beams are compared in **Fig. 13**. The crack patterns of these beams at the
 23 end of the analysis (at the end of the last converged load increment) are presented in **Fig. 14**. These
 24 two figures show that the numerical model is able to capture with good accuracy not only the

deformational response of the beams, but also the localization, opening and profile of the shear failure crack (compare with **Fig. 4**).

Parametric Study

The computer program, whose good predictive performance for the simulation of the behavior of the type of structures under consideration was confirmed in the previous section, was adopted to perform a parametric study for the evaluation of the influence on the load carrying capacity and failure mode of the following parameters: shear strengthening ratio, elasticity modulus and tensile strength of FRP shear strengthening systems. For this study, and for comparison purposes, the results obtained in the 4LI-P2 beam were adopted. The arrangement of the steel reinforcement, the concrete material properties, support and load conditions, and the finite element mesh, were the same ones adopted in the numerical simulations of the previous section.

Effect of shear strengthening ratio

The shear strengthening ratio was increased by using laminates of higher cross sectional thickness: $1.4 \times 20 \text{ mm}^2$ [$0.05 \times 0.79 \text{ in}^2$] (corresponding to the 4LI-P2 beam to be compared), $2.8 \times 20 \text{ mm}^2$ [$0.11 \times 0.79 \text{ in}^2$], and $5.6 \times 20 \text{ mm}^2$ [$0.22 \times 0.79 \text{ in}^2$]. In **Fig. 15a** the load-deflection curves for the three different laminates, hence different shear strengthening ratios, are compared. As expected, by increasing the shear strengthening ratio of the CFRP laminates the load carrying capacity and ultimate deflection have increased, and in the case of CFRP laminates of $5.6 \times 20 \text{ mm}^2$ [$0.22 \times 0.79 \text{ in}^2$] cross section a flexural failure mode was obtained.

Effect of ultimate tensile strength of the FRP laminates

The influence of ultimate tensile stress of the FRP laminates on the load-deflection behavior of the RC beams was investigated. For this purpose, two other ultimate tensile strength values of the FRP laminates were assumed: 1400 MPa (203 ksi) and 4300 MPa (623 ksi). These values are, respectively, lower and higher than the ultimate tensile strength of the CFRP laminates adopted previously in the 4LI-P2 beam (2577 MPa [373.8 ksi]). The relationship between load and deflection at loaded section for the simulated beams are compared in **Fig. 15b**, where the high

influence of the tensile strength of FRP systems on the maximum load carrying capacity of the strengthened RC beams, is evident (the higher the former, the higher the later). All the simulated beams failed in shear with the rupture of the FRP laminates.

Effect of elasticity modulus of the FRP laminates

The elasticity modulus of the FRP laminates was changed, by adopting the following values: 60 GPa (8.7 Msi) (corresponding to GFRP - glass fiber reinforced polymer) and 250 (36.3 Msi) (CFRP of high modulus). The obtained results, illustrated in **Fig. 15c**, show that the elastic modulus of the FRP laminates has a relatively negligible effect on the ultimate load carrying capacity of shear strengthened RC beams. Such observation is in agreement with the parametric studies carried out by Bianco et al.²¹ with an analytical approach for predicting the contribution of NSM-FRP laminates for the shear strengthening of RC beams. This demonstration can stimulate the manufacturers of FRP systems for the development of more cost competitive and technically effective solutions for the shear strengthening of RC structures.

CONCLUSIONS

An experimental program was carried out with T cross section RC beams of relatively high cross section height (600 mm) that were shear strengthened with CFRP laminates applied according to the NSM technique²². The laminates were positioned at different depth into slits opened on the side faces of the beam's web. From the obtained experimental results, it can be concluded that NSM technique with CFRP laminates is very effective in RC beams of relatively high cross section height, not only in terms of increasing the overall behavior of the RC beams, but also in assuring higher mobilization of the tensile properties of the CFRP. The CFRP shear strengthening configurations provided an increase in terms of maximum load that ranged between 66% and 81%. The maximum tensile strain recorded in the laminates up to the maximum load has ranged between 12.2‰ and 16.3‰, which indicates that was exceeded 69% of the ultimate tensile strain of the adopted CFRP laminates. The obtained results have confirmed that the deeper the laminates are installed into the beam's web, the higher the shear strengthening effectiveness they can provide.

The shear strengthening configuration constituted by two independent laminates in the same slit, as well as the strengthening configuration where laminates of two distinct inclinations (52° and 90°) are installed at different depth inside the slit, have provided a more ductile behavior after peak load for the beams shear strengthened with these configurations.

Dias and Barros' model for the prediction of the contribution of NSM CFRP laminates for the shear strengthening of RC beams was generalized in order to simulate the favorable effect of locating the laminates deeper into the beam's web cross section. More experimental data is needed to better calibrate the equation that takes into account this effect.

Finally, a 3D multi-directional smeared crack model available in FEM-based FEMIX computer program was used to demonstrate that this type of model, which simulates the opening and sliding softening characteristics of the 3D concrete fracture process, is capable of capturing the relevant behavioral aspects of the type of RC structural elements tested in the experimental program. By using this model, a parametric study was executed to demonstrate that the tensile strength of FRP laminates has a more favorable effect on the load carrying capacity of NSM shear strengthened RC beams than its elasticity modulus..

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NOTATIONS

A_{fv}	cross sectional area of a “CFRP laminate stirrup” formed by two side laminates
A_{sl}	cross sectional area of the longitudinal tensile steel reinforcement
A_{sw}	cross sectional area of the arms of a steel stirrup
C_1	coefficient obtained as a function of the CFRP orientation
C_2	coefficient obtained as a function of the CFRP orientation
D_n^{cr}	fracture mode I stiffness modulus
$D_{t_i}^{cr}$	sliding stiffness modulus in the $\hat{t}_i$ direction
E	elasticity modulus
E_f	elasticity modulus of the CFRP
E_s	elasticity modulus of the steel stirrups
F_{max}	load carrying capacity (maximum force) of the CFRP shear strengthened beam
F_{max}^{Ref}	load carrying capacity (maximum force) of the reference beam
F^{Ref}	load capacity of the reference beam
$G_{f,n}$	mode I fracture energy
$G_{f,s}$	shear fracture energy
R	coefficient that defines the effect of installing CFRP laminates deeper into the slits
V_f	shear resistance provided by the CFRP
V_{ref}	shear resistance of the reference beam
V_t	shear resistance of the strengthened beam
a	monitored shear span of the beam (smaller beam shear span)
a_f	smaller cross-sectional dimension of the CFRP laminates

b	shear span of the beam without CFRP
b_f	larger cross-sectional dimension of the CFRP laminates
b_w	width of the beam's web
d	distance from extreme compression fiber to the centroid of tensile reinforcement
f_{cm}	average value of the compressive strength of the concrete
f_{ct}	concrete tensile strength
f_{fu}	maximum tensile strength of the CFRP
h_w	height of the beam's web
l_b	crack band width
s_f	spacing between CFRP laminates
s_w	spacing between steel stirrups
u_{LS}	deflection at the loaded section for the tested beams
$u_{F_{max}}$	deflection at loaded section corresponding to the maximum force
z	distance from the centroid of the cross section of the laminate to the centroid of the cross section of a laminate of 10mm cross section depth
α	orientation of the shear failure crack
α_i	fracture parameters used to define the trilinear stress-strain softening diagram
β	shear retention factor
γ_f	reduction factor
$\gamma_{t,p}^{cr}$	crack peak shear strain
ΔF	increase of the load provided by the shear strengthening system
ΔF_{max}	difference between the maximum load of shear strengthened beam with CFRP and the maximum load of reference beam
$\varepsilon_{CFRP}^{SG\_Li}$	strains measured in the monitored laminates
ε_{CFRP}^{max}	maximum strain in monitored laminates up to F_{max}
ε_{fe}	CFRP effective strain
ε_{fe}^*	CFRP effective strain considering the effect of installing CFRP laminates deeper into the slits
ε_{fu}	maximum value of the CFRP strain
$\varepsilon_{n,i}^{cr}$	crack normal strain used to define point i in the trilinear stress-strain softening diagram
$\varepsilon_{n,u}^{cr}$	ultimate crack normal strain
ε_{sh}	strain corresponding to point 2 (PT2) of the steel stress-strain relationship
ε_{su}	strain corresponding to point 3 (PT3) of the steel stress-strain relationship
ε_{sy}	strain corresponding to point 1 (PT1) of the steel stress-strain relationship
θ_f	CFRP orientation with respect to the longitudinal axis of the beam
ν	Poisson's coefficient
ξ_i	fracture parameters used to define the trilinear stress-strain softening diagram
ρ_f	percentage of the CFRP shear strengthening

ρ_{sl}	percentage of the longitudinal tensile reinforcement
ρ_{sw}	percentage of steel stirrups
$\sigma_{n,i}^{cr}$	crack normal stress used to define point i in the trilinear stress-strain softening diagram
σ_{sh}	stress corresponding to point 2 (PT2) of the steel stress-strain relationship
σ_{su}	stress corresponding to point 3 (PT3) of the steel stress-strain relationship
σ_{sy}	stress corresponding to point 1 (PT1) of the steel stress-strain relationship
$\tau_{t,p}^{cr}$	crack peak shear strength

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6

Table 1 - General information about the series of the tested RC beams.

Beam	ρ_{sl} [%] (1)	a/d	Shear reinforcement system in the shear span a					
			Material	Quantity	CFRP position (2)	Percentage [%] (3)	Spacing, mm (in)	Angle [°]
4LI-S2	2.0	2.5	Steel stirrups	3 ϕ 6	-	0.09	350 (13.78)	90
			CFRP laminates	2×4 laminates (1.4×20 mm ² [[0.05×0.79 in ²]])	S	0.113	350 (13.78)	52
4LI-P2			Steel stirrups	3 ϕ 6	-	0.09	350 (13.78)	90
			CFRP laminates	2×4 laminates (1.4×20 mm ² [[0.05×0.79 in ²]])	D	0.113	350 (13.78)	52
4LI4LI-SP1			Steel stirrups	3 ϕ 6	-	0.09	350 (13.78)	90
			CFRP laminates	2×(2×4) laminates (1.4×10 mm ² [0.05×0.39 in ²]])	S+D	0.113	350 (13.78)	52
4LI4LV-SP1			Steel stirrups	3 ϕ 6	-	0.09	350 (13.78)	90
			CFRP laminates	2×4 laminates (1.4×10 mm ² [[0.05×0.39 in ²]])	S	0.056	350 (13.78)	52
			CFRP laminates	2×4 laminates (1.4×10 mm ² [[0.05×0.39 in ²]])	D	0.044	350 (13.78)	90

(1) The percentage of the longitudinal tensile reinforcement was obtained from $\rho_{sl} = (A_{sl} / (b_w \times d)) \times 100$ where A_{sl} is the cross sectional area of the longitudinal tensile steel reinforcement (see **Fig. 1a**), $b_w = 180$ mm (7.09 in) is the width of the beam's web and d is the distance from extreme compression fiber to the centroid of tensile reinforcement. (2) S: Surface, D: Deeper, S+D: Surface [(2 $\times$ 4) laminates] + Deeper [(2 $\times$ 4) laminates] (see **Fig. 1b**). (3) The percentage of the vertical steel stirrups was obtained from $\rho_{sw} = (A_{sw} / (b_w \times s_w)) \times 100$ where A_{sw} is the cross sectional of the arms of a steel stirrup, and s_w is the spacing of the stirrups. The CFRP percentage was obtained from $\rho_f = ((2 \times a_f \times b_f) / (b_w \times s_f \times \sin \theta_f)) \times 100$ where a_f and b_f are the dimensions of the laminate cross section, s_f is the CFRP spacing and θ_f is the CFRP orientation.

1

Table 2 - Experimental results of the tested beams.

Beam	F_{max} , kN (kips)	$\Delta F_{max} / F_{max}^{Ref}$ [%]	$u_{F_{max}}$, mm (in)	Strain values in the CFRP laminates					
				Monitored laminate ⁽¹⁾	$\epsilon_{CFRP}^{SG\_L1}$ [‰]	$\epsilon_{CFRP}^{SG\_L2}$ [‰]	$\epsilon_{CFRP}^{SG\_L3}$ [‰]	$\epsilon_{CFRP}^{SG\_L4}$ [‰]	ϵ_{CFRP}^{max} [‰]
Ref	331 (74)	0	10.7 (0.42)	-	-	-	-	-	-
4LI-S2	556 (125)	68	16.7 (0.66)	2	10.15	16.33	9.91	0.13	16.33
				3	0.30 (0.30)	7.21 (7.37)	8.19 (8.62)	4.80 (6.65)	
4LI-P2	599 (134)	81	17.5 (0.69)	2	12.35	13.70	12.49	0.24	13.70
				3	0.17	0.27	3.84	4.55	
4LI4LI-SP1	550 (123)	66	20.3 (0.80)	2S	5.33 (6.29)	7.63	11.46	8.60	14.28
				3S	13.31	14.28	10.11	3.69 (5.67)	
				2D	6.55	7.37	12.28	9.37	
				3D	11.32	12.99	12.46	8.78	
4LI4LV-SP1	566 (127)	71	16.0 (0.63)	2I	11.27	11.59	10.71	9.23	12.24
				3I	0.21	7.21	8.85	6.75 (9.30)	
				2V	2.81	12.24	8.47	NA ⁽²⁾	
				3V	0.57	6.89	10.81	NA ⁽²⁾	

(1) See **Fig. 1b**. (2) This SG did not work. Note: The strain values in brackets are referred to the load, at which occurred the maximum strain in the laminates before the maximum load.

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Table 3 - Effective strain of the analyzed CFRP shear strengthening configurations.

Beam	CFRP shear reinforcement system			ε_{fe} [‰] ⁽¹⁾	ε_{je} [‰] ⁽²⁾	ε_{fe} [‰]	$R^{(4)}$	R	ε_{fe}^* [‰] ⁽⁶⁾
	Number and type of CFRP	ρ_f [%]	θ_f [°]						
4LI-S2	2×4 laminates (1.4×20 mm ²)	0.113	52	8.01	5.92	5.92	1.0925	1.0925	6.47
4LI-P2	2×4 laminates (1.4×20 mm ²)	0.113	52	11.51	5.92	5.92	1.3699	1.3699	8.11
4LI4LI-SP1	2×4 laminates (1.4×10 mm ²)	0.056	52	8.39	8.21	8.21 ⁽³⁾	1.0000	1.1669 ⁽⁵⁾	9.58
	2×4 laminates (1.4×10 mm ²)	0.056	52		8.21		1.3337		
4LI4LV-SP1	2×4 laminates (1.4×10 mm ²)	0.056	52	10.21	8.21	6.92 ⁽³⁾	1.0000	1.2986 ⁽⁵⁾	8.99
	2×4 laminates (1.4×10 mm ²)	0.044	90		5.28		1.6775		

(1) Value of ε_{fe} obtained from equation (2). (2) Value of ε_{je} obtained from equation (7). (3) Average value of ε_{je} obtained from equation (7) considering the percentage ρ_f (example for 4LI4LV-SP1 beam: $6.92 = (8.21 \times 0.056 + 5.28 \times 0.044) / (0.056 + 0.044)$). (4) Value of R obtained from equation (9). (5) Average value of R obtained from equation (9) considering the percentage ρ_f (example for 4LI4LI-SP1 beam: $1.1669 = (1.0000 \times 0.056 + 1.3337 \times 0.056) / (0.056 + 0.056)$). (6) Value ε_{fe}^* obtained from equation (8).

1

Table 4 - Values of the parameters of the constitutive model (concrete and steel).

Concrete	ν_c	E_c (MPa [ksi])	f_c (MPa [ksi])	f_{ct} (MPa [ksi])	$G_{f,n}$ (N/mm [lb/in])	ξ_1	α_1	ξ_2	α_2	$\tau_{t,p}^{cr}$ (MPa [ksi])	$G_{f,s}$ (N/mm [lb/in])	β
	0.19	32400 (4699)	36.40 (5.28)	2.2 (0.32)	0.086 (0.49)	0.008	0.4	0.2	0.1	1.2 (0.17)	0.07 (0.40)	0.6
Steel	Property				$\phi 6$		$\phi 12$		$\phi 16$		$\phi 32$	
	f_{sym} (MPa [ksi])				565 (82.0)		566 (82.1)		560 (81.2)		654 (94.8)	
	f_{sum} (MPa [ksi])				682 (98.9)		661 (95.9)		675 (97.9)		782 (113.4)	
	ϵ_{sy} (‰)				2.8		2.8		2.8		3.2	
	σ_{sy} (MPa [ksi])				565 (82.0)		566 (82.1)		560 (81.2)		654 (94.8)	
	ϵ_{sh} (‰)				20		30		30		10	
	σ_{sh} (MPa [ksi])				566 (82.1)		567 (82.1)		561 (81.2)		661 (95.9)	
	ϵ_{su} (‰)				45		215		220		50	
	σ_{su} (MPa [ksi])				682 (98.9)		661 (95.9)		675 (97.9)		782 (113.4)	
	Third branch exponent				1		1		1		1	

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Fig. 2 - Position of the strain gauges in the: a) CFRP laminates; b) steel stirrups (dimensions in mm [1 mm= 0.04 in]).

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Fig. 3 - a) Force vs deflection at loaded section of the tested beams and b) $\Delta F/F^{Ref}$ vs deflection at loaded section of the NSM CFRP tested beams.

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Fig. 4 - Crack patterns of the tested beams at failure (CFRP laminates identification: S: surface; D: deeper; V: vertical; and I: inclined).

a) Beam 4LI-S2

b) Beam 4LI-P2

Monitored laminates installed near the surface

Monitored laminates installed deeper into the slits

c) Beam 4LI4LI-SP1

Fig. 5 - Variation of strains in the monitored CFRP laminates of the tested beams.

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Fig. 6 - Variation of strains in the monitored steel stirrup of: a) beam 4LI-S2; b) beam 4LI-P2.

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Fig. 7 - Comparison of the shear strengthening effectiveness of the NSM technique in terms of: a) V_f and b) ϵ_{fe} .

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Fig. 8 - Effective strain of the CFRP (ε_{fe}) vs $(E_f \rho_f + E_s \rho_{sw}) / (f_{cm}^{2/3})$.

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Fig. 9 - Geometric parameters of equation (9).

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3 Fig. 10 - Diagram to simulate: a) the fracture mode I crack propagation ($\sigma_{n,1}^{cr} = f_{ct}$, $\sigma_{n,2}^{cr} = \alpha_1 \sigma_{n,1}^{cr}$,
4 $\sigma_{n,3}^{cr} = \alpha_2 \sigma_{n,1}^{cr}$, $\varepsilon_{n,2}^{cr} = \xi_1 \varepsilon_{n,u}^{cr}$, $\varepsilon_{n,3}^{cr} = \xi_2 \varepsilon_{n,u}^{cr}$), b) the relationship between each crack shear stress and
5 crack shear strain component.
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1 *Fig. 11 - Geometry, finite element mesh, and support conditions of the reference beam (dimensions*
2 *in mm [1 mm = 0.04 in]).*
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Fig. 12 - Constitutive model to simulate the tensile and compressive behavior of the steel bars.

Fig. 13 - Comparison between experimental and numerical relationships of force vs. deflection at the loaded section of tested RC beams.

Fig. 14 - Crack pattern of the beams at failure (pink: crack completely open; red: crack in the opening process; cyan: crack in the reopening process; green: crack in the closing process; blue: closed crack).

Fig. 15 - Influence of the: a) shear strengthening ratio, b) ultimate tensile strength of the FRP laminates, and c) elasticity modulus of the FRP.