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Onsite assessment of structural timber members by means of hierarchical models and probabilistic methods

Hélder S. Sousa [1*], José S. Machado [2], Jorge M. Branco [1], Paulo B. Lourenço [1]

[1] ISISE, Department of Civil Engineering, University of Minho, 4800-058 Guimarães, Portugal

[2] Laboratório Nacional de Engenharia Civil, Núcleo de Estruturas de Madeira, Av. Do Brasil, 101, 1700-066 Lisboa, Portugal

e-mail addresses: Hélder S. Sousa - sousa.hms@gmail.com
José S. Machado - saporiti@lnec.pt
Jorge M. Branco - jbranco@civil.uminho.pt
Paulo B. Lourenço - pbl@civil.uminho.pt

[*] corresponding author: ISISE, University of Minho, Department of Civil Engineering, Azurém, 4800-058 Guimarães, Portugal; e-mail: sousa.hms@gmail.com
Tel: +351 253510200; Fax: +351 253510217

ABSTRACT

One of the main motivations for hierarchical modelling is to understand how properties, composition and structure at lower scale levels may influence and be used to predict the material properties at macroscopic and structural engineering scales. Structural timber is, in most cases, characterized by three parameters usually designated as reference properties: density, bending modulus of elasticity and bending strength.

The present paper addresses a review on different possibilities for obtaining reliable data about the mechanical behaviour of timber elements by collecting information at different levels and by organizing that information into a hierarchy of sequential levels (from lowest to highest). The applicability and limitations of statistic and probabilistic methods on the prediction and inference of timber's reference material properties are discussed and exemplified.

KEYWORDS: Bayesian methods; hierarchical modelling; timber reference properties; updating

1. INTRODUCTION

The quality (reliability) of a probabilistic structural analysis process is highly dependent on the quality of the information used for the input variables. Structural timber is, in general case, characterized by three parameters usually designated as reference properties: density, bending modulus of elasticity and bending strength.

The onsite assessment of these properties is being done following different approaches, which often consider the hierarchical structure of wood. This hierarchical structure can be seen at different scales, from nanostructure to macroscale, [1], similarly to other natural materials such as bone [2].

It is recognized that the structural performance of timber elements is dependent on variables that operate at different material's scale. This dependence influences the results obtained through the different tools and methods used for onsite assessment of structural timber elements. From the inclination of the microfibrils of cellulose inside the cells walls (micro) through the characteristics of the growth rings (meso) until the effect of gross defects (macroscale), different models have been built to deal with the requirement of more reliable models for the prediction of structural timber elements' performance.

Hierarchical or multilevel modelling is therefore suitable for this material, since it reflects the necessity of acquiring knowledge from timber's variables at different levels. The complexity of timber and the restrictions existing when performing onsite assessment make Bayesian statistics an excellent tool for combining information from multiple sources (non-destructive tests, NDT; semi-destructive tests, SDT; or even destructive tests, DT), to update information when new data is available and to include expert opinion (qualitative information).

Hierarchical Bayesian modelling requires the awareness and the distinction of different scales, such that a homogenization step may be taken to each of those scales as to define similar properties for each scale. The different hierarchical models applied to wood properties, such as density, strength or stiffness, are defined according to the study's purpose. If the adopted main unit is the growth ring then the levels can comprise a macrolevel (multilayer material with alternative layer of earlywood and latewood) and can end at a very low level as the nanostructure, where the layers of the cell's secondary wall are considered as unidirectional fibre-reinforced composites and middle lamella and primary wall are considered as random short-fibre/particle reinforced composites [1][3]. If the main unit is clear wood (macroscale), then growth ring (mesoscale) and cell level (microscale) can define the hierarchical model [4]. The main unit of analysis can also be the structural member with three levels defined as micro (timber board or beam), meso (local) and macro (global) [5]. In the present paper, focus is given to the models where the macrolevel is defined at the material level.

Several attempts were made to hierarchically model the stiffness and strength of timber elements, by considering the presence of weak sections separated by segments of clear wood [6][7][8][9]. In [10], Bayesian methods were used to update the mechanical properties of existing timber elements and the assessment was performed using First Order Reliability Methods (FORM). The results of that work evidenced that different degrees of belief in the new data may significantly influence the reliability level. Usually for in-service timber elements, new data are derived from NDT results obtained with ultrasound, resistance drilling and penetration resistance equipment. In [10], NDT tests were made to chestnut wood specimens and combined with results from compressive strength parallel to the grain tests. The uncertainty of the different NDT results was modelled by Maximum Likelihood estimates.

Hierarchical modelling has also been carried out using Bayesian Probabilistic Networks (BPN) for the analysis of variability of timber mechanical properties [11][12][13]. BPNs are used to represent knowledge based on Bayesian regression analysis describing the causal interrelationships and the logical arrangement of the network variables. In [11], a hierarchical model was used to determine the influence of the origins (different tree growth locations) and cross-sectional dimensions of new timber elements on the probability distribution of its material properties. On that work, BPNs using information of machine grading indicators were used to describe and infer on the dependence of different origins and dimensions of sawn structural timber on the relevant timber material properties. The parameters of the prior probability distribution functions, as well as the regression parameters, were estimated as random variables with mean values, standard deviations and correlations through the Maximum Likelihood method.

The present paper addresses different possibilities for obtaining reliable data about the reference properties of timber elements by collecting information at different levels and by organizing that information into a hierarchy of sequential levels (from lowest to highest).

2. BAYESIAN PROBABILISTIC METHODOLOGY

Bayesian statistics is an inference method based in the Bayes' rule allowing to estimate the updated probability, given an additional evidence is provided. Bayesian probability, therefore belongs to the category of evidential probability analysis that is used to evaluate the probability of a new information or hypothesis. For that aim, Bayesian probabilistic methods first specifies a given prior probability, which is then updated when new relevant data are made available. The prior probability distribution expresses the uncertainty about a given parameter before evidence is taken into account. The posterior probability function, which is the conditional distribution of the uncertainty, may be obtained by considering the Bayes' theorem, multiplying the prior distribution by the likelihood function and then normalizing it.

As the prior distribution probability is often only the subjective assessment of an expert, in Bayesian methods probabilities are considered as the best possible expression of the degree of belief in the occurrence of a certain event. The Bayesian probabilistic approach does not consider that probabilities are direct and unbiased predictors of occurrence frequencies that can be observed in practice. The only consideration is that, if the analysis is carried out carefully, the probabilities will be correct if averaged over a large number of decision situations [14]. To fulfil that consideration, it is necessary that the subjective and purely intuitive part is neither systematically over conservative, nor over confident. Therefore, calibration to common practice and to empirical data may be considered as an adequate path to that aim.

The JCSS Probabilistic Model Code [15] concludes that, compared to the frequentistic interpretation the Bayesian interpretation is the only one that makes sense in the end, as it overcomes the difficulties of updating distributions when more statistical data is available.

When uncertainties are present, The Bayesian interpretation overcomes these difficulties and provides the most logical and useful framework for consistent decision making [14].

Bayesian methods allow quantifying an approximation about the statistical uncertainty related to the estimated parameters, regarding both the physical uncertainty of the considered variable, as well as the statistical uncertainty related to the model parameters. Therefore, they offer a suitable method for parameter estimation and model updating. However, for making this possible, it is necessary to take into account the measurement and the model uncertainties in the probabilistic model formulation. Since Bayesian methods grant the opportunity to incorporate different considerations about the uncertainty of models in the updated probabilistic model, the comparison between different assessment experts' results may be regarded as a problem, as consensus about a comparison basis has not yet been established.

2.1. Maximum Likelihood method

In a probability paper, the vertical scale is changed by means of a non-linear transformation such that the cumulative distribution curve plotted in that graph is represented by a straight line. Attending to the configuration of that line (location and slope) it is possible to assess the parameters of the inherent distribution. This method is useful for normality tests [16] and to determine if a given data sample is well defined by a specific type of probability distribution. However, a more efficient and accurate method is the Maximum Likelihood method, which is based on finding the set of parameters of an assumed probability distribution function which most likely characterizes the underlying data sample. Although the Maximum Likelihood method is not a full Bayesian approach and it can also be used in a frequentistic approach, it is commonly used to find the distribution parameters of the prior information in a Bayesian methodology, and thus it will be briefly described here. In general, for a fixed set of data and an underlying statistical model, the Maximum Likelihood method allows to select the set of values of the model parameters that maximizes the likelihood function. The general procedure on how to implement the Maximum Likelihood method can be found in [5][16] and also in [18], where a parameter that describes the model uncertainty is also implemented.

In Bayesian statistics, the maximum likelihood estimator coincides with the most probable Bayesian estimator, given that the parameters of the prior distribution are uniformly distributed, meaning that the maximum posterior estimate is the parameter that maximizes the probability of that parameter given the analyzed data. Therefore, the Bayesian estimator coincides with the maximum likelihood estimator for a uniform prior distribution.

In probabilistic analysis, as the inference on characteristic values is of special interest in the field of structural safety assessment, it is also recommended that special focus is given to the extreme values of the distributions. Therefore, a scheme for estimating the parameters of probability distributions focusing on the tail behaviour should also be addressed, as considered in [19] where a censored Maximum Likelihood estimation technique was used.

2.2. Bayesian Probabilistic Networks

Bayesian Probabilistic Networks (BPN) are used to represent knowledge upon a system, based on Bayesian regression analysis describing the causal interrelationships and the logical arrangement of the network variables. BPNs are represented by directed acyclic graphs (DAG), composed by a set of nodes representing each system variable, connected by a set of directed edges linking the variables according to their dependency or cause-effect relationship. The causal relationship structure of a BPN is often described by family relations that differentiates child node variables with ingoing edges (effects), from parent node variables with outgoing edges (causes) [20]. A (parent) node without any ingoing edges, thus without any parent node converging to it, is often called a root node. The direction-dependent criterion of connectivity evidences the induced dependency relationship between variables and is classified as converging, diverging or serial (or cascade), according to its arrangement

[21]. Each variable node represents a random variable, either defined as a continuous random variable or as a finite set of mutually exclusive discrete intervals.

The main objective of a BPN is to calculate the distribution probabilities regarding a certain target variable, by considering the factorization of the variables' joint distribution based on the conditional relations within the developed generic algorithm. In this light, the DAG is the qualitative part of a BPN, whereas the conditional probability functions serve as the quantitative part. Therefore, the algorithms themselves are indifferent to the scope for which the BPN is employed, and thus have been employed in several different real-world problems, besides the hierarchical modelling of timber reference properties.

In the case of BPNs, it is recommended that the parent nodes are composed by indicators with strong correlation with the child node (reference property in analysis) expressed, for instance, by high coefficients of determination. The strength of the correlation may be qualitatively described as proposed in [15] where coefficients of correlation of 0.8, 0.6, 0.4 and 0.2 indicate high, average, low and very low correlation, respectively. After determining the indicators with higher predictive power, the dependencies within the DAG are created with different levels of hierarchy according to expert decision. The levels of hierarchy should attend to the source of the data, its relevance and both its size and material scale.

When considering decay in timber elements, dynamic BPNs should be implemented as to incorporate a time dimension through the addition of a direct mechanism for representing temporal dependencies among the variables, see *e.g.* [22][23]. Dynamic BPNs have been extended to the modelling of deterioration [24], while aspects of optimization of inspection and decision making for maintenance regarding deterioration have been addressed by BPN analysis in *e.g.* [25][26][27].

A limitation of Bayesian methods is the overall requirement of a sufficient large sample for a reliable analysis. It should also be noted that, in a parallel BPN, a small sample may lead to the impossibility of factorization of the joint probability due to the non occurrence of a given intersection of evidences. In this case, the construction of a BPN is highly dependent of the choice of the intervals' range when using discrete variables.

3. PREDICTION OF REFERENCE PROPERTIES

Reference properties can be predicted from information collected at different scales. Figure 1 illustrates the variables and their interaction for the inference of the reference properties (in gray). The wood species information can be considered a cross-scale variable that influences decisively some other variables, as for instance visual strength grading.

3.1. Density

Wood density is a basic quality indicator of the mechanical properties of timber. Density is used for establishing clear wood strength values which is afterwards adjusted accounting the presence of defects (strength ratio) presented by the timber element to deliver the final strength (allowable strength) [28]. Several indicators can be used for predicting the density of a structural timber element (visual grading/strength class mean value, core drill, penetration resistance, drill resistance, pull-out resistance), Figure 2.

An initial (first level) prediction of density of a timber element is provided by the onsite visual strength grading (VSG) process. This first prediction is generally the value obtained from the preliminary visual survey and allocated to part or all timber elements in the structure (broader concept of macrolevel) for the assessment of the timber element or, in an even more macrolevel, a general value given to all timber elements of the structure in order to proceed to a first structural analysis of the timber structure [29].

At a mesolevel, the element can be considered as being composed of clear wood zones and knot zones [9]. Density of clear wood zones can be determined using different NDT and SDT methods, some indicated in Figure 2. Core drilling is a direct method and one that could also provide information about the wood species, superficial decay and moisture content [29], although the process of extraction of the wood core may substantially influence the moisture content due to the drill friction.

Density prediction is also necessary for predicting the modulus of elasticity (MOE) through the determination of the dynamic modulus of elasticity. Matters related to the importance of density and difficulties/uncertainties related with its prediction can be found in [31].

Accuracy and precision of density's prediction model is strongly dependent on the variability showed by each individual timber element. Density's variation occurs along the length and within the cross-section (width and depth) of a timber beam. The error of prediction can be partially dealt if the NDT or SDT method is applied taking into consideration important characteristics of the member (namely wood species, growth ring pattern and spatial variation inside the member) and if a sufficient number of readings are collected. The determination of penetration resistance along the length of the beam can provide more reliable data by incorporating possible density's lengthwise variation pattern [32]. As this method only considers the surface layers of the timber element, it does not take into account the cross-section variation that can be much larger that the longitudinal variation. The uncertainty in the prediction of density within a cross-section is considerably high given the difficulties of the NDT/SDT methods to take into account the growth rate variations and the effect of juvenile wood [33].

Density data obtained from core drilling, penetration resistance or drilling resistance can be balanced with expert estimation through Bayesian analysis.

To illustrate the application of hierarchical Bayesian analysis, data from thirty different new timber beams with a cross-section of 90 x 160 mm^2 and length of 2400 mm was used. The Portuguese standard for visual grading was applied [34] as the beams were identified as maritime pine (*Pinus pinaster* Ait.). The standard specifies two grades, being the density defined in terms of rate of growth – lower or equal to 6 mm for the upper grade (mean density 610 kg/m^3), or lower or equal to 10 mm for the lower grade (mean density 580 kg/m^3). A clear wood zone, 551 mm long, was cut from the beams, following a probabilistic framework for analysing the bending behaviour of timber beams as a heterogeneous material composed of clear and weak zones [9].

The mean value of each visual grade was used as prior information on the density of each beam. Apparent density was determined according to [35]. In the second step a core drilling method was applied to get new information. Two wood cores were taken from one edge of the specimen, at each end (to take into account a possible lengthwise variation). The length of the cores was 1/4 of the depth of the beam (what is called marginal areas in most visual strength grading standards).

The employed Bayesian hierarchical model adopted a normal distribution as priori distribution [36] and although the mean and standard deviation are considered known (resulting from visual grading) a certain level of uncertainty was adopted. Therefore, the hyperparameters distribution's (hyperprior) are assumed as normal (location parameter) and gamma (scale parameter) distributions. The Bayesian model posterior distribution was estimated through a Markov Chain Monte Carlo (MCMC) method. MCMC was carried out by running Winbugs inside R software trough R2WinBUGS package (1 chain, 1 000 burn-in iterations, 90 000 used iterations).

Figure 3 shows the correlation obtained by density inference using the information from visual grade core drilling and the combination of both (visual grading and core drilling) by Bayesian inference.

The obtained results, Figure 3 and Figure 4a, showed that Bayesian inference resulted in a non-significant adjustment on the density values obtained from core sampling. The average error (underestimation) in density prediction was -3.06% and -3.68%, respectively using solely core drilling or applying the Bayesian hierarchical model (BHM). Nevertheless, although minor, an increase of the coefficient of determination, r^2, was obtained (Figure 3) and of precision (Figure 4b) by the BHM.

The high scatter of individual errors shown in Figure 4a.2 (core drilling) can be explained by the fact that the core extraction did not take into account the variation of density inside the cross-section. Variation of wood density across the cross-section can be taken into account (at least partially) by including in the model information from type of annual ring patterns exhibited by each beam and information from other NDT methods as the drilling resistance.

In the present case although Bayesian inference did not provide a clear improvement of the reliability of the density prediction provided by core drilling, it was able to combine information from expert and new data providing a more robust prediction model.

It should be stressed that the use of hierarchical Bayesian models is strongly affected by the choices made (using non-informative or informative prior distributions for instance) and in the case of small sample size (which is almost always the case) this effect can be stronger [37].

3.2. Modulus of elasticity in bending

Modulus of elasticity in bending is often determined considering information obtained through NDT made onsite, for example using flexural and longitudinal vibrational tests, stress wave transmission time, penetration depth, and its correlation to laboratory tests [32]. However, the information obtained should always be complemented with visual grading of the timber element, since as mentioned in [38] for ancient timber elements, the knot incidence (knot diameter to depth/width ratio) and slope of grain, are important influencing parameters that may lead to significant MOE reduction. In [8], a hierarchical model was built for the multi-scale variability of MOE which included an explicit representation of the stiffness' variability between timber boards and the stiffness' variability within boards. All parameters of the hierarchical stiffness model were estimated based on a sample of 30 randomly selected new timber boards within the strength class L25 of Norway spruce from southern Germany. The elements were differentiated along its length in weak sections and in clear wood sections. After, a model was proposed by defining the mean modulus of elasticity within an element and the differences between that mean and the results within sections of the same element and between other elements. These values were modelled by probabilistic distributions with parameters obtained using the Maximum Likelihood method.

In [9], a hierarchical model for inferring on the reference properties of existing timber elements was proposed by also considering the distinction between clear and knot wood zones. This work, however, presented a framework for timber elements in-service and was thus different from the previously mentioned. The model procedure was based in three main steps: i) visual identification of clear and knot wood zones; ii) non-destructive prediction of the properties of clear wood zones; iii) prediction of the reference materials using clear wood properties and applying a knot factor for predicting the strength reduction effect of knots on clear wood properties. The application of this procedure to maritime pine beams evidenced an average to high correlation between experimental and predicted global modulus of elasticity (coefficient of determination, r^2, between 0.76 and 0.55 with p-value $\approx 1\times10^{-8}$). Nevertheless, weaker results were obtained for bending strength, evidencing the need to improve the method for determining the strength reduction effect of weak zones.

Attending the need to consider defects in the prediction of modulus of elasticity and taking into account the scale effect, a Bayesian Probabilistic Network is used in this example. The BPN considers the data from a multi-scale experimental campaign described in [13][16] which collected the results from bending tests in old chestnut timber floor beams in different structural sizes (beams and boards) and for segments with different visual grading. The network is presented in Figure 5, where visual grading was considered as parent node, because it provides a link between scales and it is a parameter commonly available in the assessment of existing timber structures. The classes for visual grade were obtained according to UNI 11119:2004 [39], which for on-site diagnosis considers three classes (I, II and III). The timber element is classified in a given class if it fulfills all the imposed requirements. Otherwise, it is graded in this study as non-classifiable (NC).

By consideration of an evidence, *e*, in a given node N of the BPN, the probability of the class C with evidence, C_e, in that node is $P(C_e) = 1$, whereas the remaining classes with no evidence, C_{n_ev}, have $P(C_{n\_ev}) = 0$. This means that the state of node N is known with certain to be from class C_e. In a BPN, this inference permits to propagate information between nodes. Based on this evidence, *e*, the probability distribution of the remaining nodes can be updated by use of Bayes' theorem. For example, if considering the scale fixed to the material scale (segments of the boards) and evidence is given in child node VI_b (visual inspection in boards), the Bayes' theorem for the probability of frequency of bending modulus of elasticity (E_m) follows as:

$$\mathrm{P}\left(E_{\mathrm{m}}, VI_{\mathrm{B}} \mid VI_{\mathrm{b}} = e\right) = \frac{\mathrm{P}\left(VI_{\mathrm{b}} = e \mid E_{\mathrm{m}}, VI_{\mathrm{B}}\right) \cdot \mathrm{P}\left(E_{\mathrm{m}}, VI_{\mathrm{B}}\right)}{\mathrm{P}\left(VI_{\mathrm{b}} = e\right)} \quad (1)$$

The results deriving from evidence in VI_b = I (local scale information) and for different considerations on the visual inspection of a beam (global scale information) are presented in Figure 6 through the cumulative frequency functions of the modulus of elasticity. It should be noted that no beam was graded as class I in the experimental campaign, and therefore no information is available for that case. For the other cases of beams' visual grades, it is clear that beams classified with lower visual grades (III and NC) will result in lower values of modulus of elasticity, while beams classified as VI_B = II will result in higher values of modulus of elasticity. Although this is an expectable result, the Bayesian Probabilistic Networks allows to more adequately combine the information from different size scales and to predict the value range for modulus of elasticity and its variation for different scenarios with the possibility of updating with different premises. The information given in Figure 6 allows to define the characteristic values of bending modulus of elasticity, E_m, (accounting to the probability of frequency) for each combination of prior information, therefore obtaining updated values for the design or assessment of the structural element. The results of Figure 6 also evidence that, even for the same local information (VI_b = I), the global visual inspection is important to accurately define the modulus of elasticity of the structural element.

3.3. Bending strength

For new timber elements, bending strength may be determined either directly or indirectly. Indirectly through machine strength grading via empirically known correlations to other reference properties, such as density and modulus of elasticity, and directly through samples tested in bending up to failure load. These procedures are often incompatible with the assessment of in-service timber elements, as its removal is either not possible or cost inefficient. In these cases, an onsite assessment method is required for predicting the strength of the timber elements. This method is normally based on a combination of NDT methods and visual inspection. When removal of small samples is possible, the information of small scale mechanical tests may also be employed for a more precise prediction of the global mechanical properties.

Earlier work [40] adopted a stochastic model of hierarchical series system to represent the bending strength of Swedish spruce (new wood) anticipating failure in a weak section with defects. The model parameters were defined regarding Maximum Likelihood estimates according to the results of 197 bending test results. Assuming that the estimated parameters are applicable in the series system model for the full uncut beams, a theoretical bending strength distribution function was obtained in dependence of number of defect clusters within the span of constant bending moment loading. A strong test of the prediction power of the model was established by experiments with 54 long beams from the same population of beams from which the small test pieces were cut.

For the example considered in this work, bending strength (f_m) of timber elements is predicted using a Bayesian Probability Network that considers prior information on the bending modulus of elasticity and visual inspection of small scale elements. This model, therefore, considers a differentiation between clear wood segments and sections with different levels of defects according to visual grading. As also seen for the previous example, when analyzing the modulus of elasticity in bending, a probabilistic framework for the hierarchical model is considered as it allows for the inference and updating of the relevant material property by combining different prior evidence.

The Bayesian Probabilistic Network is built from the same database considered for the previous bending modulus of elasticity prediction example [17]. In this case, a sample of 51 tests (four point bending tests) made to old chestnut (*Castanea sativa* Mill.) boards was considered. Bending stiffness and strength were measured globally as to include the presence and influence of natural defects. This example considers only the scale concerning the boards global scale, but assumes a division according to visual grading since it was noticed, during the experimental campaign, that the failure mode was directly influenced by local defects. Although modulus of elasticity in bending and bending strength are both considered as individual reference properties of timber, in this example information on the modulus of elasticity is used as prior data for the prediction of bending strength, since a high correlation between these two properties was found for the analyzed sample (coefficient of determination of $r^2 = 0.69$ and p-value $\approx 4.5\times10^{-14}$).

Considering the modulus of elasticity in bending (E_m) obtained by the results of the mechanical tests in different boards (E_m with classes from 5 kN/mm^2 up to 17.5 kN/mm^2 with class interval of 2.5 kN/mm^2) and the visual inspection (VI) of boards (VI with visual grades I, II, III and NC of UNI 11119:2004 [39]) as parent nodes, the bending strength (f_m) was taken as a child node in a simplified parallel network (Figure 7). Conditional probabilities are given for the child node and updating of nodes within the network is made by knowledge upon the parent nodes using Bayes theorem.

In this case, if evidence is given in parent node VI, the Bayes' theorem follows as in Equation (2), whereas if evidence is given in parent node E_m , the theorem follows as in Equation (3).

$$\mathrm{P}\left(f_m, E_m \mid \mathrm{VI}=e\right)=\frac{\mathrm{P}\left(\mathrm{VI}=e \mid f_m, E_m\right)\cdot \mathrm{P}\left(f_m, E_m\right)}{\mathrm{P}\left(\mathrm{VI}=e\right)} \tag{2}$$

$$\mathrm{P}\left(f_m, \mathrm{VI} \mid E_m=e\right)=\frac{\mathrm{P}\left(E_m=e \mid f_m, \mathrm{VI}\right)\cdot \mathrm{P}\left(f_m, \mathrm{VI}\right)}{\mathrm{P}\left(E_m=e\right)} \tag{3}$$

These equations lead to the posterior joint probabilities, or updated probabilities, of the bending strength f_m. Figure 8 presents the comparison between distribution curves in the same stiffness class but with updated information regarding different visual grade, whereas Figure 9 presents the comparison within the same visual inspection grade and with updated information regarding different stiffness classes. The results are fitted to Lognormal probability distributions taking into account the indicators obtained through Maximum Likelihood Estimates. Lognormal distributions were chosen, due to their good fit to the lower tail of the results and also regarding the recommendations of the Probabilistic Model Code [36] for timber. Each case is plotted against the initial distribution without any evidence, which corresponds to the full sample of the experimental results without any posterior information.

In this example, it should be noted that for lower values of E_m (0 to 7.5 kN/mm^2) no higher classes of visual inspection (I and II) were found, and that higher values of E_m (more than 10 kN/mm^2) have no case of non-classifiable (NC) segments. This is in accordance to real practice assessment, as it not expected to find high values of E_m for low visual graded elements or, on the other hand, to find low values of E_m for high visual graded elements.

Between different classes of E_m an increase of bending strength for higher values of stiffness is visible, whereas, within the same stiffness class, a reduction of bending strength is found for lower visual grade classes, as expected. In all cases, the lower tail of the distribution with evidence is significantly different from the distribution without any information, which leads to significant differences in the characteristic value. As example, when comparing the 5th percentile value of the prior distribution without any information and the calculated values for the distributions resulting from evidence of E_m = [10;12.5] kN/mm^2 an increase of 22% is found when VI = I, whereas decreases of 23% and 30% are found respectively for VI = II and VI = III.

By this hierarchical model, it was possible to update the prior distributions of a reference property regarding information obtained by different sources, namely mechanical tests and visual inspection. This model evidenced that differences above 20% for design values may be obtained depending on different posterior information, therefore showing the importance of the updating process for a better definition and assessment of reference properties.

4. CONCLUSIONS

Bayesian methods are statistic tools that may consider data from different sources, such as different NDTs, SDTs or DTs, or even their combination. However, these data must be classified and arranged with respect to its relevance and dependability, in order to obtain an adequate hierarchical modelling and inference for different reference properties of timber.

This paper addressed different possibilities for obtaining reliable data about the mechanical behaviour of timber elements by collecting information at different levels and by organizing that information into a hierarchy of sequential levels. The applicability and limitations of statistic and probabilistic methods were presented for the prediction and inference of timber's reference material properties, namely density, bending modulus of elasticity and bending strength, by means of examples. On those examples it was noted that the construction of the hierarchical Bayesian models is strongly affected by expert decision regarding the arrangement of nodes, its dependency and selection of scale levels. It was also noted that the predicted reference properties were highly dependent on the information that was inputted into the model as prior and posterior information.

The presented examples also evidenced that Bayesian methods are able to combine information obtained from different scales and sources, for the prediction the value, variation and distribution of updated parameters in the assessment of timber reference properties.

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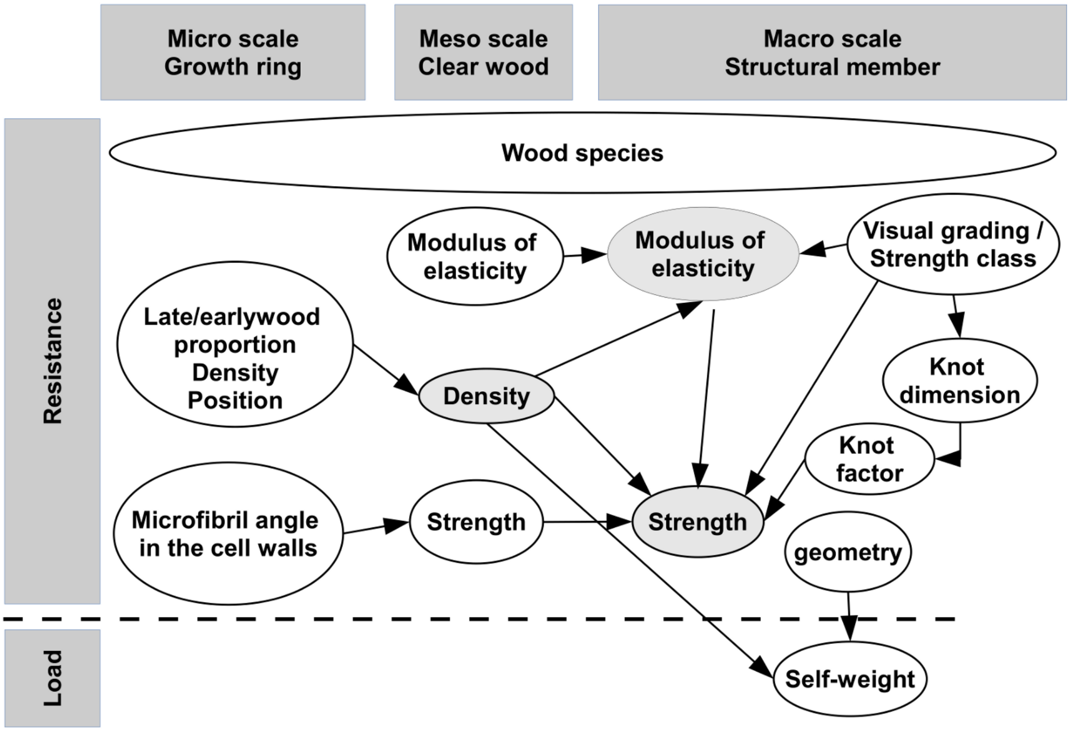


Figure 1: Interaction of different variables for the inference of timber reference properties.

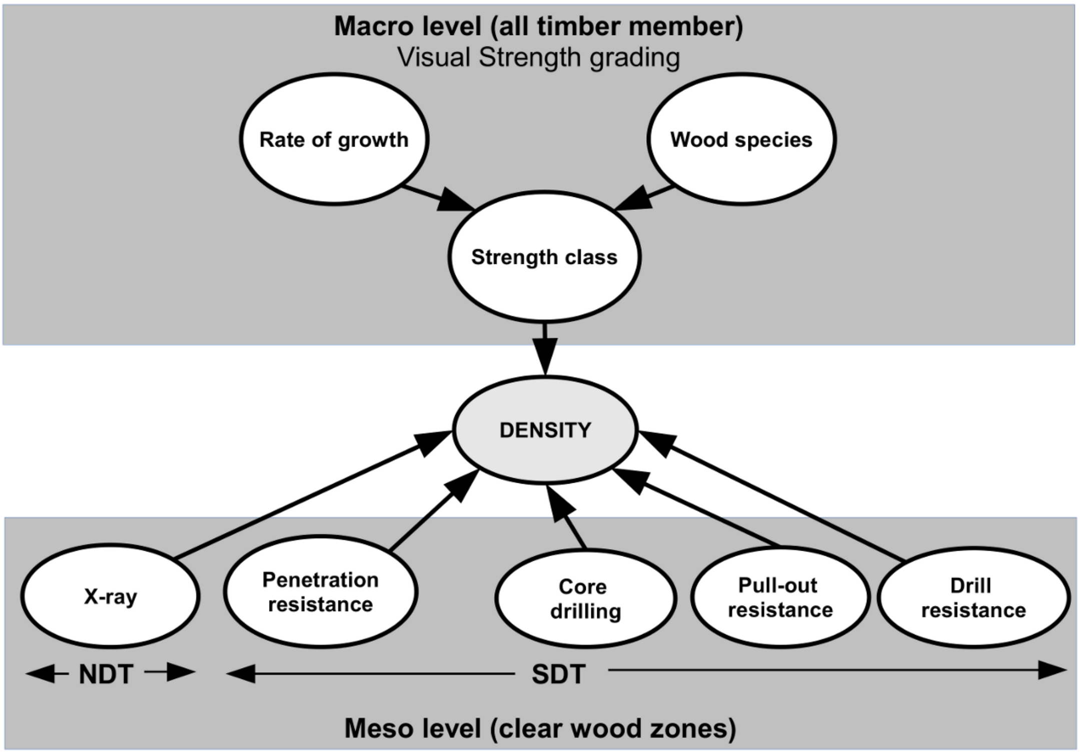


Figure 2: Different levels and indicators used to predict wood's density.

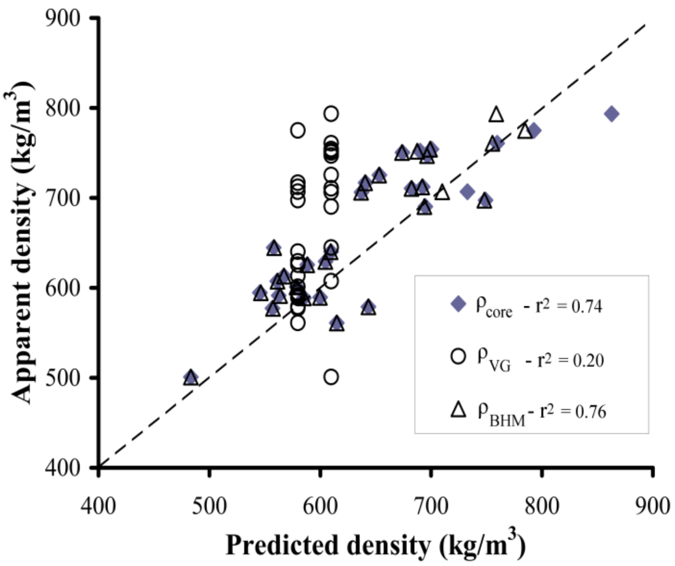


Figure 3: Correlation obtained between apparent density and density obtained from visual grading (ρ_{VS}), core drilling (ρ_{core}) and Bayesian hierarchical modelling (ρ_{BHM}) using visual and core drilling as prior and new data, respectively.

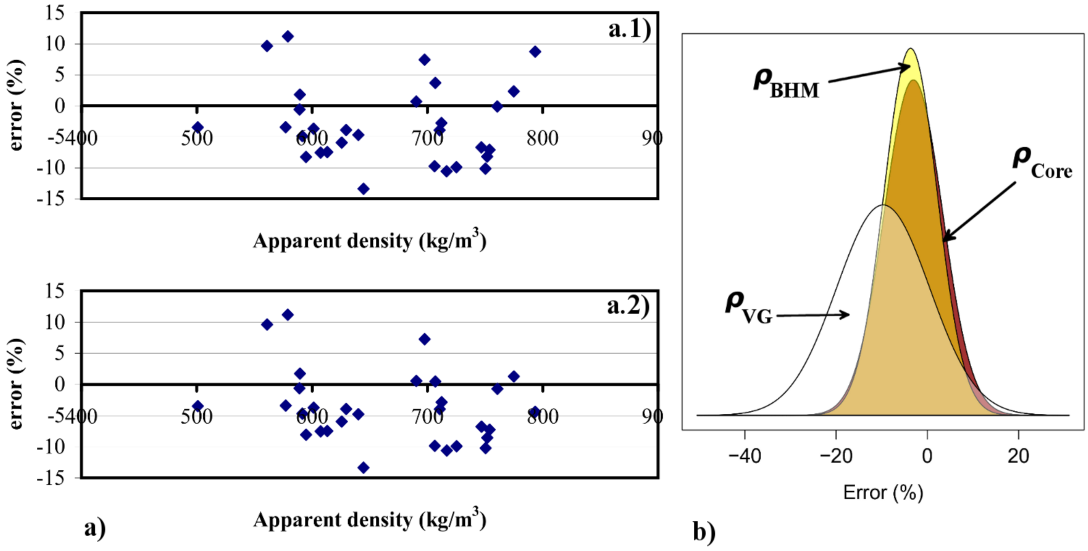


Figure 4: a) Error (as function of apparent density) due to core drilling (a.1) and Bayesian hierarchical modelling (a.2); Normal distribution curves fit to density values predicted by visual grading (ρ_{VS}), core drilling (ρ_{core}) and Bayesian hierarchical modelling (ρ_{BHM}).

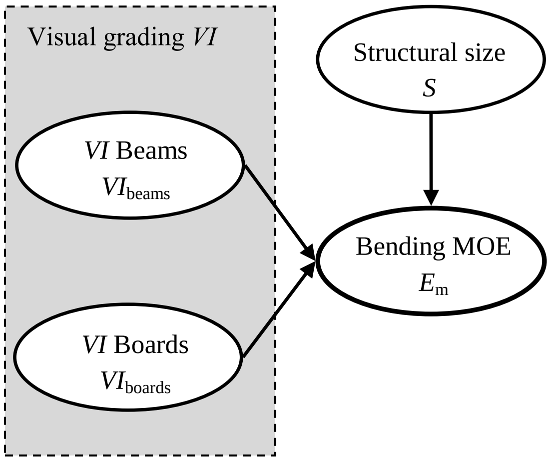


Figure 5: Example of a proposed hierarchical model for inference on bending modulus of elasticity (MOE) regarding different scales on visual grading, adapted from [16].

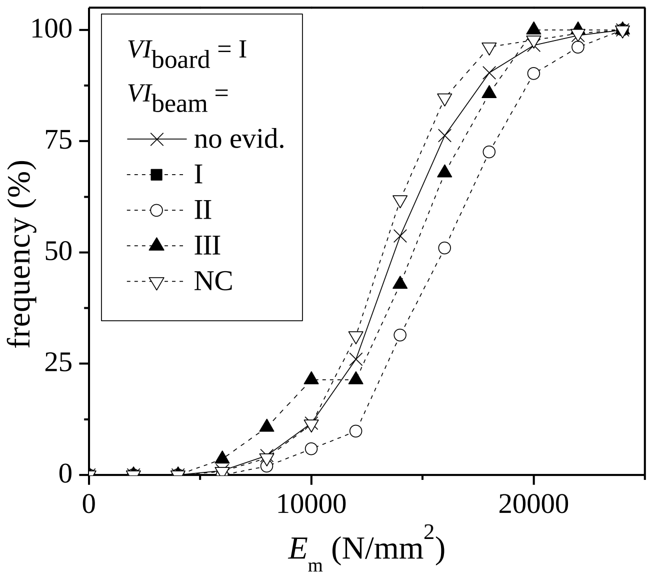


Figure 6: Cumulative frequency distributions results obtained through a hierarchical BPN inferring on bending stiffness regarding different evidences on visual grading, adapted from [16].

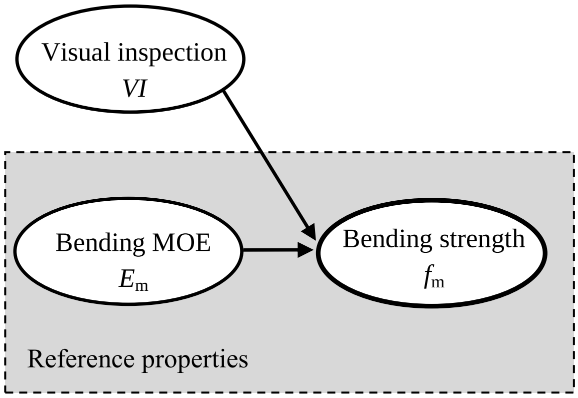
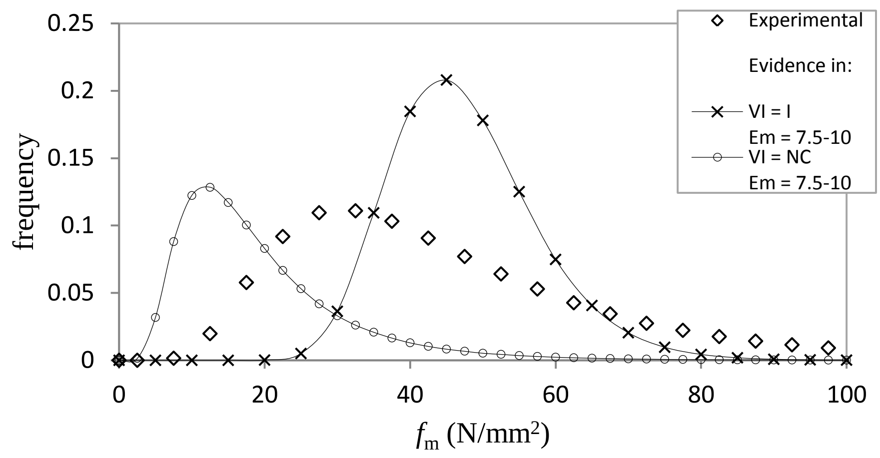


Figure 7: Simplified parallel model for bending strength inference.

a)



b)

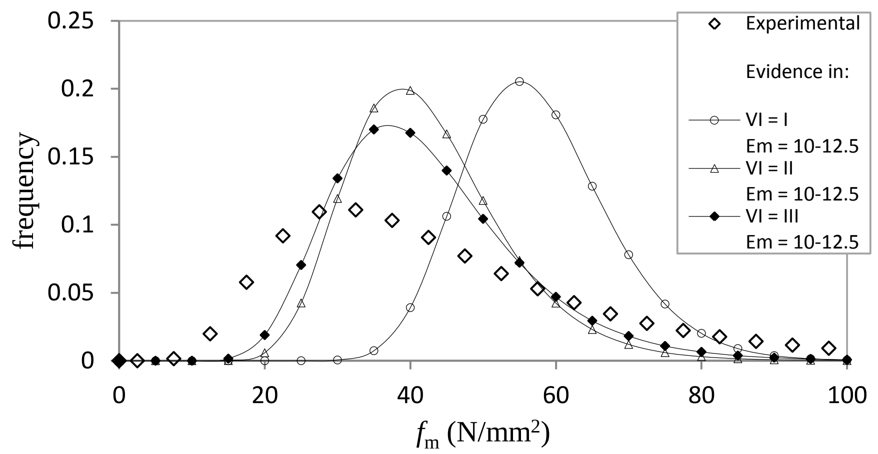


Figure 8: Distribution curves for bending strength (N/mm^2) within the same stiffness class and updated information for different visual inspection grades: a) 7.5 to 10 kN/mm^2; b) 10 to 12.5 kN/mm^2.

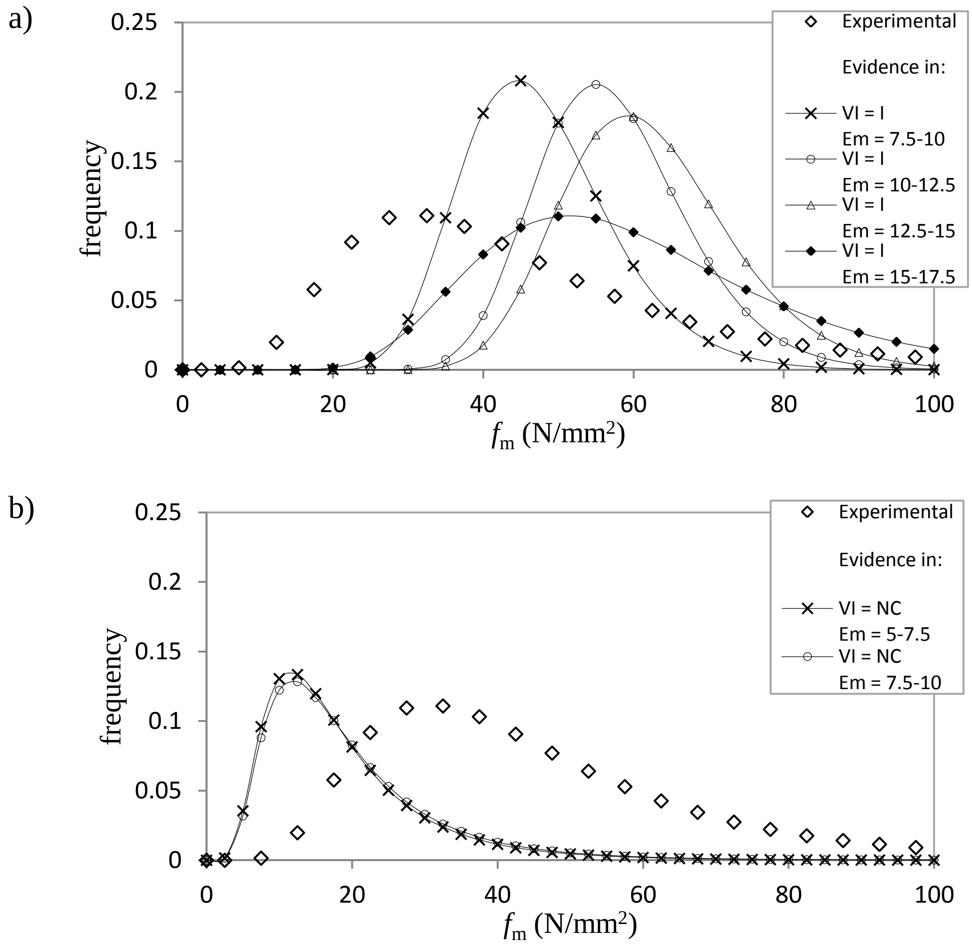


Figure 9: Distribution curves for bending strength (N/mm^2) within the same visual inspection grade and updated information for different stiffness classes: a) class I; b) class NC.