Creep Behaviour of Cracked Steel Fibre Reinforced Self-Compacting Concrete Laminar Structures
Amin Abrishambaf

Creep Behaviour of Cracked Steel Fibre Reinforced Self-Compacting Concrete Laminar Structures

Universidade do Minho
Escola de Engenharia

Tese de Doutoramento
Civil Engineering

Trabalho efectuado sob a orientação do Professor Doutor Joaquim António Oliveira Barros e coorientação do Professor Doutor Vítor Manuel do Couto Fernandes da Cunha

May 2015
STATEMENT OF INTEGRITY

I hereby declare having conducted my thesis with integrity. I confirm that I have not used plagiarism or any form of falsification of results in the process of the thesis elaboration.

I further declare that I have fully acknowledged the Code of Ethical Conduct of the University of Minho.

University of Minho, _______________________________

Full name:  Amin Abrishambaf

Signature:  ____________________________________________
Acknowledgement

I would like to express my special appreciation and thanks to my advisors: Prof. Doctor Joaquim António Oliveira Barros and Prof. Doctor Vítor Manuel do Couto Fernandes da Cunha, you have been a tremendous mentor for me. I would like to thanks you for encouraging my research and for allowing me to grow as a research scientist. Your advice on both research as well as on my career have been priceless. It was a pleasure working with you.

The experimental work would not have been finalized so straightforwardly without the enthusiastic dedication of the laboratory technicians, especially, Sr. José Matos and Marco Jorge. I am grateful for their helps during performing of the experimental test. I would also like to acknowledge the help of Delfina Gonçalves, from Civitest company, on the experimental program.

A special thanks to my family. Words cannot express how grateful I am to my father (Mohammad Hossein), mother (Mahin) and my brothers (Reza and Omid) for all of the sacrifices that you have made on my behalf. Your prayer for me was what sustained me thus far. At the end I would like express deeply appreciation to my friends who supported me in this program, and also for their delightful friendship.
Abstract

Fibre reinforced self-compacting concrete (FRSCC) is a novel type of concrete containing discrete elements, which enhance the concrete’s post-cracking properties. For structural applications, commonly, it contains short discrete steel fibres that are randomly distributed and oriented. Fibre reinforcement can be consider as an alternative to conventional steel bars in order to improve the structural efficiency as well as the in-situ working conditions. Although, in the last decades, fibres were mainly applied in non-structural elements or even to control early thermal cracking, and plastic or drying shrinkage, they can be used to reduce or even replace the ordinary steel reinforcements in structural concrete elements. In the laminar structures with lower safety requirements such as grade slabs, nowadays, fibres can already be considered as an alternative in order to replace the ordinary bars completely.

One of the most important benefits of steel fibre reinforced concrete, SFRC, is the durability improvement, as a consequence of its improved post-cracking properties. For a cracked section, under a sustained load, the time-dependent crack widening has been attributed to two mechanisms: fibre pull-out process and time-dependent fibre creep. Creep is a visco-elastic phenomenon, which mainly occurs in the viscous hydrated cement paste. This may be a concern, since steel fibre reinforced self-compacting concrete (SFRSCC) has a high binder content, in part to attain its self-compactibility. Thus SFRSCC may exhibit a relatively high deformability due to long-term loads.

The present work aims to increase knowledge within this research area. The main purpose was to achieve as much as possible a consistent comprehension of the behaviour of this material under monotonic and long-term (in the cracked state) loading conditions. Therefore, in the first phase, an experimental campaign was executed in order to understand how fibres were distributed and oriented in SFRSCC laminar structures, and, furthermore, how these parameters influence the overall composite behaviour at a macro-level. Then, the micro-mechanical aspects of fibre reinforcements were analysed by performing a series of monotonic and long-term fibre pull-out tests. Finally, based on the fibre’s micro-mechanical properties, an integral approach was used to predict the flexural behaviour of SFRSCC laminar structures under monotonic and long-term loading conditions.
Resumo

Betão auto-compactável reforçado com fibras (BACRF) é um material relativamente recente contendo elementos de reforço discretos, os quais contribuem para o melhoramento das propriedades pós-fissuradas do material. Para aplicações estruturais, usualmente, são utilizadas fibras de aço distribuídas e orientadas aleatoriamente. A aplicação das fibras discretas pode ser encarada como uma alternativa viável ao uso de armadura convencional, fomentando a eficiência estrutural e melhorando as condições de trabalho in-situ. No entanto, nas últimas décadas, as fibras têm sido principalmente utilizadas com um propósito não-estrutural, para o controle da fissuração devida ao calor de hidratação, retração plástica e por secagem; as fibras podem ser uma alternativa plausível para a redução ou eliminação da armadura convencional em estruturas de betão armado. Em estruturas laminares com menores exigências do ponto de vista estrutural, como por exemplo lajes / pavimentos térreos, o uso de fibras, atualmente, já é encarado como uma alternativa viável à armadura convencional.

Uma das maiores vantagens do betão reforçado com fibras aço, BRFA, é a sua durabilidade, que advém do seu comportamento melhorado no regime pós-fissurado. Para uma secção fissurada submetida a uma carga constante, a evolução da abertura da fissura com o tempo é atribuída a dois mecanismos, nomeadamente, o arrancamento da fibra e a fluência da mesma ao longo do tempo. A fluência é um fenómeno visco-elástico que ocorre principalmente na pasta de cimento hidratada. No caso de betões auto-compactáveis, tal facto pode ser uma preocupação dado que este tipo de betões têm uma quantidade elevada de ligante, em parte para cumprir requisitos de auto-compactibilidade. Por esta razão, os betões auto-compactáveis poderão exibir uma elevada deformabilidade ao comportamento diferido quando sujeitos a cargas constantes.

Com este trabalho pretende-se aumentar o conhecimento do comportamento diferido de BACRF no regime pós-fissurado. Numa primeira fase estudou-se a distribuição e orientação das fibras em estruturas laminares de BACRF, e como esses parâmetros influenciam o comportamento mecânico do compósito. Posteriormente, o comportamento micro-mecânico das fibras foi estudado através de uma série de ensaios de arrancamento monotónicos e diferidos no tempo. Finalmente, com base nos ensaios de arrancamento, é proposta uma abordagem integrada para prever o comportamento de estruturas laminares de BACRF em flexão sob ação de cargas monotónicas e diferidas no tempo.
Notations and Symbols

Chapter 2 – Literature Overview

Greek Letters

\( \bar{\delta} \) \hspace{1cm} \text{Average signal of the displacement transducers} \hspace{1cm} \text{mm}

\( \bar{\delta}_s \) \hspace{1cm} \text{Average displacement at peak stress} \hspace{1cm} \text{mm}

\( \varepsilon_c (t) \) \hspace{1cm} \text{Total strain} \hspace{1cm} \text{---}

\( \varepsilon_{cc} (t) \) \hspace{1cm} \text{Strain due to the creep} \hspace{1cm} \text{---}

\( \varepsilon_{ci} (t_0) \) \hspace{1cm} \text{Instantaneous strain} \hspace{1cm} \text{---}

\( \varepsilon_{cs} (t) \) \hspace{1cm} \text{Strain related to the shrinkage} \hspace{1cm} \text{---}

\( \varepsilon_{fc} \) \hspace{1cm} \text{Creep deformation of fibre reinforced concrete} \hspace{1cm} \text{---}

\( \varepsilon_{oc} \) \hspace{1cm} \text{Creep deformation of ordinary concrete} \hspace{1cm} \text{---}

\( \vartheta \) \hspace{1cm} \text{Inclination angle of wedge grooves in splitting test} \hspace{1cm} \text{degree}

\( \mu \) \hspace{1cm} \text{Friction coefficient} \hspace{1cm} \text{---}

\( \sigma_x \) \hspace{1cm} \text{Maximum horizontal stress} \hspace{1cm} \text{MPa}

\( \sigma_y \) \hspace{1cm} \text{Maximum vertical stress} \hspace{1cm} \text{MPa}

\( \sigma_w \) \hspace{1cm} \text{Tensile stress} \hspace{1cm} \text{MPa}

\( \Delta(t) \) \hspace{1cm} \text{Mid-span deflection at time} \ t \hspace{1cm} \text{mm}

\( \Delta(t_0) \) \hspace{1cm} \text{Mid-span deflection at the end of loading phase} \hspace{1cm} \text{mm}

Roman Letters

\( b \) \hspace{1cm} \text{Width of the specimen} \hspace{1cm} \text{mm}

\( d \) \hspace{1cm} \text{Diameter of the cylinder in splitting test} \hspace{1cm} \text{mm}

\( f_{ck} \) \hspace{1cm} \text{Characteristic tensile strength} \hspace{1cm} \text{MPa}

\( f_{cm} \) \hspace{1cm} \text{Compressive strength} \hspace{1cm} \text{MPa}

\( f_{eq} \) \hspace{1cm} \text{Equivalent flexural tensile strength} \hspace{1cm} \text{MPa}

\( f_{ct, \text{min}} \) \hspace{1cm} \text{Lower values of the characteristic tensile strength} \hspace{1cm} \text{MPa}

\( f_{ct, \text{mean}} \) \hspace{1cm} \text{Mean values of the characteristic tensile strength} \hspace{1cm} \text{MPa}

\( f_{ct, \text{max}} \) \hspace{1cm} \text{Upper values of the characteristic tensile strength} \hspace{1cm} \text{MPa}

\( f_{R0} \) \hspace{1cm} \text{Residual flexural tensile strength} \hspace{1cm} \text{MPa}

\( h_{sp} \) \hspace{1cm} \text{Displacement between tip of the notch and top of cross section} \hspace{1cm} \text{mm}

\( l \) \hspace{1cm} \text{Thickness of the net area in the notched plane in splitting test} \hspace{1cm} \text{mm}

\( w \) \hspace{1cm} \text{Crack opening} \hspace{1cm} \text{mm}
\( w_m \)  
Ultimate crack opening  
mm

\( v_f \)  
Fibre volume  
\%

\( A_n \)  
Cross section area at the notch  
mm\(^2\)

\( D_{Ez} \)  
Energy absorption capacity due to the concrete contribution  
kN/mm

\( D_{Ez}' \)  
Energy absorption capacity due to the fibre contribution  
kN/mm

\( F_L \)  
Limit of proportionality  
kN

\( F_{\text{max}} \)  
Maximum load  
kN

\( F_{SP} \)  
Transversal splitting force  
kN

**Chapter 3 – Fibre Structure in the SFRSCC**

**Greek Letters**

\( \alpha \)  
Fibre aspect ratio  
---

\( \beta \)  
Angle between the direction of the concrete flow and the notched plane direction  
degree

\( \gamma \)  
Effective reduction of the fibre content  
---

\( \eta \)  
Fibre orientation factor  
---

\( \eta_{1D} \)  
The fibre orientation factor in one-dimensional case  
---

\( \eta_{2D} \)  
The fibre orientation factor in two-dimensional case  
---

\( \eta_{3D} \)  
The fibre orientation factor in three-dimensional case  
---

\( \bar{\eta}_{b3} \)  
Stress transfer efficiency parameter in the boundary layers  
---

\( \theta \)  
Angle between fibre and crack line  
degree

\( \theta_m \)  
Average fibre orientation angle  
degree

\( \theta_s \)  
Standard Gaussian orientation angle  
degree

\( \theta' \)  
Local out-of-plane angle  
degree

\( \theta'_c \)  
Cutoff angle  
degree

\( \xi_{\text{seg}} \)  
Fibre segregation parameter  
---

\( \rho_s \)  
Density of the steel of the fibres  
kg/m\(^3\)

\( \tau \)  
Interface bond strength  
MPa

\( \tau_f \)  
Fibre and matrix friction resistance  
MPa

\( \varphi \)  
Orientation of fibres in space  
degree

\( \omega \)  
Degree of orientation  
---

\( \omega_{2,3} \)  
Degree of orientation  
---

\( \bar{\Lambda}_2 \)  
Average nearest neighbour distances between randomly distributed points in a plane  
mm

\( \bar{\Lambda}_3 \)  
Average nearest neighbour distances between randomly distributed points in a space  
mm

\( \sigma(\theta) \)  
Standard deviation of orientation angle  
degree
\[ \sigma_m \] Tensile strength of the plain concrete \hspace{1cm} \text{MPa}

Roman Letters

\[ a_f \] Major axis length of the fibre elliptical cross-section \hspace{1cm} \text{mm}

\[ a_{ij} \] Component of the orientation tensor

\[ b \] Width of the beam section \hspace{1cm} \text{mm}

\[ b_f \] Minor axis length of the fibre elliptical cross-section \hspace{1cm} \text{mm}

\[ d \] Fibre diameter \hspace{1cm} \text{mm}

\[ h \] Height of the analysed cross-section parallel to the direction of vibration \hspace{1cm} \text{mm}

\[ h_{rp} \] Height of the remaining ligament of the beam section \hspace{1cm} \text{mm}

\[ l_f \] Length of a fibre \hspace{1cm} \text{mm}

\[ t_c \] Distance to the nearest exterior surface of concrete prism \hspace{1cm} \text{mm}

\[ t \] Thickness of the specimen \hspace{1cm} \text{mm}

\[ y \] Distances of the gravity point of fibre from the edge of the mould \hspace{1cm} \text{mm}

\[ \bar{y} \] Coordinate of the center of gravity of the reinforcement \hspace{1cm} \text{mm}

\[ z \] Distances of the gravity point of fibre from the edge of the mould \hspace{1cm} \text{mm}

\[ A_f \] Cross-section of a fibre \hspace{1cm} \text{mm}^2

\[ F(\theta_{av}) \] Cumulative distribution values of the standardized Gaussian law \hspace{1cm} \% 

\[ F(\phi) \] Cumulative values of Gaussian distribution \hspace{1cm} \% 

\[ L_V \] Total fibre length in a unit volume of concrete \hspace{1cm} \text{mm}

\[ N \] Theoretical number of fibres in the cross section \hspace{1cm} \text{fibres}

\[ ND \] Notch depth \hspace{1cm} \text{mm}

\[ N_f \] Number of fibres per unit surface \hspace{1cm} \text{Fibre/cm}^2

\[ N_f^1 \] Effective number of fibre per unit area parallel to the direction of the vibration/flow \hspace{1cm} \text{fibre/cm}^2

\[ \bar{N}_f \] Minimum fibre content \hspace{1cm} \text{fibres}

\[ N_f^2 \] Effective number of fibre per unit area perpendicular to the direction of the vibration/flow \hspace{1cm} \text{fibre/cm}^2

\[ N_f^{3D} \] Effective number of fibre intersections \hspace{1cm} \text{fibres}

\[ P(\theta_{ia}) \] Probability of orientation profile \hspace{1cm} \% 

\[ V_f \] Fibre volume fraction \hspace{1cm} \text{kg/m}^3

---

**Chapter 4 – Post-Cracking Behaviour of SFRSCC**

Greek Letters

\[ \delta_{av} \] Average displacement of specimen in uniaxial test \hspace{1cm} \text{mm}

\[ \varepsilon \] Strain \hspace{1cm} ---
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{pl}$</td>
<td>Plastic strain tensor</td>
</tr>
<tr>
<td>$\varepsilon_{c1}$</td>
<td>Strain at compressive strength</td>
</tr>
<tr>
<td>$\varepsilon_{c10}$</td>
<td>Strain at compressive strength of plain concrete</td>
</tr>
<tr>
<td>$\varepsilon_{el}$</td>
<td>Elastic compressive strain</td>
</tr>
<tr>
<td>$\varepsilon_{in}$</td>
<td>Compressive inelastic strain</td>
</tr>
<tr>
<td>$\varepsilon_{pl}$</td>
<td>Plastic strain in compression</td>
</tr>
<tr>
<td>$\varepsilon_{pl}$</td>
<td>Plastic strain in tension</td>
</tr>
<tr>
<td>$\epsilon_{c}$</td>
<td>Cracking strain</td>
</tr>
<tr>
<td>$\eta_\theta$</td>
<td>Fibre orientation factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle between the notched plane and the direction of the concrete flow (degree)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress (MPa)</td>
</tr>
<tr>
<td>$\sigma_{b0}$</td>
<td>Initial biaxial compressive strength (MPa)</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Compressive stress (MPa)</td>
</tr>
<tr>
<td>$\sigma_{e0}$</td>
<td>Initial biaxial tensile strength (MPa)</td>
</tr>
<tr>
<td>$\sigma_{max}$</td>
<td>Maximum tensile stress (MPa)</td>
</tr>
<tr>
<td>$\sigma_{peak}$</td>
<td>Stress at peak load (MPa)</td>
</tr>
<tr>
<td>$\sigma_{t0}$</td>
<td>Uniaxial tensile strength (MPa)</td>
</tr>
<tr>
<td>$\sigma_{t0}$</td>
<td>Tensile strength (MPa)</td>
</tr>
<tr>
<td>$\sigma_N$</td>
<td>Normal stress component (MPa)</td>
</tr>
<tr>
<td>$\sigma_{SP}$</td>
<td>Nominal tensile stress at the centre of the specimen in splitting test (MPa)</td>
</tr>
<tr>
<td>$\sigma_{0.3}$</td>
<td>Residual stresses at a crack opening width of 0.3 mm (MPa)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Residual stresses at a crack opening width of 1 mm (MPa)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Residual stresses at a crack opening width of 2 mm (MPa)</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>Effective stress (MPa)</td>
</tr>
<tr>
<td>$\hat{\sigma}_{\max}$</td>
<td>Maximum principal effective stress (MPa)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{el}$</td>
<td>Effective compressive cohesive stresses (MPa)</td>
</tr>
<tr>
<td>$\bar{\sigma}_{pl}$</td>
<td>Effective tensile cohesive stresses (MPa)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress (MPa)</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>Shear strength (MPa)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Frictional angle (degree)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Dilation angle (degree)</td>
</tr>
</tbody>
</table>

**Roman Letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_c$</td>
<td>Damage parameter in compression</td>
</tr>
<tr>
<td>$d_f$</td>
<td>Diameter of fibre (mm)</td>
</tr>
</tbody>
</table>
$d_t$ Damage parameter in tension ---
$e$ Eccentricity ---
$err$ Normalized error %
$f_{cm}$ Compressive strength MPa
$k_c$ Initial yield surface ---
$l_f$ Length of fibre mm
$\bar{p}$ Hydrostatic stress MPa
$\bar{q}$ Von Mises equivalent effective stress MPa
$w$ Crack opening mm
$w_{peak}$ Crack width corresponding to $\sigma_{peak}$ mm
$D$ Diameter of the cylinder mm
$D_0^{el}$ Initial (undamaged) elastic constitutive tensor ---
$E_{ci}$ Tangent modulus of elasticity GPa
$E_{cm}$ Concrete Young’s modulus GPa
$E_{c1}$ Secant modulus of elasticity GPa
$F$ Applied line load kN
$F_{EXP}$ Experimental load value at $i$ crack width kN
$F_{NUM}$ Numerical load value at $i$ crack width kN
$G_{r1}$ Dissipated energy up to a crack width of 1 mm N/mm
$G_{r2}$ Dissipated energy up to a crack width of 2 mm N/mm
$I_1$ Stress invariant in direction 1 MPa
$J_2$ Stress invariant in direction 2 MPa
$J_3$ Stress invariant in direction 3 MPa
$L$ Thickness of the net area in the notched plane in splitting test mm
$N^f_{eff}$ Number of effective fibres Fibre/cm$^2$
$S_1$ Principal stresses in direction 1 MPa
$S_2$ Principal stresses in direction 2 MPa
$S_3$ Principal stresses in direction 3 MPa
$\bar{S}$ Deviatoric part of the effective stress tensor $\bar{\sigma}$ MPa
$W_F$ Fibre weight percentage Kg/m$^3$

Chapter 5 – Time – Dependent Pull – Out Behaviour

Greek Letters

$\varphi^C$ Creep coefficient at creep stage ---

xiii
\( \varphi^0 \)  Creep coefficient at origin

Roman Letters

\( d_f \) Fibre’s diameter  \[ \text{mm} \]
\( l_f \) Fibre’s length  \[ \text{mm} \]
\( s \) Fibre slip  \[ \text{mm} \]
\( s_{\text{inst}} \) Instantaneous slip  \[ \text{mm} \]
\( s_{\text{lt}} \) Long-term slip  \[ \text{mm} \]
\( s_{\text{pr}} \) Fibre pre-slip level  \[ \text{mm} \]
\( s_{\text{p}} \) Slip at maximum load recorded during the test  \[ \text{mm} \]
\( s_{\text{rec}}^{\text{inst}} \) Instantaneous slip recovery at the beginning of unloading process  \[ \text{mm} \]
\( s_{\text{rec}}^{\text{lt}} \) Long-term slip recovery  \[ \text{mm} \]
\( s_{\text{rec}}^{\text{total}} \) Total slip recovery  \[ \text{mm} \]
\( s_{\text{re}}^{\text{res}} \) Residual slip after unloading stage  \[ \text{mm} \]
\( s_{\text{pr}}^{\text{res}} \) Residual slip after unloading of the specimen  \[ \text{mm} \]
\( s_{\text{lt}}^{n} \) Long-term slip at time \( t \)  \[ \text{mm} \]
\( t \) Time  \[ \text{Hours} \]
\( F \) Force  \[ \text{kN} \]
\( F_a \) Applied load  \[ \text{kN} \]
\( F_l \) Load at debonding step  \[ \text{kN} \]
\( F_{\text{pr}} \) Load and once the target slip reached  \[ \text{kN} \]
\( F_{\text{p}}^{\text{max}} \) Maximum load recorded during the test  \[ \text{kN} \]
\( K_{\text{sec,inst}} \) Instantaneous secondary stiffness at re-loading process  \[ \text{kN} / \text{mm} \]
\( K_{\text{sec,lt}} \) Long-term secondary stiffness at reloading process of post-creep test  \[ \text{kN} / \text{mm} \]
\( R_m \) Centre of curvature  \[ \text{mm} \]
\( SR_{t_2 - t_1} \) Slip rate between time \( t_2 \) to \( t_1 \)  \[ \text{m} / \text{day} \]

Chapter 6 – Time – Dependent Flexural Behaviour of Cracked SFRSCC Elements

Greek Letters

\( \alpha_i \) Normalized stress  \[ \text{---} \]
\( \beta \) Angle between concrete flow and notch direction  \[ \text{Degree} \]
\( \varepsilon_{cc} \left( t, t_0 \right) \) Creep deformation of concrete under compression  \[ \text{---} \]
\[ \varepsilon_i \] Normalized strain
\[ \varepsilon_{ult} \] Ultimate tensile strain
\[ \mu \] Perimeter of the member, mm
\[ \nu \] Poisson ratio
\[ \rho \] Concrete density, N/mm\(^3\)
\[ \sigma \] Nominal stress, N / mm\(^2\)
\[ \sigma_c \] Compressive stress, MPa
\[ \sigma_{n,lt} \] Tensile strength of concrete, MPa
\[ \varphi \] Creep coefficient
\[ \varphi(t,t_0) \] Creep coefficient at time \( t \)
\[ \varphi^c \] Creep coefficient in the creep stage
\[ \varphi^o \] Creep coefficient in origin
\[ \eta \] Fibre orientation factor
\[ \xi_{seg} \] Fibre segregation parameter

Roman Letters

\[ f_{cm} \] Compressive strength of concrete, MPa
\[ f_{ct} \] Tensile strength of concrete, MPa
\[ f_{ct,avg} \] Average tensile strength of concrete, MPa
\[ f_{rt} \] Residual flexural tensile strength, N / mm\(^2\)
\[ h \] Total depth of the specimen, mm
\[ h_0 \] Notional size of the member, mm
\[ h_{tp} \] Distance between the tip of the notch and top of the specimen, mm
\[ l_b \] Crack band width
\[ t \] Time, hour
\[ w \] Crack width, mm
\[ w_{cr} \] Pre-cracking width, mm
\[ w_{inst} \] Instantaneous crack opening, mm
\[ w_{lt} \] Long-term crack opening, mm
\[ w_{res} \] Residual crack opening after unloading stage, mm
\[ w_{rec} \] Instantaneous crack opening recovery at the beginning of unloading process, mm
\[ w_{rec}^{max} \] Crack opening width at maximum load, mm
\[ w_{rec}^t \] Long-term crack opening at time \( t \), mm
\[ w_{rec}^{lt} \] Long-term crack opening recovery, mm
\[ w_{res} \] Residual crack opening after unloading of the specimen, mm
\[ w_{rec} \] Total crack opening recovery, mm
\begin{itemize}
\item \( y \) Distance between the location of LVDT and bottom of specimen  \( \text{mm} \)
\item \( A_c \) Cross section of the member  \( \text{mm}^2 \)
\item \( \text{COR} \) Crack opening rate  \( \mu\text{m/day} \)
\item \( E_{ci} \) Initial young modulus  \( \text{GPa} \)
\item \( F \) Force  \( \text{kN} \)
\item \( F_a \) Applied load level in creep test  \( \text{kN} \)
\item \( F_{cr} \) Load at correspondent crack opening \( w_{cr} \)  \( \text{kN} \)
\item \( F_j \) Load corresponding to CMOD\(_j\)  \( \text{kN} \)
\item \( F_L \) Load at crack initiation  \( \text{kN} \)
\item \( F_{p \text{max}} \) Maximum load recorded during the test  \( \text{kN} \)
\item \( G_{f}^{I} \) Fracture energy in mode I  \( \text{N/mm} \)
\item \( K_{\text{sec}} \) Secondary stiffness  \( \text{kN/mm} \)
\item \( K_0 \) Initial stiffness of concrete  \( \text{kN/mm} \)
\item \( L \) Spam length  \( \text{mm} \)
\item \( L_s \) Length of loading spam  \( \text{mm} \)
\item \( N_{\text{eff}}^f \) The number of the effective fibres per unit area  \( \text{fibre/cm}^2 \)
\item \( N_f \) The number of the fibres per unit area  \( \text{fibre/cm}^2 \)
\end{itemize}

**Chapter 7 – Modelling Instantaneous and Long – term Flexural Behaviour of SFRSCC with Short Discrete Embedded Fibres**

Greek Letters

\begin{itemize}
\item \( \beta \) Shear retention factor  ---
\item \( \gamma_{1}^{cr} \) Crack shear strain in \( \hat{t}_1 \) direction  ---
\item \( \gamma_{2}^{cr} \) Crack shear strain in \( \hat{t}_2 \) direction  ---
\item \( \varepsilon_f \) Strain of the fibre  ---
\item \( \varepsilon_{n}^{cr} \) Crack normal strain  ---
\item \( \varepsilon_{n,\text{ult}}^{cr} \) Ultimate crack normal strain  ---
\item \( \eta_{\theta} \) Average fibre orientation factor  ---
\item \( \theta \) Fibre inclination angle  degree
\item \( \hat{n} \) Plane’s normal versor  ---
\item \( \xi_i \) Parameters that define the shape of the crack normal stress versus strain diagram  ---
\item \( \xi_x \) x coordinate of a random number  ---
\item \( \xi_y \) y coordinate of a random number  ---
\item \( \xi_z \) z coordinate of a random number  ---
\end{itemize}
\( \sigma_f \) Stress of the fibre \( \text{N} / \text{mm}^2 \)

\( \Sigma_f \) Fibre’s stress vector with two shear components \( \text{N} / \text{mm}^2 \)

\( \Delta \varepsilon \) Strain increment ---

\( \Delta \varepsilon_{cr} \) Incremental strain vector regarding to the crack ---

\( \Delta \varepsilon_{co} \) Incremental strain vector of uncracked concrete ---

\( \Delta \varepsilon_{cr} \) Incremental local crack strain vector ---

\( \Delta \sigma \) Stress increment \( \text{N} / \text{mm}^2 \)

\( \Delta \sigma_{cr} \) Incremental local stress vector \( \text{N} / \text{mm}^2 \)

\( \Delta \sigma_{cr} \) Mode I incremental crack normal stress \( \text{N} / \text{mm}^2 \)

\( \Delta \tau_{11} \) Sliding mode incremental crack shear strain in \( \hat{t}_1 \) ---

\( \Delta \tau_{22} \) Sliding mode incremental crack shear strain in \( \hat{t}_2 \) ---

\( \Delta w \) Increment in crack opening mm

Roman Letters

\( \alpha_i \) Parameters that define the shape of the crack normal stress versus strain diagram ---

\( b_{rect} \) Width of the prismatic specimen mm

\( d \) Vector with the solid element’s nodal displacements ---

\( f_{ct} \) Material tensile strength \( \text{N} / \text{mm}^2 \)

\( h_{rect} \) Height of the prismatic specimen mm

\( l_b \) Crack band width mm

\( l_{rect} \) Length of the prismatic specimen mm

\( m_f \) Mass weight of a single fibre gr

\( \hat{n}_i \) Fibre orientation versor ---

\( q^e \) Equivalent nodal force vector ---

\( s_1 \) Sliding displacement in \( \hat{t}_1 \) direction mm

\( s_2 \) Sliding displacement in \( \hat{t}_2 \) direction mm

\( v \) Poisson’s ratio ---

\( w \) Crack opening mm

\( w_{ct} \) Crack opening at tensile strength mm

\( w_{re} \) Returning crack opening mm

\( w_{un} \) Unloading crack opening mm

\( w_0 \) Plastic crack opening mm

\( A_f \) Cross section area of the fibre mm\(^2\)

\( \overline{A_f} \) Reduced shear area for circular sections mm\(^2\)
Strain-displacement matrix of a solid element

\( C_f \)  
Fibre content  
\( D \)  
Tangential constitutive matrix  
\( D^{cr} \)  
Crack constitutive matrix  
\( D^n^{cr} \)  
Crack normal constitutive matrix  
\( D^{cr}_{t_i} \)  
Crack constitutive matrix in \( \hat{t}_i \)  
\( D^{cr}_{t_2} \)  
Crack constitutive matrix in \( \hat{t}_2 \)  
\( D^{cr}_i \)  
Crack constitutive matrix of the \( i \)-th crack  
\( D^{creo} \)  
The sliding fracture modes II stiffness modulus  
\( D^{creo} \)  
The sliding fracture modes III stiffness modulus  
\( E \)  
Young’s modulus  
\( F_{\text{new}} \)  
Force on the reloading branch  
\( F_{\text{re}} \)  
Force corresponding to the returning crack opening  
\( F_{\text{un}} \)  
Unloading force on the monotonic curve at \( w_{un} \)  
\( G \)  
Fibre’s elastic shear modulus  
\( G_c \)  
Concrete elastic shear modulus  
\( G_f \)  
Mode I fracture energy  
\( J \)  
Jacobian at the sampling point of the integration scheme  
\( K^{rc} \)  
Stiffness matrix of an element  
\( K^{creco} \)  
Concrete element stiffness matrix  
\( K_c \)  
Initial stiffness of composite  
\( K_{\text{sec, re}} \)  
Secondary stiffness at reloading branch  
\( K_{\text{sec, un}} \)  
Secondary stiffness at the time of unloading  
\( K_{un} \)  
Unloading stiffness  
\( K_0 \)  
Target stiffness  
\( K^{f}_i \)  
Stiffness matrix of the \( i \)-th fibre  
\( K^{f}_0 \)  
Stiffness matrix component with the contribution of fibre’s axial  
\( N^f_T \)  
Total number of fibres  
\( N^f_{\text{vol.}} \)  
Total number of fibres  
\( T^{f}_{1} \)  
Vector corresponding to the first line of the transformation matrix  
\( T^{f}_{2} \)  
Vectors corresponding to the second line of the transformation matrix  
\( T^{f}_{3} \)  
Vectors corresponding to the third line of the transformation matrix  
\( T^{cr}_i \)  
Crack transformation matrix of the \( i \)-th crack
\( T^c \)  
Transformation matrix

\( V_v \)  
Volume of specimen  
\( \text{mm}^3 \)

\( W_a \)  
Axial contribution of the fibre reinforcement to the stiffness matrix
CONTENTS

Chapter 1 – Introduction ................................................................. 1
  1.1 Motivation .............................................................................. 1
  1.2 Amis and scopes .................................................................... 3

Chapter 2 – Literature Overview .................................................... 7
  2.1 Introduction ............................................................................ 7
  2.2 Pull-out behaviour of steel fibres .............................................. 8
    2.2.1 Pull-out mechanism of aligned fibres .................................. 9
    2.2.2 Pull-out mechanism of inclined fibres ................................. 11
    2.2.3 Influence of fibre orientation ............................................ 12
    2.2.4 Influence of fibre embedment length ................................. 14
  2.3 Post-cracking behaviour of SFRC .......................................... 15
    2.3.1 Test methods ................................................................. 19
  2.4 Long-term behaviour of fibre reinforced concrete .................... 31
    2.4.1 Micro – composite mechanical behaviour .......................... 32
    2.4.2 Macro – composite mechanical behaviour .......................... 35
  2.5 Conclusion ............................................................................. 44

Chapter 3 – Fibre Structure in the SFRSCC ................................... 47
  3.1 Introduction ............................................................................. 47
  3.2 Parameters affecting fibre dispersion and orientation ................ 48
    3.2.1 Dynamic effects ............................................................... 48
    3.2.2 Casting condition ............................................................. 51
  3.3 Fibre orientation and efficiency .............................................. 54
3.3.1 Fibre orientation factors ................................................................. 54
3.3.2 Fibre orientation profile ................................................................. 57
3.3.3 Inhomogeneity and anisometry ....................................................... 58
3.3.4 Geometrical boundary effects on fibre distribution and orientation .... 59

3.4 Parameters of the fibre structure ....................................................... 64
3.4.1 Effective fibre content ................................................................. 64
3.4.2 Minimum fibre content ................................................................. 65
3.4.3 Degree of orientation of the fibre structure ..................................... 65
3.4.4 Segregation of the fibres ............................................................... 66
3.4.5 Fibres spacing ................................................................................. 66

3.5 Methods for analysing the structure of FRC .................................... 68
3.5.1 Destructive methods ................................................................. 68
3.5.2 Non-destructive methods ............................................................. 70

3.6 Orientation and distribution measurement by image analysis .......... 72

3.7 Assessment of fibre distribution/orientation profile in SFRSCC panel .... 77
3.7.1 Procedure to assess fibre distribution in SFRSCC panel ................. 78
3.7.2 Analysis procedure ...................................................................... 78
3.7.3 Selection of the specimens ........................................................... 80
3.7.4 Experimental results ................................................................... 83
3.7.5 An analytical investigation of fibre distribution parameters ........... 90

3.8 Conclusions ...................................................................................... 94

Chapter 4 – Post-Cracking Behaviour of SFRSCC ......................... 97

4.1 Introduction ..................................................................................... 97
4.2 Experimental research ................................................................. 99
4.2.1 Concrete mixture ...................................................................... 99
4.2.2 Specimens ............................................................................... 100
4.2.3 Test set-up .............................................................................. 103
4.2.4 Analysis of results and discussion .......................................... 105
4.2.5 Correlation between the fracture and fibre distribution parameters 111

4.3 Determination of the $\sigma - w$ law by inverse analysis (IA) ......... 115
4.3.1 Numerical simulation ................................................................ 118
4.3.2 Concrete constitutive model .................................................... 119
4.3.3 Inverse analysis procedure .................................................... 126
4.3.4 Numerical results, validation and discussion ................................................. 127
4.3.5 Uniaxial tensile response vs. tensile law obtained from IA ......................... 130
4.4 Conclusions .............................................................................................................. 137

Chapter 5 – Time – Dependent Pull – Out Behaviour ............................. 141

5.1 Introduction ............................................................................................................. 141
5.2 Experimental program .......................................................................................... 143
  5.2.1 Concrete mixture .............................................................................................. 143
  5.2.2 Specimens ........................................................................................................ 144
  5.2.3 Test set-up ........................................................................................................ 146
5.3 Results and discussions ........................................................................................ 153
  5.3.1 Failure modes ................................................................................................... 153
  5.3.2 Monotonic fibre pull-out test .......................................................................... 155
  5.3.3 Long-term fibre pull-out test ........................................................................... 161
  5.3.4 Comparison between monotonic and long-term results .................................. 167
5.4 Analytical approach to predict time-dependent fibre pull-out behaviour .......... 169
  5.4.1 Long-term slip .................................................................................................. 169
  5.4.2 Creep coefficient ............................................................................................. 172
5.5 Conclusions ............................................................................................................ 174

Chapter 6 – Time – Dependent Flexural Behaviour of Cracked
SFRSCC Elements ........................................................................................................ 177

6.1 Introduction ............................................................................................................ 177
6.2 Experimental program .......................................................................................... 180
  6.2.1 Concrete mixture .............................................................................................. 180
  6.2.2 Specimens ........................................................................................................ 181
  6.2.3 Test set-up ........................................................................................................ 183
6.3 Discussion of the experimental results ................................................................. 189
  6.3.1 Monotonic four-point bending test ................................................................. 189
  6.3.2 Long-term four-point bending test ................................................................. 199
  6.3.3 Comparison between monotonic and long-term results .................................. 209
6.4 Analytical approach to predict creep behaviour of cracked SFRSCC ............. 216
6.5 Conclusions ............................................................................................................ 219
Chapter 7 – Modelling Instantaneous and Long – Term Flexural Behaviour of SFRSCC with Short Discrete Embedded Fibres

7.1 Introduction ........................................................................................................................................ 222
7.2 Fibre structure modelling ................................................................................................................... 225
  7.2.1 Algorithm ...................................................................................................................................... 225
  7.2.2 Fibre distribution ............................................................................................................................ 228
7.3 Determination of instantaneous flexural behaviour by a finite element model ......................... 230
  7.3.1 Concrete material model ................................................................................................................. 230
  7.3.2 Constitutive model for embedded cables ......................................................................................... 234
  7.3.3 Evaluation of the stiffness matrix of concrete and embedded cables ....................................... 235
  7.3.4 Numerical simulation ..................................................................................................................... 238
  7.3.5 Numerical results .......................................................................................................................... 240
7.4 Determination of long-term flexural behaviour by an analytical model ...................................... 245
  7.4.1 Description of the method and analytical simulation ................................................................. 246
  7.4.2 Result’s validation and discussion ............................................................................................... 255
7.5 Conclusions ........................................................................................................................................ 257

Chapter 8 – Conclusions .......................................................................................................................... 260

Chapter 9 – References ............................................................................................................................ 265

Appendix I – Fibre Orientation Profile .................................................................................................. 279
Appendix II – Experimental results of splitting and uniaxial tensile tests ....................................... 282
Appendix III – Experimental results of monotonic and cyclic fibre pull-out tests .......................... 289
Appendix IV – Experimental results of long-term fibre pull-out tests .............................................. 292
Appendix V – Experimental results of monotonic and cyclic fibre pull-out tests .......................... 295
Appendix VI – Experimental results of long-term bending tests ...................................................... 300
Chapter 2 – Literature Overview

Figure 2.1: Examples of fibre pull-out test configurations (V. M. C. F. Cunha, 2010)......9
Figure 2.2: Pull-out load – slip relationship of a smooth fibre. ...........................................10
Figure 2.3: Pull-out load – slip relationship of a hooked end fibre.................................11
Figure 2.4: An inclined fibre crossing a crack section......................................................12
Figure 2.5: Pull-out response of hooked steel fibres at different inclination angles
(Robins, 2002). ..................................................................................................................13
Figure 2.6: Influence of snubbing and spalling on the pull-out force for different fibre
inclination angles (Van Gysel, 2000). ..............................................................................14
Figure 2.7: Influence of bond length on the (a) peak pull-out load and (b) slip at peak
load (V. M. C. F. Cunha, 2010). ......................................................................................15
Figure 2.8: Schematic description of stress-crack width relationship for conventional
concrete and fibre reinforced concrete.................................................................16
Figure 2.9: Fracture process of conventional concrete and FRC (Löfgren, 2005).........16
Figure 2.10: (a) Fictitious crack model in steel fibre reinforced concrete, (b) examples of
σ – w relationships. ...........................................................................................................18
Figure 2.11: Scheme of distinct approaches to estimate the stress – crack width law of
SFRC.........................................................................................................................18
Figure 2.12: Underestimation of tensile strength due to the stress-gradient.................20
Figure 2.13: Calculation of crack opening w from averaged, measured displacement over
the notch (RILEM-TC162-TDF, 2001). ..............................................................................22
Figure 2.14: Specimen dimensions and loading position of three point bending test
(RILEM-TC162-TDF, 2002b). .......................................................................................24
Figure 2.15: Load – deflection diagram for the determination of the equivalent flexural
tensile strength ...............................................................................................................25
Figure 2.16: Load – CMOD diagram for or the determination of the residual flexural tensile strength.................................................................26
Figure 2.17: (a) Geometry and (b) instrumentation of the notched specimen for the bending test according to (UNI-11039, 2003). ..................................................27
Figure 2.18: (a) Stress-distribution in the loading plane, (b) force-crack opening relationship in splitting tensile test. .................................................................29
Figure 2.19: (a) Strut-and-tie model used for the study of the pre-cracking regime, (b) propagation in the post-peak regime, (c) final cracked state of the specimen and (d) approximate strut-and tie model for the final cracked state of the cylinder. .......................29
Figure 2.20: (a) Force diagram in double wedge splitting test specimen and (b) measuring devices arrangement for double wedge splitting test (di Prisco, 2013). .........................30
Figure 2.21: (a) Long-term crack opening as a function of time due to constant stress and (b) typical long-term crack opening profile for concrete under sustained stress............32
Figure 2.22: The fibre pull-out displacements over time show for (a) non-debonded and (b) debonded fibres (W. P. Boshoff, Mechtcherine, V., Zijil, G.P.A.G., 2009). ..............33
Figure 2.23: Fibre pull-out displacement versus time (Babafemi, 2014). ..............................34
Figure 2.24: Mechanisms of basic creep (Ulm, 1998): (a) short-term micro-diffusion of water between capillary pores and (b) sliding of C-S-H sheets............................................36
Figure 2.25: Uniaxial tensile creep response of: (a) steel fibre reinforced concrete (Mouton, 2012) and (b) macro-synthetic fibre reinforced concrete (Babafemi, 2014) ....39
Figure 2.26: Average time dependent strains for each loading level (W. P. Boshoff, Mechtcherine, V., Zijil, G.P.A.G., 2009). .................................................................40
Figure 2.27: Time dependent deflection of the specimens (Nakova, 2012). .........................41
Figure 2.28: (a) Creep coefficient in terms of mid-span deflection and (b) correlation between the creep coefficient and number of fibres in the crack sections (Buratti, 2012). ..............................................................................................42
Figure 2.29: Time dependent deflection development for long-time tests (Kanstad, 2012). .................................................................................................................43

Chapter 3 – Fibre structures in the SFRSCC

Figure 3.1: Effect of vibration on fibre alignment...............................................................49
Figure 3.2: Longitudinal centre sections of the prisms for each of the three tested mixtures (Stähli, 2007). ............................................................................................50
Figure 3.3: Explanation for fibre alignment in flowing concrete (Stähli, 2007)..................50
Figure 3.4: Rotation of fibres while moving from the centre of the slabs to the edges (Blanco, 2015). ..................................................................................................................................................51
Figure 3.5: Load – CMOD response from flexure tests: (a) 50 mm fibre (b) 30 mm fibre (Torrijos, 2010). ..................................................................................................................................................52
Figure 3.6: Schematic showing flow of concrete according to casting method; (a) centre; (b) perimeter; (c) random (Barnett, 2010). ..................................................................................................................................................53
Figure 3.7: 2D fibre bridging crack for stress transferring over the leading crack.............55
Figure 3.8: 3D fibre bridging crack for Stress transferring over the leading crack..........56
Figure 3.9: Determination of three zones in a prismatic specimen, where \( b, h, l_f \) and \( ND \) are the width, the height of beam, the fibre length and the notch depth: (a) cross section of a beam; (b) notched cross section ..................................................................................................................................................60
Figure 3.10: Fibre spacing: (a) 2D and (b) 3D ........................................................................67
Figure 3.11: Reinforced-concrete CT scan has been processed with VGStudio Max 2.0: (a) original; (b) CT scan; (c) extracted steel fibres; (d) slice along cylinder axis; (e) slice perpendicular to cylinder (Krause, 2010)..................................................................................................................................................69
Figure 3.12: Image analysis procedure of the concrete’s surface: (a) pre-processing, i.e. applied mask; (b) segmentation procedure; and (c) classification procedure (V. M. C. F. Cunha, 2010)..................................................................................................................................................69
Figure 3.13: View of the measurement device and probes position (Lataste, 2008)........70
Figure 3.14: Schematic of measurement directions and approximate current paths for X and Z direction (Ozyurt, 2006). ..................................................................................................................................................71
Figure 3.15: (a) Ultrasound transmission of cube with embedded fibre (adapted from Reinhardt et al. (2001)); (b) simulation of the flux density of an example frame (Faifer, 2011). ..................................................................................................................................................72
Figure 3.16: The orientation of a single fibre can be expressed in polar coordinates by the two angles \( (\theta, \varphi) \) or in Cartesian coordinates by the components of a vector \( p \), \( (p_1, p_2, p_3) \). ..................................................................................................................................................72
Figure 3.17: (a) The orientation of an ellipse in to a cross-section; (b) the probability that a fibre intersect a random plane section.........................................................75
Figure 3.18: The proposed line system for counting the number of fibres......................77
Figure 3.19: Image processing steps: (a) converting a coloured image to grayscale (b) adjusting a threshold, (c) defining mask, noise (remove small noises) and watershed
(separated fibres that are stuck together) functions, (d) fitting the best ellipse to each fibre.

Figure 3.20: Classification process: (a) initial image, (b) image after applying median filter and watershed function. ................................................................. 79

Figure 3.21: Extracted cores used for the execution of image analysis: (a) panel A, (b) panel B. .................................................................................................................. 80

Figure 3.22: Localization of the plane surface considered in the fibre distribution assessment of prismatic specimens (dimensions are in mm). ........................................... 81

Figure 3.23: Classification method of the prismatic specimens .......................................................................................................................... 82

Figure 3.24: Prismatic specimen’s used for the execution of image analysis in plan C. . 82

Figure 3.25: Localization of the plane surface considered in the fibre distribution assessment of beams (dimensions are in mm). ................................................................. 83

Figure 3.26: Digital images of fibre distribution for A6 specimens: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$ ........................................................................................................ 86

Figure 3.27: Explanation for fibre alignment in flowing concrete of a panel: (a) casting from the centre (top view), (b) through the cross section. .............................................. 88

Figure 3.28: Relationship between the number of fibres in the fracture surfaces and the distance from the centre. ......................................................................................... 89

Figure 3.29: Orientation factor, $\eta_\theta$, versus number of the effective fibres, $N_{eff}$: (a) panels A and B, (b) panel C. .......................................................................................... 89

Figure 3.30: Predicted orientation probability functions for specimens in panels A and B: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$. .............................................................................................................. 91

Figure 3.31: Predicted orientation probability functions for specimens in panel C and $\beta$ in the intervals: (a) $[0-15^\circ]$, (b) $[15-45^\circ]$, (c) $[45-75^\circ]$ and (d) $[75-90^\circ]$. ........................................... 91

Chapter 4 – Post-Cracking Behaviour of SFRSCC

Figure 4.1: Slump test of steel fibre reinforced self-compacting concrete. ................. 100

Figure 4.2: (a) The designed casting mould; (b), (c) and (d) the casting process of the panels. ...................................................................................................................... 101

Figure 4.3: Core extracting plan: (a) panel A, (b) panel B ........................................ 102

Figure 4.4: (a) Extracted cores, (b) panel after cores were extracted ...................... 102
Figure 4.5: Schematic representation of the prismatic specimen production from an extracted core (dimensions are in mm).................................103

Figure 4.6: Geometry of the specimen and set-up of the splitting tensile test (dimensions are in mm): (a) specimen front view (top of the panel), (b) specimen lateral view and (c) LVDT connection detail .................................................................104

Figure 4.7: Uniaxial tensile test set-up: (a) specimen front view, (b) specimen lateral view (units in mm), (c) LVDT connection detail. .................................................................105

Figure 4.8: Nominal tensile stress – crack opening width relationship, $\sigma - w$, obtained from splitting tensile test for: (a) $\theta=0^\circ$ and (b) $\theta=90^\circ$ .................................................................107

Figure 4.9: Nominal tensile stress – crack opening width relationship, $\sigma - w$, obtained from splitting tensile test for the top and bottom sides of the specimens: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$ .................................................................................108

Figure 4.10: Uniaxial tensile stress – crack width relationship, $\sigma - w$: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$ .................................................................................110

Figure 4.11: Uniaxial tensile post-cracking parameters versus splitting tensile post cracking parameters: (a) residual stress and (b) fracture energy. .................................................................111

Figure 4.12: Relationship between the fracture parameters and the number of effective fibres found in the notched fracture surfaces of specimens: (a) tensile residual stresses and (b) fracture absorption. .................................................................................112

Figure 4.13: Relationship between the fibre distribution, the fibre orientation factor and the post-cracking parameters: (a) peak stress, (b), (c) and (d) stress at a 0.3, 1 and 2 mm crack width, respectively; (e) and (f) energy absorption up to 1 and 2 mm crack width, respectively. .................................................................................114

Figure 4.14: Categories of uniaxial tensile stress – crack width relationships, $\sigma - w$, : (a) $\eta_\theta \geq 0.80$ and $N^{\ell}_{\text{eff}} \geq 1.20$, (b) $0.68 < \eta_\theta < 0.80$ and $0.41 < N^{\ell}_{\text{eff}} < 1.20$, (c) $\eta_\theta \leq 0.68$ and $N^{\ell}_{\text{eff}} \leq 0.41$. .................................................................................115

Figure 4.15: Three-dimensional view of numerical model: (a) geometry, constraints and prescribed displacement, (b) finite element mesh.................................................................119

Figure 4.16: Yield surface under biaxial stress used in the concrete damage plasticity model. .................................................................................120

Figure 4.17: Definition of inelastic compressive strain in the CDP model. ..................123

Figure 4.18: Strain definition after cracking-tension stiffening..................................126

Figure 4.19: Experimental and numerical force – crack width relationship, $F-w$, for: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$. .................................................................................127
Figure 4.20: Numerical uniaxial stress – crack width relationship, $\sigma - w$, obtained from inverse analysis for: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$...

Figure 4.21: Strain distribution in numerical model...

Figure 4.22: Comparison of the uniaxial stress – crack width relationship, $\sigma - w$, for $\theta = 0^\circ$...

Figure 4.23: Comparison of the stress – crack width relationship, $\sigma - w$, for $\theta = 90^\circ$...

Figure 4.24: Stress distribution in numerical modeling of $\theta = 0^\circ$ specimen: (a) horizontal direction at the time of the crack initiation, (b) vertical direction at the time of the crack initiation, (c) horizontal direction and $w = 0.5$ mm, (d) vertical direction and $w = 0.5$ mm, (e) horizontal direction and $w = 1$ mm, (f) vertical direction and $w = 1$ mm...

Figure 4.25: Stress components acting on an active fibre in the splitting tensile test...

Figure 4.26: Variation of cohesion and frictional angle by increasing fibre slip...

Figure 4.27: The influence of the normal stress on the bond shear stress-slip relationship...

Figure 4.28: Numerical splitting tensile post-cracking parameters versus experimental uniaxial tensile post-cracking parameters: (a) residual stress and (b) fracture energy...

Chapter 5 – Time – Dependent Pull – Out Behaviour

Figure 5.1: Production of the specimens for fibre pull-out test: (a) casting device, (b) fibres installation in mould, (c) casting panel and (d) extracted pull-out specimen...

Figure 5.2: Preparation of the specimens for pullout test...

Figure 5.3: Configuration of single fibre pull-out test: (a) details of the test, (b) connection details of the LVDTs, (c) a fibre being pulled out and (d) comparison between $F - s$ curves measured by LVDT and microscope...

Figure 5.4: Response obtained from the instantaneous pull-out test with one cycle of unloading and re-loading...

Figure 5.5: Scheme presentation of the fibre pull-out pre-slip test...

Figure 5.6: Fibre pull-out creep test: (a) general view, (b) loading details, (c) specimen installation details and (d) fibre close-up...

Figure 5.7: Definition of the slip parameters in pull-out creep response: (a) slip-time relationship, $s - t$, (b) force-slip, $F - s$...

Figure 5.8: Scheme representation of the post-creep test...

Figure 5.9: Representation of the assembled long-term force-slip curve.
Figure 5.10: Typical failure modes of pull-out test: fibres with orientation angle of (a) 0°, (b) 30°, (c) and (d) 60° ................................................................. 154

Figure 5.11: Pull-out mechanism of an inclined fibre ......................................................... 155

Figure 5.12: Average monotonic pull-out load-slip relationships for fibre inclination angles of: (a) 0°, (b) 30° and (c) 60° ................................................................. 157

Figure 5.13: Pictures of pull-out process of specimens with 60° fibre inclination angle: (a), (b) and (c) correspondent to points A, B and C in Figure 5.12(c), respectively. ..... 157

Figure 5.14: The effect of fibre inclination angle on: (a) pull-out peak load and (b) slip at peak load. ......................................................................................... 158

Figure 5.15: Contribution of the end hook to the overall pull-out behaviour in fibres with inclination angle of 0° ................................................................. 159

Figure 5.16: Pulled-out fibres: (a) hooked end type and (b) smooth type. ......................... 159

Figure 5.17: The average cyclic pull-out load – slip relationships for fibres with inclination angle of 0°: (a) \( s_{pr} = 0.3 \) mm and (b) \( s_{pr} = 0.5 \) mm ................................................................. 160

Figure 5.18: Development of long-term slip along time during fibre pull-out creep test: (a) and (b) 0 degree, (c) and (d) 30 degree, (e) and (f) 60 degree; (a), (c), (e) \( s_{pr} = 0.3 \) mm and (b), (d), (f) \( s_{pr} = 0.5 \) mm ................................................................. 162

Figure 5.19: Development of long-term slip along time for different fibre orientations: (a) \( s_{pr} = 0.3 \) mm and (b) \( s_{pr} = 0.5 \) mm ................................................................. 164

Figure 5.20: Influence of fibre orientation angle on the long-term slip: \( s_{pr} = 0.3 \) mm and (b) \( s_{pr} = 0.5 \) mm ................................................................. 165

Figure 5.21: Influence of pre-slip level on the development of long-term slip along time. ................................................................................................. 167

Figure 5.22: Comparison of monotonic and long-term assembled curves: (a) and (b) 0 degree, (c) and (d) 30 degree, (e) and (f) 60 degree; (a), (c), (e) \( s_{pr} = 0.3 \) mm and (b), (d), (f) \( s_{pr} = 0.5 \) mm ................................................................. 168

Figure 5.23: Long-term assembled curve for a specimen with 30 degree fibre inclination angle ................................................................................................. 168

Figure 5.24: Comparison between experimental and analytical long-term slip vs. time relationship for series with fibre inclination angle: (a) 0°, (b) 30° and (c) 60° .......... 171

Figure 5.25: Influence of \( F_a/F_{pr} \) load level on the long-term slip vs. time response for \( s_{pr} \) equal to: (a), (c), (d) 0.3 mm and (b), (e) 0.5 mm; fibre inclination angle of: (a), (b) 0° and (c) 30° and (d), (e) 60° ................................................................. 172
Chapter 6 – Time – Dependent Flexural Behaviour of Cracked SFRSCC Elements

**Figure 6.1:** (a) Specimen’s extracting plane, (b) definition of $\beta$ angle. ........................................ 182

**Figure 6.2:** Test set-up of monotonic four-point bending test (dimensions are in mm): (a) geometry of the specimen and LVDT for measuring the deflection (b) LVDTs to record CTOD, (c) LVDT connection details for measuring CTOD. ........................................ 184

**Figure 6.3:** Scheme representation of cyclic test ........................................ 184

**Figure 6.4:** Scheme representation of the cracking test ........................................ 185

**Figure 6.5:** Creep test set-up: (a) general view, (b) position and connection details of the LVDT. ........................................ 186

**Figure 6.6:** Definition of crack opening parameters in creep test: (a) crack opening-time relationship, $w-t$, (b) force-crack opening curve, $F-w$. ........................................ 187

**Figure 6.7:** Scheme representation of the post-creep test. ........................................ 188

**Figure 6.8:** A graphical representation of assembled long-term force-crack width curve. ........................................ 189

**Figure 6.9:** Monotonic force-crack tip opening displacement relationship for $\beta$ in the intervals: (a) $[0-15^\circ]$, (b) $[15-45^\circ]$, (c) $[45-75^\circ]$ and (d) $[75-90^\circ]$. ........................................ 190

**Figure 6.10:** Comparison of monotonic $F$-CTOD relationship for different series. .... 192

**Figure 6.11:** Relation between the flexural residual strength and the number of effective fibres: (a) $f_{r1}$, (b) $f_{r2}$, (c) $f_{r3}$ and (d) $f_{r4}$. ........................................ 195

**Figure 6.12:** Finite element mesh used in the simulation of four-point bending test. ..... 196

**Figure 6.13:** Quadrilinear stress – strain relationship for modeling fracture mode I ($\sigma^{cr}_{n,2} = \alpha \sigma^{cr}_{n,1}, \sigma^{cr}_{n,3} = \alpha \sigma^{cr}_{n,1}, \sigma^{cr}_{n,4} = \alpha \sigma^{cr}_{n,1}, \varepsilon^{cr}_{n,2} = \xi \varepsilon^{cr}_{n,1}, \varepsilon^{cr}_{n,3} = \xi \varepsilon^{cr}_{n,1}, \varepsilon^{cr}_{n,4} = \xi \varepsilon^{cr}_{n,1}$) ........................................ 196

**Figure 6.14:** Experimental and numerical force – crack tip opening displacement relationship, $F$-CTOD, for $\beta$ in the intervals: (a) $[0-15^\circ]$, (b) $[15-45^\circ]$, (c) $[45-75^\circ]$ and (d) $[75-90^\circ]$. ........................................ 198

**Figure 6.15:** Stress – strain relationships obtained by inverse analysis. ........................................ 199

**Figure 6.16:** Crack opening rate as a function of $F_a / F_{cr}$: (a) specimens pre-cracked up to 0.3 mm, $w_{cr}=0.3$ mm, (b) specimens pre-cracked up to 0.5 mm, $w_{cr}=0.5$ mm. ........................................ 200
Figure 6.17: Relationship between creep coefficient in creep stage and $F_a / F_{cr}$: (a) $w_{cr}=0.3$ mm, (b) $w_{cr}=0.5$ mm.

Figure 6.18: Relationship between creep coefficient at the origin and $F_a / F_{cr}$: (a) $w_{cr}=0.3$ mm, (b) $w_{cr}=0.5$ mm.

Figure 6.19: Creep coefficient versus time in the creep tests for the two pre-crack width levels grouped in low and high $F_a / F_{cr}$ ratio.

Figure 6.20: Experimental and numerical crack patterns in the loading span of monotonic four-point bending test simulations for the $\beta=[0-15^\circ]$ series at CTOD of: (a), (b) 0.3 mm and (c), (d) 0.5 mm.

Figure 6.21: Instantaneous crack opening, $w_{inst}$, vs. distance from the casting point: $F_a / F_{cr}$ = (a) low and (b) high.

Figure 6.22: Long-term residual crack opening, $w_{res}$, vs. distance from the casting point: $F_a / F_{cr}$ = (a) low and (b) high.

Figure 6.23: Creep coefficient versus time for specimens at different distances from casting point in the series of: (a) $w_{cr}=0.3$ mm and (b) $w_{cr}=0.5$ mm.

Figure 6.24: Creep coefficient versus time for different orientation of the notched plane in the series of: $w_{cr}=(a) 0.3$ mm and (b) $0.5$ mm.

Figure 6.25: Comparison of the monotonic and long-term assembled curves for $w_{cr}=0.3$ mm: (a), (b) $\beta = [0-15^\circ]$; (c), (d) $\beta = [15-45^\circ]$; (e), (f) $\beta = [45-75^\circ]$; (g), (h) $\beta = [75-90^\circ]$.

Figure 6.26: Comparison of the monotonic and long-term assembled curves for $w_{cr}=0.5$ mm: (a), (b) $\beta = [0-15^\circ]$; (c), (d) $\beta = [15-45^\circ]$; (e), (f) $\beta = [45-75^\circ]$; (g), (h) $\beta = [75-90^\circ]$.

Figure 6.27: Relationship between secondary stiffness, $K_{sec}$, and: (a) long-term residual crack opening width, $w_{cr}^{res}$, (b) ratio of the applied load, $F_a / F_{cr}$.

Figure 6.28: Influence of long-term residual crack opening on the secondary stiffness: (a), (b) $\beta = [0-15^\circ]$; (c), (d) $\beta = [15-45^\circ]$; (e), (f) $\beta = [45-75^\circ]$; (g), (h) $\beta = [75-90^\circ]$.

Figure 6.29: Comparison between experimental and analytical creep coefficient vs. time relationship for series: (a) $w_{cr}=0.3$ mm and (b) $w_{cr}=0.5$ mm.

Figure 6.30: Influence of $F_a / F_{cr}$ load level on the creep coefficient vs. time response for $w_{cr}$ equal to: (a) 0.1 mm, (b) 0.2 mm, (c) 0.3 mm and (d) 0.5 mm.
**Figure 6.31**: Influence of $F_a / F_{cr}$ load level on the long-term crack opening vs. time response for $w_{cr}$ equal to: (a) 0.3 mm and (b) 0.5 mm. ................................................................. 219

**Chapter 7 – Modelling Instantaneous and Long-Term Flexural Behaviour of SFRSCC with Short Discrete Embedded Fibres**

**Figure 7.1**: Distribution of the steel fibres in the specimen: (a) before and (b) after eliminating violate fibres, respectively. ................................................................. 227

**Figure 7.2**: A methodology to distribute steel fibres in a prismatic specimen (the lateral face of the specimen coincides with the bottom surface of the SFRSCC panel in contact with the mould): $\beta =$ (a) [0-15°], (b) [15-45°], (c) [45-75°] and (d) [75-90°]. ............... 229

**Figure 7.3**: Crack stress components, displacements and local coordinate system of a crack................................................................. 232

**Figure 7.4**: Stress-strain relation based on the Tri-linear law. ............................ 234

**Figure 7.5**: Determination of the embedded cable’s stress-strain diagram based on the experimental pull-out force – slip relation................................................................. 235

**Figure 7.6**: Three-dimensional scheme of the embedded fibre intersecting an active crack ($n$ is the vector normal to the crack plane). ................................................................. 235

**Figure 7.7**: Three-dimensional mesh of bulk concrete. ................................. 239

**Figure 7.8**: Comparison between the experimental $F$ – CTOD relationship and numerical simulation for $\beta =$ (a) [0-15°], (b) [15-45°], (c) [45-75°] and (d) [75-90°]. ............... 241

**Figure 7.9**: (a) and (b) Fibres at the crack plane for $\beta=$[0-15°] and [15-45°] series, respectively, (the gray area shows the notch location); (c) and (d) normal stresses for a CTOD correspondent to maximum load at post-crack branch; (e) and (f) normal stresses for a CTOD=2.5 and $\beta=$[0-15°], [15-45°].................................................. 243

**Figure 7.10**: (a) and (b) Fibres at the crack plane for $\beta=$[45-75°] and [75-90°] series, respectively (the gray area shows the notch location); (c) and (d) normal stresses for a CTOD correspondent to maximum post-cracking load for $\beta=$[45-75°] and [75-90°] series. ................................................................................................. 245

**Figure 7.11**: Scheme representation of unloading curve................................................. 247

**Figure 7.12**: The influence of number of the effective fibres on the unloading stiffness: (a) $K_0$ and (b) $K_{sec}$................................................................. 249

**Figure 7.13**: Schematic illustration of completed unloading and reloading curves....... 250
Figure 7.14: Comparison between the experimental average $F$ – CTOD relationship and analytical approach for the specimens with one cycle of unloading and reloading at CTOD=0.3 mm: $\beta= (a) [0-15^\circ]$, (b) [15-45^\circ], (c) [45-75^\circ] and (d) [75-90^\circ]. ..........................252

Figure 7.15: Comparison between the experimental average $F$ – CTOD relationship and analytical approach for the specimens with one cycle of unloading and reloading at CTOD=0.5 mm: $\beta= (a) [0-15^\circ]$, (b) [15-45^\circ], (c) [45-75^\circ] and (d) [75-90^\circ]. ..........................252

Figure 7.16: Flowchart of the algorithm adopted in the developed model. ..........................254

Figure 7.17: Comparison between the experimental bending and analytical (obtained from the fibre pull-out creep test) creep coefficient vs. time relationship for the specimens with $w_{cr}=0.3$ mm, $F_a / F_{cr} =100\%$ and $\beta= (a) [0-15^\circ]$, (b) [15-45^\circ], [45-75^\circ] and [75-90^\circ]. ...........................................................................................................256

Figure 7.18: Comparison between the experimental bending and analytical (obtained from the fibre pull-out creep test) creep coefficient vs. time relationship for the specimens with $w_{cr}=0.5$ mm, $F_a / F_{cr} =100\%$ and $\beta= (a) [0-15^\circ]$, (b) [15-45^\circ], [45-75^\circ] and [75-90^\circ]. ...........................................................................................................257
Table 2.1: Geometry of the fibres used (Buratti and Mazzotti, 2012)..............................42
Table 3.1: The influence of casting method on the fibre orientation and flexural strength (Barnett et al., 2010).........................................................................................................................53
Table 3.2: Increments of η for any η1 and common cross section geometries..............63
Table 3.3: Calculation of global fibre orientation angles.................................................74
Table 3.4: Fibre distribution parameters for the specimens after uniaxial tension test (panels A and B). ..........................................................................................................................85
Table 3.5: Fibre distribution parameters for the specimens after four-point bending test (panel C). .........................................................................................................................87
Table 3.6: Comparison between the experimental and analytical fibre orientation factors. ..........................................................................................................................93
Table 3.7: Comparison between the experimental and analytical number of fibres [fibres/cm²]. .........................................................................................................................94
Table 4.1: Mix proportions of steel fibre reinforced self-compacting concrete per m³ ...99
Table 4.2: Residual stress and toughness parameters obtained from splitting and direct tensile tests. ..................................................................................................................106
Table 4.3: The constitutive parameters of CDP model. ......................................................122
Table 4.4: Mechanical properties adopted in the numerical simulations......................124
Table 4.5: Comparison of the FE analysis and experimental results..............................128
Table 4.6: Residual stress and toughness parameters obtained from different analysis..136
Table 5.1: Mix design of steel fibre reinforced self-compacting concrete per m³ ............144
Table 5.2: Average slip rate for each pre-slip level (numbers in parenthesis present CoV in %)..........................................................................................................................161
Table 5.3: Average creep coefficient at creep stage for each pre-slip level (numbers in parenthesis show CoV in %). .......................................................... 163

Table 5.4: Average creep coefficient at origin for each pre-slip level (numbers in parenthesis show CoV in %). .......................................................... 164

Table 5.5: The average applied load in creep test ($F_a$) and maximum load recorded during the pull-out test ($F_{max}$) for each series of specimens. ................................................................. 166

Table 6.1: Mix design of steel fibre reinforced self-compacting concrete per m$^3$. ........ 180

Table 6.2: Average and characteristic results of the monotonic bending test. .............. 193

Table 6.3: Mechanical properties adopted in the numerical simulation. ............................. 197

Table 6.4: Values of the fracture parameters defining stress – strain softening laws. .... 199

Table 7.1: Comparison between image analysis result and numerical fibre distribution. .......................................................................................................................... 229

Table 7.2: Plain concrete properties used in the numerical simulation. ......................... 239

Table 7.3: Tri-linear stress-strain relationships used for simulating the fibres’ bond-slip behaviour. .......................................................................................................................... 240
CHAPTER 1

Introduction

1.1 Motivation

Concrete is the most widely used construction material in the world. Although it was primarily conceived more than two thousand years ago and played a great role in the impressive development of Roman infrastructure, its application in modern time just dates back to the middle of the nineteenth century (Roth, 2012) and was often the protagonist of the most significant achievements in civil engineering and architecture (Collard-Wexler, 2013). In spite of this long running progress, novel concrete materials have been recently designed in research laboratories (Mehta and Monteiro, 2006) and brought to the market by innovation-driven companies with the aim of responding to the new challenges of the modern construction industry (Rezgui, 2010).

Fibre reinforced concrete (FRC) is a novel type of concrete containing fibrous material which increases its structural integrity. It contains short discrete fibres that are distributed and oriented randomly. Fibre reinforcement can consider as an alternative to conventional steel bars in order to improve the efficiency and working conditions on the construction sites and particularly in the prefabricated elements. Although, in the last decades, the fibres were mainly applied in the non-structural elements or even to control early thermal contraction cracking and plastic or drying shrinkage, but they can be also
used to reduce or even replace the ordinary steel reinforcements in the structural elements. In this case, for concrete structures, the labour cost reduces roughly since the main part related to the reinforcement work decreases if ordinary steel bars replaced by the short discrete steel fibres. In some type of structures with relatively low reliability for the safety of the structure such as grade slabs, foundations and walls, the fibres can be considered as an alternative in order to replace the ordinary bars completely. However, in the load carrying structural elements, discrete fibres can be used with a combination of the conventional or prestressed reinforcements.

Addition of fibres to a cementicious matrix contributes more to the energy absorption and crack control of structural elements rather than enhancing load transfer capacity (Lataste et al., 2008). Crack opening in concrete is assumed counteracted by the bond stresses that develop at the fibres / matrix interface. On the other hand, one of the most important properties of fibre reinforced concrete is its ability to transfer stresses across a cracked section rather uniformly, which is dependent on the fibre reinforcement effectiveness, i.e. fibre properties (their strength, bond, stiffness), fibre orientation and distribution (Vandewalle and Dupont, 2003). The stress transfer capability enhances mainly the composite’s toughness, which is a parameter for measuring the energy absorption during monotonic or cyclic loading (RILEM-TC162-TDF, 2002b). However, variation of fibre distribution in large scale elements may result in significant inconsistency in mechanical behaviours within distinct sections. For example, concrete presents a higher post-cracking behaviour if fibres orient according to the direction of the applied stress properly. One can prove that only fibres are active in concrete which are parallel to the direction of the principal applied stress (Lee and Barr, 2002). Therefore, these effects should be considered for structural design especially where variation of fibre distribution and orientation in a section results in affecting mechanical properties significantly.

The dispersion and orientation of fibres in the hardened-state results from a series of stages that steel fibre reinforced concrete (SFRC) passes from mixing to hardening state, namely (Laranjeira, 2010): fresh-state properties after mixing; casting conditions into the formwork; flowability characteristics; vibration and wall-effect introduced by the formwork. Having in mind that mechanical properties are significantly related to the fibre orientation and dispersion, which are affected by concrete’s flow in the fresh state, it is
important to control those parameters properly (Ferrara and Meda, 2006; Kim et al., 2008; Pansuk et al., 2008).

Another important benefit of SFRC is the durability improvement. In general, the creep deformation of the material could lead to the failure mechanism of the structural element at a load lower than static ultimate load (Boshoff et al., 2009a). This makes creep deformation an important factor for consideration from the medium to long-term. Addition of the steel fibres to concrete could limit the long-term crack widening in a cracked section since fibres provide a resistance to the time-dependent sliding action (Tan et al., 1994; Tan and Saha, 2005). The time-dependent deformation of a cracked element under flexure can be due to the following effects: drying shrinkage of the compression zone, creep in the compression zone or uncracked section which produce basic creep, time-dependent bond failure and bond creep strains between concrete and fibres crossing the crack and finally creep in the fibre material.

Steel fibre reinforced self-compacting concrete (SFRSCC) may exhibit a relatively high deformability due to long-term loads. In some structural systems, the long-term deformation of a structural element can be beneficial, since it enforces stresses to redistribute, which can limit the crack propagation. From another point of view, if the creep deformation damages significantly affect the fibre/matrix interface bond, it will lead to an undesirable excessive decrease on the post-cracking strength, thus the influence of creep will be adverse (Arango et al., 2012). Despite being available some standards for designing SFRC structures (UNI-11039, 2003; CEB-FIP, 2010), it seems that they still not take into account the long-term behaviour under cracked conditions. Therefore, information regarding the long-term behaviour of cracked SFRSCC elements, particularly planar structures, is still limited. Consequently, understanding the behaviour of cracked SFRSCC elements under a sustained load will help towards a more rational design and accurate prediction of the composite behaviour under serviceability conditions.

1.2 Amis and scopes

The main purpose of this work is to achieve, as much as possible, a consistent comprehension of the behaviour of SFRSCC applied in the laminar structures not only
under monotonic loading condition but also when a cracked SFRSCC subjected to a sustained load. Therefore the following tasks were addressed in this thesis:

1. A literature review regarding to the relevant subjects in the thesis was performed from the micro-level to macro-composite mechanical behaviour. The first part was focused on the monotonic loading condition, whereas in the second part, the behaviour of the specimen was described under a sustained load. This section was depicted in Chapter 2.

2. A survey regarding to the fibre distribution structures at meso-level, parameters that defines fibre distribution and factors that influence it, was described. Then, in order to comprehend how fibres were distributed and oriented in a SFRSCC laminar structure, the fibre distribution parameters were ascertained in terms of fibre density, orientation parameter and segregation factor. This information is included in Chapter 3.

3. The influence of the fibre distribution and orientation on the post-cracking behaviour of a SFRSCC laminar structure was studied. The post-cracking behaviour was assessed by both splitting tensile tests and uniaxial tensile tests. Then, mode I fracture parameters of SFRSCC were derived from a comprehensive nonlinear 3D finite element modeling of indirect splitting tensile tests. The combined experimental and numerical research allowed a comparison between the stress-crack width ($\sigma - w$) relationship acquired straightforwardly from direct tensile tests, and the $\sigma - w$ response derived from inverse analysis of the splitting tensile tests results. In Chapter 4 were described these results.

4. Effectiveness of a fibre as an element for transferring stresses under monotonic and sustained loads across cracks was assessed. Firstly, single fibre pull-out creep tests were performed, where fibre slip was monitored as a function of the time. The influence of fibre orientation angle as well as pre-imposed fibre slip level on the creep response was investigated. After the end of the creep pull-out tests, the specimens were then subjected to monotonic fibre pull-out tests. Secondly, a series of monotonic fibre pull-out tests were also carried out in order to quantify the effects of the pull-out creep behaviour. Finally, the monotonic force-slip curve was then compared with the correspondent assembled creep force-slip curve. This section was included in Chapter 5.
5. An extensive experimental program that aims to study the long-term behaviour of cracked SFRSCC applied in laminar structures was performed. In a first stage, the influence of the initial crack opening level, applied stress level, fibre orientation/dispersion and distance from the casting point, on the flexural creep behaviour of SFRSCC was investigated. Moreover, in order to evaluate the effects of the creep phenomenon on the residual flexural strengths, a series of monotonic tests were also executed. In Chapter 6 were presented the relevant results.

6. In the first phase, three-dimensional numerical simulations of monotonic four-point flexural tests of SFRSCC specimens were carried out. In this simulation, SFRSCC was assumed as a two phase material, i.e. plain concrete and discrete steel fibres. The nonlinear material behaviour of SCC matrix was simulated using 3D smeared crack model, while the fibre reinforcement mechanisms were modelled using the micro-mechanical behaviour laws determined from experimental fibre pull-out tests. Then, in the second part, an analytical approach was developed based on the cross sectional analysis in order to determine flexural long-term behaviour of the specimens considering long-term fibre/matrix interface laws achieved from experimental fibre pull-out creep tests. This section was included in Chapter 7.
Literature Overview

2.1 Introduction

Concrete is the most widely used construction material in the world. Although it was primarily conceived more than two thousand years ago and played a great role in the impressive development of Roman infrastructure, its application in modern time just dates back to the middle of the nineteenth century (Kirby and Laurson, 1932) and was often the protagonist of the most significant achievements in civil engineering and architecture (Roth, 2012; Collard-Wexler, 2013). In spite of this long running progress, novel concrete materials have been recently designed in research laboratories (Mehta and Monteiro, 2006) and brought to the market by innovation-driven companies with the aim of responding to the new challenges of the modern construction industry (Rezgui, 2010).

Fibre reinforced concrete (FRC) is a novel type of concrete containing fibrous material which increases its structural integrity. It contains short discrete fibres that are distributed and oriented randomly. Fibre reinforcement can consider as an alternative to conventional steel bars in order to improve the efficiency and working conditions on the construction sites and particularly in the prefabricated elements. Although, in the last decades, the fibres were mainly applied in the non-structural element or even to control early thermal contraction cracking and plastic or drying shrinkage, but they can be also
used to reduce or even replace the ordinary steel reinforcements in the structural elements. In this case, for concrete structures, the labour cost reduces roughly since the main part related to the reinforcement work decreases if ordinary steel bars replaced by the short discrete steel fibres. In some type of structures with relatively low reliability for the safety of the structure such as grade slabs, foundations and walls, the fibres can be considered as an alternative in order to replace the ordinary bars completely. However, in the load carrying structural elements, discrete fibres can be used with a combination of the conventional or prestressed reinforcements.

A wide range of the fibres are available in market which made of various materials and geometries. Based on their applications, some of them are used in order to improve toughness and reduce crack widths, while other can be applied to control plastic shrinkage cracking or to reduce spalling of concrete in high temperature (Löfgren, 2005). Considering the size of the fibres, they are divided to the micro or macro scale fibres. In general, the micro fibres are used to bridge the micro-cracks in concrete whereas the macro ones are used to control the widen cracks. Macro fibres are often characterised by their aspect ratio, i.e the length to the diameter ratio.

2.2 Pull-out behaviour of steel fibres

Adding steel fibres to cementitious matrices increases ductility and also enhances the crack width control. These beneficial effects arise due to the crack bridging capacity provided by fibres. This is achieved through the development of bond-slip mechanism with the surrounding matrix since it is recognized as an important factor in composite action (Naaman and Najm, 1991). Hence, in order to predict the behaviour of fibre reinforced composites, it is important to understand the fibre-matrix interface (Li and Stang, 1997). However, although extensive researches were carried out in this research area, but still there is no standard method to determine bond in fibre reinforced composite materials (Guerrero and Naaman, 2000). In general, this test can be executed in a single fibre level or in a group of fibres. In the former test configuration, the contribution of a single fibre is determined, while in the latter one the load-slip relationship of multiple fibres is measured. However, execution of fibre pull-out test in single fibre level is more preferable since it is easier and results with a reasonable accuracy can be produced. This test has been performed by means of several test configurations as depicted in Figure 2.1.
2.2.1 Pull-out mechanism of aligned fibres

The pull-out process is simplified divided into different stages depending on the fibre geometry. In the case of the smooth fibres, the pull-out mechanism consists of two stages: debonding of surrounding interface and frictional slip of the fibre. These mechanisms are detailed in Figure 2.2. In the first phase, the pull-out load increases almost linearly with the slip. On the other hand, this linear ascending part is associated with the elastic behaviour of bond. In the second phase of pre-peak branch, a nonlinear response is observed which is correspondent to the initiation of debonding process where micro-cracks start to be formed in the interfacial zone of fibre-matrix. After the peak load is attained, the load sharply decreases with increases of slip which corresponds to unstable interfacial crack growth on the post-peak behaviour and full debonding occurs, see phase 3 in Figure 2.2. Subsequently, in the last phase, the fibre pulls-out under frictional slip which coincides with reduction in the pull-out load progressively. In this stage, the pull-out load decreases gradually with the increase of slip since the available frictional area and also the roughness of the failure surface decrease. In this type of fibres, if the pullout load attains the force corresponding to the fibre tensile strength without being exceeded the bond strength, it will be observed the fibre rupturing. However, since the material in the interface zone is less hard than surrounding matrix due to the higher porosity, and also considering the smooth surface of the steel fibres, a smooth fibre normally has a low probability to be ruptured.
Similar to the smooth fibres, the pull-out procedure of hooked end fibres also consists of two stages: debonding of surrounding interface and frictional slip of the fibre. In fact, the main difference is correspondent to the frictional pull-out component. On the other hand, in the hooked end fibres frictional pull-out is also accompanied by a mechanical bond mechanism correspondent to the plastic deformation of the hook. Figure 2.3 shows a scheme presentation of pull-out load – slip relationship of a hooked end fibre. Similar to the smooth fibres, phases 1 and 2 correspondent to elastic deformation of the bond and debonding of adhesive, respectively. However, unlike smooth fibres, after phase 2, the pull-out load still follows an ascending trend and increases, see phase 3 in Figure 2.3. This is associated to a combination mechanism of debonding of adhesive and also mechanical anchorage provided by the fibre hook. After the peak load is attained, both curvatures of the hooked part are deformed progressively and thus, the pull-out load starts to decrease (phase 4). This phase is follows by a smooth branch which coincide with the last fibre curvature passing the last corner of the fibre imprint made in the matrix. Finally, after the hook is fully straightened, the pullout process occurs under frictional resistance (phase 6) similar to the smooth fibres.

Figure 2.2: Pull-out load – slip relationship of a smooth fibre.
2.2.2 Pull-out mechanism of inclined fibres

Regarding to the pull-out procedure of an inclined fibre, debonding of the fibre-matrix interface and also sliding of the fibre in the debonded interface are also observed. However, in the inclined fibres, other additional mechanisms are also included: bending of the fibre between the two laps of the crack surface and also spalling of the matrix at fibre exit point due to the bending of the fibre. It was shown that the toughness of hooked end and smooth fibres maximizes at a non-zero inclination angle, mainly between 15 – 45 degrees (Robins et al., 2002). This is principally due to the snubbing effect occurs in the higher pull-out load (Dupont, 2003). This effect force the segment of the fibre which is bridging an existing crack to remain perpendicular to the crack plane when the crack opens (Stang and Li, 2005). At this time, the fibre is bent and the stress concentrated at the point where the fibre is bent. This stress concentration may cause local concrete spalling since the concrete between the fibre and crack plane is crushed or pulled off, see Figure 2.4. Therefore, when the inclination of the fibre towards the crack plane increases, the pull-out resistance is less influenced by the matrix strength and mainly influenced by the fibre strength (Brandt, 1985) because the snubbing effect leads to the higher probability of fibre rupturing. From another point of view, by increasing the inclination of the fibre, the probability of matrix spalling increases, hence the matrix strength influences the pull-out resistance at higher angles. Furthermore, it was also shown that the scattering in the experimental results acquired from the single fibre pull-out test increase at higher
fibre inclination angles (Li et al., 2006) due to the variation of matrix stiffness and whether or not there are aggregates at the fibre exit point which influence spalling of the matrix at the crack surface (Van Gysel, 2000).

![Image](image_url)

*Figure 2.4: An inclined fibre crossing a crack section.*

### 2.2.3 Influence of fibre orientation

In fibre reinforced concrete, fibres are distributed and also oriented randomly in the bulk material. This is a consequence of the production technology of FRC. Therefore, it is logical to assume that fibres crossing a crack section with various orientations. More details regarding to this aspect will be discussed in chapter 3.

An extensive experimental research was performed on the pull-out behaviour of inclined hooked end fibres. These works investigated the influence of the fibre orientation angle on the pull-out response (Banthia and Teriottier, 1994; Armelin and Banthia, 1997; Van Gysel, 2000; Robins et al., 2002; Cunha, 2010). In some other researches, different matrix strength (Banthia and Teriottier, 1994; Van Gysel, 2000; Robins et al., 2002) as well as fibre yield strength (Van Gysel, 2000) were also applied. Figure 2.5 presents the pull-out response of hooked steel fibres at different inclination angles. From the results is was shown that, up to the 10°, the maximum pull-out load tended to increase. However, after 10° inclination angle it started to reduce slightly. Considering slip at the peak load, it always increased when the inclination angle enhanced. Regarding to failure mechanism, it
was observed that up to the 10° fibre orientation angle, the fibres were completely pulled-out (the mechanical anchorage of fibre was mobilized) while in the higher inclination angles the fibre rupturing occurred. Van Gysel (2000) concluded that up to 10° inclination angle, due to the snubbing effects, the maximum pull-out load increased. After this orientation angle, the peak load reduced and also the failure mode was changed to fibre rupturing due to the matrix spalling, see Figure 2.6. As it was discussed in the previous section, in the higher inclination angle, a relatively higher stress concentration at the fibre bending point happens which lead to the concrete spalling at fibre exit point and finally the fibre will rupture. Furthermore, Naaman and Shah (1976) and Banthia and Teriottier (1994) illustrated that the maximum pull-out load may increase with the fibre inclination up to an angle of 45°. However, it should be mentioned that this scattering in the results depends on many parameters such as: concrete matrix strength, fibre embedment length and fibre cross section dimensions, fibre strength, dimensions of the fibre mechanical anchorage, etc.

![Figure 2.5: Pull-out response of hooked steel fibres at different inclination angles (Robins et al., 2002).](image)

In conclusion, generally, the pull-out peak load and toughness are maximized for inclination angles between 0° to 20°. Higher orientation angles leads to a reduction in the peak load and toughness which may be justified by the fact that for higher inclination angles the fibre generally fractures in an earlier stage.
2.2.4 Influence of fibre embedment length

A fibre can be considered as effective if the hook becomes fully mobilised. However, it will not happen if the embedment length is too short. It is shown that if the embedment is less than the length of the mechanical anchorage, the fibre is not fully mobilised (Robins et al., 2002). It is expected that by increasing the fibre embedment length, the pull-out response is improved since a larger fibre area is in contact with concrete matrix, particularly in the case of smooth fibres. However for the hooked end fibres, the maximum pull-out load may not influence by the embedment length significantly. This may be justified by the fact that, in the hooked end fibres, the influence of the mechanical anchorage plastification is more significant than the loose of adherence between fibre and matrix component which is affected by the fibre embedment length (Naaman and Najm, 1991; Gräunewald, 2004). This fact was studied by Cunha (2010) experimentally. Figure 2.7 illustrates the influence of fibre embedment length on the peak pull-out load and slip at peak load for smooth and hooked end fibres in different orientation angles. It is shown that the higher bond length only led to an important increase of the peak load for the aligned smooth fibre while in the other cases no significant improvement was found. Moreover, for the smooth and hooked end aligned fibres a slight increase of slip at peak load was achieved, whereas for the inclined fibres no clear relevance was observed.

Figure 2.6: Influence of snubbing and spalling on the pull-out force for different fibre inclination angles (Van Gysel, 2000).
2.3 Post-cracking behaviour of SFRC

The addition of randomly distributed steel fibres improve the fracture characteristics and structural behaviour, i.e. the fracture toughness, ductility, impact resistance, reduced crack spacing and crack widths, increased flexural stiffness and spalling resistance (Shah and Skarendahl, 1985; Clarke et al., 2007) since the fibres are able to bridge cracks.

Concrete is a heterogeneous material with pores and micro-cracks existed by shrinkage and thermal strains, which have been restrained by coarse aggregates and fibres in the case of FRC. During loading, the matrix transfers a part of load to the fibre until micro-cracks initiated. Up to the this level, the response of conventional concrete and fibre reinforced concrete are similar, since it was shown that addition of fibres to concrete have marginal influence on the tensile strength of composite (Barros and Figueiras, 1999). Once the tensile strength of material reached, where all micro-cracks grow and eventually lead to a macro-crack, the behaviour of conventional concrete and fibre reinforced concrete is different. In the conventional concrete an abrupt reduction of stress obtained since the crack is only bridged by the contribution of coarse aggregates, therefore low fracture energy is expected, see Figure 2.8 and Figure 2.9.
In the case of the FRC, once the crack initiated, the fibres start to bridge the two laps of an existing crack by debonding between fibre and concrete which happened in the shorter fibre embedded length until full debonding occurs. The bridging of the fibres across cracks provides a post-cracking tensile strength to concrete and, thus the dissipated energy is much higher than the conventional concrete, see Figure 2.8. Depending on the amount of the fibres bridging the crack and also fibre-matrix bond strength, the post-cracking stress can be larger than tensile strength of concrete, resulting in a so call strain hardening behaviour where multiple cracking occurs. Nevertheless, for the normal content of fibres (lower than 1%) the damage localises immediately after initiation of the first crack and the material exhibits strain softening behaviour.

**Figure 2.8:** Schematic description of stress-crack width relationship for conventional concrete and fibre reinforced concrete.

**Figure 2.9:** Fracture process of conventional concrete and FRC (Löfgren, 2005).
The post-cracking tensile behaviour of random discrete fibre reinforced concrete can be simulated either by a stress – crack width relationship, \( \sigma - w \), or a stress – strain relationship, \( \sigma - \varepsilon \). However, the use of stress – crack width or stress – strain relationships has been subjected to a major discussion among researchers since both methods have their own advantages or limitations (CEB-FIP, 1999; CNR-DT204, 2006). In fact, the main advantage of using stress – crack width relationship is that it presents an actual post-cracking response of composite materials. Furthermore, it can be also compared directly with the experimental results at material level such as the one obtained from the uniaxial tensile test. From another point of view, if post-cracking response is defined by the stress – strain diagram, it can be easily introduced in the structural design applications, thus it is the most desirable. However, it should be mentioned that it does not show the actual post-cracking behaviour of FRC and also cannot be obtained directly from a test method (Montaignac et al., 2011).

A crack in the plain concrete can be modeled using the fictitious crack model which was developed by (Hillerborg, 1980). In this model, the crack propagation in concrete is modeled by a zone of diffuse micro-cracking, named as the process zone, and a localized crack. For plain concrete, the tension stress drops to zero when the aggregate interlock ends, whereas in the case of the fibre reinforced concrete, the fibres crossing the crack will provide more resistance by pulling-out from the matrix. The stresses provided by fibres across the crack are described as a function of the crack opening where the tensile stress at the top of the crack is assumed to be equal to the concrete tensile strength, see Figure 2.10(a). To characterize post-cracking behaviour, in terms of stress – crack width relationship, different approaches are available. A rigid plastic model is proposed by Model code 2010. In this model, the concrete stress drops from the tensile strength to a lower residual stress just after cracking. Kooiman (2000) proposed a bilinear diagram for the softening phase of cementitious materials. Similar to the bilinear diagram, Du et al. (1990) and Hordijk (1991) proposed the power shape \( \sigma - w \) diagrams. In general, the bilinear or power shape \( \sigma - w \) relationships render a reasonable estimation of plain concrete post-cracking behaviour since the shape of the diagram is not too dependent on the concrete strength class. However, in the case of FRC, the shape and response of composite depends on the fibre content, fibre type and quality of the matrix. Therefore, it is necessary that, for each FRC composite, a tensile response that characterizes the post-cracking response appropriately should be determined. Barragán (2002), Barros et al.
(2005), Cominoli et al. (2007) and Dozio (2008) proposed a trilinear diagram which can present the post-cracking response of FRC properly, see Figure 2.10(b). From another point of view, $\sigma - w$ relationship can be also determined directly throughout uniaxial tensile test or in indirect fashion. In indirect tensile tests, the $\sigma - w$ response of FRC is assessed by an inverse analysis procedure (Roelfstra and Wittmann, 1986) that takes into account the various experimental test results, such as: splitting tensile test; three-point notched beam bending test; wedge splitting test. Figure 2.11 depicts the different approaches to estimate the stress – crack width law of SFRC. In this procedure, a step-wise inverse analysis is carried out using segments of either load – deflection or load – crack mouth opening width to achieve a piece-wise polylinear $\sigma - w$ relationship.

Figure 2.10: (a) Fictitious crack model in steel fibre reinforced concrete, (b) examples of $\sigma - w$ relationships.

Figure 2.11: Scheme of distinct approaches to estimate the stress – crack width law of SFRC.
2.3.1 Test methods

2.3.1.1 Uniaxial tensile test

The uniaxial tensile tests were regarded to be necessary in order to evaluate the tensile load bearing capacity of the mixture and obtain appropriate material input parameters for modelling the structural behaviour. Probability, this testing method is the only way to achieve stress – crack width relationship and also all the relevant fracture parameters from the experiment directly (Van Mier and Van Vliet, 2002). However, despite many difficulties involve in the execution of the test, it is necessary that this test should be performed under a well control condition. Many researchers investigated the influence of the specimen’s geometry, size and boundary conditions on either plane concrete (Bazant and Pfeiffe, 1987; Vliet, 2000; Carpinteri et al., 2002) or fibre reinforced concrete (Barragán, 2002). However, among these can be pointed out the perfect manufacturing and alignment of the concrete specimen with the loading actuator, in order to avoid undesired secondary flexure; the adopted attachment between the specimen and loading platens; and the performance of the testing control system. However, it was shown that the crack propagation in the specimen depends on the rotation of the specimen’s boundaries (Ostergaard, 2003). Some of the researchers advise that the specimen’s boundaries conditions should be free to rotate (Van-Mier et al., 1996; Van-Mier and Van-Vliet, 2002) whereas other indicated that they should be prevent from the rotation (Hillerborg, 1980; Ostergaard, 2003). RILEM-TC162-TDF (2001) advises that the flexural stiffness of machine and grip should be sufficient to prevent the specimen from any significant rotation.

In the case of using rotatable boundaries conditions, during the execution of the test, it is expected that the specimen is cracked in the notched part where the tensile strength of the section is lower. The crack normally happens in one side of the notch but not covers the entire notch section once. Afterward, most of the load is carried by the uncracked part in the ligament. With regarding to this uncracked section, the tensile load which applied to the centre of the specimen will have an eccentricity. This eccentricity exist a bending moment which speed-up the crack growth, see Figure 2.12. Hence, in order to avoid the influence of secondary flexure, the connection between the specimen and machine should be very stiff without a hinge connection.
In this test, the tensile stresses are measured directly without the influence of the stress gradient. However, it is not recommended to use this testing method for determining tensile strength since the top and bottom surfaces of the specimen can never be perfectly levelled. Therefore during the test execution, it might cause stress gradient in the specimen which could lead to the crack initiation outside of the notch plane. Considering that the overall maximum load is divided by the area of the notch cross section, therefore the average tensile stress which is determined in this way is usually smaller than the tensile stress presented at a place where the section shows the first crack, see Figure 2.12. Once the crack has been propagated through the section, these effects disappeared, thus the stress distribution can be considered as uniform. On the other hand, for a cracked specimen, the non-uniformity of the stress distribution could be justified by a non-uniform fibre distribution through the fracture section. It should be noted that this influence is in a smaller order than in the pre-cracked concrete (Dupont, 2003).

\[ F \]  

\[ M \]

\[ \text{bending moment induced by the non-uniform stress distribution} \]

\[ \text{assumed stress distribution} \]

\[ \text{real stress distribution} \]

*Figure 2.12: Underestimation of tensile strength due to the stress-gradient.*

The tensile strength of concrete depends on the shape and also the matrix and aggregate interface which can be influenced by the environmental conditions. CEB-FIP Model Code (1990) proposed that the lower, mean and upper values of the characteristic tensile strength could be determined based on the characteristic compressive strength, \( f_{ck} \), by Eqs 2.1-2.3 respectively,
where, $f_{ck}$ is computed as compressive strength, $f_{cm}$, deduced by 8 MPa ($f_{ck} = f_{cm} - 8$ MPa).

It should be remarked that, in the SFRC, the concrete tensile strength is only marginally changed if the amount of the fibres used ($V_f$) were smaller than 1.5 % (Barros, 1999). In uniaxial tension test, the stress-crack opening relationship ($\sigma - w$), see Figure 2.13, according to the RILEM-TC162-TDF (2001) recommendation, can be determined from the following equations simply:

$$\sigma = \frac{P}{A_n}$$  

(2.4)

$$w = \bar{\delta} - \bar{\delta}_p$$  

(2.5)

where $P$ is the load which is recorded during the test, $A_n$ is the cross section area at the notch, $\bar{\delta}$ denoting the average signal of the displacement transducers and $\bar{\delta}_p$ representing the average displacement at peak stress. The dissipated energy also can be calculated between two crack openings $w_i$ and $w_m$ from Eq. 2.6 easily, where $w_i$ is the crack opening corresponding to the situation where the crack has crossed the failure plane and $w_m$ maybe denote ultimate crack opening.

$$W_F = \int_{w_i}^{w_m} \sigma (w) dw$$  

(2.6)
Figure 2.13: Calculation of crack opening $w$ from averaged, measured displacement over the notch (RILEM-TC162-TDF, 2001).

2.3.1.2 Three and four point bending tests

There are several international recommendations which proposed different test configurations to determine the tensile behaviour of fibre reinforced concrete under flexure. These configurations can be divided in two major groups namely: three-point bending test and four-point bending test. The bending test in three point configuration is proposed in international codes such as: CEB-FIP (1999), RILEM-TC162-TDF (2002b) and EN-14651 (2007), whereas the four point bending test is recommended in JSCE-SF4 (1984) and ASTM-C1018-89 (1991). The major aspect which differentiates these two test methods is related to the position of the fracture surface in the specimen. On the other hand, in three point bending test, the cross section has a pre-defined notch at its mid span, while in four point configuration, the section is flush (without notch). In the case of three point bending test a single crack is formed at the tip of the notch whereas in four point bending test the crack is initiated in the weakest section between two loading points. It is recommended to used bending test in four point configuration to determine post-cracking behaviour of fibre reinforced concrete since it is capable to show the capacity of FRC in favouring multiple cracking (Groth, 2000). However, in some other standards like UNI-11039 (2003) (loading spam is 450 mm), DBV (2001) (loading spam is 550 mm) and CNR-DT-204 (2006) they also proposed to perform four point bending test on the specimen with a notch at its mid span.
The main advantage of bending test is simplification in the execution, particularly preparation of the specimens as well as test setup comparing to uniaxial tensile test. Furthermore, if the test controlled with CMOD rate, the unstable crack formation could be prevented even in the case of the plain concrete. Moreover, the scattering of the results is also lower. From another point of view, Cunha (2010) showed that the stress – crack width diagram determined from the inverse analysis of bending test results overestimated the tensile post-peak behaviour of steel fibre reinforced self-compacting concrete, SFRSCC, when compared to the one obtained from uniaxial tensile tests. This tendency was also observed, even if the influence of fibre distribution and orientation was taken into consideration.

Three point bending test (RILEM-TC-162-TDF, 2002b)

This test method is used to determine the post-cracking behaviour of steel fibre reinforced concrete. It can be evaluated in terms of areas under the load – deflection curves, load – bearing capacity at a certain deflection or crack mouth opening displacement (CMOD) achieved by testing a simple supported notched beam in three point bending test. The standard discussed two concepts for measuring post-cracking response of FRC. The first one is based on the concept of equivalent flexural tensile strength, $f_{eq}$, which depends on the energy dissipated up to a certain level of deflection. The other one known as residual flexural tensile strength, $f_{Ri}$, corresponded to the load values determined in various deflections or CMOD levels.

Concrete beams with dimensions of 550 mm in length and 150 × 150 mm cross section are selected as standard specimen for this test. A notch with a depth of 25 mm and maximum width of 5 mm should be cut at the specimen mid span, see Figure 2.14. Two LVDTs are used for measuring the mid-span deflection and also CMOD at the mid-width of the notch.
The load at limit of proportionality \( F_L \) is determined according to Figure 2.15. \( F_L \) is the highest value of the load at deflection or CMOD interval 0.05 mm. By assuming a linear stress distribution through the section, the stress at limit of proportionality, \( f_{ct,L} \), can be determined by:

\[
f_{ct,L} = \frac{3F_L L}{2bh_{sp}}
\]

where \( b \) is the width of the specimen and \( h_{sp} \) is the displacement between tip of the notch and top of cross section (mm). Using \( F_L \), its corresponding deflection, \( \delta_L \), and assuming \( \delta_L + 0.3 \text{ mm} \) as ultimate deflection, the energy absorption capacity due to the concrete contribution, \( D_{hz}^c \), can be isolated from the fibre contribution. This can be simplified as a straight line which connecting \( F_L \) to the point on the abscissa \( \delta_L + 0.3 \text{ mm} \) (Figure 2.15). Afterward, the influence of adding fibres to concrete in terms of energy absorption capacity, \( D_{hz}^f \), can be determined up to a certain deflection by calculating area under load-deflection curve and, then, subtracting it from \( D_{hz}^c \). The contribution of fibres is evaluated at two deflection limits: \( \delta_i + 0.65 \text{ mm} \) which is designated as \( \delta_2 \) and \( \delta_i + 2.65 \text{ mm} \) which is designated as \( \delta_3 \).
mm known as $\delta_3$ in order to determine $D_{BZ,2}$ and $D_{BZ,3}$. Finally, based upon these parameters and assuming a linear stress distribution in the notch cross section, $f_{eq,2}$ and $f_{eq,3}$ (N/mm$^2$) are determined as follow:

$$f_{eq,2} = \frac{3}{2} \left( \frac{D_{BZ,2}}{0.5} \right) \frac{L}{b h_{sp}^2}$$  \hspace{1cm} (2.8)

$$f_{eq,3} = \frac{3}{2} \left( \frac{D_{BZ,3}}{2.5} \right) \frac{L}{b h_{sp}^2}$$  \hspace{1cm} (2.9)

Furthermore, the residual flexural tensile strength, $f_{Ri}$, is determined by the following equation.

$$f_{R,i} = \frac{3 F_{R,i} L}{2 b h_{sp}^2}$$  \hspace{1cm} (2.10)

In this equation $f_{Ri}$ should be determined at force values correspondent to the deflection levels: $\delta_{R1} = 0.46 \text{ mm } (CMOD_1 = 0.5 \text{ mm})$, $\delta_{R2} = 1.31 \text{ mm } (CMOD_2 = 1.5 \text{ mm})$, $\delta_{R3} = 2.15 \text{ mm } (CMOD_3 = 2.5 \text{ mm})$ and $\delta_{R4} = 3.00 \text{ mm } (CMOD_4 = 3.5 \text{ mm})$, see Figure 2.16. However, $f_{R,i}$ and $f_{eq,2}$ are used in the design at serviceability states, while $f_{R,4}$ and $f_{eq,3}$ are implemented in the design at ultimate limit states.

Figure 2.15: Load – deflection diagram for the determination of the equivalent flexural tensile strength.
Figure 2.16: Load – CMOD diagram for or the determination of the residual flexural tensile strength.

**Four point bending test (UNI-11039, 2003)**

According to Italian standard (UNI-11039, 2003) the geometry of the specimen in four point bending test have dimensions of 150×150×600 mm with a notch depth of 45 mm. The load is applied on the $L/3$ of the specimen ($L$ is the loading span = 450 mm) according to Figure 2.17(a). The deflection and CMOD are measured similar to three point bending test, see Figure 2.17(b).

In order to identify the post-cracking behaviour of FRC, three parameters are proposed by UNI-11039 (2003), namely the first crack strength, $f_{If}$, and two equivalent flexural strengths, $f_{eq,0-0.6}$ and $f_{eq,0.6-3.0}$. The first flexural strength which is significant for the serviceability limit state correspondent to the CMOD rang from the 0 to 0.6 mm. The second one which is used for designing at ultimate limit state related to the CMOD range from the 0.6 to 3.0 mm.
2.3.1.3 Splitting tensile test

Splitting tensile test, also known as Brazilian test, is well disseminated to estimate the concrete tensile strength. The test is standardised in different international codes such as: ACI-318-R (1993), CEB-FIP (1999), ASTM-C496 (2004), and EN-1992-1-1 (2004). The main advantages of the splitting tensile test are that it is quite cheap and simple to be performed on either cylinders (e.g. extracted cores from real structural elements) or cube specimens. Moreover, it only requires a testing rig capable of applying compressive loading. Unlike three-point beam bending test, it is expected that the result should be closer to uniaxial tensile test, since most of the bulk concrete along and across the potential fracture plane is subjected to a uniform tensile stress (Carmona and Aguado, 2012). However, besides the transversal tensile stresses, longitudinal compressive stresses also appear. One disadvantage of this test method is that suitable data on the post-cracking regime hardly can be obtained from it due to the unstable crack propagation. However, by performing this test with a closed-loop crack width control, a stable response can be achieved, thus this problem can be easily overcome (Rocco et al., 1999; Carmona and Aguado, 2012).

In splitting tensile test, a compressive load is applied along two diametrically opposite line loads of a cylindrical or prismatic specimen. The compressive load induces a linear tensile stress state perpendicular to the loading plane at the central part of the specimen (Timoshenko and Goodier, 1991). After reaching the tensile strength in this zone, the micro-cracks coalescence and the crack starts to grow towards the loading
strips. The stress distribution along the loading plane at the centre of the specimen is shown in Figure 2.18(a) where the tensile stress is almost uniform in nearly more than 70% of the specimen’s diameter. The maximum horizontal ($\sigma_x$) and vertical ($\sigma_y$) tensile stress for an element where is located at the centre of the cylindrical specimen, see Figure 2.18(a), are achieved by the following equations which are also available in the design codes.

$$\sigma_{x,max} = \frac{2F}{\pi ld}$$

$$\sigma_{y,max} = -\frac{6F}{\pi ld}$$

where $F$ is the applied line load, $d$ is the diameter of the cylinder and $l$ is the thickness of the net area in the notched plane.

In the case of determining the post-peak behaviour of concrete with crack opening control splitting tensile test, a typical force-crack opening ($F - w$) curve similar to Figure 2.18(b) is observed. The curve consists of four main stages, namely, pre-cracking regime, pre-peak non-linearity, softening and plateau. The pre-cracking regime is obviously a linear stage and continues up to around 85% of the maximum load ($F_{max}$). It follows by pre-peak non-linearity branch which is happened due to the loss of stiffness of the matrix. In this phase, due to the Poisson ratio, concrete starts to deformed under the applied diametrical load horizontally and compressive stresses are transmitted by two mirror arcs between the loading plane which interacted by tension ties perpendicular to the notch plane, see Figure 2.19(a). After this stage, micro cracks are formed in the centre of the specimen and spread vertically towards the supports. This is accompanied by a gradual reduction in the force which is called softening phase, see Figure 2.18(b) and Figure 2.19(b). Finally, the last phase corresponded to the plateau stage of the curve. In this stretch, the crack gets wider, wedges appears near two supports and the two halves of concrete start to be separated. These halves subjected to a constant compression load of approximately 40% of the maximum load and produce the plateau branch of the curve (Figure 2.18(b), Figure 2.19(c) and (d)) (Carmona and Aguado, 2012).
In the case of plain concrete, since it is relatively a homogeneous material, the stress distribution along the ligament is a nearly uniform tensile stress. Nevertheless, fibre reinforced concrete is a pseudo-ductile material with a toughness level depends on fibre content, fibre type and concrete matrix, thus assuming a uniform stress distribution in the fracture plane is far from the reality. However, it should be noted that this drawback is also observable in uniaxial tensile test or even bending test. Recently, different types of tests based on the latter have been proposed, namely, double-punch test (Barcelona test) (Chen, 1970; Molins et al., 2009) and wedge splitting test (Ostergaard, 2003). In wedge splitting test, some authors estimated stress – crack opening relationship successfully by
performing inverse analysis (Ostergaard, 2003; Löfgren, 2005). Recently, di Prisco et al. (2013) proposed a modified version of splitting test named as double edge wedge splitting test (DEWS) in order to determine stress – crack opening relationship of fibre reinforced cementitious composites directly from the experiment. On the other hand, unlike splitting test, in DEWS test the compressive stresses are deviated far from the ligament and, hence pure mode I fracture (uniaxial tensile stress state) is likely to be induced along the ligament, see Figure 2.20(a). Then, the transversal splitting force $F_{sp}$ induced by the applied vertical load $P$ can be determined as follow:

$$F_{sp} = P \frac{\cos \theta - \mu \sin \theta}{\sin \theta + \mu \cos \theta}$$

(2.13)

where $\theta$ is the inclination angle of wedge grooves ($\theta = 45^\circ$), $\mu$ is the friction coefficient depends on the contact details. In the case of using Brass plates to the groove edges, the friction coefficient is proposed as 0.06. All details are depicted in Figure 2.20(b). The validation of the proposed test methodology experimentally and numerically can be found in L. Ferrara, di Prisco, M., Lamperti, M.G.L. (2010) and di Prisco (2013).

![Figure 2.20](image)

*Figure 2.20:* (a) Force diagram in double wedge splitting test specimen and (b) measuring devices arrangement for double wedge splitting test (di Prisco et al., 2013).
2.4 Long-term behaviour of fibre reinforced concrete

Volume changes of concrete including shrinkage and creep depend on the mix composition, environmental condition and the level of permanent stress in the element. In general case, the total strain, \( \varepsilon_c(t) \), at time \( t \) for a concrete element loaded at time \( t_0 \) under a constant stress \( \sigma_c(t_0) \) and also constant temperature can be presented as sum of the separated strains:

\[
\varepsilon_c(t) = \varepsilon_{ci}(t_0) + \varepsilon_{cc}(t) + \varepsilon_{cs}(t)
\]  

whereas, \( \varepsilon_{ci}(t_0) \) is the instantaneous strain in concrete at time \( t \), \( \varepsilon_{cc}(t) \) present strain due to the creep at time \( t \) and \( \varepsilon_{cs}(t) \) shows strain related to the shrinkage at time \( t \).

Shrinkage of concrete is a combination of several types, namely: plastic, autogenous, drying, thermal and carbonic shrinkage. The most important type is the drying shrinkage which appears because of the movement of the water through the harden concrete, i.e. evaporation of the internal water in the external environment.

When concrete is subjected to permanent load, instantaneous deformation appears which gradually increases with time due to the creep of concrete. Creep is a visco-elastic phenomenon, mainly occurs in the viscous hydrated cement paste. This may be a concern, since self-compacting concrete (SCC) has a high binder content, in part to attain its self-compactibility, thus SCC may exhibit a relatively high deformability due to long-term loads. Creep of a cracked section consists of three main stages, see Figure 2.21(a), namely: primary, secondary and tertiary stages. When the load is applied, the instantaneous crack opening occurred. During the primary stage, under a constant load, the crack opening increases with time, however with a higher rate. Afterward, in the next step, the crack opening rate reaches a minimum and approximately constant value called secondary or steady-state creep. Eventually, if the tertiary stage is attained, the crack opening rate will increase exponentially and consequently lead to unstable structural failure. It should be noted that the later stage happens if the specimen is loaded in a relatively high level of loading. Consequently, creep in a crack section can basically be regarded as the additional crack opening above the initial instantaneous crack opening caused by constant loading, see Figure 2.21(b) at time \( t_1 \). In case of removing the sustained load (time \( t_2 \)), crack opening is instantly reduced, but generally this diminution
will be lower than the instantaneous crack opening on loading stage (time \( t_1 \)). This reduction in crack opening is called instantaneous recovery, which is followed by a gradual reduction in opening named as creep recovery, see Figure 2.21 after time \( t_2 \). However, since creep is not a completely reversible phenomenon, application of any sustained load, during a significant period of time, results in an unrecoverable residual deformation called flow creep (Neville, 1959). Since instantaneous deformation of steel fibre reinforced concretes is rather unaffected by fibre reinforcement, therefore employment of steel fibres does not significantly influences the delayed component of creep (Mangat and Azari, 1985). On the other hand, the use of steel fibres reduces the flow component of creep since they provide restrain to the sliding action of matrix. This restrain occurs through the fibre/matrix interfacial bond strength, which depends on several factors, such as: fibre embedment length, fibre geometry, mechanical anchorage, fibre orientation, etc. Furthermore, creep of concrete is mainly affected by environmental conditions, size and geometry of the structural elements, water/cement ratio, cement’s type, curing conditions and the stress/strength ratio.

\[ \text{(a)} \quad \text{(b)} \]

*Figure 2.21:* (a) Long-term crack opening as a function of time due to constant stress and (b) typical long-term crack opening profile for concrete under sustained stress.

### 2.4.1 Micro – composite mechanical behaviour

One of the most important benefits of SFRC is the durability improvement. Under a sustained load, the time-dependent crack widening has been attributed to be caused by two mechanisms: fibre pull-out process and time-dependent fibre creep. The latter
parameter could be important if synthetic fibres were employed. On the other hand, the long-term crack widening could be considered mainly in the fibre/matrix adherence which depends on several factors, such as: quality of matrix, bond strength, fibre embedment length, fibre geometry, mechanical anchorage, fibre orientation, etc.

In the author’s knowledge, few works reported on the time-dependent fibre pull-out behaviour are limited to those reported by Boshoff et al. (2009a) and Babafemi and Boshoff (2014) which investigated synthetic fibres. Boshoff et al. (2009a) studied the time-dependent behaviour of bond between fibre/matrix in strain hardening cement-based composites employing micro PVA fibres (length of 12 mm and 40 μm in diameter) by maintaining a single fibre pull-out specimen under a sustained load. The creep tests were executed on two series of the specimens non-debonded and debonded fibres. For the later set, before the creep test was introduced, the fibres were debonded from the matrix in the monotonic single fibre pull-out test until a slight load drop or change of gradient occurred in the force-slip response. The creep load was selected as 50% of the average shear strength. Figure 2.22 shows the fibre pull-out displacement over time for the non-debonded and debonded fibres. It was observed that all the fibres were pulled-out before 70 hours. The average time to pull-out for non-debonded specimens was 57 hours, whereas for the debonded series it was 16 hours. Therefore, after a full debonding of a fibre from the matrix, the resistance to creep deformation reduced significantly.

![Figure 2.22: The fibre pull-out displacements over time show for (a) non-debonded and (b) debonded fibres (Boshoff et al., 2009a).](image-url)
In a similar study, Babafemi and Boshoff (2014) repeated the creep fibre pullout test but, this time, using macro polypropylene fibres (length and diameter of 40 and 0.8 mm, respectively) in a fibre reinforced concrete. The creep tests were performed on only non-debonded fibres which loaded in 50, 60, 70 and 80% of the load correspondent to the average interfacial shear strength. The result of the time-dependent fibre pull-out is shown in Figure 2.23. It was concluded that the time-dependent pull-out displacement of a single fibre was affected by the magnitude of the stress level. On the other hand, the higher the stress, the quicker the pull-out. Furthermore, in order to determine elongation of the fibre under a sustained load, they also maintained a single fibre to a load of 30% corresponding to the fibre tensile strength. It was indicated that a fibre can lengthen with up to 40% after 4 days at a load as low as 30% of its capacity. However, it is worth noting that in the case of using steel fibres in composite, the contribution of fibre elongation under a sustained load could be negligible.

\[ \text{Figure 2.23: Fibre pull-out displacement versus time (Babafemi and Boshoff, 2014).} \]

It should be mentioned that in these works synthetic fibres with only 0 degree inclination angle (simulate fibres perpendicular to the crack plane) was studied, whereas it was depicted that orientation of the fibres towards the crack plane has an important effect on the tensile behaviour of FRC (Stahli et al., 2008; Boulekbache et al., 2010; Torrijos et al., 2010). Therefore, information regarding to the effectiveness of a fibre as a medium to prevent the widening of crack under a sustained load in the single fibre level are very scarce in literature. In fact, understanding the time dependent alteration in the fibre and matrix interfacial zone in the single fibre level (meso-level) could be used
towards more details on the creep behaviour of steel fibre reinforced concrete, SFRC, elements in macro-level.

2.4.2 Macro – composite mechanical behaviour

The creep deformation of the material could ultimately lead to the failure mechanism of the structural element at a lower load than static ultimate load (Boshoff et al., 2009b). On the other hand, in some structural systems, the long-term deformation of the structural element can be beneficial, since it enforces stresses to redistribute, which can limit the crack propagation. From another point of view, if the creep deformation damages significantly affect the fibre/matrix interface bond, it will lead to an undesirable excessive decrease on the post-cracking strength, thus the influence of creep will be adverse (Arango et al., 2012).

The time-dependent deformation of the cracked elements under flexure can be due to the following effects: drying shrinkage of the compression zone, creep in the compression zone or uncracked section which produce basic creep, time-dependent bond failure and bond creep strains between concrete and fibres crossing the crack (creep in tension zone) and finally creep in the fibre material. However, if a cracked specimen subjected to the long-term uniaxial tensile loading, thus the influence of creep and shrinkage in the compression zone should be ignored.

2.4.2.1 Creep in compression

Creep in compression, known as basic creep, is explained by two physical mechanisms with different kinetics (Ulm and Acker, 1998): the short-term micro-diffusion of water between capillary pores and the long-term sliding of C-S-H sheets characterized by a non-asymptotic aging (Figure 2.24). Compressive creep is sensitive to a lot of parameters. On the other hand, creep decreases with the age of loading and increases with water to cement ratio (Østergaard et al., 2001; Bissonnette et al., 2007). Creep remains in linear viscoelastic region for a stress to strength ratio up to 60 percent (Domone, 1974; Atrushi, 2003;). Østergaard et al. (2001) showed that specific creep strain is not proportion to the stresses when a specimen is loaded up to 45% instead of 25% of the tensile strength. The strength of concrete has a considerable effect of creep. On the other hand, within a wide
range, creep is oppositely proportional to the strength of concrete at the time of application of loading. Therefore, it is indicated that creep can be expressed as a linear function of the stress/strength ratio (Neville, 1959). However, the influence of concrete age at the time of loading should be ignored. Maia and Figueiras (2012) illustrated that for loadings at the age of 12 and 24h, the relationship between the creep strain and the stress was not linear. Westman (1995) concluded that early age creep in compression has a strong age dependency when sample was loaded within the first 24 h after set.

![Mechanisms of basic creep](image)

*Figure 2.24: Mechanisms of basic creep (Ulm and Acker, 1998): (a) short-term micro-diffusion of water between capillary pores and (b) sliding of C-S-H sheets.*

Different discussions are available in literature regarding to the influence of steel fibres on the basic creep of concrete. Balaguru and Ramakrishnan (1988) and Velasco et al. (2008) depicted that creep is higher for the concrete reinforced with steel fibres. Bissonnette et al. (2007) reported that steel fibres have negative influence on the creep of concrete since the fibres can possibly create voids in the cement paste structure as a consequence of the reduction of the workability. Another aspect is related to the creation of a porous region around the fibres, similar to the transition zone between aggregate and cement paste. On the contrary, Zhang (2003) illustrated that addition of the steel fibres can reduce basic creep. Mangat and Azari (1985) carried out an experimental program to determine the effects of steel fibres on the creep of cement matrices under compressive loads. The results indicated that steel fibres restrain the creep of cement matrix at different stress/strength ratio, and this restrain is greater at lower stresses and at higher fibre contents. The addition of steel fibres does not significantly influence the
instantaneous and delayed elastic component of creep, but they provide restrain to the sliding action of the matrix due to the flow component of creep, see Figure 2.21(b). Steel fibres become more effective in restraining creep as the age under load increases. This is due to the fact that they affect only the flow component, which is important at the later ages, while the instantaneous and delayed elastic components are dominant at early ages.

Marangon et al. (2012) studied the influence of steel fibres on the compressive creep behaviour of a steel fibre reinforced self-compacting concrete mixture. The self-compacting concretes were reinforced with volume fractions of 0, 1 and 1.25% of steel fibres with two different aspect ratios of 65 and 80. The obtained results indicated that application of the steel fibres, for the aspect ratios and volume fraction studied, did not change considerably the compressive creep behaviour of the self-compacting concrete. The same conclusion was also made by Weiss et al. (1999). They reported that if the fibre volume content does not exceed 1%, the creep behaviour in compression of steel fibre reinforced self-compacting concrete is similar to the plain self-compacting concrete.

Various codes are available in order to predict creep behaviour of the plain concrete such as: ACI-Committee-209 (1997), CEB-FIP (1999), EN-1992-1-1 (2004) and JSCE (2005). However, in all the mentioned design codes, in the case of fibre reinforced concrete, the influence of the fibres on creep response was not taken into the consideration. The only available expression is limited to Mangat and Azari (1985). They proposed a theoretical expression to predict the creep strain of fibre reinforced concrete at a stress-strength ratio of 0.3, based on knowledge of the creep of ordinary concrete ε_{oc}, coefficient of friction μ, fibre volume ν_f and aspect ratio of the fibres l/d:

$$\varepsilon_{fc} = \varepsilon_{co} \left(1 - 1.96 \mu \nu_f \frac{l}{d}\right)$$

(2.15)

With this expression, the decrease of creep of steel fibre reinforced concrete, when compare to plain concrete, is ranging from 0 to 30%.

### 2.4.2.2 Creep in tension

The positive influence of steel fibres in concrete can be only maintained if the crack does not significantly widen under a sustained load. It was shown that addition of steel fibres to the concrete could limit the long-term crack widening in a cracked section since fibres
provide a resistance to the time-dependent sliding action. Few works reported on the uniaxial tensile creep of cracked FRC. However, the focus of these studies was either on steel fibres (Mouton and Boshoff, 2012; Zhao et al., 2012) or synthetic fibres (Boshoff et al., 2009a; Babafemi and Boshoff, 2014). Mouton and Boshoff (2012) performed the uniaxial tensile creep test on the pre-cracked specimens of fibre reinforced concrete (compressive strength = 44.3 MPa) with 40 kg/m³ of steel fibres. In general, determination of the time-dependent behaviour of a cracked section experimentally, consists of three main stages: firstly, the beams are pre-cracked; secondly, the creep tests are carried out on the pre-cracked beams; finally, post-creep bending tests are performed until failure of the specimen. In this study, the pre-cracking test was terminated once the crack was formed in the specimen. The specimens were loaded in creep test equal to the 50% of the post-peak residual strength obtained from the monotonic test. Figure 2.25(a) shows the creep response for each specimen. It was observed a very negligible long-term crack opening in the specimens (0.02 mm after a period of 3 months). However, in the specimen number 2, once the specimen was loaded in creep test, a sudden increase of crack opening was achieved which led to a significant increment in the creep response.

In another study, Babafemi and Boshoff (2014) executed the same experiment on the macro- synthetic fibre reinforced concrete (compressive strength = 40.19 MPa) using 1% by volume of polypropylene fibres. In their study, they also investigated the influence of the loading level. On the other hand, 30, 40, 50, 60 and 70% of the post-peak residual strength were selected as creep loads. The time dependent behaviour of the specimens at various stress levels is depicted in Figure 2.25(b). It is shown that with increase in the stress level, the long-term CMOD tended to increase as well. From the results, an indication of sustainable creep up to the 50% of the post-peak strength is observed. After this stress level, all the specimens had entered in the tertiary creep stage before the unloading. By comparing the results of both studies (Figure 2.25(a) and (b)) it is obviously observable that, for the same level of loading (50%), the concrete with steel fibres showed a much lower long-term crack widening in compare to the concrete with syntethic fibres. On the other hand, application of the steel fibres to the concrete, provide more resistance to the long-term loading and thus, a more improvement in the durability can be acheived. From one point of view, it should be mentioned that pull-out mechanism of a steel fibre consists of the mobilization of the mechanical anchorage and frictional slip of the fibre, whereas in the case of synthetic fibres only frictional slip of the fibre is
predominant. Therefore, when the bond between a synthetic fibre and matrix is damaged, as it was observed in Figure 2.23, this fibre can be easily pulled-out under a sustained load. From another point of view, the synthetic fibres showed also a relatively high elongation in the creep test, while it is very negligible in the case of the steel fibres.

![Figure 2.25: Uniaxial tensile creep response of: (a) steel fibre reinforced concrete (Mouton and Boshoff, 2012) and (b) macro-synthetic fibre reinforced concrete (Babafemi and Boshoff, 2014).](image)

Boshoff et al., (2009a) studied the creep response of the cracked strain hardening cement based composites. The specimens were pre-cracked up to 1% strain and loaded at 30, 50, 70 and 80% of the tensile strength. Figure 2.26 illustrates the average time dependent strains for each loading level. Similar to the previous researches, creep strain enhanced with increasing loading level being more pronounced in in the higher loading levels. On the other hand, the creep increases almost exponentially at the higher loads. During the creep test non of the specimens passed to the tertiary creep stage. On the other hand, even in the case of the specimen with 80% stress level, the load was sustained after 24 months. Infact, the increased creep of the cracked specimens can only ascripted to three possible sources, namely: the initiation of the new cracks, the widening of the existing cracks over time and the creep of the fibres bridging the cracks.
2.4.2.3 Creep in flexure

Some information is available in literature regarding to the time-dependent behaviour of FRC in the cracked state (MacKay and Trottier, 2004; Tan and Saha, 2005; Arango et al., 2012; Kanstad and Zirgulis, 2012; Zerbino and Barragan, 2012). However, many of them mainly assessed the creep behaviour of concrete reinforced with synthetic fibres (Al-Khaja, 1995; Kurt and Balaguru, 2000; Oh et al., 2005). It was reported that the cracked micro/macro-synthetic fibre reinforced concrete presented significant crack widening over time under sustained load. There are also some works regarding the creep evaluation of steel fibre reinforced concrete under flexural loading (Arango et al., 2012; Zerbino and Barragan, 2012). It was showed that application of steel fibres in concrete limited long-term crack widening considerably (Tan et al., 1994; Tan and Saha, 2005). Nakova and Markovski (2012) studied the influence of the steel fibres on controlling the long-term crack widening over time. A series of the full scale reinforced concrete beam (RC) were used as the control specimen, whereas, in other specimens, the reinforcements kept same but the plain concrete was replaced by steel fibre reinforced concrete with 30 and 60 kg/m³ steel fibres named as SFRC1 and SFRC2, respectively. As it is shown in Figure 2.27, addition of the steel fibres in concrete led to a reduction in the long-term deflection. On the other hand, the time dependent deflections were 17.7 and 27.8% smaller for SFRC1 and SFRC2 when compared to the RC beams. Finally, after termination of the creep test, the specimens were tested until failure and, consequently, RC series depicted a steeper degradation in their stiffness.

Figure 2.26: Average time dependent strains for each loading level (Boshoff et al., 2009a).
Buratti and Mazzotti (2012) discussed a comparison between the long-term behaviour of a plan self-compacting concrete (SCC) beam containing conventional reinforcement and that of some fibre reinforced self compacting concrete (FRSCC) beams, containing different dosages of steel or synthetic fibres without any reinforcement. Two types of steel fibres were used as presented in Table 2.1. Six different types of SCC were tested using either fibre or rebar as reinforcement, namely: a combination of 17 kg/m$^3$ steel fibres A and 3 kg/m$^3$ fibre B and 0.3 kg/m$^3$ synthetic fibre (M), 25 kg/m$^3$ of steel fibre A (25a), 25 kg/m$^3$ of steel fibre B (25b), 35 kg/m$^3$ of steel fibre A (35a), 35 kg/m$^3$ of steel fibre B (35b) and the last one contained of only steel rebar (R). All the specimens pre-cracked up to 0.2 mm as representative of the crack width of concrete at serviceability limit state. In the creep test, it was chosen to apply a loading value able to produce a bending moment in the cracked section equal to the 50% of that achieved at CMOD=0.2 mm. The time dependent of the creep coefficient in terms of mid-span deflection is given in Figure 2.28(a). The creep coefficient was determined from the following equation:

$$\delta(t) = \frac{\Delta(t) - \Delta(t_0)}{\Delta(t_0)}$$

(2.16)

where $\Delta(t)$ indicated the mid-span deflection at time $t$ and $\Delta(t_0)$ is the mid-span deflection at the end of loading phase. It was concluded that the beam R, without fibre, showed the largest deformation and deformation speed. Also the beam M indicated significant deformation that increased with an almost steady state rate. However, the beam with smallest deformation was 25b. Furthermore, it was also concluded that after 260 days from the beginning of the test, the mid-span deflection increased by 115% on
beam R and by 60 to 80% only on the FRSCC which showed the fact that using fibres in concrete reduced creep deformation. Figure 2.28(b) shows the relationship between the creep coefficients in terms of mid-span deflection at the end of 260 and the number of fibres. Although it was not very clear, but it was possible to notice that creep deformation decreased when the effective number of fibres increased.

Table 2.1: Geometry of the fibres used (Buratti and Mazzotti, 2012).

<table>
<thead>
<tr>
<th>Fibre type</th>
<th>Diameter [mm]</th>
<th>Length [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel A</td>
<td>0.75</td>
<td>50</td>
</tr>
<tr>
<td>Steel B</td>
<td>0.6</td>
<td>33</td>
</tr>
<tr>
<td>Synthetic</td>
<td>$18 \times 10^3$</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 2.28: (a) Creep coefficient in terms of mid-span deflection and (b) correlation between the creep coefficient and number of fibres in the crack sections (Buratti and Mazzotti, 2012).

Kanstad and Zirgulis (2012) reported a comparison between pre-cracked synthetic and steel fibre reinforced concrete beams tested in five years with constant load. Four series of the specimens with different fibres were tested as follow: 0.5% by volume of steel hooked end fibres (0.5Steel-1), 0.7 and 1.0% of plastic straight fibres (0.7Synth-3 and 1.0Synth-3) and 1.0% by volume of basalt twisted fibres (1.0Synth-1). All the specimens were pre-cracked up to 1.5 mm and loaded at 50% of the residual strength at
1.5 mm crack width. In Figure 2.29 is depicted a comparison of the creep results in terms of long-term mid span deflection. It is shown that the basalt fibres presented the highest long-term deflection, whereas the beam with 1% plastic fibres illustrated the lowest one. Furthermore, the long-term deflection tends to reduce by increasing in the percentage of the plastic fibres in the composite. After five years under loading, the time dependent deflections were in the range 2 – 3 times the initial deflections. Finally, by comparing residual strength test results of beams before and after the long-term tests, it could be concluded that the long-term deflection does not have any significant influence on the residual strength. It should be mentioned that, in this study, the specimens with steel fibres showed a relatively close long-term deflection in compare to the other synthetic fibres. However, as it was shown in the previous sections, many authors confirmed that synthetic fibre reinforced concrete presented higher creep response comparing to the steel fibre reinforced concrete. Maybe this inconsistency could be correspondent to the different synthetic fibre materials applied, dosage of the fibres, compressive strength of concrete which improve bond between fibre and matrix, various studied pre-cracking and loading levels, etc.

![Figure 2.29: Time dependent deflection development for long-time tests (Kanstad and Zirgulis, 2012).](image)

Barragan and Zerbino, (2012) investigated the influence of the pre-crack width on the creep response. For this purpose, a SFRC with 40 kg/m$^3$ hook ended steel fibres (50 mm in length and 1mm diameter) and a compressive strength of 45 MPa was designed. The following pre-cracking levels were studied: 0.2, 0.5, 1.0, 1.5 and 3.5 mm. In the
creep test, beams were loaded at various stress levels according to the final stress level reached at the end of the cracking tests. For the small pre-crack level 0.2 mm, stable response was obtained even when applying stress levels equal to the 100%. If the stress level was lower than 50%, then there is almost no increase of the crack opening due to the creep. A stable response could still be observed for the beams pre-cracked until 0.5 mm although in a relatively high crack opening rate. In the case of the specimens pre-cracked up to 1 mm, which is close to the highest residual strength of SFRC, the specimen with 78% level of loading led to a creep failure at 83 days. However, for the other specimen with lower percentage of loading, a relatively stable condition was achieved. Considering beams with 1.5 mm pre-cracking levels, the specimen which was loaded in 100% failed during the initial loading part, while the one with lower loading level of 66% depicted a stable creep response. A similar behaviour was observed also for the beams pre-cracked until 3.5 mm. Finally, since the specimens in each cracking level were subjected to the different level of loadings, a precise conclusion regarding to the influence of the pre-cracking width on the creep behaviour could not be obtained.

2.5 Conclusion

According to the all studies available in literature, it is worth noting that the tensile behaviour of fibre reinforced concrete under monotonic loading condition, from the micro-scale to macro-scale levels, was studied widely. On the contrary, information regarding to the creep response of fibre reinforced concrete in the cracked condition are very scarce, particularly in the micro-scale level. However, in order to predict the creep behaviour of a cracked fibre reinforced concrete element, it is important to understand the long-term bond response between each single fibre and matrix considering the fibre inclination angle. Regarding to the available studied in the macro-scale level, it should be mentioned that the focus of the mentioned researches was principally on the beams where, in the case of using fibre reinforced self-compacting concrete, the rotation of the fibres due to the concrete flow was completely distinct of planar structures. In the case of casting beams or panels, fibres tend to be aligned parallel or perpendicular to the concrete flow direction, respectively. Therefore, information regarding the long-term behaviour of cracked SFRC elements, particularly planar structures, is still limited. Consequently,
understanding the behaviour of cracked SFRC elements under a sustained load will help towards a more rational design and accurate prediction of the composite behaviour under serviceability conditions.
CHAPTER 3

Fibre Structure in the SFRSCC

3.1 Introduction

The addition of fibres to a cementitious matrix contributes more to the energy absorption and crack control of structural elements rather than enhancing load transfer capacity (Lataste et al., 2008). Crack opening in concrete is assumed counteracted by the bond stresses that develop at the fibres / matrix interface. On the other hand, one of the most important properties of fibre-reinforced concrete (FRC) is its ability to transfer stresses across a cracked section rather uniformly, which is dependent on the fibre reinforcement effectiveness, i.e. fibre properties (their strength, bond, stiffness), fibre orientation and distribution (Vandewalle and Dupont, 2003). The stress transfer capability enhances mainly the composite’s toughness, which is a parameter for measuring the energy absorption during monotonic or cyclic loading (RILEM-TC162-TDF, 2002b). Recent research has shown that mechanical properties of FRC are affected by distribution and orientation of steel fibres (Ferrara and Meda, 2005; Kim et al., 2008). It is shown that variation of fibre distribution in large-scale elements may result in significant inconsistency in mechanical behaviours within distinct sections. For example, concrete represents a higher post-cracking behaviour if fibres orient according to the direction of the applied stress properly. One can prove that only fibres are active in concrete which are
parallel to the direction of the principal applied stress (Lee and Barr, 2002). Therefore, these effects should be considered for structural design especially where variation of fibre distribution and orientation in a section results in affecting mechanical properties significantly.

### 3.2 Parameters affecting fibre dispersion and orientation

The dispersion and orientation of fibres in the hardened-state are the final result of a series of stages that FRC passes from mixing to hardening state, namely (Laranjeira, 2010): fresh-state properties after mixing; casting conditions into the formwork; flowability characteristics; vibration and wall-effect introduced by the formwork. Among these factors, wall effects introduced by the moulds, and the properties of SFSCC in the fresh state, especially its flowability, are the most important ones (Dupont, 2005; Martinie et al., 2009; Laranjeira, 2010). Workability and flowability affect both the fibre segregation and scattering of the fibres in a high degree. On the other hand, a more workable concrete represents a more likelihood of the vertical settlement of fibres. Application of high dosage of fibres during mixing increases the probability of fibre interaction, so affecting workability of concrete as well as balling. Since fibres tend to align along moulded walls, specimen size is another factor which influences dispersion and orientation of fibres indirectly (Dupont, 2005).

#### 3.2.1 Dynamic effects

During mixing of FRC and discharging from the mixture, fibres may be oriented in three dimensions in matrix randomly but this will not necessarily true after the composite material has been flowed or compacted into a framework. On the other hand, after casting, orientation of fibres is influenced by dynamic effects which are happened in order to fill the framework properly. In the case of conventional FRC, external and internal vibrations are applied while the flowability of SFRSCC is a significant privilege. In the case of applying vertical vibration, fibres tend to align in a plane perpendicular to the direction of vibration, see Figure 3.1, so they acquire a 2D orientation in horizontal planes (Edgington and Hannant, 1997; Soroushian and Lee, 1990).
Fresh state properties influence segregation of fibres and, therefore mechanical behaviour of FRC. SFRSCC was found to represent superior performance compared to conventional concrete in terms of high segregation resistance, good placeability and high mechanical performance (Ozyurt et al., 2007). The fresh state performance of the concrete should represent adequate viscosity in order to not only prevent fibres from segregation but also to orient them in the flow direction. Khayat and Roussel (1997) and Ferrara and Meda (2005) represented that application of viscous modified admixture enhances rheological stability of fresh concrete, therefore reduction in vibration time and a good homogeneity of fibre distribution along the casting direction can be achieved. But, from another point of view, poor placeability and low mechanical performance will be expected.

Application of steel fibres in high percentage may affect mechanical properties of concrete positively, but since all fibres cannot be aligned in the direction of the applied stress, this effectiveness is arguable. Stähli and Custer (2007) studied the effects of flow behaviour of steel fibre self-compacting concrete on fibre distribution and orientation as well as on mechanical properties in the beams. In this experiment, three mixtures with various viscosities were designed by changing the percentage of super-plasticizer. It was observed that the lower viscosity favours a preferential fibre alignment. In mixture 1, with lower slump and higher viscosity, more fibres were perpendicular to the flow direction, while in mixtures 2 and 3 (medium and low viscosity, respectively) fibres aligned properly parallel to the direction of concrete flow (Figure 3.2). However, as it is shown in Figure 3.2 clearly, segregation of fibres happened in mixture 3. Segregation can have negative effect on the mechanical behaviour of FRC where uniform stress is applied over the whole cross-section of structural element. From another point of view, it has also affirmative effect if a flexural beam subjected to the positive bending moment since most
fibres concentrate in the tension part. However, it should not be ignored that fibre segregation improves positive bending moment since, for negative moment the post-cracking response decreases.

![Mixture 1, longitudinal section](#)

![Mixture 2, longitudinal section](#)

![Mixture 3, longitudinal section](#)

*Figure 3.2: Longitudinal centre sections of the prisms for each of the three tested mixtures (Stähli and Custer, 2007).*

The results of four-point bending test clearly showed that the bending strength improved with the increasing flowability. This was due to the preferential alignment of a higher number of fibres parallel to the tensile stresses’ direction. On the other hand, there were more effective fibres to bridge the crack formed in the specimen. Finally the authors suggested a simplified justification for the reason of the fibres alignment, see Figure 3.3. An especial flow profile was developed when fresh concrete flows through a rectangular shape mould. In this flow profile, the velocity of concrete flow was reduced along the walls of the mould due to the frictional restraint.

![Flowing concrete](#)

*Figure 3.3: Explanation for fibre alignment in flowing concrete (Stähli and Custer, 2007).*
Recently, CEB-FIP (2010) proposed an orientation factor ($K$) that considers the influence of fibre orientation in the structural response of the elements. In general, if an isotropic fibre distribution is assumed, then the fibre orientation factor $K=1.0$. However, in the SCC, several factors affect fibre orientation state such as flow length, flow thickness, dimensions of the mould and fibre length (Sanal and Zihnioglu, 2013). Therefore, for the favourable or unfavourable effects, this factor could be lower or greater than 1. Blanco et al. (2015) estimated values of $K$ ranging from 2.0 to 3.2 for the slabs casting from their centre point. Furthermore, they concluded that the velocity profile exerted by the movement of concrete generates forces that cause fibres to drift and rotate perpendicular to the direction of the flow. Consequently, they tend to change their orientation while moving from the casting point towards the edge of the slab as depicted in Figure 3.4. This becomes more evident when the flow distance covered by SCC increases.

*Figure 3.4: Rotation of fibres while moving from the centre of the slabs to the edges (Blanco et al., 2015).*

### 3.2.2 Casting condition

The casting process plays a significant role in the orientation of fibre. It must be carefully designed to make the flow direction of fresh concrete and, therefore, the alignment of fibres as close as possible to the direction of the applied principle tensile stress in order to increase the number of effective fibres in the structure. Ferrara et al. (2011) studied the role of casting methods on fibre orientation. Slabs were cast according to two different methods either pouring concrete from the shorter side of the mould, so it flowed parallel
to the longer edge or casting centrally along the longer side which concrete spread through the formwork like a radial pattern. It was shown that the fresh state properties of concrete and the casting method can influence orientation of fibres in a high degree. In details, the variation in the mechanical behaviours of the extracted beams from the slab which casted in the direction of the shorter edge is higher than those obtained by concrete purring from the larger side of the framework. Torrijos et al. (2010) investigated the influence of not only placing conditions but also using steel fibres of different lengths on the post-cracking response of SFRSCC. Beams were casted in three different ways namely, filling the moulds from centre (C) and end (T) in horizontal condition and also casting vertically (V). Hooked-end steel fibres with 30 and 50 mm length were used in the same mix proportions. It is noticeably observed that similar to the conventional vibrated FRC, fibres oriented mainly in horizontal planes. Consequently, fibres become less effective in the case of casting beams vertically (like a column) and tested horizontally. Mix proportion with 50 mm long steel fibres represented a more acceptable hardening along the post-peak regime but important wall effects were evidenced. Casting beams from the end of the mould achieved the highest post-peak strength with residual strengths completely above the first peak stress. In the case of filling the moulds from centre, a slightly less hardening after the first-peak stress and lower residual strength were reported. On the contrary, if specimens were casted vertically, they presented softening behaviours because fibres aligned in the direction of the applied stress (Figure 3.5).

Figure 3.5: Load – CMOD response from flexure tests: (a) 50 mm fibre (b) 30 mm fibre (Torrijos et al., 2010).
In another study, Barnett et al. (2010) evaluated the influence of fibre distribution and orientation on flexural strength in a series of round panel slabs. The specimens were casted from different directions namely: centre, perimeter and random (Figure 3.6). The results of the experiment are summarized in Table 3.1. It was observed that when concrete was poured from centre, fibres tended to align perpendicular to the flow of the fresh concrete. In random and perimeter direction of casting, fibres aligned more random and parallel to radius of the slabs, respectively. Consequently, round panel specimens casted from the centre represented higher flexural strength compared to the panels poured by other methods, because more fibres perpendicular to radius of the slab are available to bridge the radial cracks formed during testing. However, near the bottom surface, fibres tended to turn more parallel to radius of the round panel specimens. Therefore, since the pattern of fibre distribution changed with depth, distribution variation of fibres in depth becomes a major factor to predict mechanical properties. In particular, the effect of flowability on fibre distribution and orientation might be affected by length of fibres, their shape and also interaction with coarse aggregate particles.

![Figure 3.6: Schematic showing flow of concrete according to casting method; (a) centre; (b) perimeter; (c) random (Barnett et al., 2010).](image)

<table>
<thead>
<tr>
<th>Direction of casting</th>
<th>Fibre orientation toward the radius of slab</th>
<th>Effects on the flexural strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre</td>
<td>Perpendicular</td>
<td>Highest strength</td>
</tr>
<tr>
<td>Perimeter</td>
<td>Parallel</td>
<td>Lowest strength</td>
</tr>
<tr>
<td>Random</td>
<td>Random</td>
<td>Medium strength</td>
</tr>
</tbody>
</table>
3.3 Fibre orientation and efficiency

3.3.1 Fibre orientation factors

To determine the mechanical properties of fibre reinforced composite materials, it is quite important to identify the fibre orientation. Depending on the fibre orientation in a certain composite, usually given by a fibre orientation factor, $\eta$, the number of fibres bridging a crack may be somehow distinct. However, this factor changes for the different dimensional cases and may also be influenced by the concrete flow (Cunha, 2010). Therefore, fibres in the one-dimension, 1D, case are aligned parallel to an axis; in two-dimension, 2D, they are randomly oriented within a plane; and finally in three-dimension, 3D, have an isotropic uniform random distribution.

3.3.1.1 One-dimensional case

The fibre orientation factor in one-dimensional case is somehow straightforward and optimal since fibres align in the direction of the load. It is shown theoretically that this factor is equal to 1 ($\eta_{1D}=1$) (Li, 2001). From another point of view, due to the preferable fibre orientation regarding the fracture surface, this value would not be an optimal one for the fibre effectiveness factor, but a slightly lower value (Cunha, 2010).

3.3.1.2 Two-dimensional case

In a two-dimensional case, it is assumed that fibres are randomly aligned in a plane. Normally, this kind of distribution is more frequent in thin-wall elements, plates and slabs where the thickness of the structural element is less than the length of fibres used. Consequently, the interval which fibres can rotate is limited to the surface of the element exclusively. It is clear that the smaller dimension in the element represents the more limitation in the free orientation of the fibre. Figure 3.7 reveals a 2D case where the fibre orientation $\theta$ is the angle between fibre longitudinal axis and the crack line (x-axis). This angle can vary between 0 and $\pi$, while the tensile stress is applied in y-axis direction. According to Kamerwara Rao (1979) and Stroeven and Hu (2006) the average projection factor (fibre orientation factor), $\eta_{2D}$, is determined by the mean of the fibre length in the
direction of tensile stress. Therefore, the fibre orientation factor in a bulk zone where the fibre can rotate freely in all directions is determined from Eq. 3.1. In the case of the fibre being positioned near the boundary zone, due to the wall-effect, the fibre orientation factor should be computed by Eq. 3.2. Furthermore, it is shown that over the boundary layer with a thickness of half the fibre length, fibre orientation factor is 55% of the bulk value (Stroeven and Hu, 2006).

![Diagram of fibre orientation](image)

*Figure 3.7: 2D fibre bridging crack for stress transferring over the leading crack.*

\[
\eta_{2D} = \frac{\int_0^{\pi/2} \cos \theta d\theta}{\int_0^{\pi/2} d\theta} = \frac{2}{\pi} \quad (3.1)
\]

\[
\eta_{2D} = \frac{\int_0^{\pi/2} \sin \theta d\theta + \int_0^{\alpha} \cos \theta \sin \theta d\theta}{\int_0^{\pi/2} d\theta} = \frac{2}{\pi} (\cos \alpha - \cos \alpha \ln \cos \alpha) \quad (3.2)
\]

with,

\[
\cos \alpha = \frac{t_c}{l_f} \quad (3.3)
\]

where, \(\theta\) is the angle between fibre and crack line, \(\alpha\) is the limiting value of fibre orientation in boundary zone and \(t_c\) is distance to the nearest exterior surface of concrete prism.

3.3.1.3 Three-dimensional case

In 3D portion, fibres are oriented with an isotropic uniform random distribution, since they can rotate freely in a three-dimensional system, see Figure 3.8. In this figure, it is
assumed that $xy$ plane represents crack plane while the tensile stress is applied in the direction of the $z$-axis. According to Stroeven and Hu (2006) the average projection factor, $\eta_{3D}$, is determined by the mean of the fibre length in the direction of tensile stress. In the bulk zone, fibre orientation factor is determined as follow:

$$\eta_{3D} = \frac{\int_0^{\pi/2} \cos \theta \sin \theta d\theta}{\int_0^{\pi/2} \sin \theta d\theta} = \frac{1}{2}$$

(3.4)

Figure 3.8: 3D fibre bridging crack for Stress transferring over the leading crack.

where, the trigonometric term $\cosine$ is the relative probability of fibre intersecting with crack, and term $sine$ represents the relative spatial frequency. Generally, in 3D bulk, the fibre is identified by two angles, namely $\theta$ and $\phi$ which $\theta$ is corresponded to the tensile stress direction while $\phi$ defines orientation of fibres in space. Since the second angle, $\phi$, will lead to the same integration in nominator and denominator, it has no effect on the orientation factor. However, similar to the 2D case, the boundary condition influences the fibre orientation factor in the same way. Therefore, Eq. 3.5 should be used for calculating fibre orientation factor in boundary zone.

$$\eta_{3D} = \frac{1}{2} \left( 1 - \frac{2\alpha}{\pi} - \frac{1}{\pi} \sin 2\alpha + \frac{4}{\pi} \cos \alpha \ln \tan \left( \frac{\alpha}{2} + \frac{\pi}{4} \right) \right)$$

(3.5)

The limit values of Eq. 3.5 are: 0.5 for the bulk case and 0 at the specimen’s surface. Most of the boundary effects are limited to a zone half the fibre length in width. It is proved experimentally that fibre orientation factor over the boundary zone with half fibre length is 61% of the bulk value (Stroeven and Hu, 2006).
3.3.2 Fibre orientation profile

In FRC, the average orientation of the fibres is generally distinguished by the so-call orientation factor, $\eta$. The limit values of the orientation factor are: 0 to 1 for the fibres parallel or orthogonal to the analyzed plane, respectively. From the investigation on the experimental data, Laranjeira et al. (2010) illustrated that fibre orientation follows a Gaussian law, therefore the distribution and average orientation of the fibres are associated with each other. According to this research, in order to determine fibre orientation profile, the following steps should be carried out:

1. Determination of the orientation factor, $\eta$, and selection of the grouping intervals, $\Delta \theta$.
2. Identification of the approximated average orientation angle, $\theta_m$ from Eq. 3.6 and the corresponded standard deviation of orientation angle through Eq. 3.7.

$$\theta_m = \arccos(\eta) \times \frac{180}{\pi}$$  \hspace{1cm} (3.6)

$$\sigma(\theta) = 90 \times \eta \times (1 - \eta)$$  \hspace{1cm} (3.7)
3. Calculation of the standard Gaussian orientation angle at the extreme values of the intervals considered.

$$\theta_s = \frac{\theta - \theta_m}{\sigma(\theta)}$$  \hspace{1cm} (3.8)
4. Determination of the cumulative distribution values of the standardized Gaussian law, $F(\theta_s)$, and adaptation into respective cumulative values of Gaussian distribution, $F(\theta)$.
5. Identification of the relative frequency of each interval, $f(\theta_{ia})$, at the respective class marks, $\theta_{ia}$.
6. Application of the factor accounting for the range error, $F_{ER}(\theta)$, over $f(\theta_{ia})$ in order to obtain the complete orientation profile, $P(\theta_{ia})$. 

57
3.3.3 Inhomogeneity and anisometry

In a FRC if it is observed anisometry of the fibre distribution and orientation, consequently, having in mind that the overall composite behaviour is affected by the fibre orientation towards the principal stresses, it is logic to assume inhomogeneity within mechanical properties, i.e. material anisotropy. For example, experimental results revealed that fibres tend to segregate in plane perpendicular to the gravity field during compaction (Stroeven, 1979). Segregation can have negative effect on the mechanical behaviour of FRC where uniform stress is applied over the whole cross-section of structural element. From another point of view, it has also positive effect in the case of the flexural beams since most fibres concentrate in the tension part. However, it should not be ignored that fibre segregation improves positive bending moment since, for negative moment, strength is decreased (Stähli and Custer, 2007).

Previous research showed that the concrete toughness is influenced by the number of effective fibres in the crack section. However, the number of the effective fibres depends on the fibre dosage, fibre orientation and the length efficiency factor (Stroeven, 1991). Krenchel (1975) showed that the number of fibres can be calculated as follow:

\[ N_f = \eta \frac{V_f}{A_f} \]  

(3.9)

where, \(N_f\) is the number of fibres per unit surface, \(\eta\) is the fibre orientation factor, \(V_f\) is the fibre volume fraction and \(A_f\) is the cross-section of a fibre.

An analytical framework, based on the geometrical probability theory, was proposed by Stroeven (1979) to facilitate the analysis of heterogeneous with respect to location (segregation) and orientation (anisometry) of random fibre structures. It was shown that the number of fibres in a volume of concrete where the projection plane is either parallel or perpendicular to the axis of symmetry can be calculated by Eqs. 3.10 and 3.11, respectively (Stroeven, 1979).

\[ N_{f}^{\parallel} = \frac{1}{2} L_{V} \left[ 1 + 2 \frac{l}{l_f} + \left( \frac{4}{\pi} - 1 \right) \omega \right] \]  

(3.10)

\[ N_{f}^{\perp} = \frac{1}{2} L_{V} \left[ 1 + 2 \frac{l}{l_f} - \omega \right] \]  

(3.11)
with
\[ L_V = \frac{4V_f}{\pi d^2} \]  
(3.12)

where, \( V_f \) is fibre volume fraction, \( d \) is the fibre diameter, \( L_V \) is the total fibre length in a unit volume of concrete, \( l_f \) is the length of a fibre and \( \omega \) is the degree of orientation.

In the case of the analysing just a cross sectional plane, \( t \to \infty \), Eqs. 3.10 and 3.11 for determining the fibres parallel or perpendicular to the axis of symmetry are simplified into Eqs. 3.13 and 3.14, respectively.

\[ N^\parallel_f = \frac{1}{2} L_V \left[ 1 + \left( \frac{4}{\pi} - 1 \right) \omega \right] \]  
(3.13)

\[ N^\perp_f = \frac{1}{2} L_V (1 - \omega) \]  
(3.14)

According to Stroeven (1999), the following two formulas can be used to estimate the composite strength:

\[ \sigma^\parallel = \sigma_m (1 - V_f) + \frac{1}{6} \alpha \tau V_f (1 + 0.5 c V_f) \]  
(3.15)

\[ \sigma^\perp = \sigma_m (1 - V_f) + \frac{1}{6} \alpha \tau V_f (1 - c V_f) \]  
(3.16)

where \( \alpha \) and \( \tau \) are fibre aspect ratio and interface bond strength, respectively. Whereas \( c \) is a constant that is equal to the ratio between the degree of orientation and the fibre volume fraction \( c = \frac{\omega}{V_f} \) and \( \sigma_m \) is the tensile strength of the plain concrete.

### 3.3.4 Geometrical boundary effects on fibre distribution and orientation

The rigid surfaces of the mould pose a physical constraint to the free fibre positioning, thus causing disturbances in fibre dispersion and orientation. Several researchers proposed numerical methods to evaluate the effects of boundary on the fibre orientation factor (Krenchel, 1975; Soroushian and Lee, 1990; Dupont, 2005). In the case of adopting a fibre with a large length compared to the structural element’s dimensions, the fibre orientation factor is strongly affected by nearby boundaries. Otherwise, if the dimensions
of the structural element become considerably larger than the fibre length, the fibres can be oriented freely in all directions without any limitation. Therefore, the effect of the near boundaries reduces significantly. It was shown that the effect of the boundary on the fibre orientation factor became more significant when the dimensions of structural element are smaller than five times of the fibre length (Krenchel, 1975; Soroushian and Lee, 1990). Furthermore, according to the investigation by Stroeven (2006) most of the boundary zones are limited to a zone half the fibre length in width.

Some researchers proposed different methods for determination of fibre orientation factor, \( \eta \) (Krenchel, 1975; Soroushian and Lee, 1990; Stroeven, 1991). The cross section of a fibre reinforced concrete beam can be divided into three distinct zones (Figure 3.9(a)). In the first zone, fibres can rotate in all directions freely that means fibre is in bulk. In the second zone exists one boundary, i.e. one mould wall that is parallel to the direction of the plane in which the orientation factor is defined. Finally, the third zone has two boundary conditions meaning that fibre is located in the corners of the mould.

\[
\begin{align*}
\eta &= \frac{2}{h} \left( \frac{l_f}{2} \right) \left( \frac{h_f}{2} \right) \left( \frac{l_f}{2} \right) \\
&= \frac{2}{h} \left( \frac{l_f}{2} \right) \left( \frac{h_f}{2} \right) \left( \frac{l_f}{2} \right)
\end{align*}
\]

Figure 3.9: Determination of three zones in a prismatic specimen, where \( b, h, \ell_f \) and ND are the width, the height of beam, the fibre length and the notch depth: (a) cross section of a beam; (b) notched cross section.

According to Krenchel (1975), the overall orientation factor of the cross section is computed as follow:
\[
\eta = \frac{\eta_1(b-l_f)(h-l_f) + \eta_2[(b-l_f)l_f + (h-l_f)l_f] + \eta_3l_f^2}{bh} \tag{3.17}
\]

where, \(\eta_1\), \(\eta_2\) and \(\eta_3\) are the orientation factors in zones 1, 2 and 3, respectively. Dupont (2005) extended the previous equation for the case of a notched cross section. In the presence of a notch two situations can arise. If the notch depth is less than \((l_f)/2\), two new areas \(\eta_4\) and \(\eta_5\) should be defined, Figure 3.9(b), so the orientation factor is determined of Eq. 3.18.

\[
\eta = \frac{\eta_1(b-l_f)(h-l_f) + \eta_2[(b-l_f)l_f/2 + (h-l_f)l_f] + \eta_3l_f^2/2 + (l_f/2 - ND)[\eta_4(b-l_f) + \eta_5l_f]}{b(h-ND)} \tag{3.18}
\]

with, \(\eta_1\), \(\eta_2\), \(\eta_3\) are 0.5, 0.6, 0.84 and independent of the fibre length and \(ND\) is the notch depth. Since the average orientation coefficients \(\eta_4\) and \(\eta_5\) are dependent on the fibre length, they should be calculated from Eqs. 3.19 and 3.20.

\[
\eta_4 = \frac{1}{(l_f/2 - ND)} \int_{s_2}^{l_f/2} \left[ \int_{0}^{\arcsin \left( \frac{2\sqrt{2}l_f}{l_f} \right)} A_4 \cos \theta d\theta + \int_{\arcsin \left( \frac{2\sqrt{2}l_f}{l_f} \right)}^{\pi/2} A_2 \cos \theta d\theta \right] dy \tag{3.19}
\]

\[
\eta_5 = \frac{\int_{s_1}^{l_f/2} \left[ \int_{0}^{\arcsin \left( \frac{2\sqrt{2}l_f}{l_f} \right)} A_4 \cos \theta d\theta + \int_{\arcsin \left( \frac{2\sqrt{2}l_f}{l_f} \right)}^{\pi/2} A_2 \cos \theta d\theta + \int_{\arcsin \left( \frac{2\sqrt{2}l_f}{l_f} \right)}^{\pi/2} Max[0; A_3] \cos \theta d\theta \right] d\zeta}{\left( \left( l_f/2 \right) \left( l_f/2 - ND \right) \right)} + \tag{3.20}
\]
with \( y \) and \( z \) are distances of the gravity point of fibre from the edge of the mould and \( \theta \) is the angle of the fibre with longitudinal axis.

On the other case, if the notch depth is larger than \((l_f)/2\), the overall orientation coefficient should be computed by:

\[
\eta = \frac{\eta_1(b-l_f)(h-l_f/2-ND)+\eta_2((b-l_f)l_f/2+(h-l_f/2-ND)l_f)+\eta_3l_f^2/2}{b(h-ND)} \tag{3.21}
\]

Finally, the theoretical number of fibres in a rectangular cross section can be calculated by Eq. 3.22:

\[
N = \frac{V_f \times \eta \times b \times h}{\pi d_f^2 \times \rho_s} \tag{3.22}
\]

where, \( N \) is the theoretical number of fibres in the cross section, \( V_f \) is the amount of fibres in concrete [kg/m\(^3\)], \( \eta \) is the orientation coefficient which should be determined from Eqs. 3.17, 3.18 or 3.21, \( b \) is the width of the beam section [mm], \( h \) is the height of the remaining ligament of the beam section [mm], \( d_f \) is the diameter of a fibre [mm], \( \rho_s \) is the density of the steel of the fibres [kg/m\(^3\)].

In spite of the relevant work by Dupont (2003), this procedure is only applicable in the case of the isotropic condition assumption. Thereby, Laranjeira (2010) proposed another method in order to determine the orientation factor in isotropic and anisotropic conditions which was adapted to any orientation factor in the bulk zone \((\eta_b)\). A summary of the results is illustrated in Table 3.2.

Basically, combination of two factors are required to evaluate the reduction in stress transfer capacity in the boundary zone, namely the reduction in the number of fibres in the crack plane and the effects of the external boundary on the embedment length. However, in the boundary zone the magnitude of strength reduction is of the order of 10%. Basically, the average values of the stress transfer capability over a boundary zone of half the fibre length for surfaces parallel and perpendicular to the orientation plane of the 2D portion is determined by (Stroeven, 2006):

\[
\sigma_\parallel = \sigma_m(1-V_f) + \frac{1}{3}aV_f\tau_f\bar{\rho}_b\left(1+\frac{1}{2}\omega\right) \tag{3.23}
\]
\[ \sigma_\perp = \sigma_m (1-V_f) + \frac{1}{3} a V_f \tau_f \bar{\eta}_{b3} (1-\omega) \]  

(3.24)

where, \( \sigma_m \) represents the plain matrix strength, \( \bar{\eta}_{b3} \) is stress transfer efficiency parameter in the boundary layers, which for surfaces parallel to the orientation plane of the 2D portion is 0.203, and \( a V_f \tau_f \) is called fibre factor, in which \( a \) is the aspect ratio, \( V_f \) is the volume fraction and \( \tau_f \) is the fibre and matrix friction resistance.

Table 3.2: Increments of \( \eta \) for any \( \eta_i \) and common cross section geometries.

<table>
<thead>
<tr>
<th>Cross-section Geometry</th>
<th>Type of concrete</th>
<th>Increment of ( \eta ) due to wall-effects (( \eta \Delta w ))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangular</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B = Width</td>
<td>Vertical SCC</td>
<td>( \Delta \eta_w = \frac{l_f}{B} \left( 0.677 - 0.730 \eta_0 \right) )</td>
</tr>
<tr>
<td>H = Height</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l_f ) = Fibre</td>
<td>Horizontal SCC</td>
<td>( \Delta \eta_W = \frac{2}{B \times H} \left[ \frac{B \times H}{l_f} \left( 0.456 - 0.730 \eta_0 \right) + 0.533 \eta_0 - 0.127 \right] )</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td></td>
</tr>
<tr>
<td><strong>Circular</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D = Diameter</td>
<td>Vertical SCC</td>
<td>( \Delta \eta_w = \frac{l^* \left( 2D - l^* \right)}{D^2} \left( 0.677 - 0.730 \eta_0 \right) )</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Horizontal SCC</td>
<td>( \Delta \eta_w = \frac{l^* \left( 2D - l^* \right)}{D^2} \left( 0.465 - 0.730 \eta_0 \right) )</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td></td>
</tr>
</tbody>
</table>

\[ l^* = l_f + \Delta l \]

\[ \Delta L = D \times \left[ 1 - \cos \left( \frac{a \sin \left( \frac{l_f}{D} \right)}{D} \right) \right] \]
In the case of the 3D portion the stress transfer capability can be calculated by Eq. 3.25:

\[ \sigma_{3D} = \eta_{b3} a V_f \tau_f \]  

where, \( \eta \) is calculated as follow:

\[ \eta_{b3} = \cos^3 a - 2 \cos^2 a + 2 \ln 2 \cos a \quad \text{for} \quad \frac{\pi}{3} \leq a \leq \frac{\pi}{2} \]  

\[ \eta_{b3} = -\frac{1}{3} \cos^3 a - 2 \cos^2 a - \cos a - 2 \cos a \ln \cos a - \frac{1}{3} \quad \text{for} \quad 0 \leq a \leq \frac{\pi}{3} \]  

### 3.4 Parameters of the fibre structure

The fibre efficiency can be improved, mainly, by two distinct ways. Within the first group, fibre efficiency can be improved by enhancing the mechanical performance of the fibre, e.g. bond properties between fibre/matrix. This can be achieved either by chemical or mechanical treatment of the fibre surface. The more emphasis way of improving the fibre efficiency lies in the fibre structure. On the other hand, the fibre structure corresponded to the distribution of the fibres along an element uniformly, segregation of the fibres over the depth of an element, minimum and effective fibre content, fibre spacing and effectiveness of fibre orientation. However, these parameters may reduce or enhance the mechanical behaviours of the composite materials considerably (Babut, 1986).

#### 3.4.1 Effective fibre content

It is recommended that the effective fibre content should be determined in the cross-sections parallel and perpendicular to the direction of vibration in the case of FRC and to the direction of flow in the case of SFRSCC. With respect to the nominal fibre content, the effective number of fibre intersections, \( N_{f}^{3D} \), can be computed from a formula as follow (Kasperkiewicz, 1977):

\[ N_{f}^{3D} = \frac{2V_f}{\pi d^2} \times 100 \]  

64
where, \( d \) and \( V_f \) are diameter of fibre [in mm] and fibre content [per percent by volume], respectively. Moreover, by assuming a three-dimensional isotropic uniform random fibre distribution, the theoretical number of fibre also can be determined by using Eq. 3.29 and accepting \( \eta=0.5 \) relating to the orientation factor for the theoretical 3D isotropic uniform random distribution. Also, \( A_f \) is the cross section of a single fibre [mm\(^2\)] (Soroushian and Lee, 1990).

\[
N_f^{3D} = \eta \frac{V_f}{A_f}
\]  

(3.29)

### 3.4.2 Minimum fibre content

One of the most important properties of fibre reinforced concrete (FRC) is its ability to transfer stresses across a cracked section uniformly (fibre reinforcement effectiveness). Therefore, in the terms of mechanical properties, minimum fibre content may have the similar meaning to that of strength in plain concrete. According to Babut (1986), the scatter in results between theoretical and numerical effective number of fibres can be described by means of coefficient of variation. On the other hand, the minimum value of effective number of fibres among the examined cross-sections parallel to the direction of vibration, or flow, in a specimen is known as minimum fibre content \((\bar{N}_{f}^{\parallel})\). Consequently, the relative distance between the minimum value \( \bar{N}_{f}^{\parallel} \) and the expected theoretical value \( N_f^{3D} \) is regarded as an effective reduction of the fibre content (\( \gamma \)) and calculated as follow:

\[
\gamma = \frac{N_f^{3D} - \bar{N}_{f}^{\parallel}}{N_f^{3D}}
\]  

(3.30)

### 3.4.3 Degree of orientation of the fibre structure

In fibre reinforced concrete, it is assumed that fibres have a spatial orientation that can be categorized in two ideal situations, namely 3-dimensional (3D) and 2-dimensional (2D) orientation. The below expression can be used to determine the degree of orientation:

\[
\omega_{2.3} = \frac{N_f^{\parallel} - N_f^{\perp}}{N_f^{\parallel} + 0.273N_f^{\perp}}
\]  

(3.31)
where, $N^\parallel_j$ and $N^\perp_j$ are the effective number of fibre per unit area determined from two orthogonal planes, e.g. parallel and perpendicular to the direction of the vibration/flow. In the case of an ideal 3-dimensional fibre orientation the value of $\omega_1$ is defined as 0.0. This specifies that all fibres are aligned with the same probability in all spatial directions. For the fully planar fibre orientation the value of $\omega_2$ is 1.0 (Babut, 1986).

### 3.4.4 Segregation of the fibres

Segregation of the fibres can have negative effect on the mechanical behaviour of fibre reinforced concrete element where uniform stress is applied over the whole cross-section. From another point of view, it has also positive effect in the case of the flexural beams since most fibres concentrate in the tension part. However, during compaction, fibres tend to segregate because of the variety in volume density of the composing materials (Stroeven, 1979). Babut (1982) proposed a very simple method to determine segregation of the fibres. In this method, the fibre contents in both top and bottom of the analyses cross section are compared. Afterward, in 1986, he offered another method which determined an indicator of the degree of segregation $\xi_{seg}$. It is based on the calculating X- and Y-coordinates of the entire fibre junctions in the image plane.

$$\xi_{seg} = \frac{\bar{y}}{h}$$  \hspace{1cm} (3.32)

where, $\bar{y}$ is the coordinate of the center of gravity of the reinforcement and $h$ is the height (or depth) of the analysed cross-section parallel to the direction of vibration. For calculating the location of the center of gravity of the steel fibres, an average value of the coordinates of entire fibre intersections in an analysed cross-section should be determined. However, the value of $\xi_{seg}$ tends to change in the interval of 0 (segregation at the top of the cross-section) and 1 (segregation at the bottom of the cross-section). In the ideal fibre distribution this value is 0.5.

### 3.4.5 Fibres spacing

Often, it is assumed that fibres are distributed in matrix uniformly. However, the mechanical properties of FRC are affected by dispersion of the fibres in a high degree.
Hence, the spacing between fibres or from another point of view distribution of the fibres in a matrix uniformly becomes significant in the fibre structure. Stroeven (1979) suggested two formulas based on the geometric probability theory to determine the fibre spacing, in which case they contain only geometric parameters and the volume fraction. For a random dispersion of fibres, the experimental specification of the spacing by means of sectioning is determined by:

\[
\overline{\Delta}_2 = 0.5N_f^{-1/2}
\]

\[
\overline{\Delta}_3 = 0.554 \left( \frac{2N_f}{l_f} \right)^{-1/3}
\]

where, \(\overline{\Delta}_2\) and \(\overline{\Delta}_3\) are the average nearest neighbour distances between randomly distributed points in a plane or, alternatively, in space respectively (Figure 3.10). \(N_f\) is the number of fibres and \(l_f\) represents the length of the fibre.

\[ \text{(3.33)} \]

\[ \text{(3.34)} \]

For design purpose, the following formulas are proposed:

\[
\overline{\Delta}_2 = \frac{0.63l_f}{(a^2V_f)^{1/2}}
\]

\[
\overline{\Delta}_3 = \frac{0.51l_f}{(a^2V_f)^{1/3}}
\]

where \(a\) and \(V_f\) are the aspect ratio and the volume fraction, respectively.

\[ \text{(3.35)} \]

\[ \text{(3.36)} \]
In the case of image analysis by means of projections (X-ray radiography) Eqs. 3.33 and 3.34 were modified as follow:

\[
\bar{X}_2 = 0.707 \left( \frac{t/l_f + 0.5}{N_f} \right)^{1/2} \tag{1.37}
\]

\[
\bar{X}_3 = 0.554 \left( \frac{t + 0.5l_f}{N_f} \right)^{1/3} \tag{1.38}
\]

with \( t \) is thickness of the specimen.

### 3.5 Methods for analysing the structure of FRC

There are several methodologies for evaluating fibre orientation and dispersion in FRC. The existing techniques to evaluate orientation of fibres in FRC are mainly grouped in to two types of methods, namely destructive and non-destructive methods, which are based on direct and indirect measurements (Barnett et al., 2010).

#### 3.5.1 Destructive methods

In the fresh concrete stage, wash out test has been proposed to evaluate fibre concentration. In this method, a sample of fresh concrete is sieved and washed out; steel fibres are separated from the aggregates using a magnet. Obviously, this technique can only be applied to metallic fibres. The number of fibres represents average fibre concentration in mixture (Vandewalle and Dupont, 2003). In a more accurate and reliable method, the number of fibres are counted in a cross-section of a hardened concrete element. This method presents low cost, since it can be performed after mechanical testing (Barros et al., 2005; Cunha, 2010; Oliviera, 2010). Direct and destructive methods for fibre orientation can be obtained through techniques such as computerized tomography (CT-scan) (Stähli and Custer, 2007); image analysis (Ferrara et al., 2011) and x-ray method (Barnett et al., 2010). Mostly, these techniques are limited to laboratory and research purpose due to the restriction in sample size, complexity and cost of these methods.
Computerized tomography or CT-scan is based on the method of radioscopy. The sample is located between an x-ray resource and a detector, so a two-dimensional radioscopy image can be achieved. One of the most important advantages of this method is producing three-dimensional (3D) reconstruction of the components inner specimen by taking two-dimensional (2D) images at various angles, see Figure 3.11. However, reconstruction is a mathematical process to combine 2D images and construct a 3D model of an object (Hausherr et al., 2006; Krause et al., 2010; Skyscan, 2001).

![Figure 3.11: Reinforced-concrete CT scan has been processed with VGStudio Max 2.0: (a) original; (b) CT scan; (c) extracted steel fibres; (d) slice along cylinder axis; (e) slice perpendicular to cylinder (Krause et al., 2010).](image)

In image analysis method, an image of a cut cross section is obtained by a digital photograph camera. Afterward, three main steps should be applied on the acquired photos, namely: pre-processing, segmentation and classification. In the pre-processing step, a reflection mask will be used to remove colour channels and obtain a grey-scale photograph. This step is not mandatory, but the segmentation procedure is simpler in grey scale images. In the next step, segmentation of the desire object will be obtained using a threshold function which is nonetheless in a grey scale limit. Finally, in the classification process, the effects of other constitutive materials which have the same reflective properties to the desire object will be eliminated, Figure 3.12. However, the main advantage of this method is simplicity and low cost (Cunha, 2010).

![Figure 3.12: Image analysis procedure of the concrete’s surface: (a) pre-processing, i.e. applied mask; (b) segmentation procedure; and (c) classification procedure (Cunha, 2010).](image)
3.5.2 Non-destructive methods

Recently, some progresses in new non-destructive methods have been achieved to determine orientation and dispersion of steel fibres in FRC. Among these methods can be pointed out the electrical resistivity (Barnett et al., 2010), ultrasound and quantitative acoustic emission technique (Reinhardt et al., 2001) and magnetic approach (Faifer et al., 2011).

Electrical resistivity method is based on the measurement of electrical resistivity of concrete which being influenced by steel fibres. Lataste et al. (2008) performed this non-destructive test with a four probes device, in which two electrodes (A and B, see Figure 3.13) input a known electrical current. This current will pass through concrete due to the potential difference created between the two types of probes. The measured resistivity depends on how the electric charge flows from electrodes A and B to the two other electrodes, M and N. Finally, by moving the device’s probes in various orientations and positions, resistivity variations are recorded, and the electrical properties of the concrete can be obtained and mapped.

![Image of measurement device](https://via.placeholder.com/150)

Figure 3.13: View of the measurement device and probes position (Lataste et al., 2008).

The alternate current-impedance spectroscopy (AC-IS) approach is based on the current conductivity of SFRC. In this method, two stainless steel circular electrodes are installed on concrete specimen and the current passing through is measured, see Figure 3.14. To obtain spectroscopy map of the specimen, this process should be repeated by changing the location of the electrodes in various directions. Finally, the direction with higher passing current represents the orientation of fibres (Ozyurt et al., 2006).
Ultrasound transmission is used to determine the Young modulus, density, crack depth and strength of harden concrete. The method consists of measuring the time of travel of an ultrasonic pulse passing through concrete’s sample (Reinhardt et al., 2001). A pulse is generated by a transducer located on one side of concrete and collected by a receiver transducer from other side (Figure 3.15(a)). Then, time of travel is measured and processed by a data acquisition system. Regarding the output results, generally, higher velocities are obtained when concrete’s quality is better in terms of density, uniformity, homogeneity, etc.

Magnetic approach, which was recently proposed, is a non-destructive in-situ test for the determination of fibre density and their average orientation. This method is based on the employment of a probe which is sensitive to magnetic properties of the steel fibres (Figure 3.15(b)).

Mathematical and semi-empirical models are also available for the evaluation of fibre orientation factor (Dupont, 2005; GrÄunewald, 2004). The evaluation of fibre orientation factor allows establishing the stress-crack opening relationship, which simulates the overall post-cracking behaviour of SFRSCC, based upon the knowledge of the micro-mechanical behaviour of a single fibre.
Figure 3.15: (a) Ultrasound transmission of cube with embedded fibre (Reinhardt et al., 2001); (b) simulation of the flux density of an example frame (Faifer et al., 2011).

3.6 Orientation and distribution measurement by image analysis

The orientation of an individual fibre can be identified in terms of the two angles $\theta$ and $\varphi$, see Figure 3.16. By the Cartesian components of a vector $p$ which is oriented parallel to the fibre, the orientation of a single fibre can be expressed. These components are defined by:

\[
\begin{align*}
p_1 &= \sin \theta \cos \varphi \\
p_2 &= \sin \theta \sin \varphi \\
p_3 &= \cos \theta
\end{align*}
\]  

(3.39)

Figure 3.16: The orientation of a single fibre can be expressed in polar coordinates by the two angles $(\theta, \varphi)$ or in Cartesian coordinates by the components of a vector $p$, $(p_1, p_2, p_3)$. 

72
Normally, the probability distribution function (PDF), $\psi(\theta, \phi)$, is used to describe the orientation state at a point in space. It means, the probability which a fibre is oriented between the angles $\theta_1$ and $\theta_1 + d\theta$, $\phi_1$ and $\phi_1 + d\phi$ is defined as follow,

$$p(\theta_1 \leq \theta \leq \theta_1 + d\theta, \phi_1 \leq \phi \leq \phi_1 + d\phi) = \psi(\theta_1, \phi_1) \sin \theta_1 d\theta d\phi$$

(3.40)

Since $\psi(\theta, \phi)$ is a density function, the following normalization condition is valid.

$$\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \psi(\theta, \phi) \sin \theta d\theta d\phi = 1$$

(3.41)

The orientation distribution function is well known as a complete method to obtain the fibre orientation profile but, from another point of view, the practical application of this function is somehow complicating and time consuming (Advani and Tucker, 1987; Eberhardt and Clarke, 2001). Therefore, Advani and Tucker (1987) suggested a more practical and simplified method to state orientation of fibres by identifying orientation tensors instead of orientation distribution function. The determinations of orientation tensors are much easier, and they give succinct information about fibre orientation density (Ferrara et al., 2011). From moments of the orientation distribution function, orientation tensors can be achieved, and, generally, the second orders are used to determine orientation density which is calculated as follow:

$$a_{ij} = \sum \frac{(p_i'p_j')}{\sum F_n} F_n$$

(3.42)

In this equation, $a_{ij}$ is the component of the orientation tensor, $p'_i$ and $p'_j$ state for the components of unit vector $p$, $F$ represents the weighting function and the subscript $n$ illustrates the number of fibres. The probability that a fibre is intercepted by the cross section under consideration is described by weighting function. Obviously, the probability of intercepting a fibre which is oriented vertically or horizontally to the cutting section is 1 or 0, respectively. In order to obtain fibre orientation state, each component should be weighted by Eqs. 3.43 and 3.44. where, $l_f$ represents fibre length, $\theta'$ shows the local out-of-plane angle, $\theta'_c$ states the cutoff angle that can be described as the angle above which a fibre has a useful projected height of fibre diameter $d$. Finally, the relationship between the local ($\theta', \phi'$) and global ($\theta, \phi$) fibre orientation angles is given in Table 3.3.
\[ F_n = \frac{1}{l_f \cos \theta'_n} \quad \text{for} \quad \theta' < \theta'_c \] (3.43)

\[ F_n = \frac{1}{d} \quad \text{for} \quad \theta' > \theta'_c \] (3.44)

\[ \theta'_c = \cos^{-1}\left( \frac{d}{l_f} \right) \] (3.45)

<table>
<thead>
<tr>
<th>Section plane</th>
<th>Local coordinates on the section plane</th>
<th>Vector components</th>
</tr>
</thead>
<tbody>
<tr>
<td>XY</td>
<td>( x' = x )</td>
<td>( p_x = \sin \theta' \cos \phi' )</td>
</tr>
<tr>
<td></td>
<td>( y' = y )</td>
<td>( p_y = \sin \theta' \sin \phi' )</td>
</tr>
<tr>
<td></td>
<td>( z' = z )</td>
<td>( p_z = \cos \theta' )</td>
</tr>
</tbody>
</table>

| XZ            | \( x' = x \)                           | \( p_x = p_z = \sin \theta' \cos \phi' \) |
|               | \( y' = -z \)                          | \( p_y = p_z = \cos \theta' \) |
|               | \( z' = y \)                           | \( p_z = -p_y = -\sin \theta' \sin \phi' \) |

| YZ            | \( x' = y \)                           | \( p_x = p_z = \cos \theta' \) |
|               | \( y' = z \)                           | \( p_y = p_z = \sin \theta' \cos \phi' \) |
|               | \( z' = x \)                           | \( p_z = p_y = \sin \theta' \sin \phi' \) |

Table 3.3: Calculation of global fibre orientation angles.

The most popular method to determine the fibre orientation factor is to provide cross-sections in various directions. In the case of providing a cross-section of FRC with circular fibres, two different shapes of cut fibres are appeared. If the section plane is
perpendicular to a single fibre, it appears like a circle while if it is at an angle to the section plane, it visible similar to an ellipse. In the case of the ellipse shape of a single fibre, the out-of-plane orientation, $\theta$, can be computed by the major ($a_f$) and minor ($b_f$) axis lengths of its elliptical cross-section, see Figure 3.17(a), (Eberhardt and Clarke, 2001):

$$\theta = \cos^{-1}(b_f / a_f)$$  \hspace{1cm} (3.46)

The probability that a randomly section plane will cross a single fibre with limit length is a function of its length ($l_f$), diameter ($d$), and the angle it makes with the section plane. As it is illustrated in Figure 16(b), the probability of intersection is computed by (Zhu et al., 1997):

$$p(\theta) = l_f \cos \theta + d \sin \theta$$  \hspace{1cm} (3.47)

![Figure 3.17](image)

*Figure 3.17:* (a) The orientation of an ellipse in to a cross-section; (b) the probability that a fibre intersect a random plane section.

Moreover, by assuming a three-dimensional isotropic uniform random fibre distribution, the theoretical fibre density also will be computed by using Eq. 3.48 and accepting $\eta = 0.5$ relating to the orientation factor for the theoretical 3D isotropic uniform random distribution (Soroushian and Lee, 1990):

$$N_f = \eta \frac{V_f}{A_f}$$  \hspace{1cm} (3.48)

with $V_f$ is fibre content per percent per volume, $\eta$ and $A_f$ are, respectively, a dimensionless orientation factor and the cross sectional area of a single fibre [mm$^2$].
Fibre orientation also is calculated by obtaining orientation factor $\eta$ in two different approaches. In the first method, the orientation factor $\eta_{\text{img}}$ is calculated by using Eq. 3.49 based on the image analysis procedure of the cross sections. On the other hand, in this method, the orientation factor $\eta_{\text{img}}$ is determined as an average orientation within a cross section.

$$\eta_{\text{img}} = \frac{1}{N_f} \sum_{i=1}^{N_f} \cos \theta_i$$

(3.49)

with $N_f$ the total number of fibres which can be determined by counting all the visible ellipses and circles at the cross section (calculated in previous step), $\theta$ the out-plane angle which is known as the angle between longitudinal axis and the loading direction.

In another method which is proposed by Soroushian and Lee (1990) computing the average orientation factor from Eq. 3.50, based on the counting number of fibres in a certain cross section. This approach is known as fibre effectiveness index by many researchers.

$$\eta_{\text{exp}} = N_f \frac{A_f}{V_f}$$

(3.50)

which, $V_f$ and $A_f$ represents the volumetric fraction of fibre (in %) and the cross sectional area of a single fibre (in mm).

Stroeven (1979) offered another method for determining $\eta$ which is based on the geometric probability concept. In the case of the image analysis by means of sectioning the factor $\eta$ is calculated by Eqs. 3.51 and 3.52:

$$\eta_{\text{exp}} = \frac{l_f}{aV_f}$$

(3.51)

$$\eta_{\text{img}} = \frac{2}{N_f}$$

(3.52)

In Eq. 3.51, $\eta$ depends on the factors which affect the overall mechanical behaviour of FRC. These parameters are volumetric fraction ($V_f$), the length ($l_f$) and aspect ratio ($a = l_f / d$) of fibres. Hence, Eq. 3.51 can be used for design proposed. Furthermore, Eq. 3.52 proposed the appropriate relationship for the evaluation of experimental results.
achieved by image analysis. In this equation, $N_j$ should be determined by counting intersection through coverage with a random system of lines. On the other hand, in this method, the obtained picture should be divided to the equal strips, see Figure 3.18, and the fibre intersections are counted in each separately.

![Figure 3.18: The proposed line system for counting the number of fibres.](image)

In the case of image analysis by means of projections, the value of $\eta$ should be calculated as follow:

$$\eta_{\text{avg}} = \frac{2t}{\pi d N_j} \quad (3.54)$$

where, $t$ is thickness of the slice and $d$ is diameter of the fibre.

### 3.7 Assessment of fibre distribution/orientation profile in SFRSCC panel

In this section, the results regarding to the fibre distribution parameters namely: fibre density, orientation factor and segregation in the SFRSCC panel will be represented and discussed. The mention parameters will be determined from the geometrical data of a well matched ellipse as well as its centre of the gravity for each fibre, by executing image analysis technique on the fracture surface of the tested specimens. Furthermore, the achieved parameters will be used to support the steel fibre reinforced self-compacting concrete’s post-cracking parameters in the subsequent chapter (Chapter 4).
3.7.1 Procedure to assess fibre distribution in SFRSCC panel

To find out correlations between fibre distribution parameters and mechanical properties, such as, residual stresses and energy absorption, it is quite important to determine fibre dispersion and fibre orientation parameters. As it was explained earlier, there are several methods for assessing the fibre distribution and orientation in fibre reinforced composites, namely: tomography (CT-scan), image analysis, x-ray method, electrical resistivity, ultrasound and quantitative acoustic emission technique, and magnetic approach. Among these methods, image analysis technique was chosen due to its simplicity and relatively low cost of the necessary equipments.

3.7.2 Analysis procedure

The adopted procedure for fibre detection comprised four main steps. Firstly, the fracture surface of the specimen was grinded. To enhance the reflective properties of the steel fibres, the surface was polished and cleaned with acetone. Secondly, a coloured image of this surface was taken using a high resolution digital photograph camera (Canon, 12 Megapixel). Afterwards, the obtained image was processed using ImageJ (Rasband, 2008) software to recognize steel fibres. This step comprises of the following tasks: Pre-processing, Segmentation and Classification.

In the first task, since the obtained image was a coloured photograph (16 bits), it was converted to an 8 bits picture in gray scale (Figure 3.19(a)). In the Segmentation task, by defining a threshold and also a mask function a block and white image was achieved, that the circles and ellipses are representative of the fibre cross section classified from the surrounded matrix (Figure 3.19(b) and (c)). In the analysis procedure for each photograph an appropriate value for threshold function was set-up manually.
Figure 3.19: Image processing steps: (a) converting a coloured image to grayscale (b) adjusting a threshold, (c) defining mask, noise (remove small noises) and watershed (separated fibres that are stuck together) functions, (d) fitting the best ellipse to each fibre.

In spite of the executing mentioned procedure in the Segmentation step, still some bright points were observed in images which in reality were not fibres and could destroy the results. Since in the composition, the coarse aggregate was crashed granite one, the mica in the constituent also reflect the flash light, therefore it was observed some non-fibre bright points in the photos. Depends on the size of these points, two different strategies were applied. These strategies eliminated in the Classification procedure. If the clusters are small, they could be eliminated by defining a median filter easily that the best value was set-up manually, see Figure 3.20. However, in the case of the exciting clusters with a size approximately close to the fibre’s diameter, the mentioned strategy cannot be effective. Therefore, by determining the area of a circle (0.237 mm$^2$) which represents a completely perpendicular fibre to the surface, in the Classification step, only points will take in to the consideration with area equal to and bigger than 0.237 mm$^2$. After the Classification process, a visual inspection of the final photograph was done and compare with the initial image. This task is necessary to be executed in order to check if any ellipses consist of more than one fibre due to the cut quality or two fibres touching themselves. Hence it is mandatory to divide the corresponded ellipse to two fibres. This task can be performed by defining watershed function in the image analysis software automatically (Figure 3.20).
Finally, in the post-processing, a best ellipse corresponded to each cluster was fitted (Figure 3.19(d)). For each image, the number of the ellipses, as well as the centre of gravity (segregation factor) and diameters of each individual fitted ellipse (orientation factor) were derived out in a data file.

3.7.3 Selection of the specimens

The image analysis was carried out on the fracture surfaces of the prismatic specimens either from uniaxial tensile test (60×102×110 mm$^3$) or four-point bending test (60×60×240 mm$^3$). The experimental results of the latter tests will be presented in Chapters 4 and 6, respectively. For all the specimens, image analysis was executed after the mechanical tests have been performed. Three series of SFRSCC panels with 60 kg/m$^3$ fibre content were studied. As it was shown in section 3.2.2, since casting panels from the centre point improve tensile behaviours (Barnett et al., 2010), this direction of casting was selected for the production of SFRSCC panels.

For the first attempt, two series of panels with the dimensions of 1600×1000 mm$^2$ in plan and 60 mm of thickness were cast, named as: panel A and B. Cores were extracted from each panel along the concrete flow directions, according to the scheme illustrated in Figure 3.21. In this figure, the pale dash lines with arrows present the supposed concrete flow directions. Then, each core was cut to produce a prismatic specimen with dimensions of 110×102×60 mm$^3$. In the next chapter, these specimens were used for determining $\sigma - w$ relationship by the means of uniaxial tensile test. The image analysis was performed on the specimens after the test had been performed. The location of the studied plane is depicted in Figure 3.22(b). The dispersion and orientation of fibres were
studied in two planes either parallel ($\theta=0^\circ$) or perpendicular ($\theta=90^\circ$) to the concrete flow direction.

![Figure 3.21: Extracted cores used for the execution of image analysis: (a) panel A, (b) panel B.](image)

![Figure 3.22: Localization of the plane surface considered in the fibre distribution assessment of prismatic specimens (dimensions are in mm).](image)

In the next series, a bigger panel with the dimensions of 1500×1500×60 mm$^3$ was cast considering the same method of casting similar to the first series. This panel was designated as panel C. The prismatic specimens were extracted according to the following strategy: by considering $\beta$ as the angle between the direction of the concrete flow and the notched plane direction, four series of prismatic specimens with different notched plane orientations towards the concrete flow directions can be defined. Figure 3.23 depicts a scheme of the adopted classification methodology based on the angle $\beta$. The four intervals established for the angle $\beta$ were [0-15°], [15-45°], [45-75°] and [75-90°]. Later, these beams were tested under four-point bending configuration in order to evaluate the long-
term response as well as the influence of the long-term crack opening on the $\sigma - w$ relationship. The results were presented in Chapter 6.

![Figure 3.23: Classification method of the prismatic specimens.](image)

For each series ($\beta=[0-15^\circ]$, $[15-45^\circ]$, $[45-75^\circ]$ and $[75-90^\circ]$) five specimens from distinct panel locations were selected, see Figure 3.24. The studied specimens were specified by the gray solid hatch. Hereinafter, each specimen was designated by a numeral string, where the first number defines the $\beta$ angle, in degrees, for four intervals of
the relative orientation between the notched plane and the SFRSCC flow lines (7.5: \([0-15^\circ]\), 30: \([15-45^\circ]\), 60: \([45-75^\circ]\) and 87.5: \([75-90^\circ]\)) and the second one shows the specimen number. The image analysis was carried out on a plane parallel to the notch plane with an offset equal to half the length of the used fibre Figure 3.25. The grinded plane was obtained by cutting the specimens after the monotonic four-point (4-P) bending tests have been carried out.

![Figure 3.25: Localization of the plane surface considered in the fibre distribution assessment of beams (dimensions are in mm).](image)

### 3.7.4 Experimental results

After computation and analysis of the image technique results, the following parameters that characterize the fibre structure were derived out:

1. The number of fibres per unit area, \(N_f\);
2. Fibre orientation factor, \(\eta_\theta\);
3. Fibre segregation parameter, \(\xi_{seg}\);

The number of fibres per unit area, \(N_f\), is the ratio between the total number of fibres counted in an image, \(N_f^I\), (counting all the visible ellipses and circles at the cross section) and the total area of the image \(A\):

\[
N_f = \frac{N_f^I}{A} \tag{3.55}
\]

The assessment of the fibre orientation degree at a certain plane can be given by a fibre orientation factor, \(\eta_\theta\), using Eq. 3.49. Based on an image analysis procedure of cut
planes, the ellipses’ axis of an intersecting fibre can be easily determined. Therefore, in this method, the orientation factor $\eta_\theta$ can be regarded as an average orientation towards a certain plane surface.

The last analysed parameter was the fibre segregation along the gravity direction, determined from:

$$\bar{\xi}_{\text{seg}} = \frac{1}{hN_T} \sum_{i=1}^{N_i} \bar{y}$$

(3.56)

where $\bar{y}$ is the coordinate in the Y axis of the centre of gravity of the fibre, and $h$ is the height (or depth) of the analysed cross-section. On the other hand, an average value of the coordinates in the Y axis of entire fibres should be determined in the analysed cross-section.

Furthermore, the number of the effective fibres per unit area, $N_{eff}^f$, was also determined. This parameter could not be assessed through the previous technique, therefore it was appraised by manually counting the fibres that intersected the notched plane of the tested beams and also had the hooked end deformed (in order to be considered a fibre that provided effective reinforcement).

Table 3.4 includes the fibre distribution parameters obtained by image analysis on the plane surface (see Figure 3.22) of the specimens subjected to uniaxial tension test. Within each panel, by assuming the casting point as origin, specimens with the same distance from casting origin are presented in the same row. For each studied distance, the number of fibres was assessed in two perpendicular planes ($\theta = 0^\circ$ and $90^\circ$). From the analysed results, $N_i^f$ and $N_{eff}^f$ were significantly higher at the specimens with $\theta = 0^\circ$, approximately 80% and 254 %, respectively, when comparing to specimens with $\theta = 90^\circ$. This high variation of the fibre distribution in two perpendicular directions could be ascribed to a preferential fibre alignment influenced by the concrete’s flowability. Moreover, the probability that a random section plane crossing a single fibre is a function of the fibre’s length ($l_f$), diameter ($d$), and also the angle that the it makes with the section plane (fibre orientation factor) (Zhu et al., 1997). Since all the fibres have the same aspect ratio, the value of $d$ and $l_f$ are constant, therefore the probability function depends on the fibre orientation factor. On the other hand, the higher orientation factor leads to a higher probability of a single fibre intersecting a section plane. In terms of the fibre orientation
factor, $\eta_\theta$, specimens from series $\theta = 0^\circ$ had higher values than the $\theta = 90^\circ$ series, which means that the fibres are more aligned perpendicular to the plane surface in the $\theta = 0^\circ$ series. It is clearly observable by comparing two pictures in obtained of specimen A6 when the cut plane was parallel and perpendicular to the concrete flow direction. Concerning the fibre segregation factor, $\xi_{seg}$, this parameter can assume values between 0 (segregation at the top of the cross-section) and 1 (segregation at the bottom of the cross-section). In a SFRC with homogeneous fibre distribution, $\xi_{seg}$ is 0.5. From Table 3.4 the obtained average values of $\xi_{seg}$ for the studied cross sectional planes were slightly higher than 0.5, approximately 7.6 to 14.6%. Thus, for the studied self-compacting concrete composition, slightly fibre segregation towards the bottom of the specimen due to the gravity action was observed.

**Table 3.4:** Fibre distribution parameters for the specimens after uniaxial tension test (panels A and B).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Distance [cm]</th>
<th>$N_f$ [fibres/cm$^2$]</th>
<th>$N_{eff}$ [fibres/cm$^2$]</th>
<th>$\eta_\theta$</th>
<th>$\xi_{seg}$</th>
<th>$N_f$ [fibres/cm$^2$]</th>
<th>$N_{eff}$ [fibres/cm$^2$]</th>
<th>$\eta_\theta$</th>
<th>$\xi_{seg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B3</td>
<td>20.0</td>
<td>2.071</td>
<td>1.291</td>
<td>0.827</td>
<td>0.580</td>
<td>1.557</td>
<td>0.405</td>
<td>0.688</td>
<td>0.476</td>
</tr>
<tr>
<td>A4</td>
<td>23.5</td>
<td>1.889</td>
<td>1.356</td>
<td>0.855</td>
<td>0.518</td>
<td>1.430</td>
<td>0.506</td>
<td>0.737</td>
<td>0.510</td>
</tr>
<tr>
<td>C4</td>
<td>32.0</td>
<td>2.036</td>
<td>1.430</td>
<td>0.851</td>
<td>0.555</td>
<td>0.665</td>
<td>0.133</td>
<td>0.630</td>
<td>0.597</td>
</tr>
<tr>
<td>D3</td>
<td>32.0</td>
<td>1.913</td>
<td>0.853</td>
<td>0.775</td>
<td>0.491</td>
<td>1.436</td>
<td>0.415</td>
<td>0.666</td>
<td>0.586</td>
</tr>
<tr>
<td>B4</td>
<td>40.0</td>
<td>1.956</td>
<td>0.851</td>
<td>0.773</td>
<td>0.530</td>
<td>0.506</td>
<td>0.074</td>
<td>0.561</td>
<td>0.643</td>
</tr>
<tr>
<td>A5</td>
<td>46.5</td>
<td>2.220</td>
<td>1.212</td>
<td>0.814</td>
<td>0.479</td>
<td>1.097</td>
<td>0.311</td>
<td>0.672</td>
<td>0.725</td>
</tr>
<tr>
<td>A6</td>
<td>69.5</td>
<td>2.304</td>
<td>1.803</td>
<td>0.866</td>
<td>0.557</td>
<td>0.967</td>
<td>0.132</td>
<td>0.604</td>
<td>0.539</td>
</tr>
<tr>
<td>C6</td>
<td>77.5</td>
<td>2.142</td>
<td>1.303</td>
<td>0.818</td>
<td>0.600</td>
<td>1.232</td>
<td>0.541</td>
<td>0.756</td>
<td>0.485</td>
</tr>
<tr>
<td>D1</td>
<td>77.5</td>
<td>1.921</td>
<td>1.089</td>
<td>0.795</td>
<td>0.532</td>
<td>1.355</td>
<td>0.631</td>
<td>0.760</td>
<td>0.594</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>2.050</td>
<td>1.24</td>
<td>0.820</td>
<td>0.538</td>
<td>1.138</td>
<td>0.35</td>
<td>0.675</td>
<td>0.573</td>
</tr>
<tr>
<td>CoV (%)</td>
<td></td>
<td>7.16</td>
<td>23.74</td>
<td>4.15</td>
<td>7.33</td>
<td>31.98</td>
<td>57.11</td>
<td>10.20</td>
<td>14.00</td>
</tr>
</tbody>
</table>
Figure 3.26: Digital images of fibre distribution for A6 specimens: (a) $\theta=0^\circ$ and (b) $\theta=90^\circ$.

Table 3.5 depicts the abovementioned parameters in the case of the prismatic specimens after testing in monotonic four-point (4-P) bending test. $N^f$ and $N_{eff}^f$ were higher in the specimens with the notched plane parallel to the expected concrete flow direction, confirming the results already obtained in this respect. In general, these values decreased as the notched plane rotated towards the perpendicular position regarding the flow direction. For the case of the notched plane orientation, $\beta$, comprised within $[0-15^\circ]$, $N_{eff}^f$ was approximately 76, 156 and 686% higher than when $\beta$ was between $[15-45^\circ]$, $[45-75^\circ]$ and $[75-90^\circ]$ intervals, respectively. Regarding to the low value of $N_{eff}^f$ compared to $N^f$ it should be mentioned that due to the high compressive strength of the concrete which exist a strong bond between fibre and matrix, many of fibres were ruptured during the execution of monotonic test. Since in the determination of $N_{eff}^f$ only fibres with mobilized hook were counted and the ruptured fibres were not taken in to the account, this parameter revealed a lower value.

Moreover, considering the fibre orientation factor, $\eta_\theta$, a similar trend to the one obtained for the number of fibres was found. A quite higher $\eta_\theta$ value was achieved for the series with a $\beta=[0-15^\circ]$, having been 8, 20 and 30% higher than the one obtained for the series $[15-45^\circ]$, $[45-75^\circ]$ and $[75-90^\circ]$, respectively. This could be ascribed to a preferential fibre alignment, which was influenced by the flowability of concrete, and induces the fibres to be reoriented and remain preferentially perpendicular to the concrete flow direction. From the results in Table 3.5, similar to the previous series, it is verified the occurrence of a slight segregation of the fibres of similar level for all the series.
considered, caused by the highest specific weight of the steel fibres amongst the constituents of the SFRSCC. However, this segregation was lower than the one reported when SFRC is cast with mechanical vibration, since this operation is not executed in SFRSCC.

*Table 3.5: Fibre distribution parameters for the specimens after four-point bending test (panel C).*

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Specimen</th>
<th>$N^f$ [fibre/cm$^2$]</th>
<th>$N^f_{eff}$ [fibre/cm$^2$]</th>
<th>$\eta_0$ [-]</th>
<th>$\xi_{seg}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5°-1</td>
<td>2.38</td>
<td>1.27</td>
<td>0.863</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>7.5°-2</td>
<td>2.56</td>
<td>1.66</td>
<td>0.884</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>7.5°-3</td>
<td>1.42</td>
<td>0.83</td>
<td>0.874</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>7.5°-4</td>
<td>2.29</td>
<td>1.18</td>
<td>0.884</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>7.5°-5</td>
<td>1.96</td>
<td>0.95</td>
<td>0.872</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>[0-15°]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>2.12</td>
<td>1.18</td>
<td>0.875</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>CoV (%)</td>
<td>21.11</td>
<td>27.54</td>
<td>1.01</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
<td>30°-1</td>
<td>1.85</td>
<td>0.72</td>
<td>0.847</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>30°-2</td>
<td>1.71</td>
<td>0.86</td>
<td>0.817</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>30°-3</td>
<td>1.71</td>
<td>0.83</td>
<td>0.821</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>30°-4</td>
<td>1.48</td>
<td>0.46</td>
<td>0.752</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>30°-5</td>
<td>2.04</td>
<td>0.47</td>
<td>0.8</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>[15-45°]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1.76</td>
<td>0.67</td>
<td>0.807</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>CoV (%)</td>
<td>11.75</td>
<td>28.65</td>
<td>4.37</td>
<td>2.56</td>
<td></td>
</tr>
<tr>
<td>60°-1</td>
<td>1.11</td>
<td>0.35</td>
<td>0.717</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>60°-2</td>
<td>1.38</td>
<td>0.53</td>
<td>0.755</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>60°-3</td>
<td>1.43</td>
<td>0.40</td>
<td>0.657</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>60°-4</td>
<td>1.58</td>
<td>0.47</td>
<td>0.779</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>60°-5</td>
<td>2.18</td>
<td>0.57</td>
<td>0.74</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>[45-75°]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1.53</td>
<td>0.46</td>
<td>0.73</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>CoV (%)</td>
<td>25.93</td>
<td>19.59</td>
<td>6.36</td>
<td>8.77</td>
<td></td>
</tr>
<tr>
<td>87.5°-1</td>
<td>0.85</td>
<td>0.17</td>
<td>0.713</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>87.5°-2</td>
<td>0.95</td>
<td>0.37</td>
<td>0.734</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>87.5°-3</td>
<td>0.90</td>
<td>0.05</td>
<td>0.695</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>87.5°-4</td>
<td>0.56</td>
<td>0.06</td>
<td>0.519</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>87.5°-5</td>
<td>1.07</td>
<td>0.17</td>
<td>0.7</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>[75-90°]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.87</td>
<td>0.15</td>
<td>0.672</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>CoV (%)</td>
<td>21.75</td>
<td>78.38</td>
<td>12.94</td>
<td>15.21</td>
<td></td>
</tr>
</tbody>
</table>
According to the obtained results, in the case of casting panels, and in particular from its centre, since the wall effects were limited mainly to the bottom surface, the flow velocity was uniform and diffused radially outwards from the centre casting point, see Figure 3.27(a). As a consequence, in the specimens with the notched plane direction parallel to the flow direction, a higher number of effective fibres was observed, than when the specimens had the notched plane perpendicular to the expected flow direction. Figure 3.27(b) shows the influence of the concrete flow velocity profile on the preferential orientation of the fibres along the thickness of the panel. According to the expected concrete flow velocity profile, due to friction between concrete and the bottom surface of the panel (wall effect), the flow velocity starts to decrease nonlinearly from the top to bottom panel’s face. Therefore, fibres tend to be more aligned in horizontal planes parallel to the bottom surface of the panel.

![Figure 3.27: Explanation for fibre alignment in flowing concrete of a panel: (a) casting from the centre (top view), (b) through the cross section.](image)

Figure 3.28 depicts the relationship between the fibre density measured at the notched fracture surfaces of the specimens extracted from panels A and B versus the distance from the casting point. In this figure $N_{f\parallel}$ and $N_{f\perp}$ are, respectively, the fibre density at a crack plane parallel and perpendicular to the concrete flow. As it is expected, due to the proper viscosity of the concrete, a good homogeneity and dispersion of the fibres were achieved all over the panels, and a higher fibre density was obtained in the fracture surfaces in the alignment of the concrete flow.
Figure 3.28: Relationship between the number of fibres in the fracture surfaces and the distance from the centre.

Figure 3.29 depicts the relation between fibre orientation factor, $\eta_{\theta}$, and number of the effective fibres, $N_{\text{eff}}^f$. It was clearly showed that by increasing the orientation factor, the number of the effective fibres tends to rise exponentially.
3.7.5 An analytical investigation of fibre distribution parameters

Figure 3.30 and Figure 3.31 illustrate the orientation probability density functions obtained for the average fibre orientation factor of each series separately, as well as the orientation probability functions for both the two-dimensional fibre distribution and three-dimensional isotropic uniform random distributions. In these figures, the distribution of the orientation angle through the cut plane was studied for each specimen in panels A and B separately, and then the experimental results were compared to Gaussian distribution. These results can be found in Appendix I for all specimens. According to this study, it was concluded that the distribution of the orientation angle follows closely a Gaussian distribution. Laranjeira (2010) had already obtained similar conclusion. Based on this method, an Excel spreadsheet was developed according to section 3.3.2 in order to determine the probability density distribution of fibre orientation. According to Figure 3.30, \( \theta = 0^\circ \) specimens show a distribution shifted to the left side, which means fibres have a tendency to be oriented more perpendicular to the cut plane (crack plane). On the other hand, the \( \theta = 90^\circ \) distribution is slightly transferred to the right side and more fibres tend to be aligned parallel to the cut plane (crack plane). The mentioned explanation could be observed more gradually if one compares distributions in panel C. Figure 3.31(a) presents the distribution of the probability of fibre orientation for the specimens comprised in the interval \( \beta = [0-15^\circ] \), which was shifted to the left side, meaning that fibres have a tendency to be aligned more perpendicular to the notched plane. When angle \( \beta \) increases, which is translated in a reduction of the orientation factor, the distribution was shifted from the left to the right side progressively, see Figure 3.31(b) to (d). Comparing the theoretical orientation factor for a two-dimensional fibre distribution, \( 2/\pi \), see section 3.3.1.2, and a three-dimensional, 0.5, see section 3.3.1.3, isotropic uniform random fibre distribution with the orientation factors for the distinct series, it was observed that \( \theta = 0^\circ \) and \( \beta = [0-15^\circ] \) series showed a completely different distribution. As the notched plane orientation increased, the fibre distribution within the panels tended to a two-dimensional distribution. Figure 3.30(b) and Figure 3.31(d) show that the orientation distribution of the \( \theta = 90^\circ \) and \( \beta = [75-90^\circ] \) series almost coincided with two-dimensional fibre distribution. Consequently, when a SFRSCC panel was cast from the centre, assuming a two-dimensional distribution could be far apart from the reality. Therefore, in SFRSCC panels, mainly, due to the high flowability of the self-compacting concrete, it was expected an anisometric fibre structure. Hence, since the material tensile behaviour...
of fibre reinforced concretes was intimately connected to the fibre distribution, an anisotropic material behaviour should be considered. In conclusion, in the present case, the fibre distribution was prominently influenced by the placing conditions and concrete flowability.

Figure 3.30: Predicted orientation probability functions for specimens in panels A and B: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$.

Figure 3.31: Predicted orientation probability functions for specimens in panel C and $\beta$ in the intervals: (a) [0-15°], (b) [15-45°], (c) [45-75°] and (d) [75-90°].
In Table 3.6, it is depicted the experimental fibre orientation factors obtained from the image analysis for the beams in panel C, compared to the ones achieved by the analytical formations available in literature. It was shown that the orientation factors predicted by Krenchel (1975) and Dupont (2003) are approximately similar but lower than the one achieved by Laranjeira (2010). However, in comparison to the image analysis orientation factor (experimental one), it seems that Laranjeira (2010) prediction is more realistic. In the analytical formulation proposed by Krenchel (1975), determination of fibre orientation factor depends on the dimensions of the mould and fibre length, see Eq. 3.17. Furthermore, as it was shown in Figure 3.9(a), three orientation parameters were also defined which stated the increment of the bulk fibre orientation factor, $\eta_1 = 0.5$, due to the wall effects, $\eta_2 = 0.6$ and $\eta_3 = 0.84$. The same method was also proposed by (Doupont, 2003), see Eq. 3.21. In a more realistic method, which was proposed by Laranjeira (2010), the increment of the bulk fibre orientation factor (for the SCC proposed as 0.6) due to the following parameters was studied: casting method and formwork geometry (Table 3.2), but the influence of concrete flow (dynamic effects) was not taken in to the consideration. However, when a FRSCC flow along a formwork, due to the frictional restraint with the walls, a special flow profile develops and fibres rotate with the flow of fresh concrete. This phenomenon depends on many parameters such as: concrete plastic viscosity, casting rate, time that fibres are submitted to the flow velocity and flow distance. Moreover, from Table 3.6 it is also observed that all the analytical approaches propose a same orientation factor for all notched plane orientations ($\beta=[0-15^\circ], [15-45^\circ], [45-75^\circ]$ and $[75-90^\circ]$), but from the image analysis results it was shown that the fibre orientation factor depends on the direction of the notched plane. On the other hand, anisometric fibre structure was not considered. Nevertheless, if the average experimental fibre orientation factor between all series consider, $n_{\text{avg}}^{\text{Exp}} = 0.771$, it was observed that this value was very close to what was predicted by Laranjeira (2010).
Table 3.6: Comparison between the experimental and analytical fibre orientation factors.

<table>
<thead>
<tr>
<th>Reference</th>
<th>[0-15°]</th>
<th>[15-45°]</th>
<th>[45-75°]</th>
<th>[75-90°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>0.875</td>
<td>0.807</td>
<td>0.730</td>
<td>0.672</td>
</tr>
<tr>
<td>Krenchel (1975)</td>
<td>0.666</td>
<td>0.666</td>
<td>0.666</td>
<td>0.666</td>
</tr>
<tr>
<td>Dupont (2003)</td>
<td>0.611</td>
<td>0.611</td>
<td>0.611</td>
<td>0.611</td>
</tr>
<tr>
<td>Laranjeira (2010)</td>
<td>0.768</td>
<td>0.768</td>
<td>0.768</td>
<td>0.768</td>
</tr>
</tbody>
</table>

Table 3.7 shows a comparison between the experimental number of fibres obtained from the image analysis for the beams in panel C and the ones achieved by the analytical formations available in literature. It is observed that Soroushian and Lee (1990), Eq. 3.29, and Vandewalle and Dupont (2003), Eq. 3.22, overestimated the number of fibres for all series highly, whereas it seems that the values determined by Kasperkiewicz (1977) are closer to the experimental one, particularly if the average experimental number of fibres between all series is considered ($N_{avg}^f=1.571$). It should be mentioned that in the determination of number of fibres, Kasperkiewicz (1977) does not consider fibre orientation factor as a parameter, see Eq. 3.28, thus, the same values were acquired for all directions of notch plane. However, the probability that a random section plane crossing a single fibre is a function of the fibre’s length ($l_f$), diameter ($d$), and also the angle that the it makes with the section plane (fibre orientation factor) (Zhu et al., 1997).

Consequently, until now, is spite of the recent advances on the link between rheological properties and casting process of FRSCC, there is no analytical formulation, particularly in the case of the planar structures, to predict realistically fibre structure (fibre orientation factor and number of fibres) considering the influence of flow as well as the dependency of this parameter to the studied plane orientation.
Table 3.7: Comparison between the experimental and analytical number of fibres [fibres/cm$^2$].

<table>
<thead>
<tr>
<th>Reference</th>
<th>Notched plane orientation ($\beta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[0-15°]</td>
</tr>
<tr>
<td>Experimental</td>
<td>2.122</td>
</tr>
<tr>
<td>Kasperkiexicz (1977)</td>
<td>1.620</td>
</tr>
<tr>
<td>Soroushian and Lee (1990)</td>
<td>2.837</td>
</tr>
</tbody>
</table>

3.8 Conclusions

In this chapter, an analysis of the fibre distribution parameters within self-compacting concrete panels was conducted. For this purpose, some parameters that give an idea of the fibre structure, such as: fibre density, orientation and segregation were ascertained. The achieved experimental results were compared with the analytical approaches available in literature. The fibre structure analysis was performed on the specimens extracted from the panels using image analysis procedure on sawn planes from the specimens as well as counting fibres manually in order to determine the number of effective fibres.

The fibre structure parameters were evaluated in planes having different angles towards the concrete flow direction. It was concluded that the number of fibres were significantly higher for the specimens with cut plane parallel to the flow direction. The same conclusion was also made by considering fibre orientation factor. On the other hand, when the studied plane was parallel to the concrete flow direction, the fibres were more perpendicular to the section. The fibre orientation factor tends to reduce by increasing angle between the cut plane and flow direction which is translated to the rotation of fibres more parallel to the cut surface. The cause of this result is correspondent to the circular movement of concrete. On the other hand, In the case of casting panels from the centre, fibres have a tendency to align perpendicular to the radial flow, mainly due to the uniform flow profile velocity that diffuses outwards radially from the centre of the panel. Consequently, the total number of the effective fibres was higher in cut planes parallel towards the concrete flow when compared to the other cases, particularly
orthogonal plane towards the concrete flow. Considering the fibre segregation parameter, it was verified the occurrence of a slight segregation of the fibres of similar level for all the series considered, caused by the highest specific weight of the steel fibres amongst the constituents of the SFRSCC.

The probabilistic distribution of the orientation angle through a cut plane follows closely a Gaussian distribution. By determining the probability density function of fibre orientation for each series separately, it was shown that the specimens with the notch plane parallel to the flow direction presented a completely different fibre distribution. On the hand, as the notched plane orientation increased, the fibre distribution within the panels tended to a two-dimensional distribution. It was shown that the orientation distribution of the $\theta = 90^\circ$ and $\beta = [75-90^\circ]$ series almost coincided with two-dimensional fibre distribution. Consequently, when a SFRSCC panel was cast from the centre, assuming a two-dimensional distribution could be far apart from the reality. Therefore, in SFRSCC panels, mainly, due to the high flowability of the self-compacting concrete, it was expected an anisometric fibre structure. Hence, since the material tensile behaviour of fibre reinforced concretes was intimately connected to the fibre distribution, an anisotropic material behaviour should be considered. In conclusion, in the present case, the fibre distribution was prominently influenced by the placing conditions and concrete flowability.

At last, the fibre distribution and orientation was assessed by the available analytical formulations in literature and compared to the ones acquired experimentally (image analysis method). None of the approaches predicted the fibre distribution parameters accurately. This could be ascribed to the high dependency of fibre orientation to the flowability of self-compacting concrete. On the other hand, anisometric fibre structure was not taken into the consideration. Therefore, in the case of the planar structures, there is no analytical formulation to predict realistically number of fibres as well as fibre orientation factor considering the influence of the concrete flow.
### 4.1 Introduction

Addition of fibres to a cementitious matrix contributes mainly to the energy absorption capacity and crack control of structural elements, as well as to the enhancement of the load bearing capacity, particularly, in structural configurations with high support redundancy (Balaguru and Shah, 1992; ACI-544–1R, 2002). The fibre reinforcement mechanisms are mainly effective after concrete cracking initiation and, mostly, improve the post-cracking behaviour, due to the stress transfer provided by fibres bridging cracked sections. Crack opening in steel fibre reinforced concrete (SFRC) is counteracted by the bond stresses that develop at the fibres / matrix interface during the fibre pull-out. On the other hand, one of the most important properties of SFRC is its ability to transfer stresses across a cracked section rather uniformly, which nonetheless is dependent on the fibre reinforcement effectiveness, i.e. fibre properties (their strength, bond, and stiffness), and fibre orientation and distribution (Vandewalle and Dupont, 2003). The stress transfer capability of fibres enhances mainly the composite’s toughness, which is a parameter related to the energy absorption during monotonic or cyclic loading (RILEM-TC162-TDF, 2002b).
The dispersion and orientation of fibres in the hardened-state results from a series of stages that SFRC passes from mixing to hardening state, namely (Laranjeira, 2010): fresh-state properties after mixing; casting conditions into the formwork; flowability characteristics; vibration and wall-effect introduced by the formwork. Among these factors, wall-effects introduced by the moulds, and the properties of SFRC in the fresh state, especially its flowability, are the most important ones (Dupont, 2005; Laranjeira, 2010; Martinie et al., 2009). Having in mind that mechanical properties are significantly related to the fibre orientation and dispersion, which are affected by concrete’s flow in the fresh state, it is important to control both those parameters (flowability and wall-effect) (Ferrara and Meda, 2006; Kim et al., 2008; Pansuk et al., 2008).

Application of steel fibres enhances the mechanical properties of concrete, but since all fibres cannot be aligned in the direction of the applied stress, the effectiveness of the fibres is dependent of the loading conditions, mainly on the directions of the principal tensile stresses. Moreover, it is shown that the fibre distribution’s scatter in large scale elements may result in a significant inconsistency of the mechanical behaviour along the structural element. Therefore, it is feasible to expect an anisometric material behaviour for this kind of composite. In addition, the fibre efficiency depends on the orientation of the fibres towards the active crack plane. Some authors agree that in steel fibre reinforced self-compacting concrete (SFRSCC) the variability in the post-cracking parameters observed in bending tests, and also in uniaxial direct tensile tests, can be justified by the dispersion and alignment of the fibres (Barragan et al., 2003; Torrijos et al., 2010). Therefore, a significant research effort has been done to achieve better mechanical performances for SFRC by conditioning the distribution and orientation of the fibres (Ozyurt et al., 2007; Boulekbache et al., 2010; Torrijos et al., 2010; Kang and Kim, 2011). However, these effects should be considered for structural design, especially when fibre distribution and orientation affect significantly the mechanical properties of SFRC.

The main objective of this chapter is to connect experimentally the influence of the distribution/orientation of fibres, which are affected by flowability of concrete, to the post-cracking behaviour of SFRSCC developed and applied on laminar structures. To perform this evaluation SFRSCC panels were casted from their centre point. For each SFRSCC panel, cylindrical specimens were extracted and notched either parallel or perpendicular to the concrete flow direction to evaluate the effects of fibre dispersion and alignment on the tensile performance. To characterize fibre density and orientation
throughout the panels, an image analysis technique was employed across the cut plane of each tested specimen. The post-cracking behaviour was assessed by both splitting tensile tests and also uniaxial tensile tests. Furthermore, the present chapter also illustrates a methodology to predict the stress-crack width ($\sigma - w$) relationship of SFRSCC of laminar structures using an inverse analysis procedure. The $\sigma - w$ relationship of the SFRSCC was obtained from the numerical simulations of the splitting tensile results with a nonlinear 3D finite element model. Finally, the $\sigma - w$ response obtained from inverse analysis was compared to the one directly derived from the uniaxial tensile tests.

4.2 Experimental research

4.2.1 Concrete mixture

The constituent materials used in the composition of the SFRSCC were: Portland cement CEM 42.5 R (C), water (W), superplasticizer Sika® 3005 (SP), limestone filler, crushed granite aggregate, fine and coarse sand, and hooked-end steel fibres (length, $l_f$, of 33 mm; diameter, $d_f$, of 0.55 mm; aspect ratio, $l_f/d_f$, of 60 and a yield stress of 1100 MPa). The adopted mix proportions are shown in Table 4.1, where W/C is the water/cement ratio. To evaluate the properties of SFRSCC in the fresh state, the inverted Abrams cone slump test was performed according to EFNARC (2005) recommendations. An average spread of 670 mm was achieved without sign of segregation of the constituents, see Figure 4.1. The compressive strength and Young’s modulus were determined using cylinders of 150 mm diameter and 300 mm height after 28 days of moist curing in a climate chamber (3 cylinders for each test). The average compressive strength ($f_{cm}$) and the average value of the Young’s modulus ($E_{cm}$) were 47.77 MPa (7.45 %) and 34.15 GPa (0.21 %), respectively, where the values in parentheses represent the coefficient of variation.

<table>
<thead>
<tr>
<th>Cement</th>
<th>Water</th>
<th>W/C</th>
<th>SP</th>
<th>Filler</th>
<th>Fine sand</th>
<th>Coarse sand</th>
<th>Coarse agg.</th>
<th>Fibre</th>
</tr>
</thead>
<tbody>
<tr>
<td>[kg]</td>
<td>[kg]</td>
<td>[-]</td>
<td>[kg]</td>
<td>[kg]</td>
<td>[kg]</td>
<td>[kg]</td>
<td>[kg]</td>
<td>[kg]</td>
</tr>
<tr>
<td>413</td>
<td>140</td>
<td>0.34</td>
<td>7.83</td>
<td>353</td>
<td>237</td>
<td>710</td>
<td>590</td>
<td>60</td>
</tr>
</tbody>
</table>
4.2.2 Specimens

According to Barnett et al. (2010), as it was shown in Chapter 3, section 3.2.2, casting a slab from its centre assures better mechanical behaviour compared to the other casting methods. Therefore this direction of casting was selected for the production of two SFRSCC panels. For this purpose, a mould was design which allowing casting panels with the desire dimensions. A 3D view of the mould is shown in Figure 4.2(a). The dimensions of the panels are $1600 \times 1000 \text{ mm}^2$ in plan, with 60 mm of thickness. The fresh concrete was poured directly from the mixing-truck by using a U-shape channel at the centre of the mould from a height of 60 cm, see Figure 4.2(b) and (c). The concrete was poured in the mould patiently in order to move and fill all parts of the mould properly. Figure 4.2(d) shows moving concrete to the corner of the mould which present a good filing ability of the proposed SFRSCC without any sign of the matrix constituent’s segregation.
The influence of fibre dispersion and orientation within the panel on the post-cracking behaviour was assessed by means of splitting (Brazilian type) and direct tensile tests. Twenty-three specimens were extracted from each panel along the concrete flow directions, according to the scheme represented in Figure 4.3. In this figure the pale dash lines with arrows represent the supposed concrete flow directions. When the drilling operations were performed, the panels were already in their harden-mature phase. The hatched cores were used for executing splitting tensile tests, while the rest were used for uniaxial tensile tests. Figure 4.4 illustrates the extracted cores and a panel after drilling. In the splitting tensile test, to localize the specimen’s fracture, two notches with a 5 mm depth were executed on cores’ opposite sides. The influence of the crack orientation towards the concrete flow was assessed in two distinct directions. By assuming $\theta$ as the angle between the notched plane and the direction of the concrete flow, the notch plane is designated parallel for $\theta = 0^\circ$ or perpendicular for $\theta = 90^\circ$. Since the core scheme was maintained for both panels, for each core location there are two cores with perpendicular notch direction. This will enable to evaluate the influence of fibre orientation at a certain distance from the casting position on the stress-crack width ($\sigma - w$) relationship. For instance, $\theta$ of A1 specimen is $90^\circ$ and $0^\circ$ in panels A and B, respectively.
The remaining cores extracted from the cast panels were sawn out from cylinders of 150 mm diameter and 60 mm thickness according to the schematic representation shown in Figure 4.5. Twenty two prismatic specimens with dimensions of $110 \times 102 \times 60$ mm$^3$ were produced for the uniaxial tensile test program. Following the same notching procedure for the splitting test specimens, the prismatic specimens were notched according to parallel ($\theta = 0^\circ$) and perpendicular ($\theta = 90^\circ$) directions to the expected concrete flow. The notch was executed in the four lateral faces of the specimen, at its mid-height, with a thickness of 2 mm and a depth of 5 mm. Special cares were given to this operation to produce a notch with precise and uniform dimensions, and also to ensure the notch plane becomes perpendicular to the direction of the applied stress.
4.2.3 Test set-up

4.2.3.1 Splitting tensile test

To determine the $\sigma - w$ relationship representative of the SFRSCC panel, splitting tensile tests based on the ASTM-C496 (2004) were executed. The tests were carried out in displacement-control using an universal testing rig with a bearing capacity of 150 kN. The tests were performed with a relatively low displacement rate of 0.001 mm/s enabling to obtain a stable response once the crack process is initiated. This low displacement rate was kept constant throughout the test execution. An external displacement transducer positioned on the actuator that measured the vertical deformation of the specimen was used to control the test.

Each specimen was positioned between two rigid supports and subjected to a diametral compressive line load applied along the thickness of the specimen. It is assumed that this applied load induces a constant tensile stress in the central part of the notched plane; therefore the results are expected to be close to the uniaxial tensile test results (Carmona and Aguado, 2012). The test set-up is depicted in Figure 4.6. In each specimen five linear variable differential transducers (LVDTs) were applied according to the configuration schematically represented in Figure 4.6(a) and (b) to record crack opening along the notched plane. The support aluminium plates of each LVDT guarantee the register of the opening of the two opposed faces of the notch, Figure 4.6(c). To assess if unsymmetric crack opening occurs, due to fibre segregation during the casting
procedure, two LVDTs were located at the specimen’s bottom surface, while the others were fixed on the top surface of the specimen.

Figure 4.6: Geometry of the specimen and set-up of the splitting tensile test (dimensions are in mm): (a) specimen front view (top of the panel), (b) specimen lateral view and (c) LVDT connection detail.

4.2.3.2 Uniaxial tensile test

After sawing and notching operations, each specimen was carefully cleaned with pressurized air and acetone. Then, two loading steel plates were glued with epoxy to the top and bottom surfaces of the specimen and subjected to a uniform pressure during three days enabling the perfect alignment of the loading plates. Sikadur®-30 Normal epoxy adhesive was used for this purpose.

A high stiff universal testing rig with a bearing capacity of 1000kN was used to execute the uniaxial tensile tests, Figure 4.7(a). This test was carried out in close-loop displacement control by averaging the signals of four displacement transducers installed on the two opposite faces of the specimen (top and bottom of the panel), Figure 4.7(b). Distinct displacement rates were applied during the test according to the following procedure: 0.005 mm/min up to a displacement of 0.05 mm, 0.02 mm/min up to a displacement of 0.1 mm, 0.08 mm/min up to a displacement of 0.5 mm/min and finally, 0.1 mm/min until the completion of the test. The adopted displacement rates comply with the recommendations of RILEM-TC162-TDF (2001).
4.2.4 Analysis of results and discussion

Table 4.2 includes the residual stresses and toughness parameters for different average crack widths. In this table, $\sigma_{\text{peak}}$ is the stress at peak load that represents the maximum tensile stress; $\sigma_{0.3}$, $\sigma_1$ and $\sigma_2$ are the residual stresses at a crack opening width of 0.3, 1 and 2 mm, respectively; $G_{F1}$ and $G_{F2}$ are the dissipated energy up to a crack width of, respectively, 1 and 2 mm. Additionally, the coefficient of variation, CoV, and the characteristic values for a confidence interval of 95%, $k_{95\%}$, are also included. From the results it is noticed that the influence of the notch orientation towards the concrete’s flow on the post-peak behaviour of the material is quite high. The series with a notch inclination of $\theta = 0^\circ$ shows higher residual tensile stresses and also larger dissipated energy than the specimens with $\theta = 90^\circ$. These increments ranged from 2.03 to 3.08 times depending on the crack width for residual stresses and between 2.40 to 2.71 times for dissipated energy. This variation in the post-cracking parameters could be ascribed to a preferential orientation of the fibres at the fracture surface. As it will be discussed in more detail further ahead, during the casting stage, fibres have the tendency to be aligned perpendicular to the direction of concrete flow, due to a uniform radial velocity profile as also observed by (Barnett et al., 2010; Boulekbache et al., 2010). Therefore, for the specimens with the notched plane parallel to the flow direction, more fibres are almost perpendicular to the crack plane and, consequently, a higher number of fibres intersect more effectively the fracture surface. Previous research on the fibre pull-out behaviour
has revealed that fibre reinforcement effectiveness is almost the same for a fibre orientation towards the normal to the crack plane lower than 30 degrees (Cunha, 2010).

Table 4.2: Residual stress and toughness parameters obtained from splitting and direct tensile tests.

<table>
<thead>
<tr>
<th>Series</th>
<th>Parameter</th>
<th>$\sigma_{\text{peak}}$ [MPa]</th>
<th>$\sigma_{0.3}$ [MPa]</th>
<th>$\sigma_1$ [MPa]</th>
<th>$\sigma_2$ [MPa]</th>
<th>$G_{F1}$ [N/mm]</th>
<th>$G_{F2}$ [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Splitting tensile test</td>
<td>$\theta = 0^\circ$ ((\sigma_{\parallel}))</td>
<td>Average</td>
<td>4.39</td>
<td>4.23</td>
<td>3.82</td>
<td>2.79</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>CoV(%)</td>
<td>25.6</td>
<td>29.7</td>
<td>24.3</td>
<td>30.2</td>
<td>27.2</td>
<td>25.2</td>
</tr>
<tr>
<td></td>
<td>$K_{95%}$</td>
<td>3.52</td>
<td>3.16</td>
<td>2.09</td>
<td>1.95</td>
<td>3.36</td>
<td>6.08</td>
</tr>
<tr>
<td></td>
<td>$\theta = 90^\circ$ ((\sigma_{\perp}))</td>
<td>Average</td>
<td>2.47</td>
<td>2.13</td>
<td>1.96</td>
<td>1.50</td>
<td>2.08</td>
</tr>
<tr>
<td></td>
<td>CoV(%)</td>
<td>33.1</td>
<td>48.6</td>
<td>37.9</td>
<td>35.3</td>
<td>35.9</td>
<td>33.2</td>
</tr>
<tr>
<td></td>
<td>$K_{95%}$</td>
<td>2.07</td>
<td>1.74</td>
<td>1.46</td>
<td>1.09</td>
<td>1.49</td>
<td>2.83</td>
</tr>
<tr>
<td>Uniaxial tensile test</td>
<td>$\theta = 0^\circ$ ((\sigma_{\parallel}))</td>
<td>Average</td>
<td>3.33</td>
<td>3.24</td>
<td>2.30</td>
<td>1.14</td>
<td>2.94</td>
</tr>
<tr>
<td></td>
<td>CoV(%)</td>
<td>19.0</td>
<td>21.4</td>
<td>27.4</td>
<td>39.8</td>
<td>24.2</td>
<td>23.7</td>
</tr>
<tr>
<td></td>
<td>$K_{95%}$</td>
<td>3.10</td>
<td>2.73</td>
<td>1.83</td>
<td>0.80</td>
<td>2.42</td>
<td>3.72</td>
</tr>
<tr>
<td></td>
<td>$\theta = 90^\circ$ ((\sigma_{\perp}))</td>
<td>Average</td>
<td>2.72</td>
<td>1.05</td>
<td>1.02</td>
<td>0.56</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>CoV(%)</td>
<td>19.1</td>
<td>64.5</td>
<td>65.4</td>
<td>57.1</td>
<td>59.6</td>
<td>59.9</td>
</tr>
<tr>
<td></td>
<td>$K_{95%}$</td>
<td>2.34</td>
<td>0.51</td>
<td>0.48</td>
<td>0.30</td>
<td>0.57</td>
<td>0.96</td>
</tr>
</tbody>
</table>

$^*$ // and $\perp$ - notch direction parallel (\(\theta = 0^\circ\)) and perpendicular (\(\theta = 90^\circ\)) to the concrete flow direction, respectively.

4.2.4.1 Splitting tensile test

Figure 4.8 depicts the nominal stress – crack opening mouth displacement relationship, $\sigma$ – $w$, for specimens extracted from distinct panels’ locations. The envelope and the correspondent average curves are presented in this figure. In Appendix II the experimental response of each specimen is shown. The crack width was determined by averaging the recorded displacements of the 5 LVDTs installed on the faces of each specimen, see Figure 4.6. The nominal tensile stress at the centre of the specimen was obtained from the following equation (ASTM-C496, 2004):

$$\sigma = \sigma_{sp} = \frac{2F}{\pi DL}$$ (4.1)
where $F$ is the applied line load, $D$ is the diameter of the cylinder (150 mm) and $L$ is the thickness of the net area in the notched plane (50 mm). Although the applicability of Eq. 4.1 is arguable in the softening phase of SFRSCC, since it is based on the theory of elasticity, it will be used to estimate the tensile stress at the cracked surface, as adopted by other researchers (Carmona et al., 2012; Cunha, 2010; Rocco et al., 1999).

The $\sigma - w$ responses are almost linear up to the stress at crack initiation. Up to this stress level, the displacements recorded by the LVDTs represent the transversal elastic deformation of the SFRSCC volume between the supports of the LVDTs (Figure 4.6(c)). Therefore, the deformability during this first phase should have been removed from the $\sigma - w$ response, but due to its negligible value this was not executed. After crack initiation, the $\sigma - w$ response is nonlinear up to peak load. Once the peak load was attained, the load has smoothly decreased being visible a softening response. Note that, for the specimens with the notch perpendicular to the flow direction ($\theta = 90^\circ$), the peak stress was equal to the stress at crack initiation.

Generally, the $\sigma - w$ responses exhibited a relatively high scatter. In SFRSCC, this type of scatter is generally high, even in specimens from the same casting and with the same testing conditions, due to the high dependence of the post-cracking behaviour on the fibre distribution and orientation. Since the specimens were extracted from distinct slab locations, at different distances from the casting point, a high scatter was expected. In fact, the viscous nature of SFRSCC affects the distribution of the concrete constituents along the flow process.

Figure 4.8: Nominal tensile stress – crack opening width relationship, $\sigma - w$, obtained from splitting tensile test for: (a) $\theta=0^\circ$ and (b) $\theta = 90^\circ$. 

107
Figure 4.9 shows the $\sigma - w$ relationships at the two sides of the specimens, representative of the top and bottom surfaces of the panels. Additionally, the average curve is also included. The crack width was determined by averaging the LVDTs readouts installed on each surface. As it is shown from the results, the LVDTs on the bottom surface registered a lower value of the crack opening than the ones at the top surface for the same load level. This means that the crack opened asymmetrically, which is justified by the fibre tendency to segregate along the depth of the element (Barros, 2011). The effect of fibre segregation was slightly higher in the $\theta = 90^\circ$ series. This aspect was corroborated and discussed in Section 3.7.4 with the determination of a fibre segregation factor.

![Figure 4.9: Nominal tensile stress – crack opening width relationship, $\sigma - w$, obtained from splitting tensile test for the top and bottom sides of the specimens: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$.](image)

**4.2.4.2 Uniaxial tensile test**

In all specimens subjected to the uniaxial tensile test, the crack was occurred in the notch plane, thus the desire failure mode was attained. Moreover, even thought the relatively high concrete’s strength, it was not observed failure at the boundaries of the specimens, confirming a good specimen’s gluing process.

Figure 4.10 depicts the average and envelope stress-crack width ($\sigma - w$) curves regarding to each series. The $\sigma - w$ relationship was obtained from $\sigma - \delta_{avg}$ curves according to the recommendation from RILEM-TC162-TDF (2001). In Appendix II is enclosed the experimental response of each specimen. For both series ($\theta = 0^\circ$ and $90^\circ$),
the $\sigma - w$ curve is almost linear up to the load at crack initiation. The concrete tensile strength was approximately 2.7 MPa. Once the tensile strength is attained, the stress suddenly decreases up to a crack width about 0.07 mm. Beyond this crack width, $\theta = 0^\circ$ and $90^\circ$ series behave in a completely distinct way. A semi-hardening and a plateau responses are observed for the $\theta = 0^\circ$ and $90^\circ$ series, respectively. Regarding the $\theta = 0^\circ$ series, Cunha (2010) have analyzed the micromechanical behaviour of hooked end fibres by performing fibre pull-out tests, and have verified that after a fibre sliding of nearby 0.1 mm, the fibre reinforcement mechanism is mainly governed by the hook plastification during the fibre pull-out process. Therefore, in this series, fibres start to be pulled-out slowly being observed a semi-hardening response. Afterwards, in $\theta = 0^\circ$ specimens, up to the crack width of about 0.6 mm, a plateau response is observed, which is then followed by a smooth drop in the residual stress. From experimental and analytical analysis, it was verified (Soroushian, 1990; P. Stroeven, Hu, J., 2006) that the average orientation angle value of the active fibres bridging a leading crack is about $35^\circ$. According to fibre pull-out tests performed by Cunha et al. (2011), in the case of the inclination angle of $30^\circ$ with the load direction, fibre rupture is the most predominant failure mode between the slip range of 0.6-1.0 mm. In fact, during the uniaxial tensile test execution, after peak load is attained the sound of the fibre rupturing was clearly noticeable that caused a rapid drop in the value of the load. This was confirmed after analysing the fracture surface by visual inspection.

In the case of $\theta = 90^\circ$, some specimens shown a pseudo-hardening behaviour, especially those located nearby the centre of the panel. After this pseudo-hardening behaviour, it is observed a small plateau followed by a reduction of the residual stress beyond a crack width of about 0.9 mm, which corresponded to the rupture of the fibres.

The pre-peak branch shows very low scattering, while in the post-cracking phase the scatter of the response was considerably higher. In the elastic phase the contribution of fibres is rather negligible. After crack initiation, the role of the fibres becomes more important in bridging the stresses across the crack surfaces. This process depends significantly on how fibres are distributed and oriented through the matrix, which means the scattering observed in the post-cracking phase is highly influenced by the variation of the fibre dispersion and orientation amongst different specimens. Hence, for the latter series it is more logical to categorize the $\sigma - w$ relationships based on distinct fibre orientation factor and distribution, which will be discussed in the next section.
4.2.4.3 Comparison of the results

Figure 4.11 shows the relationship between the ratio of the splitting tensile post-cracking parameters, $\sigma_{SPLT}$, $G_{F\,SPLT}$ and the uniaxial tensile post-cracking parameters, $\sigma_{UTT}$, $G_{F\,UTT}$ for the crack width corresponding to $\sigma_{peak}$ that is known as $w_{peak}$, and at crack width values of 0.3, 1.0 and 2.0 mm. In Figure 4.11(a) for $\theta = 0^\circ$ series, $w_{peak}$ does not represent the same value for splitting tensile test (0.44 mm) and uniaxial tensile test (0.34 mm), therefore this interval is represented as a hatched vertical strip. For the $\theta = 90^\circ$ series this problem is not crucial since $\sigma_{peak}$ coincides with the stress at crack initiation, which happened for a negligible crack opening ($w_{peak}$). The data plotted in Figure 4.11(a) clearly shows that $\sigma_{SPLT}$ is larger than $\sigma_{UTT}$ for almost all $w\ (CMOD)$ values considered except at $w = w_{peak}$ for the $\theta=90^\circ$ series. Therefore, splitting tensile test overestimates the tensile residual strength. The average tensile stress at peak load for the splitting and uniaxial tensile test was 4.39 and 3.30 MPa for $\theta = 0^\circ$ specimens, and 2.47 and 2.72 MPa for $\theta=90^\circ$ series. With the increase of the crack opening, the $\sigma_{SPLT}/\sigma_{UTT}$ ratio became higher, since in the softening phase fibres started being mobilized as they bridge the stresses across the crack surfaces.

Figure 4.11(b) depicts the relationship between the energy absorbed during the fracture process in both test set-ups, up to a crack width of 0.3, 1 and 2 mm. Both series presented a similar tendency, an increase of $G_f$ with the crack width was observed. On the other hand, in the average term, for 0.3 mm crack width, the $G_{F\,SPLT}/G_{F\,UTT}$ ratio is 1.33 and 1.94 for $\theta = 0^\circ$ and $\theta = 90^\circ$ series, respectively. This ratio
has increased up to 1.62 and 2.05 for 2 mm crack width, respectively, for $\theta = 0^\circ$ and $\theta = 90^\circ$ series.

![Graph](image)

**Figure 4.11:** Uniaxial tensile post-cracking parameters versus splitting tensile post cracking parameters: (a) residual stress and (b) fracture energy.

### 4.2.5 Correlation between the fracture and fibre distribution parameters

Figure 4.12(a) depicts the relationship between maximum residual stress, $\sigma_{\text{peak}}$, stresses up to 1 and 3 mm crack opening, $\sigma_1$ and $\sigma_3$, and the number of the effective fibres counted manually at the notch fracture surface of the specimens after performing the splitting tensile test. It was observed that the residual stresses have a tendency to increase with the number of fibres bridging the fracture surface, being this effect more pronounced in the $\sigma_{\text{peak}}$ and $\sigma_1$. The residual stress for a 3 mm crack width ($\sigma_3$) increased at a considerably lower rate comparing to others. This is mainly due to the fibre rupture that has occurred more often before 3 mm crack width. Figure 4.12(b) shows the relationship between fracture energy up to 1 and 3 mm crack opening named as $G_1$ and $G_3$, and the number of the effective fibres. Similar to the residual tensile stresses, the dissipated energy has shown a linear increase with $N_{\text{eff}}^f$ for both limits of the crack opening studied. However, $G_3$ was influenced by $N_{\text{eff}}^f$ in a lower rate, due to the reason already discussed.
Figure 4.12: Relationship between the fracture parameters and the number of effective fibres found in the notched fracture surfaces of specimens: (a) tensile residual stresses and (b) fracture absorption.

Figure 4.13 depicts the relationships between the number of effective fibre, $N_{eff}^f$, the fibre orientation factor ($\eta_\theta$) and the aforementioned post-cracking parameters of uniaxial tensile test in Table 4.2, as well as their projection for both series obtained from the uniaxial tensile test. Since post-cracking parameters were affected by not only the fibre distribution but also the fibre orientation, it is more logical to plot these parameters versus both factors.

Regarding to the peak stress, $\sigma_{peak}$, for $\theta=0^\circ$ series, it was observed an upward trends that mean the peak stress tended to increase with growing $N_{eff}^f$ and $\eta_\theta$. However, no significant relation was observed for the $\theta=90^\circ$ specimens. It is worth noting that since $\theta=0^\circ$ series presented a semi-hardening response, the peak stress is corresponded to the stress in post-cracking stage at a crack width of nearby 0.4 mm where the contribution of the fibres was significant. However, in $\theta=90^\circ$ specimens the $\sigma_{peak}$ coincide with the crack initiation of matrix (tensile strength of concrete) which the fibres distribution parameters have a null influence in this stage and it is more dependent to the matrix skeleton and paste. For the residual stresses at a crack opening width of 0.3 and 1.0 mm, $\sigma_{0.3}$ and $\sigma_1$, respectively, it was clearly observable a linearly ascending relationship for both series between these stresses, $N_{eff}^f$ and $\eta_\theta$. This was expected, since the residual stress sustained by the crack was intimately related to the number of mobilized fibres. To investigate the
influence of each factor \((N_{\text{eff}}^f \text{ or } \eta_\theta)\) on the post-cracking parameters, independently, in each figure, the projection of the results in the corresponding plane was executed. In all figures, \(\theta=0^\circ\) specimens showed a clear jump in post-cracking parameters and also lower scattering. As it was proved from the image processing results in the previous chapter, the CoVs of \(N_{\text{eff}}^f\) and \(\eta_\theta\) for the \(\theta=90^\circ\) series were considerably higher than for the \(\theta=0^\circ\) series.

Figure 4.13(d) illustrates the relation between the residual stress at a crack opening of 2 mm, \(\sigma_2\), \(N_{\text{eff}}^f\) and \(\eta_\theta\). Similar to the previous series, it is visible a linear increase of \(\sigma_2\) with \(N_{\text{eff}}^f\) and \(\eta_\theta\) but in a lower rate. It should be mentioned that after a crack opening width of 2 mm, many fibres were ruptured particularly in the case of \(\theta=0^\circ\) series resulting in a high reduction of the transfer stress by the fibres after peak load. Nevertheless, it is evident a higher residual stress for \(\theta=0^\circ\) specimens.

Finally, Figure 4.13(e) and (f) depict the influence of \(N_{\text{eff}}^f\) and \(\eta_\theta\) on the dissipated energy up to the 1 and 2 mm crack opening width, \(G_1\) and \(G_2\), respectively. In the overall, the dissipated energy has shown a linear increase with \(N_{\text{eff}}^f\) and \(\eta_\theta\) for both limits of the crack opening studied. Similar to the previous parameters, the dissipated energies for the \(\theta=0^\circ\) series were higher comparing to the other series due to the higher number of effective fibres.

The \(\sigma - w\) relationships previously obtained (see Figure 4.10) have shown a high scatter due to the distinct fibre distributions. In order to reduce the scatter of the results and also study the influence of \(\eta_\theta\) and \(N_{\text{eff}}^f\), the \(\sigma - w\) relationships were separated in three different categories, see Figure 4.14. From this figure, it is concluded that the post-cracking parameters depend not only in \(\eta_\theta\) but also in \(N_{\text{eff}}^f\).
Figure 4.13: Relationship between the fibre distribution, the fibre orientation factor and the post-cracking parameters: (a) peak stress, (b), (c) and (d) stress at a 0.3, 1 and 2 mm crack width, respectively; (e) and (f) energy absorption up to 1 and 2 mm crack width, respectively.
**Figure 4.14:** Categories of uniaxial tensile stress – crack width relationships, $\sigma - w$, : (a) $\eta_0 \geq 0.80$ and $N_{eff}^f \geq 1.20$, (b) $0.68 < \eta_0 < 0.80$ and $0.41 < N_{eff}^f < 1.20$, (c) $\eta_0 \leq 0.68$ and $N_{eff}^f \leq 0.41$.

### 4.3 Determination of the $\sigma - w$ law by inverse analysis (IA)

The post-cracking tensile behaviour of random discrete fibre reinforced concrete can be simulated either by a stress–crack width relationship, $\sigma - w$, or a stress–strain relationship, $\sigma - \varepsilon$. Since the latter can be used directly in the design of reinforced concrete elements, it is thus often used for design purposes. In the case of steel fibre reinforced concretes, SFRC, of low fibre content where multiple cracking does not occur, the $\sigma - \varepsilon$ response is usually correlated with the $\sigma - w$ diagram by adopting a characteristic length parameter. The $\sigma - w$ is the most adequate to simulate the post-cracking behaviour of low fibre content SFRC (Montaignac et al., 2011), and can be directly obtained from uniaxial tensile tests (RILEM-TC162-TDF, 2001). On the other hand, in indirect tensile tests, the
σ – w response of SFRC is assessed by an inverse analysis procedure that takes into account the experimental test results, such as: splitting tensile test (ASTM-C496, 2004); three-point notched beam bending test (RILEM-TC162-TDF, 2002b); wedge splitting test (Skocek and Stang, 2008).

It would be expectable that the σ – w relationships obtained from distinct tensile test methods would render close material σ – w relationships, but, actually, this does not occur (Cunha, 2010). From a conceptual point of view, the uniaxial tensile test is the most appropriate method to obtain the σ – w relationship, since it can provide directly a stress – crack width relationship. From the aforementioned relationship, all the fracture mode I parameters can be derived, namely, the stress at crack initiation, the fracture energy and the shape of the stress – crack width relationship. However, executing this test involves some economic and logistic difficulties, such as the necessity of specialized and expensive equipment, sophisticated test set-up to avoid detrimental interferences, like load eccentricity, since it decreases the stress at the onset of crack initiation (Zhou, 1988), therefore, this test is not used so often. Furthermore, the obtained results are quite sensitive to the geometry, size and boundary conditions of the specimens (Barragán, 2002; Bazant and Pfeiffer, 1987; Carpinteri et al., 2002). Due to the aforesaid disadvantages, other more simple and economic test methods are being used to determine mode I fracture parameters.

Splitting tensile test, also known as Brazilian test, is well disseminated to estimate the concrete tensile strength. Recently, different types of tests based on the latter have been proposed, namely, double-punch test (Barcelona test) (Chen, 1970; Molins et al., 2009) and wedge splitting test (Ostergaard, 2003). The main advantages of the splitting tensile test are that it is very simple and economic of executing, can be performed with either cylindrical (e.g. extracted cores from real structural elements) or cubic specimens. Moreover, it is only required a testing rig capable of performing compressive loading. Unlike three-point beam bending test, it is expected that the result should be closer to uniaxial tensile test, since most of the concrete bulk in the loading plane is subjected to a constant tensile stress (Carmona and Aguado, 2012). One disadvantage of this test method lies in disability to obtain suitable data on the post-cracking regime due to the unstable crack propagation. However, by performing this test with a closed-loop crack width control, a stable response can be achieved, thus this problem can be easily overcome (Rocco et al., 1999).
In the case of the three-point beam bending test, Cunha (2010) showed that the stress – crack width diagram determined from the inverse analysis of three-point beam bending test results overestimated the tensile post-peak behaviour of steel fibre reinforced self-compacting concrete, SFRSCC, when compared to the one obtained from uniaxial tensile tests. This tendency was also observed, even if the influence of fibre distribution and orientation was taken into consideration. From another point of view, having in mind that concrete’s flowability in the fresh state affects significantly fibre orientation / dispersion and, consequently, the mechanical properties of SFRSCC, designing planar structural elements like panels, shells and walls from constitutive laws derived from the results of this test can conduct to unrealistic predictions. In Chapter 3, it was shown that, in the case of casting beams and panels, fibres tend to be oriented parallel and perpendicular to the concrete flow direction, respectively. Therefore, in order to determine a constitutive tensile law that simulates, as close as possible, the material behaviour in the real structure, recently some design documents such as the UNI-11039 (2003) and CEB-FIP (2010) suggest that the testing specimen has to be cast in such a way that its fibre orientation profile is similar to the one in the structural element. Consequently, although bending tests are suggested by recommendations and test standards, such as RILEM-TC162-TDF (2002b) and EN-14651 (2007), researchers prefer to estimate the residual tensile strength of SFRC by means of other methods like uniaxial tensile test, splitting tensile test or centrally loaded round panels as described in ASTM-C1550 (2004).

Since mechanical behaviour of SFRC is strongly affected by fibre distribution and orientation, a simple methodology to realistically predict the fibre orientation dependent response of fibre reinforced composites in tension is needed. Splitting tensile test could be considered for this purpose because it is cheap, does not need sophisticated testing equipment, and can be executed easily also on cores extracted from structural elements, thus maintaining the same fibre orientation profile as in the structure. Nevertheless, it is well known that splitting test may not be the most adequate option to ascertain the SFRC tensile parameters, mainly due to the biaxial stress-state along the crack plane. In spite of that, in this section, the possibility of employing splitting tensile test to derive in a realistic fashion the uniaxial tensile stress – crack width relationship was checked. The present section illustrates a methodology to predict the stress – crack width ($\sigma - w$) relationship of SFRSCC of laminar structures using an inverse analysis procedure taking
the experimental results of splitting tensile tests. The $\sigma - w$ relationship of the SFRSCC was obtained from the numerical simulations of the splitting tensile results with a nonlinear 3D finite element model. Finally, the $\sigma - w$ response obtained from inverse analysis was compared to the one directly derived from the uniaxial tensile tests.

4.3.1 Numerical simulation

In this section, the methodology is presented for obtaining the post-cracking behaviour of SFRSCC by inverse analysis, IA, of the splitting tensile test results. The experimental force – crack width responses were simulated adopting distinct sets of parameters for the tensile stress – crack width law. For this purpose, the ABAQUS® finite element software (Abaqus, 2009a) was used. To check the accuracy of the proposed methodology, further ahead, the $\sigma - w$ relationship obtained from the inverse analysis of the splitting tensile tests will be compared to the $\sigma - w$ directly obtained from the uniaxial tensile test.

The geometry of the specimen and the material behaviour were simulated using an appropriate element and material model available in the ABAQUS® program library (Abaqus, 2009b, 2009c). Due to the symmetry of specimen geometry, supports and loading conditions used in the splitting tensile test set-up, a quarter of the specimen was modeled, Figure 4.15(a). The geometry model comprises two main parts: notch and un-notch (flush) parts since they have different thicknesses. Preliminary analyses were carried out in order to obtain a mesh refinement that does not compromise both the accuracy of the numerical simulations and the computational cost. The assembled mesh parts are shown in Figure 4.15(b). In the present mesh, 8-noded solid elements with hexahedral shape and 8-integration points were used. The total number of elements was 5674 with maximum and minimum volumes equal to 82 mm$^3$ and 39 mm$^3$, respectively. In addition, the distortion of the finite elements was also checked to avoid modeling inaccuracies. To model the softening behaviour of SFRSCC, the numerical analyses were carried out under displacement control. The non-zero prescribed displacement constraints were applied on the top of the notch part of the model, in similarity to the experiment.
Figure 4.15: Three-dimensional view of numerical model: (a) geometry, constraints and prescribed displacement, (b) finite element mesh.

4.3.2 Concrete constitutive model

The concrete damage plasticity (CDP) model was used to simulate the mechanical behaviour of concrete, since it is capable of simulating the damage due to concrete cracking and plastic deformations in compression. In other words, this model uses the concept of isotropic damage elasticity in combination with isotropic compression and tension plasticity to simulate the inelastic behaviour of concrete under compressive and tensile stresses. The model is a modification of the Drucker–Prager hypothesis.

In general, the flow potential surface and the yield surface make use of the principal stresses \((S_1, S_2, S_3)\) or the stress invariants \((I_1, J_2, J_3)\). The yield function defines a surface in the effective stress space in order to represent the states of failure or damage (Figure 4.16). The effective stress tensor is determined as follow:

\[
\bar{\sigma} = D_0^{el} : (\varepsilon - \varepsilon^{pl})
\]  

(4.2)

where \(D_0^{el}\) is the initial (undamaged) elastic constitutive tensor of the material, \(\varepsilon\) is the strain tensor and \(\varepsilon^{pl}\) is the plastic strain tensor. On other hand, the yield function and the flow potential surface use two stress invariants of the effective stress tensor, namely the hydrostatic stress \((\bar{p})\) and the Von Mises equivalent effective stress \((\bar{q})\) that can be determined from Eqs. 4.3 and 4.4, respectively.
\[
\bar{p} = -I_1/3 = -1/3 \text{trace} (\sigma) \tag{4.3}
\]

\[
\bar{q} = \sqrt{3J_2} = \sqrt{3/2 (\bar{S} : \bar{S})} \tag{4.4}
\]

where \( \bar{S} \) is the deviatoric part of the effective stress tensor \( \bar{\sigma} \). The potential flow and yield function are defined with four parameters. These parameters are the dilation angle \( (\psi) \) and the eccentricity \( (e) \) that determines the shape of the potential flow surface, the ratio between the initial biaxial compressive strength and the initial uniaxial compressive strength \( (\sigma_{ib}/\sigma_{ic}) \), the \( k \) parameter that defines the initial yield surface and will be detailed subsequently. The potential plastic flow surface in CDP model is the same as Drucker – Prager hyperbolic function:

\[
G = \sqrt{(e \sigma_{ic} \tan \psi)^2 + \bar{q}^2} - \bar{p} \tan \psi \tag{4.5}
\]

Since the potential plastic flow surface is defined in the \( p-q \) plane (effective meridional plane), therefore, the dilation angle \( (\psi) \) should be measured in the same plane \( (p-q \) plane) at high confining pressure. However, the \( \psi \) is simulated as concrete’s internal friction angle, which usually ranges between 34° to 43° (Kmiecik and Kaminski, 2011; Lee et al., 2008) depending on the concrete type. According to Jankowiak and Lodygowski (2005), the eccentricity factor in Eq. 4.5, \( e \), which represents the eccentricity of the potential plastic surface, can be determined by the ratio between the uniaxial tensile strength \( (\sigma_{it}) \) and compressive strength \( (\sigma_{ic}) \).

![Figure 4.16: Yield surface under biaxial stress used in the concrete damage plasticity model.](image)

120
The CDP model uses a yield surface that is defined as the loading function proposed by Lubliner et al. (1989), Figure 4.16. The evaluation of the yield surface is controlled by two hardening variables, namely, the plastic strain in tension ($\tilde{\varepsilon}_{pt}^c$) and the plastic strain in compression ($\tilde{\varepsilon}_{pc}^c$). In the case of the effective stress, the yield function is determined as follow:

$$
F = \frac{1}{1-\alpha} \left( q - 3\alpha \bar{p} + \beta \left( \tilde{\varepsilon}^c \right) \left( \tilde{\sigma}_{\max} \right) - \gamma \left( \tilde{\sigma}_{\max} \right) - \tilde{\sigma}_f \left( \tilde{\varepsilon}^c \right) \right) \quad \text{(4.6)}
$$

where:

$$
\alpha = \frac{(\sigma_{s0}/\sigma_{so}) - 1}{2(\sigma_{s0}/\sigma_{so}) - 1}, \quad 0 \leq \alpha \leq 0.5 \quad \text{(4.7)}
$$

$$
\beta = \frac{\tilde{\sigma}_f \left( \tilde{\varepsilon}^c \right)}{\tilde{\sigma}_f \left( \tilde{\varepsilon}^c \right)} (1 - \alpha) - (1 + \alpha) \quad \text{(4.8)}
$$

$$
\gamma = \frac{3(1 - k_c)}{2k_c - 1} \quad \text{(4.9)}
$$

In these equations, $\tilde{\sigma}_{\max}$ stands for the maximum principal effective stress and is the algebraic maximum eigen value of the effective stress $\tilde{\sigma}$ indicated in Eq. 4.2 (Jankowiak et al., 2005), $\langle x \rangle$ represents Macauley bracket $= 1/2(\|x\| + x)$, $\tilde{\sigma}_f \left( \tilde{\varepsilon}^c \right)$ and $\tilde{\sigma}_f \left( \tilde{\varepsilon}^c \right)$ are the effective tensile and compressive cohesive stresses, respectively, and will be discussed in the subsequent section. Parameter $k_c$ is physically assumed as a ratio of the distances between, respectively, the compressive meridian and the tensile meridian with hydrostatic axis in the deviatoric cross section. If this ratio tends to 1, the deviatoric cross section of the failure surface becomes a circle similar to the Drucker – Prager yielding surface. However, definition of this parameter is only possible if the full triaxial compressive tests are executed on concrete specimens (Kmiecyk and Kaminski, 2011). Table 4.3 includes the constitutive parameters of CDP model used to simulate the concrete behaviour in the splitting tensile specimens.
Table 4.3: The constitutive parameters of CDP model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilatation angle [degrees]</td>
<td>40</td>
</tr>
<tr>
<td>Eccentricity, e [-]</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma_{0c}/\sigma_{co}$ [-]</td>
<td>1.16</td>
</tr>
<tr>
<td>$K_c$ [-]</td>
<td>0.667</td>
</tr>
</tbody>
</table>

4.3.2.1 Stress – strain relationship for modeling the SFRSCC uniaxial compressive behaviour

In CDP model, when the value of the compressive stress ($\sigma_c$) reaches the compressive strength ($\sigma_{cm} = f_{cm}$), the concrete shifts to the softening phase, as depicted in Figure 4.17. The compressive inelastic strain, $\varepsilon_c^{in}$, used in CDP model is defined by subtracting the elastic strain component, $\varepsilon_c^{el}$, from the total strain, $\varepsilon_c$, in the uniaxial compressive test.

\[
\varepsilon_c^{in} = \varepsilon_c - \varepsilon_c^{el} \quad (4.10)
\]

\[
\varepsilon_c^{el} = \sigma_c/E_0 \quad (4.11)
\]

According to what is proposed by CEB-FIP (2010), in the pre-peak phase concrete starts to show nonlinear behaviour above a stress level corresponding to 40% of $f_{cm}$. In the CDP model, from the stress – inelastic strain relationship ($\sigma_c - \varepsilon_c^{in}$) that is provided by the user, the stress versus strain response ($\sigma_c - \varepsilon_c$) is converted into the stress – plastic strain curve ($\sigma_c - \varepsilon_c^{pl}$) automatically by the software.

When unloading during the softening stage, the CDP model uses Eq. 4.12 to convert inelastic strain to plastic strain ($\varepsilon_c^{pl}$). In this equation, $d_c$, which is known as damage parameter in compression (Figure 4.17), depends on the concrete damage level, and ranges from zero to one, respectively for undamaged and fully damaged material.

\[
\varepsilon_c^{pl} = \varepsilon_c^{in} - \frac{d_c}{(1-d_c)} \frac{\sigma_c}{E_0} \quad (4.12)
\]
By assuming $E_0$ as the concrete initial modulus of elasticity in the undamaged phase, it can be determined the compressive stress and also the effective cohesive stress by Eqs. 4.13 and 4.14.

$$\sigma_c = (1-d_c)E_0(e_c - \bar{\varepsilon}_c^{pl})$$  \hspace{1cm} (4.13)

$$\bar{\sigma}_c = \sigma_c / (1-d_c) = E_0(e_c - \bar{\varepsilon}_c^{pl})$$  \hspace{1cm} (4.14)

![Diagram](image)

**Figure 4.17:** Definition of inelastic compressive strain in the CDP model.

Available research (Balaguru and Shah, 1992; Barros, 1995) shows that the post-peak behaviour of fibre reinforced composites, FRC, in compression cannot be simulated by the models proposed for plain concrete (Wee et al., 1996), due to the higher post-peak compressive residual strength provided by fibre reinforcement. Therefore, among some available expressions in literature for FRC (Barros and Figueiras, 1999; Cunha et al., 2008; Fanella, 1985), the model proposed by Barros and Figueiras (1999) was used to derive the stress – inelastic strain relationship ($\sigma_c - \bar{\varepsilon}_c^{in}$) for SFRSCC. This relationship was defined and input into the software taking into account the compressive strength from the experimental part. This relationship is dependent on two parameters, namely, the compressive strength, $f_{cm}$, and the fibre weight percentage, $W_f$ and is determined by:

$$\sigma_c = f_{cm} \frac{\varepsilon_c / \varepsilon_{c1}}{(1-p-q) + q(\varepsilon_c / \varepsilon_{c1}) + p(\varepsilon_c / \varepsilon_{c1})^{(1-q)/p}}$$  \hspace{1cm} (4.15)
with

\[ q = 1 - p - \frac{E_{c1}}{E_{ci}} , \quad p + q \in \{0, 1\} , \quad \frac{1-q}{p} > 0 \]  \hspace{1cm} (4.16)

and

\[ p = 1.0 - 0.919 \exp(-0.394W_f) \]  \hspace{1cm} (4.17)

In Eq. 4.15, \( \varepsilon_{c1} \) is the strain at compressive strength, and for concretes reinforced with hooked-end steel fibres of an aspect ratio of 60 (the ones used in the present experimental programs) is given by:

\[ \varepsilon_{c1} = \varepsilon_{c10} + 0.0002W_f \]  \hspace{1cm} (4.18)

According to CEB-FIP (2010), \( \varepsilon_{c10} \), which is the strain at compressive strength of plain concrete, is equal to 2.2×10\(^{-3}\), \( E_{ci} \) is the tangent modulus of elasticity, obtained from 21500\([f_{cm}/10]^{3/2}\), and \( E_{c1} \) is the secant modulus of elasticity given by \( f_{cm}/\varepsilon_{c1} \). From the aforementioned equations, the compressive mechanical properties and the stress – strain relationship used in the numerical simulation were determined. Table 4.4 includes the values of the model parameters used for the compressive behaviour.

Table 4.4: Mechanical properties adopted in the numerical simulations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, ( \rho )</td>
<td>2.4×10^6 N/mm(^3)</td>
</tr>
<tr>
<td>Poisson ratio, ( \nu )</td>
<td>0.2</td>
</tr>
<tr>
<td>Initial young modulus, ( E_{ci} )</td>
<td>34.15 N/mm(^2)</td>
</tr>
<tr>
<td>Compressive strength, ( f_{cm} )</td>
<td>47.77 N/mm(^2)</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>Inverse analysis</td>
</tr>
<tr>
<td>Post-cracking parameters</td>
<td>Inverse analysis</td>
</tr>
</tbody>
</table>
4.3.2.2 Stress – strain relationship for modeling the SFRSCC uniaxial tensile behaviour

The stress – strain response under uniaxial tension follows a linear elastic behaviour until it reaches the tensile strength \( \sigma_{t0} \), which corresponds to the coalescence of micro-cracking into a macro-crack. Once the tensile strength is attained, the stress starts to decrease, Figure 4.18. If strain softening of SFRC is considered, the stress reduction is controlled by the reinforcement mechanisms of fibres bridging the active crack plane. The SFRC softening phase is expressed as a function of cracking strain, \( \tilde{\varepsilon}_t^{ck} \), which can be determined by subtracting the elastic strain corresponding to the undamaged part from the total strain:

\[
\tilde{\varepsilon}_t^{ck} = \varepsilon_t - \varepsilon_{0t}^{el}
\]  
\[\varepsilon_{0t}^{el} = \sigma_t / E_0 \tag{4.19} \]

From the stress versus cracking strain response \( (\sigma_t - \tilde{\varepsilon}_t^{ck}) \) defined by the user, the stress – strain curve \( (\sigma_t - \varepsilon_t) \) is converted to stress – plastic strain relationship \( (\sigma_t - \tilde{\varepsilon}_t^{pl}) \). When unloading during the softening stage, the CDP model uses Eq. 4.21 to convert inelastic strain to plastic strain \( (\tilde{\varepsilon}_t^{pl}) \). In this equation, \( d_t \), which is known as damage parameter in tension (Figure 4.18), depends on the concrete damage level, and ranges from zero to one, respectively for undamaged and fully damage material.

\[
\tilde{\varepsilon}_t^{pl} = \tilde{\varepsilon}_t^{in} - \frac{d_t \sigma_t}{(1-d_t) E_0} \tag{4.21}
\]

By assuming \( E_0 \) as the concrete initial modulus of elasticity in the undamaged phase, it can be determined the tension stress and also the effective cohesive stress by Eqs. 4.22 and 4.23.

\[
\sigma_c = (1-d_c) E_0 \left( \varepsilon_c - \tilde{\varepsilon}_c^{pl} \right) \tag{4.22}
\]
\[
\sigma_c' = \sigma_c / (1-d_c) = E_0 \left( \varepsilon_c - \tilde{\varepsilon}_c^{pl} \right) \tag{4.23}
\]

In order to guarantee that results are not dependent on the refinement of the finite element mesh, it is preferable to implement fracture energy cracking criterion to define tension stiffening of concrete. In this case, the concrete tensile response is modeled by a
stress – crack width displacement curve ($\sigma_t - w$) rather than stress – strain relationship ($\sigma_t - \varepsilon_t$). However, implementation of this concept requires the adoption of a characteristic length parameter ($L$) associated to an integration point. This parameter depends on the element geometry, dimensions of the element and of the adopted integration scheme (Abaqus, 2009b).

![Diagram of strain definition after cracking-tension stiffening.](image)

*Figure 4.18: Strain definition after cracking-tension stiffening.*

### 4.3.3 Inverse analysis procedure

The values $\sigma_i$ and $w_i$ that define the tensile stress – crack width law were determined by fitting the numerical load – crack width curve to the correspondent experimental average curve. The applied inverse analysis methodology followed up what was proposed by Roelfstra and Wittmann (1986). This procedure can be divided into three main steps. In the first stage, a preliminary set of the parameters that define the $\sigma - w$ relationship were initialized and set as input of the uniaxial tensile behaviour in the model (section 4.1.2), and the error of the numerical simulation was also initialized ($\text{err}_f = 5\%$). In the second stage, the numerical load – crack width response, $F_{\text{NUM}} - w$, was obtained from the nonlinear finite element analysis. In the last step, the computed numerical $F_{\text{NUM}} - w$ response was compared to the experimental one, $F_{\text{EXP}} - w$. The value of the force at
distinct crack widths were computed, and the normalized error, $err$, was determined as follows:

$$
err = \frac{\sum_{i=0}^{n} |F_{i}\text{EXP} - F_{i}\text{NUM}|}{\sum_{i=0}^{n} F_{i}\text{EXP}}
$$

(4.24)

where $F_{i}\text{EXP}$ and $F_{i}\text{NUM}$ were the experimental and the numerical load value at $i$ crack width value, respectively. If the $err$ is smaller than the pre-defined value the iterative procedure stops, otherwise a new set of $\sigma - w$ parameters is updated and checked out.

The final SFRSCC $\sigma - w$ relationship was defined by the parameter set that leads to a lowest normalised error between the experimental and numerical compressive force versus crack width curves.

### 4.3.4 Numerical results, validation and discussion

Figure 4.19 shows the experimental compressive force – crack width curves obtained from the splitting tensile tests, when the notch plane is parallel ($\theta=0^\circ$) and perpendicular ($\theta=90^\circ$) to the concrete flow direction. The plotted crack width was calculated as the average of the values measured by the five LVDTs. In this figure, $\text{EXP}_{\text{SPLT}}\text{Avg}$ and $\text{EXP}_{\text{SPLT}}\text{Envelope}$ are, respectively, the average and envelope relations. In addition was comprised the numerical response ($\text{NUM}_{\text{SPLT}}$) obtained from inverse analysis. The numerical analysis was carried out up to a crack width of 2 mm measured at the centre height of the cylinder. A good accuracy between the numerical and experimental responses was observed up to this crack width limit.

![Graphs showing experimental and numerical force-crack width relationship](image)

*Figure 4.19: Experimental and numerical force – crack width relationship, $F_{w}$, for: (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$.***
Table 4.5 includes the residual numerical and experimental forces and the normalized fitting error (\(\text{err}\)) obtained by Eq. 2.24. In this table, \(F_{cr}\) and \(F_{\text{peak}}\) are forces at crack initiation and peak load, respectively; \(F_{0.3}, F_1\) and \(F_2\) are the post-cracking residual forces at a crack width of 0.3, 1 and 2 mm, respectively. When analyzing the abovementioned residual forces, a negligible difference between the numerical and experimental results was observed. If one compares the determined error (\(\text{err}\)) for each series, they were smaller than the initialized error (5 %).

<table>
<thead>
<tr>
<th>(\theta = 0^\circ)</th>
<th>(\theta = 90^\circ)</th>
<th>(F_{cr})</th>
<th>(F_{\text{peak}})</th>
<th>(F_{0.3})</th>
<th>(F_1)</th>
<th>(F_2)</th>
<th>(\text{err})</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUM\text{SPLT}</td>
<td>EXP\text{SPLT}</td>
<td>40</td>
<td>58.67</td>
<td>57.53</td>
<td>48.03</td>
<td>40.51</td>
<td>3.92</td>
</tr>
<tr>
<td>NUM\text{SPLT}</td>
<td>EXP\text{SPLT}</td>
<td>27.12</td>
<td>32.25</td>
<td>28.32</td>
<td>24.62</td>
<td>22.13</td>
<td>3.82</td>
</tr>
</tbody>
</table>

\(\parallel\) and \(\perp\) – notch direction parallel (\(\theta = 0^\circ\)) and perpendicular (\(\theta = 90^\circ\)) to the concrete flow direction, respectively.

Figure 4.20: Numerical uniaxial stress – crack width relationship, \(\sigma – w\), obtained from inverse analysis for: (a) \(\theta = 0^\circ\) and (b) \(\theta = 90^\circ\).
cracking residual stresses in the case of the $\theta = 0^0$ series were considerable higher. This was due to the fibres tendency to be oriented perpendicular to the concrete flow direction, when panels are casted from the centre. As shown in Figure 3.27, there is a higher probability of more fibres intersecting the fracture surface in specimens with a notch plane coinciding with the concrete flow lines, ($\theta = 0^0$), than in the $\theta = 90^0$ series. This was also supported on the number of the effective fibres at the fracture surface, details regarding this aspect can be found in Chapter 3.

Figure 4.21(a)-(e) show the strains at various loading phases for the numerical modeling of the $\theta = 90^0$ series. Figure 4.21(a) shows the strain distribution just before the crack formation (pre-cracking stage). In this stage, the tensile strain field mainly raised at mid height of the notch’s region. Up to this strain level, the strains were due to the transversal elastic deformation of the bulk SFRSCC. Once the concrete tensile stress was reached, which coincided with attaining the peak load, the crack localization happened at the mid height of the notch part as illustrated in Figure 4.21(b). In the latter figure, the strain value was converted from the elastic strain to the cracking strain ($\varepsilon_{cr}$). After this step, the material showed a softening response and the crack progressed from the opposite notch tips to the centre of the specimen and towards the loading supports, Figure 4.21(c) and (d). Finally, this stage was followed by the crack widening at the centre of the specimen (see the deformed mesh in the Figure 4.21(e)) and decrease of the load bearing capacity of the specimen.
Figure 4.21: Strain distribution in numerical model.

4.3.5 Uniaxial tensile response vs. tensile law obtained from IA

Figure 4.22 and Figure 4.23 show for the $\theta = 0^\circ$ and $\theta = 90^\circ$ series, respectively, the uniaxial $\sigma – w$ relationships obtained from the inverse analysis procedure of the splitting tensile test ($\text{NUMSPLT}$), the envelope and average curves from uniaxial tensile test ($\text{EXP}_{\text{UTT Envelope, EXP}_{\text{UTT Avg.}}}$) carried out according to the RILEM-TC162-TDF (2001) recommendations. Hereinafter, the $\text{EXP}_{\text{UTT Avg}}$ curves will be considered as reference curves, since the uniaxial tensile test is the only test that could render the accurate tensile stress – crack width relationship. Consequently, they will be used for the validation of the presented numerical methodology. Additionally is presented the $\sigma – w$ relation with the stress calculated according to Eq. 4.1 as proposed by ASTM-C496 (2004) standard ($\text{EXP}_{\text{SPLT}}$):

\[
\sigma = \begin{cases} 
0.024e^{11} & \text{for } 0^\circ \\
0.21e^{11} & \text{for } 0^\circ \\
0.58e^{11} & \text{for } 0^\circ 
\end{cases}
\]
The $\sigma - w$ relationship obtained from the inverse analysis procedure rendered a relatively good approximation of the uniaxial tensile response, principally, for the series $\theta = 90^\circ$. As shown in Figure 4.22, $NUM_{SPLT}$ and $EXP_{SPLT}$ approaches provided similar tensile strengths, 3.6 and 3.3 MPa, respectively, which were higher than $EXP_{UTT}$ Avg. The obtained tensile strength from the $NUM_{SPLT}$ (3.6MPa) was in accordance with the mean value suggested by CEB-FIP (2010) (3.5MPa). At the early cracking stages ($w < 0.6$ mm), in $\theta =0^\circ$ series, $NUM_{SPLT}$ and $EXP_{SPLT}$ approaches gave a stress – crack width relationship nearby the upper bound limit of the $EXP_{UTT}$ envelope. However, this overestimation could be ascribed to the effects of the compressive stress along the notch plane.
Figure 4.24: Stress distribution in numerical modeling of $\theta = 0^\circ$ specimen: (a) horizontal direction at the time of the crack initiation, (b) vertical direction at the time of the crack initiation, (c) horizontal direction and $w = 0.5$ mm, (d) vertical direction and $w = 0.5$ mm, (e) horizontal direction and $w = 1$ mm, (f) vertical direction and $w = 1$ mm.

Figure 4.24(a)-(f) show the stress field in the case of the $\theta = 0^\circ$ simulation accordingly to the horizontal ($S_{11}$) and vertical ($S_{22}$) directions: on the cracking onset; at $w = 0.5$ mm corresponding to the maximum tensile stress, see Figure 4.20; at $w = 1$ mm
within the softening phase. Figure 4.24(a) and (b) show that once the stress in the S11 direction reached the tensile strength of the material, all the elements at the notch zone were subjected to compressive stresses in the S22 direction. Therefore, this biaxial stress state led to an overestimation of the tensile strength in the splitting tests. After the cracking initiation, at a crack width of 0.5 mm, the tensile stress along S11 direction reached its maximum value. Nevertheless, the compressive stress field along the S22 direction is considerable, see Figure 4.24(d), which provides some confinement to the fibres crossing the fracture plane, conducting to an improvement of the fibre-matrix bond behaviour, thus increasing the fibre pull-out resistance. Consequently, the $\sigma - w$ relationship, derived from the inverse analysis up to the $w = 0.5$ mm, was overestimated.

Figure 4.25 shows the stress components acting on an active fibre crossing the notched section where the crack is propagating. The load applied to the specimen, $F$, contributes for the increase of the stress component that improves the fibre confinement, $\sigma_N$. For a fibre that is subjected to the compressive force in the fracture plane of the splitting tensile test, see Figure 4.25, the stress components consist of, namely: shear stress, $\tau$, and normal stress, $\sigma_N$. According to the Mohr-Coulomb failure criterion, it could be illustrated that:

$$\tau_m = c + \sigma_N \tan(\phi) \quad (4.25)$$

where $\tau_m$ is the shear strength, $\sigma_N$ is the normal stress, $c$ is the cohesion of the fibre-matrix interface, and $\phi$ is the frictional angle of the microstructure of this interface.

![Figure 4.25: Stress components acting on an active fibre in the splitting tensile test.](image-url)
When the fibre was debonded and started to slip, cohesion and frictional angle tend to decrease due to the increase of the damage on this fibre-matrix interface (Naaman and Najm, 1991; Naaman et al., 1992), see Figure 4.26. From another point of view, the shear stress and normal stress present a direct relationship. On the other hand, by increasing normal stress, $\sigma_N$, shear stress tends to increase as well. This procedure is schematically represented in Figure 4.27. In conclusion, for the $\theta=0^\circ$ series, the compressive stress field along the crack plane imposes a higher normal stress component, $\sigma_N$, which provides a larger confinement to the fibre, resulting higher bond strength and energy absorption during the fibre pull-out process (Figure 4.27). These effects have the consequence of overestimating the $\sigma - w$ relationship achieved from the inverse analysis. However, once the compressive stress field along the fracture plane reduces, the normal stress component decreases, and the fibre pull-out is marginally affected by this confinement effect.

![Figure 4.26](image-url)  
*Figure 4.26: Variation of cohesion and frictional angle by increasing fibre slip.*

![Figure 4.27](image-url)  
*Figure 4.27: The influence of the normal stress on the bond shear stress-slip relationship.*
From another point of view, in the inverse analysis procedure for both series, the stress-strain response in compression was derived using a model proposed by Barros and Figueiras (1999) with an assumption that the fibres were distributed and oriented randomly. It is shown that in the case of FRC the compressive post-peak response depends on the contribution of the fibres to bridge cracks. In reality, the compressive behaviour of $\theta = 0^\circ$ specimen could provide a higher post-peak response due to the greater number of the fibres and their preferential orientation towards the direction of the applied stress.

Once the crack gets wider, $\text{EXP}^{\text{SPLT}}$ method is unable to correctly predict the tensile behaviour, since this method assumes a linear elastic stress distribution. On the other hand, $\text{NUM}^{\text{SPLT}}$ starts to get closer to the response obtained from the uniaxial tensile test. This trend could be justified by analysis of Figure 4.24(e) and (f). In these figures, all the elements in the central zone of the notch plane were subjected to the tensile stress in the both directions since the compressive stress area along the fracture plane was decreased and shifted. The accuracy of these methods for predicting the experimental $\sigma - w$ curves was also quantified by its fracture parameters. Table 4.6 includes the residual stresses and toughness parameters for different average crack widths obtained from distinct methods. In this table, $\sigma_{\text{peak}}$ is the stress at peak load; $\sigma_{0.3}$, $\sigma_1$ and $\sigma_2$ are the residual stresses at a crack width of 0.3, 1 and 2 mm, respectively; $G_{F1}$ and $G_{F2}$ are the dissipated energy up to a crack width of, respectively, 1 and 2 mm. By comparing the determined fracture parameters for the $\text{NUM}^{\text{SPLT}}$ and $\text{EXP}^{\text{SPLT}}$ relations, regarding the $\theta = 0^\circ$ series, they showed higher values than the $\text{EXP}^{\text{UTT}}$Avg, respectively, 13%, 66% in the case of $\sigma_1$, 9.65 %, 144.74 % for $\sigma_2$ and 42 %, 64 % in $G_{F2}$.

With reference to the tensile strength in $\theta = 90^\circ$ series (see Figure 4.23), like the previous series, inverse analysis procedure of the splitting tensile tests overestimated it when comparing to the tensile strength obtained from the uniaxial tests, although it was within the experimental envelope. According to the $\text{EXP}^{\text{UTT}}$ results, abrupt load decay occurred at crack initiation due to the brittle nature of concrete fracture and lower fibre content. The loss of stress is interrupted when hooked fibre reinforcement mechanisms become more effective, which happens at a crack width in-between [0.1 - 0.3] mm and pseudo-hardening phase is initiated. The result of the inverse analysis method reproduced the $\text{EXP}^{\text{UTT}}$ response with an acceptable accuracy, because in this series, during all steps a major number of the elements in the fracture plane were subjected to tensile stress in both
directions. Since the tensile residual stresses of this series were much lower than the previous series (θ= 0°), the load bearing capacity of the specimen was decreased and the compressive stresses were not so preponderant in the overall response.

The experimental splitting σ – w method, EXP_{SPLT}, clearly overestimated the post-cracking behaviour for the same reasons pointed out previously for the θ=0° series. These differences become more visible if one compares the fracture parameters included in Table 4.6. This comparison could be performed by estimating an error for each parameter separately. Hence, for NUM_{SPLT} and EXP_{SPLT} methods, regarding the θ = 90° series, they both showed a higher value than EXP_{UTT}Avg, respectively, of 0.95 %, 102 % in the case of σ\(_{0.3}\); 37 %, 92 % for σ\(_1\); 15 %, 91 % in G\(_{F1}\) and finally 17 %, 106 % in the case of G\(_{F2}\).

Table 4.6: Residual stress and toughness parameters obtained from different analysis.

<table>
<thead>
<tr>
<th>Series</th>
<th>Parameter</th>
<th>(\sigma_{\text{peak}}) [MPa]</th>
<th>(\sigma_{0.3}) [MPa]</th>
<th>(\sigma_1) [MPa]</th>
<th>(\sigma_2) [MPa]</th>
<th>(G_{F1}) [N/mm]</th>
<th>(G_{F2}) [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ = 0° (σ(_{</td>
<td></td>
<td>}))</td>
<td>NUM_{SPLT}</td>
<td>4.50</td>
<td>4.10</td>
<td>2.60</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>EXP_{SPLT}</td>
<td>4.39</td>
<td>4.23</td>
<td>3.82</td>
<td>2.79</td>
<td>4.07</td>
<td>7.32</td>
</tr>
<tr>
<td></td>
<td>EXP_{UTT}</td>
<td>3.33</td>
<td>3.24</td>
<td>2.30</td>
<td>1.14</td>
<td>2.94</td>
<td>4.47</td>
</tr>
<tr>
<td>θ = 90° (σ(_{\perp}))</td>
<td>NUM_{SPLT}</td>
<td>3.20</td>
<td>1.06</td>
<td>1.40</td>
<td>0.47</td>
<td>1.26</td>
<td>2.18</td>
</tr>
<tr>
<td></td>
<td>EXP_{SPLT}</td>
<td>2.47</td>
<td>2.13</td>
<td>1.96</td>
<td>1.50</td>
<td>2.08</td>
<td>3.83</td>
</tr>
<tr>
<td></td>
<td>EXP_{UTT}</td>
<td>2.72</td>
<td>1.05</td>
<td>1.02</td>
<td>0.56</td>
<td>1.09</td>
<td>1.86</td>
</tr>
</tbody>
</table>

Figure 4.28 shows the relationships between the ratio of the splitting tensile post-cracking parameters obtained from inverse analysis procedure, \(\sigma_{\text{SPLT}}\) and \(G_{F_{SPLT}}\), and the uniaxial tensile post-cracking parameters, \(\sigma_{\text{UTT}}\) and \(G_{F_{UTT}}\), respectively, for distinct crack widths. The data plotted in Figure 4.28 clearly showed that \(\sigma_{\text{SPLT}}\) was generally higher than \(\sigma_{\text{UTT}}\) for almost all \(w\) (CMOD) values. Therefore, the inverse analysis of the splitting tensile test overestimated the tensile residual strength, being this effect more preponderant on the specimens with more effective fibres at the crack surface. In spite of that, it should be noticed that in most cases the residual parameters, which define the \(\sigma - w\) law, were within the experimental envelope of the uniaxial tensile tests. The overall results were
predominantly satisfactory if considering the anisotropic material behaviour due to the anisometry of the fibre orientation and distribution induced by the flowability of concrete.

![Graph](image)

**Figure 4.28:** Numerical splitting tensile post-cracking parameters versus experimental uniaxial tensile post-cracking parameters: (a) residual stress and (b) fracture energy.

### 4.4 Conclusions

In this chapter, the influence of fibre distribution / orientation on the tensile performance of steel fibre reinforced self-compacting concrete (SFRSCC) was characterized by performing splitting and uniaxial tensile tests on cored specimens extracted from different panel locations. Furthermore, the uniaxial tensile behaviour of steel fibre reinforced self-compacting concrete was obtained indirectly by performing inverse analysis on the experimental results of splitting tensile tests. To validate the presented methodology, the obtained tensile stress versus crack width relationship ($\sigma - w$) was then compared to the one directly obtained from the uniaxial tensile test. For this purpose, a comprehensive nonlinear 3D finite element model was used to simulate the splitting tensile tests.

Fibre distribution and orientation have a strong impact on the tensile behaviour of specimens drilled from the panels. In the case of the series with crack plane parallel to the concrete flow direction ($\theta = 0^\circ$), specimens shown significantly higher post-cracking parameters than the other studied case with a perpendicular crack plane to the flow direction ($\theta = 90^\circ$). When a panel is cast from the centre, fibres have a tendency to line up perpendicularly to the radial flow, mainly due to the uniform flow profile velocity that
diffuses outwards radially from the centre of the panel. Hence, the total number of the effective fibres intersecting the parallel crack plane \((\theta = 0^\circ)\) was higher than the one registered in the orthogonal crack plane \((\theta = 90^\circ)\).

Roughly, a linear relationship between number of the effective fibres, orientation factor and post-cracking parameters were observed. It was shown that by increasing the number of effective fibres as well as their orientation, fracture parameters tend to raise. This strong dependency could explain that in \(\theta=0^\circ\) series due to the appearing higher number of effective fibres which were mainly perpendicular to the crack plane, the concrete represented a semi-hardening behaviour, while in the other series a high stress decay was achieved.

The application of the inverse analysis procedure of the splitting tensile test experimental results predicted successfully the tensile post-cracking parameters of SFRSCC. On the other hand, the prediction of the fracture parameters directly from the experimental behaviour of the splitting tests, \(\text{EXP}_\text{SPLT}\), i.e. assuming a linear stress distribution at the notch section, as expected, rendered not so good results. The \(\sigma - w\) responses determined by the inverse analysis technique reproduced all the distinct phases observed during the uniaxial tensile test, particularly, the reduction in the strength due to the loss of the matrix stiffness once the crack initiated and also the semi-hardening phase at the early cracking stages. Considering the obtained tensile strength from the three mentioned methods, \(\text{NUM}_\text{SPLT}\) and \(\text{EXP}_\text{SPLT}\) tend to overestimate the tensile strength obtained from uniaxial tensile tests, \(\text{EXP}_\text{UTT}\). However, the determined tensile strength by inverse analysis, \(\text{NUM}_\text{SPLT}\), is reasonably in accordance with the suggested by CEB-FIP (2010).

The predicted post-cracking response from inverse analysis of splitting tensile test tends to slightly overestimate the response acquired from the uniaxial tensile tests. However, this can be diluted by the scatter of SFRSCC composites, due to the dispersion and orientation of the fibres. Moreover, one has to account with the distinct nature of the two testing methods, which will induce completely distinct stress states at the fracture surface. In the uniaxial tensile tests, the concrete in the fracture plane is subjected to the pure tensile stress, while in the splitting tensile test a high compressive stress localizes near the supports will dissipate some energy. In general, in the case of using high fibre content in FRC, especially when the tensile behaviour shows a partial or complete hardening behaviour, the presented methodology could somehow overestimate the
constitutive $\sigma - w$ response. Therefore in this case it is preferable to execute another indirect tensile test configuration, by e.g. the one proposed by di Prisco (2013).

In conclusion, the overall results are predominantly satisfactory if considering the distinct nature of the tests and of the SFRSCC anisometry. The inverse analysis of the splitting tensile response can predict with a relatively good accuracy the uniaxial tensile post-cracking behaviour, in particular, for low fibre contents.
CHAPTER 5

Time – Dependent Pull – Out Behaviour

5.1 Introduction

The application of short and randomly distributed steel fibres is becoming increasingly more popular in the concrete technology, since they can attenuate the drawback deal from the concrete brittleness. In concrete reinforced with a relatively low fibre contents, the effectiveness of the fibre reinforcement becomes more predominant after cracking of the matrix, since fibres are the elements that transfer stress across the crack’s surfaces. It is widely acknowledge that the mechanical behaviour of steel fibre reinforced concrete, SFRC, depends not only on the bond behaviour of the fibre/matrix interface, but also on how fibres are distributed and oriented through the matrix (Barnett et al., 2010; Kang and Kim, 2011). Consequently, considering that fibres and matrix are bonded by a weak interface, understanding this interfacial behaviour is of utmost importance for a reliable predictive of the mechanical performance of such composites.

The creep deformation can compromise functionality of a structure due to abnormal high long-term deflection. The durability of cracked concrete structure can be also significantly affected if crack opening evaluation due to creep effect exceeds the limit recommended by codes concerning of conventional steel bars. Predicting the deflection
evaluation due to creep effect is even more important in the cracked structures exclusively reinforced by fibres (Boshoff et al., 2009b). It was shown that the addition of steel fibres to the concrete can limit the long-term crack widening in a cracked section, since fibres restrain the crack growth (Tan et al., 1994; Tan and Saha, 2005). However, the crack restraint effectiveness depends on the time evaluation of the sliding process of the fibre pull-out. Under a sustained load, the time-dependent crack widening has been attributed to two main mechanisms: creep behaviour of fibre/matrix interface, and time-dependent fibre creep. In the case of using steel fibres in the composite, the latter contribution can be negligible due to the relatively low shear level installed in the fibre and high axial stiffness. On the other hand, the long-term crack widening can be ascribed mainly to the long-term fibre/matrix bond mechanisms, which depends on several factors such as: quality of matrix, bond strength, fibre embedment length, fibre geometry, fibre anchorage mechanism, fibre orientation, etc.

Intensive research is currently undertaken to characterize the serviceability performance of cracked SFRC elements. The durability improvement of the cracked SFRC can be maintained if the crack does not significantly widen under a sustained load. In literature there are some studies regarding the time-dependent behaviour of FRC, in the cracked state, using either synthetic (Al-Khaja, 1995; Kurt and Balaguru, 2000) or steel (Mouton and Boshoff, 2012; Zhao et al., 2012) fibres. In the case of FRC reinforced with synthetic fibres, a relatively large crack widening over time under a sustained load was reported (Babafemi and Boshoff, 2014; Boshoff et al., 2009a), while in FRC with steel fibres a considerable lower crack opening was obtained (Tan et al., 1994; Tan and Saha, 2005). To the best of authors’ knowledge, only a limited number of works are dedicated to the time-dependent fibre pull-out behaviour. In the works of Babafemi and Boshoff (2014) and Boshoff et al. (2009a) the used aligned synthetic fibres (i.e. fibres perpendicular to the crack plane) were pulled-out just in a few hours after the loading initiation, even when they were loaded with just 50% of the fibre’s bond strength. It is well acknowledged that orientation of the fibres towards the crack plane has an important influence on the pull-out behaviour of a single fibre and, consequently, on the FRC tensile behaviour, see Chapter 4. In fact, understanding the time-dependent modifications at the fibre / matrix interfacial zone of a single fibre (meso-level) can render a better understanding of the creep behaviour of SFRC at the composite level.
In this chapter, an experimental programme that aims to study the time-dependent pull-out behaviour of a single fibre is presented and discussed. For this purpose, cylindrical concrete specimens containing a single fibre with an embedment length of \( l_f / 4 \), being \( l_f \) the fibre’s length, and distinct inclinations were produced. For each series, the influence of the fibre orientation angle (0, 30 and 60º), as well as the pre-imposed fibre slip levels, \( s_{pr} \), 0.3 and 0.5 mm (representative of slip at serviceability limit state and slip near to peak load, respectively) on the pull-out behaviour are appraised. In a first stage, the fibres were subjected to monotonic pull-out test up to the desired pre-slip level, \( s_{pr} \). Afterwards, the creep pull-out test was carried out under a sustained load until the stabilization of the long-term slip. The specimens used in the pull-out creep tests were then subjected to a quasi instantaneous pull-out test until failure. Finally, a series of quasi instantaneous single fibre pull-out tests were also executed on undamaged specimens (without any pre-slip level), in order to quantify the influence of the long-term term slip on the pull-out residual force evolution.

5.2 Experimental program

5.2.1 Concrete mixture

A self-compacting concrete composition was developed with mixture constituents presented in Table 5.1. To guarantee the required workability, a third generation Superplasticizer Sika® 3005 (SP) has been used. Crushed granite coarse aggregates with a maximum aggregate size of 12 mm were used. The concrete rheology properties in the fresh state were determined by the Abrams cone slump test in the inverted position according to the recommendation of EFNARC (2005). The average spread of the mixture was around 590 mm. Hooked-end steel fibres with a length, \( l_f \), of 33 mm; diameter, \( d_f \), of 0.55 mm; aspect ratio, \( l_f / d_f \), of 60 and a yield stress of 1100 MPa were used.

The compressive strength of the hardened concrete was assessed by testing six cylinders with a diameter of 150 mm and a height of 300 mm at the age of 28 days. The specimens were cast and stored in a climatic chamber room under 20ºC and 60% relative humidity. Moreover, these specimens were stored in the same climatic conditions in
which the creep pull-out tests were performed until the age of 28 days. The average compressive strength was 72 MPa with a coefficient of variation (CoV) of 8.23 %, while the average Young’s modulus was 42.15 GPa with CoV of 0.26 %.

Table 5.1: Mix design of steel fibre reinforced self-compacting concrete per m³.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>412</td>
<td>124</td>
<td>0.30</td>
<td>7.83</td>
<td>353</td>
<td>237</td>
<td>710</td>
<td>590</td>
</tr>
</tbody>
</table>

5.2.2 Specimens

The pull-out tests were executed on single sided cylindrical concrete specimens. For this purpose, a special mould was used allowing the production of 81 specimens at the same time with the desired fibre’s embedment length and orientation, see Figure 5.1(a). The bottom surface of the mould consists of ten steel strip shape pieces. Before assembling the mould, fibres were accommodated with the desired embedment length and inclination in the grooves of the lateral face of the steel strip pieces, respectively with 0, 30 and 60° (Figure 5.1(b)). Afterward, the mould was cast as shown in Figure 5.1(c). The casting procedure was performed carefully, from the centre of panel, in order to prevent any undesired fibre’s warping. After curing of concrete, the panel was demoulded, turned over, and cylindrical cores comprising a single fibre were extracted. Figure 5.1(d) depicts the final produced cylindrical specimen, with actual height and diameter of 80 mm.
Prior to testing, two aluminium plates were attached to the extremity of the fibre, in order to avoid undesired fibre rupture at the grip, since the specimens that were used for assessing long-term pull-out behaviour needed to be fasten three times (for pre-slip, creep and post-creep stages). The aluminium plates were glued using epoxy Sikadur®-32N and stored for at least three days until the curing process of epoxy was completed, see Figure 5.2. Additionally, these aluminium plates also guarantee a null slip between fibre and grip, since the grip could be tighten firmly, and therefore assure the use of a more sophisticated test set-up (Cunha, 2010).

In this study, inclination angles of 0, 30 and 60° were investigated to appraise the influence of fibre orientation on the instantaneous pull-out load – slip (\(F - s\)) and long-term slip – time (\(s_t - t\)) relationships. Cunha (2010) concluded that the influence of the
fibre embedment length is not as significant as the orientation angle. Therefore, considering that the theoretical average value of a fibre’s embedded length, \( l_{fb} \), bridging an active crack is \( l_f / 4 \) (Stroeven and Hu, 2006), in the present study a \( l_{fb} = 8.25 \) mm, was selected in the production of all the pull-out specimens. For each orientation angle, six monotonic and six long-term pull-out tests were executed (three for each pre-imposed fibre slip level, \( s_{pr} \)). For the series with an inclination angle of 0°, additional specimens were also prepared, namely, six specimens (three for each \( s_{pr} \)) for executing monotonic tests with one cycle of unloading and re-loading (this test will be detailed further ahead); and three supplementary specimens with smooth fibres to be tested in monotonic configuration in order to determine the grade of the hooked end mechanical anchorage mobilization. Therefore, a total of forty five specimens were tested.

5.2.3 Test set-up

5.2.3.1 Instantaneous fibre pull-out test

Monotonic test

The instantaneous fibre pull-out load – slip relationship, \( F - s \), was acquired by using a universal testing rig with a load carrying capacity of 50 kN. Since the peak pull-out loads were relatively low, a load cell with high accuracy was used, namely, a HBM® type S9 load cell with a capacity of 5kN. This load cell was interposed between the servo-actuator and the grip, see Figure 5.3(a).

The single sided pull-out specimen was accommodated in a steel frame. This frame incorporated one bottom steel plate bolted to the machine testing frame and a steel ring mounted on the upper side of the specimen to fix it. The steel ring was connected to the bottom steel plate using three screws disposed around the specimen forming an angle of 120°. The diameter of both specimen (80 mm) and steel ring hole (60 mm) were designed to prevent the application of confinement to the fibre due to development of compressive stress held in the concrete medium surrounding fibre, which could influence the pull-out response.
Figure 5.3: Configuration of single fibre pull-out test: (a) details of the test, (b) connection details of the LVDTs, (c) a fibre being pulled out and (d) comparison between F – s curves measured by LVDT and microscope.

For measuring the fibre’s slip during the pull-out procedure, three Linear Variable Differential Transformers (LVDTs) with linear stroke +/- 5 mm were installed on the back side of grip using aluminium cubic supports to exclude measuring deformation of the test rig, Figure 5.3(b) and (c). However, to guarantee a null slip of fibre in grip, firstly, two monotonic tests were executed on specimens with fibre orientations of 30° and 60° measuring slip with not only LVDTs installed on the back side of the grip but also using a VMS-004D-400x USB Microscope with 2 Mega Pixels camera implemented to obtain consequence pictures from the fibre during the pull-out process. Figure 5.3(d) compared the F – s relationship of the obtained by two methods. The results confirmed a very negligible fibre’s slip at the grip. Nevertheless, the Microscope was also maintained to obtain
consequence pictures from the fibre during the pull-out process. Afterwards, the pictures were analysed. The test was controlled by an LVDT installed on the actuator, and adopting a displacement rate of 1 m/s up to the slip of 2 mm, and 4 m/s until the end of the test. This test control procedure guaranteed a stable response during the test, especially during the debonding process of the fibre.

**Test with one unloading and re-loading cycle**

To evaluate if an unloading (in pre-slipping test) / re-loading (in post-creept test) cycle will affect peak load and, furthermore to obtain the influence of the long-term slip on the secondary stiffness, a series of tests with one unloading / re-loading cycle were also conducted. In this test, fibres were loaded until the nominal pre-slipping value, $s_{pr}$, i.e. 0.3 or 0.5 mm. Then, the specimens were unloaded followed by a monotonic loading until the failure of the specimen (Figure 5.4). Hereinafter, $s_{res}^{pr}$ is denoted as the residual slip after unloading of the specimen, $K_{sec,inst}$ is the instantaneous secondary stiffness at re-loading process, $F_p^{max}$ and $s_p^{max}$ are the maximum load recorded during the test and its correspondent slip, respectively.

![Figure 5.4: Schematic response obtained from the instantaneous pull-out test with one unloading / re-loading cycle.](image)

---

148
5.2.3.2 Long-term fibre pull-out test

To obtain the long-term $F - s$ relationship, experimental tests were carried out in three stages: firstly, the fibres were exposed to a pre-slip level of either 0.3 or 0.5 mm; secondly, the creep tests were carried out for the pre-imposed fibre slip specimens until the stabilization of the long-term slip; finally, pull-out tests were performed until the fibre was fully pulled-out.

Pre-slip of the fibre

Prior to the execution of the long-term pull-out tests, each specimen’s fibre was subjected to a certain pre-slip level, $s_{pr}$. The monotonic pull-out test set-up was adopted for pursuing this objective and the fibre was loaded until the average slip in the LVDTs reached the target $s_{pr}$, and then the specimen was unloaded. Note that the latter slip also comprises a parcel related to the fibre elongation. In this study, two nominal slip levels were investigated: 0.3 mm that can be correlated to the crack opening width in serviceability limit states, and 0.5 mm that was close to the peak pull-out load. After imposing the pre-slip, the specimen was unloaded. Finally the specimen was carefully removed, in order to avoid any introduction of bending or torsion when unfastening the fibre from the grip. Figure 5.5 shows an idealization scheme of the pre-slip process, where $F_L$ is load at debonding step, $F_{pr}$ and $s_{pr}$ are load and correspondent slip once the target slip reached, $s_{pr}^{res}$ is residual slip after unloading of the specimen.

![Figure 5.5: Scheme presentation of the fibre pull-out pre-slip test.](image)
Pull-out creep test

The fibre pull-out creep tests were performed in a climatic chamber room with a fixed temperature of 20°C ± 0.5 and 60% relative humidity ± 5% as recommended by ASTM C512 (2004) for the assessment of creep under compression. Before positioning the specimens on the creep table, they were kept in the chamber room for around five hours to acclimatize. For evaluating the long-term performance of fibre/matrix interface, an innovative test set-up was designed allowing testing five specimens simultaneously (Figure 5.6(a)). The test set-up was composed of a creep table, five LVDTs, a data acquisition system and a computer. In the creep table, for each specimen, it was available a grip, a steel arm, that was withheld by a support at a quarter of its length, and a steel bar for supporting the applied weight loads, see Figure 5.6(b) and (c). The same steel rings used in the monotonic tests were accommodated on the top of the specimen, connected to the table by three bolts. Furthermore, a hinge was used to connect the grip to the steel arm in order to prevent the introduction of bending moment at the fibre, and also to assure that the specimen was loaded in the fibre’s axis, see Figure 5.6(c).

During the creep test, the fibre slip was acquired with an LVDT installed on each grip. The same connection details of the monotonic test set-up were adopted. To evaluate the level of fibre slip age in the grip, a USB Microscope was also used, see Figure 5.6(c). The fibre slippage at grip was also marginal, in similarity to what happened for the monotonic tests. In this test at the time of loading and unloading procedures, the slip was measured within a relatively short time period (one reading per 2 sec) since the slip variation was significant at this stage. To reduce the size of the recorded data, the read-outs were saved with a higher time period, one read-out per 500 and 1000 seconds for the first month and until the completion of the test, respectively.

After the initiation of the creep test, the value of the applied load, $F_a$, Figure 5.7(b), was maintained constant until the long-term slip was stabilized. When the variation of the long-term slip was smaller than one micrometer for three consecutive days, it was assumed that the slip was stabilized, and the test was finalized. In the present research, it was opted to apply a load level ($F_a / F_{pr}$) equal to 100 or 80% of that observed at $s_{pr}=0.3$ or 0.5 mm in the pre-slip tests, respectively. It should be mentioned that for the specimens with 0° fibre inclination angle, the pre-slip level 0.5 mm was very close to the slip correspondent to the peak load. Therefore, in $s_{pr}=0.5$ mm series, to prevent the premature...
fibre’s rupturing or pulling-out at initial stage of creep test, the specimens were subjected to a lower loading level.

![General view of fibre pull-out creep test](image1.jpg)

![Loading details](image2.jpg)

![Specimen installation details](image3.jpg)

![Fibre close-up](image4.jpg)

**Figure 5.6:** Fibre pull-out creep test: (a) general view, (b) loading details, (c) specimen installation details and (d) fibre close-up.

Afterward the conclusion of the creep tests, the specimens were unloaded, but the data acquisition system was kept active for at least a period of one week, enabling to record the recoverable slip component due to the creep recovery process. Figure 5.7(a) and (b) include a schematic representation of slip versus time and force – slip relationship. In these figures, $s_{\text{inst}}$ is the instantaneous slip, $s_l$ represents the long-term slip, $s_{\text{inst}}^{\text{rec}}$ states the instantaneous slip recovery at the beginning of the unloading process, $s_l^{\text{rec}}$...
represents long-term slip recovery, $s_{\text{total}}^{\text{rec}}$ is the total slip recovery, $s_{\text{inst}}^{\text{rec}}$ is the residual slip after complete unloading stage, and $F_a$ is the applied load level in creep test.

![Diagram](a)

![Diagram](b)

*Figure 5.7:* Definition of the slip parameters in pull-out creep response: (a) slip – time relationship, $s – t$, (b) force – slip, $F – s$.

**Post-creep test**

One week after the creep tests have ended (a waiting period for creep deformation recovery), the specimens were then subjected to monotonic fibre pull-out test. The monotonic test set-up was similar to the one used for pre-slippping the fibres, which was previously detailed. A graphical presentation of $F – s$ curve is shown in Figure 5.8, where $F_p^{\text{max}}$ and $s_p^{\text{max}}$ are the maximum load recorded during the test with its correspondent slip, respectively.

![Diagram](c)

*Figure 5.8:* Scheme representation of the post-creep test.
Assembled long-term force-slip curve

Finally, the complete $F - s$ curves were assembled with the individual $F - s$ curves from the pre-slipping, creep and post-creep responses, as schematically presented in Figure 5.9. The push curve from this assembled response will be compared with the correspondent curve from the instantaneous monotonic pull-out tests. In Figure 5.9, $F_{pr}$ and $s_{pr}$ are the pre-slipping load and correspondent slip value, respectively. $s_{pr}^{res}$ is the residual slip after unloading of the specimen, $K_{sec,lt}$ is the long-term secondary stiffness at re-loading process of post-creep test, $F_{p}^{max}$ and $s_{p}^{max}$ are the maximum load recorded during the test and its correspondent slip, respectively.

$F_{pr}$

Pre-slipping

Creep

Post-creep

$s_{pr}$

$s_{pr}^{res}$

$s_{pr}^{max}$

$s_{p}^{max}$

$F_{p}^{max}$

$K_{sec,lt}$

Figure 5.9: Schematic representation of the assembled long-term force – slip curve.

5.3 Results and discussions

5.3.1 Failure modes

Figure 5.10 presents the observed typical failure modes happened in the execution of fibre pull-out tests. In the case of orientation angle of 0°, the fibre was totally pulled-out. On the other hand, after debonding of fibre/matrix interface, the hook was fully straightened and the deformed fibre moved through the mortar channel formed by initial geometry of the steel fibre. This failure mode was illustrated in Figure 5.10(a) as well as the trace of
the fibre on matrix. In the inclined fibres, the failure mode was completely different. These modes were always proceeded by matrix spalling and finally fibre’s rupturing. In the inclination angle of $30^\circ$, after debonding process, spalling of matrix was observed at the fibre bending point followed by rupturing of the fibre, see Figure 5.10(b). In another failure mode which happened when the inclination angle was $60^\circ$, matrix spalling was formed in two stages. In the first step, concrete was spalled at the fibre exit point which followed by detaching the first volume of concrete parcel. Then, the inclined part of the fibre located under the detached portion of concrete was straighted and a bigger portion of matrix parcel appeared. Lastly the fibre was ruptured, see Figure 5.10(c) and (d). The fibre rupture happened for inclined fibres was a results of strong and compact concrete matrix resulting in a good fibre anchorage in it.

![Figure 5.10](image)

*Figure 5.10: Typical failure modes of pull-out test: fibres with orientation angle of (a) $0^\circ$, (b) $30^\circ$, (c) and (d) $60^\circ$.*

In the inclined fibres, the failure criterion happens if the spalling compressive strength is reached, hence a concrete wedge is crushed. Figure 5.11 shows this situation graphically where in this figure, $R_m$ shows the centre of curvature and the hatch part depicts the volume of concrete prone to be crushed. Depends on the pullout force and concrete compressive strength, the radius of curvature could change (Soetens et al., 2013). This crushing of concrete could be processed progressively which finally followed
by fibre’s rupturing. The determined average value of fibre rupture stress for 30° inclination was 993 MPa whereas for the fibres with inclination angle of 60° this value was 840 MPa. This could be justified by the fact that the fibre subjected to a combination of tensile-bending mode. As a fibre ruptured in a value of stress lower than the tensile strength of fibre, it means that the stress concentration was appeared in the bending part of the fibre. Moreover, when the bending moment increases, the ultimate tensile capacity of the section must decrease. On the other hand, as the fibre inclination angle increases, the bending moment will increase resulting in a decrease of the tensile rupture stress.

\[ \text{Figure 5.11: Pull-out mechanism of an inclined fibre.} \]

5.3.2 Monotonic fibre pull-out test

5.3.2.1 Pull-out load – slip curves

Figure 5.12 indicates the average pull-out load versus slip curves and correspondent envelope for the adopted fibre inclinations. The slip was determined by averaging the readouts recorded in the three LVDTs installed on the grip. In all curves, the pre-peak branch was composed of a linear and nonlinear part. The first part is associated to the elastic behaviour of the bond. For the fibres with an inclination angle of 0°, the nonlinear part in the pre-peak branch can be associated to a non-recoverable degradation of the adhesion between the fibre and surrounding matrix, as well as the beginning of the end hook’s plastic deformation, Figure 5.12(a). When the peak load was attained, a relatively smooth load decay was observed. During this softening stage, the fibre mechanical anchorage is progressively mobilized. After a slip of about 3.5 mm when the hook was completely straightened, the fibre pull-out force is mainly governed by fibre / matrix
friction. Due to the reduction of the fibre’s embedment length, this stage is characterised by a small decrease of the pull-out force during the imposed slip process.

Figure 5.12(b) shows the load – slip curve for fibres with an inclination angle of 30°. The average curve was depicted up to the slip where the fibre rupture has occurred. The pre-peak branch is similar to the fibres with 0°angle, but a slightly lower value of peak load was attained. The nonlinear part was slightly less stiff due to the cracking and spalling of the matrix occurred at this fibre exit point due to the deviation force component that fibre applies to this medium (Laranjeira, 2010).

Figure 5.12(c) indicates the pull-out load – slip response for the 60° series. In similarity to the 30° series, the specimens failed by fibre rupture. When the fibre inclination angle increases, the matrix at the fibre exit point is more prone to cracking and spalling due to the higher deviation force component applied by the fibre. The nonlinear component of the first branch is of small significance until the first evidence of matrix spalling, see Figure 5.13(a). Afterwards, a matrix wedge was formed and gradually detached. The fibre segment embedded under this concrete portion was straightened, which corresponds in the pull-out – slip relationship to the plateau where the slip increases under an approximately constant load, Figure 5.13(b). However, in some specimens a sudden reduction in the load was also recorded once the matrix has spall. After the completion of wedge spalling off, a new equilibrium was attained and the pull-out fibre process has continued with a smaller stiffness until the fibre rupture, which occurred at a load level lower than in the two previous series, Figure 5.13(c).
Figure 5.12: Average monotonic pull-out load versus slip relationships for fibre inclination angles of: (a) 0°, (b) 30° and (c) 60°.

Figure 5.13: Pictures of pull-out process of specimens with 60° fibre inclination angle: (a), (b) and (c) correspondent to points A, B and C in Figure 5.12(c), respectively.

Figure 5.14(a) depicts the influence of fibre inclination angle on the maximum pull-out load, $F_p^{\text{max}}$. The peak load decreases as the inclination angle increases, and the maximum pull-out load was observed for inclination angle of 0° which was 7% and 21% higher in compare to 30° and 60° aligned fibres, respectively. As it was discussed earlier, in the case of the inclined fibre due to the stress concentration at the fibre bending point, it was subjected to a combination of stress-bending mode which caused rupturing of fibre in a lower load than ultimate load capacity of section. Regarding the slip at peak load, Figure 5.14(b), it was observed that $s_p^{\text{max}}$ has decreased slightly up to an inclination angle of 30°, and then has increased significantly for 60° specimens. In fact for 60° series, $s_p^{\text{max}}$ was 64% and 54% higher than for 0° and 30° orientation angles, respectively. This significant increase of $s_p^{\text{max}}$ is caused by the spalling of the matrix at the fibre exit point.
(Soetens et al., 2013). On the other hand, when the orientation angle increases, a bigger portion of concrete is pushed off at the fibre’s exit point, hence a larger fibre’s protruding length is straighten, leads to an additional slip component. Consequently, in the case of large inclination angles, similar to 60°, the slip included an important increment due to the fibre deformation. Furthermore, in 30° specimens only a very low portion of concrete spalled during the test execution which justifies a small increase of \( s^\text{max}_p \) when compared to the 0° series.

![Graphs showing pull-out peak load and slip at peak load vs. fibre orientation angle.](image)

*Figure 5.14:* The effect of fibre inclination angle on: (a) pull-out peak load and (b) slip at peak load.

### 5.3.2.2 Mechanical contribution of the hooked end

In general, for the hooked end fibres, fibre/matrix bond consists of three components: the first one is related to the adherence between paste and fibre (hereinafter known as chemical bond), the other is concerned with the mechanical component due to the plastification of the hook during slipping throughout the concrete channel, and the last one is fibre/matrix friction. Figure 5.15 includes the average load – slip relationships of the hooked end and smooth fibres for the 0° series. Moreover, the estimated contribution of the end hook to the overall pull-out behaviour is also depicted, which was determined by subtracting the smooth fibre average curve from the hooked fibre average curve. This assessment was performed exclusively for the 0° series, since for inclined fibres other factors are involved such as a supplementary frictional resistance due to the force component normal to the fibre axis, bending and plastification on the exiting point of the fibre; spalling of the concrete near to the fibre bending point (Cunha, 2010). The
component correspondent to the chemical adherence between paste and fibre played a small role in the overall pull-out response, which was principally governed by the mechanical bond. Considering the maximum pull-out load, the contribution of the chemical and mechanical bonds were, respectively, 16% and 84%. Furthermore, by considering the response of smooth fibres, it was also observed that the chemical bond was exhausted at a slip of 0.06 mm. Thus, for the investigated pre-slip levels, \( s_{pr} = 0.3 \) and 0.5 mm, the chemical bond was already fully exhausted, and fibre pull-out mechanism is governed by mechanical bond provided by the hook anchorage mechanism.

![Graph showing force vs. slip for hooked and smooth fibres](image)

Figure 5.15: Contribution of the end hook to the overall pull-out behaviour in fibres with inclination angle of 0°.

It is expected that for smooth and hooked end fibres when the fibre and matrix friction governing the pull-out process (in the case of the hooked end fibres, after the hook was completely mobilized), the two responses should be match together. In fact, this was not happened and the hooked end series showed a higher level of load in friction stage. In the author opinion, when the fibre is completely deboned, the whole fibre starts to slip through the mortar channel and the hook becomes more or less straight. This quasi straightened part of the hook is then providing more frictional resistance between the steel fibre and the surrounding matrix than the straight fibre, see Figure 5.16.

![Image of pulled-out fibres](image)

(a) quasi straightened part  (b)

Figure 5.16: Pulled-out fibres: (a) hooked end type and (b) smooth type.
5.3.2.3 Instantaneous test with one unloading / reloading cycle

Figure 5.17 shows the average cyclic pull-out load – slip relationships for fibres with inclination angle of 0°. A peak pull-out load of 251 and 257 N for pre-slip levels of 0.3 and 0.5 mm, respectively, was observed. Comparing these values with the average values obtained from the monotonic tests (254 N), see Figure 5.12(a), it can be concluded that the maximum pull-out load of fibre is not influenced by occurrence of an unloading and re-loading cycle. The average instantaneous secondary stiffness, $K_{sec,inst}$, was also determined according to the schematic representation in Figure 5.9. Values of 1.70 and 1.61 kN/mm for pre-slip levels of 0.3 and 0.5 mm, respectively, were determined. As it was expected, $K_{sec,inst}$ has decreased with the increase of the pre-slip level but quite moderately. By comparing $s_{pr}^{res}$ for both series, it is worth noting that for $s_{pr} = 0.3$ mm series 52% of $s_{pr}$ was recovered, while in the case of $s_{pr} = 0.5$ mm specimens, only 18% of $s_{pr}$ was recovered. However, in the pre-slip level of 0.5 mm, the chemical bond was already destroyed, see Figure 5.16, and also the hook of fibre was completely deboned (mechanical debonding). Since this process was a plastic and non-recoverable phenomenon, therefore, when specimens were unloaded, only a few portion of slip was recovered and a relatively high residual slip, $s_{pr}^{res}$, was achieved.

![Figure 5.17](image)

*Figure 5.17:* The average cyclic pull-out load – slip relationships for fibres with inclination angle of 0° and $s_{pr}$ equals to: (a) 0.3 mm, and (b) 0.5 mm.
5.3.3 Long-term fibre pull-out test

5.3.3.1 Creep parameters

Table 5.2 shows the average slip rate, $SR$, for each $s_{pr}$ at 7, 15, 30 and 60 days from starting time of the creep test. The slip rate was calculated from Eq. 5.1:

$$SR_{t_2-t_1} = \frac{s_{ht}^{t_2} - s_{ht}^{t_1}}{t_2 - t_1}$$

(5.1)

where, $SR_{t_2-t_1}$ is slip rate between time $t_2$ to $t_1$ and $s_{ht}^{t_i}$ shows long-term slip at time $t_i$. For instance, the slip rate between 15 to 7 days ($SR_{15-7}$) is determined as $(s_{ht}^{15} - s_{ht}^{7})/(15-7)$. For $s_{pr} = 0.5$ mm series, the creep test could not be performed on the specimens with a fibre inclination angle of 30°, because the fibres have ruptured during the execution of the instantaneous pull-out tests at an average slip of 0.46 mm. For all specimens, it was observed that the highest slip rate occurred during the first week, and continuously decreased over time. At 60 days of testing, the slip rate became smaller than the test stopping criteria, which means that the long-term slip stabilized after a period of two months.

Table 5.2: Average slip rate for each pre-slip level (numbers in parenthesis present CoV in %).

<table>
<thead>
<tr>
<th>Pre-slip level</th>
<th>Orientation</th>
<th>7 days</th>
<th>15 days</th>
<th>30 days</th>
<th>60 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3 mm</td>
<td>0</td>
<td>6.12 (8)</td>
<td>2.15 (34)</td>
<td>0.61 (23)</td>
<td>0.27 (16)</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>7.15 (46)</td>
<td>1.75 (39)</td>
<td>0.63 (10)</td>
<td>0.22 (23)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>3.71 (32)</td>
<td>1.19 (23)</td>
<td>0.39 (14)</td>
<td>0.14 (31)</td>
</tr>
<tr>
<td>0.5 mm</td>
<td>0</td>
<td>10.97 (15)</td>
<td>1.99 (18)</td>
<td>0.97 (22)</td>
<td>0.36 (7)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>3.57 (6)</td>
<td>1.15 (32)</td>
<td>0.71 (26)</td>
<td>0.18 (11)</td>
</tr>
</tbody>
</table>

Figure 5.18 includes the average and envelope of the long-term slip, $s_{lt}$, versus time values for each fibre orientation angle and pre-slip level, up to a period of two months. All series have shown a two-stage creep response, namely, primary and secondary stages, in which the long-term slip was increased importantly with time and then followed by a steady state part where the increment in slip was not so predominant. However, in $s_{pr} =$
0.5 mm series, the slip was stabilized in a higher time period. It should be worth noted that the former specimens subjected to a higher level of damage in the bond between fibre/matrix, therefore slip was stabilized in a higher time period. Under the sustained load, none of the specimens passed to the tertiary stage of the creep behaviour, i.e. unstable creep response, where long-term slip increases in time sharply until fully extraction of the fibre.

Figure 5.18: Long-term slip versus time during fibre pull-out creep test: (a) and (b) 0 degree, (c) and (d) 30 degree, (e) and (f) 60 degree; (a), (c), (e) $s_{pr} = 0.3$ mm and (b), (d), (f) $s_{pr} = 0.5$ mm.
Furthermore, the creep coefficient parameter was also introduced to consider the long-term behaviour of the specimens. This parameter was considered for two distinct stages, namely: only in the creep stage ($\varphi^C$) and at the origin ($\varphi^O$), as being calculated by the following equations (EN-1992-1-1, 2004):

$$\varphi^C = \frac{s_{lt}}{s_{inst}}$$  \hspace{1cm} (5.2)

$$\varphi^O = \frac{s_{lt}}{s_{inst} + s_{res}^{cr}}$$  \hspace{1cm} (5.3)

where $s_{inst}$ is instantaneous slip at the time of loading, $s_{lt}$ presents long-term slip and $s_{res}^{cr}$ states the residual slip after unloading specimen in pre-slip test. Table 5.3 and Table 5.4 show the creep coefficient at creep level and origin, respectively, for each $s_{pr}$ at 7, 15, 30 and 60 days from starting time of the creep test. The same conclusion similar to the long-term slip parameter was also attained. It is worth noting that by introducing creep coefficient parameter, a lower coefficient of variation, CoV, was achieved in comparison to the slip rate, see Table 5.2, since in the definition of the creep coefficient parameters, $s_{inst}$ was also considered. On the other hand, in some specimens, during the reloading procedure in the creep test, a slightly higher increase in $s_{inst}$ was observed which lead to a higher $s_{lt}$. The same conclusion was also made by Buratti and Mazzotti (2012).

**Table 5.3:** Average creep coefficient at creep stage for each pre-slip level (numbers in parenthesis show CoV in %).

<table>
<thead>
<tr>
<th>Pre-slip level</th>
<th>Orientation</th>
<th>7 days</th>
<th>15 days</th>
<th>30 days</th>
<th>60 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 mm</td>
<td>0</td>
<td>0.259 (11)</td>
<td>0.358 (2)</td>
<td>0.412 (0.6)</td>
<td>0.463 (3)</td>
</tr>
<tr>
<td>0.3 mm</td>
<td>30</td>
<td>0.240 (9)</td>
<td>0.309 (10)</td>
<td>0.365 (11)</td>
<td>0.432 (19)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.198 (8)</td>
<td>0.269 (3)</td>
<td>0.352 (4)</td>
<td>0.394 (3)</td>
</tr>
<tr>
<td>0.5 mm</td>
<td>0</td>
<td>0.458 (0.4)</td>
<td>0.553 (2)</td>
<td>0.640 (4)</td>
<td>0.706 (4)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.230 (4)</td>
<td>0.317 (4)</td>
<td>0.373 (8)</td>
<td>0.410 (7)</td>
</tr>
</tbody>
</table>
Table 5.4: Average creep coefficient at origin for each pre-slip level (numbers in parenthesis show CoV in %).

<table>
<thead>
<tr>
<th>Pre-slip level</th>
<th>Orientation</th>
<th>7 days</th>
<th>15 days</th>
<th>30 days</th>
<th>60 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3 mm</td>
<td>0</td>
<td>0.118 (8)</td>
<td>0.166 (4)</td>
<td>0.191 (0.5)</td>
<td>0.213 (2)</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.116 (7)</td>
<td>0.154 (5)</td>
<td>0.180 (6)</td>
<td>0.212 (4)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.045 (5)</td>
<td>0.061 (2)</td>
<td>0.079 (3)</td>
<td>0.090 (4)</td>
</tr>
<tr>
<td>0.5 mm</td>
<td>0</td>
<td>0.135 (0.6)</td>
<td>0.163 (2)</td>
<td>0.188 (5)</td>
<td>0.208 (3)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.080 (5)</td>
<td>0.109 (6)</td>
<td>0.125 (6)</td>
<td>0.138 (5)</td>
</tr>
</tbody>
</table>

5.3.3.2 Influence of fibre orientation on the creep parameter

Figure 5.19 shows the long-term slip versus time relationship for distinct fibre orientation angles. Fibre orientation has an important effect on the long-term slip, since a clear increase of $s_{lt}$ with the decrease of fibre inclination was obtained. This tendency seems to increase with the $s_{pr}$ applied. This can be clearly concluded from the results showed in Figure 5.20(a). In the series of $s_{pr} = 0.3$ mm and fibre inclination angle of $0^\circ$ at 60 days time lapse, the long-term slip was 19% and 69% higher than in the case of fibre orientation angles of 30 and 60°, respectively.

![Figure 5.19](image)

*Figure 5.19: Long-term slip versus time for different fibre orientations and $s_{pr}$ equal to: (a) 0.3 mm, and (b) 0.5 mm.*
Regarding the series $s_{pr} = 0.5$ mm, the fibre orientation effect is even more pronounced (Figure 5.20(b)) since at 60 days time lapse, the long-term slip of fibre at $0^\circ$ was 135% larger than at $60^\circ$. The different failure modes observed in the specimens with fibres differently oriented can contribute for this behaviour. As it was explained earlier, in Section 5.3.1, for specimens with inclination angle of $0^\circ$, the fibre was fully pulled-out whereas for $60^\circ$ orientation angle, the pull-out procedure was mainly governed by matrix spalling and finally fibre rupturing. In the case of inclined fibres ($30$ and $60^\circ$), a compressive stress field exists at the fibre bending point, which applied by the deviational force component of the fibre. Considering the Mohr-Coulomb effects, the fibre slippage is also resisted by the shear friction that increases with a normal stress field installed in the fibre exit point. However, for a fibre with $0^\circ$ inclination angle since it is subjected to only direct tensile loading (without any deviational force component), therefore it is more susceptible to be pulled-out under a sustained load than other series. From another point of view, for $0^\circ$ series, $s_{pr} = 0.5$ mm was very close to the slip correspondent to the peak load, $s_{p,\text{max}}$. On the other hand, by comparing the ratio of the applied load in creep test to the maximum load recorded during pull-out test, $F_a / F_{p,\text{max}}$, see Table 5.5, in average terms, this coefficient was 81% and 39% for $0^\circ$ and $60^\circ$ orientation angles, respectively. Therefore, loading specimens in a level close to the maximum load carrying capacity of the specimen provides the development of higher long-term slip under a sustained load.

![Figure 5.20](image_url)

*Figure 5.20: Influence of fibre orientation angle on the long-term slip for $s_{pr}$ equal to: (a) $0.3$ mm, and (b) $0.5$ mm.*
Table 5.5: The average applied load in creep test \((F_a)\) and maximum load recorded during the pull-out test \((F_{max})\) for each series of specimens.

| \(s_{pr}\) (mm) | \(F_a\) (N) | | | \(F_{max}\) (N) | | |
|------------------|-------------|------------------|------------------|
|                  | 0°          | 30°              | 60°              | 0°              | 30°              | 60°              |
| 0.3              | 179         | 187              | 87               | 255             | 227              | 160              |
| 0.5              | 205         | ---              | 74               | 253             | ---              | 190              |

5.3.3.3 Influence of pre-slip level on the creep parameters

Figure 5.21 estimates the influence of the pre-slip level on the long-term slip – time responses. Considering specimens with 0° fibre orientation angle, the long-term slip increased significantly with \(s_{pr}\). From Figure 5.15, it was observed that for the both studied pre-slip values, the chemical bond was completely exhausted and the debonding of the hooked end has already started. Moreover, it is also observed that for a slip of 0.3 mm the fibre’s hook was partially debonded, whereas for \(s_{pr}=0.5\) mm it is expected a higher debond at the fibre’s hook. Therefore, in \(s_{pr} = 0.5\) mm specimens, the bond between the fibre/matrix had already a higher level of damage, consequently it was observed a higher long-term slip due to the creep behaviour. An increase of 50% on the long-term slip was observed for the fibres pre-slipped up to 0.5 mm when compared to the \(s_{pr} = 0.3\) mm series. On the other hand, the series with a 60° angle have shown a distinct behaviour. By increasing the pre-slip from 0.3 mm to 0.5 mm, the long-term slip had a slight increase. From Figure 5.12(c) it is observed that both studied pre-slip levels were positioned in the plateau part of the monotonic pull-out response of the 60° series, in which the slip increased at an almost constant load. Therefore, the achieved pre-slipping load, \(F_{pr}\), for both investigated pre-slip levels was relatively similar. In the long-term pull-out tests, considering the selected loading levels for each series, the specimens of the 0.5 mm series were subjected to a lower load, \(F_a\), see Table 5.5. Hence, it is rational to expect that the creep response for this series exhibited a similar response to the 0.3 mm series, even though \(s_{pr} =0.5\) mm series endured a higher level of damage at the fibre/matrix interface. From another point of view, when having in mind the maximum monotonic
pull-out load, the $s_{pr} = 0.3$ mm series were subjected to a loading level of $F_a / F_p^{max} = 54\%$, whereas the $s_{pr} = 0.5$ mm series the loading level was just 39\%.

**Figure 5.21**: Influence of pre-slip level on the development of long-term slip along time.

### 5.3.4 Comparison between monotonic and long-term results

Considering fibres with a $0^\circ$ degree inclination, a small reduction on the instantaneous secondary stiffness, $K_{sec,inst}$, obtained from cyclic test (section 5.3.2.3) can be observed when compared to the long-term secondary stiffness, $K_{sec,lt}$, determined from the post-creep test. The average $K_{sec,lt}$ was 1.51 and 1.41 kN/mm for the series with pre-slip levels of 0.3 and 0.5 mm, respectively. When comparing to the $K_{sec,inst}$ a 12.6\% and 14.2\% reduction was observed for the 0.3 and 0.5 mm series, respectively. This could corroborate that the creep phenomenon had a minor detrimental influence on the interfacial bond between fibre and matrix.

Figure 5.22 includes both the monotonic and long-term pull-out assembled curves for each fibre orientation category and $s_{pr}$. These responses were obtained by averaging all the correspondent curves. The long-term assembled relationships consist of overlaying the force – slip curves from the tests corresponding to: the specimen’s pre-slip, creep, and post-creep tests, as schematically represented in Figure 5.9. The development of the long-term slip has a minor influence on the post-creep residual forces. It is evident that the assembled curves resemble quite well the average responses of the monotonic tests. Nevertheless, in some cases, due to the scatter of the test results, the assembled responses
did not follow so closely the average monotonic curves, nonetheless, they are yet comprised within the experimental envelope of the monotonic pull-out tests.

![Force vs Slip Graphs](image)

Figure 5.22: Comparison of monotonic and long-term assembled curves: (a) and (b) 0 degree, (c) and (d) 30 degree, (e) and (f) 60 degree; (a), (c), (e) $s_{pr} = 0.3$ mm and (b), (d), (f) $s_{pr} = 0.5$ mm.
Regarding to series with 30° fibre inclination angle, the behaviour of one specimen was quite different. For this specimen, during reloading process in the creep test, a portion of concrete spalled which lead to a jump in the registered instantaneous slip. This response could be observed in Figure 5.23. After the specimen was unloaded in the creep test, a higher long-term residual slip, \( s_{pr}^{res} \), was recorded, which lead to a reduction of \( F_{p}^{\text{max}} \) when the specimen was tested in post-creep stage.

![Figure 5.23: Long-term assembled curve for a specimen with 30 degree fibre inclination angle.](image)

5.4 Analytical approach to predict time-dependent fibre pull-out behaviour

5.4.1 Long-term slip

In this section, the experimental results presented in section 5.3.3 were used to calibrate empirical equations to predict the long-term response of the pre-slipped fibre pull-out specimens. For each fibre inclination angle series, the influence of \( s_{pr} \) and \( F_{a} / F_{pr} \) parameters were taken into account in the present approach. A combined power and hyperbolic equation was used, since similar equations were already proposed by ACI-Committee-209 (1997) and CEB-FIP (1999) to predict creep behaviour of plain concrete under compression behaviour. The following equation is proposed for predicting the time-dependent fibre pull-out behaviour:
\[ s_{lt} = \frac{t^{A}}{B + t^{A}} \times c \]  \hspace{1cm} (5.4)

where \( s_{lt} \) is long-term slip (in mm), and \( t \) presents the time duration of loading (in hours). According to the experimental data, parameters A and B are determined by:

\[ A = s_{pr} \left( 1.5 - \frac{1}{F} \right) + d \]  \hspace{1cm} (5.5)

\[ B = 1/\left( 0.047 s_{pr} \right) \]  \hspace{1cm} (5.6)

where \( s_{pr} \) is the pre-slip value (in mm) and \( F \) presents level of loading (\( F_a / F_{pr} \)). In Eqs. 5.4 and 5.5 constants \( c \) and \( d \) were determined by nonlinear curve fitting analysis procedures. The coefficient \( c \) depends on the fibre orientation angle and the following values were proposed: 1, 0.72 and 0.45 (R-square values: 0.96, 0.95 and 0.97) for fibre orientation angles 0\(^\circ\), 30\(^\circ\) and 60\(^\circ\), respectively. Furthermore, coefficient \( d \) was also determined as 0.11 (R-square value of 0.96) for all series. It should be mentioned that Eq. 5.4 is only valid for \( 0.6 < F_a / F_{pr} \leq 1.0 \). Figure 5.24 compares the long-term slip versus time obtained from the empirical expressions and the correspondent experimental ones. Since the equations where calibrated by assigning the obtained experimental data and considering that relatively high R-square values where achieved, it is natural that high predictive performance is obtained when compared the experimental and empirical results. However, to properly validate the proposed equations more experimental data is required, with of the pre-slip values, load levels, and higher time periods.

Figure 5.25 (a) to (e) depict the results of a parametric study to investigate the influence of pre-slip value (\( s_{pr} = 0.3 \) and 0.5 mm) and also the load level (\( F_a / F_{pr} = 0.6, 0.7, 0.8, 0.9 \) and 1.0) on the long-term slip, \( s_{lt} \), versus time response up to a period of one year. In general, the long-term slip increased with the increase of the \( F_a / F_{pr} \) ratio, and this increase was more significant for higher values of \( s_{pr} \). Considering now the influence of \( s_{pr} \) on the long-term slip, for the same fibre inclination angle and \( F_a / F_{pr} \) it was observed that the increase of \( s_{pr} \) led to an increase of the long-term slip, mainly for the higher load levels.
Figure 5.24: Comparison between experimental and analytical long-term slip vs. time relationship for series with fibre inclination angle: (a) 0°, (b) 30° and (c) 60°.
In this section, the experimental results were used to propose equations to predict the long-term response of the pre-slipped fibre pull-out specimens in terms of creep coefficient. In Chapter 7, these equations were used to predict the long-term response of the specimens in macro-level. The equations have the same basis similar to the previous section:

\[ \varphi^C = \frac{t^A}{b + t^A} \times c \]  

(5.8)

where \( \varphi^C \) is analytical creep coefficient, and \( t \) presents the time duration of loading (in hours). According to the experimental data parameter \( A \) was determined by Eq. 5.9:

Figure 5.25: Influence of \( F_a/F_{pr} \) on the long-term slip vs. time response for \( s_{pr} \) equal to: (a), (c), (d) 0.3 mm and (b), (e) 0.5 mm; fibre inclination angle of: (a), (b) 0° and (c) 30° and (d), (e) 60°.
where $s_{pr}$ is the pre-slip value (in mm) and $F$ presents level of loading ($F_a / F_{pr}$). In Eqs. 5.8 and 5.9 constants $b$, $c$ and $d$ were determined by nonlinear curve fitting analysis procedures. Parameter $b$ was proposed as 15 (R-square values: 0.96) for all series. The coefficient $c$ depends on the fibre orientation angle and the following values were proposed: 1, 0.87 and 0.63 (R-square values: 0.96, 0.94 and 0.97) for fibre orientation angles 0°, 30° and 60°, respectively. Furthermore, constant $d$ was also achieved as 0.1 for all specimens. Figure 5.26 compares the long-term slip versus time obtained from the empirical expressions and the correspondent experimental ones, where it was be concluded that the proposed equation predicts with high accuracy the registered experimental data.

\[
A = s_{pr} \left(1.4 - 1/(2F)\right) + d
\]  
(5.9)

Figure 5.26: Comparison between experimental and analytical creep coefficient vs. time relationship for series with fibre inclination angle: (a) 0°, (b) 30° and (c) 60°.
In the present chapter, the effectiveness of a fibre as an element for transferring stresses under a sustained load across a crack was assessed by performing single fibre pull-out creep tests, in which the fibre slip was monitored as a function of time and pull-out load. For this purpose, cylindrical specimens with a single fibre located in the centre were cast with distinct fibre inclination angle, and monotonic and long-term pull-out tests were carried out. The influence of the fibre inclination angle and the pre-imposed fibre slip levels on the long-term pull-out behaviour was studied. Finally, the assembled long-term curves comprising the pre-slipping, creep and post-creep responses were compared to the correspondent monotonic ones.

In general, two fibre failure modes were observed during monotonic pull-out tests. In the case of the fibres with inclination angle of 0°, the complete fibre pull-out was observed, whereas for the inclined fibres, fibre rupture was the predominant failure mode. The maximum monotonic pull-out load decreased with the increase of the inclination angle, while the slip at peak load, in general, increased with the angle. However, a slight reduction on the slip at the peak load was observed for the 30° series, whereas for a 60° series the slip at the peak load increased considerably. This significant increase could be ascribed to other additional mechanisms in the pull-out process, which include the matrix spalling and fibre deformation.

Considering the long-term pull-out tests for both investigated pre-slip levels, i.e. 0.3 and 0.5 mm, stable responses were obtained for all series. Regarding the influence of the pre-slip levels, $s_{pr} = 0.5$ mm series showed a higher long-term slip comparing to the $s_{pr} = 0.3$ mm series. Since in the applied pre-slip of 0.5 mm, the interface bond between fibre and matrix was more deteriorated, therefore the long-term slip has increased in a higher rate.

The creep behaviour of the specimens was influenced by the fibre orientation angle. For the time period considered (60 days), fibres with 0° orientation angle showed the highest long-term slip, whereas the lowest long-term slip was observed for the 60° specimens. This was due to the different failure modes of these specimens. In the case of inclined fibres, the fibre slippage is also resisted by the shear friction that increases with a normal stress field installed in the fibre exit point. However, for a fibre with 0° inclination angle since it is subjected to only direct tensile loading (without any deviational force
component), therefore it is more susceptible to be pulled-out under a sustained load than other series. From another point of view, by comparing the ratio of the applied load in creep test to the maximum load recorded during the test, $F_a / F_p^{\text{max}}$, $0^\circ$ specimens were subjected to a higher loading level.

In general, the fibre pull-out behaviour registered after the specimens have been subjected to long-term pull-out tests was not significantly affected, when the behaviour obtained from instantaneous monotonic tests is taken for the comparison purpose, even when the specimens were pre-slipped nearby the slip corresponding to the maximum pull-out load.

Based on the results obtained from the creep tests, an empirical equation was proposed to predict the evolution of the long-term slip with time. The proposed equation has predicted with high accuracy the experimental results, and seems capable of estimating the $s_{lt} - t$ for hooked end steel fibres up to 0.5 mm and loaded within the interval $0.6 < F_a / F_{pr} \leq 1.0$. However, further research is required to appraise its capability to other steel fibres, pre-slip and $F_a / F_{pr}$ values, fibre orientation and embedded length and concrete strength class.
6.1 Introduction

Discrete fibres are increasingly being used in the construction industry to overcome the brittle nature of plain concrete under tension, and either to avoid or reduce the use of conventional steel reinforcement. In fibre reinforced concrete, FRC, macro-cracks that arise within the cementitious matrix are bridged by fibres randomly distributed in the concrete. The fibres are able to transfer the stresses across the crack’s surfaces improving the tensile post-cracking strength that enhances the composite’s toughness and crack growth control, resulting favourable effects in terms of load carrying capacity, ductility and durability of structures made by FRCs. It has been investigated extensively that mechanical properties of steel fibre reinforced concrete, SFRC, depend on both the fibre orientation and distribution (Ferrara and Meda, 2006; Kang and Kim, 2011). Moreover, discrete fibres are more effective when preferentially aligned along the directions of the principal tensile stresses. In order to achieve the maximum benefit of fibre’s contribution, an appropriate design of concrete properties in the fresh state, as well as a careful selection of the fibre type and casting procedure should be considered.
The effectiveness of a fibre as a reinforcing element becomes more predominant after it has been crossed by a crack. Determination of the tensile post-cracking behaviour of SFRC has been widely studied, either by direct or indirect tests (Barros and Figueiras, 1999; Bencardino et al., 2010; Carmona and Aguado, 2012; Denneman et al., 2011; Prisco et al., 2013). However, regarding the long-term response of this composite, the available knowledge in literature is still somehow quite scarce. Creep is a visco-elastic phenomenon, mainly occurs in the viscous hydrated cement paste. This may be a concern, since steel fibre reinforced self-compacting concrete (SFRSCC) has high binder content, in part to attain self-compactibility. Thus SFRSCC may exhibit a relatively high deformability due to long-term loads. The creep deformation of the material could ultimately lead to the failure mechanism of the structural element at a lower load than static ultimate load (Boshoff et al., 2009b). On the other hand, in some structural systems, the long-term deformation of the structural element can be beneficial, since it enforces stresses to redistribute, which can limit the crack propagation. From another point of view, if the creep deformation damages significantly affect the fibre/matrix interface bond, it will lead to an undesirable excessive decrease on the post-cracking strength, thus the influence of creep will be adverse (Arango et al., 2012).

Some information is available in literature regarding to the time-dependent behaviour of FRC in the cracked state (MacKay and Trottier, 2004; Arango et al., 2012; Kanstad and Zirgulis, 2012, Zerbino and Barragan, 2012). However, many of them mainly assessed the creep behaviour of concrete reinforced with synthetic fibres (Al-Khaja, 1995; Kurt and Balaguru, 2000; Oh et al., 2005). It was reported that the cracked micro/macro-synthetic fibre reinforced concrete presented significant crack widening over time under sustained uniaxial tensile load (Babafemi and Boshoff, 2014). There are also some works regarding the creep evaluation of steel fibre reinforced concrete under uniaxial tensile loading (Mouton and Boshoff, 2012; Zhao et al., 2012) and flexural loading (Arango et al., 2012; Zerbino and Barragan, 2012). It was showed that application of steel fibres in concrete limited long-term crack widening considerably (Tan et al., 1994; Tan and Saha, 2005). It is worth noting that focus of the mentioned studies was principally on the beams where, in the case of using steel fibre reinforced self-compacting concrete, as it was shown in Chapter 3, the rotation of the fibres due to the concrete flow was completely distinct of planar structures (Boulekbache et al., 2010; Martinie and Roussel, 2011).
Creep in bending of a cracked SFRC element is the result of the following phenomena: concrete creep in compression (produce basic creep); fibres creep at material level in tension; loss of fibre-matrix adherence and subsequent fibre free-sliding. Creep of fibres is only significant in fibres susceptible to thermo-hygrometric effects, not in the case of the steel fibres. It was shown that the time-dependent alterations in the fibre-matrix interface zone influence significantly the long-term fibre reinforcement effectiveness, and, consequently, the creep behaviour of cracked fibre reinforced concrete (Boshoff et al., 2009a; Babafemi and Boshoff, 2014). The crack width opening and progression with time are strongly dependent of the long-term behaviour of concrete, load and environmental conditions. Therefore, it is important to evaluate the concrete capability to maintain the crack opening width relatively low under a sustained load, in order to guarantee the effectiveness of fibres under serviceability conditions. Moreover, despite being available some standards for designing SFRC structures (UNI-11039, 2003; CEB-FIP, 2010), it seems that they still not take into account the long-term behaviour under cracked conditions. Therefore, information regarding the long-term behaviour of cracked SFRC elements, particularly planar structures, is still limited. Consequently, understanding the behaviour of cracked SFRC elements under a sustained load will help towards a more rational design and accurate prediction of the composite behaviour under serviceability conditions.

In the present work an extensive experimental program that aims to study the long-term behaviour of cracked steel fibre reinforced self-compacting concrete, SFRSCC, is described. For this purpose, prismatic specimens were extracted from a SFRSCC panel cast from its centre, and the relevant results obtained from the creep tests with these specimens are presented and discussed. The influence of fibre orientation and dispersion on the creep behaviour of cracked SFRSCC elements was appraised. This was achieved based on the angle between the extracted specimen’s notch plane and the expected concrete flow direction, since extracted specimens were notched in distinct directions. The prismatic specimens were previously subjected to a four-point bending test up to a certain crack width opening \(w_{cr} = 0.3\) and \(0.5\) mm). Then, the bending test was carried out under a sustained load until the stabilization of the crack width opening. In a first stage the influence of the initial crack opening level, applied stress level [50-100%], fibre orientation/dispersion, and distance from the casting point on the flexural creep behaviour was investigated. Afterwards, the specimens used in the flexural creep tests were
subjected to an instantaneous four-point bending test until failure. Finally, a series of instantaneous four-point bending tests were also executed on uncracked prismatic specimens, in order to quantify the influence of the creep phenomenon on the evolution of the flexural residual strength.

6.2 Experimental program

6.2.1 Concrete mixture

A SFRSCC was produced according to the mixture composition given in Table 6.1. W/C abbreviates water (W) to cement (C) ratio. Superplasticizer Sika® 3005 (SP) has been used to assure the required self-compactibility requirements due to the low water content. A crushed granite coarse aggregate was used with a maximum aggregate size of 12 mm. A fibre content of 60 kg/m$^3$ of hooked-end steel fibres was used. The fibres had the following characteristics: 33 mm of length, $l_f$; 0.55 mm of diameter, $d_f$; aspect ratio, $l_f/d_f$, of 60 and a yield stress of about 1100 MPa. The fresh concrete behaviour was determined by the Abrams cone slump test in the inverted position according to EFNARC (2005) recommendations. The spread diameter was approximately 590 mm.

The SFRSCC’s compressive strength was assessed by testing 6 cylinders with a diameter of 150 mm and a height of 300 mm. After casting, the specimens were stored in a climatic chamber room at constant temperature of 20 °C and relative humidity of 60%. The cylinders were tested at the age of 28 days, and an average compressive strength and an average young modulus of 72 MPa and 42.15 GPa was obtained, respectively, with a coefficient of variation (CoV) of 8.23% and 0.26%, respectively.

Table 6.1: Mix design of steel fibre reinforced self-compacting concrete per m$^3$.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>412</td>
<td>124</td>
<td>0.30</td>
<td>7.83</td>
<td>353</td>
<td>237</td>
<td>710</td>
<td>590</td>
<td>60</td>
</tr>
</tbody>
</table>
6.2.2 Specimens

The casting of this panel for the SFRSCC cracked creep tests was performed from its geometrical centre. However, previous research has revealed that this casting procedure improves the residual tensile behaviour since due to the circular distribution of fibres through the panel, a higher number of effective fibres exist in order to bridge the radial cracks formed when the panel was loaded from its centre (Barnett et al., 2010). The fresh concrete was poured directly from a mixing truck by a U-shape channel almost in the vertical position into a mould with the dimensions of 1500×1500 mm$^2$ in plan and 60 mm of thickness. A total of one hundred and twelve prismatic specimens with the dimensions of 240×60×60 mm$^3$ were extracted from distinct locations of the panel. The orientation of the extracted prismatic specimens within the panel was established having in mind the expected concrete flow direction, see Figure 6.1(a). In this scheme, the light gray solid hatched specimens were used for assessing the long-term behaviour, the dark gray solid hatched beams were tested under instantaneous monotonic load conditions, whereas the rest of the specimens were stored to be tested under instantaneous cyclic loading (loading up to $w_{cr}$, unloading and reloading until failure) at the same age of the specimens cracked for the creep test. Since the panel was cast in its centre point, the symmetry of the panel assures that for each specimen in the creep test, there will be a mirror specimen on the other side of the panel for the execution of the instantaneous cyclic test to estimate the effects of creep crack width propagation on the residual strength. For instance, in Fig. 1(a), specimen L-0.5-7.5°-8 was tested under cyclic load condition while specimen L-0.5-7.5°-5 was used for the creep test. After the extraction of the prismatic specimens, a notch was executed at its middle length. The notch depth and thickness were 10 and 2 mm, respectively.

The influence of the crack plane orientation towards the expected concrete flow was assessed in four different directions. The orientation of the notched plane was defined accordingly to the following strategy: by considering $\beta$ as the angle between the direction of the concrete flow and the notched plane direction, four series of prismatic specimens with different notched plane orientations towards the concrete flow directions can be defined. Figure 6.1(b) depicts a scheme of the adopted classification methodology based on the angle $\beta$. The four intervals established for the angle $\beta$ were [0-15°], [15-45°], [45-75°] and [75-90°]. Since in Chapters 3 and 4 it was found that the radial flow of the
SFRSCC promotes a preferential fibre alignment perpendicularly to the flow direction, the present adopted strategy enables to appraise the influence of fibre orientation, at a certain distance from the casting position, on the instantaneous force-crack width ($\sigma - w$) and on the creep coefficient-time ($\varphi - t$) relationships. Hereinafter, each series was designated by an alphanumeric string according to: the first character represents the distance from the casing point (L-Low: [0-375 mm]; A-Average: [375-565 mm] and H-High: [565-750 mm]); the second numeral refers to the two studied pre-crack widths ($w_{cr}$), 0.3 and 0.5 mm; the third numeral defines the $\beta$ angle, in degrees, for four intervals of the relative orientation between the notched plane and the SFRSCC flow lines (7.5: [0-15°], 30: [15-45°], 60: [45-75°] and 87.5: [75-90°]), and the last numeral represents the number of the series’ specimen. For instance, L-0.3-7.5º-1 represents the specimen number 1 located at a low distance from the casting point, with a pre-crack width of 0.3 mm, and with an angle $\beta$ in the [0-15°] interval. A similar strategy for the designation of the instantaneous monotonic test specimens was followed, however the second character that indicates the pre-crack width was ignored, since these tests were performed on uncracked specimens in opposition to the creep tests.

*Figure 6.1:* (a) Specimen’s extracting plane, (b) definition of $\beta$ angle.
6.2.3 Test set-up

6.2.3.1 Monotonic four-point bending tests

The instantaneous $F$-CTOD (crack tip opening displacement) relationship representative of the SFRSCC panel was determined with a total of sixty four monotonic four-point bending tests. These tests were carried out on notched beams extracted from the panel following the recommendations of Italian standard (UNI-11039, 2003). The monotonic tests were carried out on a universal testing machine of 50 kN load bearing capacity and at a displacement rate of 0.002 mm/s.

In this test set-up, two line loads were applied at the beam length thirds. Both the deflection at mid-span as well as the crack mouth opening displacement (CMOD) were recorded. Figure 6.2 depicts the test set-up and geometry of the specimen. To determine the deflection during the test, a LVDT (Linear Variable Differential Transformers) was mounted on an aluminium bar supported at mid height of the sections coinciding with the supports of the specimen in order to exclusively record the deflection of the specimen, see Figure 6.2(a). In order to follow the recommendations of UNI-11039 (2003) this lateral face of the specimen coincides with the surface of the SFRSCC panel in contact with the mould, and the notch was executed in the face parallel to the casting direction. Therefore the bottom face of the specimen in the test set-up corresponds to a lateral surface of the extracted beam from the panel, i.e. the beam was rotated 90° along its longitudinal axis for testing. To assess an eventual asymmetric crack opening, due to fibre segregation along the panel’s thickness, CTOD was measured by installing three LVDTs on the beam’s bottom surface, see Figure 6.2(b). These transducers recorded the crack opening width at about 10 mm from the top and bottom surfaces of the panel, as well as at its middle height. The adopted disposition of the aluminium LVDT supports mitigates the measurement of the elastic deformations surrounding the mouth of the notch plane, see Figure 6.2(c).

To obtain the influence of the long-term crack opening on the secondary stiffness, $K_{sec}$, and also to investigate if the unloading and reloading process impose the residual load decay in the $F$-CTOD relationship, a series of cyclic tests adopting the test set-up in Figure 6.2 were also implemented. In this test, beams were loaded until the nominal crack opening, $w_{cr}$, value achieved (0.3 or 0.5 mm). Then, the specimens were unloaded. Afterward, a monotonic test was carried out by maintaining the test set-up previously
presented. Finally, the $F$-CTOD relationship was compared to the ones obtained from the monotonic configuration. A graphical representation of the force-crack opening relationship was depicted in Figure 6.3, where $F_L$ is load at crack initiation, $F_{cr}$ and $w_{cr}$ are the load and correspondent crack opening once the target crack opening reached, respectively, $w_{cr}^{res}$ is the residual crack opening after unloading of the specimen, $F_p^{max}$ and $w_p^{max}$ are the maximum load recorded during the test and its correspondent crack opening width, respectively.

![Test set-up of monotonic four-point bending test](image)

**Figure 6.2:** Test set-up of monotonic four-point bending test (dimensions are in mm): (a) geometry of the specimen and LVDT for measuring the deflection (b) LVDTs to record CTOD, (c) LVDT connection details for measuring CTOD.

![Scheme representation of cyclic test](image)

**Figure 6.3:** Scheme representation of cyclic test.
6.2.3.2 Long-term four-point bending tests

In order to obtain the long-term $F$-$w$ (crack width) relationship, experimental tests were carried out in three stages: firstly, the beams were pre-cracked up to a crack width of either 0.3 or 0.5 mm; secondly, the creep tests were carried out on the pre-cracked beams until the stabilization of the creep crack width has been achieved; finally, post-creep bending tests were performed until failure of the specimen.

Pre-cracking of the specimens

The creep four-point bending tests were performed in pre-cracked specimens. Therefore, in a first stage, the monotonic test set-up previously described was used to achieve the desired crack opening width ($w_{cr}$). In this research, two initial crack opening widths were studied, namely, 0.3 mm, recommended for the serviceability limit state, and 0.5 mm that is coincide to the crack opening width corresponding to the one used to compute the residual flexural strength, $f_{R,1}$ (CEB-FIP, 2010). Afterward, the test was stopped and the specimen unloaded. During execution of the test, load and crack opening width were recorded. Figure 6.4 shows an schematization of the cracking process, where $F_L$ is load at crack initiation, $F_{cr}$ and $w_{cr}$ are the load and correspondent crack opening once the target crack opening reached, respectively, $w_{cr}^{res}$ is the residual crack opening after unloading of the specimen.

![Figure 6.4: Scheme representation of the cracking test.](image)

185
Creep tests

After finalizing the pre-cracking procedure, the specimens were carefully moved to the climate chamber room, at a temperature of 20°C ± 0.5 and a 60% relative humidity ± 5%. The creep testing rigs were located inside this room, thus the creep tests were carried out under controlled thermo-hygrometric conditions. The creep testing rig allowed to simultaneously loading three specimens. Figure 6.5(a) shows the adopted creep test set-up. Before mounting the specimens in the creep testing frame, they were kept in the chamber room for five hours to acclimatize. The test was executed under force control. Specimens were loaded by injecting oil into the hydraulic actuator until the desired value of the load has been reached. The specimens’ loading procedure was carried out very carefully in order to attain the target load level with the minimum deviation as possible.

To measure the crack opening width, for each specimen, one LVDT was installed at the middle of the notched surface, see Figure 6.5(b). The LVDTs and load cell were connected to a computerized data acquisition system. The data was recorded in distinct time-steps according to the following procedure: during loading and unloading of specimen: one sample per 5 seconds; in the first and second month after loading: one sample per 500 seconds and one sample per 1000 seconds, respectively.

Figure 6.5: Creep test set-up: (a) general view, (b) position and connection details of the LVDT.
After the initiation of the creep test, the value of the applied load, $F_a$, was fixed and maintained constant until the crack opening width was stabilized. When the variation of the crack opening value was smaller than one micrometer for three consecutive days, it was assumed that the crack opening was stabilized, and then the test was finalized. In the present research, for each creep frame, beams where positioned and loaded according to the following methodology: the beam with the highest value of $F_{cr}$ (the load corresponding to $w_{cr}$) was placed on the bottom, with a moderate $F_{cr}$ was localized in the middle, and with the lowest $F_{cr}$ was placed on the top. Then, they were loaded simultaneously with the smallest $F_{cr}$ value of these three tested specimens. In other words, it was opted to apply a load level that produces a bending moment in the notched cross section of the beam placed on top of the frame equal to 100% of that observed at $w_{cr}=0.3$ or 0.5 mm in the pre-cracking test. By following this methodology, the influence of different loading levels ($F_a / F_{cr}$) on the long-term behaviour of the adopted SFRSCC could be investigated as well.

Figure 6.6: Definition of crack opening parameters in creep test: (a) crack opening-time relationship, $w$-$t$, (b) force-crack opening curve, $F$-$w$.

Afterward the conclusion of the creep tests, the specimens were unloaded, but the data acquisition system was kept active for at least a period of one week, enabling to record the closing of the crack width due to the creep recovery process. Figure 6.6(a) and (b) show schematically the variation of crack opening versus time and force-crack opening relationship, respectively. In these figures, $w_{inst}$ is the instantaneous crack
opening, $w_p$ is the long-term crack opening, $w_{\text{inst}}^{\text{rec}}$ is the instantaneous crack opening recovery at the beginning of unloading process, $w_{\text{lt}}^{\text{rec}}$ represents the long-term crack opening recovery, $w_{\text{total}}^{\text{rec}}$ is total crack opening recovery, $w_0^{\text{rec}}$ indicates the residual crack opening after unloading stage, and $F_a$ depicts applied load level in creep test.

**Post-creep tests**

After the end of the creep tests and one week waiting period in order to enable creep deformation recovery, the specimens were then subjected to monotonic four-point bending tests until a 4 mm crack mouth opening width has been reached. The monotonic test set-up was similar to the one used for pre-cracking the beams, which was previously presented. A constant displacement rate of 0.002 mm/s was also imposed. A schematic representation of $F - w$ curve is depicted in Figure 6.7, where $F_p^\text{max}$ and $w_p^\text{max}$ are the maximum load recorded during the test and its correspondent crack opening width, respectively.

![Figure 6.7: Scheme representation of the post-creep test.](image)

**Assembled long-term force-crack width curve**

Finally, the complete $F - w$ curves were assembled with the individual $F - w$ curves from the pre-cracking, creep and post-creep responses, as shown in Figure 6.8. The push curve from this assembled response will be compared with the correspondent curve from the monotonic test on uncracked beams. In Figure 6.8, hereinafter, $F_L$ is the load at crack opening.
initiation, $F_{cr}$ and $w_{cr}$ are the load and correspondent crack opening, respectively, for the defined pre-crack levels, i.e. 0.3 and 0.5 mm, $w_{cr}^{res}$ is the residual crack opening after unloading of the specimen, $K_{sec}$ is secondary stiffness at re-loading process of post-creep test, $F_{p}^{max}$ and $w_{p}^{max}$ are the maximum load at post-cracking branch and its correspondent crack opening width, respectively.

![Figure 6.8: A graphical representation of assembled long-term force-crack width curve.](image)

6.3 Discussion of the experimental results

6.3.1 Monotonic four-point bending tests

In the first stage, a preliminary study was carried out to compare the $F$-CTOD relationships achieved from the cyclic and monotonic tests in order to investigate if the unloading and reloading process influences the post-cracking residual strength. It was observed that the cyclic push curves were matched with the ones from the monotonic tests, concluding in no influence of the residual forces due to the one cycle of unloading and reloading process. Figure 6.9(a) to (d) depicts the envelope and average force-crack tip opening displacement relationships, $F$-CTOD, obtained from the monotonic four-point bending tests (in combination with push curves of cyclic tests) when $\beta=[0-15^\circ]$, $[15-45^\circ]$, $[45-75^\circ]$ and $[75-90^\circ]$, respectively. The experimental results of the tests can be found in Appendix V for each specimen individually. The CTOD was determined by averaging the

189
displacements recorded in the three LVDTs fixed at the bottom face of the specimen (Figure 6.2). The recorded value of crack opening width (CMOD$_y$) was corrected to the real crack tip opening width (CTOD) as follows:

$$CTOD = CMOD_y \frac{h}{h + y}$$

(6.1)

where, CMOD$_y$ is the recorded value in the LVDTs at a distance $y$ below the notch mouth of the specimen, and $h$ is the total depth of the specimen.

![Figure 6.9](image)

*Figure 6.9:* Monotonic force-crack tip opening displacement relationship for $\beta$ in the intervals: (a) [0-15°], (b) [15-45°], (c) [45-75°] and (d) [75-90°].

In general, the specimens have shown a linear behaviour up to the load correspondent to the crack initiation. Up to this load, the value measured by the LVDTs corresponds to the elastic deformation of the SFRSCC bulk in-between the LVDT’s supports. Therefore, the results exhibited a low scatter, since in this stage, the
contribution of the fibres was almost null as the material response was mainly dependent on the elastic behaviour of the concrete constituents. After the cracking onset, the fibres started to be mobilized bridging the stresses across the crack surfaces. In general, for the post-cracking stage the scatter was relatively high, because after the mobilization of fibres the composite performance was extremely dependent of the fibre dispersion and orientation. Since the beams were extracted from different locations of the panel at distinct distances from the casting point, a high scatter of the tests results was expected. This aspect can be ascribed to the reduction of the concrete flow velocity with the increase of the distance to the casting point. This variation of the flow velocity will influence both the fibre dispersion and orientation along the flow profile, and consequently will lead to quite different fibre structures of the specimens at distinct distances from the casting point.

Regarding the first two series, namely, $\beta=\{0-15^\circ\}$ and $\{15-45^\circ\}$, they have shown a similar force value at the limit of proportionality. Once the tensile strength of material was attained, both series revealed a deflection-hardening response up to a CTOD of around 0.6 mm, but for $\{15-45^\circ\}$ specimens a slightly lower peak load value was attained. From the micromechanical point of view, after the adhesion of fibre and surrounding matrix has been exceeded, the fibre reinforcement effectiveness was mainly governed by the plastification process of the hooked end, which provides the highest contribution for this deflection-hardening capacity. Afterward, a softening stage was observed for both series. In the $\beta=\{0-15^\circ\}$ case, since the specimens contained more fibres that intersected the cracked plane with a lower angle, i.e. $29^\circ$ in Table 3.5, the residual force decay was smoother, whereas for the $\beta=\{15-45^\circ\}$ series, a higher load decay was observed between 0.74 and 1.5 mm of CTOD. In the latter specimens, fibres have intersected the cracked plane with a higher average orientation angle, i.e. $36^\circ$, see Figure 3.31(b). Fibre pull-out tests carried out have shown that fibres with an orientation angle of $30^\circ$ ruptured for a slip value nearby 0.5 mm. Therefore, in this series, once the peak load was attained, the fibres are more prone to be ruptured within the CTOD range of 0.74-1.5 mm. Actually, during the execution of the test, the strident sound of the fibres rupturing was clearly differentiated. In general, during execution of bending test, because the crack wide opening is different along the crack surface, since the fibres along the notch plane mobilize at various stages regarding on their position towards the neutral axis. On the other hand, the fibres that are farther from the neutral axis have a higher probability of
rupturing firstly. When the fibres start to rupture, mainly those close to the bottom face, the residual load starts to decay rapidly, because a part of the energy that is released during the fibre rupturing will be sustained by other fibres, thus the pull-out process of fibres positioned above will promptly accelerate.

Regarding the cases of $\beta=[45-75^\circ]$ and $[75-90^\circ]$ series, after the crack initiation, the load decreased suddenly, followed by a small plateau, and beyond a CTOD of about 0.75 mm the load decreased smoothly. This could be ascribed to a higher probability of fibre rupture, since in these two series, fibres have a tendency to be oriented with a higher orientation angle towards the notched plane, see Figure 3.31(c) and (d), with average orientation angles of $43^\circ$ and $48^\circ$, respectively.

By comparing all $F$-CTOD relationships, Figure 6.10, could be concluded that the post-peak behaviour of the SFRSCC was highly dependent of the direction in which the mechanical properties were assessed. The series with the notched plane more parallel towards to the concrete flow direction, i.e. [0-15$^\circ$] and [15-45$^\circ$] series, showed a higher residual force, and therefore a larger energy absorption capacity than the specimens with a fracture surface more perpendicular to the flow direction. When $\beta = [0-15^\circ]$, [15-45$^\circ$], [45-75$^\circ$] and [75-90$^\circ$], it was illustrated that the average orientation angle of the fibres towards the notch plane were 29, 36, 43 and 48$^\circ$, respectively, see Table 3.5. However, from the monotonic single fibre pull-out test it was verified that the average orientation angle value of an effective fibre which bridging a leading crack was about 30 degrees. Therefore, regarding to the first two series, the number of the effective fibres in the crack surface was much higher than the last two intervals. This gap becomes more momentous if one compares the flexural residual strength in different average crack widths.

![Figure 6.10: Comparison of monotonic F-CTOD relationship for different series.](image)
6.3.1.1 Flexural residual strengths

Table 6.2 comprises the average and characteristic residual flexural strengths obtained from the monotonic bending tests. In this table, \( f_{r1}, f_{r2}, f_{r3} \) and \( f_{r4} \) are the residual flexural strength at CTOD of 0.5, 1.5, 2.5 and 3.5 mm, respectively, computed as follows:

\[
f_{r} = \frac{3F_{j}(L - L_{i})}{2bh_{sp}^{2}}
\]  

(6.2)

where, \( f_{ri} \) is the residual flexural tensile strength corresponding with CTOD \( j \), \( F_{j} \) is the load corresponding to CTOD \( j \), \( L \) is the span length (180 mm), \( L_{i} \) is the length of loading span (60 mm), \( b \) is the width of the specimen (60 mm) and \( h_{sp} \) is the distance between the tip of the notch and top of the specimen (50 mm). Furthermore, the coefficient of variation, CoV, and the characteristic values for a confidence interval of 95%, \( k_{95\%} \), are also included.

**Table 6.2:** Average and characteristic results of the monotonic bending test.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( f_{r1} ) [MPa]</th>
<th>( f_{r2} ) [MPa]</th>
<th>( f_{r3} ) [MPa]</th>
<th>( f_{r4} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg.</td>
<td>11.06</td>
<td>6.73</td>
<td>4.35</td>
<td>3.27</td>
</tr>
<tr>
<td>[0-15°]</td>
<td>CoV(%)</td>
<td>20.36</td>
<td>31.26</td>
<td>46.63</td>
</tr>
<tr>
<td>( k_{95%} )</td>
<td>9.9</td>
<td>5.65</td>
<td>3.31</td>
<td>2.48</td>
</tr>
<tr>
<td>Avg.</td>
<td>10.90</td>
<td>5.55</td>
<td>3.25</td>
<td>2.21</td>
</tr>
<tr>
<td>[15-45°]</td>
<td>CoV(%)</td>
<td>11.46</td>
<td>25.38</td>
<td>25.75</td>
</tr>
<tr>
<td>( k_{95%} )</td>
<td>9.86</td>
<td>4.37</td>
<td>2.55</td>
<td>1.58</td>
</tr>
<tr>
<td>Avg.</td>
<td>5.92</td>
<td>3.34</td>
<td>2.01</td>
<td>1.36</td>
</tr>
<tr>
<td>[45-75°]</td>
<td>CoV(%)</td>
<td>21.19</td>
<td>35.46</td>
<td>45.56</td>
</tr>
<tr>
<td>( k_{95%} )</td>
<td>5.16</td>
<td>2.62</td>
<td>1.45</td>
<td>0.93</td>
</tr>
<tr>
<td>Avg.</td>
<td>4.83</td>
<td>2.87</td>
<td>1.67</td>
<td>1.17</td>
</tr>
<tr>
<td>[75-90°]</td>
<td>CoV(%)</td>
<td>37.31</td>
<td>37.01</td>
<td>43.74</td>
</tr>
<tr>
<td>( k_{95%} )</td>
<td>3.62</td>
<td>2.16</td>
<td>1.18</td>
<td>0.82</td>
</tr>
</tbody>
</table>

From the results it was noticed that the influence of the notch orientation towards the concrete’s flow on the post-peak behaviour of the material was quite high. The series with the notch plane more perpendicular towards to the concrete flow direction, i.e. [0-
15°[ and [15-45°] series, showed a higher residual strength and therefore a larger dissipated energy than the specimens with a fracture surface more aligned towards flow direction. On the other hand, by comparing the [0-15°] and [75-90°] series a significant reduction on the residual stresses was observed, in particular, 56, 57, 62 and 64% for \( f_{r1} \), \( f_{r2} \), \( f_{r3} \) and \( f_{r4} \), respectively. This reduction in the post-cracking parameters could be ascribed to a preferential orientation of the fibres regarding the fracture plane due to the radial flow profile. As it was discussed and detailed in Chapter 3, fibres have a tendency to be reoriented perpendicular to the concrete flow direction.

### 6.3.1.2 Influence of the number of effective fibres on the flexural tensile strengths

Throughout Figure 6.11(a) to (d) are represented the relationships between the flexural tensile strength, respectively, \( f_{r1} \), \( f_{r2} \), \( f_{r3} \) and \( f_{r4} \), and the number of effective fibres found at the specimen’s fracture surface through Chapter 3 for each series separately. In spite of the results scatter, in general, a linear trend could be observed between \( f_{ri} \) with the number of effective fibres. In all figures, the flexural tensile strength tended to increase when the number of the effective fibres raised, being more pronounce in \( f_{r1} \) and \( f_{r2} \). This was expected, since the residual strength sustained by the crack was intimately related to number of mobilized fibres. However, regarding to \( f_{r3} \) and \( f_{r4} \), they also followed a same trend but in a marginal influence since after 2 mm CMOD, many fibres were already ruptured. Considering the influence of notch plane orientation towards the concrete flow direction, it was illustrated that [0-15°] specimens presented higher flexural tensile strengths since they had a higher number of effective fibres. For the other series, by increasing angle \( \beta \), the number of effective fibres reduced which led to a reduction in \( f_{ri} \).
Figure 6.11: Relation between the flexural residual strength and the number of effective fibres: (a) $f_{r1}$, (b) $f_{r2}$, (c) $f_{r3}$ and (d) $f_{r4}$.

6.3.1.3 Determination of the $\sigma - w$ law by inverse analysis (IA)

In this section, the post-cracking behaviour of the proposed SFRSCC is evaluated by inverse analysis (IA) taking the results of the four-point bending tests. For this purpose, the experimental force-crack tip opening displacement curves were simulated using FEMIX finite element program. Furthermore, this simulation will be also used in order to determine stresses for computing basic creep component in the long-term bending tests. The same inverse analysis procedure as discussed in Section 4.3.3 was adopted.

Due to the geometry, support and loading conditions used in the test set-up proposed in the four-point bending test for the characterization of the tensile behaviour under flexure of SFRSCC, a plane stress state installed in the beam was assumed. The geometry model comprises two main parts: notch and un-notch parts. The proposed mesh
is illustrated in Figure 6.12. In this mesh, 4-noded elements with Gauss-Legendre integration schemes of 1×2 and 2×2 were used for the elements in the notch and un-notch parts, respectively. The total number of elements was 234. A smeared crack model (Sena-Cruz, 2005) was assigned to all elements to simulate the concrete behaviour. The fracture mode I propagation of SFRSCC was simulated by the quadrilinear stress – strain diagram as shown in Figure 6.13, which is defined by the normalized stress, \( \sigma \), and strain, \( \varepsilon \). In this figure, \( \sigma_{n,1}^{cr} = f_{cr}^{n} \) is the tensile strength of concrete, \( \varepsilon_{n,ult}^{cr} \) states ultimate tensile strain, \( G_f^I \) is fracture energy in mode I and \( l_b \) shows the crack band width. To model the softening behaviour of SFRSCC, the numerical analyses were carried out under displacement control. The non-zero prescribed displacement constraints were applied on the top of the model as depicted in Figure 6.12. The values of the material properties used in the IA are comprised in Table 6.3.

![Figure 6.12: Finite element mesh used in the simulation of four-point bending test.](image)

![Figure 6.13: Quadrilinear stress – strain relationship for modeling fracture mode I (\( \sigma_{n,2}^{cr} = \alpha_{1}\sigma_{n,1}^{cr} \), \( \sigma_{n,3}^{cr} = \alpha_{2}\sigma_{n,1}^{cr} \), \( \sigma_{n,4}^{cr} = \alpha_{3}\sigma_{n,1}^{cr} \), \( \varepsilon_{n,1}^{cr} = \xi_{n,ult} \), \( \varepsilon_{n,2}^{cr} = \xi_{n,ult}^{2}\varepsilon_{n,ult}^{cr} \), \( \varepsilon_{n,3}^{cr} = \xi_{n,ult}^{3}\varepsilon_{n,ult}^{cr} \), \( \varepsilon_{n,4}^{cr} = \xi_{n,ult}^{4}\varepsilon_{n,ult}^{cr} \)).](image)
Table 6.3: Mechanical properties adopted in the numerical simulation.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$</td>
<td>$2.4 \times 10^6$ N/mm$^3$</td>
</tr>
<tr>
<td>Poisson ratio, $\nu$</td>
<td>0.2</td>
</tr>
<tr>
<td>Initial young modulus, $E_{ci}$</td>
<td>42.15 N/mm$^2$</td>
</tr>
<tr>
<td>Compressive strength, $f_{cm}$</td>
<td>72 N/mm$^2$</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>Inverse analysis</td>
</tr>
<tr>
<td>Post-cracking parameters</td>
<td>Inverse analysis</td>
</tr>
</tbody>
</table>

Figure 6.14 shows the experimental and numerical force – crack tip opening displacement obtained from the monotonic four-point bending tests for all series. In this figure, $\text{EXP}_{\text{Avg}}$ and $\text{EXP}_{\text{Envelope}}$ are, respectively, the experimental average and envelope relations. The numerical analysis was carried out up to a CTOD of 4 mm measured at the centre of the notch since after this value of CTOD the residual forces were low due to the high number of fibres pulled-out or fractured. A considerably good accuracy between the numerical and experimental responses was observed for all series which indicates that the quadrilinear $\sigma - \varepsilon$ relationship is capable of simulating accurately the post-cracking behaviour of SFRSCC.

Table 6.4 depicts the values of the fracture parameters defining the stress – strain softening laws for each series. Regarding the values found for $f_{ct}$, a scattering in the results is observable. However, since specimens were extracted from the different distances of the casting point, due to the variation in the matrix skeleton this scattering is expectable. In the first looking, it seems that the first two series depicted a higher tensile strength since they contain a higher number of fibres, but it was shown that addition of steel fibres to concrete have a marginal influence on the tensile strength of composite (Barros and Figueiras, 1999). In fact, when comparing the obtained $f_{ct}$ values from IA with the one suggested by CEB-FIP (2010) (proposed a mean value of 4.66, lower and upper bound values of 3.26 and 6.10 MPa), it was concluded that they were overestimated. On the other hand, in the average terms between all series, the tensile strength achieved from the IA ($f_{ct,avg} = 6.14$ MPa) was close to the upper limit proposed by CEB-FIP (2010). Comparing the fracture energy for each series, it was shown that $\beta=0$-15°[ specimens dissipated 18, 122 and 172% higher energy comparing to [15-45°].
[45-75°] and [75-90°] series, respectively. As it was already discussed, the first series illustrated the highest number of effective fibres.

Figure 6.14: Experimental and numerical force – crack tip opening displacement relationship, \(F-CTOD\), for \(\beta\) in the intervals: (a) [0-15°], (b) [15-45°], (c) [45-75°] and (d) [75-90°].

Figure 6.15 depicts a graphical presentation of the stress – strain laws acquired by IA. Comparing the first two series (\(\beta=\) [0-15°] and [15-45°]), it was shown that after the crack initiation, an upward trend was observed until a strain nearby 0.27%. Afterwards, a sharper reduction in the stress was observed for [15-45°] specimens due to the higher number of fibres ruptured. Comparing all series, it was found that [0-15°] series presented higher residual stresses in compared to the other series. The minimum residual stresses were achieved for [75-90°] specimens. This was due to the tendency of fibres to align more parallel to the crack plane by increasing in \(\beta\). More details were already discussed in the previous sections.
Table 6.4: Values of the fracture parameters defining stress – strain softening laws.

<table>
<thead>
<tr>
<th>β</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
<th>$\xi_3$</th>
<th>$f_{ct}$ [MPa]</th>
<th>$G_f^t$ [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0-15°]</td>
<td>0.72</td>
<td>0.85</td>
<td>0.33</td>
<td>0.014</td>
<td>0.18</td>
<td>0.46</td>
<td>6.77</td>
<td>6.00</td>
</tr>
<tr>
<td>[15-45°]</td>
<td>0.68</td>
<td>0.86</td>
<td>0.20</td>
<td>0.014</td>
<td>0.18</td>
<td>0.38</td>
<td>6.50</td>
<td>5.10</td>
</tr>
<tr>
<td>[45-75°]</td>
<td>0.44</td>
<td>0.46</td>
<td>0.10</td>
<td>0.024</td>
<td>0.18</td>
<td>0.35</td>
<td>5.65</td>
<td>2.70</td>
</tr>
<tr>
<td>[75-90°]</td>
<td>0.29</td>
<td>0.35</td>
<td>0.10</td>
<td>0.032</td>
<td>0.25</td>
<td>0.35</td>
<td>5.64</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Figure 6.15: Stress – strain relationships obtained by inverse analysis.

6.3.2 Long-term four-point bending test

The long-term deformation in a cracked SFRSCC element under flexure consists of two main phenomena: creep in the cracked section under tension (time-dependent modifications in the fibre/matrix interface zone) and long-term deformation in uncracked section under compression which produce basic creep.
6.3.2.1 Creep in tension

Figure 6.20 shows the relationship between the crack opening rate, COR, [µm/day] and the applied load level \((F_a / F_{cr})\) for the two pre-crack widths investigated (i.e. 0.3 and 0.5 mm). The crack opening rate was determined from Eq. 6.3 at 7, 15, 30 and 60 days:

\[
COR_{t_2-t_1} = \frac{w_{lt}^{t_2} - w_{lt}^{t_1}}{t_2 - t_1}
\]

(6.3)

where \(COR_{t_2-t_1}\) is the crack opening rate between time \(t_2\) and \(t_1\), and \(w_{lt}\) is the long-term crack opening at time \(t_i\). For both \(w_{cr}=0.3\) and 0.5 mm series, at 7 and 15 days, the COR increased with the load level. This increase of COR was more significant in the \(w_{cr}=0.5\) mm series and for \(F_a / F_{cr}\) ratios higher than 80%. Considering the COR evolution with \(F_a / F_{cr}\) in the first two weeks of loading, it was observed for \(w_{cr}=0.3\) mm a linear trend, while for \(w_{cr}=0.5\) mm it was detected a nonlinear increase of the COR with \(F_a / F_{cr}\). In the case of \(F_a / F_{cr} = 100\%\), COR was in average 50% higher for the larger pre-cracking width, i.e. \(w_{cr}=0.5\) mm. The COR at 30 and 60 days’ time was not significantly influenced by the \(F_a / F_{cr}\) ratio since, in generally, after a period of one month the crack opening width tended to become stabilized.

![Graphs showing crack opening rate as a function of \(F_a / F_{cr}\).](image)

*Figure 6.16: Crack opening rate as a function of \(F_a / F_{cr}\): (a) specimens pre-cracked up to 0.3 mm, \(w_{cr}=0.3\) mm, (b) specimens pre-cracked up to 0.5 mm, \(w_{cr}=0.5\) mm.*

To consider the long-term crack opening and also to take into account the loading levels of the specimens, the creep coefficient parameter is introduced. This parameter was
considered for two distinct stages, namely: only in the creep stage \((\phi^C)\) and at the origin \((\phi^O)\), as being calculated by the following equations (EN-1992-1-1, 2004):

\[
\phi^C = \frac{W_{\text{lt}}}{W_{\text{inst}}}
\]

\[
\phi^O = \frac{W_{\text{lt}}}{W_{\text{inst}} + W_{\text{cr}}^{\text{res}}}
\]

where, \(W_{\text{inst}}\) is instantaneous crack opening at the time of loading, \(W_{\text{lt}}\) represents long-term crack opening and \(W_{\text{cr}}^{\text{res}}\) is the residual crack opening after unloading specimen in cracking test (Figure 6.6). Figure 6.17 and Figure 6.18 illustrate the variation of the creep coefficients in the creep stage \((\phi^C)\) and origin \((\phi^O)\), respectively, for different ratios of \(F_a/F_{cr}\). In both figures, as it was expected, curves followed upward trends, which mean these coefficients increased when beams were loaded with a higher \(F_a/F_{cr}\) ratio. Figure 6.18 shows a lower scattering in the results since in the calculation of \(\phi^O\), the residual crack opening after unloading specimen in cracking process, \(W_{cr}^{\text{res}}\), was also took into the consideration. In general, after unloading specimens in cracking test, the recovery of the crack opening was somewhat different in various specimens. Therefore, the long-term crack opening of these specimens could be slightly affected.

Considering the influence of pre-crack width, the creep coefficient in \(w_{cr} = 0.5\) mm series has increased with the \(F_a/F_{cr}\) at a higher rate than in the case of the series of \(w_{cr} = 0.3\) mm. By considering the micromechanical behaviour of a single fibre, the \(w_{cr} = 0.5\) series was submitted to higher damage level of fibre/matrix interface, which led to a higher increase of the crack opening width under a creep load. This influence was more meaningful for a load level of 100%. From the monotonic tests’ results, see Figure 6.9, it can be observed that the pre-cracking level of 0.5 mm was close to the CTOD correspondent to the maximum load at post-cracking branch, \(w_{p}^{\text{max}}\). Therefore, subjecting beams to a sustained load in a level near to the maximum bearing capacity load of the specimen at post-cracking branch, \(F_{p}^{\text{max}}\), can lead to a significant increase in the values of the creep coefficient. This was observed in the \(w_{cr} = 0.5\) mm series at the higher load level.
Figure 6.17: Relationship between creep coefficient in creep stage and $F_a/F_{cr}$: (a) $w_{cr}=0.3$ mm, (b) $w_{cr}=0.5$ mm.

Figure 6.18: Relationship between creep coefficient at the origin and $F_a/F_{cr}$: (a) $w_{cr}=0.3$ mm, (b) $w_{cr}=0.5$ mm.

For the simplicity sake, it was decided to categorize the $F_a/F_{cr}$ ratios into two distinct intervals [50-75]% and [75-100%] with an average values of 62.5 and 87.5%, designated as low and high grade $F_a/F_{cr}$ ratios, respectively. Figure 6.19 depicts the relationship between creep coefficient and time for the studied $w_{cr}$ and $F_a/F_{cr}$ ratios, up to a period of two months. The creep curves were obtained by averaging the responses in each correspondent category. All series showed a two-stage creep response, namely, the so-called primary and secondary stages. The creep coefficient became stabilized within the studied time period, considering the criterion for stabilized creep, already indicated. During the execution of the test, none of the beams entered into the tertiary creep stage, in
which a specimen fails due to creep. In general, the creep coefficient was approximately 48% and 64% higher for the high load level, respectively, regarding \( w_{cr} = 0.3 \) and 0.5 mm series. On the other hand, the influence of the pre-crack width on the creep coefficient was smaller. Series with a \( w_{cr} = 0.5 \) mm, loaded with a high \( F_a / F_{cr} \) ratio had 30% higher creep coefficient than the one of the \( w_{cr} = 0.3 \) mm series for the same level of loading. As explained earlier, this would be feasible since \( w_{cr} = 0.5 \) mm was close to \( w_p^{\text{max}} \), in particular for the specimens with a load level of \( F_a / F_{cr} = 100\% \). It is noteworthy that even in this case, a stable creep response was achieved.

![Figure 6.19](image)

*Figure 6.19:* Creep coefficient versus time in the creep tests for the two pre-crack width levels grouped in low and high \( F_a / F_{cr} \) ratio.

### 6.3.2.2 Creep in compression

The basic creep deformations were determined using Eurocode (EN-1992-1-1, 2004) considering the loading levels in Figure 6.19. Because the fibre volume content in this research does not exceed 1%, the creep behaviour in compression of FRC is similar to the plain concrete (Weiss et al., 1999). According to Eurocode (EN-1992-1-1, 2004), the creep deformation of concrete \( \varepsilon_{cc} (t, t_0) \) at time \( t = t_0 \) for a constant compressive stress \( \sigma_c \) at the concrete age \( t_0 \), is given by:

\[
\varepsilon_{cc} (t, t_0) = \varphi (t, t_0) \left( \frac{\sigma_c}{E_c} \right) \quad (6.3)
\]
where $\varphi(t, t_0)$ is the creep coefficient at time $t$, $t_0$ is the age of concrete at time of loading and $E_t$ represents the tangent modulus. The creep coefficient $\varphi(t, t_0)$ is determined by:

$$\varphi(t, t_0) = \varphi_0 \beta_c(t, t_0)$$  \hspace{1cm} (6.4)

$$\varphi_0 = \varphi_{RH} \beta(f_{cm}) \beta(t_0)$$  \hspace{1cm} (6.5)

In Eq. 6.5, $\varphi_{RH}$, $\beta(f_{cm})$ and $\beta(t_0)$ are factors to allow for the effects of the relative humidity (Eq. 6.6), compressive stress (Eq. 6.7) and concrete age at loading (Eq. 6.8), respectively.

$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1} \frac{h_0}{100}ight] \alpha_2$$  \hspace{1cm} (6.6)

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$$  \hspace{1cm} (6.7)

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.2})}$$  \hspace{1cm} (6.8)

whereas $RH$ is relative humidity of the ambient environment, $h_0$ shows the notional size of the member ($h_0 = 2A_c/u$, $A_c$ and $u$ are the cross section and perimeter of the member), $f_{cm}$ depicts compressive strength of concrete. Moreover, $\beta_c(t, t_0)$ is a coefficient to describe the development of creep with time after loading which is determined as follow:

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{\beta_H + t - t_0}\right]^{0.3}$$  \hspace{1cm} (6.9)

$$\beta_H = 1.5 \left[1 + (0.012RH)^{18}\right] h_0 + 250 \alpha_3$$  \hspace{1cm} (6.10)

$$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7}, \quad \alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2}, \quad \alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5}$$  \hspace{1cm} (6.11)

For this purpose, the compressive stresses, $\sigma_c$, were assessed from finite element numerical simulations in Section 6.3.1.3 of the four-point bending tests. In the case of $w_{cr} = 0.3$ and 0.5 mm, at the end of two months, the basic creep strains ranged between
4.22 \times 10^{-5} to 5.91 \times 10^{-5} and 7.29 \times 10^{-5} to 9.45 \times 10^{-5}, respectively, depending on the applied load level. However, for the specimens with the proposed dimensions, the influence of the basic creep was neglected due to the following reasons:

1. When a specimen with the proposed dimensions is subjected to the pre-cracking four-point bending test, a critical crack is formed at the notch tip, and propagates upwards. Afterward, the two almost rigid halves of the specimen start to rotate around the hinge, which is formed at the top of the specimen and lead to the opening of the crack. In both pre-crack levels studied, when CTOD reached 0.3 or 0.5 mm, a major part of the section above the notch was cracked. On the other hand, the location of the neutral axis was moved very close to the top of the specimen and thus, remaining a small part of the section subjected to the compressive stress. Figure 6.20 depicts the crack pattern of a specimen under four-point loading when the CTOD reached to 0.3 and 0.5 mm. This figure only includes the results of $\beta=[0-15^\circ]$ series, since this series showed the highest load value due to the highest number of effective fibres. From this figure, it is clearly shown that, approximately, all the section is cracked. This conclusion was supported experimentally and numerically.

![Image](a)

![Image](b)

![Image](c)

![Image](d)

*Figure 6.20:* Experimental and numerical crack patterns in the loading span of monotonic four-point bending test simulations for the $\beta=[0-15^\circ]$ series at CTOD of: (a), (b) 0.3 mm and (c), (d) 0.5 mm.
2. The selected loads in the creep tests produced a very low compressive stresses in this uncracked zone and by considering the relatively high concrete compressive strength, therefore a very low basic creep deformation is expected. However, according to the numerical simulation, the compressive stress/strength ratios were obtained as follow: 4.53 to 6.35% for $w_{cr}=0.3$ mm and 7.22 to 10.11% for $w_{cr}=0.5$ mm in the loading levels low to high, respectively.

3. It is worth noting that performing all creep tests took more than one and half years and the first series of the specimens were subjected to the creep test two months after casting panel. According to Eurocode (EN-1992-1-1, 2004), development of the creep coefficient under a constant compressive stress is significant in the early age of concrete. On the other hand, after 50 days of casting, and for the proposed type of concrete, development of this coefficient is insignificant.

Since for the specimens with proposed dimensions, the influence of basic creep was marginal, consequently, hereinafter, the long-term widening of the crack refers to the time-dependent sliding action between fibre and surrounding matrix.

6.3.2.3 Influence of distance from the casting point on the creep parameters

Figure 6.21 reveals the instantaneous crack opening at the time of loading in creep frame ($w_{inst}$) versus distance from the centre of the panel (casting point) for the low and high $F_a/F_{cr}$ series. It was shown that by getting farther from the casting point, $w_{inst}$ has a tendency to increase. This could be ascribed to the distribution of the fibre as well as quality of matrix by getting distance from the centre. Figure 6.22 presents the effects of the distance from the casting point on the long-term residual crack opening ($w_{res}^{ret}$) after unloading and creep recovery stage. Similar to the previous figure, $w_{res}^{ret}$ followed an ascending pattern predominantly when $F_a/F_{cr}$ was high. Considering the influence of $w_{cr}$ on $w_{res}^{ret}$, as it was expected, series with $w_{cr}=0.5$ mm revealed a higher value in the long-term residual crack opening than $w_{cr}=0.3$ mm specimens.
Figure 6.21: Instantaneous crack opening, $w_{\text{inst}}$, vs. distance from the casting point: $F_a / F_{cr} = (a)$ low and (b) high.

Figure 6.22: Long-term residual crack opening, $w_{\text{res}}$, vs. distance from the casting point: $F_a / F_{cr} = (a)$ low and (b) high.

Figure 6.23 shows the influence of the specimens’ distance to the casting point on the creep coefficient versus time relationship for specimens with $F_a / F_{cr} = 100\%$. It was opted to only present these relationships for the maximum $F_a / F_{cr}$, since they were considered as the most critical ones. Specimens positioned near the centre of the panel (i.e. casting point) presented a lower creep coefficient, while those located in the corner of the panel had the highest creep coefficients. This aspect could be ascribed to the decrease of the concrete flow velocity with the increase of the distance from the casting point. This decrease of the flow velocity influences both the fibre dispersion and orientation along the flow profile, and, consequently, leads to quite different fibre structures within the
specimens at distinct distances from the casting point. From another point of view, this observation could be also attributed to a decrease of the matrix strength with the increase of the distance from the casting point due to some segregation of the aggregate skeleton, resulting a weaker fibre/matrix interfacial bond strength that decreases the fibre reinforcement effectiveness.

![Figure 6.23: Creep coefficient versus time for specimens at different distances from casting point in the series of: (a) \( w_{cr} = 0.3 \) mm and (b) \( w_{cr} = 0.5 \) mm.](image)

6.3.2.4 Influence of notch plane orientation on the creep parameters

Figure 6.24 presents the influence of the notched plane orientation (regarded to the flow direction of SFRSCC) on the relationship of creep coefficient versus time. These relationships were determined by averaging the response of the specimens with \( F_a / F_{cr} = 100\% \). For the \( w_{cr} = 0.3 \) mm series, the variation of creep with time was slightly affected by the direction of the notched plane regarding the expected concrete flow, while in the \( w_{cr} = 0.5 \) mm series this influence was more significant. In the case of the \( \beta = [0-15^\circ] \) series, the creep coefficient at the end of two months was 13 and 31% higher than in \( \beta = [75-90^\circ] \) for the \( w_{cr} = 0.3 \) and 0.5 mm series, respectively. It is worth noting that the \( \beta = [0-15^\circ] \) specimens contain effective fibres perpendicular to the crack plane, which were pulled-out under a sustain load progressively, while the \( \beta = [75-90^\circ] \) specimens have fibres with a higher orientation angle towards the notch plane. In this case, the fibre reinforcement mechanism of the specimen was mainly governed by matrix spalling at the fibres’ exit points instead of fibres pulled-out. Therefore, the development of creep
coefficient along time was influenced by the orientation of the crack plane within the SFRSCC panel, and this influence seems as larger as higher is the crack width.

Figure 6.24: Creep coefficient versus time for different orientation of the notched plane in the series of: \( w_{cr} \) = (a) 0.3 mm and (b) 0.5 mm.

6.3.3 Comparison between monotonic and long-term results

6.3.3.1 Influence of long-term residual crack opening on flexural residual strengths

Figure 6.25 and Figure 6.26 summarize the comparison between the long-term assembled curves (Figure 6.8) obtained in four-point bending tests and the correspondent ones determined in the monotonic tests, for \( w_{cr} = 0.3 \) and 0.5 mm, respectively. These curves were obtained by averaging all responses. Regarding the monotonic tests, the experimental envelope was also included. The long-term assembled curves consist of overlaying the force-CTOD curves from the tests corresponding to: specimen’s pre-cracking, creep, and post-creep. Moreover, each \( w_{cr} \) series was divided into eight subcategories, according to the \( \beta \) orientation factor and the \( F_a/F_{cr} \) ratio. The results of the series \( w_{cr}=0.3 \) mm with a low \( F_a/F_{cr} \) and \( \beta=[75-90^\circ] \) are not presented (Figure 6.25(g)), due to technical problems during the execution of the tests of this series. From the analysis of these results, in general, it was concluded that the crack growth during the creep tests has a minor influence on the post-creep flexural behaviour. In fact, it is evident that the assembled curves resemble quite well the average response from the monotonic
tests. Nevertheless, in some cases, due to the scattering in the results, as consequence of distinct fibre distributions, the assembled responses did not follow so closely the average monotonic curves, but nonetheless, they were yet comprised within the experimental envelope of the monotonic flexural tests.
Figure 6.25: Comparison of the monotonic and long-term assembled curves for \( w_{cr} = 0.3 \) mm: (a), (b) \( \beta = [0-15^\circ] \); (c), (d) \( \beta = [15-45^\circ] \); (e), (f) \( \beta = [45-75^\circ] \); (g), (h) \( \beta = [75-90^\circ] \). (a), (c), (e), (g) 50% \( \leq F_a / F_{cr} \leq 75\% \) and (b), (d), (f), (h) 75% \( \leq F_a / F_{cr} \leq 100\% \).
Figure 6.26: Comparison of the monotonic and long-term assembled curves for $w_{cr}=0.5$ mm: (a), (b) $\beta = [0-15^\circ]$; (c), (d) $\beta = [15-45^\circ]$; (e), (f) $\beta = [45-75^\circ]$; (g), (h) $\beta = [75-90^\circ]$. (a), (c), (e), (g) $50\% \leq F_a/F_{cr} \leq 75\%$ and (b), (d), (f), (h) $75\% \leq F_a/F_{cr} \leq 100\%$.

6.3.3.2 Influence of long-term residual crack opening on secondary stiffness

Figure 6.27(a) illustrates the relationship between long-term residual crack opening width, $w_{rel}^{\infty}$ (Figure 6.6), and secondary stiffness in the re-loading of post-creep test, $K_{sec}$ (Figure 6.7). It is observed that the stiffness of the re-loading branch of the post-creep curve decreases with the increase of the long-term residual crack width. Furthermore, the following equation was also proposed to estimate the influence of $w_{rel}^{\infty}$ on the $K_{sec}$:

$$k_{sec} = k_0 \left( \frac{c_1}{w_{rel}^{\infty} c_2} \right)$$  \hspace{1cm} (6.12)
where $K_0$ is the initial stiffness determined by averaging the values obtained from force-CTOD relationships in Figure 6.9 ($K_0=565.53$ kN/mm with a CoV of 12%), and constants $c1$ and $c2$ were achieved by nonlinear curve fitting analysis procedures, having been obtained the following values: $c1=0.012$ and $c2=0.480$. Figure 6.27(a) shows that Eq. 6.16 fits with reasonable accuracy the obtained experimental results (R-square equal to 0.71). Figure 6.27(b) depicts the relationship between secondary stiffness, $K_{sec}$, and the ratio of the applied load for each series individually. From the results it was observable that, for both series, with the increase in the level of the applied load, the secondary stiffness tended to reduce. It could happen since, as it was explained earlier, when the level of the applied load increased, higher creep coefficient and, therefore, higher long-term residual crack opening width was achieved.

![Graph](image1)

**Figure 6.27:** Relationship between secondary stiffness, $K_{sec}$, and: (a) long-term residual crack opening width, $w_{cr}^{x'}$, (b) ratio of the applied load, $F_a / F_{cr}$.

In order to evaluate if the flexural residual strengths were affected by the long-term residual crack opening in the creep test, a comparison was also performed based on the influence of secondary stiffness, $K_{sec}$. Although this method was based on the some assumptions and simplifications, but it could render a more clear vision regarding to the effects of the long-term crack opening on the residual flexural strengths. In the first stage, for specimens located near to the upper or lower limits of the experimental envelope curves in each series of Figure 6.9, a preliminary analysis was executed to consider if there is a high variation in the value of $K_{sec}$, but the differences were negligible. Then, the
procedure was executed as follow: firstly, for the simplicity sake, the residual crack opening at time of unloading ($w_{cr}^{res}$) was determined by averaging this parameter from the cyclic tests for each $w_{cr}$ (0.3 or 0.5 mm), separately. Afterwards, from the obtained average $w_{cr}^{res}$, lines were drew with gradients equal to $K_{sec}$ obtained from the re-loading branch of cyclic test (Figure 6.3) or post-creep tests (Figure 6.7) until they intersected the average $f_r$ - CTOD curves. Finally, by a comparison between the intersection points achieved from the cyclic or long-term lines with $f_r$ - CTOD average curve, it could be concluded if residual strength at CTODs of 0.3 or 0.5 mm was shifted due to the long-term crack opening.
Figure 6.28: Influence of long-term residual crack opening on the secondary stiffness: (a), (b) $\beta = [0-15^\circ]$; (c), (d) $\beta = [15-45^\circ]$; (e), (f) $\beta = [45-75^\circ]$; (g), (h) $\beta = [75-90^\circ]$. (a), (c), (e), (g) $w_{cr}=0.3$ mm, and (b), (d), (f), (h) $w_{cr}=0.5$ mm.

Through Figure 6.28(a) to (h) are depict the influence of the long-term residual crack opening on the secondary stiffness. In this figure, $f_c$ was determined by Eq. 6.2. To compare results more easily, each curve was magnified up to a CTOD of 1.5 mm. However, the original curve was also presented in the corner till a CTOD of 4 mm. In each figure, the black line presented the average re-loading response obtained from cyclic test whereas the dark and light lines depict re-loading of specimens in the post-creep stage when they were subjected to low and high level of loading in creep test, respectively. As it was shown, for all series, the secondary stiffness and therefore residual strength at CTOD=0.3 mm was not influenced considerably even for the high level of loading. Therefore, for the serviceability limit state, where the crack width should be limited, the creep phenomenon was not concern. Regarding to the $w_{cr}=0.5$ mm specimens with $\beta = [0-15^\circ]$ and $[75-90^\circ]$, a reduction of 24 to 85% and 15 to 102% in $K_{sec}$ was registered if
specimens loaded in low to high levels, respectively, which could also lead to a destructive influence in the residual strength, $f_{r1}$. Therefore, in the case of $\beta = [0-15^\circ]$ and $[15-45^\circ]$ series, when the specimens were loaded in the creep test with a high level of $F_a/F_{cr}$, $f_{r1}$ was shifted from the deflection-hardening stage into the softening phase due to the long-term residual crack opening. Considering $\beta = [45-75^\circ]$ and $[75-90^\circ]$ series, the same conclusion can also be made since $f_{r1}$ was shifted from the plateau part to the softening stage of the curves.

6.4 Analytical approach to predict creep behaviour of cracked SFRSCC

In this section, the experimental results presented in section 6.3.2 were used to propose an equation to predict the long-term response of the cracked SFRSCC. The influence of $w_{cr}$ and $F_a/F_{cr}$ parameters were taken into account in the present approach. A combined power and hyperbolic equation was used, since similar equations were already proposed by ACI-Committee-209 (1997) and CEB-FIP (2010) to predict creep behaviour of plain concrete. Therefore, the simple following equation is proposed for predicting the long-term behaviour of cracked SFRSCC:

$$\phi^c = \frac{t^A}{b + t^d}$$  \hspace{1cm} (6.13)

where $\phi^c$ is creep coefficient, and $t$ represents the time duration of loading (in hours). According to the experimental data, for each $w_{cr}$, the coefficient $A$ is determined by:

$$A = w_{cr} \left(1 - 1/(2F)\right) + d$$  \hspace{1cm} (6.14)

where $w_{cr}$ is the pre-crack width (in mm) and $F$ represents level of loading ($F_a/F_{cr}$). In Eqs. 6.13 and 6.14 the constants $b$ and $d$ were determined by nonlinear curve fitting analysis procedures, and the following values were obtained: $b=15$ (R-square value of 0.94) and $d=0.17$ (R-square value of 0.88). It should be mentioned that Eq. 6.13 is only valid for $F_a/F_{cr} > 0.5$. Furthermore, the functions used may capture the shape of the phenomenon, but not quantify its physical aspects. Figure 6.29 compares the creep coefficient versus time obtained analytically and experimentally for each series of
different $w_{cr}$ and loading level, where it was be concluded that the proposed equation predicts with high accuracy the registered experimental data.

Figure 6.29: Comparison between experimental and analytical creep coefficient vs. time relationship for series: (a) $w_{cr}=0.3$ mm and (b) $w_{cr}=0.5$ mm.

Figure 6.30(a) to (d) depict the results of a parametric study to investigate the influence of pre-crack width ($w_{cr} = 0.3$ and 0.5 mm) and also the load level ($F_a/F_{cr} = 0.5$, 0.6, 0.7, 0.8, 0.9 and 1.0) on the creep coefficient, $\phi'$, versus time response up to a period of one year. However, it should be mentioned that to properly validate the proposed equation for more generalized pre-crack widths, load levels, and higher time periods, more experimental data is required. In general, the creep coefficient increased with the increase of the $F_a/F_{cr}$ ratio, and this increase was more significant for higher values of $w_{cr}$. Considering now the influence of $w_{cr}$ on creep coefficient, for the same load level it was observed that the increase of $w_{cr}$ lead to a significant increase of creep coefficient, mainly for the higher load levels.

After determination of creep coefficient from Eq. 6.13, in order to determine the long-term crack opening analytically, $w_{lt}$, Eq. 6.4 could be arranged as follows: $w_{lt} = \phi' \times w_{inst}$. Figure 6.31 depicts the results of a parametric study to investigate the influence of pre-crack width ($w_{cr} = 0.3$ and 0.5 mm) and also the load level ($F_a/F_{cr} = 0.5$, 0.6, 0.7, 0.8, 0.9 and 1.0) on the long-term crack opening, $w_{lt}$, versus time response up to a period of one year. In this study, since analytical determination of $w_{inst}$ needs more experimental results, this parameter was determined using experimental values for each
studied \( w_{cr} \) and \( F_a/F_{cr} \). Similar to the previous figure, the long-term crack opening increased with the increase of the \( F_a/F_{cr} \) ratio, which this was more predominant for higher values of \( w_{cr} \).

![Graphs showing creep coefficient vs. time response for different values of \( w_{cr} \).](image)

*Figure 6.30:* Influence of \( F_a/F_{cr} \) load level on the creep coefficient vs. time response for \( w_{cr} \) equal to: (a) 0.1 mm, (b) 0.2 mm, (c) 0.3 mm and (d) 0.5 mm.
Figure 6.31: Influence of $F_a/F_cr$ load level on the long-term crack opening vs. time response for $w_{cr}$ equal to: (a) 0.3 mm and (b) 0.5 mm.

6.5 Conclusions

The present work reported the results of an experimental program to investigate the long-term behaviour of pre-cracked SFRSCC laminar structures (of relatively small thickness). One hundred and twelve prismatic specimens were extracted from a SFRSCC panel. These specimens were notched with different orientations regarding to the expected SFRSCC flow direction, and were submitted to four-point flexural tests under a sustained load. The influence of the following parameters on the creep behaviour was studied: initial crack opening level (0.3 mm and 0.5 mm), applied stress level, fibre orientation/dispersion, and distance from the casting point. Moreover, to evaluate the effect of long-term residual crack opening on the flexural post-cracking strength as well as secondary stiffness, a series of instantaneous monotonic and cyclic tests were carried out and the corresponding force vs crack mouth opening displacement ($F$-CTOD) curves were compared to the ones obtained by assembling the $F$-CTOD curves determined in pre-crack monotonic tests, creep tests and post-creep monotonic tests.

From the monotonic tests, it was concluded that the post-cracking flexural tensile behaviour of the adopted SFRSCC was considerably influenced by both the fibre dispersion and orientation. It is worthy to note that specimens with notched plane parallel to concrete flow direction have shown the highest post-cracking strength. This was a
direct consequence of a preferential fibre orientation perpendicular to those fracture planes, due to the concrete flow profile. Therefore, when a SFRSCC panel is cast from its centre, fibres tend to be aligned perpendicular to flow direction.

Concerning the long-term creep tests, two pre-cracking levels \( (w_{cr}) \) were considered. Stable responses were observed for all specimens. However, as it was expected, by increasing the level of the applied load, higher values of the creep coefficient were achieved. Regarding the influence of the pre-cracking levels, \( w_{cr}=0.5 \) mm series conducted to higher values of the creep coefficient than in the other studied series \( (w_{cr}=0.3 \) mm), especially, if they were loaded with a high \( F_a/F_{cr} \). On the other hand, since \( w_{cr}=0.5 \) mm was very close to the CTOD correspondent to the maximum load at post-cracking branch of the monotonic responses, the bond interface between fibre and matrix was more damaged, therefore the creep crack width increased with a higher rate. However, even in this case, still a stable response was obtained, although requiring a higher time period for this stabilization.

Specimens located nearer to the panel’s corner showed a higher increase of creep coefficient with time. This aspect could be ascribed to the decrease of the concrete flow velocity with the increase of the distance from the casting point which leads to different fibre distribution as well as matrix constituents through the panel.

The creep tests also revealed that the SFRSCC was influenced by the orientation of the notch plane regarding the expected concrete flow (defined by the \( \beta \) angle). In fact, \( \beta=[0-15^\circ] \) specimens presented the highest creep coefficients, whereas \( \beta=[75-90^\circ] \) series showed the lowest ones. This fact was a direct consequence of the fibre orientation within the panel.

In general, the post-cracking strength after the long-term loading was not influenced significantly when compared to the one obtained from instantaneous monotonic tests, even when the specimens were pre-cracked up to close the CTOD corresponding to the maximum load at post-cracking branch, and when loaded with the higher \( F_a/F_{cr} \) ratios.

Based on the results obtained from the creep tests, an equation was proposed to predict the creep coefficient for the developed SFRSCC when cracked up to 0.5 mm and loaded in the interval \( 0.5 < F_a/F_{cr} \leq 1.0 \).
CHAPTER 7

Modelling Instantaneous and Long – Term Flexural Behaviour of SFRSCC with Short Discrete Embedded Fibres

7.1 Introduction

In the last decades, the development of steel fibre reinforced self-compacting concrete (SFRSCC) technology for structural applications has been pushed forward by increasingly higher industry requirements. The adoption of this material, can partially or even fully exclude the use of conventional steel reinforcement, since steel fibres are able to provide enhanced residual strength. Several researchers have shown that the incorporation of discrete fibres can improve both toughness and durability of concrete due to the crack width restraint (Barros et al., 2005; Zerbino and Barragan, 2012; Michels et al., 2013). However, the tensile performance of SFRSCC, and in particular the post-cracking strength, depends, among other factors, on how fibres are distributed and oriented in the matrix, since it will also contribute to the grade of fibre efficiency. In laminar structures, a proper knowledge and better understanding of the fibre distribution/orientation parameters can enable a better estimation of the material post-
cracking strength, and consequently, contribute to the material properties scatter, see Chapter 4. Therefore, in order to predict the mechanical behaviour of SFRC accurately, it is crucial to understand how fibres are distributed, as well as oriented in a composite. In the case of SFRSCC, fibre distribution is influenced, mainly, by the rheological properties in the fresh state of concrete (flowability), wall effects and casting procedure (Boulekbache et al., 2010; Ferrara et al., 2011). Therefore, having in the mind the aforementioned aspects and factors, which affect and contribute to the post-cracking response, assuming SFRSCC as anisotropic material could lead to an unrealistic estimation of the mechanical performance in a certain structural element.

In general, the stress-crack opening displacement relationship, $\sigma - w$, can be used to estimate the post-cracking response of low content fibre reinforced concrete. Several analytical micro-mechanical models are available in the literature to predict the tensile performance of steel fibre reinforced concrete, SFRC, assuming a random fibre distribution (Tanigawa and Hatanaka, 1983; Marti et al., 1999; Voo and Fooster, 2003; Lee et al., 2011). These models are principally based on averaging the contribution of the individual fibres transferring stresses across a crack plane. Lim et al. (1987) proposed an analytical model for the tensile behaviour of SFRC. In this model, they simulated the elastic behaviour from the un-cracked specimens while the post-peak response was modeled by pre-cracked samples. However, in this macro-scale based model, the action of all fibres bridging a crack was taking in to the account as whole without considering the behaviour of each single fibre. In the meso-scale based models, the bridging of macro-cracking is based on the distribution and orientation of the fibres as well as their individual pullout response obtained experimentally. Marti et al. (1999) and Voo and Fooster (2003) proposed a model for the tensile behaviour of the SFRC according to the fibre pull-out response. However, in these models they assumed an isotropic random fibre distribution without considering orientation of fibres. However, in the other studies, the orientation of fibres was also taken in to the consideration by executing fibre pull-out test on the fibres with various inclinations (Tanigawa and Hatanaka, 1983; Maalej et al., 1995; Lee et al., 2011). In these models, a fibre orientation profile was determined and according to their angle with a crack plane, a specific pullout response was assigned. However, as it was concluded in Chapter 3, in the case of SFRSCC laminar structures, assuming a two-dimensional fibre distribution, $2/\pi$, (KamerwaraRao, 1979) or a three-dimensional, 0.5, (Stroeven and Hu, 2006) isotropic uniform random fibre distribution is
far apart from the reality. In addition, the post-cracking behaviour of the composite is also strongly influenced by the micro-mechanical properties of a single fibre. In fact, the knowledge of the bond-slip behaviour between fibre/matrix conjugated with an accurate fibre distribution can render a good estimation of the composite mechanical properties (Cunha et al., 2011). Therefore, SFRSCC can be assumed as a two-phase material, namely, the plain concrete phase (aggregate and paste) and the fibre phase. The first one is considered as a homogeneous phase, while the latter one comprises the information regarding to the fibre density and orientation considering the influence of concrete flowability and also wall effects. It is expected that this approach could enhance the accuracy and reliability of the predicted material behaviour laws comparing to other methods.

In this chapter, firstly, a numerical approach for modeling SFRSCC in laminar structural elements is presented. SFRSCC was assumed as a two-phase material comprising one homogeneous phase representative of aggregates and paste, and a second phase that includes all the discrete fibres. The random fibre distribution was obtained with an algorithm based on the Monte Carlo method (Cunha, 2010), which provided a realistic distribution of fibres through the bulk. The unreinforced concrete phase was modeled using solid finite elements, while fibres were modeled using 3D embedded elements. The geometry, position and orientation of the fibres were inserted into the solid finite element mesh to generate a heterogeneous mesh. A 3D multi-fixed smeared crack model was used to simulate the fracture process of the cementitious matrix. Furthermore, in order to simulate the fibre/matrix interface properties, a nonlinear behaviour was assigned to fibres. This interface behaviour laws were obtained from fibre pull-out tests with distinct fibre’s inclination angles. Then, in the second part, an analytical approach was developed based on analyzing the cracked cross section in order to determine flexural long-term behaviour of the specimens considering long-term fibre/matrix interface properties.
7.2 Fibre structure modelling

7.2.1 Algorithm

In order to simulate the distribution of fibres in the SFRSCC some considerations and simplifications were made which are: the fibre have a null thickness and assumed as one dimensional element, the shape of the end-hooks was also dissembled. Furthermore, it was assumed that the fibres could overlap each other. In this numerical simulation, the fibre structure which represented the distribution of the fibres in the bulk concrete was randomly generated implementing a Monte Carlo procedure. This algorithm is summarized as follow (Cunha, 2010):

1. The initial parameters were defined. The content and geometrical data of fibres, the geometry of specimen as well as its dimensions were inputted and finally type of the distribution and correlated parameters were selected.
2. Initializing the random number generation engine using Mersenne-Twister (MT) procedure which generates seeds randomly.
3. The volume of specimen, $V_s$, was determined, then the total number of fibres contained in the specimen was computed from the following well known equation:

$$N_{f^\text{vol}} = V_s \frac{C_f}{m_f}$$  \hspace{1cm} (7.1)

where, $C_f$ is the fibre content and $m_f$ shows mass weight of a single fibre. Hereinafter, considering also the next steps, this algorithm was repeated until $N_{f^\text{vol}}$ was attained.
4. The fibre localization coordinates as well as its orientation were generated randomly. Depends on the geometry of the specimens, the coordinates were created in either Cartesian or spherical coordinate system for prismatic or cylindrical specimen, respectively. The fibre orientation could be distributed by selecting the available statistical distribution functions. From previous experimental work, it was observed that the Gaussian distribution provided an accurate prediction for the fibre orientation probability in laminar specimens. Therefore, in the numerical modeling, this distribution function was adopted to generate the orientations of the fibres with the follow procedure:
4.1 A set of the three uniform random numbers, \((\xi_x, \xi_y, \xi_z)\), were generated and the centre of gravity coordinates were determined, \((x_i, y_i, z_i)\), for the \(i^{th}\) fibre:

\[
\begin{align*}
    x_i &= \xi_x b_{rc} \\
    y_i &= \xi_y l_{rc} \\
    z_i &= \xi_z h_{rc}
\end{align*}
\] (7.2)

where, \(b_{rc}\), \(l_{rc}\) and \(h_{rc}\) are width, length and height of the prismatic specimen.

4.2 According to the specified values of mean and standard deviation provided for Gaussian distribution in step (1), the cumulative frequency Gaussian function will be solved using Newton-Raphson method in which the fibre orientation was set as independent variable, \(x = \cos \theta_i\).

4.3 The fibre orientation versor \(\hat{n}\) was computed by randomly generating the first two components and determining the other component by Eq. 7.3:

\[
\cos(\theta_i) = \hat{\mu} \otimes \hat{\nu}
\] (7.3)

which, \(\cos(\theta_i)\) is fibre orientation towards the selected studied plane based on the cumulative Gaussian distribution function calculated in the previous stage, and \(\hat{\nu}\) is the considered plane’s normal versor.

4.4 The end-nodes coordinates for the \(i^{th}\) fibre \((j = 1, 2)\) were computed by:

\[
\begin{align*}
    x_j^i &= x_i \pm l_j \mu_i / 2 \\
    y_j^i &= y_i \pm l_j \nu_i / 2 \\
    z_j^i &= z_i \pm l_j m_i / 2
\end{align*}
\] (7.4)

4.5 In this step, it is necessary to check if the position of the \(i^{th}\) fibre’s gravity centre \((x_i, y_i, z_i)\) was valid to be placed in the prismatic mould. In addition, the \(i^{th}\) fibre end-nodes coordinates must also convince the specimen’s boundary condition. Since in this work, the prismatic specimens were extracted from a panel, it should be noted that the wall effects were mainly limited to the top and bottom surfaces of the specimens.

4.6 In this step, those fibres which did not satisfy the following boundary conditions, see Eqs. 7.5, were considered as the violate ones. In fact, when beams were extracted from the slab, the protrusive parts of these fibres were cut, thus they will not be considered as active fibres in the specimen.
Therefore, since the violate fibres were not effective and could cause appearing some errors in the analysis procedure, for simplicity sake, they were completely eliminated, see Figure 7.1.

\[
0 \leq x_i^l \leq b_{rec} \\
0 \leq y_i^l \leq l_{rec}
\] (7.5)

4.7 Finally the total fibre’s end-nodes coordinates, the number of fibres intersecting a specified plane as well as their orientation towards the plane were all saved.

*Figure 7.1:* Distribution of the steel fibres in the specimen: (a) before and (b) after eliminating violate fibres, respectively.
7.2.2 Fibre distribution

When casting a panel from its centre, fibres tended to reorient and maintain preferentially perpendicular to the concrete flow direction. This can be justified due to the uniform velocity profile of concrete, which diffuses radially and outwards from the centre of panel as illustrated in Chapter 3. Thus, in order to have a realistic numerical fibre distribution, a Gaussian probability function based on the mean value and standard deviation of the fibre orientations (obtained from the image analysis results) was adopted. Figure 7.2 depicts the fibre structure mesh for the series $\beta=[0-15^\circ]$, $[15-45^\circ]$, $[45-75^\circ]$ and $[75-90^\circ]$. In this figure it is possible to visualize the preferential fibre alignment due to the influence of the concrete flow. In order to find out the accuracy level of this numerical fibre distribution, the latter distributions were compared with the results from image analysis, at a same cross section, in terms of the total number of fibres, $N_f$, and average fibre orientation factor, $\eta_o$, see Table 7.1. The real fibre distribution was accurately modeled with the aforementioned approach.
Figure 7.2: A methodology to distribute steel fibres in a prismatic specimen (the lateral face of the specimen coincides with the bottom surface of the SFRSCC panel in contact with the mould): $\beta =$ (a) [0-15°], (b) [15-45°], (c) [45-75°] and (d) [75-90°].

Table 7.1: Comparison between image analysis result and numerical fibre distribution.

<table>
<thead>
<tr>
<th>Series</th>
<th>$N_f^f$ [fibre]</th>
<th>$\eta_0$ [-]</th>
<th>$N_f^f$ [fibre]</th>
<th>$\eta_0$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0-15°]</td>
<td>76</td>
<td>0.875</td>
<td>80</td>
<td>0.872</td>
</tr>
<tr>
<td>[15-45°]</td>
<td>70</td>
<td>0.807</td>
<td>76</td>
<td>0.807</td>
</tr>
<tr>
<td>[45-75°]</td>
<td>56</td>
<td>0.730</td>
<td>52</td>
<td>0.735</td>
</tr>
<tr>
<td>[75-90°]</td>
<td>31</td>
<td>0.672</td>
<td>38</td>
<td>0.687</td>
</tr>
</tbody>
</table>
7.3 Determination of instantaneous flexural behaviour by a finite element model

7.3.1 Concrete material model

In this numerical approach, the nonlinear behaviour of concrete was modeled with a 3D multi-fixed smeared crack model available in the FEM-based computer program, FEMIX (Sena-Cruz, 2005; Ventura-Gouveia, 2008). In the present case, it was assumed that in each integration point only one crack could be formed. To solve the nonlinear equation system, an incremental iterative procedure was adopted. The relationship between the incremental strain and stress is defined as follow:

\[ \Delta \sigma = D \Delta \varepsilon \]  

(7.6)

in which \( \Delta \sigma \) is the stress vector increment, \( \Delta \varepsilon \) is the strain vector increment and \( D \) is the tangential constitutive matrix.

In a smeared approach, the total incremental strain can be decomposed in an incremental crack strain vector, \( \Delta \varepsilon^c \), as well as an incremental strain vector of corresponding to the uncracked concrete in between cracks, \( \Delta \varepsilon^{co} \) (Bazant, 1980; de Barst and Nauta, 1985).

\[ \Delta \varepsilon = \Delta \varepsilon^c + \Delta \varepsilon^{co} \]  

(7.7)

7.3.1.1 Crack strain and stress vectors

Figure 7.3 schematically presents a crack plane crossing a solid finite element. According to fracture mechanics, three distinct fracture modes could be considered, namely, Mode I-crack opening mode; Mode II- in-plane shear mode and Mode III- out-of-plane shear mode (Bazant and Planas, 1998; Shah et al., 1995). In the three dimensional case, modes II and III can be regarded as shear modes perpendicular to the crack plane. To each fracture mode I, II and III corresponds, respectively, the crack opening displacement, \( w \); sliding displacement in \( \hat{t}_1 \) direction, \( s_1 \), and the sliding displacement in \( \hat{t}_2 \) direction, \( s_2 \). In this model, \( w \) was correlated with a crack normal strain using crack band width \( l_b \) (
\(\varepsilon'_{n} = \text{w/l}_{b}\) whereas \(s_{1}\) and \(s_{2}\) sliding components were replaced with the crack shear strains \(\gamma_{s}^{1}\) and \(\gamma_{s}^{2}\), respectively. Therefore, the local and global incremental crack strain vectors, \(\Delta \varepsilon_{l}^{cr}\) and \(\Delta \varepsilon_{g}^{cr}\), respectively, are given by Eqs. 7.8 and 7.9:

\[
\Delta \varepsilon_{l}^{cr} = \left[ \Delta \varepsilon_{n}^{cr}, \Delta \gamma_{s}^{1}, \Delta \gamma_{s}^{2} \right]^{T}
\]  (7.8)

\[
\Delta \varepsilon_{g}^{cr} = \left[ \Delta \varepsilon_{1}^{cr}, \Delta \varepsilon_{2}^{cr}, \Delta \varepsilon_{3}^{cr}, \Delta \gamma_{23}^{cr}, \Delta \gamma_{31}^{cr}, \Delta \gamma_{12}^{cr} \right]^{T}
\]  (7.9)

Eq. 7.10 gives the relationship between \(\Delta \varepsilon_{l}^{cr}\) and \(\Delta \varepsilon_{g}^{cr}\):

\[
\Delta \varepsilon_{g}^{cr} = \left[ T^{cr} \right]^{T} \Delta \varepsilon_{l}^{cr}
\]  (7.10)

where \(T^{cr}\) is the transformation matrix determined by:

\[
T^{cr} = \begin{bmatrix}
    a_{11}^2 & a_{12}^2 & a_{13}^2 & 2a_{12}a_{13} & 2a_{11}a_{13} & 2a_{11}a_{12} \\
    a_{11}a_{21} & a_{12}a_{22} & a_{13}a_{23} & a_{12}a_{23} + a_{13}a_{22} & a_{11}a_{23} + a_{13}a_{21} & a_{11}a_{22} + a_{12}a_{21} \\
    a_{11}a_{31} & a_{12}a_{32} & a_{13}a_{33} & a_{13}a_{32} + a_{13}a_{33} & a_{11}a_{33} + a_{13}a_{31} & a_{12}a_{31} + a_{13}a_{32} \\
\end{bmatrix}
\]  (7.11)

The components \(a_{11}\), \(a_{12}\) and \(a_{13}\) form a vector that follows the direction of the \(n\) local axis; \(a_{21}\), \(a_{22}\) and \(a_{23}\) form a vector that defines the \(t_{1}\) local axis; \(a_{31}\), \(a_{32}\) and \(a_{33}\) form a vector that defines the \(t_{2}\) local axis. All these vectors are defined in the global coordinate system. Furthermore, the local incremental stress vector, \(\Delta \sigma_{l}^{cr}\), is defined as follows:

\[
\Delta \sigma_{l}^{cr} = \left[ \Delta \sigma_{n}^{cr}, \Delta \tau_{s}^{1}, \Delta \tau_{s}^{2} \right]^{T}
\]  (7.12)

in which \(\Delta \sigma_{n}^{cr}\) is the incremental crack normal stress, \(\Delta \tau_{s}^{1}\) and \(\Delta \tau_{s}^{2}\) are the incremental crack shear stresses along \(\hat{i}_{1}\) and \(\hat{i}_{2}\) directions, respectively.

The incremental stress components in the global coordinate system are:

\[
\Delta \sigma = \left[ \Delta \sigma_{1}, \Delta \sigma_{2}, \Delta \sigma_{3}, \Delta \tau_{23}, \Delta \tau_{31}, \Delta \tau_{12} \right]^{T}
\]  (7.13)

The relationship between \(\Delta \sigma_{n}^{cr}\) and \(\Delta \sigma\) is given by:

\[
\Delta \sigma_{l}^{cr} = T^{cr} \Delta \sigma
\]  (7.14)
7.3.1.2 Concrete constitutive law

In the smeared crack model, it was assumed that the concrete has a linear elastic behaviour before the tensile strength of material is reached. Thus, the relation between $\Delta \varepsilon^{co}$ and $\Delta \sigma$ can be established by:

$$\Delta \sigma = D^{co} \Delta \varepsilon^{co}$$

(7.15)

where $D^{co}$ is the elastic constitutive matrix of the un-cracked concrete bulk (Zienkiewicz and Taylor, 2000) which determined by:

$$D^{co} = \frac{E}{(1+\nu)+(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

(7.16)

In this equation, $E$ is the Young's modulus and $\nu$ states the Poisson’s ratio of the undamaged concrete. When the tensile strength of material is reached, the concrete is shifted to the cracked stage, thus the crack opening and shear sliding behaviour can be determined by:
with the crack constitutive matrix, $D^{cr}$, combining the uncoupled Modes I, II, III fracture stiffness modulus:

$$D^{cr} = \begin{bmatrix} D_{h}^{cr} & 0 & 0 \\ 0 & D_{t}^{cr} & 0 \\ 0 & 0 & D_{z}^{cr} \end{bmatrix}$$  \hspace{1cm} (7.18)$$

Combining Eqs. 7.6 to 7.18 the following constitutive law for the cracked concrete is obtained (Sena-Cruz, 2005):

$$\Delta \sigma = D^{corc} \Delta \varepsilon$$ \hspace{1cm} (7.19)$$

where

$$D^{corc} = D^{co} - D^{co} \left[ \hat{L}^{cr} \right]^{T} \left( \hat{L}^{cr} + \hat{T}^{cr} D^{co} \left[ \hat{L}^{cr} \right]^{T} \right)^{-1} \hat{T}^{cr} D^{co}$$ \hspace{1cm} (7.20)$$

Figure 7.4 illustrates the tri-linear stress-strain law, which was defined for the fracture crack opening Mode I. The ultimate crack normal strain, $\varepsilon_{n,ult}^{cr}$, is given by Eq. 7.21 and is defined by: $\alpha_i$ and $\xi_i$ parameters; Mode I fracture energy, $G_f$; material tensile strength, $f_{ct} = \sigma_{n,1}^{cr}$; and crack band width, $l_b$ (Sena-Cruz, 2005).

$$\varepsilon_{n,ult}^{cr} = \frac{1}{\xi_1 + \alpha_1 \xi_2 - \alpha_2 \xi_1 + \alpha_2} \frac{G_f}{f_{ct} l_b}$$ \hspace{1cm} (7.21)$$

In Eq. 7.21, $\alpha_i$ and $\xi_i$ are the parameters that define the shape of the crack normal stress versus strain diagram and are determined by:

$$\alpha_1 = \frac{\sigma_{n,2}^{cr}}{\sigma_{n,1}^{cr}} \hspace{1cm} \alpha_2 = \frac{\sigma_{n,3}^{cr}}{\sigma_{n,3}^{cr}} \hspace{1cm} \xi_1 = \frac{\varepsilon_{n,2}^{cr}}{\varepsilon_{n,ult}^{cr}} \hspace{1cm} \xi_2 = \frac{\varepsilon_{n,3}^{cr}}{\varepsilon_{n,ult}^{cr}}$$ \hspace{1cm} (7.22)$$

The sliding fracture modes II and III stiffness modulus, $D_{I_2}^{cr}$ and $D_{I_2}^{cr}$, respectively, are derived from:

$$D_{I_1}^{cr} = D_{I_2}^{cr} = \frac{\beta}{1 - \beta} G_c$$ \hspace{1cm} (7.23)$$
in which, $G_i$ and $\beta$ are the concrete elastic shear modulus and shear retention factor, respectively. The shear degradation of concrete with the increment of the crack normal strain is defined by a linear softening constitutive law as follows:

$$\beta = 1 - \frac{\varepsilon_{cr}}{\varepsilon_{n,ult}}$$  \hspace{1cm} (7.24)

![Figure 7.4: Stress-strain relation based on the Tri-linear law.](image)

### 7.3.2 Constitutive model for embedded cables

The embedded elements were modeled assuming a perfect bond between fibre and matrix. Therefore, the bond-slip behaviour was modeled in an indirect fashion by converting the fibre load – slip, $F - s$, relation into a stress – strain, $\sigma_f - \varepsilon_f$, relationship. The latter relationship was determined by fitting three-linear curves of the fibre pull-out test average curves performed for three distinct inclination angles $0^\circ$, $30^\circ$ and $60^\circ$. Figure 7.5 depicts this procedure, where $\varepsilon_f$, $s$ and $l_b$ are the fibre’s strain, fibre’s slip and the crack band width of the solid finite element (width of the finite elements above the notch where the crack was forced to be formed, $l_b=2$ mm), whereas $\sigma_f$ is the stress calculated from the pull-out force, $F$, divided by the fibre’s cross section area, $A_f$. The tensile stress – strain relationship was assigned to each fibre depending on its inclination angle, $\theta$, between the fibre and the normal vector of the active crack surface $\hat{n}$ (Figure 7.6). Since assigning a single $\sigma_f - \varepsilon_f$ law to each fibre with a particular orientation angle and embedded length
was not feasible, the three laws derived out from the fibre pull-out tests with inclination angles, $\alpha$, of 0°, 30° and 60° were used (Figure 7.6). The fibre pull-out responses for the series with $\alpha$ of 0°, 30° and 60° were ascribed to the embedded elements with $\theta$ ranging from [0, 15°], [15, 45°] and [45, 75°], respectively. It was assumed that the fibres with an inclination $\theta$ between [75, 90°] were not mobilized.

Figure 7.5: Determination of the embedded cable’s stress-strain diagram based on the experimental pull-out force – slip relation.

Figure 7.6: Three – dimensional scheme of the embedded fibre intersecting an active crack ($n$ is the vector normal to the crack plane).

7.3.3 Evaluation of the stiffness matrix of concrete and embedded cables

The stiffness matrix of an element is a summation of the concrete bulk and fibre reinforcement stiffness matrixes and determined as follow:
\[ K_{rc}^{f} = K_{rc}^{crea} + \sum_{i=1}^{n_f} K_{fi} \]  

(7.25)

in which \( K_{rc}^{crea} \), \( K_{fi} \) and \( n_f \) are the solid finite element stiffness matrix (concrete), the stiffness matrix of the \( i^{th} \) fibre embedded in the concrete mother-element, and the total number of the embedded fibres in the mother-element, respectively. The concrete element stiffness matrix was calculated by:

\[ K_{rc}^{crea} = \int_{V} B^{T} D_{rc}^{crea} B dV \]  

(7.26)

with \( D_{rc}^{crea} \) is the cracked concrete’s constitutive matrix and \( B \) states the strain-displacement matrix of a solid element.

By using the internal work of the axial component, the axial contribution of the fibre reinforcement to the stiffness matrix can be determined as follow:

\[ W_a = \int_{V} \delta \varepsilon_{f}^{T} \sigma_{f} dV = \int_{L} \delta \varepsilon_{f}^{T} E_{f} \varepsilon_{f} A_{f} dL \]  

(7.27)

where,

\[ dL = \sqrt{\left( \frac{dx_{1}}{ds} \right)^{2} + \left( \frac{dx_{2}}{ds} \right)^{2} + \left( \frac{dx_{3}}{ds} \right)^{2}} ds = J ds \]  

(7.28)

whereas, \( \varepsilon_{f} \), \( \sigma_{f} \), and \( A_{f} \) show strain, stress and cross section area of the fibre, and \( J \) depicts the Jacobian at the sampling point of the integration scheme adopted in the numerical evaluation of \( K_{fi} \). Therefore, by substituting Eq. 7.28 in 7.27, the internal work could be determined in natural coordinates by:

\[ W_a = \int_{L}^{1} \delta \varepsilon_{f}^{T} E_{f} \varepsilon_{f} A_{f} J ds \]  

(7.29)

\[ \varepsilon_{f} = T_{i}^{f} B d \]  

(7.30)

By substituting Eq. 7.30 in 7.29, the stiffness matrix was achieved. In Eq. 7.30 \( d \) represents the vector with the solid element’s nodal displacements and \( T_{i}^{f} \) is the vector corresponding to the first line of the transformation matrix from the fibre’s local coordinate system to the global coordinate system. \( T_{i}^{f} \) is given by:
\[
T^f = \begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
    a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\
    a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36}
\end{bmatrix}
\]

where, \( a_{ij} \) are the components of matrix \( \bar{a} \) containing the direction cosines, i.e. the projection of fibre’s local coordinate system \((x'_1, x'_2, x'_3)\) versors towards the global coordinate system \((x_1, x_2, x_3)\) versors, see Figure 7.6:

\[
\bar{a}^f = \begin{bmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix} = \begin{bmatrix}
    \cos(x'_1, x_1) & \cos(x'_2, x_2) & \cos(x'_3, x_3) \\
    \cos(x'_2, x_1) & \cos(x'_3, x_2) & \cos(x'_1, x_3) \\
    \cos(x'_3, x_1) & \cos(x'_1, x_2) & \cos(x'_2, x_3)
\end{bmatrix}
\]

Therefore, the stiffness matrix component with the contribution of fibre’s axial is depicted by:

\[
K^f_a = \int_1^2 B^T T^f \mathbf{K} \mathbf{B} dV, A_j, Jds
\]

Considering the shear contribution, in a similar manner, the components of the fibre stiffness matrix is given by:

\[
K^f_{s,1} = \int_1^2 B^T T^f_1 \mathbf{K} \mathbf{B} \bar{A}_j, Jds
\]

\[
K^f_{s,2} = \int_1^2 B^T T^f_2 \mathbf{K} \mathbf{B} \bar{A}_j, Jds
\]

where, \( G \) is fibre’s elastic shear modulus, \( \bar{A}_j \) shows reduced shear area for circular sections (Timoshenko, 1991), \( T^f_1 \) and \( T^f_2 \) are vectors corresponding to the second and third lines of the transformation matrix, respectively, see Eq. 7.31.

Finally, the equivalent nodal force vector, \( \mathbf{q}^e \), was comprise of concrete contribution and fibre contribution in which the latter one included of axial and shear components of fibre, and given by:

\[
\mathbf{q}^e = \int_1^2 B^T \sigma dV + \int_1^2 B^T T^f \mathbf{K} \mathbf{B} \bar{A}_j, Jds + \int_1^2 B^T T^f \mathbf{K} \mathbf{B} \bar{A}_j, Jds
\]

where \( \sigma \) depicts the fibre stress with axial component determined from the adopted stress-strain relationship considering the procedure illustrated in Figure 7.5. Furthermore, \( \mathbf{\tau} \) is the fibre’s stress vector with two shear components. For modeling the fibre shear
behaviour an elasto-plastic behaviour was adopted. Moreover, the shear contribution of the fibre was only considered for crack width smaller than 0.5 mm.

7.3.4 Numerical simulation

7.3.4.1 Concrete phase

In order to appraise the performance of the proposed model, a series of monotonic four-point bending tests in Chapter 6 were simulated. Figure 7.7 depicts the three-dimensional mesh which was used for modeling the fracture process of the concrete matrix. In this figure, it is also appended the loading and boundary conditions. In the present mesh, a total of 5304 Lagrangian 8-noded solid elements with minimum and maximum volumes of 50 and 250 mm$^3$, respectively, were used. Preliminary analyses were carried out in order to obtain a mesh refinement that does not compromise both the accuracy of the numerical simulations and the computational cost. The distortion of the finite elements was also checked to avoid modeling inaccuracies. Similar to the experiment, the load was applied on the third of the specimen’s length. Regarding to the boundary conditions, on the one side of the specimen, the translation movements were restrained in three directions (x, y and z), whereas on the other side the specimen is free to the movement in x direction, see Figure 7.7. In order to localize the crack formation at the notch region, the nonlinear behaviour was assigned to the elements above the notch zone (at the mid-length of the specimen), while to the remaining solid elements was assigned a linear elastic behaviour. Furthermore, 1×2×2 and 2×2×2 Gauss-Legendre integration schemes were adapted to the elements above the notch, and the rest of elements, respectively. A tri-linear stress-strain law (Figure 7.4) was used to simulate the post-cracking behaviour of plain SCC. The material properties to model the fracture process in the plain concrete matrix are included in Table 7.2. The numerical analysis was performed by displacement control.
Table 7.2: Plain concrete properties used in the numerical simulation.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$2.5 \times 10^6$ N/mm$^3$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$41.5 \times 10^3$ N/mm$^2$</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>72 N/mm$^2$</td>
</tr>
<tr>
<td>Tri-linear tension-softening diagram</td>
<td>$f_{ct}=4.66$ N/mm$^2$; $\xi_1=0.09$; $a_1=0.2; \xi_2=0.23$; $a_2=0.17$</td>
</tr>
<tr>
<td>Fracture energy</td>
<td>0.118 N/mm</td>
</tr>
<tr>
<td>Crack band-width</td>
<td>$l_p=2$ mm</td>
</tr>
</tbody>
</table>

7.3.4.2 Fibre phase

After the mesh representative of fibre distribution was generated, Figure 7.2, it was inserted into the solid three-dimensional mesh, which modeled the plain concrete, Figure 7.7. A single fibre was modeled as single embedded element that could be defined by two-end-nodes. Since an embedded element corresponding to a fibre could intersect one or several solid elements, it was necessary to compute these intersection points and divide the original finite element into distinct embedded elements to ensure compatibility between fibre and solid meshes. For this purpose, an inverse mapping technique was employed to determine the coordinates of these intersection points (Cunha et al., 2012).

Three-dimensional embedded elements with two integration points were used for simulating fibres. A Gauss-Legendre integration scheme was adopted. All the embedded
elements were ascribed with nonlinear behaviour. Nevertheless, only the fibres intersecting an active crack will be mobilized. Therefore, the remaining fibres remain in the elastic regime. As it was explained earlier, three stress-strain laws derived out from the fibre pull-out test with fibre’s inclination angles of 0°, 30° and 60° degrees were assigned to the embedded fibres with θ inclination angles ranging from [0, 15°], [15, 45°] and [45, 75°]. The strain in an embedded element, εf, was obtained from the strain field of its mother-element and the corresponding slip was determined multiplying εf by the crack band – width, l. Table 7.3 includes the parameters of the tri-linear σf–εf laws of the embedded elements with an angle θ.

Table 7.3: Tri-linear stress-strain relationships used for simulating the fibres’ bond-slip behaviour.

<table>
<thead>
<tr>
<th>θ [°]</th>
<th>Failure mode</th>
<th>σf,1 [MPa]</th>
<th>σf,2 [MPa]</th>
<th>σf,3 [MPa]</th>
<th>εf,1 [-]</th>
<th>εf,2 [-]</th>
<th>εf,3 [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>[0-15] Pullout</td>
<td>1060</td>
<td>558</td>
<td>240</td>
<td>0.33</td>
<td>1.23</td>
<td>2.4</td>
</tr>
<tr>
<td>30°</td>
<td>[15-45] Rupture</td>
<td>200</td>
<td>770</td>
<td>1000</td>
<td>0.015</td>
<td>0.08</td>
<td>0.32</td>
</tr>
<tr>
<td>60°</td>
<td>[45-75] Rupture</td>
<td>600</td>
<td>585</td>
<td>830</td>
<td>0.15</td>
<td>0.36</td>
<td>0.68</td>
</tr>
</tbody>
</table>

7.3.5 Numerical results

Figure 7.8 depicts the experimental average force-crack tip opening displacement relationships, Avg. EXP, the experimental envelopes, Envelope EXP, and numerical simulations, NUM. The experimental force-crack tip opening displacements were determined according to Section 6.3.1. A good agreement between the numerical and experimental responses was obtained for all series. The force corresponding to the crack onset was predicted with a high accuracy for all inclination angles of β. This was expected, since for low content fibre reinforced composites, the tensile strength is not significantly influenced by the fibre’s distribution, orientation and mobilization. Regarding the series with β = [0-15°] and [15-45°], although the numerical simulations slightly underestimated the average peak load, they were still located within the experimental envelope.
Figure 7.8: Comparison between the experimental $F$ – CTOD relationship and numerical simulation for $\beta = (a) [0\text{-}15^\circ], (b) [15\text{-}45^\circ], (c) [45\text{-}75^\circ]$ and (d) [75\text{-}90^\circ]$.

In general, the specimens have shown a linear behaviour up to the load where the crack was initiated in matrix and they have depicted a similar load value at limit of proportionality. When the crack was initiated in composite, series with $\beta=[0\text{-}15^\circ]$ and $[15\text{-}45^\circ]$ revealed a deflection-hardening response up to a CTOD nearby 0.65 mm, although $\beta=[15\text{-}45^\circ]$ revealed a slightly lower peak load. From the micromechanical point of view, after the adhesion of fibre and surrounding matrix has been debonded, the fibre reinforcement effectiveness was mainly governed by the mobilization process of the hooked end, which provides the highest contribution for this deflection-hardening capacity. Figure 7.9(a) and (b) show the location of the embedded elements (fibres) that intersected the crack plane for the $[0\text{-}15^\circ]$ and $[15\text{-}45^\circ]$ series, respectively. Note that the fibres intersected the notch were removed from the mesh. Figure 7.9(c) and (d) depict the normal stresses at the crack plane for a CTOD correspondent to maximum post-cracking load for $\beta=[0\text{-}15^\circ]$ and $[15\text{-}45^\circ]$ series, respectively. It should be mentioned that due to
the high heterogeneity of the mesh (fibre orientation and distribution), the crack lost orthogonality and started to rotate nearby the neutral axis (at the crack tip). Consequently, the normal tensile stresses increased unrealistically due to numerical stress locking. However, in the present case, this will not influence significantly the flexural response since it is very close to the neutral axis. The numerical stress locking was more pronounced for the [0-15°] and [15-45°] series, since they had higher number of fibres intersecting the crack plane, in contrast to the [45-75°] and [75-90°] series. In Figure 7.9(c) and (d), it was clearly shown that both series presented close stress values transferred by the fibres which led to a close estimation of the peak load. After the peak load was attained, a softening response was observed for both series. In the case of [0-15°] series, because the specimens contained more fibres that crossed the cracked plane with a lower orientation angle, the residual load decay was smoother, while for the β=[15-45°] specimens, a higher load decay was observable between 0.74 and 1.95 mm of CTOD. In the latter specimens, fibres have intersected the cracked plane with a higher average numerical orientation angle, i.e. 36°. Therefore, a stress-strain law correspondent to the fibres with inclination angle of 30° degree from the pullout test was assigned to a major number of fibres intersecting the crack plane where they were ruptured in a slip of 0.64 mm. Consequently, in this series, once the peak load was attained, the fibres are more prone to be ruptured. Actually, this can be observed by comparing Figure 7.9(e) and (f). These figures depict the normal stresses at the crack plane for a CTOD=2.5 mm. From the figures, it was observed that for [0-15°] series, there are some areas with higher stresses transferred by fibres in the bottom section of the specimen than [15-45°] series. Therefore, in a CTOD of 2.5 mm, the numerical residual forces for [0-15°] series was 82% higher than [15-45°] categories.

Regarding the $F$ – CTOD relationships for the [45-75°] and [75-90°] series, after the crack localization, the force dropped sharply up to a crack opening width between 0.18 – 0.20 mm, which then was followed by a plateau. Afterwards, a softening phase was observed in which the residual forces gradually decayed due to both fibre pull-out and rupture. Figure 7.10(a) and (b) show the embedded elements (fibres) that intersected the crack plane for the [45-75°] and [75-90°] series, respectively. Moreover, Figure 7.10(c) and (d) depict the normal stress correspondent to the maximum post-cracking load. As expected, the [45-75°] series exhibited higher residual strength than the [75-90°] series, mainly, due to the higher number of fibres at the fracture surface.
Figure 7.9: (a) and (b) Fibres at the crack plane for $\beta=[0-15^\circ]$ and $[15-45^\circ]$ series, respectively, (the gray area shows the notch location); (c) and (d) normal stresses for a CTOD correspondent to maximum load at post-crack branch; (e) and (f) normal stresses for a CTOD=2.5 and $\beta=[0-15^\circ]$, $[15-45^\circ]$. 
Comparing the $F$ – CMOD relationships for all series in Figure 7.8, it was observed that after the coalescence of micro-cracking into a macro-crack, $[0-15^\circ]$ and $[15-45^\circ]$ series showed a deflection hardening behaviour, whereas an abrupt reduction in the load was achieved for $[45-75^\circ]$ and $[75-90^\circ]$. On the other hand, higher residual forces were observed for the first two series than the two other categories. In terms of stress transfer capacity at the crack plane this can be also observed by comparing Figure 7.9(c), (d) and Figure 7.10(c) and (d). It was shown that higher stress levels were transferred by the fibres when the notch direction was more parallel to the flow direction. In $\beta= [0-15^\circ]$ case, the specimens contained more fibres that intersected the cracked plane with a lower angle, i.e. $29^\circ$. However, when angle $\beta$ increases, the fibre orientation factor tend to reduce which is translated to the rotation of fibres more parallel to the crack plane. The cause of this result is correspondent to the circular movement of concrete. On the other hand, when a panel is cast from the centre, the velocity profile exerted by the movement of concrete forces the fibres to align perpendicularly to the concrete flow direction and hence, there is a higher probability of fibres intersecting the fracture surface being more effective in the specimens with notch planes of $[0-15^\circ]$ and $[15-45^\circ]$, than with $[45-75^\circ]$ and $[75-90^\circ]$. In addition, it was also observed that the aligned fibres (with a $0^\circ$ inclination angle) exhibited the highest pull-out stress, whereas for $30^\circ$ and $60^\circ$ inclination angles the maximum stress tended to decrease with the increase of the orientation angle, see Table 7.3. Having in the mind this explanation and also due to the higher number of effective fibres intersecting the crack plane a higher stress transfer level for $[0-15^\circ]$ and $[15-45^\circ]$ series was clearly visible, which was translated into an overall maximum post-cracking force of 8.65 kN and 8.25 kN in opposition to the 4.52 kN and 3.77 kN observed for $[45-75^\circ]$ and $[75-90^\circ]$ series, respectively.
Figure 7.10: (a) and (b) Fibres at the crack plane for $\beta=[45-75^\circ]$ and $[75-90^\circ]$ series, respectively (the gray area shows the notch location); (c) and (d) normal stresses for a CTOD correspondent to maximum post-cracking load for $\beta=[45-75^\circ]$ and $[75-90^\circ]$ series.

7.4 Determination of long-term flexural behaviour by an analytical model

In fact, fibre and matrix are bonded through a weak interface, which understanding the behaviour of interfacial zone between fibre/matrix is important in micro-scale to predict the overall mechanical properties of SFRC in macro-scale. As it was shown in previous section, by averaging the contribution of each fibre, the overall response of composite
could be predicted with a high accuracy. In this section, it was proposed a simple analytical approach based on the cross sectional analysis in order to determine the long-term flexural behaviour of the cracked specimens in Chapter 6 taking the results of fibre pull-out creep tests from Chapter 5.

7.4.1 Description of the method and analytical simulation

The method consists of two main phases. Firstly, according to the applied load in the creep test, a value for the total opening in the crack (residual crack opening after unloading of specimen in cracking test, $w_{cr}^{res}$, + instantaneous crack opening in the creep test, $w_{inst}$) should be determined. This parameter is termed returning crack opening. On the other hand, from the monotonic $F$ − CTOD relationship achieved by the numerical simulation of four-point bending test in the previous section, up to a certain crack width opening, $w_{cr}$, the unloading stage (representative of cracking test) and then from the residual crack opening, $w_{cr}^{res}$, the reloading stage of the specimen in the creep test were simulated analytically. The proposed model was validated by simulating the unloading and reloading stages of the performed cyclic tests in Chapter 6. In the next phase, for each fibre, according to its orientation angle with a crack plane, and also its pre-slip level a specific creep pull-out law was assigned. Then, by averaging the creep response of the individual fibre for each time step, the overall long-term response of the specimen was predicted.

7.4.1.1 Determination of the returning crack opening

In this stage, in order to determine the returning crack opening when a specimen is reloaded in the creep test, the $F$ − CTOD relationships of cyclic tests (in unloading and reloading stages) from Chapter 6 were simulated analytically. Further ahead, according to the applied load level in the flexural creep test, the returning crack opening will be used to determine a pre-slip level for each fibre.

Whenever a reversal from the envelope monotonic $F$ − CTOD curve happened, the subsequent curve is termed as unloading curve, and reversal from this later curve is
named as reloading curve. The curves corresponding to unloading and reloading branches are depicted by a transition curve (Varma, 2012), which starts from an already known starting point and a slop that needs to be evaluated, and ends at a target point, whose coordinates and also the slop at this point should be determined. A scheme presentation of unloading curve (from point A to B) is depicted in Figure 7.11 (cracking test). If \( w_{un} (= w_{cr}) \) is the unloading crack opening and \( F_{un} \) illustrates its corresponding unloading force on the monotonic curve, then unloading from point \( A(w_{un}, F_{un}) \) with reversal slope \( K_{un} \), will target point \( B(w_{0}, 0) \) with target slope \( K_{0} \). For this stage, the proposed equations to determine the transition curve are as follow:

\[
F_{un} = (w - w_{0})(K_{0} + A[w - w_{0}]^{R}) \quad \text{for} \quad w_{0} \leq w \leq w_{un} \tag{7.37}
\]

\[
R = \frac{K_{un} - K_{sec,un}}{K_{sec,un} - K_{0}} \tag{7.38}
\]

\[
A = \frac{K_{sec,un} - K_{0}}{|w_{un} - w_{0}|^{R}} \tag{7.39}
\]

where \( w_{0} (= w_{cr}^{ret}) \) is the plastic crack opening, \( K_{0} \) states the tangential stiffness of unloading curve at \( w_{0} \) and \( K_{sec,un} \) shows the secondary stiffness at the time of unloading.

*Figure 7.11: Scheme representation of unloading curve.*
Figure 7.12 shows the influence of number of effective fibres obtained from Chapter 3 on the $K_0$ and $K_{sec,un}$ for each series of specimens ($\beta=[0-15^\circ]$, $[15-45^\circ]$, $[45-75^\circ]$ and $[75-90^\circ]$). The experimental $K_0$ and $K_{sec,un}$ were determined by averaging the responses of the performed cyclic tests. It is concluded that $K_0$ and $K_{sec}$ was influenced by number of effective fibres and tend to increase for the higher number of effective fibres. Therefore, the following equations were proposed to predict $K_0$, $K_{sec}$ and $K_{un}$ for each series of the specimens (Chang & Mander, 1994):

\[
K_0 = \left[ \frac{K_c}{w_{ct}^{1.1}} + B \right] \quad (7.40)
\]

\[
K_{sec,un} = K_c \left[ \frac{F_{un}}{K_c \times w_{ct}} + C \right] \quad (7.41)
\]

\[
K_{un} = K_c \quad (7.42)
\]

where $w_{ct}$ is crack opening at tensile strength and $K_c$ states initial stiffness of composite (see Figure 7.11). Furthermore, according to the experimental data, coefficients $B$ and $C$ were determined as 1.1 and 0.8 for $0 \leq \beta < 45^\circ$, 1.25 and 0.45 for $45 \leq \beta \leq 90^\circ$, respectively. Since $K_{un}$ is mainly depends on the concrete stiffness contribution, this parameter was assumed equal to $K_c$ for all series. Finally, $w_0$ was achieved from Eq. 7.43:

\[
w_0 = w_{un} - \frac{F_{un}}{K_{sec,un}} \quad (7.43)
\]
Figure 7.12: The influence of number of the effective fibres on the unloading stiffness: (a) $K_0$ and (b) $K_{sec}$.

Figure 7.13 shows a schematic illustration of completed unloading and reloading curves. It should be noted that for the unloading crack opening, $w_{un}$, the reloading curve predicts new force (point C and $F_{new}$). Due to the difference in unloading and reloading stiffness, the reloading branch always joins the envelope monotonic curve at a higher crack opening compare to the unloading crack opening on the monotonic curve, given rise to a crack opening shift, $\Delta w$. Based on the experimental data:

$$\Delta w = 0.28w_{un}$$  \hspace{1cm} (7.44)

$$\Delta F = 0.14F_{un}$$  \hspace{1cm} (7.45)

$$k_{sec, re} = \frac{F_{new}}{w_{un} - w_0}$$  \hspace{1cm} (7.46)

where, $K_{sec, re}$ shows the secondary stiffness at reloading branch and $F_{new} = F_{un} - \Delta F$.
Figure 7.13: Schematic illustration of completed unloading and reloading curves.

The reloading curve is defined by three points and two connecting transition curves: the initial B, intermediate C and target D points. The first transition curve connects point B \((w_0, 0)\) with a starting slope \(K_c\) to an intermediate point \(C(w_{un}, F_{new})\) with a slope \(K_{new,BC}\), where \(F_{new}\) is force on the reloading branch, \(K_{new,BC}\) states its tangential stiffness which based on the experimental data obtained as \(0.82K_{sec,re}\). For branch BC, the equations for the transition curve were determined as follow:

\[
F_{BC} = (w - w_0)\left(K_c + A\left|w-w_0\right|^R\right) \text{ for } w_0 \leq w \leq w_{un}
\]  
(7.47)

\[
R = \frac{K_{new,BC} - K_{sec,re}}{K_{sec,re} - K_c}
\]  
(7.48)

\[
A = \frac{K_{sec,re} - K_c}{\left|w_{un} - w_0\right|^R}
\]  
(7.49)

Similarly, the second transition curve connects intermediate point \(C(w_{un}, F_{new})\) with a slope \(K_{new,BC}\) to return point \(D(w_{re}, F_{re})\) with a target slope \(K_{re}\), where \(F_{re}\) and \(K_{re}\) are the force corresponding to the returning crack opening \((w_{re})\) and its tangential stiffness calculated on the envelope monotonic curve. The equations for determining the transition curve in branch CD are as follow:

\[
F_{CD} = F_{BC} + (w - w_{un})\left(K_{new,BC} + A\left|w-w_{un}\right|^R\right) \text{ for } w_{un} \leq w \leq w_{re}
\]  
(7.50)
\[
R = \frac{K_{re} - K_{\text{new,CD}}}{K_{\text{new,CD}} - K_{\text{new,BC}}} \quad (7.51)
\]

\[
A = \frac{K_{\text{new,CD}} - K_{\text{new,BC}}}{|w_{re} - w_{un}|^R} \quad (7.52)
\]

\[
K_{\text{new,CD}} = \frac{F_{re} - F_{\text{new}}}{w_{re} - w_{un}} \quad (7.53)
\]

\[
w_{re} = w_{un} + \Delta w \quad (7.54)
\]

Figure 7.14 and Figure 7.15 show a comparison between the experimental average \( F \rightarrow \text{CTOD} \) relationship and the analytical approach for the specimens with one cycle of unloaded and reloaded at \( \text{CTODs of 0.3 and 0.5 mm} \). The experimental \( F \rightarrow \text{CTOD} \) relationships were achieved by averaging the responses of the executed cyclic tests on the beams in Chapter 6 for each series of \( \beta \) ([0-15°], [15-45°], [45-75°] and [75-90°]), individually. The curves were magnified in order to present a more clear vision regarding the comparison between experimental and analytical \( F \rightarrow \text{CTOD} \) relationships. It was shown that the analytical approach predicted the experimental \( F \rightarrow \text{CTOD} \) relationship at the unloading and reloading stages with a high accuracy. Although, in some specimens, the analytical \( F \rightarrow \text{CTOD} \) curves did not follow the average experimental data perfectly, but in all cases the deviations were lower than 10%. 

(a)  
(b)
Figure 7.14: Comparison between the experimental average $F$ – CTOD relationship and analytical approach for the specimens with one cycle of unloading and reloading at CTOD=0.3 mm: $\beta= (a) [0-15^\circ], (b) [15-45^\circ], (c) [45-75^\circ]$ and (d) [75-90$^\circ$].

Figure 7.15: Comparison between the experimental average $F$ – CTOD relationship and analytical approach for the specimens with one cycle of unloading and reloading at CTOD=0.5 mm: $\beta= (a) [0-15^\circ], (b) [15-45^\circ], (c) [45-75^\circ]$ and (d) [75-90$^\circ$].
7.4.1.2 Simulation of the creep behaviour

The flowchart of the algorithm in the developed model is shown in Figure 7.16. In this figure, \( t_r \) is the total time that the specimen kept under sustain load in creep test (hours), \( F \) states the level of the applied load in creep test in percent \( (F = F_a / F_{cr}) \), \( w_{re} \) shows returning crack opening (mm), \( h \) is the height of the cracked cross section (mm), \( y \) illustrates the height of the notch (mm) and \( N_f^T \) depicts the total number of fibres crossing the section. Furthermore, for a desire value of pre-crack level, the height of the cracked section, \( h \), can be determined from the numerical simulation of four-point bending test similar to the previous section. This algorithm consists of two internal and external loops. The internal loop determines the creep coefficient for each fibre based on its position in the fracture surface, whereas the external loop used to obtain the average creep coefficient between all fibres for a step of time. It should be mentioned that the creep coefficient parameter was assigned to each fibre in order to predict creep behaviour of beams since, as it was shown in Chapter 5, this parameter presented a lower scattering compare to the long-term slip. On the other hand, since the creep tests were performed in force control, in some specimens a slightly higher instantaneous slip obtained which led to a higher long-term slip. Therefore, by defining the creep coefficient parameter this influence can be marginal. However, when the overall creep behaviour (creep coefficient vs. time) of the beam was predicted, this curve can be converted to the long-term crack opening vs. time easily by determining instantaneous crack opening \( (w_{ti} = \varphi \times w_{inst}) \) based on the applied load level in the creep test from the analytical formulations in the previous section \( (w_{inst} = w_{re} - w_0) \). The idealised following equation was used to predict a pre-slip value for each fibre according to its position in the crack surface (RILEM-TC162-TDF, 2002a):

\[
s_{pr} = \frac{w_{re}(h-z)}{h-y}
\] (7.55)

where, \( z \) is the distance between the fibre centre of gravity and bottom of the specimen in the cracked cross section. The validation of this equation was also checked by the numerical simulation of monotonic four-point bending test. Then, for each fibre, equations in section 5.6 was applied to assign a creep pull-out law considering its orientation angle with the crack plane, \( \theta \), and also determining a pre-slip value, \( s_{pr} \), by Eq.
Finally, for a step of time, an average creep coefficient between all fibres was obtained and saved in an output data file.

Figure 7.16: Flowchart of the algorithm adopted in the developed model.
7.4.2 Result’s validation and discussion

In order to validate the proposed model, the development of creep coefficient with time of the beams in Figure 6.24 was simulated since for each series of β some specimens are available. In these figures, the specimens were loaded in 100% of $F_u / F_{cr}$ and pre-cracked up to 0.3 and 0.5 mm. For each series, the fibre structure which modeled in Section 7.2.2 was used, because from the previous analysis it was concluded that with the proposed fibre distribution methodology, the monotonic $F – CTOD$ relationships could be predicted with a high accuracy. Due to the impossibilities of having a creep coefficient – time law for every possible inclination angle, the creep coefficient – time laws obtained from the creep pull-out tests with an inclination angle of 0°, 30° and 60° was assigned to the embedded cables with an orientation towards the active crack surface ranging from [0-15°], [15-45°] and [45-90°], respectively. Moreover, as it was shown in section 6.3.2.2, for the two studied pre-crack levels 0.3 and 0.5 mm, a significant part of the section above the notch was cracked. Therefore, it was assumed that all fibres in the section were activated.

Figure 7.17 and Figure 7.18 show a comparison between the experimental and analytical development of creep coefficient versus time in the creep four-point bending tests, for the beams pre-cracked up to 0.3 and 0.5 mm, respectively. It was shown that the proposed methodology could predict the creep behaviour of the specimens in flexure reasonably, particularly for the specimens with $w_{cr} = 0.3$ mm. In the case of the $w_{cr} = 0.5$ mm specimens, the analytical approach slightly underestimated the experimental curves. This could be referred to some simplifications and assumptions made in this model, being more pronounce in the higher pre-cracking levels since specimens subjected to a higher load level. Firstly, the creep was assumed in the side of fibre with lower embedded length where its bond was damaged in the cracking test. However, in the other side of the fibre, under a sustained load, some micro-cracks could also propagate in the fibre/matrix interfacial zone which increases the long-term crack widening. Secondly, in this methodology, the influence of creep in the compression of the uncracked section was not considered, although it was proved to be very low. Thirdly, in the modeling of creep in flexure, it was assumed that the fibres were subjected to the axial loading while the influence of shear component was ignored. Lastly, the assigned creep coefficient – time laws to the fibres were achieved for only one embedded length $l_f / 4$ ($l_f$ is length of the
fibre in mm) and limit fibre orientation angles. In spite of these, it is worth noting that, for all series of $\beta$, the proposed model predicted more than 90 and 82% of the experimental results for the beams with $w_{cr} = 0.3$ and 0.5 mm, respectively. However, this section was investigated a preliminary study of this methodology and in order to develop and calibrate the model, more experimental data are needed.

Figure 7.17: Comparison between the experimental bending and analytical (obtained from the fibre pull-out creep test) creep coefficient vs. time relationship for the specimens with $w_{cr}=0.3$ mm, $F_a/F_{cr}=100\%$ and $\beta= (a) [0-15\^\circ]$, (b) [15-45\^\circ], [45-75\^\circ] and [75-90\^\circ].
Figure 7.18: Comparison between the experimental bending and analytical (obtained from the fibre pull-out creep test) creep coefficient vs. time relationship for the specimens with $w_{cr}=0.5$ mm, $F_a/F_{cr}=100\%$ and $\beta= (a) [0-15^\circ]$, (b) [15-45^\circ]$, [45-75^\circ]$ and [75-90^\circ]$.

### 7.5 Conclusions

In this chapter, a numerical approach was proposed in order to model the behaviour of steel fibre reinforced self-compacting concrete, SFRSCC. The fibres structure was randomly generated considering both the flow and wall effects influence, moreover the micro-mechanical behaviour of a single fibre was considered. The micro-mechanical behavior of the fibre was evaluated by performing fibre pull-out tests for distinct orientation angles. The SFRSCC was modeled as a two-phase material, i.e. the fracture process of the concrete bulk was simulated with a 3D multi-directional smeared crack model, whereas the fibre phase was comprised by discrete randomly distributed
embedded elements. In order to simulate the fibre reinforcement mechanism, the force – slip laws obtained from the pull-out tests were assigned to each fibre depending on its orientation angle towards the active crack plane. Furthermore, following the same methodology, the long-term behaviour of a cracked element under flexure was also predicted based on the cross sectional analysis.

Considering the aforementioned methodology, the experimental results of distinct four-point bending tests were modeled. The prismatic specimens were extracted from a panel, and the notch was performed according to the expected concrete flow direction. The finite element simulations of the four-point-bending tests predicted with a high level of accuracy the experimental force – crack tip opening displacement. Therefore, with a realistic prediction of the actual fibre distribution/orientation and having the knowledge of the micro-mechanical behaviour of the fibres, the heterogeneity mechanical behaviour of the SFRSCC composite could be predicted.

In spite of the good performance of the numerical model, in some series, after the crack initiation, the numerical response slightly underestimated the experimental average response. This could be ascribed to the fibres’ constitutive laws determined in an indirect fashion from the fibre’s load – slip relationships. Nevertheless, it should be mentioned that all numerical responses were comprised within the experimental envelope part.

Furthermore, the flexural long-term behaviour of a cracked SFRSCC section could be also predicted by having the knowledge regarding to the long-term micro-mechanical behaviour of a fibre, particularly for the low values of pre-crack levels. Even thought, the results were satisfactory, but, however, it is well known that due to the assumptions and simplifications, this methodology still needs to be improved based on the more experimental results.
The main purpose of this thesis was to achieve as much as possible a consistent comprehension of the behaviour of steel fibre reinforced self-compacting concrete, SFRSCC, applied in laminar structures under monotonic and long-term (in cracked state) loading conditions. Therefore, in the first phase, researches were executed in order to understand how fibres were distributed and oriented in a SFRSCC laminar structure, and, furthermore, how these parameters influence the overall composite behaviour at a macro-level. Then, the micro-mechanical aspects of fibre reinforcements were analysed by performing a series of monotonic and long-term fibre pull-out tests. Finally, based on the fibre’s micro-mechanical properties, an integral approach was used to predict flexural behaviour of the SFRSCC laminar structures under monotonic and long-term loading conditions. In the scope of this thesis the following conclusions were derived out:

The fibre structure parameters were evaluated in planes having different angles towards the concrete flow direction. It was concluded that the number of fibres were significantly higher for the specimens with cut plane parallel to the flow direction. The same conclusion was also made by considering fibre orientation factor. On the other hand, when the studied plane was parallel to the concrete flow direction, the fibres were more perpendicular to the section. The fibre orientation factor tends to reduce by increasing angle between the cut plane and flow direction which is translated to the rotation of fibres more parallel to the cut surface. The cause of this result is correspondent to the circular movement of concrete. On the other hand, in the case of casting panels
from the centre, fibres have a tendency to align perpendicular to the radial flow, mainly due to the uniform flow profile velocity that diffuses outwards radially from the centre of the panel. Consequently, the total number of the effective fibres was higher in cut planes parallel towards the concrete flow when compared to the other cases, particularly orthogonal plane towards the concrete flow. Considering the fibre segregation parameter, it was verified the occurrence of a slight segregation of the fibres of similar level for all the series considered, caused by the highest specific weight of the steel fibres amongst the constituents of the SFRSCC.

Fibre distribution and orientation have a strong impact on the tensile behaviour of specimens drilled from the panels. In the case of the series with crack plane parallel to the concrete flow direction, specimens showed significantly higher post-cracking parameters than the other studied case with a perpendicular crack plane to the flow direction. Furthermore, a linear relationship between number of the effective fibres, orientation factor and post-cracking parameters were observed. It was shown that by increasing the number of effective fibres as well as their orientation, fracture parameters tend to raise. This strong dependency could explain that when the notch plane was parallel to the concrete flow direction due to the appearing higher number of effective fibres which were mainly perpendicular to the crack plane, the concrete represented a semi-hardening behaviour, while in the other series a high stress decay was achieved.

Application of inverse analysis procedure of the splitting tensile test experimental results predicted successfully the tensile post-cracking parameters of SFRSCC. The $\sigma - w$ responses determined by the inverse analysis technique reproduced all the distinct phases observed during the uniaxial tensile test, particularly, the reduction in the strength due to the loss of the matrix stiffness once the crack initiated and also the semi-hardening phase at the early cracking stages. Considering the obtained tensile strength, inverse analysis of splitting test overestimated the tensile strength obtained from uniaxial tensile tests.

Regarding to the micro-mechanical behaviour, in general, two fibre failure modes were observed during monotonic pull-out tests. In the case of the fibres with inclination angle of $0^\circ$, the complete fibre pull-out was obtained, whereas for the inclined fibres, the fibre rupturing was the failure mode. Regarding to the influence of orientation angle, the maximum pull-out load reduced with increasing in inclination angle while the relation between slip at peak load and fibre orientation followed a reverse trend. On the other hand, a slight reduction on the slip at the peak load was presented for a $30^\circ$ angle,
whereas for a 60° angle the slip at the peak load increased considerably. This significant increase corresponded to other additional mechanisms in the pull-out process includes a parcel that related to fibre deformation and matrix spalling.

Considering the fibre pull-out creep tests for both investigated pre-slip levels ($s_{pr} = 0.3$ and $0.5$ mm), stable responses were attained for all series. Regarding to the influence of the pre-slip levels, $s_{pr} = 0.5$ mm series showed a higher long-term slip comparing to $s_{pr} = 0.3$ mm series. On the other hand, since $s_{pr} = 0.5$ mm was very close to the slip at peak load, the interface bond between fibre and matrix was more deteriorated, therefore the long-term slip was increased in a higher rate. But even in that case, still a stable response was obtained although in a higher time period. For the specimens with $s_{pr} = 0.3$ mm, a reduction in the long-term slip was observed when the fibre orientation angle increased, while for the $s_{pr} = 0.5$ mm series, this reduction was more significant. However, in the case of the specimens with orientation angle of 60°, the ratio between the applied load and maximum load, $F_a / F_{max}$, was much lower than other two studied fibre inclination angles, therefore a lower long-term slip was obtained.

Concerning the long-term flexural creep tests, two pre-cracking levels were considered ($w_{cr} = 0.3$ and $0.5$ mm). Stable responses were observed for all specimens. However, as it was expected, by increasing the level of the applied load, higher values of the creep coefficient were achieved. Regarding the influence of the pre-cracking levels, $w_{cr} = 0.5$ mm series conducted to higher values of the creep coefficient than in the other studied series ($w_{cr} = 0.3$ mm), especially, if they were loaded with a high $F_a / F_{cr}$. On the other hand, since $w_{cr} = 0.5$ mm was very close to the CTOD correspondent to the maximum load at post-cracking branch of the monotonic responses, the bond interface between fibre and matrix was more damaged, therefore the creep crack width increased with a higher rate. However, even in this case, still a stable response was obtained, although requiring a higher time period for this stabilization. Moreover, specimens located nearer to the panel’s corner showed a higher increase of creep coefficient with time. This aspect could be ascribed to the decrease of the concrete flow velocity with the increase of the distance from the casting point which leads to different fibre distribution as well as matrix constituents through the panel.

Finally, a numerical approach was proposed in order to model the behaviour of steel fibre reinforced self-compacting concrete, SFRSCC. The fibres structure was randomly
generated considering both the flow and wall effects influence, moreover the micro-
mechanical behaviour of a single fibre was considered. The micro-mechanical behavior of
the fibre was evaluated by performing fibre pull-out tests for distinct orientation angles.
The SFRSCC was modeled as a two-phase material, i.e. the fracture process of the
concrete bulk was simulated with a 3D multi-directional smeared crack model, whereas
the fibre phase was comprised by discrete randomly distributed embedded elements. In
order to simulate the fibre reinforcement mechanism, the force – slip laws obtained from
the pull-out tests were assigned to each fibre depending on its orientation angle towards
the active crack plane. Furthermore, following the same methodology, the long-term
behaviour of a cracked element under flexure was also predicted based on the cross
sectional analysis. The finite element simulations of the monotonic four-point bending
tests predicted with a high level of accuracy the experimental force – crack tip opening
displacement. Therefore, with a realistic prediction of the actual fibre distribution
/orientation and having the knowledge of the micro-mechanical behaviour of the fibres,
the heterogeneity mechanical behaviour of the SFRSCC composite could be predicted.
Furthermore, the flexural long-term behaviour of a cracked SFRSCC section could be
also predicted by having the knowledge regarding to the long-term micro-mechanical
behaviour of a fibre, particularly for the low values of pre-crack levels.
CHAPTER 9

References


In Figure I.1 is depicted the fibre orientation profile for each specimen extracted from panels A and B.

*Figure I.1:* Fibre orientation profile for each specimen extracted from panels A and B.
A5 and $\theta = 0^\circ$

A5 and $\theta = 90^\circ$

A6 and $\theta = 0^\circ$

A6 and $\theta = 90^\circ$

B3 and $\theta = 0^\circ$

B3 and $\theta = 90^\circ$

B4 and $\theta = 0^\circ$

B4 and $\theta = 90^\circ$
Appendix – II

Experimental results of splitting and uniaxial tensile tests

In Figures II.1 and II.2 the experimental results of splitting tensile tests and uniaxial tensile tests for each specimen extracted from the panels are shown, respectively.

Figure II.1: Experimental results of splitting tensile test for each specimen.
A2 and $\theta=0^\circ$

A2 and $\theta=90^\circ$

A3 and $\theta=0^\circ$

A3 and $\theta=90^\circ$

B1 and $\theta=0^\circ$

B1 and $\theta=90^\circ$

B2 and $\theta=0^\circ$

B2 and $\theta=90^\circ$
Figure II.2: Experimental results of uniaxial tensile test for each specimen.
D3 and $\theta=0^\circ$

D3 and $\theta=90^\circ$
Appendix – III

Experimental results of monotonic and cyclic fibre pull-out tests

In Figures III.1 and III.2, are depicted the results of the monotonic and cyclic fibre pull-out tests for each specimen individually. In each figure, the results are shown in terms of the slip for three LVDTs installed on the grip. Each specimen named according to the following methodology: from the left to the right, each character shows type of the test (M: Monotonic or Cy: Cyclic); type of fibre (S: Smooth or H: Hooked end); fibre orientation angle (0, 30 or 60 degree) and number of the specimen.

*Figure III.1:* Experimental results of monotonic fibre pull-out tests.
**Figure III.2**: Experimental results of cyclic fibre pull-out tests.

Cy-0.3-0-1  
Cy-0.3-0-2  
Cy-0.3-0-3  
Cy-0.5-0-1  
Cy-0.5-0-2  
Cy-0.5-0-3
Appendix – IV

Experimental results of long-term fibre pull-out tests

Each specimen named according to the following methodology: from the left to the right, each character shows type of the test (C: Creep or A: Assembled curve); pre slip level (0.3 or 0.5 mm) fibre orientation angle (0, 30 or 60 degree) and number of the specimen. Figures IV.1 and IV.2 show the creep behaviour in terms of total long-term slip versus time and the assembled curves of the specimens pre-slipped up to 0.3 mm.

Figure IV.1: Total slip of the specimens pre-cracked up to 0.3 mm.
Figure VI.2: The assembled curves of the specimens pre-cracked up to 0.3 mm.
Figures IV.3 and IV.4 show the creep behaviour in terms of total slip versus time and the assembled curves of the specimens pre-cracked up to 0.5 mm.

**Figure IV.3:** Total slip of the specimens pre-cracked up to 0.5 mm.

**Figure VI.4:** The assembled curves of the specimens pre-cracked up to 0.5 mm.
Appendix – V

Experimental results of monotonic and cyclic fibre pull-out tests

In Figures V.1 and V.2 are depicted the experimental results of the monotonic and cyclic four-point bending tests, respectively, for each beam individually. In each figure the results are shown for three LVDTs measured CMOD at top, middle and bottom of the panel as well as the one recorded deflection, see Figure 6.2.

*Figure V.1:* Experimental results of monotonic four-point bending test.
Figure V.2: Experimental results of cyclic four-point bending test.

L-0.3-7.5°-9
L-0.3-7.5°-10
L-0.3-7.5°-11
L-0.3-30°-13
L-0.3-30°-14
L-0.3-30°-15
L-0.5-87.5°-1
L-0.5-87.5°-2
L-0.5-87.5°-3
L-0.5-87.5°-4
A-0.3-7.5°-9
A-0.3-7.5°-10
A-0.3-7.5°-11
A-0.3-7.5°-12
A-0.3-87.5°-13
A-0.3-87.5°-15
A-0.3-60°-16
A-0.5-87.5°-1
A-0.5-87.5°-2
A-0.5-87.5°-3
A-0.5-7.5°-5
A-0.5-7.5°-6
A-0.5-7.5°-7
H-0.3-30°-9
H-0.3-30°-10
H-0.3-30°-11
H-0.3-60°-13
H-0.3-60°-14
H-0.3-60°-15
H-0.3-60°-16
Appendix – VI

Experimental results of long-term bending tests

Figures VI-1 and VI-2 show the creep behaviour in terms of total long-term crack opening versus time and the assembled curves of the specimens pre-cracked up to 0.3 mm.

Figure VI.1: Total long-term crack opening of the specimens pre-cracked up to 0.3 mm.

L-0.3-7.5°-2, \( F_a / F_{cr} = 100\% \)  
L-0.3-7.5°-3, \( F_a / F_{cr} = 59\% \)  
L-0.3-7.5°-4, \( F_a / F_{cr} = 59\% \)  
L-0.3-87.5°-5, \( F_a / F_{cr} = 67\% \)  
L-0.3-87.5°-6, \( F_a / F_{cr} = 62\% \)  
L-0.3-87.5°-7, \( F_a / F_{cr} = 100\% \)
A-0.3-60°-1, $F_a / F_{cr} = 63\%$
A-0.3-60°-2, $F_a / F_{cr} = 73\%$
A-0.3-30°-3, $F_a / F_{cr} = 71\%$

H-0.3-30°-1, $F_a / F_{cr} = 84\%$
H-0.3-30°-2, $F_a / F_{cr} = 81\%$
H-0.3-30°-3, $F_a / F_{cr} = 100\%$
H-0.3-30°-5, $F_a / F_{cr} = 89\%$

H-0.3-60°-6, $F_a / F_{cr} = 100\%$
H-0.3-60°-7, $F_a / F_{cr} = 99\%$
Figure VI.2: The assembled curves of the specimens pre-cracked up to 0.3 mm.
Figures VI-3 and VI-4 show the creep behaviour in terms of total long-term crack opening versus time and the assembled curves of the specimens pre-cracked up to 0.5 mm.

Figure VI.3: Total long-term crack opening of the specimens pre-cracked up to 0.5 mm.
H-0.5-60°-2, \( F_a / F_{cr} = 93\% \)

H-0.5-60°-3, \( F_a / F_{cr} = 100\% \)

H-0.5-30°-5, \( F_a / F_{cr} = 90\% \)

H-0.5-30°-6, \( F_a / F_{cr} = 100\% \)

H-0.5-30°-7, \( F_a / F_{cr} = 67\% \)
Figure VI.2: The assembled curves of the specimens pre-cracked up to 0.5 mm.